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Uncertainty in Climate Change Modelling: Can Global Sensitivity Analysis Be of Help?

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UNCERTAINTY IN CLIMATE CHANGE MODELLING: CAN GLOBAL SENSITIVITY ANALYSIS BE OF HELP?

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Abstract. The complexity of integrated assessment models (IAMs) prevents the direct appreciation of the impact of uncertainty on the model predictions. However, for a full understanding and corroboration of model results, analysts might be willing, and ought to identify the model inputs that influence the model results the most (key drivers), appraise the relevance of interactions and the direction of change associated with the simultaneous variation of the model inputs. We show that such information is already contained in the data set produced by Monte Carlo simulations and that it can be extracted without additional calculations. Our discussion is guided by an application of the proposed methodologies to the well-known DICE model of William Nordhaus (2008). A comparison of the proposed methodology to approaches previously applied on the same model shows that robust insights concerning the dependence of future atmospheric temperature, global emissions and current carbon costs and taxes on the model's exogenous inputs can be obtained. The method avoids the fallacy of a priori deeming the important factors based on sole intuition.

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1 INTRODUCTION

Climate change is a complex phenomenon which impacts our society in a multiplicity of ways. The growing pressure of legislators and consumers is requiring firms to manufacture products with sustainability and environmental ethics as part of their modus operandi (Tang and Zhou, 2012). Climate policy and the promotion of renewable energies are among the sources of change in the regulation of energy markets (Most and Keles, 2010). However, climate change is characterized by ample uncertainties as to its causes as well as its impacts (among others see Baker, 2009, and Baker and Solak, 2011).

When environmental and climate change issues are considered, integrated assessment models (IAMs) play a central role in aiding policy makers during the formulation of mitigating strategies and risk

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management plans. This practice is not free from criticism (see Tol, 2003; Weitzman, 2009; Pindyck, 2012); yet, scientific models are today's intermediaries between science and policy. These plans are very complicated machines due to the intricacy of the phenomena under investigation, their space and time scales and the variety of features they capture, ranging from physical laws to socio-economic aspects. This makes it impossible to have a direct understanding of the relationship between the endogenous and exogenous variables. Climate scientists and decision-makers are then exposed to the risk of drawing conclusions without a full appreciation of the model's behavior and of the most critical assumptions. This generates an issue of trust in model results (Risbey et al., 2005). In these circumstances, the literature highlights that "*the standard of quality for models must be high, lest model use falls into disrepute and stakeholders reject the use of models altogether*" (Saltelli and D'Hombres, 2010, p. 302). The problem is perceived in the climate change community (Oppenheimer et al., 2007): for instance, Swart et al. (2009) underline that "*dealing consistently with risk and uncertainty across the Intergovernmental Panel on Climate Change (IPCC) reports is a difficult challenge*" (p.3) and that "*observed differences in handling uncertainties by the three IPCC working groups emerge*" (p.1).

How can we overcome these problems? Webster (2009) suggests that, independently of the IPCC working group affinity, it is appropriate for the community to produce more instances of rigorous analysis of uncertainty for their respective models and projections. The US Environmental Protection Agency recommends that model developers and users perform sensitivity and uncertainty analysis to help determine when a model can be appropriately used to inform a decision (US EPA, 2009). However, although sensitivity analysis (SA) techniques are the key ingredient needed to draw out the maximum capabilities of mathematical modeling (Rabitz, 1989), surveys show that the application of the most recently developed methods is quite limited in the field of climate change economics. Saltelli and Annoni (2010) review several papers published in prominent scientific journals such as *Science* and *Nature* and conclude that the most widely utilized methods are one-factor-at-a-time (OFAT) techniques. Generally defined, OFAT are methods of designing computational experiments involving the testing of factors, or causes, one at a time. These methods are quite inadequate for identifying the factors on which to focus scientists' or decision-makers' attention in the presence of uncertainty; furthermore, they do not allow analysts to appreciate the relevance of interactions.

OFAT methods are used in the series of rebuttals among Dietz et al. (2007a, 2007b), Nordhaus (2007a, 2007b), Stern and Taylor (2007) and Tol and Yohe (2006, 2007). Weitzman (2007) focused on the sensitivity of model outputs primarily to the choice of the discount rate and to a few other selected model inputs. Saltelli and d'Hombres (2010) offer a detailed analysis of the debate and conclude that, because SA was not used in a systematic way, SA did not help the analysts in sustaining their deductions.

Moreover, a methodological deficiency is often represented by the pre-selection of the factors on which to focus attention for further modelling and future research. This pre-selection is usually performed for reducing the burden of the analysis. However, it is likely to lead modelers to forego important factors, with the consequence of focusing additional efforts in data collection, modelling and research in a sub-optimal fashion. Also, if the sensitivity analysis results are used to direct decision-makers towards the factors on which to focus managerial attention (Eschenbach, 1992), these factors have to be identified in a systematic way for avoiding misleading indications.

Exploiting the informational content of a complex scientific code is, nonetheless, challenging. On the one hand, one needs methods that minimize computational burden. On the other hand, the same methods must be robust and take all sources of uncertainty into account.

In this work, we argue that the answer to this challenge requires a combination of global sensitivity methods. Our methodology is based on a set of recent advances in the areas of sensitivity analysis of model output. Our goal is, also, to demonstrate that insights concerning direction of change, model structure and key uncertainty drivers can be directly extracted from the sample generated by a traditional Monte Carlo uncertainty propagation procedure, without the need of ad-hoc sampling plans. The key is to base the analysis on the high-dimensional model representation (HDMR) theory as developed in Rabitz and Alis (1999). HDMR grants us with the understanding of whether the endogenous variable response to changes in the exogenous variables is equal to the superimposition of their individual effects or whether interactions are relevant (model structure). Also, it allows us to appraise direction of change in a global sense, as opposed to the traditional local information of comparative statics. The methodology is then complemented by the use of density-based methods for the identification of key uncertainty drivers in the presence of both correlated and uncorrelated exogenous variables.

Numerical experiments are performed using one of the best known IAMs, Nordhaus' DICE model. The results show that a systematic application of these methods provides several crucial insights to both analysts and policy-makers. Furthermore, one avoids pitfalls in the identification of the variables and areas on which to focus additional information collection and/or modeling efforts.

The remainder of the paper is organized as follows. Section 2 reviews the existing literature and provides a brief snapshot of how global sensitivity analysis methods are, or are not, being used. Section 3 presents our proposed methodology, whose estimation and computation aspects are considered in Section 4. The global SA is applied on the DICE and the results of this exercise are presented in Section 5. Concluding remarks close the paper.

2 A CURSORY LITERATURE REVIEW ON INTEGRATED ASSESSMENT MODELS AND THEIR SENSITIVITY ANALYSIS

Most and Keles (2010) highlight that policy-makers increasingly benefit from the utilization of decision-support models. Their observations are in line with the earlier statements of Jannsen (1997) who underlined that the IPCC "*is placing increasing emphasis on the use of dynamic or time-dependent simulation models to assess the effects of global climate change*" (p. 22). The models developed to support decision-making in the climate change arena are numerous. Game-theoretic models for climate negotiations are discussed in Forgó et al. (2005). We recall the role of the market allocation (MARKAL) model of Fishbone and Abilock (1981), a linear programming model for energy market planning. A modified version of MARKAL is used in Kanudia and Loulou (1998) to find optimal responses in long-term energy planning in view of alternative climate change patterns for the Quebec region. An extension of MARKAL is used in Barreto and Kypreos (2004) for a global model discussing the interaction between climate change and technology learning. The authors consider five regions that cover both industrialized and developing countries. Masui (2005) describes an application of a general equilibrium model for the selection of policies for abating CO_2 and best managing solid waste. Baker and Solak (2011) develop a model for the robust determination of optimal energy technology R&D investment programs in consideration of the effects of climate change.
A prominent class of models utilized is represented by IAMs. Aside their traditional use of evaluating the long term implications of climate-economy interactions, IAMs are also becoming increasingly used as a tool to study how uncertainty and ambiguity affect policy makers' decisions regarding climate change. Golub et al. (2011) provide a comprehensive overview of different approaches used to model uncertainty when applying IAMs. Millner et al. (2010), Lemoine and Traeger (2011), and Iverson and Perrings (2012) are recent examples of applications using the DICE model to study these areas of decision science.

The list of studies covered by Tol (2008) in his meta-analysis of the range of estimates of the social cost of carbon, let three IAMs emerge as the most widely applied and commonly cited in the literature: Richard Tol's FUND, Chris Hope's PAGE and the DICE model of William Nordhaus.

Many of the cited studies in Tol (2008) acknowledge the existence of uncertainty and attempt to perform some type of SA. This is usually accomplished by altering the values of a certain targeted inputs, often the discount rate and/or climate sensitivity, to test outcomes under different scenarios. The tendency is, therefore, to perform specific sensitivity questions, and not to let the model undergo a systematic investigation through SA methods.

Monte Carlo simulation to propagate uncertainty in model inputs is becoming part of best practices in the IAM literature. It has been used for an uncertainty analysis of the DICE model (Nordhaus, 1994,

2008) and in different vintages of the PAGE model (Hope, 2006). It is employed in a recent study by Dietz (2011) in an assessment of catastrophic climate change based on the PAGE model and by Nordhaus and Popp (1997) using DICE and Popp (2004) using ENTICE, an extension of the DICE model. It is also used in Dietz and Asheim (2012) in their work on sustainable discounted utilitarism, where Monte Carlo simulation accompanies the risk analysis of a modified version of DICE. Monte Carlo propagation (sometimes called uncertainty analysis) conveys to decision makers the uncertainty in model predictions, avoiding the risk of overconfidence in model forecasts. However, for a full understanding and corroboration of model results, analysts might be willing to (and ought to) identify the model inputs that influence the model results the most (key drivers), the direction of change associated with the variation of a given input and the overall model structure (interaction analysis).

We are aware of only three studies devoted to the application of methods similar to the ones proposed in this paper to study the effects of uncertainty on IAMs. van Vuuren et al. (2008) apply a probabilistic approach to an energy model, Hof et al. (2008) use the FAIR IAM and Anthoff and Tol (2011) explicitly address the effects of uncertainty on the social cost of carbon (current damages caused by each unit of emissions) using the FUND model. In all cases, Monte Carlo simulations are used to propagate uncertainty and the results of those simulations are post-processed using either raw correlations or standardized regression coefficients to signal the magnitude of the impact that parameter uncertainty has on model outputs. The SA literature clearly describes the weaknesses of using correlations or standardized regression coefficients as a methodology for post-processing the Monte Carlo results. These limitations are mainly linked to their poor performance in the presence of non-linearities and interactions (Campolongo and Saltelli, 1997) so that several authors have argued in favor of the utilization of more robust methods (Sobol', 1993; Rabitz and Alis, 1999; Saltelli et al. 2008).

3 GLOBAL SENSITIVITY ANALYSIS: SETTINGS AND METHODS

By global sensitivity analysis one means the probabilistic evaluation of a model sensitivity, in the presence of uncertainty in the model inputs. Formally, let $\Omega_X \subseteq R^n$ be the set of possible values that the model inputs can assume and $(\Omega_X, B(\Omega_X), P_X)$ denote the corresponding probability space. $F_{\mathbf{X}}(x)$ denotes the joint cumulative distribution function (CDF) of the model inputs and $f_{\mathbf{X}}(x)$ their density. $F_{\mathbf{X}}(x)$ is assigned by the analyst based on her state-of-knowledge about the model inputs. **x** denotes one of the possible realizations of the random vector **X**. We denote by:

$$y = g(\mathbf{x}) : \Omega_X \subseteq \mathbf{R}^n \to \mathbf{R} \quad (1)$$

the relationship that links the model inputs to the model output. The analytic expression of g is, usually, not explicitly known, being the result of elaborate calculations of complex computer codes.

Because **X** is uncertain, *y* becomes a random variable, denoted by *Y*. The associated probability space is (Ω_Y, B_Y, P_Y), $F_Y(y)$ and $f_Y(y)$ denote the CDF and density of *Y*, respectively.

Performing a global SA means propagating uncertainty through the model, either analytically or numerically, to obtain *y* (Reilly et al., 2001; Forest et al., 2002; Bernstein et al., 2009; Webster, 2009). Numerical uncertainty propagation goes under the heading of Monte Carlo simulation, which covers the various sampling generation methods (Sobol' quasi random sequences, Latin Hypercube sampling, etc.). Independently of the random number generation algorithm, a sample of size *N* is produced and the model is evaluated *N* times. The cost of the analysis is *C*=*N* model runs, as noted in the next section.

An integral part of a global SA is the statement of the goals of the analysis in order to identify the most appropriate methods and avoid misleading conclusions. In this respect Saltelli and Tarantola (2002) introduce the concept of SA setting (see also Saltelli et al. 2008; Borgonovo, 2010). A setting is a way to frame the SA quest so as to clearly identify its objectives. In this paper we make use of the following settings:

1. Model structure: to determine whether the endogenous variable behavior is the result of the superimposition of individual effects or it is driven by interactions;
2. Direction of change: to determine what is the expected direction of change in the endogenous variable due to individual or simultaneous changes in the exogenous model inputs;
3. Factor Prioritization: to determine the key uncertainty drivers, namely the factors on which resources should be focused in data and information collection to most effectively reduce variability in a model's predictions.

We now discuss each of these settings in turn.

3.1 MODEL STRUCTURE

The understanding of the structure of a model input-output mapping requires the assessment of interactions, as shown below. Assume that the model mapping $g(\mathbf{x})$ is integrable (thus, in principle even non-smooth). Then, $g(\mathbf{x})$ can be written exactly as (Efron and Stein, 1981); Sobol', 1993; Rabitz and Alis, 1999):

$$g(\mathbf{x}) = g_0 + \sum_{i=1}^{n} g_i(x_i) + \sum_{i<j}^{n} g_{i,j}(x_i, x_j) + ... + g_{1,2,...,n}(x_1, x_2, ..., x_n) \qquad (2)$$

where:

$$\begin{cases} g_0 = \mathbb{E}_{\mathbf{X}}[g(\mathbf{x})] = \int \ldots \int g(\mathbf{x}) \prod_{i=1}^{n} dF_i \\ g_i(x_i) = \mathbb{E}_{\mathbf{X}}[g(\mathbf{x}) \mid X_i = x_i] - g_0 = \int \ldots \int g(\mathbf{x}) \prod_{s=1, s \neq i}^{n} dF_s \\ g_{i,j}(x_i, x_j) = \mathbb{E}_{\mathbf{X}}[g(\mathbf{x}) \mid X_i = x_i, X_j = x_j] - g_i(x_i) - g_j(x_j) - g_0 = \int \ldots \int g(\mathbf{x}) \prod_{s=1, s \neq i,j}^{n} dF_s \\ \ldots\ldots \end{cases} \quad (3)$$

Eq. (2) is called the high-dimensional model representation (HDMR) of $g(\mathbf{x})$ (Rabitz and Alis, 1999). In (3) g_0 is the average value of *y* over Ω_Y ; $g_i(x_i)$ accounts for the individual effect of X_i, $g_{i,j}(x_i, x_j)$ accounts for the residual interactions of model inputs X_i, X_j, and so on. Eq. (2) states that $g(\mathbf{x})$ is exactly reconstructed by the sum of the functions in the right hand side of (3). Eqs. (2) and (3) provide the multivariate "integral" expansion of $g(\mathbf{x})$.

Assume now that $g(\mathbf{x})$ is square integrable. Then, by the orthogonality of the functions in eq. (3), by subtracting g_0 from $g(\mathbf{x})$ one obtains the complete decomposition of the variance of Y:

$$\mathbb{V}_{\mathbf{X}}[Y] = \sum_{i=1}^{n} V_i + \sum_{i<j}^{n} V_{ij} + \ldots + V_{1,2,\ldots,n} \quad (4)$$

where the generic term of order *r* in eq. (4) is given by:

$$V_{i_1,i_2,\ldots,i_r} = \int g^2_{i_1,i_2,\ldots,i_r} \mathrm{d}F_{i_1} \mathrm{d}F_{i_2} \ldots \mathrm{d}F_{i_r} \quad (5)$$

On the basis of (4) and (5), Sobol' (1993) introduced the sensitivity indices of order *r* defined as:

$$S_{i_1,i_2,\ldots i_r} \equiv \frac{V_{i_1,i_2,\ldots i_r}}{\mathbb{V}_{\boldsymbol{X}}[Y]} \quad (6)$$

Special attention is deserved by the first and the total order sensitivity indices, defined respectively as:

$$S_l^1 \equiv \frac{V_l}{\mathbb{V}_{\boldsymbol{X}}[Y]} = \frac{\mathbb{V}_{X_l}[\mathbb{E}\{Y \mid X_l\}]}{\mathbb{V}_{\boldsymbol{X}}[Y]} \qquad (l = 1, \ldots, n) \quad (7)$$

and:

$$S_l^1 \equiv \sum_{r=2}^{n} \sum_{l,i_2,\ldots,i_r}^{n} \frac{V_{l,i_2,\ldots,i_r}}{\mathbb{V}_{\boldsymbol{X}}[Y]} = \frac{\mathbb{E}_{\mathbf{X}_{\sim l}}[\mathbb{V}_{X_l}\{Y \mid X_l\}]}{\mathbb{V}_{\boldsymbol{X}}[Y]} \qquad (l = 1, \ldots, n) \quad (8)$$

In eq. (8), the symbol $\mathbf{X}_{\sim l}$ denotes all factors but X_l. Disentangling the contribution of single variables and of interactions to the overall model variability (Setting 1) is quite naturally addressed by applying the functional ANOVA decomposition and the associated sensitivity measures reported in (6), (7) and (8). The sum of these variance-based sensitivity measures provides indications on model structure. In the case $\sum_{i=1}^{n} S_i^1 = 1$ the model is additive, that is, its response is the exact superimposition of the individual effects of the exogenous variable. Conversely, if $\sum_{i=1}^{n} S_i^1 < 1$ interaction effects are present. The lower the sum of the first order indices is, the higher the relevance of interactions.

3.2 DIRECTION OF CHANGE

Setting 2, the expected direction of change in the endogenous variable, can be addressed through the investigation of functions $g_0 + g_i(x_i)$. Note that, from the second equation in (3), we have:

$$\mathbb{E}[g(\mathbf{x}) \mid X_i = x_i] = g_i(x_i) + g_0 \tag{9}$$

Thus, $g_0 + g_i(x_i)$ represents the conditional expectation of $g(\mathbf{x})$ as a function of x_i. In particular, if $g(\mathbf{x})$ is additive, then $g_i(x_i) + g_0$ displays the exact dependence of Y on X_i. Thus, we are able to understand whether Y is a monotonic function of X_i with no approximation and for all values of X_i. If $g(\mathbf{x})$ is not additive, then eq. (9) is a trend line that allows us to understand the dependence of Y on X_i as all possible values of the remaining model inputs are averaged. Thus, there is a difference between comparative statics in the sense of Samuelson (1947) and comparative statics performed using an integral approach, like the one adopted here. By differential comparative statics, one obtains a local information, namely the variation rate of *Y* around one given point in the input parameter space for a small variation in X_i. On the basis of eq. (9) one obtains a global information about what happens to $g(\mathbf{x})$ as X_i varies over its entire range.

3.3 FACTOR PRIORITIZATION

The identification of key uncertainty drivers (setting 3) may appear to be linked to the discussion above on variance decomposition, suggesting that a critical parameter could be the one which has a significant impact on the endogenous variable(s) variance. However, it is well known that variance is not a good summary measure of uncertainty, especially when the distributions are skewed or multimodal, and when inputs are correlated, which is likely the to be the case in many natural phenomena, including climate change. In this case, Borgonovo (2007) proposes a better suited sensitivity measure, defined as follows:

$$\delta_l = \frac{1}{2} \mathbb{E}_l[s_l(x_l)] \tag{10}$$

where:

$$s_I(x_I) = \left| f_Y(y) - f_{Y|X_I=x_I}(y) \right| \tag{11}$$

$s_i(x_i)$ measures the separation between the unconditional distribution of the model output [$f_Y(y)$] and the conditional model output distribution given that model input X_i is fixed at x_i [$f_{Y|X_i=x_i}(y)$]. Geometrically, $s_i(x_i)$ is the area enclosed between $f_Y(y)$ and $f_{Y|X_i=x_i}(y)$.

It can be shown that δ possesses the following convenient properties: (i) normalization to unity, i.e. $0 \le \delta_i \le 1, \quad i = 1, 2, ..., n$; (ii) joint normalization: $\delta_{1,2,...,n} = 1$; (iii) scale invariance: if $u(Y)$ and $t(Y)$ are two monotonic functions, then $\delta_i^{u(Y)} = \delta_i^{t(Y)} = \delta_i^Y$. The first property states that each exogenous model input has an ”importance index”', which lies between 0 and 1. In particular, an exogenous model input X_i has null importance if Y and X_i are independent. The second property states that the joint importance of all model inputs is unity. The third property of scale invariance is desirable for two aspects. The first one emerges in numerical estimation. In several applications the output of a Monte Carlo simulation is sparse or spans a wide range. This could bring about inaccurate estimation of sensitivity measures. To improve numerical precision, analysts often resort to a transformation of the model output (usually, a log-transformation). Scale invariance insures that, after any monotonic transformation the results of SA remain unaltered. The second reason is that in many applications the model output is valued through a utility function. It is a well-known principle of economic theory that utility functions have an ordinal, not cardinal, meaning, so that they can be freely modified through monotonic transformations. Scale invariance, then, insures that results of the sensitivity analysis remain valid for any chosen monotonic utility function. For further discussion on the decision-making implications of this result see Baucells and Borgonovo (2012).

4 ESTIMATION AND COMPUTATIONAL COST

The estimation of the sensitivity measures proposed above is analytically feasible only in very few instances and with simple mathematical expressions that usually do not represent an environmental or economic problem. For IAMS, which are complex simulation tools encoded in dedicated software, the estimation is forcedly numerical.

An algorithm that strictly reproduces the definitions in eqs (7), (8) and (10) - brute force estimation - is associated with a computational cost equal to:

$$C = Nn^2 \tag{12}$$

model runs, where *N* is the sample size of Monte Carlo simulation and *n* the number of factors. *N* should be chosen in such a way as to ensure estimation accuracy. At *N*=1000 *C* is greater than one million model runs, making the estimation prohibitive for any IAM.

However, computation reduction results have been reached in the global SA literature. They have led to a drastic reduction in the estimation of variance-based indices, lowering *C* to:

$$C = N(n+2) \tag{13}$$

model runs for estimating all first and total order sensitivity measures (Saltelli, 2002; Saltelli et al., 2010; Campolongo et al., 2011).

The sampling plans in Castaings et al. (2012) lower the computational cost of the δ-importance measure to:

$$C = N \cdot r \tag{14}$$

where *r* is the number of replicates.

Note that an analyst pursuing these estimation strategies has possibly to run two different sets of numerical experiments, one to estimate S_l^1 and S_l^T and one to estimate δ_l . Moreover, in both cases, the sampling plans would differ from the utilization of a simple Monte Carlo uncertainty propagation.

In this paper we pursue an alternative strategy which enables us to obtain all sensitivity measures from the same dataset and at the lowest possible computational cost. Recent work has produced notable advances in this respect, lowering the computational cost to:

$$C = N \tag{15}$$

There are two main ways to proceed. The first foresees making use of a meta-model. We recall Kriging (see Kleijnen, 2009), Gaussian emulation (Oakely and O'Hagan, 2004), Cut-HDMR (Rabitz and Alis, 1999; Ziehn and Tomlin, 2010), polynomial chaos expansion (Sudret, 2008), and state-dependent parameter modelling (Ratto and Pagano, 2010). Here we make use of the GUI-HDMR software of Ziehn and Tomlin (2009). The software allows the estimation of Sobol' sensitivity measures of orders 1 and 2 from the component functions $g_i(x_i)$, $g_{i,j}(x_i, x)$ which are obtained by fitting orthonormal bases, through a system of equations of the type:

$$\begin{gathered} g_i(x_i) \approx \sum_{r=1}^{h} \alpha_r^i \phi_r(x_i) \\ g_{i,j}(x_i, x_j) \approx \sum_{p=1}^{h'} \sum_{q=1}^{h''} \beta_{p,q}^{i,j} \phi_p(x_i) \phi_q(x_i) \end{gathered} \tag{16}$$

where $\phi_r(x_i)$ is an element of a family of orthonormal polynomials, α_r^i , $\beta_{p,q}^{i,j}$ are the corresponding coefficients, h , h' and h'' determine the order of the expansion (see for further details Ziehn and Tomlin, 2009).

Following the Cut-HDMR approach (Rabitz and Alıs, 1999) one then obtains insights on model structure, through knowledge of the variance-based sensitivity indices, and on monotonicity, by plotting the $g_i(x_i)$ functions.

The second way is to utilize orthogonal projections and is used in Plischke et al. (2012). This technique allows one to estimate variance-based sensitivity measures and δ_l . The method consists of a reordering of the data set to form a scatterplot $X_i \oplus y$, followed by a partitioning of the data set. The method works as a post-processing algorithm and the estimation is direct, without the need of a meta-model. We shall make use of both the Cut-HDMR meta-model and Plischke et al's method in our analysis. The advantage of combining the proposed approaches is that one retrieves all the discussed insights without having to utilize an ad-hoc sampling scheme and using the dataset produced by Monte Carlo simulation. Thus, we add to current practice where Monte Carlo propagation has become part of the standard way of operating. In the next section, we discuss the application of the proposed approach to the DICE model.

5 GLOBAL SENSITIVITY ANALYSIS OF THE DICE MODEL

To illustrate the proposed methodology we have chosen the DICE model to perform a global SA. DICE is one of the most widely acknowledged IAMs due to the expertise of William Nordhaus, "*whose careful pragmatic modeling throughout his DICE series of IAMs has long set a standard*" (Weitzman, 2007, p.713). Nordhaus (2008) characterizes the DICE model as "*a global model that aggregates different countries into a single level of output, capital stock, technology, and emissions. The estimates for the global aggregates are built up from data that include all major countries, and the specification allows for differentiated responses and technological growth*" (p.33). DICE has been evolving since the early 1990s with many refinements and adaptations to answer specific research questions. A few examples of the diffusion and utilization of DICE in the scientific debate and its impact in climate change analysis are offered next. Jannsen (1997) combines the economic part of DICE with the mathematical part of IMAGE - integrated model to assess greenhouse effect - to obtain the OMEGA code (optimization model for economic and greenhouse assessment). Keller et al. (2004) modify DICE for assessing the impact of uncertainties and learning about climate thresholds. Baker and Solak (2011) use DICE as a benchmark to calibrate their analysis. The utilization of DICE in Dietz and Asheim (2012) has been discussed in Section 2.

We use Version 2007.delta.8 of DICE (Version 2007.delta.8 can be downloaded from Nordhaus' website at http://nordhaus.econ.yale.edu/DICE2007_short.gms). We do not go into details concerning the nature and general structure of the model. It is comprehensively described in Nordhaus (2008) and one can find synthetic descriptions in Jannsen (1997), Keller et al. (2004) and Dietz and Asheim (2012). We limit ourselves to note that the input-output mapping is composed of a series of interconnected equations (or submodels), that generate a multiplicity of outputs. These outputs depend on 51 model inputs, which are reported in Table A1 of the Appendix. Among the many outputs produced by DICE we focus on inter-generational welfare (utility), the social cost of carbon in 2005, global atmospheric temperature in 2105, global emission level in 2105, and the optimal carbon tax for

2015, because they are relevant for policy purposes and grant comparison with previous SA performed using the same model.

The presentation of the results of our global SA exercise in divided in two parts. The first part describes the set of results stemming from a comparison of our methodology with the sensitivity of the DICE model directly performed by Nordhaus (2008), where only certain pre-selected model inputs were subjected to uncertainty and sensitivity analysis. The second set presents results for the dataset obtained when uncertainties in all model inputs are considered.

5.1 A COMPARATIVE ANALYSIS

Our reference point is the sensitivity analysis of the DICE model performed in Chapter 7 of Nordhaus (2008). It relies on a pre-screening exercise performed in Nordhaus (1994) and identifies 8 inputs as key uncertainty drivers, which should be subjected to increased scrutiny. We take the outcome of the pre-screening exercise for granted and use the same probability distributions for the 8 inputs as in Nordhaus (2008) in order to offer a comparison of the insights that can be obtained by applying the methods discussed in this paper. We will remove the restriction on the number of factors later on in the section. Results obtained when all model inputs are varied are then compared to results obtained when the subset of the a priori selected factors is considered.

Table 1 displays the results of an OFAT analysis of the DICE model originally presented in Tables 7-2 and 7-3 of Nordhaus (2008). It conveys the impact that the value of a given model input has on a model output as the input moves from one to six standard deviations from the assumed mean value.

Let $\mathbf{x^0}$ denote the mean value of the model inputs and $(x_i^0 + k\sigma_i, \mathbf{x}^0_{\sim i})$ the point obtained by moving only X_i by k standard deviations (k=1, 2, 3, 4, 5, 6). The results are shown only when the parameters move away from the mean value in the positive direction (k=[1,6]) rather than in both ones (k=[-3,3]) since, "the results are sufficiently linear that this displays the patterns accurately" (Nordhaus, 2008, p.129). The percentage changes from $g(\mathbf{x}^0)$, $\Delta_l^k = \dfrac{g(x_i^0 + k\sigma_i, \mathbf{x}^0_{\sim i}) - g(\mathbf{x}^0)}{g(\mathbf{x}^0)}$, are taken by Nordhaus (2008) sensitivity measures and displayed both in absolute and relative terms. We are then in an OFAT framework. The numerical values in the Table 1 display the value of the social cost of carbon in 2005 (top panel) and of global emissions in 2105 (bottom panel) when the value of the parameters is altered. For example, when the model inputs are at their mean value, the social cost of carbon in 2005 is $g(\mathbf{x^0})$=$28.10. When the value of GA0, the growth in total factor productivity, is altered by one standard deviation the value of social cost of carbon increases to $36.07, a 28% increase from the mean value. The table shows that for the social cost of carbon in 2005 the quadratic coefficient in the damage function (A2) has the largest effect as k (i.e., the distance from the mean value) varies.

Table 1: Summary of Nordhaus (2008)' SA Results

SOCIAL COST OF CARBON 2005								
Standard Deviation	GA0	GSIGMA	T2XCO2	A2	PBACK	POPASYM	b12	FOSSLIM
0	28.1 (0)	28.1 (0)	28.1 (0)	28.1 (0)	28.1 (0)	28.1 (0)	28.1 (0)	28.1 (0)
1	36.07 (28)	28.27 (1)	38.07 (35)	40.99 (35)	28.1 (0)	32.14 (14)	29.16 (4)	28.1 (0)
2	48.08 (71)	28.43 (1)	46.44 (65)	53.89 (65)	28.1 (0)	35.91 (28)	30.32 (8)	28.1 (0)
3	51.21 (82)	28.6 (2)	53.49 (90)	66.8 (90)	28.1 (0)	39.44 (40)	31.61 (12)	28.1 (0)
4	54.68 (95)	28.76 (2)	59.47 (112)	79.73 (112)	28.1 (0)	42.75 (52)	33.04 (18)	28.1 (0)
5	58.52 (108)	28.92 (3)	64.59 (130)	92.66 (130)	28.1 (0)	45.84 (63)	34.62 (23)	28.1 (0)
6	62.8 (123)	29.09 (4)	69.03 (146)	105.61 (146)	28.11 (0)	48.75 (73)	36.39 (30)	28.1 (0)
GLOBAL EMISSIONS 2105								
0	19.08 (0)	19.08 (0)	19.08 (0)	19.08 (0)	19.08 (0)	19.08 (0)	19.08 (0)	19.08 (0)
1	30.99 (62)	21.95 (15)	19.18 (1)	19.18 (1)	19.08 (0)	22.84 (20)	19.08 (0)	19.08 (0)
2	50.19 (163)	25.19 (32)	19.28 (1)	19.28 (1)	19.08 (0)	26.42 (38)	19.09 (0)	19.08 (0)
3	78.2 (310)	28.83 (51)	19.38 (2)	19.38 (2)	19.08 (0)	29.84 (56)	19.1 (0)	19.08 (0)
4	103.92 (445)	32.91 (72)	19.48 (2)	19.48 (2)	19.08 (0)	33.06 (73)	19.1 (0)	19.08 (0)
5	65.19 (242)	37.36 (96)	19.59 (3)	19.59 (3)	19.07 (0)	36.08 (89)	19.1 (0)	19.08 (0)
6	24.61 (29)	42.22 (121)	19.7 (3)	19.7 (3)	19.07 (0)	38.9 (104)	19.11 (0)	19.08 (0)

Source: Nordhaus (2008) and our own calculations.

For the social cost of carbon in 2005 the coefficient in the damage function (A2) is always the most influential input, regardless of the distance from the mean value. Conversely, at one standard deviation (k =1) climate sensitivity (T2XCO2) is ranked third behind the growth rate of total factor productivity (GA0) in the magnitude of the change from the mean value, but the ranking is reversed at two standard deviations from the mean value (k=2) and then again as k gets larger. Thus, this sensitivity exercise does not grant a robust identification of the key uncertainty drivers due to the instability of the implied sensitivity rankings with respect to the variation range.

The reason is that the method does not account for the simultaneous variation of all factors over their entire variation ranges. Rather, it varies them one-at-a-time at pre-determined values, leaving the remaining fixed. This limitation is overcome by the use of a global method, which makes the analysis robust over the variation range. Based on the preselected inputs, Nordhaus (2008) performs an uncertainty analysis using Monte Carlo simulation. We have argued that one can assess the key-problem drivers robustly by post-processing this data. Using the algorithm of Plischke et al. (2012), we obtain the δ_l [eq. (10)] importance of all factors. For the pre-selected model inputs with respect to the social cost of carbon and global emissions results are reported in Figure 1. The sample size is N=10000. At the top of each bar the 90% confidence intervals obtained from 500 bootstrap replicates using the bias-reducing estimator proposed in Plischke et al. (2012) are displayed.

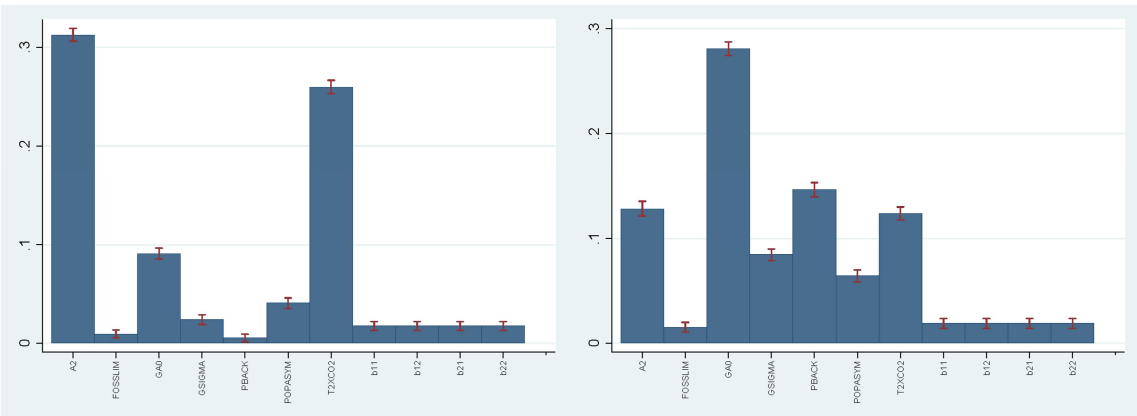

Figure 1: δ_l ‘s with Pre-Selected Model Inputs for the Social Cost of Carbon (left) and Global Emissions (right).

For the social cost of carbon, the coefficient in the damage function (A2), climate sensitivity (T2XCO2) and the growth rate of total factor productivity (GA0) are the most influential inputs. Observe that the narrow bootstrap intervals allow one to state that such conclusion is robust also with respect to uncertainty in the estimates of δ_l. Table 1 provides the message that the growth rate of total factor productivity is the most important, with respect to the level of global emissions in 2105, with the other factors having a much lower or negligible influence. We see from Figure 1 that while δ_l for this model input is still the greatest by magnitude, uncertainty in other factors is not insignificant in influencing uncertainty in future emissions.

A further limitation of the analysis in Table 1 is that OFAT methods do not reveal interactions, but these can also be extracted from the same dataset produced by the uncertainty analysis. To that end we apply the GUI-HDMR Matlab code of Ziehn and Tomlin (2009b). By the analysis of the data set generated from Monte Carlo simulations we obtain values of the second order sensitivity indices of $\sum_{i,j=1}^{n} S_{i,j}$ =0.4233, when the output is the social cost of carbon in 2005 and of $\sum_{i,j=1}^{n} S_{i,j}$ =0.6053 when the output is global emissions in 2105. These values indicate that the model responds non-additively to the inputs and that interaction effects are relevant. The interaction between the growth in the rate of factor productivity (GA0) and the price of the backstop technology (PBACK) is the most influential on the social cost of carbon in 2005, while the interaction between the coefficient in the damage function (A2) and the climate sensitivity parameter (T2XCO2) have the strongest effect on the level of global emissions in 2105. Figure 2 displays the HDMR of the most influential interactions for the social cost of carbon in 2005.

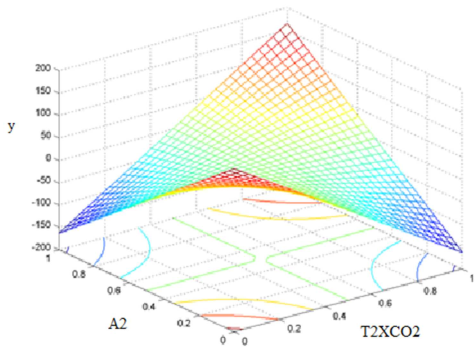

Figure 2: Input Interactions for the Social Cost of Carbon

The figure shows the plot of the bivariate function $g_{A2,T2XC02}\left(A2,T2XC02\right)$ representing the interactions between A2 and T2XCO2, when the output is the social cost of carbon. This second order function is neither convex nor concave and non-monotone. Also, note that the functions are not strictly positive or negative across the entire uncertainty ranges for the interacting inputs. When both inputs are at the upper end of their uncertainty ranges, the interactive effect is a negative one, while at the lower end of the ranges the interactive effect has the opposite sign. As a result the second order effects can have either an amplifying or dampening effect on the first order individual effects. The application of these methods thus provides a quantitative dimension to Nordhaus (2008)' statement that "*an examination of all the uncertain model inputs taken together ... may produce unexpected results because of the interactions among the model inputs and the non-linearity in the DICE model*" (p.134).

By plotting the first order terms in the HDMR decomposition one gathers insights about the direction of change and monotonicity when factors vary individually. Figure 3 shows the direction of change in the social cost of carbon (left panel) due to increase in climate sensitivity (T2XCO2), of the level of global emissions due to an increase in the initial growth rate of the technology (GA0), the two model inputs with most significant HDMR effects.

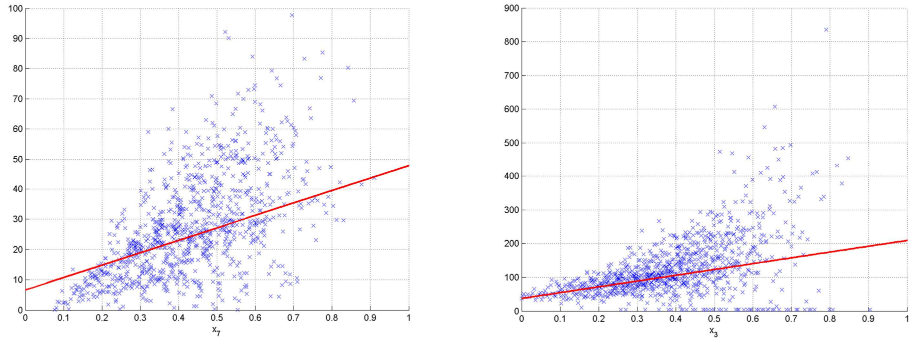

Figure 3: $g_{T2XCO2}(T2XCO2)$ on Social Cost of Carbon (left), and $g_{GA0}(GA0)$ on Global Emissions (right).

The trend lines in Figure 3, show that as the value of the climate sensitivity parameter rises so does the social cost of carbon, and the same relationship holds for the effect of the rate of technological growth on total emissions at the end of the century. For both model outputs the first order functions $g_i(x_i)$ are monotonic. In particular, one can determine whether they are increasing or decreasing for all factors (this information is not reported here for brevity). However, by looking at these graphs a decision-maker can gain insights about whether a factor tends to increase the social cost of carbon (or global emissions) on average.

In the next section we expand the analysis considering all model inputs.

5.2 RESULTS WHEN UNCERTAINTY IN ALL MODEL INPUTS IS CONSIDERED

We have seen that the cost for performing a global sensitivity analysis according to the methods we propose is independent of the number of factors. That is, there is no additional computational burden from subjecting all model inputs to uncertainty analysis, as the DICE model runs the same number of times in Monte Carlo simulations regardless of the number of model inputs that are altered. We can drop the restriction on the number of inputs and allow all 51 DICE 2007 model inputs to vary. Our goal is to see whether the preselected factors of the previous analysis are indeed the most important ones. For demonstration purposes, we assign all factors the range of ± 10% the original value, using a uniform distribution. Choosing the width of the interval is admittedly arbitrary, so that we repeated the analysis using intervals of 5% and 20% with consistent results that are not reported here but are available upon request.

A Monte Carlo sample of 10000 simulations is propagated using quasi-random sampling and post-processed using the method of Plischke et al. (2012). We also calculate the bootstrapped confidence intervals for δ_l, which is displayed as a bar at the top of each column in the histogram of Figure 4.

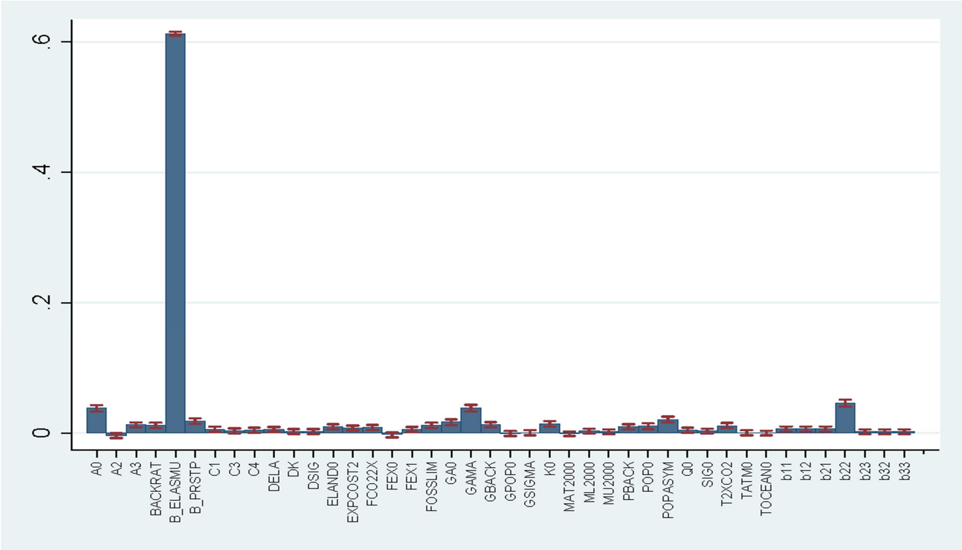

Figure 4: δ_l 's with Intergenerational Utility as Model Output

Figure 4 displays the δ-importance of the model inputs when the output of interest is inter-generational utility. The bars over the histogram present the uncertainty in the estimates obtained using 500 bootstrap replicates. Let us compare these results to the findings of Nordhaus and Popp (1997). The comparison is offered in Table 2.

Table 2: Nordhaus (2008)' Pre-Selected Parameters SA Rank

Pre-Selected Variables	Social Cost of Carbon (2005)	Global Emissions (2105)	Atmospheric Temperature (2105)	Carbon Tax (2015)
A2	22	5	6	10
GA0	7	11	11	3
FOSSLIM	42	26	30	34
GSIGMA	16	36	26	6
PBACK	17	32	29	12
POPASYM	10	12	8	8
T2XCO2	2	3	5	7
B12	12	13	12	21

From Table 2, it is clear that the pre-selection of model inputs performed in Nordhaus (1994) was correct in including the climate sensitivity parameter (T2XCO2), the coefficient in the damage function (A2) and the initial growth rate in total factor productivity (GA0) since they each rank in the top ten factors for at least two of the policy relevant model outcomes of our approach. The remaining preselected inputs are not nearly as influential and for optimal uncertainty management more useful information could have been obtained if different inputs had been pre-selected and subjected to increased analysis, or given priority in information and data collection if that is a previously limiting factor.

Figure 4 shows that the dominant input factor driving variation in the model output is the elasticity of the marginal utility of consumption (B_ELASMU). We note that B_ELASMU appears in the DICE model as the variable μ in the Ramsey's equation:

$$r = \rho + \mu \cdot b \tag{17}$$

where r is the social discount rate, ρ is the rate of pure time preference, b is the growth rate of consumption per capita and μ is the elasticity of marginal utility of consumption. μ is also known as the coefficient of relative risk aversion, because the DICE model uses a constant elasticity of substitution utility function:

$$U\left[c(t), L(t)\right] = L(t)\left[c(t)^{1-\mu} / (1-\mu)\right] \tag{18}$$

where $L(t)$ denotes labor force or population. B_ELASMU (μ) determines the shape of the utility function and the relationship between consumption increases and utility or welfare. Sterner and Persson (2008) succinctly explain the economic logic behind assumptions related to values for μ: "*the higher the value of μ, the less we care for a dollar more of consumption as we become richer. Since we expect that we will be richer in the future, when climate damages will be felt, a higher μ also implies that damages will be valued lower. Thus, a higher value of μ implies less greenhouse gas abatement today, unless for some reason we will be poorer rather than richer in the future. In this case, a higher μ would give higher damage values, which would justify more abatement*" (p.66).

The findings displayed in Figure 4 provide key support to the argument that numerical decisions affecting the parameters of the Ramsey equation and implicitly the discount rate are of primary importance in IAM exercises. However, a philosophical discussion on these aspects is beyond the scope of this paper and interested readers have plenty of well-conceived studies to consult on this topic. Nordhaus (2008) uses model inputs for the Ramsey equation that sum up to coincide with observed market rates of return on capital. However, Newell and Pizer (2003) show that market rates of return are not stable over longer periods of time, and the effects of minor changes in the relevant model inputs can have significant effects on model outcomes, as documented in Figure 4. Our analysis confirms that this model input is key for results but also allows one to understand how important this parameter is. It is also useful to understand how this factor interacts with other factors in the model. The strongest interaction is with the exponent in the Cobb-Douglas production function (GAMA) and the images of the HDMR first and second order terms $g_i(x_i)$ are similar to those of Figures 2 and 3 where at certain combinations of the model inputs in their uncertainty ranges the interactive effect can be either positive or negative.

Economists and policy makers are not only interested in drivers of inter-generational utility, which in itself is an abstract concept, but they focus also on pragmatic, relevant calculations that are of concern - such as the level of total emissions at the end of the century – and actionable – such as magnitudes of carbon taxes. Thus, we now turn to the results of a SA performed along the lines just followed, when the outputs are social cost of carbon in 2005, global emissions at the end of the century, and the rise of global atmospheric temperature at the end of the century relative to 1900, as done in Nordhaus (2008). We will also consider the effects of uncertainty on the optimal carbon tax in 2015, since that should be of concern to policy makers in the near term. Table 3 contains descriptive information about how uncertainty affects these outcomes. The ranges of outcomes are non-negligible, as a world that is 3.5 degrees warmer is likely very different than one that is 2 degrees warmer. They are also in line with the IPCC best estimates of what is required if excessive negative consequences from climate change are to be avoided. The same can be said for the range of damages current emissions are inflicting, global emissions in 2105 and the appropriate carbon tax in 2015 that would put us on an optimal trajectory, as calculated by the DICE model.

Table 3: Results from Monte Carlo Simulations

Variable	Observations	Mean	Standard Deviation	Min	Max
Global Average Temperature Rise by 2105	10,000	2.708	0.215	2.014	3.542
Social Cost of Carbon in 2005	10,000	27.252	6.126	13.130	55.170
Global Emissions in 2015	10,000	120.664	18.580	75.083	207.545
Carbon Tax in 2015	10,000	40.044	9.113	18.159	81.988

The table conveys information similar to that presented in Figure 7-2 in Nordhaus (2008) or to the finding of Arigoni Ortiz et al. (2011), who vary specific model inputs probabilistically in a SA performed on an adaptation of DICE. By going one step further and post processing the results of the probabilistic uncertainty analysis much insight is gained. Figure 5 displays the point estimates for δ_i and the bootstrapped confidence interval when the outcome of interest is the social cost of carbon in 2005 (left panel) and global emissions in 2105 (right panel).

Uncertainty in the elasticity of capital in the production function (GAMA) has the strongest influence on the social cost of carbon in 2005, followed by the exponent in the damage function (A3). The climate sensitivity parameter (T2XCO2), the elasticity of marginal utility (B_ELASMU) and the coefficient in the damage function (A2) are all roughly equally, but much less influential. The same cannot be said for the model inputs that influence the level of global emissions at the end of the century. Of primary importance is the exponent in the damage function (A3), followed by the emissions intensity of the economy in 2005 (SIG0), the initial growth rate of technological progress per decade (GA0) and the exponent in the cost control function (EXPCOST2).

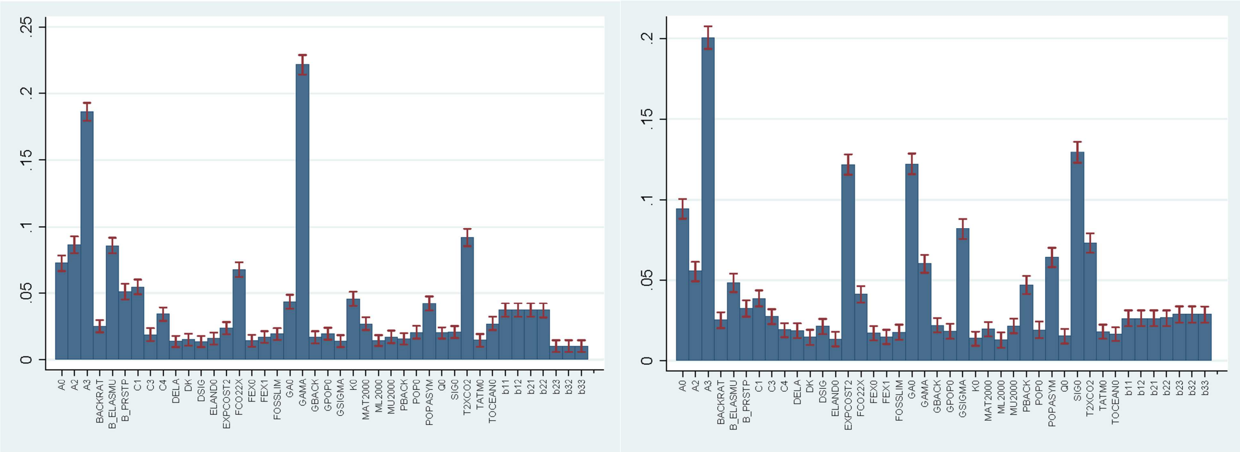

Figure 5: δ_l ‘s with Social Cost of Carbon (left panel) and Global Emissions (right panel) as Model Outputs

The left panel of Figure 6 shows that uncertainty in the initial levels of total factor productivity (A0) influence atmospheric temperature in 2105 most, followed closely by climate sensitivity (T2XCO2), emissions intensity of the economy in 2005 (SIG0), capital elasticity in the production function (GAMA) and the exponent in the cost control function (EXPCOST2). The same cannot be said of the effects of uncertainty on the calculation of the optimal carbon tax level for 2015 where the exponent in the damage function (A3) is the most influential, followed by capital elasticity (GAMA), initial levels of total factor productivity (A0), elasticity of marginal utility of consumption (B_ELASMU), and the climate sensitivity parameter (T2XCO2).

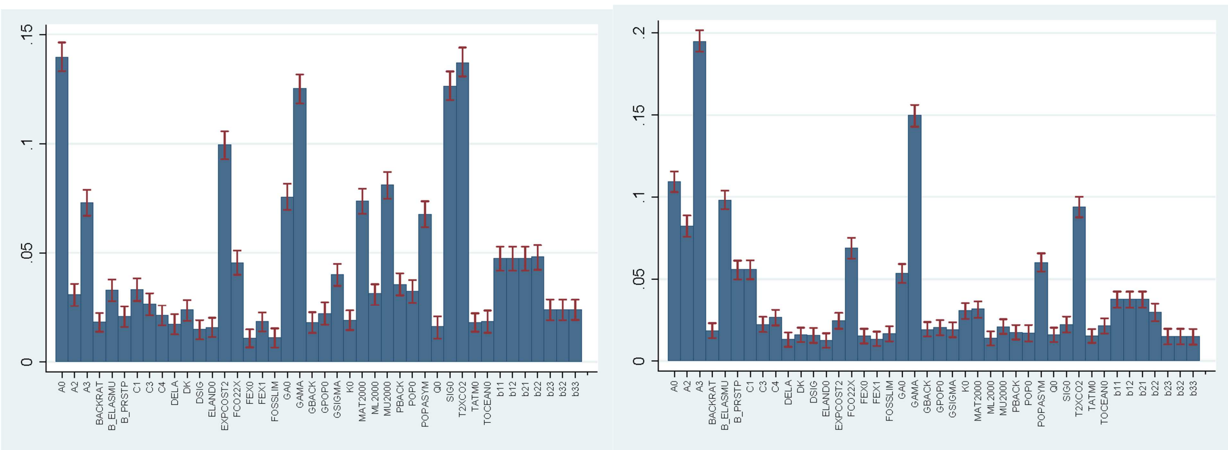

Figure 6: δ_l ‘s with Atmospheric Temperature (left panel) and Optimal Carbon Tax (right panel) as Model Outputs

As a next step, we consider the rankings of all the delta scores for each uncertain model input when different outcomes are considered. The full list of rankings is provided in Table A2 of the Appendix. To conserve on space, Table 4 summarizes this information reporting the rank correlations and Savage score correlations for each of the output. The rank correlations are computed considering the vector of the ranks of X_i with respect to each of the output. A correlation equal to unity implies that that for the two model outputs under consideration the most and least relevant factors are exactly the same. Each entry in Table 4 displays the raw correlations as first entry and the Savage score separated by a vertical bar. Savage scores place emphasis on the agreement of the key (higher ranked) uncertainty drivers, while raw correlations indicate the relationship between all model inputs (Iman and Conover, 1987; Campolongo and Saltelli, 1997; Borgonovo et al., 2010.) Let R_i be the rank of X_i. Then, the Savage Score of X_i is: $SS_i = \sum_{h=R_i}^{n} \frac{1}{h}$. For instance, a factor ranking first out of 51 has a Savage score of 4.52, a factor ranking second a score of 3.52, and so on.

Table 4: Overall Correlations and Savage Scores for Model Outputs

	Atmospheric Temperature	Social Cost of Carbon (2005)	Carbon Tax (2015)	Global Emissions (2015)	Utility
Atmospheric Temperature	1				
Social Cost of Carbon (2005)	0.709|0.613	1			
Carbon Tax (2015)	0.789|0.669	0.958|0.951	1		
Global Emissions (2015)	0.816|0.718	0.657|0.599	0.774|0.717	1	
Utility	0.545|0.430	0.731|0.645	0.669|0.622	0.554|0.388	1

When comparing rank correlations to Savage Score correlations we have two cases: a) if the rank correlation value is smaller than the corresponding Savage score correlation value, then there is higher agreement on the most important model inputs rather than across all inputs; b) the converse is true if the rank correlations are higher than the Savage score ones.

Overall, Table 4 indicates that the important factors for utility are not the same as those affecting other policy relevant outcomes. Among the policy relevant outcomes, the social cost of carbon in 2005 and the optimal carbon tax share the strongest correlation of common drivers of variation, followed by global emissions and atmospheric temperature in 2105.

In general the model inputs can be split into the group of speculative parameters where the value is not empirically known and calculated through projections and the group of inputs that are econometric in nature and depend on statistical analysis. It is common to take the econometric group of model inputs as given and instead focus on the speculative inputs when performing sensitivity or scenario analysis. However, we have shown that uncertainty in both types of inputs proves to be influential in affecting model outcomes. Thus, global SA should be performed considering all model inputs, before deeming a set of inputs as influential, since assessing key drivers without an extensive quantitative analysis might lead to misleading conclusions.

We also post-processed the data to obtain information on direction of change and interactions when all 51 inputs vary. The HDMR images when all model inputs are remain similar to those in Figures 2 and 3 where first order effects are monotonic and second order interaction effects are significant and non-monotonic (The images are available from the authors upon request).

6 CONCLUSIONS

This paper has demonstrated the usefulness of global sensitivity analysis methods in the area of integrated assessment modeling for climate change economics. It has shown that at the same computational cost of a standard uncertainty analysis one can obtain robust insights on direction of change, model structure (interactions) and key uncertainty drivers by applying recently developed methods. These insights provide analysts with a deeper understanding of a model's behavior and allow them to robustly identify the factors on which to focus additional data collection.

A further advantage of the methods proposed and described in this paper, in that significant interactions can be identified explicitly, rather than simply acknowledged or speculated upon, and the direction of the interactive effect can be observed. This simple comparative exercise is an indicator of the potential advantages of using global SA methods over simplified approaches, especially because information is extracted at no additional cost than the one of Monte Carlo simulation.

We have discussed both numerical and methodological aspects of the approach using DICE, one of the most popular models for climate change policy analysis. The results show that uncertainty in the elasticity of the marginal utility of consumption, which influences the discount rate applied, is by far the most influential parameter in affecting the dependent variable in the objective function of the model. The key uncertainty drivers have been also identified with respect to more pragmatic policy relevant model outputs. Differences in ranking of inputs with respect to the model outputs have been analyzed.

The results of this paper highlight the merits of performing global sensitivity analysis alongside other types of scenario analysis to explore different outcomes given different parameter values in the model. The most highly visible recent analysis of IAMs in the climate change literature revolved around what type of scenario should be considered as a most reasonable informer for policy. The authors of the Stern Review claim that a scenario with low discount rates and strong inter-generational equity is the correct basis, while others avoid the 'normative' discussions of discount rates, using instead observable market rates of return and arriving at much different conclusions and policy recommendations. While this highlights the usefulness of varied modeling strategies for different policy or scientific questions, our exercise has shown the benefits of using global sensitivity analysis methods since the two approaches are not interchangeable and important information can be taken from both. Lastly, global sensitivity analysis along the lines presented here could be fruitfully conducted on other classes of models routinely used in climate change policy analysis, from computable general equilibrium models for impact assessment to energy system techno-economic models.

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7 APPENDIX A: DICE MODEL INPUTS AND THEIR RANKINGS

Table A1: DICE Model Inputs

SA Number	Parameter	Description
1	A0	Initial level of total factor productivity
2	A1	Damage intercept
3	A2	Damage quadratic term
4	A3	Damage exponent
5	BACKRAT	Ratio initial to final backstop cost
6	ELASMU	Elasticity of marginal utility of consumption
7	B_PRSTP	Initial rate of social time preference per year
8	C1	Climate-equation coefficient for upper level
9	C3	Transfer coefficient upper to lower stratum
10	C4	Transfer coeffic for lower level
11	DELA	Decline rate of technological change per decade
12	DK	Depreciation rate on capital per year
13	DPARTFRACT	Decline rate of participation
14	DSIG	Decline rate of decarbonization per decade
15	DSIG2	Quadratic term in decarbonization
16	ELAND0	Carbon emissions from land 2005(GtC per decade)
17	EXPCOST2	Exponent of control cost function
18	FCO22X	Estimated forcings of equilibrium co2 doubling
19	FEX0	Estimate of 2000 forcings of non-CO2 GHG
20	FEX1	Estimate of 2100 forcings of non-CO2 GHG
21	FOSSLIM	Maximum cumulative extraction fossil fuels
22	GA0	Initial growth rate for technology per decade
23	GAMA	Capital elasticity in production function
24	GBACK	Initial cost decline backstop pc per decade
25	GPOP0	Growth rate of population per decade
26	GSIGMA	Initial growth of sigma per decade
27	K0	2005 value capital trill 2005 US dollars
28	LIMMIU	Upper limit on control rate
29	MAT2000	Concentration in atmosphere 2005 (GtC)
30	ML2000	Concentration in lower strata 2005 (GtC)
31	MU2000	Concentration in upper strata 2005 (GtC)
32	PARTFRACT1	Fraction of emissions under control regime 2005
33	PARTFRACT2	Fraction of emissions under control regime 2015
34	PARTFRACT21	Fraction of emissions under control regime 2205
35	PBACK	Cost of backstop 2005 per tC 2005
36	POP0	2005 world population millions
37	POPASYM	Asymptotic population
38	Q0	2005 world gross output trillion 2005 US dollars
39	SIG0	CO2-equivalent emissions-GNP ratio 2005
40	T2XCO2	Equilibrium temperature impact of CO2 doubling C
41	TATM0	2000 atmospheric temperature change (C) from 1900
42	TOCEAN0	2000 lower stratospheric temperature change (C) from 1900
43	b11	Carbon cycle transition matrix
44	b12	Carbon cycle transition matrix
45	b21	Carbon cycle transition matrix
46	b22	Carbon cycle transition matrix
47	b23	Carbon cycle transition matrix
48	b32	Carbon cycle transition matrix
49	b33	Carbon cycle transition matrix
50	scale1	Scaling coefficient in the objective function
51	scale2	Scaling coefficient in the objective function

Table A2: All Delta Rankings

Model Input	Atmospheric Temperature	Social Cost of Carbon 2005	Carbon Tax	Global Emissions	Utility
A0	1	6	3	5	4
A1	43	43	43	43	43
A2	22	5	6	10	42
A3	8	2	1	1	10
BACKRAT	34	20	27	24	12
ELASMU	19	4	4	11	1
B_PRSTP	29	9	10	15	6
C1	18	8	9	14	23
C3	23	27	20	19	27
C4	30	17	18	28	25
DELA	37	39	40	31	22
DK	24	33	33	38	31
DPARTFRACT	43	43	43	43	43
DSIG	40	37	31	26	32
DSIG2	43	43	43	43	43
ELAND0	39	31	42	42	15
EXPCOST2	5	21	19	4	21
FCO22X	15	7	7	13	17
FEX0	41	38	35	35	41
FEX1	33	30	41	39	24
FOSSLIM	42	26	30	34	13
GA0	7	11	11	3	7
GAMA	4	1	2	9	3
GBACK	36	29	25	25	9
GPOP0	28	25	24	32	39
GSIGMA	16	36	26	6	38
K0	31	10	16	40	8
LIMMIU	43	43	43	43	43
MAT2000	9	19	15	29	40
ML2000	21	35	39	41	30
MU2000	6	28	23	27	29
PARTFRACT1	43	43	43	43	43
PARTFRACT2	43	43	43	43	43
PARTFRACT21	43	43	43	43	43
PBACK	17	32	29	12	16
POP0	20	23	28	30	11
POPASYM	10	12	8	8	5
Q0	38	24	32	36	26
SIG0	3	22	21	2	28
T2XCO2	2	3	5	7	14
TATM0	35	34	34	33	36
TOCEAN0	32	18	22	37	37
b11	12	13	12	21	18
b12	12	13	12	21	18
b21	12	13	12	21	18
b22	11	16	17	20	2
b23	26	40	36	16	34
b32	26	40	36	16	34
b33	25	42	38	18	33
scale1	43	43	43	43	43
scale2	43	43	43	43	43