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RESEARCH ARTICLE

Exchange Rate Policy and Firm Heterogeneity

Masashige Hamano[1] · **Francesco Pappadà**[2]

Abstract
This paper examines the exchange rate policy in a tractable framework with heterogeneous firms, incomplete financial markets and nominal rigidities. External demand shocks generate exchange rate movements leading to uncertainty in the labor demand of exporter firms. When exporter firms are homogeneous in terms of productivity, a monetary policy response to external demand shocks stabilizes the export market and improves welfare, thus providing a rationale for *managed* exchange rate policies.

Mathematics Subject Classification F32 · F41 · E40

1 Introduction

In a recent contribution, Maurice Obstfeld (2020) looks back at "The Case for Flexible Exchange Rates" made by Harry G. Johnson in 1969, and explores whether his argument survives the most recent academic critiques of exchange rate flexibility. He concludes that none of the arguments against exchange rate flexibility convincingly undermines the case for a flexible exchange rate. Nonetheless, policymakers have recently adopted exchange rate policies aimed at limiting the fluctuations of the exchange rate, as documented in Ilzetzki et al. (2019). In this paper, we provide a rationale for managed exchange rate policies that protect industries and workers in the export market from exchange rate fluctuations.

The main contribution of this paper is to highlight the unexplored role of firm heterogeneity and nominal rigidities on the exchange rate policy trade-offs. In our economy, external demand shocks produce fluctuations in the nominal exchange rate that modify the selection of exporter firms. When firms are small on average

✉ Francesco Pappadà
francesco.pappada@psemail.eu

Masashige Hamano
masashige.hamano@waseda.jp

1 Waseda University, Tokyo, Japan

2 Paris School of Economics, Banque de France, Paris, France

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and homogeneous in terms of productivity, the fluctuations on external demand may induce a large fraction of firms to enter or exit the export market. In presence of wage rigidity, large fluctuations in external demand translate in high wage mark-ups. In this context, the optimal exchange rate policy reduces the fluctuations of the nominal exchange rate and hence the uncertainty in the export market. Our results therefore suggest that a managed exchange rate is welfare improving when firm heterogeneity is low, that is when many firms are subject to fluctuations in external demand. Instead, when firms are large on average and more heterogeneous, the benefits of dampened fluctuations in the exchange rate do not compensate for the costs associated with the high wage mark-ups of domestic firms. The optimal monetary policy therefore responds less to external demand shocks, letting the exchange rate free to float.

We examine the exchange rate policy trade-offs in a tractable framework where firm dynamics respond to external demand shocks under incomplete financial markets. Our setting features a two-country dynamic stochastic general equilibrium model with nominal wage rigidities, endogenous entry and heterogeneous firms. In our model, the exchange rate policy trade-offs arise from the presence of incomplete financial markets which distort the allocation under flexible prices. For this reason, a flexible exchange rate may not be efficient despite its ability to replicate the allocation of flexible prices. On the other hand, a fixed exchange rate may improve welfare by increasing the comovement between the demand shock and the production of preferred goods, getting closer to the allocation of the social planner. Importantly, the degree of firm heterogeneity determines the welfare ranking across different exchange rate policies. A managed exchange rate policy dominates the flexible exchange rate when firms are more homogeneous because it reduces the uncertainty associated with the larger fluctuations in the export market.

This paper belongs to the literature on the "shock absorber" role played by a flexible exchange rate established in the original contributions by Friedman (1953) and Mundell (1961) and recently highlighted by Obstfeld (2020). In presence of firm heterogeneity, exchange rate fluctuations partially absorb the shock for the domestic economy, while inducing a substantial volatility for exporters' profits. A monetary policy intervention aimed at dampening these fluctuations therefore acts as a powerful macroeconomic stabilization tool for the export market. This paper also relates to the literature which emphasizes the adjustments occurring at both the intensive and extensive margins of trade with or without firm heterogeneity—see among others Ghironi and Melitz (2005), Pappadà (2011), Corsetti et al. (2013), Cacciatore (2014), di Mauro and Pappadà (2014) and Hamano (2014). In this paper, we introduce nominal rigidities and discuss the exchange rate policy with endogenous entry of heterogeneous firms.[1] As we assume nominal rigidity in wage setting and let firms adjust freely their prices, the producer currency pricing ensures the "expenditure switching effect" at individual firm price level. Our modeling setup is therefore similar to Rodriguez-Lopez (2011) which analyzes the expenditure switching

[1] Hamano and Zanetti (2020) also explore the link between selection of firms and monetary policy but in a closed economy setting.

effect with heterogeneous firms and its resulting bias in aggregate price. However, the scope of our paper is different as we focus on the optimal monetary policy and the exchange rate policy under incomplete financial markets.[2]

Our paper is also related to the recent debate on trade integration or protectionism—see Auray et al. (2019), Erceg et al. (2018) and Lindé and Pescatori (2019). While we focus on exchange rate policy, Costinot et al. (2020) provide a normative analysis of the optimal trade policy in a similar framework with heterogeneous firms and selection into the export market. Cacciatore and Ghironi (2021) focus on the consequences of trade linkages on the Ramsey cooperative monetary policy in an open economy with firm heterogeneity and search and matching frictions in labor market. In a similar setting, Barattieri et al. (2018) study a temporary tariff shock and cast a doubt for its effectiveness as a macroeconomic stabilization tool. While this literature studies the interplay between monetary policy and trade policies, we rather focus on the ability of the monetary policy to act as a powerful macroeconomic stabilization tool depending on the exchange rate policy. In this respect, our paper is reminiscent of Bergin and Corsetti (2020), which study the impact of monetary policy on the comparative advantage of countries. In their paper, a perfect consumption risk-sharing is guaranteed and nominal rigidities are the only source of distortion, thus the flexible price allocation is efficient. While Bergin and Corsetti (2020) focus on the stabilization of marginal costs under complete financial markets, we discuss their stabilization under incomplete financial markets. Moreover, in their paper the optimal monetary policy affects the sectoral reallocation fostering firm entry in the differentiated goods sector. In our paper instead, the reallocation takes place within our tradable sector with heterogeneous firms.

The paper is structured as follows. In the next section, we introduce a two country model with external demand shocks, nominal rigidities and firm heterogeneity and provide an analytical solution. In Sect. 3, we derive the allocation of the social planner as a reference point to evaluate the welfare of different exchange rate policies. In Sect. 4, we first provide a welfare comparison of polar exchange rate policies, and then derive the optimal monetary policy as a function of the fundamentals of the economy. Section 5 concludes.

2 The Model

In this section, we introduce a two country dynamic stochastic general equilibrium model with firm heterogeneity. There are two important frictions in our model. First, we introduce a nominal rigidity, as households set wages one period in advance based on their expectations of future labor demand. Second, the international asset

[2] Incomplete financial markets introduce distortions in the flexible price allocation, and provide a case for the fixed exchange rate to dominate the flexible one. While Devereux (2004) highlights the role of the elasticity of labor supply and Hamano and Picard (2017) the preference for product variety in ranking the exchange rate policy, we study how the heterogeneity in firm productivity shapes the response of the economy to demand shocks.

markets are incomplete, as Home household cannot hold Foreign assets and vice versa.[3]

Both Home and Foreign countries are inhabited by a unit mass of households which provide imperfectly-substituted labor. All goods are tradable but only a fraction of them are exported by firms operating in monopolistic competition, and the number of exporters is determined endogenously. We introduce demand shock to each countries' goods, and study how these shocks interacts with firm dynamics according to the conduct of monetary policy. Finally, we show a closed form solution of our dynamic stochastic general equilibrium model without relying on any approximation method.

2.1 Households

The representative household maximizes her life time utility, $E_t \sum_{s=t}^{\infty} \beta^{s-t} U_t(j)$, where β is the exogenous discount factor. The utility of individual household j at time t depends on consumption $C_t(j)$ and labor supply $L_t(j)$ as follows:

$$U_t(j) = \ln C_t(j) + \chi \ln \frac{M_t(j)}{P_t} - \eta \frac{\left[L_t(j)\right]^{1+\varphi}}{1+\varphi}. \tag{1}$$

Households derive utility from consumption and real money holdings (to a degree χ), and experience disutility from working (to a degree η), while φ is the inverse of the Frisch elasticity of labor supply.

The basket of goods $C_t(j)$ is defined as

$$C_t(j) = \left(\frac{C_{H,t}(j)}{\alpha_t}\right)^{\alpha_t} \left(\frac{C_{F,t}(j)}{\alpha_t^*}\right)^{\alpha_t^*},$$

where α_t and α_t^* are the stochastic preferences attached to the bundle of Home goods $C_{H,t}(j)$ and imported goods $(C_{F,t}(j))$, as we denote Foreign variables with an asterisk (*). These baskets are defined over a continuum of goods Ω as

$$C_{H,t}(j) = \left(\int_{\varsigma\in\Omega} c_{D,t}(j,\varsigma)^{1-\frac{1}{\sigma}} \mathrm{d}\varsigma\right)^{\frac{1}{1-\frac{1}{\sigma}}},\ C_{F,t}(j) = \left(\int_{\varsigma^*\in\Omega} c_{X,t}(j,\varsigma^*)^{1-\frac{1}{\sigma}} \mathrm{d}\varsigma^*\right)^{\frac{1}{1-\frac{1}{\sigma}}}.$$

In each time period, only a subset of variety of goods is available from the total universe of variety of goods Ω. We denote $N_{D,t}$ and $N_{X,t}^*$ as the number of domestic and imported product varieties, respectively. $c_{D,t}(j,\varsigma)$ and $c_{X,t}(j,\varsigma^*)$ represent the demand addressed to the individual product variety indexed by ς and ς^*. σ denotes the elasticity of substitution among differentiated goods and is greater than 1.

The optimal consumption for each domestic basket, imported basket and individual product variety are found to be

[3] In line with related literature, for the sake of tractability we opt for this extreme source of market incompleteness which implies financial autarky.

$$C_{H,t}(j) = \left(\frac{P_{H,t}}{P_t}\right)^{-1} \alpha_t C_t(j), \quad C_{F,t}(j) = \left(\frac{P_{F,t}}{P_t}\right)^{-1} \alpha_t^* C_t(j),$$
$$c_{D,t}(j,\varsigma) = \left(\frac{p_{D,t}(\varsigma)}{P_{H,t}}\right)^{-\sigma} C_{H,t}(j), \quad c_{X,t}(j,\varsigma^*) = \left(\frac{p^*_{X,t}(\varsigma^*)}{P_{F,t}}\right)^{-\sigma} C_{F,t}(j).$$

In the above expressions, $p_{D,t}(\varsigma)$ stands for the price of product variety ς which is domestically produced. In particular, $p^*_{X,t}(\varsigma^*)$ denotes the price of imported product variety ς^*, denominated in Home currency units. $P_{H,t}$ and $P_{F,t}$ are the price of the basket of Home produced and imported goods, respectively. P_t is the price of the aggregated basket. Price indexes that minimize expenditures on each consumption basket are

$$P_t = P_{H,t}^{\alpha_t} P_{F,t}^{\alpha_t^*},$$
$$P_{H,t} = \left(\int_{\varsigma\in\Omega} p_{D,t}(\varsigma)^{1-\sigma} d\varsigma\right)^{\frac{1}{1-\sigma}}, \quad P_{F,t} = \left(\int_{\varsigma^*\in\Omega} p^*_{X,t}(\varsigma^*)^{1-\sigma} d\varsigma^*\right)^{\frac{1}{1-\sigma}}.$$

Similar expressions hold for Foreign. Crucially, the subset of goods available to Foreign during period t, $\Omega_t^* \in \Omega$, can be different from the subset of goods available to Home $\Omega_t \in \Omega$.

2.2 Firms

2.2.1 Production, Pricing and the Export Decision

There is a mass of $N_{D,t}$ number of firms in Home. Upon entry, firms draw their productivity level z from a distribution $G(z)$ on $[z_{\min}, \infty)$. Since there are no fixed production costs and hence no selection into domestic market, $G(z)$ also represents the productivity distribution of all producing firms. Prior to entry, however, these firms are identical and face a sunk entry cost $f_{E,t} = l_{E,t}$ units of labor.[4] The sunk cost is composed of imperfectly differentiated labor services provided by households (indexed by i) such that

$$l_{E,t} = \left(\int_0^1 l_{E,t}(j)^{1-\frac{1}{\theta}} dj\right)^{\frac{1}{1-\frac{1}{\theta}}}, \tag{2}$$

[4] As an alternative, entry cost could be paid in terms of consumption goods as in Corsetti et al. (2010). In that case, monetary policy has an impact on the number of entrants combined with price rigidity. In our setting, we choose to express entry costs in labor units because it is closely related to our source of nominal rigidity which concerns wages. As shown in the model solution in Table 1, with wage rigidity, a positive (negative) monetary shock directly increases (decreases) the entry of firms, in the same fashion as in Corsetti et al. (2010).

where θ represents the elasticity of substitution among different labor services. We consider $f_{E,t}$ to be exogenous. By defining the nominal wage for type j labor as $W_t(j)$, the total cost for a firm to setup is thus $\int_0^1 l_{E,t}(j)W_t(j)dj$. The cost minimization yields the following labor demand for type j labor service:

$$l_{E,t}(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\theta} l_{E,t}, \tag{3}$$

where W_t denotes the corresponding wage index, which is

$$W_t = \left(\int_0^1 W_t(j)^{1-\theta} dj\right)^{\frac{1}{1-\theta}}.$$

Exporting requires an operational fixed cost of $f_{X,t} = l_{f_X,t}$ units of labor defined in a similar way as in Eq. (2). The cost minimization provides a similar demand for each specific labor service as in Eq. (3).[5]

For the production of each good variety, only composite labor basket is required as input. Thus the production function of firm with productivity z is given by $y_t(z) = zl_t(z)$ where

$$l_t(z) = \left(\int_0^1 l_t(z,j)^{1-\frac{1}{\theta}} dj\right)^{\frac{1}{1-\frac{1}{\theta}}}.$$

The cost minimization yields the demand for type j labor for production as

$$l_t(z,j) = \left(\frac{W_t(j)}{W_t}\right)^{-\theta} l_t(z).$$

The firm faces a residual demand curve with constant elasticity σ. The production scale is thus determined by the demand addressed to the firm under monopolistic competition. Profit maximization yields the following optimal price $p_{D,t}(z)$ by firm with productivity z:

$$p_{D,t}(z) = \frac{\sigma}{\sigma-1}\frac{W_t}{z}.$$

If the firm exports, its price of export is $p_{X,t}(z) = \tau p_{D,t}(z)\varepsilon_t^{-1}$ where ε_t is the nominal exchange rate defined as the price of one unit of Foreign currency in terms of Home currency units, and $\tau > 1$ is an iceberg trade cost. In our definition, $p_{X,t}(z)$ is thus denominated in terms of Foreign currency units.[6]

[5] The labor demand for exporting are $l_{f_X,t} = \left(\int_0^1 l_{f_X,t}(j)^{1-\frac{1}{\theta}} dj\right)^{\frac{1}{1-\frac{1}{\theta}}}$ and $l_{f_X,t}(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\theta} l_{f_X,t}$.

[6] The practice of pricing to market and dollar pricing has also been emphasized in the literature and become a motivation to limit the fluctuations in the nominal exchange rate (see Betts and Devereux (1996), Devereux and Engel (2003), Corsetti et al. (2010) and Gopinath et al. (2020) among others). Instead of price rigidity in the export market, we introduce wage rigidity and focus on financial market incompleteness as an additional distortion in the economy.

Total firm profits $D_t(z)$ can be decomposed into those from domestic sales $D_{D,t}(z)$ and those from exporting sales $D_{X,t}(z)$ (if the firm exports) as $D_t(z) = D_{D,t}(z) + D_{X,t}(z)$. Using the demand functions found previously and the aggregate consumption defined as $C_t = \left(\int_0^1 C_t^{1-\frac{1}{\sigma}}(j)dj\right)^{\frac{1}{1-\frac{1}{\sigma}}}$, we can write the profits from each market as

$$\begin{aligned} D_{D,t}(z) &= \frac{1}{\sigma}\left(\frac{p_{D,t}(z)}{P_{H,t}}\right)^{1-\sigma}\alpha_t P_t C_t, \\ D_{X,t}(z) &= \frac{\varepsilon_t}{\sigma}\left(\frac{p_{X,t}(z)}{P^*_{H,t}}\right)^{1-\sigma}\alpha_t P^*_t C^*_t - W_t f_X. \end{aligned} \tag{4}$$

Equation (4) implies that a firm exports when z is larger than $z_{X,t}$, the cut-off level of productivity for exporting. Thus, the share of non-traded goods in the economy arises endogenously with changes in the productivity cutoff $z_{X,t}$.

2.2.2 Firm Averages

Given a distribution $G(z)$, the productivity level of a mass of $N_{D,t}$ domestically producing firms is distributed over $[z_{\min}, \infty)$. Among these firms, there are $N_{X,t} = \left[1 - G\left(z_{X,t}\right)\right]N_{D,t}$ exporters in Home. Following Melitz (2003) and Ghironi and Melitz (2005), we define two average productivity levels, $\widetilde{z}_D$ for domestically producing firms and $\widetilde{z}_{X,t}$ for exporters as follows:

$$\widetilde{z}_D \equiv \left[\int_{z_{\min}}^{\infty} z^{\sigma-1}\mathrm{d}G(z)\right]^{\frac{1}{\sigma-1}}, \quad \widetilde{z}_{X,t} \equiv \left[\frac{1}{1-G(z_{X,t})}\int_{z_{X,t}}^{\infty} z^{\sigma-1}\mathrm{d}G(z)\right]^{\frac{1}{\sigma-1}}.$$

These average productivity levels summarize all the information about the distribution of firm productivity. Given these averages, we define the average real domestic and export prices as $\widetilde{p}_{D,t} \equiv p_{D,t}\left(\widetilde{z}_D\right)$ and $\widetilde{p}_{X,t} \equiv p_{X,t}\left(\widetilde{z}_{X,t}\right)$, respectively. We also define average profits from domestic sales and export sales as $\widetilde{D}_{D,t} \equiv D_{D,t}\left(\widetilde{z}_D\right)$ and $\widetilde{D}_{X,t} \equiv D_{X,t}\left(\widetilde{z}_{X,t}\right)$. Finally, the average profit among all firms is given by $\widetilde{D}_t = \widetilde{D}_{D,t} + \left(N_{X,t}/N_{D,t}\right)\widetilde{D}_{X,t}$.

2.2.3 Firm Entry and Exit

Firm entry takes place until the expected value of entry equals the entry cost, leading to the following free entry condition:

$$\widetilde{V}_t = f_{E,t}W_t,$$

where $\widetilde{V}_t$ is the expected value of entry which is discussed below. In what follows, we assume (i) that entrants at time t only start producing at time $t+1$ (*one-period to build*), and (ii) that firms' production plants fully depreciate after one period.

2.2.4 Parametrization of Productivity Draws

We assume the following Pareto distribution for $G(z)$:

$$G(z) = 1 - \left(\frac{z_{\min}}{z}\right)^{\kappa},$$

where $z_{\min}$ is the minimum productivity level and $\kappa > \sigma - 1$ is the shape parameter.[7] With this parametrization, we have

$$\widetilde{z}_D = z_{\min}\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}, \qquad \widetilde{z}_{X,t} = z_{X,t}\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}.$$

The share of exporters in the total number of domestic firms is then given by

$$\frac{N_{X,t}}{N_{D,t}} = z_{\min}^{\kappa}\left(\widetilde{z}_{X,t}\right)^{-\kappa}\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{\kappa}{\sigma-1}}. \tag{5}$$

Finally, there exists a firm with a specific productivity cutoff $z_{X,t}$ that earns zero profits from exporting, as $D_{X,t}\left(z_{X,t}\right) = 0$. With the above Pareto distribution, this implies that the average profits of exporter firms are

$$\widetilde{D}_{X,t} = W_t f_{X,t}\frac{\sigma-1}{\kappa-(\sigma-1)}.$$

Note that there is no feedback of the cutoff level productivity to the initial distribution $G(z)$, which is fixed and time invariant. However, the equilibrium cutoff level $z_{X,t}$ and hence the average productivity of exporters $\widetilde{z}_{X,t}$ change over time.

2.3 Nominal Rigidities and Household Intertemporal Choices

We now introduce nominal wage rigidities, as we assume that wages are sticky for one time period.[8] This implies that the household j sets wages at $t-1$ and maximize her expected utility at t knowing the following demand for her labor:

[7] The assumption of a Pareto shape of firm productivity distribution $\kappa > \sigma - 1$ ensures a finite mean for the sales of the firms.

[8] While there is a strand of literature that models a downward wage rigidity (see for instance, Schmitt-Grohé and Uribe (2016), our setup employs a Calvo wage stickiness hence wages are rigid both upward and downward.

$$L_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\theta} L_t.$$

The first-order condition with respect to $W_t(j)$ yields

$$W_t(j) = \frac{\eta\theta}{(\theta-1)(1+\xi)} \frac{E_{t-1}\left[L_t(j)^{1+\varphi}\right]}{E_{t-1}\left[\frac{L_t(j)}{P_t C_t(j)}\right]}, \tag{6}$$

where $1+\xi$ is the labor subsidy which eliminates distortions due to monopolistic power in labor markets. Households set the wage so that the expected marginal cost of supplying additional labor services equals the expected marginal revenue.[9]

Along with the wage setting, the household j also chooses her share holdings of mutual funds $x_t(j)$ and bond holdings $B_t(j)$ while facing the following budget constraint:

$$\begin{aligned} &P_t C_t(j) + B_t(j) + M_t(j) + x_t(j) N_{D,t+1} \widetilde{V}_t \\ &\quad = (1+\xi) W_t(j) L_t(j) + (1+i_{t-1}) B_{t-1}(j) + M_{t-1}(j) + x_{t-1}(j) N_{D,t} \widetilde{D}_t + T_t^f, \end{aligned}$$

where i_t represents nominal interest rate between t and $t+1$ and T_t^f represents a transfer from domestic government, which can be positive or negative.

The first-order conditions with respect to share and bond holdings yields, respectively

$$\begin{aligned} \widetilde{V}_t =& E_t\left[Q_{t,t+1}(j)\widetilde{D}_{t+1}\right], \\ 1 =& (1+i_t) E_t\left[Q_{t,t+1}(j)\right]. \end{aligned}$$

where $Q_{t,t+1}$ is the stochastic discount factor defined as $Q_{t,t+1}(j) = E_t\left[\frac{\beta P_t C_t(j)}{P_{t+1} C_{t+1}(j)}\right]$.

Finally, the household j maximizes her consumption and real money holdings. As a result, we have

$$P_t C_t(j) = \frac{M_t}{\chi}\left(\frac{i_t}{1+i_t}\right), \tag{7}$$

which implies that nominal spending $P_t C_t(j)$ is tied down to the money supply M_t.

2.4 Balanced Trade and Equilibrium

At equilibrium, there is a symmetry across households so that $C_t(j) = C_t$, $L_t(j) = L_t$, $M_t(j) = M_t$ and $W_t(j) = W_t$. Furthermore, we follow Corsetti et al. (2010) and Bergin

[9] The marginal cost of one additional unit of labor supply is $\eta\theta W_t(j)^{-1} \mathrm{E}_{t-1}\left[L_t(j)^{1+\varphi}\right]$, and its marginal revenue is $(\theta-1)(1+\xi)\mathrm{E}_{t-1}\left[\frac{L_t(j)}{P_t C_t(j)}\right]$.

and Corsetti (2020) and define the monetary stance as proportional to monetary expenditure:[10]

$$\mu_t \equiv P_t C_t.$$

The government has no power to directly control private lending and borrowing and the balanced budget rule is

$$M_t - M_{t-1} = T_t^f + \xi W_t L_t.$$

We assume that trade is balanced, thus the value of Home exports is equal to the value of Home imports once they are converted to the same unit of currency: $\varepsilon_t P^*_{H,t} C^*_{H,t} = P_{F,t} C_{F,t}$. Combined with the demand of goods found previously, this implies

$$\varepsilon_t = \frac{\alpha_t^*}{\alpha_t} \frac{\mu_t}{\mu_t^*}.$$

Note that the general expression for the terms of trade (defined as the price of average Foreign exported goods in average Home exported goods) is independent of the monetary policy rule:

$$\text{TOT} \equiv \frac{\widetilde{p}^*_{X,t}}{\varepsilon_t \widetilde{p}_{X,t}} = \frac{\alpha_t^*}{\alpha_t} \frac{\mu_t}{\varepsilon_t \mu_t^*} \frac{N_{X,t} \widetilde{y}_{X,t}}{N^*_{X,t} \widetilde{y}^*_{X,t}}.$$

Under nominal wage rigidity, the aggregate labor supply L_t adjusts to its demand and the labor market clears as

$$L_t = N_{D,t} \frac{\widetilde{y}_{D,t}}{\widetilde{z}_D} + N_{X,t} \left(\frac{\widetilde{y}_{X,t}}{\widetilde{z}_{X,t}} + f_{X,t} \right) + N_{D,t+1} f_{E,t}, \tag{8}$$

where $\widetilde{y}_{D,t}$ and $\widetilde{y}_{X,t}$ stand for production scale of each average domestic firms and average exporters. The labor demand comes from producers selling their goods in the domestic and export markets (including export fixed costs), and from resources used for the creation of new firms. A similar expression holds for the Foreign country.

We can now determine the equilibrium wage using the wage setting equation (6) and the labor market clearing condition (8):

[10] When combining the monetary stance with the Euler equation on bond holdings, one gets $\frac{1}{\mu_t} = \mathrm{E}_t \lim_{s\to\infty} \beta^s \frac{1}{\mu_{t+s}} \prod_{\tau=0}^{s-1} (1 + i_{t+\tau})$. The monetary stance μ_t may therefore be expressed as a function of the future expected path of interest rates or as a money supply rule M_t as in Eq. (7).

Table 1 The closed form solution of the model

Nb of entrants	$N_{D,t+1} = \frac{\beta}{\sigma}\frac{\mu_t}{W_t f_{E,t}} E_t\left[\alpha_{t+1} + \frac{\sigma-1}{\kappa}\alpha^*_{t+1}\right]$	$N^*_{D,t+1} = \frac{\beta}{\sigma}\frac{\mu^*_t}{W^*_t f^*_{E,t}} E_t\left[\alpha^*_{t+1} + \frac{\sigma-1}{\kappa}\alpha_{t+1}\right]$
Nb of exporters	$N_{X,t} = \frac{\sigma-1}{\sigma}\left(\frac{1}{\sigma-1} - \frac{1}{\kappa}\right)\frac{\alpha^*_t \mu_t}{W_t f_{X,t}}$	$N^*_{X,t} = \frac{\sigma-1}{\sigma}\left(\frac{1}{\sigma-1} - \frac{1}{\kappa}\right)\frac{\alpha_t \mu^*_t}{W^*_t f^*_{X,t}}$
Av. exporters	$\widetilde{z}_{X,t} = \left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}\left(\frac{N_{X,t}}{N_{D,t}}\right)^{-\frac{1}{\kappa}}$	$\widetilde{z}^*_{X,t} = \left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}\left(\frac{N^*_{X,t}}{N^*_{D,t}}\right)^{-\frac{1}{\kappa}}$
Production	$\widetilde{y}_{D,t} = \frac{\sigma-1}{\sigma}\frac{\alpha_t \mu_t \widetilde{z}_D}{N_{D,t} W_t}, \quad \widetilde{y}_{X,t} = \frac{\sigma-1}{\sigma}\frac{\alpha^*_t \mu_t \widetilde{z}_{X,t}}{N_{X,t} W_t}$	$\widetilde{y}^*_{D,t} = \frac{\sigma-1}{\sigma}\frac{\alpha^*_t \mu^*_t \widetilde{z}^*_D}{N^*_{D,t} W^*_t}, \quad \widetilde{y}^*_{X,t} = \frac{\sigma-1}{\sigma}\frac{\alpha_t \mu^*_t \widetilde{z}^*_{X,t}}{N^*_{X,t} W^*_t}$
Average price	$\widetilde{p}_{D,t} = \frac{\sigma}{\sigma-1}\frac{W_t}{\widetilde{z}_D}, \quad \widetilde{p}_{X,t} = \frac{\sigma}{\sigma-1}\frac{\tau_t \varepsilon_t^{-1} W_t}{\widetilde{z}_{X,t}}$	$\widetilde{p}^*_{D,t} = \frac{\sigma}{\sigma-1}\frac{W^*_t}{\widetilde{z}^*_D}, \quad \widetilde{p}^*_{X,t} = \frac{\sigma}{\sigma-1}\frac{\tau_t \varepsilon_t W^*_t}{\widetilde{z}^*_{X,t}}$
Price indices	$P_{H,t} = N_{D,t}^{-\frac{1}{\sigma-1}}\widetilde{p}_{D,t}, \quad P_{F,t} = N_{X,t}^{*-\frac{1}{\sigma-1}}\widetilde{p}^*_{X,t}, \quad P_t = P_{H,t}^{\alpha_t} P_{F,t}^{\alpha^*_t}$	$P^*_{F,t} = N_{D,t}^{*-\frac{1}{\sigma-1}}\widetilde{p}^*_{D,t}, \quad P^*_{H,t} = N_{X,t}^{-\frac{1}{\sigma-1}}\widetilde{p}_{X,t}, \quad P^*_t = P_{F,t}^{*\alpha^*_t} P_{H,t}^{*\alpha_t}$
Consumption	$C_t = \left(\frac{C_{H,t}}{\alpha_t}\right)^{\alpha_t}\left(\frac{C_{F,t}}{\alpha^*_t}\right)^{\alpha^*_t}$	$C^*_t = \left(\frac{C^*_{F,t}}{\alpha^*_t}\right)^{\alpha^*_t}\left(\frac{C^*_{H,t}}{\alpha_t}\right)^{\alpha_t}$
Profits	$\widetilde{D}_{D,t} = \frac{\alpha_t}{\sigma}\frac{\mu_t}{N_{D,t}}, \quad \widetilde{D}_{X,t} = \frac{\sigma-1}{\kappa}\frac{\alpha_t}{\sigma}\frac{\varepsilon_t \mu^*_t}{N_{X,t}}, \quad \widetilde{D}_t = \widetilde{D}_{D,t} + \frac{N_{X,t}}{N_{D,t}}\widetilde{D}_{X,t}$	$\widetilde{D}^*_{D,t} = \frac{\alpha^*_t}{\sigma}\frac{\mu^*_t}{N^*_{D,t}}, \quad \widetilde{D}^*_{X,t} = \frac{\sigma-1}{\kappa}\frac{\alpha^*_t}{\sigma}\frac{\varepsilon_t^{-1}\mu_t}{N^*_{X,t}}, \quad \widetilde{D}^*_t = \widetilde{D}^*_{D,t} + \frac{N^*_{X,t}}{N^*_{D,t}}\widetilde{D}^*_{X,t}$
ZPC	$\widetilde{D}_{X,t} = W_t f_{X,t}\frac{\sigma-1}{\kappa-(\sigma-1)}$	$\widetilde{D}^*_{X,t} = W^*_t f^*_{X,t}\frac{\sigma-1}{\kappa-(\sigma-1)}$
Share price	$\widetilde{V}_t = f_{E,t} W_t$	$\widetilde{V}^*_t = f^*_{E,t} W^*_t$
Labor supply	$L_t = (\sigma-1)\frac{N_{D,t}\widetilde{D}_t}{W_t} + \sigma N_{X,t} f_{X,t} + N_{D,t+1} f_{E,t}$	$L^*_t = (\sigma-1)\frac{N^*_{D,t}\widetilde{D}^*_t}{W^*_t} + \sigma N^*_{X,t} f^*_{X,t} + N^*_{D,t+1} f^*_{E,t}$
Monetary stance	$\mu_t = P_t C_t$	$\mu^*_t = P^*_t C^*_t$
Wages	$W_t = \Gamma\left\{\frac{E_{t-1}\left[(A_t \mu_t)^{1+\varphi}\right]}{E_{t-1}[A_t]}\right\}^{\frac{1}{1+\varphi}}$	$W^*_t = \Gamma\left\{\frac{E_{t-1}\left[(A^*_t \mu^*_t)^{1+\varphi}\right]}{E_{t-1}[A^*_t]}\right\}^{\frac{1}{1+\varphi}}$
Exchange rate	$\varepsilon_t = \frac{\alpha^*_t}{\alpha_t}\frac{\mu_t}{\mu^*_t}$	
Definition of A_t	$A_t = \frac{\sigma-1}{\sigma}\alpha_t + \left(1 - \frac{\sigma-1}{\sigma\kappa}\right)\alpha^*_t + \frac{\beta}{\sigma}E_{t-1}\left[\alpha_{t+1} + \frac{\sigma-1}{\kappa}\alpha^*_{t+1}\right]$	$A^*_t = \frac{\sigma-1}{\sigma}\alpha^*_t + \left(1 - \frac{\sigma-1}{\sigma\kappa}\right)\alpha_t + \frac{\beta}{\sigma}E_{t-1}\left[\alpha^*_{t+1} + \frac{\sigma-1}{\kappa}\alpha_{t+1}\right]$
Shock process	$\alpha_t = \frac{1}{2}\upsilon_t, \quad \alpha^*_t = \frac{1}{2}\upsilon^*_t, \quad E_{t-1}[\upsilon_t] = E_{t-1}[\upsilon^*_t] = 1, \quad E_{t-1}[\upsilon_t \upsilon_{t+1}] = 1, \quad \upsilon_t + \upsilon^*_t = 2$	

$$W_t = \Gamma \left\{ \frac{E_{t-1}\left[\left(A_t \mu_t \right)^{1+\varphi} \right]}{E_{t-1}\left[A_t \right]} \right\}^{\frac{1}{1+\varphi}}, \tag{9}$$

where $\Gamma \equiv \left[\frac{\eta\theta}{(\theta-1)(1+\xi)} \right]^{\frac{1}{1+\varphi}}$ and

$$A_t \equiv \frac{\sigma-1}{\sigma}\left[\alpha_t + \left(1 + \frac{1}{\sigma-1} - \frac{1}{\kappa} \right)\alpha_t^* + \beta E_t\left[\frac{1}{\sigma-1}\alpha_{t+1} + \frac{1}{\kappa}\alpha_{t+1}^* \right] \right]. \tag{10}$$

The equilibrium wage thus depends on the expected interaction between labor demand fluctuations and the monetary stance captured by $E_{t-1}\left[\left(A_t \mu_t \right)^{1+\varphi} \right]$.

Finally we assume the following process for the preference shift:

$$\alpha_t = \frac{1}{2}\upsilon_t, \quad \alpha_t^* = \frac{1}{2}\upsilon_t^*.$$

The preference shocks υ_t is i.i.d. with $E_{t-1}\left[\upsilon_t\right] = E_{t-1}\left[\upsilon_t^*\right] = 1$ and $\upsilon_t + \upsilon_t^* = 2$. We report in Table 1 the closed form solution of the model which can be obtained without relying on any approximation methods. We refer to Appendix 1 for the derivation of all the endogenous variables.

3 Social Planner Allocation

In this section, we highlight the role of the two distortions in our model: (i) the nominal wage rigidity, and (ii) the incomplete financial markets. We first derive the first best allocation of the social planner, which represents the reference standpoint to evaluate the welfare impact of different exchange rate policies. We then show that the planner solution is close to the solution of our model when we allow for complete financial markets and flexible wages.

3.1 Planner Solution

The social planner is not subject to nominal wage rigidities by definition, but faces the technological constraints of one time to build and produce, has to clear the labor and goods markets and takes as given the distribution of firm productivity. Even though the expected discounted sum of utility is defined over an infinite horizon of time, the intervention of the social planner at time t has an impact only for two consecutive time periods due to the assumption of one period to build and produce. In deriving the welfare metrics, we therefore express the expected utility without loss of generality as $E_{t-1}[\mathcal{U}] \equiv E_{t-1}\left[U_t\right] + \beta E_{t-1}\left[U_{t+1}\right]$. Plugging the consumption bundle into the utility function we get

$$E_{t-1}[\mathcal{U}] = E_{t-1}\left[\alpha_t\left(lnN_{D,t}^{\frac{\sigma}{\sigma-1}}\widetilde{y}_{D,t}\right) + \alpha_t^*\left(lnN_{X,t}^{*\frac{\sigma}{\sigma-1}}\frac{\widetilde{y}^*_{X,t}}{\tau}\right)\right] + \beta E_{t-1}\left[\alpha_{t+1}\left(lnN_{D,t+1}^{\frac{\sigma}{\sigma-1}}\widetilde{y}_{D,t+1}\right) + \alpha_{t+1}^*\left(lnN_{X,t+1}^{*\frac{\sigma}{\sigma-1}}\frac{\widetilde{y}^*_{X,t+1}}{\tau_t}\right)\right]. \tag{11}$$

Equation (11) shows that the expected utility is a function of the expected (log of) extensive and intensive margins, and their covariance with demand shocks. A large *level* of extensive ($N_{D,t}$) and intensive ($\widetilde{y}_{D,t}$) margins improves welfare, whereas a large *volatility* of these margins is detrimental for welfare. On the other hand, a positive covariance between demand shifts and both the extensive and intensive margins also improves welfare. This is the case both for domestic and imported goods.[11]

The planner maximizes the sum of equally weighted utility in Home and Foreign $E_{t-1}[\mathcal{U}] + E_{t-1}\left[\mathcal{U}^*\right]$ by choosing directly labor supply, the average scale of production and the number of firms in both domestic and export market subject to the labor market clearing condition (8) in both countries, and taking as given the Pareto distribution of productivity. Table 2 provides the solution of the social planner.[12]

Proposition 1 Social Planner *When the preference attached to goods produced in Home is larger* ($\alpha_t > \alpha_t^*$), *the allocation of the social planner is such that*:

(a) *In the export market, the number of Home exporters is higher than in Foreign country* ($N_{X,t} > N^*_{X,t}$) *while their average productivity and scale of production are smaller* ($\widetilde{z}_{X,t} < \widetilde{z}^*_{X,t}$ *and* $\widetilde{y}_{X,t} < \widetilde{y}^*_{X,t}$).

(b) *In the domestic market, the average domestic production in Home is larger than in Foreign* ($\widetilde{y}_{D,t} > \widetilde{y}^*_{D,t}$), *and the future number of firms (entry) in both countries is constant.*

Corollary 1 *The allocation of the social planner is isomorphic to the allocation under complete financial markets and flexible wages.*

Proof See Appendix 3. □

The allocation with flexible wages and complete financial markets represents the benchmark against which we measure the performance of different exchange rate policies in the next section. In fact, under complete financial markets and flexible

[11] For instance, the first argument in $E_{t-1}[\mathcal{U}]$ can be written as $E_{t-1}\left[\alpha_t\left(\ln N_{D,t}^{\frac{\sigma}{\sigma-1}}\widetilde{y}_{D,t}\right)\right] = E_{t-1}\left[\alpha_t\right]\left\{E_{t-1}\left[\ln N_{D,t}\right] + E_{t-1}\left[\ln\widetilde{y}_{D,t}\right]\right\} + \left(1+\frac{1}{\sigma-1}\right)cov\left(\alpha_t, \ln N_{D,t}\right) + cov\left(\alpha_t, \ln\widetilde{y}_{D,t}\right)$. The same decomposition applies for the other terms in $E_{t-1}[\mathcal{U}]$.

[12] The detailed derivation of the planner problem is provided in Appendix 2.

Table 2 The solution of the social planner

Nb of entrants	$N_{D,t+1} = \beta(\frac{1}{\sigma-1} + \frac{1}{\kappa})\frac{E_t[\alpha_{t+1}]}{\eta L_t^{\varphi} f_{E,t}}$	$N^*_{D,t+1} = \beta(\frac{1}{\sigma-1} + \frac{1}{\kappa})\frac{E_t[\alpha^*_{t+1}]}{\eta L_t^{*\varphi} f^*_{E,t}}$
Nb of Exporters	$N_{X,t} = (\frac{1}{\sigma-1} - \frac{1}{\kappa})\frac{\alpha_t}{\eta L_t^{\varphi} f_{X,t}}$	$N^*_{X,t} = (\frac{1}{\sigma-1} - \frac{1}{\kappa})\frac{\alpha^*_t}{\eta L_t^{*\varphi} f^*_{X,t}}$
Av. exporters	$\widetilde{z}_{X,t} = \left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}} \left(\frac{N_{X,t}}{N_{D,t}}\right)^{-\frac{1}{\kappa}}$	$\widetilde{z}^*_{X,t} = \left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}} \left(\frac{N^*_{X,t}}{N^*_{D,t}}\right)^{-\frac{1}{\kappa}}$
Production	$\widetilde{y}_{D,t} = \frac{\alpha_t \widetilde{z}_D}{\eta L_t^{\varphi} N_{D,t}}, \quad \widetilde{y}_{X,t} = (\frac{1}{\sigma-1} - \frac{1}{\kappa})^{-1}\widetilde{z}_{X,t} f_{X,t}$	$\widetilde{y}^*_{D,t} = \frac{\alpha_t^* \widetilde{z}^*_D}{\eta L_t^{*\varphi} N^*_{D,t}}, \quad \widetilde{y}^*_{X,t} = (\frac{1}{\sigma-1} - \frac{1}{\kappa})^{-1}\widetilde{z}^*_{X,t} f^*_{X,t}$
Consumption	$C_t = N_{D,t}^{\left(1+\frac{1}{\sigma-1}\right)\alpha_t} N_{X,t}^{*\left(1+\frac{1}{\sigma-1}\right)\alpha^*_t} \left(\frac{\widetilde{y}_{D,t}}{\alpha_t}\right)^{\alpha_t} \left(\frac{\widetilde{y}^*_{X,t}/\tau_t}{\alpha^*_t}\right)^{\alpha^*_t}$	$C^*_t = N_{D,t}^{*\left(1+\frac{1}{\sigma-1}\right)\alpha^*_t} N_{X,t}^{\left(1+\frac{1}{\sigma-1}\right)\alpha_t} \left(\frac{\widetilde{y}^*_{D,t}}{\alpha^*_t}\right)^{\alpha^*_t} \left(\frac{\widetilde{y}_{X,t}/\tau_t}{\alpha_t}\right)^{\alpha_t}$
Labor supply	$L_t = \left\{ \frac{1}{\eta}\left[\left(2 + \frac{1}{\sigma-1} - \frac{1}{\kappa}\right)\alpha_t + \beta(\frac{1}{\sigma-1} + \frac{1}{\kappa})E_t\left[\alpha_{t+1}\right]\right]\right\}^{\frac{1}{1+\varphi}}$	$L^*_t = \left\{ \frac{1}{\eta}\left[\left(2 + \frac{1}{\sigma-1} - \frac{1}{\kappa}\right)\alpha^*_t + \beta(\frac{1}{\sigma-1} + \frac{1}{\kappa})E_t\left[\alpha^*_{t+1}\right]\right]\right\}^{\frac{1}{1+\varphi}}$
Shock process	$\alpha_t = \frac{1}{2}\upsilon_t, \quad \alpha^*_t = \frac{1}{2}\upsilon^*_t, \quad E_{t-1}\left[\upsilon_t\right] = E_{t-1}\left[\upsilon^*_t\right] = 1, \quad E_{t-1}\left[\upsilon_t \upsilon_{t+1}\right] = 1, \quad \upsilon_t + \upsilon^*_t = 2$	

wages, the monopolistic distortions in the goods and labor markets represent the only difference between the two allocations, making the planner's solution a superior one.

4 Exchange Rate Policy

In this section, we evaluate the impact of different exchange rate policies on welfare in our economy with nominal rigidities and incomplete financial markets. We first define two polar cases of exchange rate policies depending on the *de facto* exchange rate fluctuations induced uniquely by the demand shocks, without any optimization problem of the central banks. We refer to a flexible exchange rate when the monetary policy follows a constant rule such as $\mu_t = \mu_t^* = \mu_0$ for all time periods. Indeed, in this case the nominal exchange rate is free to fluctuate following demand shocks. Instead, when the monetary policy follows a cooperative peg system such as $\mu_t = 2\mu_0\alpha_t$ and $\mu_t^* = 2\mu_0\alpha_t^*$, the monetary stances fully respond to demand shocks offsetting their impact on the exchange rate. We refer to this polar case as a fixed exchange rate. We then derive the optimal monetary policy, where central banks maximize the household welfare and determine the desired variability of the exchange rate.

We show below that a managed exchange rate policy may dominate the flexible exchange rate policy under incomplete financial markets. Namely, the welfare ranking across different exchange rate policies depends upon the fluctuations in the export market, which are determined by the degree of firm heterogeneity.

4.1 Flexible Exchange Rate

The following proposition describes the allocation under a flexible exchange rate policy.

Proposition 2 Flexible exchange rate *When the preference attached to goods produced in Home is larger* ($\alpha_t > \alpha_t^*$), *the allocation under flexible exchange rate is such that*:

(a) *In the export market, the number of Home exporters is lower than in Foreign country* ($N_{X,t} < N_{X,t}^*$) *while their average productivity and scale of production are higher* ($\widetilde{z}_{X,t} > \widetilde{z}_{X,t}^*$ *and* $\widetilde{y}_{X,t} > \widetilde{y}_{X,t}^*$).
(b) *In the domestic market, the average domestic production in Home is higher than in Foreign* ($\widetilde{y}_{D,t} > \widetilde{y}_{D,t}^*$), *and the future number of firms* (*entry*) *in both countries is constant. The equilibrium wage reflects the uncertainty about future labor demand* (*a similar expression holds in Foreign*):

$$W_t^{FL} = \Gamma\mu_0 \left\{ \frac{E_{t-1}\left[A_t^{1+\varphi}\right]}{E_{t-1}\left[A_t\right]} \right\}^{\frac{1}{1+\varphi}}. \tag{12}$$

Under a flexible exchange rate policy, the monetary stance follows a constant rule $\mu_t = \mu_t^* = \mu_0$, thus the nominal exchange rate appreciates (decrease in ε_t) following a positive demand shock for goods produced in the Home country ($\alpha_t > \alpha_t^*$). Under producer currency pricing, the nominal appreciation increases the profits of Foreign exporters in units of domestic currency while it decreases the profits of Home exporters. Balanced trade implies an adjustment both at the intensive ($\widetilde{z}_{X,t} > \widetilde{z}_{X,t}^*$ and $\widetilde{y}_{X,t} > \widetilde{y}_{X,t}^*$) and the extensive margin of trade ($N_{X,t} < N_{X,t}^*$) with respect to the initial symmetric equilibrium. With respect to the domestic economy, the allocation under flexible exchange rate is remarkably similar to the one of the social planner. As shown in Table 1, the average production of domestic firms in both countries ($\widetilde{y}_{D,t}$ and $\widetilde{y}_{D,t}^*$) is proportionally affected by the current demand shock (α_t and α_t^*), whereas the future number of firms ($N_{D,t+1}$ and $N_{D,t+1}^*$) depend only upon the expected demand shifts ($E_t\left[\alpha_{t+1}\right]$ and $E_t\left[\alpha_{t+1}^*\right]$).

The equilibrium wage under flexible exchange rate policy reflects the uncertainty about labor demand (A_t) due to the nominal wage rigidity (workers set wages one period in advance). The flexible exchange rate therefore fails to stabilize wages. Why is it the case? As shown in Eq. (10), the fluctuations in labor demand in response to demand shocks are a function of firm heterogeneity (the Pareto shape κ). Consider the extreme case of $\kappa = \infty$: firms are homogeneous and less productive on average. In this case, there is a large adjustment of the extensive margin of trade in response to the demand shock, which translates in large fluctuations of labor demand. Instead, for a larger firm heterogeneity, the number of exporters is smaller and they are more productive on average. As a consequence, there is a limited reallocation of firms in the export market and smaller fluctuations in labor demand. This finding is shown in the following corollary.

Corollary 2 *In response to an external demand shock, the uncertainty about the future labor demand is smaller, the more heterogeneous are firms in terms of productivity. In the limit case of $\kappa = \sigma - 1$, the flexible exchange rate policy completely stabilizes the wage.*

Proof When $\kappa = \sigma - 1$, $W_t = \Gamma\mu_0 A_t^{\frac{\varphi}{1+\varphi}}$ and $W_t^* = \Gamma\mu_0 A_t^{*\frac{\varphi}{1+\varphi}}$ with $A_t = A_t^* = \frac{\sigma-1}{\sigma}\left(1 + \frac{\beta}{\sigma-1}\right)$. □

Contrary to models without firm heterogeneity - e.g., Devereux (2004) and Hamano and Picard (2017) - the nominal exchange rate only partially absorbs the shock and generates substantial adjustments in the export market. In our setting with heterogeneous firms, the nominal exchange rate therefore does not act as a "shock absorber" as argued in Friedman (1953) and Mundell (1961), in particular for firms in the export market.[13]

In the following proposition, we describe the differences between the allocation under flexible exchange rate and the allocation of the social planner.

Proposition 3 Non-optimality of the flexible exchange rate *The allocation under flexible exchange rate policy is isomorphic to the allocation under flexible wages. However, it does not replicate the allocation of the social planner because of incomplete financial markets.*

Proof See Appendix 4. □

Proposition 3 clearly states the distortions in our benchmark economy: the nominal wage rigidity *and* the incomplete financial markets. In the presence of incomplete financial markets, alternative exchange rate policies may dominate the flexible exchange rate policy, as pointed out by Devereux (2004). Note that this result holds true even when the allocation under flexible exchange rate is equivalent to the flexible wage allocation, that is for an infinite elasticity of labor supply ($\varphi = 0$).

4.2 Fixed Exchange Rate

We now study the opposite polar case of exchange rate policy. Under fixed exchange rate, the allocation of our economy dramatically changes, as described in the following proposition.

Proposition 4 Fixed exchange rate *When the preference attached to goods produced in Home is larger ($\alpha_t > \alpha_t^*$), the allocation under fixed exchange rate is such that:*

(a) *In the export market, the number of Home and Foreign exporters ($N_{X,t}$ and $N_{X,t}^*$), their average productivity ($\tilde{z}_{X,t}$ and $\tilde{z}_{X,t}^*$) and scale of production ($\tilde{y}_{X,t}$ and $\tilde{y}_{X,t}^*$) are constant.*

(b) *In the domestic market, both the average domestic production and the future number of firms in Home are higher than in Foreign ($\tilde{y}_{D,t} > \tilde{y}_{D,t}^*$, and $N_{D,t+1} > N_{D,t+1}^*$). The equilibrium wage is determined by the response of the monetary stance to demand shocks (a similar expression holds in Foreign):*

[13] The characteristic adjustment of the export market in our model also shows up in the terms of trade $\mathrm{TOT}_t = \frac{\alpha_t^*}{\alpha_t}\frac{W_t^*}{W_t}\frac{\tilde{z}_{X,t}}{\tilde{z}_{X,t}^*}$. Following a positive demand shock for Home produced goods, the terms of trade appreciate. However, the fall in terms of trade is dampened by the higher relative average productivity of Home exporters (a rise in $\tilde{z}_{X,t}^{FL}/\tilde{z}_{X,t}^{*FL}$). Because of the selection into the export market, a nominal appreciation of Home currency coexists with a higher average export price for the Home country. This result is similar to what Rodriguez-Lopez (2011) dubs a "negative expenditure switching effect". As it can be shown easily, the terms of trade under the fixed exchange rate policy are instead constant.

$$W_t^{FX} = 2\Gamma\mu_0 \left\{ \frac{E_{t-1}\left[\left(A_t\alpha_t\right)^{1+\varphi}\right]}{E_{t-1}\left[A_t\right]} \right\}^{\frac{1}{1+\varphi}}. \tag{13}$$

The monetary stance counteracts the impact of a demand shock as $\mu_t = 2\mu_0\alpha_t$ and $\mu_t^* = 2\mu_0\alpha_t^*$ and the exchange rate is constant as $\epsilon_t = 1$. Limiting the nominal exchange rate fluctuations mitigates the profit fluctuations in the export market. As a result, there is no variability in the selection into the export market, and the number of exporters ($N_{X,t}$ and $N_{X,t}^*$), their average productivity ($\widetilde{z}_{X,t}$ and $\widetilde{z}_{X,t}^*$) and scale of production ($\widetilde{y}_{X,t}$ and $\widetilde{y}_{X,t}^*$) are constant in both countries. While the export market is completely insulated from demand shocks, the fixed exchange rate policy transfers the burden of adjustment into the domestic economy. Despite the higher volatility in the domestic economy, the allocation under the fixed exchange rate policy is not necessarily welfare detrimental. Indeed, it ensures a better match with the preference shifts and may therefore be closer to the social planner allocation.

Finally, the comparison of the wages under flexible and fixed exchange rate—Eqs. (12) and (13)—shows that the polar exchange rate policies have the same first-order mean effect on wages, but a different higher order effect.[14] Put differently, the exchange rate policy has an impact on the risk component of the future demand shocks and influences the wage setting decision of workers. We highlight this feature of the polar exchange rate policies in the following welfare analysis.

4.3 Welfare Under Polar Exchange Rate Policies

We now compare the welfare for the polar exchange rate policies described above. Replacing the model solution (summarized in Table 1) into the utility function, we can write the welfare difference between the polar exchange rate policies as

$$\begin{aligned} E_{t-1}\left[\mathcal{U}^{FX}\right] - E_{t-1}\left[\mathcal{U}^{FL}\right] =& \left[1 + \frac{1}{2}\left(\frac{1}{\sigma-1} - \frac{1}{\kappa}\right)\right]\left\{E_{t-1}\left[\upsilon_t \ln \upsilon_t\right] - \Delta \ln W_t\right\} \\ &+ \frac{\beta}{2}\left(\frac{1}{\sigma-1} + \frac{1}{\kappa}\right)\left\{E_{t-1}\left[\ln \upsilon_t\right] - \Delta \ln W_t\right\}, \end{aligned} \tag{14}$$

where $\Delta \ln W_t \equiv \ln W_t^{FX} - \ln W_t^{FL}$ represents the wage difference between the fixed and flexible exchange rate policy:[15]

$$\Delta \ln W_t \equiv \ln W_t^{FX} - \ln W_t^{FL} = \frac{1}{1+\varphi}\left[\ln E_{t-1}\left[\left(A_t\upsilon_t\right)^{1+\varphi}\right] - \ln E_{t-1}\left[A_t^{1+\varphi}\right]\right].$$

[14] A Taylor expansion of the wage evaluated at $\alpha_t = 1/2$ gives $W_{i,t} = \Gamma\mu_0 + \frac{f_i''(1/2)}{2} Var(\alpha_t) + \cdots$, where $f_i''(\alpha_t)$ stands for the second-order derivative of the policy i.

[15] See Appendix 5.1 for more details.

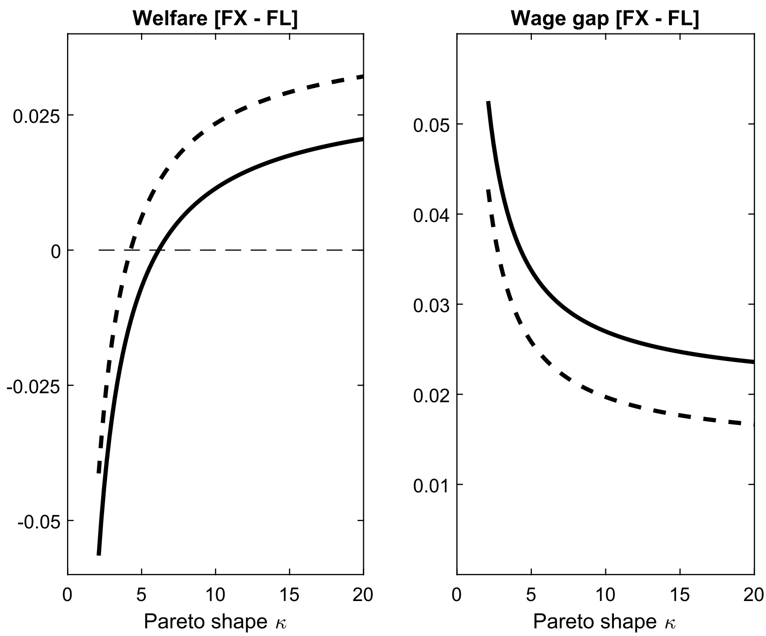

Fig. 1 Polar exchange rate policies. *Notes*: This figure reports the difference between the expected utility under fixed and flexible exchange rate $E_{t-1}[\mathcal{U}^{FX}] - E_{t-1}[\mathcal{U}^{FL}]$, and the log-difference of wages $\ln(W^{FX}) - \ln(W^{FL})$. In the benchmark calibration (solid line), we set the value of elasticity of substitution $\sigma = 3$, the discount factor $\beta = 0.9$ and the elasticity of Labor supply $\varphi^{-1} = 0.9$. The dashed line refers to an economy with higher elasticity of Labor supply: $\varphi^{-1} = 1.1$

In the expression of welfare ranking (14), both $E_{t-1}\left[v_t \ln v_t\right]$ and $E_{t-1}\left[\ln v_t\right]$ are greater than zero. These terms capture the welfare gain stemming from the better congruence between the preference shock and the amount of both domestic and imported goods under a fixed exchange rate (similar to the allocation of the social planner). However, the fluctuations of monetary stance in response to the stochastic preference shocks come at the cost of higher wages ($\Delta \ln W_t > 0$). The following proposition highlights the role of firm heterogeneity on the welfare ranking between the polar exchange rate policies.

Proposition 5 Welfare and firm heterogeneity *The wage gap between polar exchange rate policies is lower when firms are more homogeneous, increasing the welfare gains of the fixed exchange rate policy.*

Proof See Appendix 5.2. □

Under the fixed exchange rate policy, the monetary response to demand shocks increases the uncertainty for labor demand in the domestic market due to volatile domestic production and investments. However, it simultaneously dampens the fluctuations of trade and hence uncertainty in the export market. The latter gain is increasing with a lower firm dispersion. The intuition behind this result is the following: a lower firm dispersion (high κ) increases the volatility of trade under a flexible exchange rate. In such a situation, a policy which limits the nominal exchange rate fluctuations is welfare-improving.

Figure 1 reports the welfare ranking and the wage gap between the fixed and flexible exchange rate policies for different values of the Pareto shape parameter κ. As previously discussed, when firms are more homogeneous (κ is high), the wage gap between the polar exchange rate policies is reduced and a fixed exchange rate provides a higher welfare. Figure 1 also shows that our results are consistent with Devereux (2004): for any degree of firm heterogeneity, a higher elasticity of labor supply tends to decrease the wage gap and increase the welfare under fixed exchange rate policy.[16]

4.4 Optimal Monetary Policy

We now depart from the cases of polar exchange rate policies and derive the optimal monetary policy in a Nash equilibrium. The policy commitment of the monetary authority is to maximize the expected utility of domestic households while taking as given the monetary stance abroad: $\max_{\mu_t} E_{t-1}[\mathcal{U}]$. This implies an optimal monetary response to the external demand shock which is a function of the fundamentals of the economy. In turn, this defines the optimal exchange rate policy chosen by the monetary authority.

Proposition 6 Optimal monetary policy *The optimal monetary policy limits the fluctuations in the nominal exchange rate in response to demand shocks, and the extent of this response is larger when firms are more homogeneous.*

Proof See Appendix 5.3. □

Proposition 6 highlights the role of firm heterogeneity for the trade-off faced by policymakers in an open economy—see Obstfeld and Rogoff (1998) and Corsetti and Pesenti (2001). As in the related literature, the price of imported goods is sensitive to the fluctuations in nominal exchange rate stemming from the conduct of monetary policy. In our setting, policymakers do not only consider the domestic intensive and extensive margins (i.e., domestic output gap stabilization by targeting inflation), but also the selection of importers and their prices. The conduct of monetary policy indeed affects the import prices because of the endogenous entry of heterogeneous firms. In this respect, the impact of firm selection on import prices represents a new dimension of the terms of trade externalities.

Figure 2 provides a numerical illustration of our analytical results: the variability of the nominal exchange rate under optimal monetary policy decreases for a lower

16 With a higher value of the elasticity of substitution σ, the monetary intervention increases welfare by a lower extent. The numerical results are available upon request. For a broader discussion of the impact of love for variety in welfare ranking, see Hamano and Picard (2017) and Appendix 1.1.

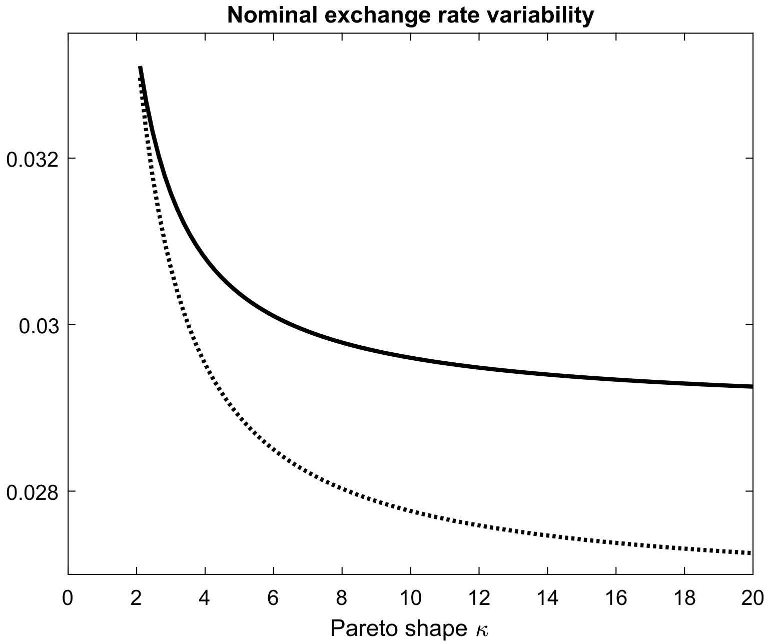

Fig. 2 Optimal monetary policy. *Notes*: This figure displays the standard deviation of the nominal exchange rate under optimal non-cooperative (solid line) and cooperative (dotted line) monetary policy. We assume that the i.i.d. demand shock υ_t takes the values 0.5, 1 and 1.5 with probability 0.25, 0.5 and 0.25, respectively, such that $E_{t-1}\left[\upsilon_t\right] = 1$

firm productivity dispersion (higher κ). As shown in Proposition 6, when firms are more homogeneous, the stronger response of the optimal monetary policy to external demand shocks further limits the fluctuations in the nominal exchange rate. This result holds true even under a cooperative optimal monetary policy, where Home and Foreign monetary authorities jointly maximize the World welfare. Figure 2 also shows that international cooperation implies lower fluctuations of the nominal exchange rate, in particular when firms are more homogeneous.[17]

5 Conclusion

This paper examines the exchange rate policy in a model with endogenous entry of heterogeneous firms and nominal rigidities. The flexible price allocation is not efficient under incomplete financial markets, raising a case about the desired exchange rate policy. We first provide an analytical solution for two polar exchange rate policies and then determine the optimal exchange rate policy.

[17] In Appendix 5.4 we derive the optimal cooperative monetary policy.

In our model, external demand shocks imply a high volatility of the extensive margin of trade when firms are homogeneous in terms of productivity. A monetary policy intervention in response to these shocks limits the fluctuations of profits for exporters and shuts down the fluctuations of firm selection in the export market. Despite generating a substantially higher volatility in the domestic market, the monetary policy intervention helps in reducing the uncertainty about future labor demand in the export market. The sub-optimally high wage markup is thus reduced when the fluctuations in the selection of exporters are relatively important. For this reason, a managed floating may represent the optimal exchange rate policy when exporter firm heterogeneity is small. At the same time, the presence of heterogeneous exporters provides a rationale for a flexible exchange rate because the selection in the export market is less sensitive to external demand shocks.

While we consider a two-country framework, our findings would hold true in a small open economy adding a new dimension to the *fear of floating* that often hits emerging markets. Finally, we focus uniquely on the impact of monetary intervention in response to external demand shocks from a qualitative standpoint, leaving for further research the quantitative implications under alternative shocks and nominal frictions.

Appendix 1: Solution of the Model

We derive here the closed form solution of the theoretical model presented in Table 1. Similar expressions hold for Foreign. First, note that using average prices and the expressions of price indices, we have $P_{H,t} = N_{D,t}^{-\frac{1}{\sigma-1}} \widetilde{p}_{D,t}$ and $P_{F,t} = N_{X,t}^{*-\frac{1}{\sigma-1}} \widetilde{p}_{X,t}^{*}$. Plugging these expressions in the expression of domestic profits, profits from exporting and total profits on average, we have $\widetilde{D}_{D,t} = \frac{\alpha t}{\sigma} \frac{\mu_t}{N_{D,t}}$, $\widetilde{D}_{X,t} = \frac{\alpha_t}{\sigma} \frac{\varepsilon_t \mu_t^*}{N_{X,t}} - f_{X,t} W_t$ and $\widetilde{D}_t = \widetilde{D}_{D,t} + \frac{N_{X,t}}{N_{D,t}} \widetilde{D}_{X,t}$. Given the zero cutoff profits (ZCP) condition, we have $\widetilde{D}_{X,t} = W_t f_{X,t} \frac{\sigma-1}{\kappa-(\sigma-1)}$. By combining these two expressions of $\widetilde{D}_{X,t}$ we have $\widetilde{D}_{X,t} = \frac{\sigma-1}{\kappa} \frac{\alpha_t}{\sigma} \frac{\varepsilon_t \mu_t^*}{N_{X,t}}$. Using the ZCP condition with the expression of $\widetilde{D}_{X,t}$ and the exchange rate implied under the balanced trade $\varepsilon_t = \frac{\alpha_t^*}{\alpha_t} \frac{\mu_t}{\mu_t^*}$, we have $N_{X,t} = \frac{1}{\sigma}(1 - \frac{\sigma-1}{\kappa}) \frac{\alpha_t^* \mu_t}{W_t f_{X,t}}$. The assumption of a Pareto distribution of firm productivity implies that $\widetilde{z}_{X,t} = \left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}} \left(\frac{N_{X,t}}{N_{D,t}}\right)^{-\frac{1}{\kappa}}$.

We are now ready to derive the number of new entrants, $N_{D,t+1}$. Free entry implies that $\widetilde{V}_t = f_{E,t} W_t$. Combined with the expression of $\widetilde{D}_{t+1}$, the Euler equation about the share holdings, $\widetilde{V}_t = E_t \left[Q_{t,t+1} \widetilde{D}_{t+1} \right]$, is expressed as

$$E_t \left[\frac{\beta P_t C_t}{P_{t+1} C_{t+1}} \left(\widetilde{D}_{D,t+1} + \frac{N_{X,t+1}}{N_{D,t+1}} \widetilde{D}_{X,t+1} \right) \right] = f_{E,t} W_t.$$

Plugging the expression of $\widetilde{D}_{D,t+1}$, $\widetilde{D}_{X,t+1}$ and the expression of the equilibrium exchange rate $\varepsilon_t = \frac{\alpha_t^*}{\alpha_t}\frac{\mu_t}{\mu_t^*}$, the previous equation is rewritten as

$$\frac{\beta}{\sigma}\frac{\mu_t}{N_{D,t+1}}E_t\left[\left(\alpha_{t+1} + \frac{\sigma-1}{\kappa}\alpha_{t+1}^*\right)\right] = f_{E,t}W_t,$$

which gives

$$N_{D,t+1} = \frac{\beta}{\sigma}\frac{\mu_t}{W_t f_{E,t}}E_t\left[\alpha_{t+1} + \frac{\sigma-1}{\kappa}\alpha_{t+1}^*\right].$$

Next we derive the labor demand in general equilibrium. Note that $\widetilde{D}_{X,t} = \frac{1}{\sigma}\frac{\varepsilon_t\widetilde{p}_{X,t}}{\tau}\widetilde{y}_{X,t} - f_{X,t}W_t$ and $\widetilde{D}_{D,t} = \frac{1}{\sigma}\widetilde{p}_{D,t}\widetilde{y}_{D,t}$. Once we plug the expression of prices into these profits, we have $\widetilde{y}_{D,t} = (\sigma-1)\frac{\widetilde{D}_{D,t}\widetilde{z}_D}{W_t}$ and $\widetilde{y}_{X,t} = (\sigma-1)\frac{\left(\widetilde{D}_{X,t}+f_{X,t}W_t\right)\widetilde{z}_{X,t}}{W_t}$. Replacing those expressions in the labor market clearings (8), we have

$$L_t = N_{D,t}(\sigma-1)\frac{\widetilde{D}_{D,t}}{W_t} + N_{X,t}\left((\sigma-1)\frac{\widetilde{D}_{X,t}+f_{X,t}W_t}{W_t} + f_{X,t}\right) + N_{D,t+1}f_{E,t}.$$

Using the expression of $\widetilde{D}_{D,t}$, $\widetilde{D}_{X,t}$, $N_{D,t+1}$, $N_{X,t}$ and the exchange rate found previously, the above expression becomes

$$L_t = \frac{\mu_t}{W_t}A_t.$$

Finally, we obtain Eq. (9) after replacing the wage setting of Eq. (6) into the above expression.

Solution of the Model Without Firm Dynamics

In our model, the monetary intervention plays a key role in mitigating the fluctuations in labor demand determined by the preference shock. In order to highlight the role of firm dynamics in labor demand fluctuations, we derive here a version of our model without selection into exporting market as described in the "lagged entry" model of Hamano and Picard (2017).

Note that by setting $f_{X,t} = 0$, all firms export despite firm heterogeneity, hence $N_{X,t} = N_{D,t}$ and $\widetilde{z}_{X,t} = \widetilde{z}_D$. In such a specific case, we have $\widetilde{D}_{D,t} = \frac{\alpha_t}{\sigma}\frac{\mu_t}{N_{D,t}}$, $\widetilde{D}_{X,t} = \frac{\alpha_t}{\sigma}\frac{\varepsilon_t\mu_t^*}{N_{D,t}}$. Once we replace these expressions in the Euler equation with free entry condition, we get

$$E_t\left[\frac{\beta P_t C_t}{P_{t+1}C_{t+1}}\left(\widetilde{D}_{D,t+1} + \widetilde{D}_{X,t+1}\right)\right] = f_{E,t}W_t,$$

which gives the number of future domestic firms: $N_{D,t+1} = \frac{\beta}{\sigma}\frac{\mu_t}{W_t f_{E,t}}E_t\left[\alpha_{t+1} + \alpha_{t+1}^*\right] = \frac{\beta}{\sigma}\frac{\mu_t}{W_t f_{E,t}}$. The labor market clearing is

$$L_t = N_{D,t}\left(\frac{\widetilde{y}_{D,t}}{\widetilde{z}_D} + \frac{\widetilde{y}_{X,t}}{\widetilde{z}_D}\right) + N_{D,t+1}f_{E,t},$$

where $\widetilde{y}_{D,t} = (\sigma - 1)\frac{\widetilde{D}_{D,t}\widetilde{z}_D}{W_t}$ and $\widetilde{y}_{X,t} = (\sigma - 1)\frac{\widetilde{D}_{X,t}\widetilde{z}_D}{W_t}$. Together with $N_{D,t+1}$, we obtain the labor demand in Hamano and Picard (2017):

$$L_t = \frac{\mu_t}{W_t}\left[\frac{\sigma - 1}{\sigma} + \frac{\beta}{\sigma}\right].$$

The equilibrium wage is

$$W_t = \Gamma\left\{E_{t-1}\left[\mu_t^{1+\varphi}\right]\right\}^{\frac{1}{1+\varphi}},$$

which corresponds to the wage in our model when $\kappa = \sigma - 1$: when firm heterogeneity is the largest as possible, the adjustment at the extensive margin due to labor demand uncertainty is the smallest.

Appendix 2: Social Planner

In this section, we show the solution of a benevolent social planner. Due to the assumption of one period to build and the full depreciation of firms after one period of production, we express the expected utility only for two consecutive periods without loss of generality as

$$\begin{aligned} E_{t-1}[\mathcal{U}] &\equiv E_{t-1}\left[U_t\right] + \beta E_{t-1}\left[U_{t+1}\right] \\ &= E_{t-1}\left[\ln C_t\right] - \frac{\eta}{1+\varphi}E_{t-1}\left[L_t^{1+\varphi}\right] + \beta\left\{E_{t-1}\left[\ln C_{t+1}\right] - \frac{\eta}{1+\varphi}E_{t-1}\left[L_{t+1}^{1+\varphi}\right]\right\}. \end{aligned}$$

Using the good market clearing conditions $\widetilde{c}_{D,t} = \widetilde{y}_{D,t}, \widetilde{c}_{X,t} = \widetilde{y}^*_{X,t}, \widetilde{c}^*_{D,t} = \widetilde{y}^*_{D,t}, \widetilde{c}^*_{X,t} = \widetilde{y}_{X,t}$, we get:

$$\begin{aligned} E_{t-1}[\mathcal{U}] =& E_{t-1}\Big[\alpha_t\left(1 + \frac{1}{\sigma - 1}\right)\ln N_{D,t} \\ &+\alpha_t \ln\widetilde{y}_{D,t} + \alpha_t^*\left(1 + \frac{1}{\sigma - 1}\right)\ln N^*_{X,t} + \alpha^*_t \ln\widetilde{y}^*_{X,t}\Big] \\ &- \frac{\eta}{1+\varphi}E_{t-1}\left[L_t^{1+\varphi}\right] \\ &+ \beta E_{t-1}\Big[\alpha_{t+1}\left(1 + \frac{1}{\sigma - 1}\right)\ln N_{D,t+1} + \alpha_{t+1}\ln\widetilde{y}_{D,t+1} \\ &+\alpha^*_{t+1}\left(1 + \frac{1}{\sigma - 1}\right)\ln N^*_{X,t+1} + \alpha^*_{t+1}\ln\widetilde{y}^*_{X,t+1}\Big] \\ &- \frac{\beta\eta}{1+\varphi}E_{t-1}\left[L_{t+1}^{1+\varphi}\right]. \end{aligned}$$

As argued in the text, the planner maximizes $E_{t-1}[\mathcal{U}] + E_{t-1}\left[\mathcal{U}^*\right]$ with respect to $\widetilde{y}_{D,t}, \widetilde{y}^*_{D,t}, N_{D,t+1}, N^*_{D,t+1}, \widetilde{y}_{X,t}, \widetilde{y}^*_{X,t}, N_{X,t}, N^*_{X,t}$ subject to two types of technological constraints, namely (5) and (8) for each country. The solution is given by Table 2. The optimal labor supply in Home is given by

$$L_t = \left(\frac{1}{\eta}\mathcal{A}_t\right)^{\frac{1}{1+\varphi}}, \tag{15}$$

where $\mathcal{A}_t \equiv \left(2 + \frac{1}{\sigma-1} - \frac{1}{\kappa}\right)\alpha_t + \beta(\frac{1}{\sigma-1} + \frac{1}{\kappa})E_t\left[\alpha_{t+1}\right]$. The planner lets Home households work more when the preference attached to goods produced in the Home country is high ($\alpha_t > \alpha^*_t$). As a result, as shown in Table 2, the number of exporters in Home is higher than in Foreign ($N_{X,t} > N^*_{X,t}$) and the average domestic production in Home ($\widetilde{y}_{D,t} > \widetilde{y}^*_{D,t}$) is higher than in Foreign. The extent of this gap depends negatively upon the marginal disutility of labor supply ηL_t^{φ} and $\eta L_t^{*\varphi}$. Further, given $N_{X,t} > N^*_{X,t}$ and noting that $N_{D,t}$ and $N^*_{D,t}$ are the state of the economy, the average productivity of Home exporters is lower than the average productivity of Foreign exporters ($\widetilde{z}_{X,t} < \widetilde{z}^*_{X,t}$). As a consequence, the average production of Home exporters is smaller than that of Foreign exporters ($\widetilde{y}_{X,t} < \widetilde{y}^*_{X,t}$). When the expected preference attached to goods produced in the Home country is high ($E_t\left[\alpha_{t+1}\right] > E_t\left[\alpha^*_{t+1}\right]$), the planner lets Home households work more. The future number of firms in the Home country is then higher than in the Foreign country ($N_{D,t+1} > N^*_{D,t+1}$).

Appendix 3: Complete Financial Markets

Let us show that the allocation of the social planner is very close to the one in our framework once we allow for complete financial markets and flexible wages. To begin with, we characterize the equilibrium exchange rate. Under complete asset markets, the marginal utility stemming from one additional unit of nominal wealth is equal across countries. Given our preferences defined in Eq. (1), this implies

$$\varepsilon_t = \frac{\mu_t}{\mu^*_t}.$$

Note that complete markets allow households to ensure against demand shocks, and as a consequence, the exchange rate is also independent from demand shocks. Table 3 reports the solution of the model under complete asset markets (with wage rigidity).

To what extent the allocation under complete markets differs from that implied by the social planner? Without wage rigidities, the equilibrium wage is $W_t = \Gamma\mu_0\mathcal{A}_t^{\frac{\varphi}{1+\varphi}}$ and $W^*_t = \Gamma\mu_0\mathcal{A}_t^{*\frac{\varphi}{1+\varphi}}$, where monetary stances serve just as the "nominal anchors" which determine the wage level in each country. As a result, the real variables are independent from monetary stances. In particular, the equilibrium labor supply under complete asset markets and flexible wages is

Table 3 The closed form solution of the model with complete financial markets

Nb of entrants	$N_{D,t+1}=\beta\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{1}{\sigma-1}+\frac{1}{\kappa}\right)\frac{\mu_t}{W_t f_{E,t}}E_t\left[\alpha_{t+1}\right]$	$N^*_{D,t+1}=\beta\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{1}{\sigma-1}+\frac{1}{\kappa}\right)\frac{\mu^*_t}{W^*_t f^*_{E,t}}E_t\left[\alpha^*_{t+1}\right]$
Nb of exporters	$N_{X,t}=\frac{\sigma-1}{\sigma}\left(\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)\frac{\alpha_t\mu_t}{W_t f_{X,t}}$	$N^*_{X,t}=\frac{\sigma-1}{\sigma}\left(\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)\frac{\alpha^*_t\mu^*_t}{W^*_t f^*_{X,t}}$
Av. exporters	$\widetilde{z}_{X,t}=\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}\left(\frac{N_{X,t}}{N_{D,t}}\right)^{-\frac{1}{\kappa}}$	$\widetilde{z}^*_{X,t}=\left[\frac{\kappa}{\kappa-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}\left(\frac{N^*_{X,t}}{N^*_{D,t}}\right)^{-\frac{1}{\kappa}}$
Production	$\widetilde{y}_{D,t}=\frac{\sigma-1}{\sigma}\frac{\alpha_t\mu_t\widetilde{z}_D}{N_{D,t}W_t},\quad \widetilde{y}_{X,t}=\left(\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)^{-1}\widetilde{z}_{X,t}f_{X,t}$	$\widetilde{y}^*_{D,t}=\frac{\sigma-1}{\sigma}\frac{\alpha^*_t\mu^*_t\widetilde{z}^*_D}{N^*_{D,t}W^*_t},\quad \widetilde{y}^*_{X,t}=\left(\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)^{-1}\widetilde{z}^*_{X,t}f^*_{X,t}$
Average price	$\widetilde{p}_{D,t}=\frac{\sigma}{\sigma-1}\frac{W_t}{\widetilde{z}_D},\quad \widetilde{p}_{X,t}=\frac{\sigma}{\sigma-1}\frac{\tau\varepsilon_t^{-1}W_t}{\widetilde{z}_{X,t}}$	$\widetilde{p}^*_{D,t}=\frac{\sigma}{\sigma-1}\frac{W^*_t}{\widetilde{z}^*_D},\quad \widetilde{p}^*_{X,t}=\frac{\sigma}{\sigma-1}\frac{\tau\varepsilon_t W^*_t}{\widetilde{z}^*_{X,t}}$
Price indices	$P_{H,t}=N_{D,t}^{-\frac{1}{\sigma-1}}\widetilde{p}_{D,t},\quad P_{F,t}=N_{X,t}^{*-\frac{1}{\sigma-1}}\widetilde{p}^*_{X,t},\quad P_t=P_{H,t}^{\alpha_t}P_{F,t}^{\alpha^*_t}$	$P^*_{F,t}=N_{D,t}^{*-\frac{1}{\sigma-1}}\widetilde{p}^*_{D,t},\quad P^*_{H,t}=N_{X,t}^{-\frac{1}{\sigma-1}}\widetilde{p}_{X,t},\quad P^*_t=P_{F,t}^{*\alpha^*_t}P_{H,t}^{*\alpha_t}$
Consumption	$C_t=N_{D,t}^{\left(1+\frac{1}{\sigma-1}\right)\alpha_t}N_{X,t}^{*\left(1+\frac{1}{\sigma-1}\right)\alpha^*_t}\left(\frac{\widetilde{y}_{D,t}}{\alpha_t}\right)^{\alpha_t}\left(\frac{\widetilde{y}^*_{X,t}/\tau_t}{\alpha^*_t}\right)^{\alpha^*_t}$	$C^*_t=N_{D,t}^{*\left(1+\frac{1}{\sigma-1}\right)\alpha^*_t}N_{X,t}^{\left(1+\frac{1}{\sigma-1}\right)\alpha_t}\left(\frac{\widetilde{y}^*_{D,t}}{\alpha^*_t}\right)^{\alpha^*_t}\left(\frac{\widetilde{y}^*_{X,t}/\tau_t}{\alpha_t}\right)^{\alpha_t}$
Profits	$\widetilde{D}_{D,t}=\frac{\alpha_t}{\sigma}\frac{\mu_t}{N_{D,t}},\quad \widetilde{D}_{X,t}=\frac{\sigma-1}{\sigma\kappa}\frac{\alpha_t\mu_t}{N_{X,t}},\quad \widetilde{D}_t=\widetilde{D}_{D,t}+\frac{N_{X,t}}{N_{D,t}}\widetilde{D}_{X,t}$	$\widetilde{D}^*_{D,t}=\frac{\alpha^*_t}{\sigma}\frac{\mu^*_t}{N^*_{D,t}},\quad \widetilde{D}^*_{X,t}=\frac{\sigma-1}{\sigma\kappa}\frac{\alpha^*_t\mu^*_t}{N^*_{X,t}},\quad \widetilde{D}^*_t=\widetilde{D}^*_{D,t}+\frac{N^*_{X,t}}{N^*_{D,t}}\widetilde{D}^*_{X,t}$
ZPC	$\widetilde{D}_{X,t}=W_t f_{X,t}\frac{\sigma-1}{\kappa-(\sigma-1)}$	$\widetilde{D}^*_{X,t}=W^*_t f^*_{X,t}\frac{\sigma-1}{\kappa-(\sigma-1)}$
Share price	$\widetilde{V}_t=f_{E,t}W_t$	$\widetilde{V}^*_t=f^*_{E,t}W^*_t$
Labor supply	$L_t=(\sigma-1)\frac{N_{D,t}\widetilde{D}_t}{W_t}+\sigma N_{X,t}f_{X,t}+N_{D,t+1}f_{E,t}$	$L^*_t=(\sigma-1)\frac{N^*_{D,t}\widetilde{D}^*_t}{W^*_t}+\sigma N^*_{X,t}f^*_{X,t}+N^*_{D,t+1}f^*_{E,t}$
Monetary stance	$\mu_t=P_tC_t$	$\mu^*_t=P^*_tC^*_t$
Wages	$W_t=\Gamma\left\{\frac{E_{t-1}\left[\left(\mathcal{A}_t\mu_t\right)^{1+\varphi}\right]}{E_{t-1}\left[\mathcal{A}_t\right]}\right\}^{\frac{1}{1+\varphi}}$	$W^*_t=\Gamma\left\{\frac{E_{t-1}\left[\left(\mathcal{A}^*_t\mu^*_t\right)^{1+\varphi}\right]}{E_{t-1}\left[\mathcal{A}^*_t\right]}\right\}^{\frac{1}{1+\varphi}}$
Exchange rate	$\varepsilon_t=\frac{\mu_t}{\mu^*_t}$	

$$L_t = \left[\frac{\sigma - 1}{\sigma} \frac{(\theta - 1)(1 + \xi)}{\eta\theta} \mathcal{A}_t \right]^{\frac{1}{1+\varphi}}.$$

Comparing the above solution with (15), we can state that the equilibrium allocation under complete financial markets and flexible wages is identical to the one implied by the social planner once monopolistic distortions both in goods and labor markets are removed. Indeed, by setting $\frac{\sigma-1}{\sigma}\frac{(\theta-1)(1+\xi)}{\theta} = 1$, the labor supply is equal to the one in the planner problem. In order to compare the allocation of the competitive equilibrium with the allocation of the social planner who does not have prices, we express the allocation with the above labor supply. Note that without wage rigidities, Eq. (6) implies that the equilibrium wage is $W_t = \Gamma^{1+\varphi}\mu_t L_t^{\varphi}$ and $W_t^* = \Gamma^{1+\varphi}\mu_t^* L_t^{*\varphi}$. By plugging the expressions for the model solution under complete asset markets of Table 3, we can show the Corollary of Proposition 1.

Finally, we can write the terms of trade under complete markets and flexible wages as

$$TOT_t = \left(\frac{L_t^*}{L_t} \right)^{\varphi} \frac{\widetilde{z}_{X,t}}{\widetilde{z}_{X,t}^*}.$$

Following a positive demand shock for Home produced goods, the Home terms of trade appreciate because L_t/L_t^* increases and $\widetilde{z}_{X,t}/\widetilde{z}_{X,t}^*$ decreases. The extent of the appreciation is higher for a lower elasticity of labor supply, $1/\varphi$. This expression is considered as the desired terms of trade by the social planner. Shutting down monopolistic power and firm heterogeneity, i.e., without variation in the cutoff level of productivity, the expression of the desired terms of trade by the social planner collapses into the one in Devereux (2004).

Appendix 4: Incomplete Financial Markets and Flexible Wages

The allocation with flexible wages under incomplete financial markets is obtained by removing the expectation operator in the solution of the benchmark economy presented in Table 1: wages are flexible and are not set one period in advance. The equilibrium wage is then $W_t = \Gamma A_t^{\frac{\varphi}{1+\varphi}} \mu_t$ and $W_t^* = \Gamma A_t^{\frac{\varphi}{1+\varphi}} \mu_t$ and the monetary stance is just a nominal anchor. Accordingly, the nominal exchange rate ε_t has no impact on the real allocation. Plugging the equilibrium flexible wage in the solution of Table 1, we prove Proposition 3.

In the peculiar case of infinite elasticity of labor supply, that is when $\varphi = 0$, the allocation with flexible wages (and incomplete financial markets) is exactly the same as under the flexible exchange rate policy. When labor supply is infinitely elastic, the flexible exchange rate can therefore compensate for the wage rigidity. However, this does not imply that a flexible exchange rate is the dominant one. This allocation is indeed far from the first best allocation. Following a positive demand shift for Home goods, the relative number of Home exporters decreases, whereas it would increase increases in the planner solution. The adjustments at the extensive and intensive margins are inefficient even when wages are flexible in our setting with incomplete financial markets. This also implies that the fluctuations in the terms of trade under a flexible exchange rate do not reproduce the complete markets allocation. The comparison between the flexible exchange rate and the flexible wage therefore highlights the role of incomplete financial markets for the choice of the exchange rate policy.

Appendix 5: Exchange Rate Policy

Polar Exchange Rate Policies

In competitive equilibrium, $E_{t-1}\left[L_{t+1}^{1+\varphi}\right]$ is constant, thus the expected utility of Home representative household for any consecutive time period is given by Eq. (11). Using the solution of Table 1 and reporting time invariant variables as a constant, we get

$$\begin{aligned}E_{t-1}[\mathcal{U}] =& E_{t-1}\left[\alpha_t \ln\mu_t\right] - E_{t-1}\left[\alpha_t \ln W_t\right] \\ &+ \left(\frac{1}{\sigma-1} + 1 - \frac{1}{\kappa}\right)\left\{E_{t-1}\left[\alpha_t^* \ln\mu_t^*\right] - E_{t-1}\left[\alpha_t^* \ln W_t^*\right]\right\} \\ &+ \frac{\beta}{\sigma-1}\left\{E_{t-1}\left[\alpha_{t+1}\ln\mu_t\right] - E_{t-1}\left[\alpha_{t+1}\ln W_t\right]\right\} \\ &+ \frac{\beta}{\kappa}\left\{E_{t-1}\left[\alpha_{t+1}^*\ln\mu_t^*\right] - E_{t-1}\left[\alpha_{t+1}^*\ln W_t^*\right]\right\} + \text{cst}.\end{aligned}$$

Recall that $\alpha_t = \frac{1}{2}\upsilon_t$ and $\alpha_t^* = \frac{1}{2}\upsilon_t^*$, and we assume zero serial correlation across shocks. Finally, plugging the expression of wages in equilibrium, we get

$$\begin{aligned}E_{t-1}[\mathcal{U}] =& \frac{1}{2}\left\{E_{t-1}\left[\upsilon_t \ln\mu_t\right] - \frac{1}{1+\varphi}\ln E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]\right\} \\ &+ \frac{1}{2}\left(\frac{1}{\sigma-1} + 1 - \frac{1}{\kappa}\right)\left\{E_{t-1}\left[\upsilon_t^*\ln\mu_t^*\right] - \frac{1}{1+\varphi}\ln E_{t-1}\left[\left(A_t^*\mu_t^*\right)^{1+\varphi}\right]\right\} \\ &+ \frac{1}{2}\frac{\beta}{\sigma-1}\left\{E_{t-1}\left[\ln\mu_t\right] - \frac{1}{1+\varphi}\ln E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]\right\} \\ &+ \frac{1}{2}\frac{\beta}{\kappa}\left\{E_{t-1}\left[\ln\mu_t^*\right] - \frac{1}{1+\varphi}\ln E_{t-1}\left[\left(A_t^*\mu_t^*\right)^{1+\varphi}\right]\right\} + \text{cst}.\end{aligned}$$

We then replace the equilibrium variables under the two polar exchange rate policies to evaluate their impact on welfare.

Welfare and Firm Heterogeneity

We provide here the proof of Proposition 5. Once we take a Taylor expansion up to the second order of $E_{t-1}\left[A_t^{1+\varphi}\right]$ and $E_{t-1}\left[\left(A_t \upsilon_t\right)^{1+\varphi}\right]$, and we evaluate these functions at $\upsilon_t = 1$, we get

$$E_{t-1}\left[A_t^{1+\varphi}\right] = A^{1+\varphi} + \frac{1}{2}(1+\varphi) f_{FL}(\kappa) Var(\upsilon_t) + \cdots$$
$$E_{t-1}\left[\left(A_t \upsilon_t\right)^{1+\varphi}\right] = A^{1+\varphi} + \frac{1}{2}(1+\varphi) f_{FX}(\kappa) Var(\upsilon_t) + \cdots$$

where $A \equiv \frac{\sigma-1}{2\sigma}\left[1+\beta\left(\frac{1}{\sigma-1}+\frac{1}{\kappa}\right)\right]$, $\Phi = \frac{\sigma-1}{2\sigma}\left(\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)$, $f_{FX}(\kappa) \equiv \left[\varphi(A-\Phi)^2 A^{\varphi-1} - 2\Phi A^{\varphi}\right]$ and $f_{FL}(\kappa) \equiv \varphi \Phi^2 A^{\varphi-1}$. We then obtain

$$\frac{2\sigma\kappa^2}{\sigma-1}\left[\frac{\partial f_{FL}(\kappa)}{\partial\kappa} - \frac{\partial f_{FX}(\kappa)}{\partial\kappa}\right] = 2\beta\varphi(A-\Phi)\Lambda^{\varphi-1} + 2(A-\Phi)^{\varphi} A^{\varphi-1} + 2A^{\varphi}$$
$$+\ 2(1-\beta)\varphi\Phi A^{\varphi-1} + \beta(\varphi-1)\varphi A^{\varphi-1}(A-2\Phi) > 0,$$

since $A - \Phi = \frac{\sigma-1}{2\sigma}\left[\frac{\sigma}{\sigma-1}+\frac{1}{\kappa}+\beta\left(\frac{1}{\sigma-1}+\frac{1}{\kappa}\right)\right] > 0$ and $A - 2\Phi = \frac{\sigma-1}{2\sigma}\left[1+\frac{2}{\kappa}+\beta\left(\frac{1}{\sigma-1}+\frac{1}{\kappa}\right)\right] > 0$. It follows that $\frac{\partial \Delta \ln W_t}{\partial \kappa} < 0$.

Optimal Non-cooperative Monetary Policy

The Home monetary authority maximizes (11) with respect to μ_t and takes μ_t^* as given:

$$\max_{\mu_t} E_{t-1}[\mathcal{U}].$$

The first-order condition with respect to μ_t is

$$\frac{1}{2}\left\{\frac{\upsilon_t}{\mu_t} - \frac{1}{E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]}\frac{\left(A_t\mu_t\right)^{1+\varphi}}{\mu_t}\right\}$$
$$+\ \frac{1}{2}\frac{\beta}{\sigma-1}\left\{\frac{1}{\mu_t} - \frac{1}{E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]}\frac{\left(A_t\mu_t\right)^{1+\varphi}}{\mu_t}\right\} = 0.$$

A similar condition holds for the monetary authority in Foreign. Replacing A_t, A_t^* and the optimal policies μ_t and μ_t^* in the expression of the exchange rate, we get

$$\varepsilon_t = \frac{\upsilon_t^*}{\upsilon_t}\frac{\mu_t}{\mu_t^*} = \frac{\upsilon_t^*}{\upsilon_t}\left[\frac{1+\frac{1}{\sigma-1}-\frac{1}{\kappa}+\beta\left(\frac{1}{\sigma-1}+\frac{1}{\kappa}\right)-\left(\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)\upsilon_t^*}{1+\frac{1}{\sigma-1}-\frac{1}{\kappa}+\beta\left(\frac{1}{\sigma-1}+\frac{1}{\kappa}\right)-\left(\frac{1}{\sigma-1}-\frac{1}{\kappa}\right)\upsilon_t}\right]\left[\frac{\upsilon_t+\frac{\beta}{\sigma-1}}{\upsilon_t^*+\frac{\beta}{\sigma-1}}\right]^{\frac{1}{1+\varphi}}.$$

It is straightforward to note that

$$\frac{\partial \epsilon_t}{\partial v_t}\Big|_{v_t=1} = -1 + \frac{\frac{1}{\sigma-1} - \frac{1}{\kappa}}{1 + \beta\left(\frac{1}{\sigma-1} + \frac{1}{\kappa}\right)} + \frac{1}{(1+\varphi)\left(1 + \frac{\beta}{\sigma-1}\right)} > -1.$$

For a given demand shock, the fluctuations of the nominal exchange rate under the optimal non-cooperative policy are therefore lower than those under the flexible exchange rate policy, which are proportional to the demand shocks. Moreover, the fluctuations under the optimal policies are more limited, the higher is the Pareto shape κ. This leads to Proposition 6.

Optimal Cooperative Monetary Policy

Under cooperation, the objective of monetary policy in the Home country is

$$\max_{\mu_t} E_{t-1}\left[\mathcal{U} + \mathcal{U}^*\right].$$

The first-order condition with respect to μ_t is

$$\frac{1}{2}\left(\frac{1}{\sigma-1} + 2 - \frac{1}{\kappa}\right)\left\{\frac{v_t}{\mu_t} - \frac{1}{E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]}\frac{\left(A_t\mu_t\right)^{1+\varphi}}{\mu_t}\right\}$$
$$+ \frac{\beta}{2}\left(\frac{1}{\sigma-1} + \frac{1}{\kappa}\right)\left\{\frac{1}{\mu_t} - \frac{1}{E_{t-1}\left[\left(A_t\mu_t\right)^{1+\varphi}\right]}\frac{\left(A_t\mu_t\right)^{1+\varphi}}{\mu_t}\right\} = 0.$$

A similar condition holds for the monetary authority in Foreign. Replacing A_t, A_t^* and the optimal policies μ_t and μ_t^* in the expression of the exchange rate, we get

$$\epsilon_t = \frac{v_t^*}{v_t}\frac{\mu_t}{\mu_t^*} = \frac{v_t^*}{v_t}\left[\frac{1 + \frac{1}{\sigma-1} - \frac{1}{\kappa} + \beta\left(\frac{1}{\sigma-1} + \frac{1}{\kappa}\right) - \left(\frac{1}{\sigma-1} - \frac{1}{\kappa}\right)v_t^*}{1 + \frac{1}{\sigma-1} - \frac{1}{\kappa} + \beta\left(\frac{1}{\sigma-1} + \frac{1}{\kappa}\right) - \left(\frac{1}{\sigma-1} - \frac{1}{\kappa}\right)v_t}\right]\left[\frac{v_t + \beta\left(\frac{\frac{1}{\sigma-1}+\frac{1}{\kappa}}{\frac{1}{\sigma-1}+2-\frac{1}{\kappa}}\right)}{v_t^* + \beta\left(\frac{\frac{1}{\sigma-1}+\frac{1}{\kappa}}{\frac{1}{\sigma-1}+2-\frac{1}{\kappa}}\right)}\right]^{\frac{1}{1+\varphi}}.$$

The non-cooperative and cooperative monetary policies are identical when $\kappa = \sigma - 1$, that is for the largest degree of firm heterogeneity. Instead, when κ is larger, the gains of international monetary policy cooperation are larger, as the coordinated response to demand shocks prevent abrupt adjustments in the extensive margins of trade.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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Masashige Hamano Associate Professor, School of Political Science and Economics at Waseda University.

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