FRACTIONAL PROGRAMMING WITH INTERVAL COEFFICIENTS

Stefania Funari, Silvio Giove
Dipartimento di Matematica Applicata
Università Ca' Foscari di Venezia

EXTENDED ABSTRACT

Many optimization problems arising in economics and in management science deal with fractional objective functions (for example, we can cite the portfolio selection and the efficiency evaluation problems).

Moreover, since in the real decision problems the data are usually imprecise and ambiguous, there has been in the specialized literature a growing interest in studying interval and fuzzy optimization problems, which are quite appropriate to treat such imprecise input data.

With regard to the linear programming with interval and fuzzy coefficients, we limit ourself to quote Shaocheng (1994) and the recent papers of Chinneck et al. (2000) and of Inuiguchi et al. (2000).

On the other hand, various approaches have been adopted to treat fuzzy fractional programming problems (see Ammar, 1998 and Li et al., 2000).

In this paper we propose an alternative formulation of a linear fractional problem with interval coefficients, which considers a special ranking function.

First of all, we remark that a total or a partial ordering can be defined among two interval or fuzzy numbers (see Bortolan et al., 1985).

In solving a fractional problem with interval coefficients, we adopt a new ranking method for comparing two interval or fuzzy numbers, which has been introduced by Detyniecki et al. (2000) and Facchinetti (1998). It is based on a simple linear inequality depending on a parameter $\lambda \in [0, 1]$, which can be interpreted, from an economic point of view, as a more or less risk aversion factor.

This ranking method compares two real interval numbers a and b, as follows:

$$a \leq b \Leftrightarrow \phi(a) \leq \phi(b) \tag{1}$$

where $a, b \in \mathcal{F}$ and $\mathcal{F}$ is the family of all the real positive closed intervals $\delta = [\delta^L, \delta^U]$, with δ^L being a positive real number representing the lower limit of the interval and δ^U being the upper limit of the interval $(0 < \delta^L < \delta^U)$. The function ϕ is defined as

$$\phi(k) = \lambda k^L + (1-\lambda)k^U, \quad \lambda \in [0,1] \tag{2}$$

for each given interval number $k \in \mathcal{F}$. For more detail, see the quoted references. The same ranking function can be applied for comparing two fuzzy numbers.

We formulate the following problem

$$\begin{aligned}
\text{maximize} \quad & \phi\left(\frac{\sum_{j=1}^n c_j x_j + \alpha}{\sum_{j=1}^n d_j x_j + \beta}\right) \\
\text{subject to} \quad & \phi\left(\sum_{j=1}^n a_{ij} x_j\right) \le b_i \quad (i = 1, ..., m) \\
& x_j \ge 0 \quad (j = 1, ..., n)
\end{aligned} \tag{3}$$

where $b_i \in \Re_0^+$ and $c_j, d_j, a_{ij}, \alpha, \beta \in \mathcal{F}$.

Using the ranking function (2) and the interval computation rules (Alefeld et al., 2000), problem (3) can be written as follows:

$$\begin{aligned}
\text{maximize} \quad & \lambda \frac{\sum_{j=1}^n c_j^L x_j + \alpha^L}{\sum_{j=1}^n d_j^U x_j + \beta^U} + (1-\lambda)\frac{\sum_{j=1}^n c_j^U x_j + \alpha^U}{\sum_{j=1}^n d_j^L x_j + \beta^L} \\
\text{subject to} \quad & \lambda \sum_{j=1}^n a_{ij}^L x_j + (1-\lambda) \sum_{j=1}^n a_{ij}^U x_j \le b_i \quad (i = 1, ..., m) \\
& x_j \ge 0 \quad (j = 1, ..., n)
\end{aligned} \tag{4}$$

Problem (4) is a mathematical programming problem whose objective function consists in the sum of two linear fractional functions. Some algorithms can be used to find an optimal solution (see, for example, Warburton, 1985 and Cambini et al., 1989).

The same procedure can be used to solve fractional problems with triangular fuzzy coefficients.

In a next step, we intend to extend the same approach to fractional problems where even the decision variables x_j are fuzzy numbers.

References

Alefeld, G., Mayer, G. (2000), “Interval Analysis: theory and applications”. *Journal of Computational and Applied Mathematics* 121, 421–464.

Ammar, E. E. (1998), “On some basic notions of fuzzy parametric nonsmooth multiobjective nonlinear fractional programming problems”. *Fuzzy Sets and Systems* 99, 291–301.

Bortolan, G., Degani, R. (1985), “A review of some methods for ranking fuzzy subsets”. *Fuzzy Sets and Systems* 15, 1–19.

Cambini, A., Martein, L. (1989), “On maximizing a sum of ratios”. *Journal of Information and Optimization Sciences* 10, 65–79.

Chinneck, J.W., Ramadan, K. (2000), “Linear programming with interval coefficients”. *Journal of the Operational Research Society* 51, 209–220.

Detyniecki, M., Yager, R.R. (2000), “Ranking fuzzy numbers using α-weighted valuations”. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems* 8, 573–591.

Facchinetti, G., Ghiselli Ricci, R., Muzzioli S. (1998), “Note on ranking fuzzy triangular numbers”. *International Journal of Intelligent systems* 13, 613–622.

Inuiguchi, M., Ramik J. (2000), “Possibilistic linear programming: a brief review of fuzzy mathematical programming and a comparison with stochastic programming in portfolio selection problem”. *Fuzzy Sets and Systems* 111, 3–28.

Li, H.L., Yu C.S., (2000), “A fuzzy multiobjective program with quasiconcave membership functions and fuzzy coefficients”. *Fuzzy Sets and Systems* 109, 59–81.

Shaocheng, T. (1994), “Interval number and fuzzy number linear programmings”. *Fuzzy Sets and Systems* 66, 301–306.

Warburton, A.R. (1985), “Parametric solution of bicriterion linear fractional programs”. *Operations Research* 33, 74–84.