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Keywords
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JEL Codes
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On the use of reserve prices in the English auction*

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20/11/2021

Abstract

We show that the introduction of a reserve price may promote entry, increase social welfare, and also induce higher revenues. The first two facts are in stark contrast with the relationship between reserve prices and entry pointed out by the literature. Our different result is obtained in a setting in which potential entrants arrive sequentially and face the risk of incurring losses conditional on winning the object on sale.

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1 Introduction

A reserve price is an instrument commonly used to modify the allocation of standard auctions and affect the price paid by the winner. Its effect is well understood: if the seller sets a reserve price she avoids selling at a low price when the second highest valuation is low and the highest one is high. At the same time, a reserve price exposes the seller to the risk of not selling. If the reserve price is optimally set, the first effect prevails over the second one and standard auctions may maximize expected revenues, see Myerson (1981), Riley and Samuelson (1981).

Another well established fact is that, if the number of bidders is not fixed, a reserve price may discourage entry. Furthermore, the negative impact on revenues of this latter effect may be of first order importance relative to the former one (see Engelbrecht-Wiggans (1987) and McAfee and McMillan (1987)). Indeed, it has been emphasized that one key aspect any market designer should care about is to find ways to promote entry (see Klemperer (2013)). Additionally, Bulow and Klemperer (1996) has shown that the effect on revenues of an extra bidder is superior to the setting of an optimally chosen reserve price.

In light of the above results, it might appear striking that we can show that a reserve price may promote entry and lead to higher efficiency (as well as revenues).

Our results hinge on the fact that a reserve price can prevent the aggregation of information, and that the possibility of aggregating information may be detrimental for entry.

The features of our setting, while specific, are realistic for relevant applications characterized by the presence of an incumbent and potential entrants. One example is the auctioning of a license for which the current incumbent may have superior information about its profitability, but one of the potential entrants may have higher value for winning. A key element in this type of environment is that the potential entrants face a rather risky situation due to the expected losses that may arise following the exit of the incumbent. In fact, the exit of the incumbent may convey adverse information regarding the profitability of winning the auction. That is, bidders may experience ex post regret along the equilibrium path, and the auction can be inefficient. For an analysis of the open ascending auction in this type of environment, see Hernando-Veciana and Michelucci (2011), and Hernando-Veciana and Michelucci (2018).[1]

[1] The first paper shows that the ascending auction is constrained efficient when the number of bidders is two. The second paper shows that with more than two bidders that is not the case due to rushes along the equilibrium path, and proposes an alternative mechanism to implement the constrained efficient allocation.

Our new insight can be understood as follows. Without a reserve price, the knowledge of the presence of an existing entrant decreases the expected gains from winning when it is profitable below the level necessary to compensate for the expected losses that arise if negative information from the exit of the incumbent is aggregated. This makes additional entry unprofitable even without assuming any direct entry cost into the auction.[2] We show a reserve price that prevents the aggregation of potentially adverse information may be used to undue the strategic advantage of being one of the first entrants to arrive, thus leveling the field and promoting both entry and efficiency.

The idea that the amount of information that is endogenously aggregated in an open ascending auction can be strategically manipulated has been introduced in Ettinger and Michelucci (2016). In that paper it is the bidders who could do so by placing jump bids. Ettinger and Michelucci (2019) provides a simple example to show that also a reserve price can be used to to profitably manipulate information. However, that example takes entry as fixed.

2 The model

Assume an ascending price auction that awards the license to operate in a market, and that an incumbent, bidder 0, is competing against a set of N potential entrants, indexed by $e : 1, ..., N$.[3] The incumbent has private information that affects the profitability of holding the license, but may not be the firm that holds the highest value for the winning the license. This situation can be described expressing bidders value function by $v_i = t_i + Q$, $i : 0, ..n$, where t_i is a private value component and Q a common value component, and assuming each bidder i is privately informed about the realization of t_i, and the incumbent is additionally informed about the realization of Q.[4] We model the key features of this environment by assuming that the sets of incumbent's private types and the common value component are binary sets with $t_0 \in \{t_l, t_h\}$, $t_h > t_l$ and $Q \in \{Q_l, Q_h\}$, $Q_h > Q_l$ and that the entrants' private type t_e is drawn from the support $[\underline{t}, \overline{t}]$, with $\underline{t} > t_l$, and $\overline{t} < t_h$, $\overline{t} > \underline{t}$.

[2]Typically models that allow for endogenous entry need to assume the presence of some cost. In our case the potential losses and the diminished profitability caused by the presence of other entrants are enough to deter further entrance.

[3]In what follows we adopt the convention to refer to the incumbent as *he* and to a generic entrant as *she*.

[4]For a setting adopting a private plus common value framework, see Goeree and Offerman (2002).

The objective of this note is to construct the simplest possible example to show that both entry and efficiency can be increased by adding a reserve price. We thus assume the following parametrization: $t_l = \frac{1}{10}$, $t_h = \frac{9}{10}$, $\underline{t} = \frac{1}{2}$, $\bar{t} = \frac{3}{5}$, $Q_l = 0$, $Q_h = 1$; with the incumbent's information drawn as follows: $Pr(t_0 = \frac{9}{10}, Q = 0) = (t_0 = \frac{9}{10}, Q = 1) = \frac{2}{7}$, $Pr(t_0 = \frac{1}{10}, Q = 1) = \frac{3}{7}$; and the entrants' types drawn as follows: $t_e \sim U[\frac{1}{2}, \frac{3}{5}]$.

An alternative way to interpret the value structure above is that entrants have sometimes a better technology than the incumbent but they face high fixed cost to enter the market for which the license is sold. In this case while it may be more efficient to allocate to one of the the entrants if the size of the mark is large ($Q = 1$), it is never the case if the size of the market is sufficiently small ($Q = 0$).

The entry process is the following (for both auctions). The incumbent is always present. The entrants arrive sequentially to the auction with each entrant e arriving in position e. An entrant when deciding whether to enter or not observes how many entrants (if any) are already present and upon this information and the knowledge of his type decides to enter or stay out. After the entry decisions of the n potential entrants are collected, all bidders are told the number of entrants present and a standard clock auction unfolds (with or without a reserve price).[5]

3 The analysis

3.1 Equilibrium analysis without reserve price

To understand why this environment exposes entrants to the *risk* of losses, it is instructive to first focus on the situation in which the incumbent is competing with one entrant only. The incumbent knows both her private value component and the common value component, thus she has a weakly dominant strategy to stay active up to $v_0 = t_0 + Q$. Given the distribution of possible valuations of the incumbent, this means that she will exit at one of the following price levels: $\frac{9}{10}$, $\frac{11}{10}$, $\frac{19}{10}$, with the exit at $\frac{9}{10}$ revealing adverse information about the common component (i.e., $Q = 0$).

The entrant's optimal strategy instead is to be active up to $t_e + 1$, $\forall t_e$. To see why, notice

[5] See Krishna (2010) for a description of the clock (or Japanese) auction. Ties are assumed to be resolved by a random device that assigns the object with equal probability to any of the bidder in the tie.

first that quitting before t_e is weakly dominated by quitting at t_e. Similarly, quitting at a price above $t_e + 1$ is weakly dominated by quitting at $t_e + 1$. What about the values between t_e and $t_e + 1$? In this range, the potential exit of the incumbent at price $\frac{9}{10}$ conveys adverse information and leads any entrant's type t_e to a loss of $\frac{9}{10} - t_e$. The risk of incurring this loss, however, is more than compensated by the gains from winning the auction in case the incumbent exits at price $\frac{11}{10}$, which are $(t_e + 1) - \frac{11}{10}$. In fact, since the positive event is $\frac{3}{2}$ as likely as the adverse event we have that $\frac{3}{2}(t_e + 1 - \frac{11}{10}) \geq \frac{9}{10} - t_e$, for any t_e.

The main result of the setting with no reserve price is presented in the proposition below.

Proposition 1. *Assume the setting introduced in the model section, and that no reserve price is introduced. Then, only the first entrant enters, the expected revenues are* $\frac{82}{70}$ *and the expected value of the winner (efficiency) is* $\frac{91}{140}$.

The proposition is based on the set-up in which the incumbent is already present, whereas the entrants arrive sequentially to the auction, and we focus on their incentive to enter. To prove the proposition notice that the previous analysis already shows that entrant number 1, regardless of his type, would find profitable to enter at least under the conjecture that no one else will. Hence, if we can show that no other entrant can find profitable to enter we are done. To do so it is enough to check the incentive to enter of entrant 2 in case he has the highest possible type $\bar{t} = \frac{3}{5}$. Given that entrant 2 can observe the presence of entrant 1, he can anticipate that the expected gains when the incumbent quits at $\frac{11}{10}$ are down to $\frac{3}{5} - t_1$ due to the presence of entrant 1. The expected losses if the incumbent quits at $\frac{9}{10}$ and both entrants are active at that price are $\frac{\frac{9}{10} - \frac{3}{5}}{2}$ (the two entrants would quit simultaneously and therefore entrant 2 would have probability $\frac{1}{2}$ of incurring loss $\frac{9}{10} - \frac{3}{5}$). Since $\frac{\frac{9}{10} - \frac{3}{5}}{2} \geq \frac{3}{2}(\frac{3}{5} - t_1)$ for all t_1 it follows that regardless of the second's entrant type it is never profitable to be active beyond price $\frac{9}{10}$ if the first entrant does. But since there is no chance to win at a price below $\frac{9}{10}$ entrant 2 may as well not enter. Since the same reasoning applies to all subsequent potential entrants, we have proved that only the first entrant enters. The expected revenues are $\frac{2}{7}\frac{9}{10} + \frac{3}{7}\frac{11}{10} + \frac{2}{7}(E(t_1) + 1) = \frac{82}{70}$, and the expected value of the private type of the winning bidder (the measure of efficiency) is $\frac{5}{7}E(t_1) + \frac{2}{7}\frac{9}{10} = \frac{91}{140}$.

3.2 Equilibrium analysis with reserve price

Lets us now discuss the effect of introducing a reserve price. We have seen that the setting introduced lead to insufficient entry. The reason was that the presence of another entrant diminished the profitability of winning when it was worth it, while still exposing the additional entrant to the risk of making a loss in case of adverse information. In other words, there was room for only one entrant to be active at the price that could convey adverse information. Obviously the first entrant to arrive need not to be the most efficient. Hence, the first entrant has a strategic advantage due to his privileged position in the queue.

A strategically set reserve price can preserve the uncertainty of whether the incumbent has drawn ($t_0 = \frac{9}{10}, Q = 0$), or ($t_0 = \frac{1}{10}, Q = 1$). By doing so it kills the above mentioned advantage and produces an environment that will induce all entrants to use the same bidding function and therefore yield higher efficiency and promote more entry.

Proposition 2. *Assume the setting introduced in the model section, and that a reserve price $r = \frac{11}{10}$ is introduced. Then, all entrants enter, the expected revenues are*

$$\frac{5}{7}\left(E(t_e^{(2)}) + E(Q|v_0 \in \{\frac{9}{10}, \frac{11}{10}\})\right) + \frac{2}{7}\left(E(t_e^{(1)} + 1)\right)$$

and the expected value of the winner (efficiency) is $\frac{5}{7}E(t_e^1) + \frac{2}{7}\frac{9}{10}$.

The optimal reserve price that does the purpose in this specific setting is $r = \frac{11}{10}$. In this case all entrants will declare to be willing to participate in the auction that starts at reservation price $r = \frac{11}{10}$. The incumbent will not participate if either $v_0 = \frac{9}{10}$ or $v_0 = \frac{11}{10}$. In this scenario, the expected value of an entrant of type t_e is $t_e + E(Q|v_0 \in \{\frac{9}{10}, \frac{11}{10}\}) = t_e + \frac{3}{5}$, which is greater or equal than $\frac{11}{10}$ for all t_e. Hence, all entrants enter and then stay active up to their expected valuations. The winner in this case is the most efficient entrant, and the price paid is the expected value of the second most efficient entrant. Instead, if the incumbent matches the reserve price, it means that $Q = 1$ for sure but also that she has the highest value and therefore she will win the auction paying the value of the most efficient entrant given that $Q = 1$.

The expected revenues are $\frac{5}{7}\left(E(t_e^{(2)}) + E(Q|v_0 \in \{\frac{9}{10}, \frac{11}{10}\})\right) + \frac{2}{7}\left(E(t_e^{(1)} + 1)\right)$, where $E(t_e^{(2)}) = \frac{1}{2} + \frac{1}{10}\frac{n-1}{n+1}$, $E(Q|v_0 \in \{\frac{9}{10}, \frac{11}{10}\}) = \frac{3}{5}$; and the expected value of the private type of the winning bidder (the measure of efficiency) is $\frac{5}{7}E(t_e^1) + \frac{2}{7}\frac{9}{10}$, where $E(t_e^{(1)}) = \frac{1}{2} + \frac{1}{10}\frac{n}{n+1}$.[6]

We can finally state our main result.

Proposition 3. *An optimally set reserve price may induce higher entry, higher efficiency, and higher revenues.*

The proposition follows directly from the comparison of the results in propositions 1 and 2. We can see that without the reserve price the first entrant wins when the incumbent's value is less or equal than $\frac{11}{10}$, while in the same scenario with the reserve price it is the most efficient of the n entrants who does. This raises both efficiency and revenues.

Clearly, the parametrization proposed made the analysis for the purpose of this note simpler and the result the most striking. Some comments are in order. First, $\underline{t}$ was set sufficiently high so that even the lowest type would enter without the reserve price. Second, the setting was constructed in such a way that the adverse information from the exit of the incumbent was strong enough to deter even the entrant with the highest type to be active if one entrant was already present in the queue. In a more general set-up is possible that more than one entrant may be active at prices that would cause all entrants to exit following adverse information from the exit of the incumbent. The equilibrium analysis of the ascending auction in that case is more complex, see Hernando-Veciana and Michelucci (2018). Also, if more than one entrant is expected to enter, each potential entrant must take into account of the effect of subsequent entry. Finally, the parametrization was such that, regardless of the entrant's type, the expected value conditional on the incumbent not matching the reserve price was as high as the reserve price. This guaranteed that all entrants would enter in the ascending auction with the optimally chosen reserve price.

4 Conclusion

We have presented a setting with an incumbent and a set of potential entrants competing in an open ascending auction where setting a reserve price increases entry, efficiency and revenues. The result is surprising because the standard effect pointed out by the literature is that reserve prices decrease entry. The different result is due to the different type of environment analyzed.

[6] $E(t_e^{(1)})$ and $\mathrm{E}(\mathrm{t}_e^{(2)})$ denote respectively the expected value of the highest and second highest type of the entrants out of n.

In the environment assumed by most of the literature on auctions, all standard auction are efficient, whereas here we assume a setting that may lead to an inefficient allocation and to ex-post losses along the equilibrium path. Those features can be present in realistic settings in which an incumbent is better informed about some common characteristics of the object for sale, and often lead to a rather different set of results from those of the canonical auction environment. For instance, using settings where those features are present Hernando-Veciana and Michelucci (2018) show that the ascending auction may fail to implement the second best efficient allocation due to rushes that occur along the equilibrium path. Ettinger and Michelucci (2016) show that the bidders' ability to place jump bids may bring interesting new strategic effects that in general lead to ambiguous results in terms of revenues and efficiency. The current note further adds to those contributions showing a surprising effect of the reserve price when entry by potential entrant in an ascending auction is sequential. We hope that future research dealing with environment where auctions fails to be efficient may contribute to unveil new strategic effect and allow to have a more comprehensive understanding of these environments.

References

Bulow, Jeremy and Paul Klemperer, "Auctions Versus Negotiations," *The American Economic Review*, March 1996, *86* (1), 180–194.

Engelbrecht-Wiggans, Richard, "On Optimal Reservation Prices in Auctions," *Management Science*, 1987, *33* (6), 763–770.

Ettinger, David and Fabio Michelucci, "Hiding Information in Open Auctions with Jump Bids," *The Economic Journal*, 2016, *126*, 1484–1502.

___ **and** ___ , "Manipulating Information Revelation with Reserve Prices," *Annals of Economics and Statistics*, 2019, *133*, 87–92.

Goeree, Jacob K. and Theo Offerman, "Efficiency in Auctions with Private and Common Values: An Experimental Study," *The American Economic Review*, 2002, *92*, 625–643.

Hernando-Veciana, Angel and Fabio Michelucci, "Second Best Efficiency and the English Auction," *Games and Economic Behavior*, 2011, *73* (2), 496–506.

___ **and** ___ , "Inefficient Rushes in Auctions," *Theoretical Economics*, 2018, *13*, 273–306.

Klemperer, Paul, "Using and Abusing Economic Theory," *Journal of the European Economic Association*, 2013, *1*, 272–300.

Krishna, Vijay, *Auction Theory*, second ed., Academic Press, 2010.

McAfee, R. Preston and John McMillan, "Auctions with Entry," *Economic Letters*, 1987, *23* (4), 343–347.

Myerson, Roger B., "Optimal Auction Design," *Mathematics of Operation Research*, February 1981, *6* (1), 58–73.

Riley, John G. and William F. Samuelson, "Optimal Auctions," *The American Economic Review*, June 1981, *71* (3), 381–392.