Introduction

History of modern logic in a new key

Lukas M. Verburgt and Matteo Cosci

This book is dedicated to the memory of John Corcoran (1937–2021), who had agreed to contribute a chapter but sadly passed away before it was completed. We like to think of the aim of the book as doing for modern logic what Corcoran did for the work of George Boole, namely to make sense of and do justice to the idea that Aristotelian syllogistic logic contributed to its creation. More specifically, the chapters show that the period between the nineteenth and early twentieth century saw a parallel development of modern logicians reshaping syllogism and reflections on syllogism shaping modern logic. This might sound odd as it stands in striking contrast to the standard narrative about the history of modern logic, which says that its creation and development happened in spite of, or in direct opposition to, the old logic. W. V. Quine, for one, wrote that Aristotelian logic is to modern logic what the 'arithmetic of primitive tribes' is to modern mathematics: not even a scientific predecessor but a 'pre-scientific fragment'.[1] And there is no denying that, in terms of scope, power and analytic rigor, the *Prior Analytics* pales in comparison to Gottlob Frege's *Begriffschrift* or A. N. Whitehead and Bertrand Russell's *Principia Mathematica*. But using today's standards to judge logic's past is, of course, not a good starting point for a history of modern logic and for capturing its historicity. When modern logic is presented as non-Aristotelian, no wonder that the role of Aristotle's syllogistic logic in its emergence disappears from view. For instance, one fact that has been neglected, and which Corcoran documented in great detail, is Boole's agreement with everything Aristotle said, his disagreements concerning what Aristotle did *not* say.[2]

Boole's position vis-à-vis Aristotle represents just one of a large number of different attitudes towards the syllogistic that modern logicians upheld. This book charts some of these attitudes and shows how their study gives rise to a more nuanced story about the creation of modern logic. Some parts of this story will be familiar. The nineteenth century was the century in which the mathematical revolution in logic achieved its breakthrough. It was also the century in which – after no less than two millennia – the hegemony of the syllogistic fell apart and Aristotle's immense achievement ceased to be logic's paradigm. It was replaced by what is today called classical or standard logic, consisting of propositional and first-order logic. All this is well known and well established.[3]

Other parts of the story that this book tells will be new. The collapse of the Aristotelian empire did not happen overnight. It was a lingering demise. Perhaps because the period between Kant – who in 1781 wrote that logic has not, will not and cannot improve upon Aristotle – and Frege – who in 1879 self-consciously moved logic far beyond Aristotle – is held to be an inactive time in the history of logic, this process of demise has hitherto received little attention. Far from being in slumber, however, the period was a highly complex one. No logician agreed with Kant's notorious claim. At the same time, logicians had 'little common ground except for their rejection of Kant's conservatism'.[4] There was an explosion of attempts to rethink logic. With this explosion came a fragmentation 'in tone, in method, in aim, in fundamental principles'[5] so extreme that logicians often did not even recognize each other's work as logic. John Venn, in his contribution to the first issue of the journal *Mind* of 1876, wrote that

> it would not be going too far to say that the principal difficulty in the way of a student of Logic at the present day … consists not so much in the fact that the chief writers upon the subject contradict one another upon many points, for an opportunity of contradiction implies agreement up to a certain stage, as in the fact that over a large region they really hardly get fairly within reach of one another at all.[6]

Many historians of logic have ignored or neglected some of the most influential logicians during the period – such as Richard Whately, John Stuart Mill, Hermann Lotze and Christoph Sigwart, for example. (Not to speak of other perhaps less influential but equally or even more interesting figures like Hugh MacColl, Lewis Carroll and E. E. Constance Jones.) The diversity of approaches to logic seems too large and the number of logical traditions too big to make sense of the work of these and other logicians, though the case could be made that there has been a process of '*Vergessenmachen*' at work in the historiography of modern logic – with pioneers of the new logic deliberately pushing older traditions into oblivion.[7]

The outcome is that historiography has been almost silent about everything there is to the nineteenth-century logic that does not 'flow directly to the waters that created the Peircean-Fregean tsunami of mathematical logic'.[8] For instance, British logic in the 1870s–90s remains a 'very confusing intermediary period',[9] where the old syllogistic was still taught, the new Boolean logic not yet established, and Whitehead and Russell's mathematical logic soon forthcoming. However, it is for much the same reason that the rise of modern logic is arguably still far from being fully understood. This can be seen from what has been called the 'Quine-Putnam muddle'.[10] Quine's view was that traditional logic ended and modern logic began with Frege's *Begriffschrift*, simply because this book contained the first system of propositional and predicate logic. Putnam, finding inspiration in David Hilbert and Wilhelm Ackermann's 1938 textbook, protested and dated the beginning of modern logic to Boole's *Mathematical Analysis of Logic* (1847) and *Laws of Thought* (1854). Unlike Quine, Putnam did justice to the two main schools, or origins, of modern logic. But by suggesting that both are part of one and the same development of 'modern mathematical logic', Putnam neglected the essential differences and tensions between the Boole-Schröder and Frege-Russell traditions. For example, as is well known, whereas Boole's system was a *calculus ratiocinator*, Frege aimed for a *lingua characteristica*. And whereas Schröder reduced Frege's system to a mere notational variant of Boole's calculus, Frege refused to call this calculus logic at all.

Another problem with the Quine-Putnam project of dating modern logic is that what could be called the 'mathematical turn in logic' was not a development beginning in the second half of the nineteenth century, let alone an accomplishment of Boole or Frege alone. It can be traced back to the final quarter of the seventeenth century, notably in the prescient work of Leibniz. Also, neither Quine nor Putnam recognized the historical fact that it took until the 1880s for Boole's work to make a mark and that Frege's work went largely unnoticed until the 1910s. It makes the exclusive focus on Boole and Frege – at the expense of other, lesser-known or even forgotten, logicians, including female logicians – all the more surprising: as has long been the case in the history of analytic philosophy, the search in the history of modern logic has traditionally been for 'founding fathers'.[11]

A related but arguably even more fundamental problem is that what Quine and Putnam both missed were the relations of what are now called the modern traditions to the older syllogistic tradition. Their search for a discrete event (or even an exact year) obscured the fact that the mathematical turn itself was part of the broader process of the lingering demise of the syllogistic, that is, of the gradual downfall of what for over two millennia had been logic's paradigm. Modern logic is without a doubt a major achievement. And it is true that it came to replace the older syllogistic logic. But it is incorrect, historically and conceptually speaking, to suggest that traditional and modern logic are opposites and that logic simply started anew upon entering the modern period. First of all, in order to establish a break with tradition, the pioneers of modern logic had to engage with that very tradition. They had to show, for example, that their logic could do everything syllogistic logic could do, and more, while the reverse was not the case. Moreover, in the first decades of the twentieth century, when the syllogism had been left behind and largely reduced to a historical curiosity, logicians continued to critically and constructively engage with it. This, at least, is what some of the chapters in the present volume show.

The book's purpose is threefold. First, it examines the role of reflections on and engagements with Aristotelian syllogistic logic in the creation of modern logic, putting the focus on the longue durée from the 1820s to the 1930s. Second, it does so by tracing how this informed the debates over the nature, scope and proper method of logic and shaped the cross-pollination of the various logical traditions in this period. Third, it presents the multifarious engagements – whether constructive, critical or destructive – of logicians with syllogistic logic as a missing link in the historiography of logic. One of this link's surprising aspects concerns the new (dis)continuities which it makes possible to uncover. For instance, it becomes clear that the process from which arose modern logic was set in motion by a revived interest in the details of traditional syllogistic logic, spurred almost single-handedly – at least in the Anglophone world – by Richard Whately. Furthermore, the logicians who initiated the mathematical turn, like Boole and Augustus De Morgan, reformed and expanded rather than abandoned the Aristotelian heritage. Also, even in and after Frege, syllogistic logic continued to play a key role, albeit in a completely different way than before. Frege himself appealed to it by way of contrast with his own new logic. It influenced the early development of set theory in the work of Bernhard Riemann, Richard Dedekind and others. And Hilbert had novel things to say about it too, just as he had about Euclidean geometry.

The nineteenth century was the century in which modern logic came into existence. The nineteenth century is also the century in which the hegemony of syllogistics fell apart. It has hitherto not been fully recognized, let alone fleshed out in depth, that these two fascinating epoch-making processes were interestingly and complexly related. The fact that modern logic replaced traditional logic, and that it was only when modern logic arrived on the scene that logic became a great discipline – to use Quine's famous phrase – has given rise to the tendency to study the former largely at the expense of the latter. This, in turn, has made that their mutual relation as well as most of what went on in logic in the nineteenth century remained poorly understood. The chapters in this volume seek to redress the balance, presenting the creation of modern logic as a long-term development for which the syllogistic tradition, in different ways and for various reasons, was formative.

Much more remains to be said about this and there is no pretension of exhaustiveness in terms of specific authors and topics or wider ramifications. We would like to think of the volume as a contribution to recent attempts at a full-blown history of 'logic' in the multifaceted ways in which it was understood in the period between the early nineteenth and beginning of the twentieth centuries.[12] Such a history would have to go far beyond deductive logic in Europe. It would also have to include, for instance, inductive logic and developments in other cultures, such as in the Arabic world.[13] It is our hope that the present volume offers starting points for such a fuller and more wide-ranging endeavour.

What follows is a short introductory overview of central aspects of the fourteen, chronologically ordered, chapters in the book. The reader is referred to the chapters themselves for more details and further discussion of the subjects.

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The book opens with Calvin Jongsma's account of the pioneering work of Richard Whately, whose entry on 'Logic' in the *Encyclopaedia Metropolitana* (1823) and especially *Elements of Logic* (1827) revived the study of traditional syllogistic logic – or, more generally, of formal logic – among English-language philosophers. Whately thus went against a long-established and widely shared preference for inductive, experimental and mathematical ways of reasoning. However, he was not completely alone in facing the influential criticism of Scottish philosophers like Thomas Reid, George Campbell and Dugald Stewart, as he was supported by his teacher Edward Copleston and aided by his student John Henry Newman. One of Whately's main arguments was that some discoveries in natural philosophy are discoveries of things 'implied in that which we already know'. Therefore, syllogistic has a role to play in the process of the advancement of knowledge. Moreover, Whately argued, even though not all sound reasoning is essentially syllogistic, any argument can easily be evaluated in terms of its soundness by expressing it in syllogistic form. The importance of Whately's achievement for the revival of the study of logic was widely acknowledged by English logicians, even those – like Boole or Mill – who aimed to expand and transform logic in ways that were at odds with or went beyond Whately's conception and treatment of logic.

In the second chapter Lukas M. Verburgt explores the creative role of the syllogism in the British tradition of inductive logic by focusing on the work of John Stuart Mill and especially on its reception by one of his most prominent followers, John Venn. Both Mill and Venn – contra Whately – held that the conclusion of a syllogism cannot state anything more than what is already implied in the premises. Mill concluded that syllogisms, considered as inferences, are essentially question-begging insofar as they involve a petitio principii. From this there followed Mill's famous view that all inference is inductive. Like Mill, Venn belonged to the inductivist school. But unlike Mill, Venn came to believe that syllogizing cannot at all be described as an inferential process. Although the premises, in the end, are bound to result from inductive generalization, the step of moving from the premises to the conclusion – especially in the case of complex mathematical reasoning – involves 'mental labour'. In this way, Venn made it possible for the inductivist tradition to engage with recent developments in the mathematization of logic associated with De Morgan and Boole.

Syllogistic could have developed into a much more comprehensive and extended field of logic earlier, considering that the Aristotelian definition of syllogism as 'a discourse in which, certain things being stated, something … follows of necessity' can be conceived in a much broader sense. As it was, for centuries no one questioned, for instance, whether the principle standing at the basis of traditional syllogistic – the famous *dictum de omni et de nullo* – is a tautological starting point. Moreover, no one took seriously the idea that the only thing which really matters in the definition of syllogism is 'that the conclusion becomes true *whenever* the premises are true'. These and other considerations stand at the heart of Bernard Bolzano's work, as shown in the third chapter by Mark Siebel. According to Bolzano's prescient *Theory of Science* (1837), the proper object of syllogistic is neither mental occurrences nor linguistic signs but 'sentences in themselves' consisting of 'ideas in themselves'. Much like Aristotle's original definition of implication, Bolzano's notion of deducibility is concerned with truth-preservation: the conclusion of an inference is deducible from the premises if they contain ideas in themselves whose substitution leads to a true conclusion if the premises are true. This notion does comprise not only formal validity but also material validity, i.e. a kind of validity being dependent on the meaning of non-logical expressions within the sentences involved in the inference. This is the stance from which Bolzano envisaged innumerable forms of logical inference beyond the limitations and constraints of traditional syllogistic.

The fourth chapter concerns George Boole's development and generalization of Aristotelian logic. Boole first used logic to argue for a new conception of mathematics, but in the process of treating logic mathematically he came to embrace novelty in logic itself with increasing confidence over time. His first step is found in *The Mathematical*

Analysis of Logic (1847), where Boole realized that a system of equations could reveal the more fundamental structure of Aristotelian syllogisms. He subjected the traditional pattern to the algebra of his calculus or 'pure analysis', as expressed by a new notational symbolism. More specifically, the syllogism became a system of two equations in three variables and, more generally, the number of possible inferences was considerably augmented. For the second step, David E. Dunning focuses on an interleaved copy of *The Mathematical Analysis of Logic*, in which Boole returned to the topic soon after having published the book. In his handwritten notes, Boole included negative subjects, doubled the number of categorical propositions to eight and offered a more complete classification of syllogisms by mathematical derivation. Moreover, he identified three algebraically derived rules of inference, which were intended to replace the traditional ones. The third step is represented by a manuscript entitled *Elementary Treatise on Logic not mathematical including philosophy of mathematical reasoning*. As the title suggests, syllogistic was treated in a 'non-mathematical' way. According to Boole, the merit of traditional syllogistic forms decreased as a result of it. The fourth step corresponds to the publication of The Laws of Thought (1854), where Boole put forward a more critical understanding of Aristotelian syllogistic, whose limitations, incompleteness and arbitrariness were made explicit to the reader. As a last step, Boole planned a book for a non-specialist audience, entitled *The Philosophy of Logic*, which was never brought to completion. All things considered, it was the rewriting of Aristotelian logic in a mathematical form that made Boole realize how logic could encompass a wider range of possible inferences and take syllogistic to a new level.

In the fifth chapter, Sun-Joo Shin analyses the transition from the dominant categorical syllogism to relational arguments as realized by Augustus De Morgan and Charles S. Peirce in their respective efforts to expand the domain of logic. De Morgan was an ardent defender of the efficacy of the syllogism and tried to extend its functionality beyond its traditional schematism, which he considered a restriction on its deductive potential. His major contribution, in this regard, was the development of a 'logic of relations' which widened the scope of syllogistic tout court. On his reading, the copula expressed every kind of linking relation between two terms and any logical relation could be considered a composition of two relations. Accordingly, a syllogism was defined as 'a composition of two relations into one'. Later, Peirce developed an extended version of Boole's algebra and restructured De Morgan's relational arguments in such a way that they could be decomposed and analysed into more than one traditional syllogism. This approach made Peirce think of relations as compositions of relations which could be multiplied to handle complex chains of inferences, going much beyond the triadic pattern of syllogistic reasoning. Thus, De Morgan and Peirce shared the idea of interpreting syllogistic as a sort of inferential subset of relational reasoning – an idea that resulted in an unprecedented and highly versatile strengthening of deductive syllogistic reasoning.

The sixth chapter discusses Ernst Schröder's algebra of logic. The main focus is on his three-volume set *Vorlesungen über die Algebra der Logik*, completed between 1890 and 1905. Within the system presented there, Volker Peckhaus singles out in particular the treatment of the 'Logic of the Ancient', as Schröder called it. In the

Vorlesungen, traditional syllogistic and early calculi of classes are replaced by a new propositional calculus, where the categorical forms are translated into algebraic expressions. The calculus is governed by two principles: the principle of propositional identity and the principle of transition, standing at the basis of the first syllogism of the system, namely the hypothetical 'inference of substitution'. Schröder used Friedrich Ueberweg's *System der Logik* (1857) as his starting point and Hermann Lotze's *Logik* (1843) as his critical target. He constructed his system by further developing Boole's elimination theorem, while also taking the work of De Morgan and of Joseph Gergonne into account. Like MacColl, Schröder advocated the requirement of existential import for categorical propositions and praised Christine Ladd-Franklin for her brilliant validation formula. On the basis of systematic combinatory calculations and elimination rules, Schröder was able to select and reorder fifteen valid forms of traditional syllogisms while recovering some of the invalid forms too. Despite his plan to develop a complete syllogistic, Schröder favoured the generality of his 'algebraic logic' and emphasized the pre-eminence of his elimination theorem.

The next chapter presents Franz Brentano and Franz Hillebrand's 'idiogenetic theory', a post-scholastic type of syllogistic theory involving acts of judging which were regarded as belonging as such to a special genus (*idios genos*) of psychical phenomena. The logical traits of the theory were first put forward by Brentano in his *Psychologie vom empirischen Standpunkt* (1874, first ed.) and then formally presented in Hillebrand's *Die neuen Theorien der kategorischen Schlüsse* (1891). The most novel aspect of the theory was that all judgements were restated in existential form as single-membered assertions, or rejections, whose subject and predicate could be *simpliciter* converted. The proposal provoked numerous reactions. Particularly the last part of Hillebrand's system, namely the extension about 'double judgments' (existential and predicative judgments bound together), was criticized by Husserl and Meinong, among others. But it also received active support from Brentano's student Anton Marty. In his chapter, Matteo Cosci recalls the Leibnizian antecedent that showed the character of supposition of the existential import holding in the traditional square of oppositions. That assumption was a matter of concern for Brentano, who may have been aware of its formulation (possibly via Leibniz's *Difficultates Quaedam Logicae*) in the process of developing his own reform of syllogistic on new, intentionalistic grounds. Aside from its intrinsic merits and originality, Brentano and Hillebrand's 'idiogenetic theory' had a considerable impact in the fields of descriptive psychology, analytic philosophy and early phenomenology towards the end of the century – not to mention its relevance for the great current in logic inaugurated by Kazimierz Twardowski, prominent student of Brentano and the standard-bearer of his reform in Poland at the beginning of the twentieth century.

In the eighth chapter Jean-Marie Chevalier presents the surprisingly rich work that Hugh MacColl carried out on the syllogism at the turn of the twentieth century. MacColl first developed his own system of propositional logic, both working with and departing from Boole's theories. He next explored the consequences of enriching propositional logic with operators, the basic one being 'strict implication', which added necessity to the simple material conditional. The operator was used to embrace all kinds of inferences such that, for instance, the relations between categorical propositions became equivalent to relations between conditional propositions. Accordingly, MacColl presented the syllogism as an inferential scheme whose nature is essentially hypothetical – or, more precisely, doubly hypothetical, as the hypothetical implication between premises and conclusion is reframed by MacColl as a second-order premise dependent on the general hypothesis which sustains the deduction as a whole. As a consequence, MacColl called into question the validity of all syllogisms expressed in the traditional form: the possibility of deducing something true had always been assumed without taking into account the underlying hypothetical character of the whole operation. For MacColl, every syllogism should be introduced by an 'If' which is antecedent to, and dominant over, the 'therefore' between the premises and the conclusion. Only under this condition are what MacColl called 'formal certainty' and 'syllogistic validity' ensured. MacColl also solved the problem of the existential import in his own way. For, MacColl argued, once one identifies different senses of existence and distinguishes a universe of existent entities from a universe of non-existent entities, '[i]n pure logic the subject, being always a *statement, must* exist – that is, it must exist as a *statement*'. Accordingly, within MacColl's system of propositional logic all statements denote something, even those concerning fictitious entities. His research also led him into the uncharted territory of non-classical, multidimensional logic. There he classified five different truth-values corresponding to five different types of syllogisms. Under his reading, syllogisms express relations of strict implication within the realm of possibility, namely a domain that is non-existent but also non-contradictory.

In the ninth chapter Erich H. Reck reconsiders the role of Gottlob Frege's *Begriffsschrift* (1879) in the transition from Aristotelian logic to modern logic. According to Frege, traditional 'term logic' did not come with a clear object-concept distinction and, more generally, was too attuned to ordinary language. For this reason, it was an inadequate tool for the analysis of the kind of mathematical concepts that Frege was interested in within his logicist project. Therefore, Frege broke with some of the basic tenets of the Aristotelian syllogistic. He rejected the Aristotelian predicative model based on the subject-predicate distinction, introduced the material conditional together with the *modus ponens* as the one sufficient kind of inference, reformulated the categorical proposition in a symbolic and more rigorous way, contrasted the idea according to which concepts are 'sums of marks' in favour of nested quantifications and, finally, distinguished multiple meanings of the copula on the basis of different notations. On closer inspection, however, while at odds with the Aristotle of the *Prior Analytics*, Frege's commitments were rather similar to Aristotle's in the *Posterior Analytics*. Put differently, Frege proved to be an Aristotelian in epistemology, if not in logic.[14] He, perhaps unwittingly, inherited Aristotle's definition-based approach, his kind-crossing prohibition rule, the axiomatic-deductive model and the awareness that not everything can be proved on pain of circularity. Another similarity between Frege and Aristotle is the adoption of a first-person perspective in logic, for a logic oriented towards universal applicability. In this sense, it can be said that Frege's trailblazing contribution to the emergence of modern logic went hand in hand with a pre-modern indebtedness to an Aristotelian model of science.

One of the most long-standing open questions concerning the syllogism was how to identify syllogistic validity in a simple and effective way. This problem was as old as the syllogism itself and highly complex. 'The Syllogism's Final Solution', as Susan Russinoff called it,[15] was found by Christine Ladd-Franklin as late as 1883. This was the year in which Ladd's test for the validity of any syllogism (or 'antilogism', as it was called) was published for the first time in a text titled 'On the Algebra of Logic', which appeared in a collection of articles by Peirce and his John Hopkins University students. Whereas Peirce had previously adopted two symmetrical types of copula (one for the propositions of existence and the other for the propositions of non-existence), Ladd-Franklin used a single type of quantified copula (positive for universal propositions always denoting non-existence and negative for particular propositions always denoting existence). The fundamental relation to be specified between two classes of sets could now be expressed by the function of exclusion (symbolized by $\bar{\mathrm{V}}$) rather than by the inclusion relation as debated by other logicians of the period.[16] Through these innovations, Ladd was able to provide a general characterization of all valid syllogisms in one single and exact formula. After many centuries, Aristotle's problem was solved by Ladd-Franklin's simple and elegant validation rule, though it took time to be accepted and even more time to be formally proved. In the tenth chapter of the book, Francine F. Abeles suggests that the idea for the solution of the problem may have come to Ladd from an 'eliminativist' reading of Aristotle himself, in particular from a certain passage of *De Interpretatione*. What is certain is that both Evert W. Beth's and Charles L. Dodgson's (i.e. Lewis Carroll's) subsequent developments in logic were indebted, at least to a certain extent, to this original result. Over the course of her life, in which she struggled for official recognition, Ladd-Franklin maintained that the most important form of reasoning is syllogistic reasoning and that scientific knowledge is nothing but a network of truths whose connections are conclusions of valid syllogisms.

The eleventh chapter studies how the syllogism was conceived by Giuseppe Peano and his school (Burali-Forti, Padoa, Vailati, Pieri). Although Aristotle and scholastic sources were never explicitly mentioned, the Peano School's study of syllogistic logic was not conducted with an obliterating approach. Indeed, syllogism was taken as the fundamental characterization of deduction as such. As soon as it was envisaged as a form of calculus of classes, syllogistic came to be regarded as a valuable part of mathematical logic, which in turn came to be seen as an extension of traditional syllogistic. Moreover, the introduction of a new mathematical symbolism in 1898 allowed the group to notice how the syllogism could operate not only as an axiom but also as a rule of logical identity and as a rule for the elimination of the common term. They realized that the syllogism could be understood as the property of transitivity holding between three terms and as the instantiation of logical transitivity itself. In addition, they realized that it could be used as a regulatory premise at the start of proofs and that the rule of inference known as *modus ponens* could be interpreted as a new kind of syllogism too. Peano's group laid down additional inferential rules – or 'various species of syllogism', as they called them – which extended the traditional forms with reference both to classes and to propositions. Syllogistic moods and forms were ultimately reduced to three general types, dubbed 'syllogism', 'singular syllogism' and 'sorites'. Such a taxonomy – as Paola Cantù explains – was the result of a collective effort to identify, simplify and, at the same time, generalize all the possible properties of deduction.

In the twelfth chapter William Ewald presents the view of syllogistic which David Hilbert presented to his students as an integral part of his systematic metamathematical study of logic in his Göttingen lectures of 1917–18. On the model of Whitehead and Russell's recent *Principia Mathematica*, and with the aid of his assistant Heinrich Behmann, Hilbert set up a logical system modelled on the requirements of consistency, independence and – most importantly – completeness. Using Russell's axiom of reducibility as a guiding principle, the calculus of predicates was extended to a calculus of classes which could also deal with the traditional syllogistic. In this framework, Hilbert's treatment of syllogistic was neither optional nor arbitrary, but rather key to proving the comprehensiveness and capacity of the overarching mathematical method. After all, the ideal of completeness that Hilbert was pursuing in his system was both systematic and historical, and the latter quality was regarded as no less important than the former. Therefore, syllogistic had to be included in the analysis with an eye to the historical completeness of the 'Grundlagen der Logik'. Aristotle had to be encompassed or '*nostrified*' – as Göttingen mathematicians used to say – within the mathematical system under development so as to achieve the general aim of 'deepening the foundations' of logic. In this sense, Hilbert's reformulation of the Aristotelian syllogistic stands as an elegant and compelling example of the exhaustiveness and rigorous systematization of that foundational program.

Early set theory emerged from innovations in mathematics and in the algebraicized logic of classes. Since the logic of classes mainly emerged from the analysis of syllogistic logic, it can be said that early set theory inherited a great deal from syllogistic logic. In fact, early set theory, as formulated by Riemann and Dedekind, basically followed this line of development. Traditional logic, after all, was an integral part of the syllabus of the German *Gymnasium* and classical logic textbooks such as those by Herbart and Drobisch were actively read and studied in the mid-nineteenth century. More specifically, early set theory was influenced by syllogistic logic through the consideration and adoption of some of its canonical elements, such as the primacy of concepts and their 'extension', the study of syllogistic figures by means of the analysis of inclusion relations, and the algebraicization of logic by a calculus of classes, as José Ferreirós explains in the thirteenth chapter of this book. Both Riemann's introduction of the concept of 'manifold', which stands at the origin of the concept of 'set' (1854), and Dedekind's development of the notion of 'system' (a synonym of 'class' or 'manifold'), which was basic to his approach to number sets and to 'ideals' (1871), bear strong traces of syllogistic theory.

The last chapter explores how the pattern of syllogistic inference was received, generalized and enhanced by the Göttingen school of logic and by Gerhard Gentzen in particular. In 1932 Gentzen put forward 'a formal definition of provability', as he called it, which brought to completion – and strongly resembled – the completeness proof that may be found in Aristotle's *Prior Analytics*. Gentzen's celebrated theorem was a rigorous formalization of a general notion of logical consequence – including the syllogism – which surpassed the traditional distinction between syntax and semantics by resorting to the so-called CUT and THINNING rules. Gentzen's result was informed in Göttingen by Paul Hertz's Aristotelianism, Hilbert's inferentialist semantics and Jacques Herbrand's notion of *champs finis*. By recasting the intuitive concept of synthetic consequence as an intrinsic feature of inference itself, Gentzen managed to find a way to reconcile the tension – first made explicit by Bolzano – between the requirement of exclusive analytical consequences and their logical 'derivability'. Moreover, his elimination theorem included the syllogism rule (that Bolzano had tried to keep out) in a more comprehensive and synthetic inference rule so that in the end a single meta-syllogistic formula could stand in place of, or 'eliminate', all syllogistic forms. To conclude with the words of Curtis Franks, 'the Göttingen logicians taught us that syllogism-free reasoning is significant because with it one can replicate the syllogism's inferential scope. The syllogism is revealed to be, not deadwood or a redundancy of logistic theory, but the gold-standard of inference against which meaning-constitutive rules are measured. [. . .] [T]he syllogism is able to capture, in concrete form, the whole abstract concept of logical consequence, so that its elimination confers on a logic the label of completeness. And this [. . .] is just the role that Aristotle saw it playing from the beginning'.[17]

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Taking the period between the 1820s and 1930s as its focus, this book looks at the central and sometimes surprising role of the syllogism in pioneering attempts to go 'under, over and beyond' Aristotle. One of its main overarching points is that these attempts played a formative part in the creation of ('non-Aristotelian') modern logic.

The Aftermath of Syllogism – which appeared in this Bloomsbury series in 2019 – ended with Hegel. The present volume basically takes things from there. It tells a more complex and arguably even more surprising story than the one told in that book. Much remains to be said and many open questions remain to be answered. And, as already emphasized, we make no claim to exhaustiveness when it comes to relevant authors, themes and topics, let alone to the historical and theoretical ramifications of and connections between the various chapters. Our focus is squarely on the interplay between the old and new logic. We take it as a sign of fruitfulness that this focus offers so much food for thought, whether regarding dominant narratives and well-known themes in the history of logic and philosophy or the new and lesser-known authors and topics it sheds light on.

Rather than ending this introduction with general remarks or conclusions, we would like to end with suggestions for further research on the topic, given in no particular order:

- Aristotelianism in the nineteenth- and early twentieth-century history of logic, history of philosophy and history of science;
- The work on philosophical and formal logic by female logicians such as E. E. Constance Jones, Sophie Bryant, and Augusta Klein;
- The transition from syllogistic to modern logic on the level of education and university curricula;
- Different kinds of influential, forgotten or otherwise minor works on syllogistic, including textbooks, new editions of older Latin tracts, didactic works, 'grey literature' (theses, dissertations, lecture notes, unpublished manuscripts), memory

aids for remembering all the different syllogistic forms and moods, and scholarly and/or philological studies of the history of syllogistic, such as Karl von Prantl's *Geschichte der Logik im Abendlande* (1855) and the monumental volumes by Heinrich Maier *Die Syllogistik des Aristoteles* (1896–1900);
- Attempts at the diagrammatic representation of syllogisms in relation to the literature on the history of visual reasoning;
- The syllogism as an object of study in fields beyond logic, such as cognitive psychology (Helmholtz, James, von Hartmann, Bain and Torrey Harris), medicine (Hector Donon) and epistemology (Ellingwood Abbot);
- The development of syllogistic logic outside Western Europe, including in the Polish, Soviet, Arabic, Chinese and Indian traditions;
- John Neville Keynes's work on the non-categorical syllogistic and its place at the crossroads of syllogistic and modern logic.

To paraphrase Umberto Eco's *The Infinity of Lists*, no list is ever complete – and the same holds for this one. All our suggestions merely serve as pointers towards a fuller understanding of the central topic of the present book.

Notes

1 Other sources of inspiration for this project have been James Van Evra, 'The Development of Logic as Reflected in the Fate of the Syllogism, 1600–1900', *History and Philosophy of Logic* 21, no. 2 (2000): 115–34, José Ferreiros, 'The Road to Modern Logic – an Interpretation', *The Bulletin of Symbolic Logic* 7, no. 4 (2001): 441–84, and Calvin Jongsma, *Richard Whately and the Revival of Syllogistic Logic in Great Britain in the Early Nineteenth Century*, PhD dissertation (advisors: Kenneth O. May and John Corcoran), University of Toronto, 1982.

2 See John Corcoran, 'Aristotle's *Prior Analytics* and Boole's *Laws of Thought*', *History and Philosophy of Logic* 24, no. 4 (2003): 261–88.

3 This part of the story of the emergence of modern logic is told particularly clearly in Ferreirós, 'The Road to Modern Logic'.

4 Jeremy Heis, 'Attempts to Rethink Logic', in *The Cambridge History of Philosophy in the Nineteenth Century (1790–1870)*, ed. Allen W. Wood and Songsuk Susan Hahn (Cambridge: Cambridge University Press, 2012), 95–132, on 96.

5 Robert Adamson, *A Short History of Logic* (London: W. Blackwood, [1882] 1911), 20.

6 John Venn, 'Consistency and Real Inference', *Mind* 1, no. 1 (1876): 43–52, on 43.

7 See, in this regard, Lorenz Demey, 'Mathematization and *Vergessenmachen* in the Historiography of Logic', *History of Humanities* 5, no. 1 (2020): 51–74.

8 Dov M. Gabbay and John Woods, 'Preface', in *Handbook of the History of Logic. Volume 4: British Logic in the Nineteenth Century*, ed. Dov M. Gabbay and John Woods (Amsterdam and Oxford: North-Holland, 2008), vii–xii, on viii.

9 Amirouche Moktefi, 'Lewis Carroll's Logic', in *Handbook of the History of Logic. Volume 4: British Logic in the Nineteenth Century*, ed. Dov M. Gabbay and John Woods (Amsterdam and Oxford: North-Holland, 2008), 457–505, on 457.

10 See, for example, Wolgang Kienzler, 'Three Types and Traditions of Logic: Syllogistic, Calculus and Predicate Logic', in *Philosophy of Logic and Mathematics*, ed. Gabriele

M. Mras, Paul Weingartner and Bernhard Ritter (Berlin and Boston: Walter de Gruyter, 2010), 133–52.

11 For a forceful critique of the historiography of analytic philosophy see, for instance, Frederique Janssen-Lauret, ‘Grandmothers of Analytic Philosophy: The Formal and Philosophical Logic of Christine Ladd-Franklin and Constance Jones’, *Minnesota Studies in Philosophy of Science*, forthcoming.

12 An important recent work, in this regard, is Sandra Lapointe (ed.), *Logic from Kant to Russell. Laying the Foundations for Analytic Philosophy* (London: Routledge, 2019).

13 See, in this regard, for example Dov M. Gabbay, Stephan Hartmann and John Woods (eds.), *Handbook of the History of Logic. Volume 10: Inductive Logic* (Oxford and Amsterdam: North Holland, 2011), and Khaled El-Rouayheb, *Relational Syllogisms and the History of Arabic Logic, 900–1900* (Leiden and Boston: Brill, 2010).

14 See Willem R. de Jong and Arianna Betti, ‘The Classical Model of Science: A Millennia-Old Model of Scientific Rationality’, *Synthese* 174 (2010): 185–203.

15 I. Susan Russinoff, ‘The Syllogism’s Final Solution’, *The Bulletin of Symbolic Logic* 5, no. 4 (1999): 451–69. See also Sara L. Uckelman, ‘What Problem Did Ladd-Franklin (Think She) Solve(d)?’, *Notre Dame Journal of Formal Logic* 62, no. 3 (2021): 527–52.

16 See Amirouche Moktefi, ‘The Social Shaping of Modern Logic’, in *Natural Arguments: A Tribute to John Woods*, ed. Dov Gabbay, Lorenzo Magnani, Woosuk Park and Ahti-Veikko Pietarinen (Rickmansworth: College Publications, 2019), 503–20.

17 See Curtis Franks’s contribution in this book, 356–80, in particular on 379.