# E Pluribus Unum

Marco LiCalzi

## 1 Introduction

*E pluribus unum* is Latin for "Out of many, one". This sentence is best known as one of three shown on the Seal of the United States, that was adopted by an Act of Congress in 1782. It appears on the obverse side of the seal, as well as on official documents and U.S. currency. Considered since long the unofficial motto of the United States (whereas the official motto since 1956 is "In God we trust"), it was originally conceived to represent the idea that a single nation would emerge out of many states.

**Fig. 1** The obverse side of the Seal of the United States

The symbolism in the seal is reinforced by a recurring motif that honors the original thirteen States in the Union with thirteen stars in the "glory" above the eagle's head, thirteen stripes on the shield, and thirteen arrows in the eagle's talon; moreover, custom has added thirteen leaves and olives on the olive branch. Numerologists will take pride in noting that the motto itself consists of thirteen letters, although this seems coincidental.

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The main theme of this article is the exploration of situations where a myriad of interactions between different people leads to the emergence of a (possibly unexpected) aggregate behavior. The best known example is the metaphor of the *invisible hand* first coined by the economist Adam Smith (1723–1790) to describe the social mechanism of the market, where the individual preferences are composed and the needs of the society are met even if each single agent is only attending his own business. However, as economists know well, such harmony is difficult to attain in practice. When dealing with human affairs, we should perhaps focus on a more neutral expression. The title of this article is my best contribution in this respect.

I would be remiss in failing to disclose that there is at least one contender that trumps the outcome of my efforts. This favorite of mine is the brilliant title of a book by Thomas C. Schelling (born 1921). He is a professor of foreign affairs who was awarded the 2005 Nobel Memorial Prize in Economic Sciences (shared with Robert Aumann) for "having enhanced our understanding of conflict and cooperation through game-theory analysis." His crowning achievement in this domain is the monograph *The Strategy of Conflict*, published in 1960, where he introduced key concepts in the analysis of conflict such as focal point and credible commitment; see [Mye09]. But the title I envy is *Micromotives and Macrobehavior*, appeared in 1978, where by a stroke of genius he let many individual heterogenous motivations adjoin the aggregate behavior of the system.

A simple example may help to illustrate how a multitude of local rules coalesce into a single aggregate behavior, sometimes in unexpected ways [Bon02]. There is a score of people in a closed space, such as a gymnasium. Consider two slightly different variants of a simple game. The first game has each person threatened by an attacker and protected by a defender: this agent moves around trying to keep the defender between the attacker and himself. The second game has each defender trying to interpose herself between the victim and the attacker. For clarity, we may dub "seek protection" the first version and "provide protection" the second version.

If we let people interact, what kind of aggregate behavior shall we expect to observe? As it turns out, "seek protection" tends to keep agents spread out and running around quite a lot; while "provide protection" gather a pretty tight crowd where people move very little. An elegant visualization is accessible at http://www.icosystem.com/demos/thegame.htm. (If a picture is worth a thousand words, a short movie must count as a million.) The point of the example is that, in spite of the remarkable similarity among the rules of the two games, they generate a completely different macrobehavior.

Examples of emergent macrobehavior abound. Closer to home (unless you live in Venice) is the case of traffic behavior. Each of the single drivers is pursuing a private objective relying on his own personal heuristics. At the macro level we observe a variety of behaviors, ranging from a smooth flow to inextricable traffic jams. For more examples, we can appropriate the blurb in the poster advertising *Unraveling Complex Systems* as the theme of the *2011 Mathematics Awareness Month*:

> We are surrounded by complex systems. Familiar examples include power grids, transportation systems, financial markets, the Internet, and structures underlying everything from the environment to the cells in our bodies. Mathematics and statistics can guide us in un-

> derstanding these systems, enhancing their reliability, and improving their performance. Mathematical models can help uncover common principles that underlie the spontaneous organization, called emergent behavior, of flocks of birds, schools of fish, self-assembling materials, social networks, and other systems made up of interacting agents.

The Mathematics Awareness Month is sponsored each year by the Joint Policy Board for Mathematics, a collaborative effort of the American Mathematical Society, the American Statistical Association, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics; see www.mathaware.org.

A final word of caution is in order. Philosophers talk about *explanandum* and *explanans*. The first is the phenomenon that needs to be explained, and the second is the explanation of that phenomenon. The study of complex systems is particularly good at generating emergent behaviors (*explanandum*), but somewhat less effective in providing the *explanans*. You may spot similar limitations in this article.

## 2 Traffic management

How do we get traffic queues? An obvious answer is that sometimes there are simply too many vehicles with respect to the capacity of the existing road network. Another recurring suspect are traffic lights, that may engender queues when their timing is not attuned to the traffic flow. Accidents, or other catastrophic events, are a third likely cause.

Some queues, however, originate in highways in the absence of any of the factors above. Imagine a smooth flow of cars moving at constant average speed over a highway. Each car travels just a few meters away from the preceding one. When, by some accidental event, the vehicle in the $n$-th position suddenly slows down, it forces the subsequent car $n+1$ to do the same. The deceleration is quite more rapid than the acceleration, so it takes some time for each vehicle to get back to the average speed.

Looking from an imaginary helicopter hovering above the traffic, we see the gap between $n-1$ and $n$ growing wider, because car $n-1$ keeps its speed while car $n$ is braking up. Meanwhile, car $n+1$ is forced to slow down to avoid collision with $n$ so the gap between these two vehicles closes, and we see $n+1$ queuing behind $n$. By induction, this generates a traffic wave moving upstream. The car in front of the wave can accelerate and move forward, while the subsequent vehicles must await until the space in front is cleared. The queue starts traveling backwards. Depending on the parameters, this may generate surprisingly long queues. Assume that car $n$ has braked because a butterfly disturbed its driver, and the "butterfly effect" takes a new meaning. For a visual animation, we recommend http://www.traffic-simulation.de, where one can also explore similar queueing phenomena due to incoming traffic from a ramp, or to a uphill road that slows down trucks, or to closing one lane in a two-lane highway.

A rather different phenomenon is known as *Braess' Paradox*. We illustrate it with a simple example borrowed from [EK10]. Consider the road network in Fig. 2,

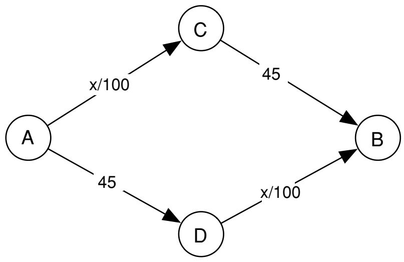


**Fig. 2** A simple road network

where each arc is labeled with the travel time (in minutes) that it takes to go from one end to the other when there are *x* cars using it. There are 4000 cars that start at A and must reach B. Each driver simultaneously chooses whether to drive through C or D, in a conscious attempt to minimize his traveling time. We say that the traffic is *at an equilibrium* when, given the distribution of vehicles over the available routes, no driver can unilaterally switch to a different path and save time.

For instance, if all drivers choose to go through C, there are $x$ =4000 cars on the arcs A-C and C-B. Each car needs 40 minutes to go from A to C and a further 45' from C to B, so the total driving time for a vehicle is 95'. Facing this situation, any single driver would rather switch to the other (empty) route where the total traveling time for one car is $45 + (1/100) = 45.01$ minutes. Hence, everybody on the same route is not an equilibrium. As symmetry suggests, it turns out the only equilibrium for this network is that cars distribute equally over the two routes. When there are 2000 vehicles on the A-C-B route and another 2000 on A-D-B, the driving time for each car is $45 + 2000/100 = 65'$; a unilateral switch to the other route would give a higher traveling time of $45 + 100/101 = 45.01$. Thus, given the traffic network in Fig. 2, we expect people to learn to split over the two routes so that each driver takes 65' to complete his route.

Suppose that some well-meaning politician, bent on improving the miserable commuting times of the local community, decides to build a new highway between C and D. To keep things simple (without loss of generality), we assume that the new highway is one-way from C to D and that it is so large and capacious that the time to cover it is zero regardless of the number of vehicles using it. We can represent the new situation by drawing a new edge from C to D and label it zero in Fig. 2. The new network now offers three ways to go from A to B: the two old routes (A-C-B and A-D-B) and the new one (A-C-D-B).

Table 1 compares a few possible configurations. For instance, the second line looks at the situation where 500 cars take the old route through C, another 500 go through the old route through D, and the remaining 3000 drive through the new highway. The last two columns report that the driving time over the old routes is $3500/100 + 45 = 80'$, while the new route takes only $3500/100 + 3500/100 = 70'$. As it turns out, in each of the examples shown (and, indeed, in any possible configuration), going through the highway is always faster than using any of the old routes.

This is not surprising: the whole purpose of opening the new road is to lower the driving times.

**Table 1** Driving times over different traffic configurations

| *Traffic on route* | | | *Driving time* | |
|---|---|---|---|---|
| *through C* | *through D* | *through CD* | *through C or D* | *through CD* |
| **0** | **0** | **4000** | 85 | **80** |
| 500 | 500 | 3000 | 80 | 70 |
| 1000 | 1000 | 2000 | 75 | 60 |
| 1500 | 1500 | 1000 | 70 | 50 |
| 2000 | 2000 | 0 | 65 | 40 |

The paradox shows up when we consider the consequences of this uniform superiority. Regardless of the traffic configuration, taking the highway is always better than trying any of the old routes. Hence, any driver ends up using the highway and all 4000 cars will be traveling on the same route. As shown in the first line of Table 1, this implies that in equilibrium the driving time is 80' for everybody. Yet, in the good old times (without the highway), the unique equilibrium had a traveling time of 65' — a whole 15' less! Paradoxically, opening up the highway makes everybody worse off. The emergence of this paradox in a general network is reasonably likely [SZ83]; see also Wikipedia for a few real-life examples. A recent result is that, if the driving time over any arc is a linear function of the number of cars using it, the worst-case scenario for a network upgrade is to have the equilibrium driving time going up by at most 33% [RT02].

The morale is that some actions may have unintended consequences if we ignore the collateral effects on individual behavior. Building the highway upgrades the infrastructure. Considered in isolation, this is an improvement. However, when the rest of the network is left unchanged, this local upgrade attracts so many drivers that it ends up clogging the old routes A-C and D-B. These routes easily suffer from congestion and thus the performance of the system degrades. Such collateral effects are called *externalities* by economists. They are wonderfully illustrated by a cartoon (ubiquitous on blogs lamenting traffic jams) where each of the drivers stuck in a huge traffic jam is thinking aloud: *If these idiots would just take the bus, I could be home by now.*

Our last foray in traffic management is borrowed from *Micromotives and Macrobehavior*:

> Standing in line at a ski lift — a long line — I overheard somebody complain that the chairs ought to go faster. It would take a bigger engine, but at those fees the management could afford one. The complaint deserves sympathy but the proposal doesn't work: speeding the lift makes the lines longer. (p. 67)

How so? Let us work out an example. There are 20 skiers in the field. It takes one minute to get in and out of the lift, ten minutes to go up, and five to get down.

Ignoring for simplicity other activities such as sipping coffee in the adjacent hut, skiers must be engaged in one of these three activities, or otherwise waiting in line. Since each of the 20 skiers needs one minute to get in and out of the lift, a full cycle that brings all skiers up lasts 20 minutes. During this time, a skier spends 1' to get in and out, 10' to go up, 5' to get down, and the remaining $20 - 16 = 4$ minutes standing in line. The queue itself has on average 4 people.

Let us see what happens if the management brings in new engines and the time to go up is halved. It still takes 20 minutes to complete a full cycle and get all skiers on board. But now a skier spends 1' to get in and out, 5' to go up, 5' to get down, and the remaining $20 - 11 = 9$ minutes standing in line. The new engines roar aloud and do their job, but there are now nine people standing in line and grumbling. Where do the extra five skiers waiting in line come from? Before the engines were installed, they used to be on the lift moving at a slower speed and enjoying the view (instead of suffering the ignominy of queueing). The reader may check that the paradox of faster lifts leading to longer queues arises when the number of skiers in the field is $n \geq 11$.

## 3 Family issues

Wedding customs vary greatly, but a piece of the western tradition seems to me a little odd. While two people are joined in marriage at the front, families and friends usually stand (or sit) behind them in two separate groups. This form of segregation during the ceremony is often imposed by seating arrangements. It may be explained in many ways, often amenable to the symbolism of wedding as an alliance between two families.

However, somewhat mysteriously, a similar segregation takes place *spontaneously* when the wedding reception is organized as a standing event where people can flow freely. Dreaded by many brides, this segregation occurs in spite of their best efforts to have people from the groom's and bride's families mix. People gather together in small groups and recombine, but it is often the case that most groups see a prevalence of one side. Why?

An important piece of the explanation is that even a tiny preference for one's neighbors to be from one's own family may lead to segregation. It is agents' preference to congregate with their relatives that tends to keep the two groups apart. This simple argument was suggested in 1969 by Schelling [Sch69] in a context inspired by racial dynamics. His argument was developed using a physical model with coins placed on graph paper, where dimes and pennies were moved around simulating people's inclinations. This approach brilliantly predates the rich computer simulations nowadays abundant in the social sciences, usually known as agent-based models. Let us illustrate Schelling's argument.

There are two kinds of agents, labelled X and O, who represent the bride's and the groom's relatives. They can move around in the reception hall, that we visualize as a grid. Each agent takes place in a cell of the grid and interacts with his immediate neighbors. For instance, the left-hand side of Table 2 depicts a grid with $6 \times 6 = 36$ cells and 11 agents for each family.

**Table 2** A configuration of agents in a grid

| | | | | | |
|---|---|---|---|---|---|
| X | X | | | | |
| X | $O_1$ | | O | | |
| X | X | O | O | O | |
| X | O | | | X | X |
| | O | O | $X_2$ | X | X |
| | | O | O | $O_3$ | |

| | | | | | |
|---|---|---|---|---|---|
| X | X | O | O | X | X |
| X | X | O | O | X | X |
| O | O | X | X | O | O |
| O | O | X | X | O | O |
| X | X | O | O | X | X |
| X | X | O | O | X | X |

The immediate neighbors of an agent are those in the eight cells around him (or less, if he is adjacent to the walls). We assume that an agent feels comfortable when his immediate neighborhood contains at least 30% people from his family. Anyone who has experienced the discomfort of being the "odd man out" in a small group entertaining a conversation should be able to relate to this assumption.

If the current arrangement makes him uncomfortable, an agent moves to another cell in the grid. For instance, starting from the situation depicted on the left of Table 2, only three agents ($O_1$, $X_2$, $O_3$) are uncomfortable and will move to a different position. Doing so, they create a new configuration where perhaps other agents feel uncomfortable and in turn decide to move. The dynamics may takes time to unfold, but in the end agents' choices lead to a stable configuration. This represents the underlying macrobehavior emerging from agents' micromotives.

Note that agents are not prejudiced against the other group: they need not be part of a majority to feel comfortable. With eight neighbors, having three of one's own relatives around is enough to stay on. Such individual preferences are compatible with perfect integration, as shown on the right-hand side of Table 2. However, when guests are randomly distributed, a different macrobehavior emerges.

This is illustrated in Fig. 3, where agents smile if they feel comfortable and frown otherwise. The initial configuration on the left is mixed up. The dynamics eventually leads to a stable configuration such as that one on the right, where a clear amount of segregation has taken place. The figures are created using the simulation tool available at http://www.rensecorten.dds.nl. This is in turn based on NetLogo, a multi-agent programmable modeling environment widely used by students, teachers and researchers worldwide. Freely accessible at http://ccl.northwestern.edu/netlogo, it offers a library with a large variety of simulative examples, ranging from biology to social sciences.

Our next example is drawn from [ML75] and takes place after marriage has been consummated. It illustrates the effect of introducing a bit of variability in individual behaviors. Suppose that all parental couples have an innate preference for males, and that they keep bearing children until their progeny has more boys than girls. The baseline case assumes that each couple has the same probability $p = 1/2$ of giving birth to a male or a female. Since each family stops growing when boys are

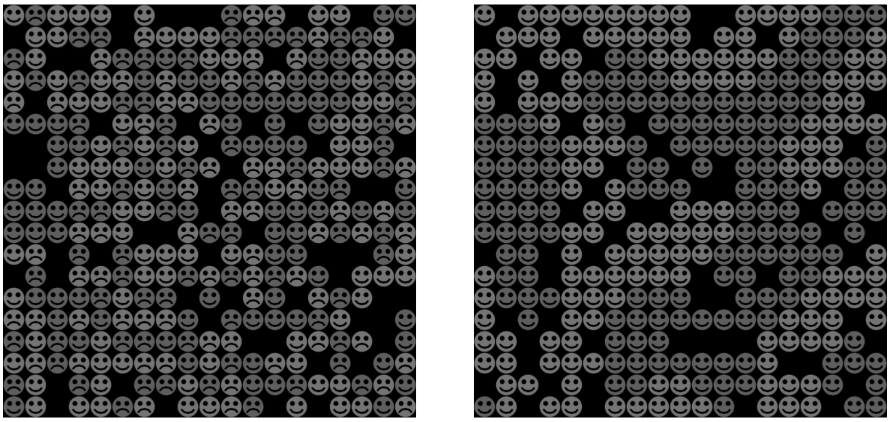

**Fig. 3** Initial (left) and final (right) configurations

a majority, it is not surprising that this leads to a situation where males are more numerous than females.

Consider now the more plausible assumption that each couple $k$ has a different probability $p_k$ of giving birth to a boy, although on average the value of $p_k$ is still $1/2$. For instance, if a couple has probability $(1/2-\varepsilon)$, there is another one for whom the probability is $(1/2+\varepsilon)$. Anything else is unchanged: each couple keeps procreating until the majority of its progeny is male. The following paradoxical effect emerges: while each family is actively seeking a majority of boys, the society as a whole end up with a majority of girls!

How is it possible? The exact argument requires some familiarity with random walks, but here is an intuitive explanation. Families with a propensity to generate boys ($p_k > 1/2$) tend to reach their target of a male majority quickly, and thus stop procreating soon. They generate more boys than girls, but the absolute numbers are small. On the other hand, families with a propensity to generate females keep chasing their target of a male majority and thus end up procreating a lot more girls. The tiny majority of boys generated by the male-biased families is rapidly overwhelmed by the large numbers of girls born in female-biased families. (My favorite simulation of this model using NetLogo is available at http://www.agsm.edu.au/bobm/teaching.)



## 4 Odds and ends

Our last section parades a few vignettes describing a wide range of applications amenable to the study of emergent behavior in the social sciences; see [Eps07] for more examples.

The Culture Model [Axe97] is a seminal study on how beliefs (or attitudes) in a population shift over time, getting closer or diverging. It has been used to explain how opinions get spatially clustered, the emergence of bandwagon effects (fashion fads), and the spontaneous division of a culture into sub-cultures. A recent application discusses how knowledgeable people can use individuals with wide social networks to disseminate information quickly and effectively, in a conscious attempt to induce “social epidemics” such as new political aggregations or outbursts of moral outrage on the internet. A curious spinoff of this line of research has looked into modeling standing ovations, when people from an audience spontaneously stand to acknowledge an outstanding performance [MP04].

A well-known, albeit controversial, foray into archeology looks at the rise and fall of the Anasazi civilization in the southwestern United States. Until its disappearance around AD 1350, the Anasazi society widely fluctuated in population and settlement size. The study combines quantitative information on environmental fluctuations with plausible behavior rules for Anasazi households and computes a detailed historical “trajectory” that matches the known facts.

Other models purport to enucleate a few key driving elements in political action. An elegant example is a study of civil violence [Eps07] considering two models. The first one illustrates how a subjugated population seemingly coordinates its rebellion against a central authority; see Rebellion in the NetLogo library for a visual animation. The reason why a repressive regime stays in place is not that his police is stronger than the people, but that the former can coordinate much better than the latter. Fiddling with its parameters, one is reminded of Tocqueville’s dictum: “It is not always when things are going from bad to worse that revolutions break out. On the contrary, it oftener happens that when a people which has put up with an oppressive rule over a long period, it takes up arms against it.” (*The Ancien Régime and the French Revolution.*) The second model describes the dynamics of inter-group violence: the analogies with recent historical examples (Hutu vs. Tutsi, or Serbs vs. Bosniaks) where local ethnic cleansing led to genocide are impressive.

A promising application for the study of emergent behavior is panic control, where one attempts to design solutions that reduce the risk associated with orderly crowds suddenly switching behavior due to panic [H00]. (For a recent example of poor design, recall the Love Parade stampede in July 2010, where 21 people lost their lives and more than 500 were injured.) Space prevents us from a longer discussion, but more information including visual animations is available at http://angel.elte.hu/ panic/.

We close with a puzzle lifted from Schelling’s *Micromotives and Macrobehaviors*. At a conference, it is often the case that seats in the first few rows are empty. Can you figure out why?



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