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US dollar/Euro exchange rate: a monthly econometric model for forecasting

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The intent of this paper is the construction of an econometric model able to produce reliable and reasonable forecasts for the US dollar/Euro real exchange rate. In order to achieve this aim, an area-wide model is analysed. The aggregation is motivated by the fact that the Euro-zone is under a single monetary policy. Furthermore, a more parsimonious parametric model enables one to consider an important source of non-stationarity given by the presence of structural breaks using the multivariate cointegration analysis. Against the Meese–Rogoff critique, the out-of-sample one-step-ahead forecasts using actual values of the exogenous produced by the estimated VECM are reasonably satisfactory.

Keywords: real exchange rates, cointegration, structural breaks, area-wide model, forecasting

1. INTRODUCTION

In this paper an econometric model of the US dollar/Euro real exchange rate is constructed for forecasting purposes. The real, rather than the nominal exchange rate, is motivated by the failure (on empirical grounds) of purchasing power parity (PPP), which states that long-run equilibrium exists between exchange rates and price levels.

The economic model is built considering the simultaneous equilibrium of exchange, money and goods markets. Therefore we take into account the joint behaviour of bilateral exchange rate, interest rate and growth rate differentials. This approach mimics the well-known Mundell–Fleming model (for an exhaustive discussion of this model see Frenkel and Razin, 1987).

In order to have better forecasts, a decision must first be made regarding the level of geographical aggregation of the economic area. Here we choose an area-wide model, which is partially motivated by the fact that the Euro-zone is under a single monetary policy. Furthermore, the more parsimonious parametric model obtained, with respect to a multi-country version, enables one to consider an important source of non-stationarity given by the presence of

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structural breaks. This particular aspect constitutes an important methodological issue, which has appeared in very recent literature on multivariate cointegration analysis.

The main results of the paper are the specification and estimation of an econometric model on US dollar/Euro real exchange rate in VECM form. Its admissibility is achieved by using cointegration tests in the presence of structural breaks. A break is introduced into the sample period (between January 1990 and December 1999) in coincidence with the crisis of the European Monetary System (September 1992). The presence of different deterministic trends in the two subsample periods is accepted and three long-run relationships are obtained. The estimates of the coefficients in the long-run relationships are consistent with the interactions suggested by economic theory.

Every time a structural economic model is built on financial market data, the Meese and Rogoff critique must be taken into consideration (Meese and Rogoff, 1983). We therefore compare the forecasting performances of the estimated econometric model with the simplest random walk forecasts as crucial benchmarks.

Here, against this critique, the out-of-sample one-step-ahead forecasts generated by the estimated VECM, by using actual values of the exogenous variables, are found reasonably satisfactory as is shown by the main statistic performance indicators.

In Section 2 the general motivation for the choice of a real exchange rate model is given; in Section 3 the US dollar/Euro real exchange rate model is presented by explaining in detail the equations for the three main markets considered, Section 4 gives the definition of the variables and the sample data used for estimating the model; in Section 5 the cointegration analysis in presence of structural breaks is explained and the empirical results are shown in Section 6; conclusions follow in Section 7.

2. MOTIVATION FOR US DOLLAR/EURO REAL EXCHANGE RATE MODEL

An important motivation in favour of the real, rather than the nominal exchange rate, is the failure (on empirical grounds) of the purchasing power parity (PPP), which states the long-run equilibrium between the exchange rates and the price levels.

Suppose S_t to be the exchange rate US dollar/Euro (price of one unit of Euro in term of US dollars) and P_t the one country's price level, then the PPP relationship is:

$$S_t = P_{\$t}/P_{€t} \tag{1}$$

or more generally:

$$S_t = Q_t \, P_{\$t}/P_{€t} \tag{2}$$

where Q_t is the US dollar/Euro real exchange rate supposed constant $\forall t$. An increment of the US inflation rate (versus that of the Euro Area) is followed by an increase of S_t, that is a depreciation of US dollar. The assumption of Q_t to be constant implies that the nominal exchange rate obeys equation (2) when monetary shocks occur. Should Q_t not be constant, as in the case of real shocks (e.g. oil shocks, productivity gaps between the two areas, etc.), then obviously the PPP relationship is no longer valid.

From equation (2) we obtain:

$$Q_t = S_t \, P_{€t}/P_{\$t} \tag{3}$$

Using the log transform, equation (3) becomes:

$$q_t = s_t + p_{€t} - p_{\$t} \tag{4}$$

where lower case notation indicates the logarithmic transformation of the variables in level. Here, an increase in q_t means a depreciation of the real US dollar followed by a depreciation of the nominal US dollar or a decrease of the US and Euro inflation rate differentials.

The *uncovered interest parity condition* (UIP) states the long-run equilibrium between the money market and the foreign exchange market, that is:

$$E_t \Delta s_{t+k} = i_{\$t} - i_{€t} \tag{5}$$

where i_t is the one country nominal interest rate and $E_t \, \Delta s_{t+k}$ is the exchange rate expected depreciation for the time $t+k$. Subtracting from both sides of equation (5) the expected inflation rate differential we obtain:

$$E_t \Delta s_{t+k} - E_t \Delta p_{\$t+k} + E_t \Delta p_{€t+k} = (i_{\$t} - E_t \Delta p_{\$t+k}) - (i_{€t} - E_t \Delta p_{€t+k}) \tag{6}$$

Using equation (4), we obtain:

$$E_t \Delta q_{t+k} = r_{\$t} - r_{€t} \tag{7}$$

where r_{jt} is the real interest rate for the jth country, with $j = \$, €$, given by:

$$r_{jt} = i_{jt} - E_t \Delta p_{jt+k} \tag{8}$$

Knowing that $E_t \Delta q_{t+k} = E_t \, q_{t+k} - q_t$, then equation (7) becomes:

$$q_t = E_t q_{t+k} - (r_{\$t} - r_{€t}) \tag{9}$$

In formula (9), we indicate the unknown $E_t q_{t+k}$, as q_t, which is known as *fundamentals exclusive of the real interest differential* (*FERID*) and is driven by fundamentals, such as productivity variables (e.g. the ratio of tradable to non-tradable goods), which should be able to capture the so-called Balassa–Samuelson effect, commodity shocks (such as the real price of oil and relative terms of trade) and budget policy (such as fiscal budget surplus or deficit and net foreign assets).

The difference $(r_{\$t} - r_{€t})$ in formula (9) is usually known as the *real interest differential* (*RID*) and it is modelled in this paper as the real long-term interest rate differential (*RRL*); therefore equation (9) can lead to the foreign exchange market relationship written in the subsequent section.

3. THE ECONOMIC MODEL

The economic model is built considering the simultaneous equalibrium of exchange, money and goods markets. We therefore take into account the joint behaviour of bilateral exchange rate, interest rate differential and growth rate differential. This approach imitates the well-known Mundell-Fleming model.

3.1 The foreign exchange market

Given the previous considerations, the real foreign exchange rate's equilibrium behaviour is described by the uncovered interest parity condition (equation (5)). Based on this condition, the real exchange rate depends on the expected real exchange rate and on the real interest rate differential (equation (9)). The expected real exchange rate, in turn, is affected in our model by the time path of several fundamental variables such as foreign trade efficiency, commodity shocks and budget policy as suggested in the recent works of MacDonald (1997) and MacDonald and Marsh (1999).

In this theoretical framework, the foreign trade efficiency is modelled as the differential between US and Euro ratio of consumer price index to the production price index (noted *LTNT*). This variable should be able to capture the Balassa–Samuelson effect, probably the best known source of systematic changes in the relative price of traded to non-traded goods across countries (Balassa, 1964; Samuelson, 1964). The Balassa–Samuelson theory states that the nominal exchange rate moves to ensure the relative price of traded goods is constant over time. Productivity differences in the production of traded goods across countries, however, usually introduce a bias into the overall real exchange rate, since productivity advances are preferably concentrated in the traded goods sector rather than the non-traded one. The price of tradable goods will rise less rapidly in the country with higher productivity in the tradable sector if the prices of all tradable and non-tradable finished products are linked to wages, which are linked to productivity and across tradable and non-tradable industries. This will cause an increase in the foreign demand for tradable goods produced in such a country (less expensive) and thus to an appreciation of the real exchange rate (a decrease of q_t). The sign we expect for *LTNT* is therefore negative.

The fiscal budget, both in terms of direct expenditure and in terms of net foreign assets (national savings), also affects the equilibrium behaviour of the real exchange rate. The first effect is described, in the MacDonald equation, by the differential between US and Euro ratio of government debt's annual rate of growth to GDP rate of growth (*FBAL*). The latter could be captured by the ratio of US to Euro ratio of net foreign asset to GDP (*NFA*). In this framework, a tight fiscal policy in United States implies, *ceteris paribus*, a decrease of *FBAL* or an increase of *NFA*. Contrary to this model, we do not take into account the *NFA* variable for two main reasons. First, aggregate data for the net foreign assets of the Euro area do not exist and they are not correctly aggregable using the individual European country variables if the information on bilateral net foreign assets among them is not available. Second, as already mentioned, both *NFA* and *FBAL* could interpret the effects of fiscal policy with an antithetic behaviour. Therefore, it is supposed that the information bearing on the fiscal policy, held in *NFA*, could be partially captured by the *FBAL* variable. Once more, in our opinion, *FBAL* is more representative of the fiscal policy interventions and it allows us to consider the country risk.

The effect of fiscal policy on the real exchange rate usually leads to the following question: 'Will a positive fiscal budget strengthen of weaken the external value of a currency?' Unfortunately, there is no one single answer. Following the traditional Mundell–Fleming two-country model, we assume that a tight fiscal policy, which increases the aggregate national savings, would lower the domestic interest rate and generate a permanent real exchange rate depreciation (an increase of q_t).

The commodity markets are the last source of shocks to the real equilibrium exchange rate. In our theoretical framework they are modelled by means of two variables: the differential between domestic and foreign ratio of export unit value to the import unit value (*LTOT*), and the real price of oil (*ROIL*). Changes in the terms of trade usually induce a shock to a country's foreign trade structure, in the sense that this will affect both the foreign demand (increase/decrease) and the domestic production structure (more or less foreign trade driven).

Changes in the real price of oil can also have an effect on the relative price of traded goods, usually through their effect on the terms of trade described above. In comparing a country that is self-sufficient in oil resources with one which needs to import oil, the latter, *ceteris paribus*, will experience a depreciation of its currency *vis-à-vis* that of the former as the price of oil rises. More generally, countries that have at least some oil (and/or other commodities) resources could find their currencies appreciating relative to countries that are net importers of oil (and/or other commodities).

The comparison of US and Euro areas, both prevailingly importers of oil, leaves the sign of *ROIL* uncertain.

Taking these considerations into account, the long-run equilibrium real exchange rate is modelled as follows:

$$Q = h(LTNT, FBAL, LTOT, ROIL, RRL) \tag{10}$$

where Q indicates the US dollar/Euro real exchange rate and *RRL* (see end of Section 1) the 10-year real interest rate differential. The precise form of the function $h(^{\cdot})$ is linear and the signs of the coefficients are derived from the description above and put in system (17).

3.2 The money market

The equation for the long-term real interest rate differential is modelled as follows:

$$RRL = g(MG, Y) \tag{11}$$

where *MG* denotes the differential between the annual growth rate of the US and Euro real money supply and Y denotes the differential between the annual growth rate of the US and Euro GDP.

In contrast with the money market's equilibrium equation, which usually describes the real money supply as a function of both the real 'policy' interest rate and the output growth, it is assumed that central banks fix the money growth target. If this is the case, given the total amount of money supply and the prices level, in which we are not interested since we are modelling the real exchange rate and Y, the only variable determined by the money market is the real interest rate. Therefore money growth can be considered as exogenous, while the markets determine the equilibrium interest rate.

Economic theory tells us that an easy monetary policy, if perceived as permanent and not just a spot increase in the monetary base, usually induces a decrease in the long-term interest rate. In fact, once liquidity has been injected in the system, banks experience the need to invest this new and large amount of liquidity and will be willing to do this even in correspondence of lower interest rates. As to the output, an increase in output levels instead induces a rise in the volume of transactions and therefore in the demand for money, which will resolve in an increase in the level of interest rates.

3.3 The goods market

If we consider the goods market equilibrium described as in the Mundell–Fleming model, then the growth rate in each country is influenced by consumption, investments, public spending and net exports. Given the variables that usually influence these income components, the growth rate in each country depends on interest rate, income expectations, taxation level, public spending and world demand of goods. Concerning the growth rate differential, the dynamic equilibrium of the goods market has been formalized in the following way:

$$Y = f(RRL, LTNT, FBAL, LTOT, ROIL) \tag{12}$$

We consider the long-term interest rate differential (*RRL*) on the basis of the hypothesis that the firms borrow money on such maturity. According to classical economic theory, the impact of a tight monetary policy on the real gross domestic product growth is negative, in the sense that higher interest rate will discourage investments and, therefore, results in lower economic growth.

The ratio of tradable to non-tradable goods prices (*LTNT*) that represents the productivity differential of the two countries, can be interpreted as a proxy of technological progress that affects the income expectations. We argue that an increase in productivity denotes an improved ability to face competition across markets. This will result in an increase in the foreign demand of the country's products and therefore in an increase in the production and finally in the output.

The taxation level and the public spending effects are captured by *FBAL*. It is observed that an easy fiscal policy (increase of *FBAL*), if directed to investments, in the first step should increase the total output, while in the long-run, this fact could be perceived as an obstacle to growth (because of the expected tight policies in the future motivated from debt repayments) and therefore having a negative impact on the latter.

In order to take into account the world demand of goods, we consider the real oil price (*ROIL*) and the terms of trade (*LTOT*). Comparing a country that is self-sufficient in oil resources with one which needs to import oil, an increase in the cost of oil leads to an increase in the output growth of the former.

The theoretical functions (10), (11) and (12) corresponding to the foreign exchange, money and goods markets equilibrium are utilized in the cointegration identification procedure. In fact, we want to verify if the constraints suggested by these functions are able to identify the well-known long-run relationships in a VECM framework (Johansen, 1995).

4. VARIABLES AND DATA DEFINITION

The model described in this work considers monthly data from January 1990 to December 1999, with the last twelve observations (from January to December 1999) used to produce ex-post forecasts. Therefore, the forecasting ability of the model is tested both in terms of evaluating the proximity of forecasted data to the observed ones (root mean square error, mean error, mean absolute error, Theil's U) and in terms of the model's ability to capture signs of the changes in the real US dollar/Euro exchange rate (percentage of signs correctly forecasted). The power of an exchange rate model could be evaluated also looking for the profit indicator from a trading rule. In this work this procedure is not followed because a monthly speculative behaviour seems not plausible. Furthermore, we can assume a strategy in which an operator buys one Euro at time t when the model forecasts an increase of the US dollar/Euro exchange rate from t to $t+1$, and sells Euro at time $t+1$, and vice versa. In this hypothesis, we prefer to use the percentage of signs correctly forecast, which tell us the number of times in which the operator behaviour has been correct.

The variables used in the model are listed in Table 1. The real US dollar/Euro exchange rate (Q) used for this analysis is the logarithm of the synthetic,[1] nominal US dollar/Euro exchange rate minus the differential between the logarithms of the Euro area consumer price index (base 1995 = 100) and the US consumer price index (base 1995 = 100).

Table 1. Description of variables utilized in the model

Variables	Description
Q	Real dollar/Euro exchange rate (logarithm)
RRL	Differential between US and Euro 10-years real interest rate
Y	Differential between US and Euro annual real GDP growth rates (logarithms)
LTNT	Differential between US and Euro ratio of consumer price index to producer price index (logarithms)
FBAL	Differential between US and Euro ratio of annual real public debt growth and the GDP growth
LTOT	Ratio of US to Euro ratio of export unit value to import unit value (logarithms)
ROIL	Real price of oil expressed in US dollar per barrel
MG	Differential between US and Euro annual real M3 growth rates (logarithms)

[1] The synthetic US dollar/Euro nominal exchange rate is the one produced by Warburg Dillon Read.

In order to take into account the well-known Balassa–Samuelson effect, a proxy of the ratio of traded to non-traded prices was built as the ratio of consumer price index to producer price index and we considered the differential between domestic (United States) and foreign logarithms of these ratios (*LTNT*)[2].

In our opinion, the fiscal policy effects are adequately captured by taking into consideration the differential between US and foreign ratios of annual real public debt growth to annual real gross domestic product growth.

Two variables have been used to model the impact of the dynamics of commodity prices on both the gross domestic product growth (*Y*) and the real exchange rate. The first variable is the terms of trade (*LTOT*). This is constructed as the ratio of US export unit value to import unit value as a proportion of the equivalent effective foreign ratio, expressed in logarithms. The second variabe is the real price of oil (*ROIL*), expressed in US dollars per barrel and defined as the ratio of nominal price of crude oil (Brent) to US producer price index.

Money markets are taken into consideration by means of two variables. The real money (*MG*) supplied to the economic system by the central banks of the areas involved in the analysis is represented by the differential between domestic and foreign annual real M3 growth (deflated by subtracting from the nominal growth rate the annual domestic inflation rate). This variable is able to capture the differences between the European Central Bank and the Federal Reserve with regards to the total amount of credit allowed to the system. The second variable used to model money markets is the rel long-term interest rate differential (*RRL*), computed as the differential between the US and European 10-year real interest rate. The real interest rate both for the Euro area and the US are computed as the ratio of nominal 10-year interest rate to a centred 13-month average of the annual inflation rate.[3]

5. THE ECONOMETRIC APPROACH

For the forecasting purposes an econometric model is used, based on the cointegration approach developed in a recent work by Johansen *et al.* (2000), in which the presence of structural breaks is considered. This approach is a slight generalization of the likelihood-based cointegration analysis in vector autoregressive models suggested by Johansen (1988, 1996). There are only few conceptual differences and the major issue for the practitioner is that new asymptotic tables are needed.

The importance of cointegration analysis in the precence of structural breaks relies on the undesired results when these breaks are ignored. In fact, when the series are trend stationary and the trend is a broken trend, if the structural

[2] Where required, seasonal variation has, as usual, been removed using ARIMA methods.

[3] The lack of data for the period between January 2000 and June 2000 has been overcome by the computation of six forecasts for the inflation rate of each country by means of ARIMA class models.

breaks are not considered, the cointegration hypothesis may be rejected. Furthermore, the forecasts using VAR might be better than a VECM, which does not consider structural breaks. On the contrary, if cointegration analysis with structural breaks is performed, VECM forecasts better than both a VAR model as usual and a VECM without structural breaks.[4]

Recent econometric literature has given considerable importance to structural breaks (Clements and Hendry, 1999). Some results regarding structural breaks in the context of univariate autoregressive time series with a unit root are well known. A time series given by stationary fluctuations around a broken constant level is better described by a random walk than a stationary time series (Perron, 1989, 1990; Rappoport and Reichlin, 1989).

Papers in special issues of the *Journal of Business & Economics Statistics*, **10**, 1990 and the *Journal of Econometrics*, **70**, 1996, discuss parameter stability in econometric models assuming known break points. Testing hypotheses for known break points in connection with cointegration testing has been suggested in recent literature by Inoue (1999), and breaks in the cointegration parameter by Kuo (1998), Seo (1998) and Hansen and Johansen (1999).

The idea here is to analyse cointegration in a Gaussian vector autoregressive model with a broken linear trend with *known* break points.

Let X_t, $t = 1, \ldots, T$ be the observed time series and let there be h pre-specified breaks at times $T_1 \leq T_2 \leq \ldots \leq T_h$, where conventionally $T_h = T$. We assume that X_t is a Gaussian VAR of order k in each subsample with the same parameters with the exception of the constant and the trend, that is the deterministic (non-stochastic) components of the multivariate process.

The unrestricted VAR model, in its first difference re-parameterization and in each subsample is:

$$\Delta X_t = (\Pi \ \ \pi_j)\binom{X_{t-1}}{t} + \mu_j + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \varepsilon_t \qquad T_{j-1} + k < t < T_j \tag{13}$$

where Π, π_j, μ_j and Γ_i are p-dimensional matrices or vectors.

If we suppose that Π is a rank reduced matrix and that π_j, for $j = 1, \ldots, h$, is expressible as a linear combination of columns of Π, therefore the matrix $(\Pi \ \pi_j \ \ldots \ \pi_h)$ can be re-written as the product of two matrices. These joined hypotheses also have other favourable consequences:

- in each subsample the deterministic component is linear both for non-stationary and cointegrating relations (Johansen *et al.*, 2000);
- the distribution of the rank cointegration test is not influenced by nuisance parameters (Nielsen and Rahbek, 2000);
- the complete model, composed of h model equations of type (13) can be re-written in a more compact form.

Formally these hypotheses are:

[4] The estimation and forecasting output, in the latter case, are available upon request.

$$\text{rank}(\Pi\ \pi_1 \ldots \pi_h) \leq r \text{ or } (\Pi\ \pi_1 \ldots \pi_h) = \alpha \begin{pmatrix} \beta \\ \gamma_1 \\ \vdots \\ \gamma_h \end{pmatrix} \tag{14}$$

where the parameters vary freely so that α and β are of dimension $(p \times r)$ and γ_j is of dimension $(1 \times r)$.

Other hypotheses are nested in inequality (14), e.g. one of no linear trend but a broken constant level or one of common trends with a broken linear trend while the cointegrating relation has a broken constant level. For example, in the latter case, the stated assumption also regards the deterministic rather than the stochastic component only, as happens in relation (14). Moreover, these alternative hypotheses are less attractive mainly for the reason that the asymptotic analysis is heavily burdened with nuisance parameters, as demonstrated by Nielsen and Rahbek (2000).

As anticipated, under the hypothesis (14) the h model equation of type (13) can be re-written in a more compact way for $t = k + 1, \ldots, T$ as:

$$\Delta X_t = \alpha(\beta' \ \gamma') \begin{pmatrix} X_{t-1} \\ tE_t \end{pmatrix} + \mu E_t + \sum_{i=1}^{k-1} \Gamma \Delta X_{t-i} + \sum_{i=1}^{k} \sum_{j=2}^{q} \kappa_{j,i} D_{j,t-i} + \varepsilon_t \tag{15}$$

with

$$D_{j,t} = \begin{cases} 1 & \text{for } t = T_{j-1} \\ 0 & \text{otherwise} \end{cases} \qquad j = 1, \ldots, h$$

which is an indicator function for the ith observation in the jth period and E_t is an h-dimensional vector whose j-element is:

$$E_{j,t} = \sum_{i=k+1}^{T_j - T_{j-1}} D_{j,t-i} = \begin{cases} 1 & \text{for } T_{j-1} + k + 1 \leq t \leq T_j \\ 0 & \text{otherwise} \end{cases} \qquad j = 1, \ldots, h$$

The dummy parameters $\kappa_{j,i}$ are p-vectors and the observations $X_1, \ldots, X_k$ are held fixed as initial observations. Note that the dummy variables $D_{j,t}, \ldots, D_{j,t-k}$ correspond to the observations $X_{Tj+1}, \ldots, X_{Tj+k}$ which are held fixed above and have the role to exclude such observations from the analysis.

The hypothesis (14) can be verified using the trace test suggested by Johansen (1996). More precisely, all the asymptotic results are given by Johansen *et al.* (2000).

In the presence of structural breaks, the asymptotic distribution of the trace test is not affected by nuisance parameters because it does not depend on the parameters of the model (15) (asymptotic similarity of the test). It depends only on the $(p-r)$ dimensions, i.e. the number of non-stationary relations, $\Delta v_i = v_i - v_{i-1}$ of the relative break points $v_j = T_j/T$, and the relative length of the sample periods Δv_i, not on their ordering.

Moreover, denoting the asymptotic distribution by $DF_h(p-r, \Delta v_1, \ldots, \Delta v_h)$, then:

$$\lim_{\Delta v_j \to 0} DF_{h+1}(p-r, \Delta v_1, \ldots, \Delta v_{h+1})$$
$$= DF_h(p-r, \Delta v_1, \ldots, \Delta v_{j-1}, \Delta v_{j+1}, \ldots, \Delta v_{h+1}) + \chi^2_{p-r}$$

where the DF_h and the χ^2 distributions are independent. The additional χ^2 term arises because the dimension of the vector (X'_{t-1}, tE'_t) is preserved although the dimension of the relative sample length vanishes, and hence the dimension of the restrictions imposed by the rank hypothesis is unaltered. On the other hand, if the dummies with vanishing sample length are taken out of the statistical analysis, the additional χ^2-distributed element disappears.

Exact analytic expressions for the asymptotic distributions are not known and the quantiles have to be determined by simulation. In order to avoid the simulations for any possible set $(v_1, \ldots, v_h)$, in Johansen *et al.* (2000) the authors show that the right tails of $DF_h(p-r, \Delta v_1, \ldots, \Delta v_h)$ and the Γ distribution are almost identical.

If r cointegrating relationships are found, it is hoped that they can be identified as economically meaningful. For this reason it is important to refer to a theoretical equilibrium model such as the one described in Section 2. From this model the linear restrictions able to uniquely identify the long-run parameters are derived. Johansen (1995) determines the conditions which, given a set of linear restrictions, must be satisfied for parameters to be identified. There is a condition concerning linear restriction (model generically identifying): the number of free parameters in matrix restrictions must be equal (system exactly identified) or greater (over-identification) than the number of estimated parameters. Furthermore, it is necessary to verify a condition concerning free parameters. These must be such that the linear combinations of the variables derived by the linear restriction are linearly independent. There is a likelihood ratio test on the over-identifying restrictions based on the ratio between the restricted and the unrestricted model likelihood functions. This test is asymptotically χ^2 distributed (Johansen, 1996).

6. EMPIRICAL RESULTS

The analysis has been performed using MALCOLM 2.4 (Mosconi, 1998).

Consider the vector of dimension $(p \times 1)$,

$$X_t = [Q_t, RRL_t, Y_t, LTNT_t, FBAL_t, LTOT_t, ROIL_t, MG_t],$$

where $p = 8$ and the variables are as defined in Table 1.

From equation (13) it is clear that this model is conditional on the first k observations in each subsample. Hence, if h breaks are introduced, then $h \times k$

observations are ignored. Given the short life of the US dollar/Euro exchange rate, our sample size allows consideration of at most one break, that we make coincide with the crisis of the European Monetary System (September 1992) so that $T_1 = 33$ and $\upsilon_1 = 0.28$. This choice seems to be strengthened by an evaluation of the time series involved in the analysis.[5] The different behaviour of trends in the first and in the second period appears more relevant for the short-term real interest rate differential in which the downward peak looks like a good indicator of the break point.

The first step of the analysis consists in the estimation of the unrestricted p-dimensional vector autoregressive model composed of two (corresponding to the number of subsamples) equation models of type (13).

In order to overcome the residuals cross-correlation, given the high VAR dimension, we choose $k = 3$, the maximum number of lags considered.

Table 2 reports the results of the Jarque–Bera normality test of the VAR model's residuals. The normality hypothesis at system level is not accepted because of problems with skewness in the *LTOT* equation and kurtosis in the *Y*, *LTNT* and *MG* equations.

With regards to the kurtosis, note that all the residuals based on misspecification tests should be modified to take into account the fact that the first k residuals of each period are set to zero by the presence of dummies $D_{j,t}$.

The trace test (Table 3) shows that there is evidence of four cointegrating vectors at the usual 5% significant level. However, we have considered only three cointegration relationships for two reasons. First, we have confidence in the theoretical model presented in Section 2 and our goal is to verify if this framework enables one to obtain reasonable and reliable forecasts of the US dollar/Euro real exchange rate. Second, the null hypothesis of three cointegrating vectors is accepted at the 1% significance level and there is evidence of ARCH disturbance of the starting unrestricted VAR model (we compute the ARCH test proposed in Bollerslev and Wooldridge, 1992).[6] A simulation exercise in Gardeazabal and Regúlez (1992) shows that, starting with a very simple VAR

Table 2. Normality test of AR Model's Residuals. For the starting unrestricted VAR residuals, the p-values of Jarque–Bera normality test are shown

Equation	Skewness	Kurtosis	Sk. + Kur
Q	0.4480	0.2910	0.4300
RRL	0.3350	0.3050	0.3710
Y	0.9320	0.0120	0.0420
LTNT	0.8750	0.0310	0.0970
FBAL	0.0570	0.2120	0.0750
LTOT	0.0010	0.3190	0.0030
ROIL	0.3260	0.4780	0.4800
MG	0.8770	0.0100	0.0370
System	0.0360	0.0260	0.0060

[5] The series are available upon request.
[6] The outputs are available upon request.

Table 3. Cointegration analysis of system—rank test

Number of lags considered:		3		
H_0: rank $\leq r$	Test statistic $-T \Sigma \log(.)$	p-value	95%	99%
$r = 0$	303.96	0.0000	223.27	236.32
$r \leq 1$	233.54	0.0000	181.86	193.72
$r \leq 2$	172.55	0.0004	144.61	155.28
$r \leq 3$	119.50	0.0128	111.39	126.86
$r \leq 4$	75.40	0.1467	82.20	90.49
$r \leq 5$	45.75	0.3316	57.06	64.18
$r \leq 6$	23.72	0.5055	35.75	41.66
$r \leq 7$	6.81	0.7677	18.08	22.70

model, it cannot be excluded that ARCH disturbance implies a restriction of the acceptance region of the trace test null hypothesis. Hence, it can be useful to expand this acceptance region, choosing the 1% significance level in order to compensate the ARCH effect. However, the validity of this assumption will be reinforced by a future simulation exercise in which the joint presence of ARCH disturbance and structural breaks is considered. In any case, the identification of the fourth cointegrating relationship could be a further chance to improve the forecast capability of this model in future research.

The results of the rank test allow one to consider the vector error correction model in the form of equation (15) where, once more, $k = 3$, and $h = 2$. As usual, the VECM separates the long-run from the short-run dynamics.

6.1 Long-run dynamics

Following the theoretical model proposed in Section 2 and writing functions (10), (11) and (12) in explicit form, we suggest these long-run relationships:

$$\begin{aligned} Q_t &= -\beta_{12}RRL_t - \beta_{14}LTNT_t - \beta_{15}FBAL_t \pm \beta_{16}LTOT_t \pm \beta_{17}ROIL_t + ecm_{1t} \\ RRS_t &= +\beta_{23}Y_t - \beta_{29}MG_t + ecm_{2t} \\ Y_t &= -\beta_{32}RRL_t + \beta_{34}LTNT_t - \beta_{35}FBAL_t \pm \beta_{36}LTOT_t \pm \beta_{37}ROIL_t + ecm_{3t} \end{aligned} \tag{17}$$

where:

- β_{ij} is the jth coefficient of the ith column of the cointegration matrix β, supposing $\beta_{ij} > 0$, $\forall i$, $i = 1, \ldots, r$ and $j = 1, \ldots, p$;
- ecm_{it} is the stationary error correction component of the ith equation, that is, using the notation introduced in model (15):

$$ecm_{t-1} = (\beta' \ \gamma') \begin{pmatrix} X_{t-1} \\ tE_t \end{pmatrix} \tag{18}$$

Table 4. Equilibrium dynamics — the real exchange rate

Equation (1) for *Q* Variable	Coefficient	Std. Error	*t*-value
RRL	−4.7978	0.2989	−16.0515
LTNT	−2.1057	0.2281	−9.2315
FBAL	−2.0330	1.1169	−1.8202
LTOT	0.6540	0.1178	5.5518
ROIL	0.1318	0.0367	3.5913
*t*E*1	0.0255	0.0039	6.5254
*t*E*2	0.0134	0.0014	9.6800

We have verified that the constraints suggested by the economic model (17) are able to identify the cointegration space, therefore the model is generically and empirically identified. The null hypothesis of the over-identification test, which places the proposed restrictions on the β matrix, is not rejected at the 95% confidence level (p-value = 0.18961).

Tables 4 to 6 present the coefficients of the estimated cointegration relationships. All the coefficients of the variables are significantly different from

Table 5. Equilibrium dynamics — the real interest rate differential

Equation (2) for *RRL* Variable	Coefficient	Std. Error	*t*-value
Y	6.0033	0.5337	11.2485
MG	−8.5467	0.5620	−15.2077
*t*E*1	0.0169	0.0029	5.7944
*t*E*2	0.0174	0.0004	44.8581

Table 6. Equilibrium dynamics — the real growth rate differential

Equation (3) for *Y* Variable	Coefficient	Std. Error	*t*-value
RRL	4.1181	0.2546	16.1748
LTNT	1.5180	0.1942	7.8167
FBAL	−1.4335	0.9608	−1.4920
LTOT	−0.4262	0.1037	−4.1099
ROIL	0.1370	0.0319	4.2947
*t*E*1	−0.0131	0.0030	−4.3202
*t*E*2	−0.0132	0.0012	−11.3529

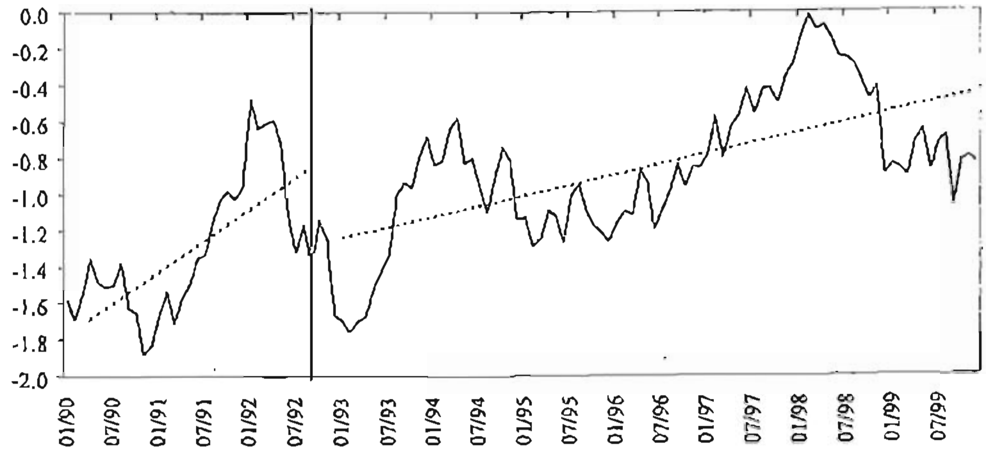

Fig. 1. Stationary component — real dollar/Euro exchange rates (the vertical line indicates the date in which we put the structural break — September 1992)

zero. Furthermore, the presence of different deterministic trends in the two subsamples is accepted for all the equations. Only in the cointegrating relationship for the real growth rate differential (Y_t) the coefficients of both the deterministic trends seem to be quite similar. Figures 1 to 3 show the error correction components (*ecm*) of the system (17): we can see that, especially in the second subsample, these residuals appear stationary around the deterministic trend.

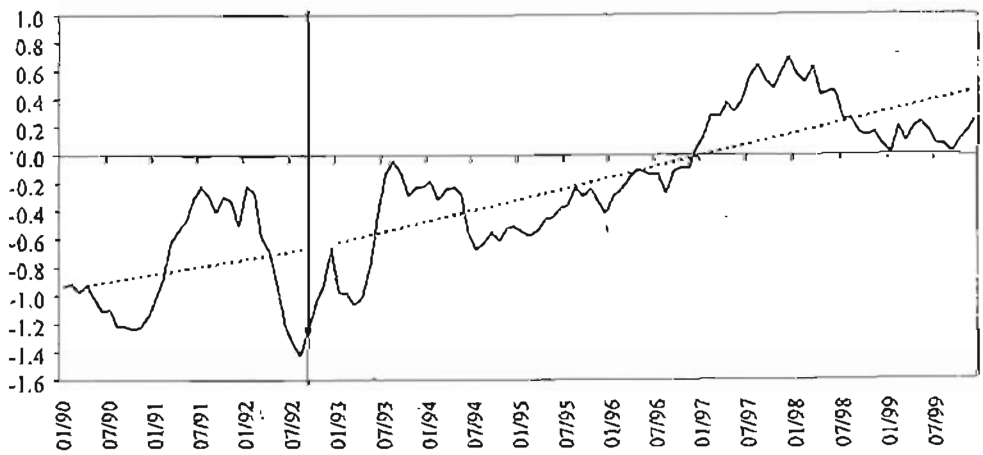

Fig. 2. Stationary component — real 10-year interest rate differential (the vertical line indicates the date in which we put the structural break — September 1992)

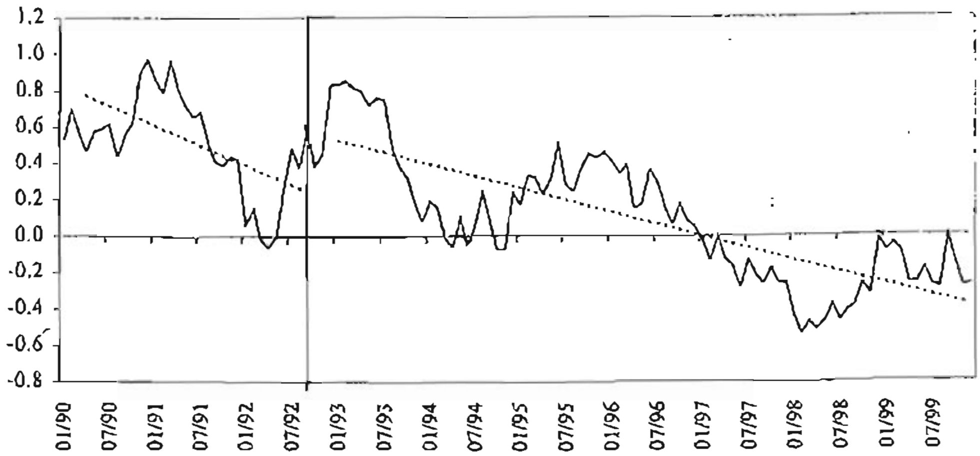

Fig. 3. Stationary component — real growth differential (the vertical line indicates the date in which we put the structural break — September 1992)

Hence, from the cointegration analysis we have obtained long-run relationships that can be used both to model the dynamic equations via the error correction mechanism and to interpret the expected interactions suggested by the economic theory.

In the equation of the real exchange rate, all the signs for the coefficients are in accordance with expectations, but the positive sign for *ROIL* is not consistent with the positive sign in the long-run equation of the GDP's growth rate differential. We can justify the second as a consequence of the stronger dependence on oil prices of the Euro-area rather than the United States.

In the real interest rate differential equation, the coefficient for *MG* is negative and for *Y* is positive in accordance with economic theory.

Finally, in the third equation, the coefficients for all variables except the long-term real interest rate differential (*RRL*) have the expected sign.

This last unsatisfactory sign of the real interest rate differential could be justified as follows: as the first long-run equation suggests, an increase in the differential between real interest rates allows for an appreciation of the US dollar. Consequently, if the appreciation is more relevant for the commodity import than goods import, then this increase could be interpreted as a sign of the expansion of real output (an increase of Y_D).

6.2 Short-run dynamics

In the previous section we determined the long-run patterns of the US dollar/Euro exchange rate, the differential between the US and European 10-year real interest rates and the differential in the GDP growth rates. Therefore, we can analyse the short-term dynamics for these variables estimating a conditional VECM in the form of equation (15).

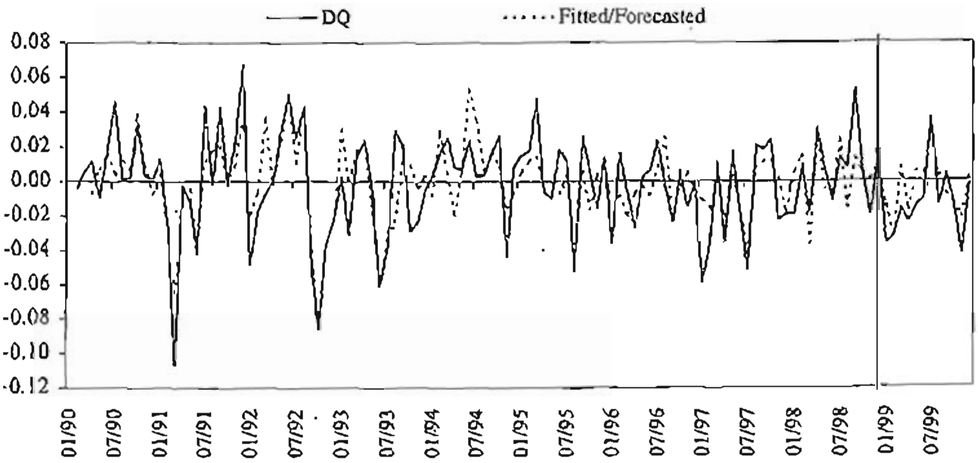

Fig. 4. Short-term dynamics — observed and fitted real dollar/Euro exchange rate changes (the vertical line indicates the start of the forecasting period)

The dynamic behaviour of fitted and observed first differences of the endogenous variables is reported in Fig. 4 to 6. The usual diagnostic test shows that the normality hypothesis of model residuals cannot be rejected for all the equations and for the entire system. Furthermore, there is no evidence of autoregrassive conditional heteroscedasticity (ARCH) effects, and statistically and economically significant autocorrelation structures, according to the standard Bartlett's band test, with the exception of the real growth differential

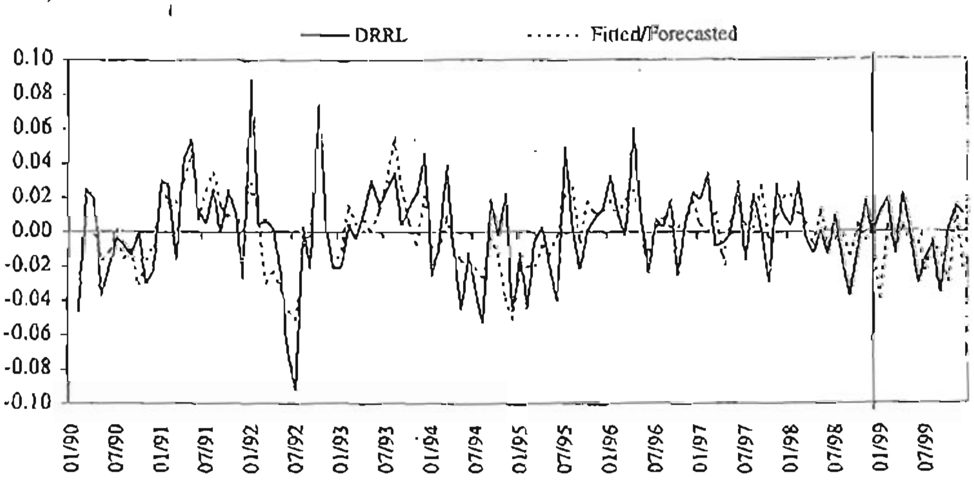

Fig. 5. Short-term dynamics — observed and fitted real 10-year interest rate differential changes (the vertical line indicates the start of the forecasting period)

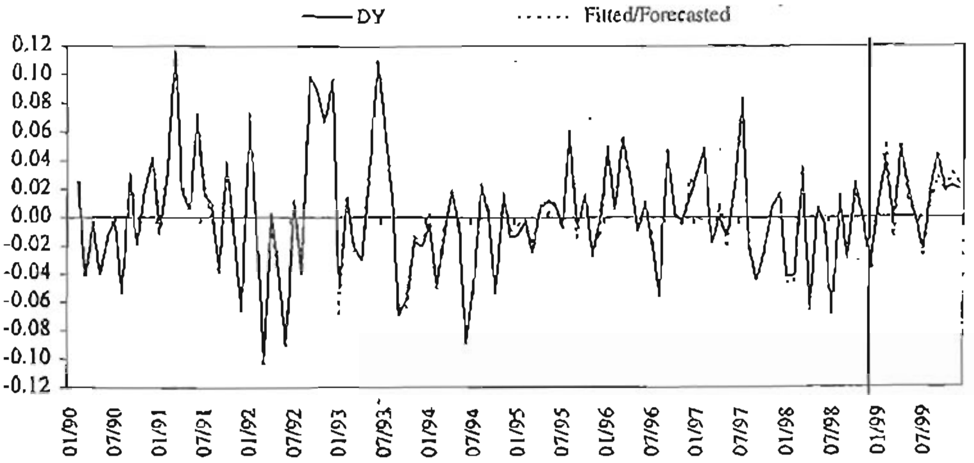

Fig. 6. Short-term dynamics — observed and fitted real growth differential changes (the vertical line indicates the first forecasted value)

changes. In this last case there is a slight indication of seasonal component in the residuals.[7]

Finally, the model's residuals are not cross-correlated[8].

6.3 Forecasting performance

After estimation as described above, out-of-sample one-step-ahead forecasts were produced for January to December 1999 using actual values of the exogenous variables.

The results obtained for monthly percentage changes of the US dollar/Euro real exchange rate are presented in Table 7 and Figure 7, while Table 8 and Figure 8 show the results for the level variable.

The forecasting performances of the US dollar/Euro exchange rate are reasonably satisfactory: the monthly changes forecasts showed in Table 7 have the indicator or correct signs at the 50% level and Theil's U statistic of 0.4839. The monthly levels slightly worsen Theil's U statistic, which rises to 0.6105 (Table 8), but reveals the good econometric model forecasts compared with random walk forecasts.

The figures are drawn without the forecast intervals for better readability. However, the computations of test statistics lead to the acceptance of the

[7] The seasonality effect could be removed by using seasonal deterministic components in the model. We do not follow this procedure for two reasons. First, if all the variables in the model are seasonally adjusted in the usual way, then the remaining seasonality effect would be removed changing the seasonally adjustment techniques. Second, the use of seasonal deterministic components in the model changes all the cointegration test statistics and their computation is not so easy in the presence of structural breaks.

[8] The extended output and the diagnostic test results of the estimated short-run equations are available upon request.

Table 7. Forecasting performance — monthly changes of real US dollar/Euro exchange rate

Date	Actual	Forecasted	Error	Actual change sign	Forecasted change sign
12/98	0.0162				
01/99	−0.0360	−0.0149	0.0211	(−)	(−)
02/99	−0.0320	−0.0313	0.0007	(−)	(−)
03/99	−0.0159	0.0078	0.0237	(−)	(+)
04/99	−0.0236	−0.0174	0.0062	(−)	(−)
05/99	−0.0141	0.0052	0.0193	(−)	(+)
06/99	−0.0104	0.0045	0.0150	(−)	(+)
07/99	0.0366	0.0290	−0.0076	(+)	(+)
08/99	−0.0141	0.0031	0.0172	(−)	(+)
09/99	0.0044	−0.0076	−0.0120	(+)	(−)
10/99	−0.0124	−0.0122	0.0002	(−)	(−)
11/99	−0.0427	−0.0210	0.0218	(−)	(−)
12/99	−0.0012	0.0023	0.0035	(−)	(+)

Mean error:	0.0091
Root mean square error:	0.0148
Mean absolute error:	0.0124
Theil's U statistic:	0.4839
Signs correctly forecasted:	50.00%

hypothesis that none of the forecast values are significantly different from the actual values, at the usual significance levels.

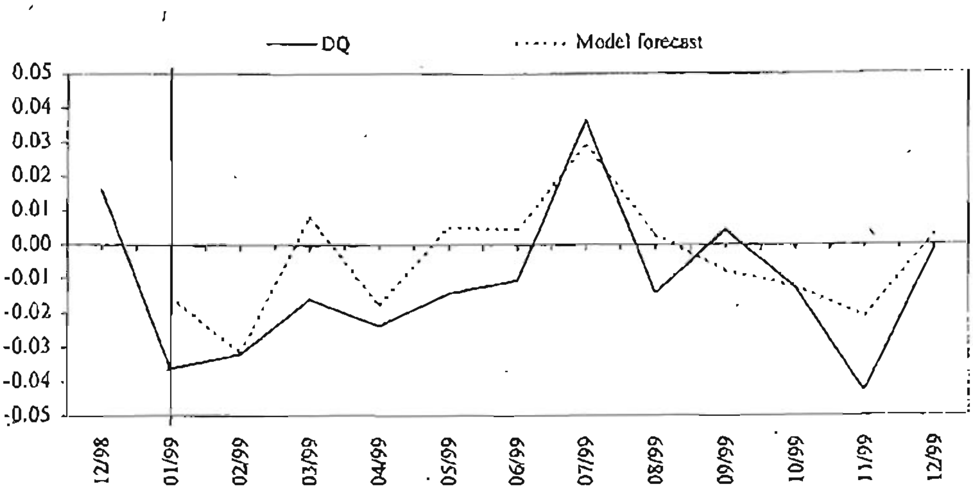

Fig. 7. Forecasting performance — observed and forecasted dollar/Euro real exchange rate changes (the vertical line indicates the first forecasted value)

Table 8. Forecasting performance – monthly levels of real US dollar/Euro exchange rate

Date	Actual	Forecasted	Error	Actual change sign	Forecasted change sign
12/98	1.2305				
01/99	1.1870	1.2123	0.0253	(–)	(–)
02/99	1.1496	1.1505	0.0009	(–)	(–)
03/99	1.1314	1.1586	0.0272	(–)	(+)
04/99	1.1051	1.1120	0.0069	(–)	(–)
05/99	1.0896	1.1108	0.0213	(–)	(+)
06/99	1.0783	1.0945	0.0163	(–)	(+)
07/99	1.1185	1.1100	-0.0085	(+)	(+)
08/99	1.1028	1.1219	0.0191	(–)	(+)
09/99	1.1077	1.0945	-0.0132	(+)	(–)
10/99	1.0940	1.0943	0.0002	(–)	(–)
11/99	1.0483	1.0713	0.0231	(–)	(–)
12/99	1.0470	1.0507	0.0037	(–)	(+)

Mean error: 0.0102
Root mean square error: 0.0166
Mean absolute error: 0.0138
Theil's U statistic: 0.6105
Signs correctly forecasted: 50.00%

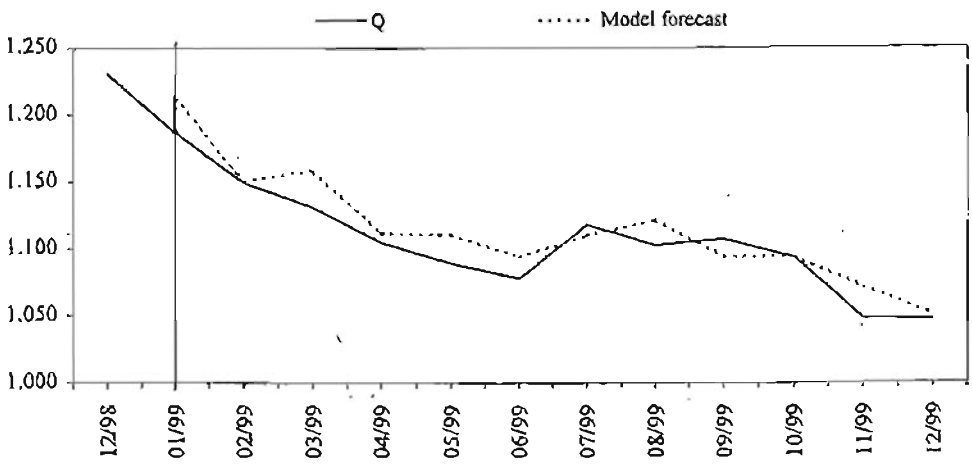

Fig. 8. Forecasting performance – observed and forecasted dollar/Euro real exchange rate (the vertical line indicates the first forecasted value)

7. CONCLUSIONS

The main results of the paper are the specification and estimation of an econometric model on US dollar/Euro real exchange rate in VECM form. For the model, the main structural feature is given by making endogenous the long-term interest rate differential and the differential between US and Euro GDP annual growth rate in addition to the exchange rate. In this way all three relevant markets, i.e. the foreign exchange market, the money market and the goods market, are modelled jointly.

Admissibility of the VECM form is achieved by using cointegration analysis in the presence of structural breaks. We introduce a break inside the sample period (between January 1990 and December 1999) to coincide with the crisis in the European Monetary System (September 1992). The presence of different deterministic trends in the two subsample periods is accepted and three long-run relationships are obtained. The estimates of the coefficients in the long-run relationships are consistent with the interactions suggested by economic theory.

The specification of three long-run relationships might be questioned because the acceptance region has been chosen at the 1% significance level. However, a simulation exercise in Gardeazabal and Regúlez (1992) shows that, starting with a very simple VAR model, it cannot be excluded that ARCH disturbance implies a narrower acceptance region of the trace test null hypothesis. Hence, it can be useful to expand this acceptance region, by choosing the 1% significance level, in order to compensate the ARCH effect. The appropriateness of this strategy will be confirmed in a future simulation exercise by considering the joint presence of ARCH disturbance and structural breaks.

The forecasting performances of US dollar/Euro exchange rate obtained are to some extent satisfactory: Theil's U statistic shows an efficiency gain in the forecasting performances with respect to the competing random walk model of 39% and 50% for the correct signs.

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