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January 2014

Abstract
This paper illustrates a methodology for analyzing bargaining games on network markets, by means of numerical models that can be calibrated with real data. Economic incentives to join or to expand a network depend on how the network surplus is being distributed, which in turn depends on a variety of factors: position of each agent (e.g., a country) in a specific network, its reliability in the cooperation scheme (e.g., geo-political stability), existence of market distortions and availability of outside options (e.g., alternative energy sources). This study is aimed at presenting a game theory methodology that can be applied to real world cases, having the potential to shed light on several political economy issues.
The methodology is presented and illustrated with application to a fictitious network structure. The method is based on a two-stage pro- cess: first, a network optimization model is used to generate payoff values under different coalitions and network structures; a second model is subsequently employed to identify cooperative game solutions. Any change in the network structure entails both a variation in the overall welfare level and in the distribution of surplus among agents, as it affects their relative bargaining power. Therefore, expected costs and benefits, at the aggregate as well as at the individual level, can be compared to assess the economic viability of any investment in network infrastructure. A number of model variants and extensions are also considered: changing demand, exogenous instability factors, market distortions, externalities and outside options.

Keywords
Network Markets, Cooperative Games, Distribution Networks, Bargaining.
JEL Codes
C63, C71, L95

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Roberto Roson* Franz Hubert†

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This paper illustrates a methodology for analyzing bargaining games on network markets, by means of numerical models that can be calibrated with real data. Economic incentives to join or to expand a network depend on how the network surplus is being distributed, which in turn depends on a variety of factors: position of each agent (e.g., a country) in a specific network, its reliability in the cooperation scheme (e.g., geo-political stability), existence of market distortions and availability of outside options (e.g., alternative energy sources). This study is aimed at presenting a game theory methodology that can be applied to real world cases, having the potential to shed light on several political economy issues.

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1 Introduction

This paper is intended to illustrate a methodology for analyzing bargaining games on network markets, by means of numerical models that can be calibrated with real data. We focus on specialized physical networks such as oil and gas pipelines, water-supply or irrigation systems, rail transport systems etc. The usage of such systems often requires the cooperation of many partners, each of whom controls only a small part of the network.

In such networks the control of dedicated resources such as switches, connections, exit and entry points defines a power structure. This power structure, in turn, is likely to affect the incentives to link up with the network, to develop it by adding new links or by increasing the capacity of existing ones. We start from the assumption that the power of a player in such a network will depend on the value of his resources for other players. For the physical networks mentioned above, it is often possible to assess with a reasonable degree of confidence how they should be used optimally, if some resources were not available. This enables us to define a cooperative game.

For a group of players, also called a coalition, we first determine which parts of the network would be accessible. Then we use a network optimization model to calculate the total payoff, achieved by the optimal usage of the sub-network under their control. This payoff is the *value* of the coalition. Obviously, the Grand coalition of all players has the whole network at its disposal and can generate the largest payoff. By repeating this optimization procedure for all possible coalitions, we obtain the so called value function of a cooperative game, which captures all the economic and technical features of the network. To obtain a measure of the players' power in the network market, we solve the game with the Shapley Value. While not the only possible theoretical solution, the Shapley value is the most widely used one (Moretti and Patrone, 2008) and already well accepted as a power index for voting games (Shapley and Shubik, 1954).[1]

The work is inspired by recent applications of cooperative game theory to gas-pipeline systems. Hubert and Ikonnikova (2011) analyze gas transit to northwestern Europe. Hubert and Cobanli (2012) consider the impact of new pipelines on the power structure in the Eurasian gas market and Hubert and Orlova (2012) investigate the liberalization of pipeline access within the European Union. These papers consider particular networks calibrated with real world data. This paper examines, instead, an abstract, fictitious network, but enlarges the scope of the analysis to consider a number of variants, which could be highly relevant in many applied settings: market imperfections, outside options, varying demand, external benefits or costs. The discussion of the different model variants highlights the flexibility of the proposed methodology.

[1] The Shapley Value calculates the share of a player as the weighted average of the value of his contribution to coalitions; the weights being determined to fulfill a number of axioms considered to be plausible and reasonable (Shapley (1953)). When applied to networks, the Shapley value is sometimes termed Myerson value (Myerson (1980); Jackson (2005)). In the appendix we also report the results for another solution concept, the nucleolus, which has been proposed by Schmeidler (1969).

Furthermore, it points out a typical feature of these models: incentives for the expansion of the network typically differ between large coalitions and smaller sub-coalitions, even when agreement between only a limited number of players is necessary to expand the network.

This paper makes a contribution by bridging network economics theory with applied, numerical models for economic policy analysis. Whereas value functions are normally taken as given in theoretical network models, we explicitly derive the surplus obtainable by the different coalitions as a consequence of market equilibrium in the network. By doing so, we can also consider a number of market imperfections, or other factors which may ultimately affect the equilibrium. The explicit consideration of such factors may be essential for the development of realistic network models, which could then be used, for example, to assess costs and benefits of realizing large network infrastructure.

The rest of the paper is organized as follows. The next section describes the basic methodology, which is illustrated in section 3 by means of a simple numerical example. Section 4 introduces a number of variants in the model, and discusses the effects of some alternative hypotheses on the example network. Section 5 concludes and provides suggestions for future research. An appendix presents the numerical results obtained by using the nucleolus instead of the Shapley value as an allocation concept.

2 Methodology

We consider a network, made of arcs and nodes. An arc connects two nodes i and j, but not all pairs of nodes are generally connected. In addition to nodes we consider supply and demand points, both of which are connected to a node in the network by an access link (directed). Each demand point is associated with a demand curve, expressing the required demand quantity volume as a (negatively sloped) function of the market delivery price or cost.

All arcs and all access links are associated with an increasing and convex function cost function C_{ij} of the flow f_{ij}. For access supply links, this could be interpreted as production cost. For demand links, this would express a final market distribution cost. For intermediate arcs, the function refers to transportation costs.

A discrete and finite number of agents operate in the network. Agents can cooperate in coalitions Γ, where an agent can joint at most one coalition. Each coalition Γ has access rights to a number of arcs, links and nodes of the networks. $A(\Gamma)$ is the set of all nodes connected by arcs controlled by the coalition Γ.

For each coalition and its associated network, we consider a network market equilibrium (NME). A network market equilibrium is found when flows in the arcs and links are determined such that:

1. Demand access links flows equal demand levels, defined by demand functions computed at marginal delivery costs (demand is served);

2. Total costs (production, transportation, distribution) are minimized;

3. Total incoming flows in each (transit) node equal total outgoing flows (flow balance constraint).

A NME is the solution of the following mathematical optimization problem:

$$\max_{f_{ij,(i,j)\in A(\Gamma)}} W_\Gamma = \sum_{d\in D} \int_0^{f_{id}} P_d(f)df - \sum_{(i,j)\in A(\Gamma)} C_{ij}(f_{ij}) \tag{1}$$

s.t.:

$$\sum_{s\in S} f_{sj} + \sum_{i\in N} f_{ij} - \sum_{i\in N} f_{ji} - \sum_{d\in D} f_{jd} = 0 \qquad \forall j \in N \tag{2}$$

where:

D is the set of demand points;

S is the set of supply points;

N is the set of transit nodes;

$A(\Gamma)$ is the set of admissible pairs of nodes/points connected by arcs/links, for which the coalition Γ possesses access rights;

f_{ij} is the flow from node/point i to node/point j;

$P_d(f)$ is the inverse demand curve at point d;

C_{ij} is the cost function of the arc/link connecting i to j.

Solving a NME problem identifies the total net welfare W obtainable from a certain network. This total welfare is virtually distributed among all parties involved in the network. For example, think of nodes, or points of supply/demand, as countries. Each country contributes to the functioning and possibly to the construction of the network infrastructure, receiving benefits is terms of consumer surplus, tax revenue or profits.

Clearly, there is no obvious way to determine how the overall pie of total welfare would be split. Therefore, to discuss the implications of surplus allocation, we make use of cooperative game theory. A cooperative game equilibrium is a normative concept applied to the distribution of benefits or costs in a group. Among the various equilibrium concepts proposed in the literature, we use here the Shapley value[2], because of its simplicity of computation and easiness of interpretation: the Shapley value assign to each agent a payoff which is proportional to her "contribution" in all possible forming coalitions, that is the difference between the overall surplus obtained by a cooperative coalition with and without the agent. The Shapley values of a game can be readily interpreted

[2]Results for an alternative equilibrium concept, the nucleolus, are presented in the appendix.

as an allocation of benefits (or costs) which reflects the relative bargaining power of each party.

In order to compute a Shapley value distribution for a network with the characteristics described above, one needs to compute the welfare associated with all possible coalitions and individual agents. Each agent is here associated with a node, therefore computing the maximum welfare for a coalition amounts to solving a NME where all arcs connect pairs of nodes belonging to the coalition[3]. In other words, we consider sub-networks obtained from the big network by removing those links where at least one of the two extremes brings to an agent not in the coalition. The smaller the coalition, the smaller the network, the lower the welfare that can be obtained. Furthermore, many coalitions may actually get zero welfare. This is true for all individuals (one member coalitions), for coalitions including only demand or only supply agents, or where demand and supply agents are disconnected.

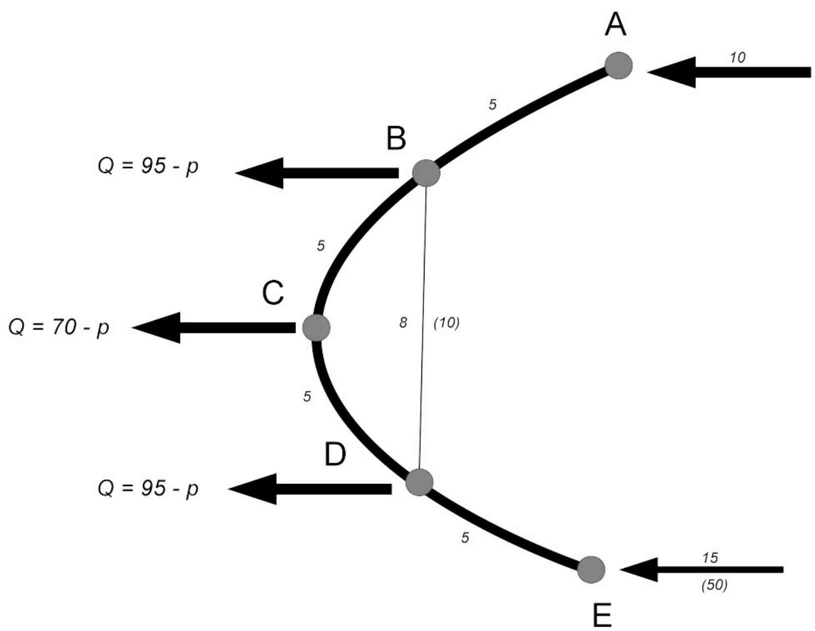
In a set of N agents, there are 2^N possible coalitions, including the grand coalition (all agents inside) and the empty one. To compute the Shapley value, or any other distribution concept of cooperative game theory (e.g., the nucleolus), the first step is solving for the NME and obtaining the welfare level associated with any sub-coalition, possibly excluding those ones which have obviously a zero welfare. This can be done with optimization software like GAMS or AIMMS, or mathematical packages like Mathematica or, if the network is not too complex, using a solver embedded into spreadsheets like Microsoft Excel or LibreOffice Calc. Once surpluses for all possible combinations of agents in the set have been obtained, the Shapley value can be computed using an algorithm, for example the one proposed by Carter (1993), based on Mathematica.

Different network structures imply, of course, different distributions of welfare. Hubert and Ikonnikova (2011), Hubert and Cobanli (2012) adopt the methodology described above to assess the distribution of surplus in gas distribution networks for Eastern Europe and the Middle East. The existing pipeline infrastructure is taken as a benchmark, to be contrasted with alternative network structures in which new links are added or the capacity of some existing links is expanded. These alternative scenarios are based on investment projects under discussion or realization.

Assessing how welfare changes and how it is differently distributed when a network is modified allows evaluating the individual incentives to undertake the proposed investment. Any network enlargement necessarily increases the overall welfare, which can be measured in monetary terms, but this could not be sufficient to cover the costs. Furthermore, some investment may not need the involvement of all parties. Think, for example, the addition of a new link, whose realization requires the involvement of only the agents located at its two sides. As a change in the network topology influences the bargaining power and the distribution of welfare, it may well be the case that a certain investment may not be globally justified, yet be locally viable.

[3]This is a simplifying assumption. More generally, a coalition would be associated to the sub-network she controls.

Figure 1: A reference network structure

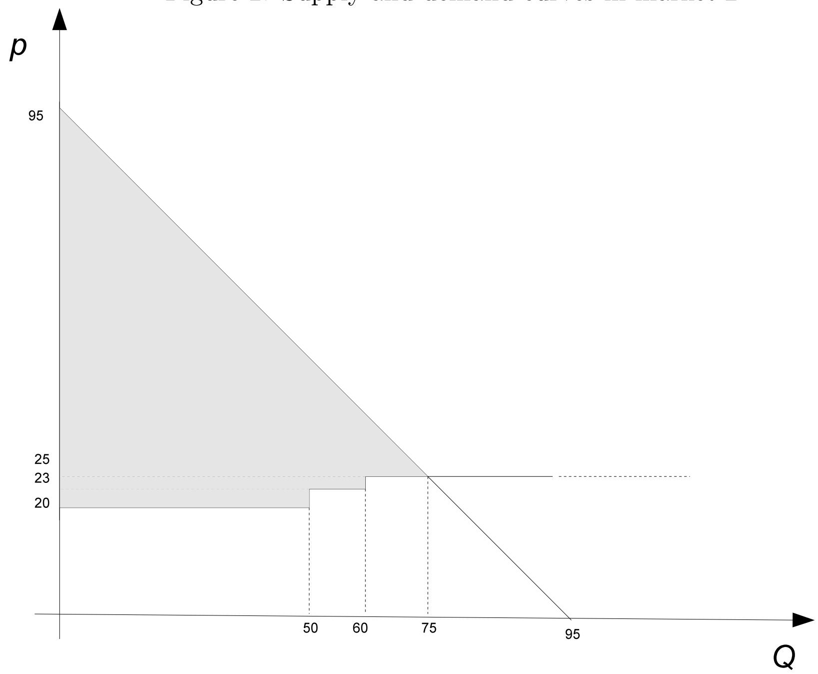


3 An Illustrative Example

To illustrate the meaning of surplus allocation in a network, let us consider a fictitious network structure as depicted in Figure 1. There are five agents (i.e., countries), each one associated with one node: A, B, C, D, E. All costs associated with arcs and links are constant, possibly up to a capacity limit. A and E are suppliers. E has a higher production cost (15 instead of 10) and it is, furthermore, affected by an upper supply capacity limit of 50 (this is indicated by a number in parentheses, otherwise there are no capacity constraints). All intermediate links have a unit transport cost of 5, except for the link connecting B to D, which has a cost of 8 but a maximum capacity of only 10. B, C and D are demand points, each one associated with a simple linear demand curve of the type $Q = \Psi$-p, where Ψ is the maximum price in the market and also a measure of the market size. There are no distribution costs.

Flows in the network of Figure 1 can be allocated by solving the mathematical program (1-2). Consider, for example, the demand market D ($Q = 95 - p$). For D, the least cost supplier is E (15+5=20). However E cannot provide more than 50 units, which is less what would be required at a price of 20 (95-20=75). The second least cost alternative is A through the A-B-D path (10+5+8=23),

Figure 2: Supply and demand curves in market D



which is also capacity constrained. Finally, it is possible to supply D trough the path A-B-C-D at a cost 10+5+5+5=25. At a price 25, 70 units are demanded and delivered at D: 50 from E, 10 from B, 10 from C. The total surplus generated in market D is 2720, corresponding to the area below the inverse demand curve but above the stepwise supply curve in Figure 2. This surplus adds to the one generated in B (3200) and C (1250) to the total welfare W (7170).

How is this total welfare going to be virtually distributed among the five agents? This depends on the relative bargaining power. Consider, for example, supplier E. The bargaining power of E has to do with what the other agents can get without E. For example, a coalition {A,B,C,D} could run a network without the D-E link. Market D, in this case, would live without its most convenient supply source, which would reduce welfare in D by 250 [(25-20)*50], lowering total welfare from 7170 to 6920.

Welfare for other sub-coalitions can be computed in a similar way, allowing to compute a Shapley distribution for the cooperative game on the network. Shapley values for this base case are reported in Table 1, in the column "Base".

Values under the heading "Ext." refer, instead, to an alternative case, where the original structure of the network as in Figure 1 has been modified, by removing the capacity limit in the link B-D. This enhancement increases the overall welfare, from 7170 to 7193, as it lowers the market price (from 25 to 23) in the D market. Furthermore, it changes the surplus distribution, actually harming

Table 1: Shapley value surplus distributions

agents \ cases	**Base**	**Ext.**	**Diff.**
A	2115	2183	+68
B	2222	2305	+83
C	530	425	-105
D	1488	1572	+84
E	815	708	-107
Total	***7170***	***7193***	***+23***

the agents C and E.

C is made worse off because it would be by-passed whenever D is served from B (or B from D). Consequently, any threat from C of not joining a coalition would be weakened, thereby reducing its bargaining power. Analogously, the threat from E of not serving D would reduce welfare in that market by an amount [(23-20)*50=150] smaller than it was before (250), because D can now revert to a fairly efficient alternative supply source.

It is interesting to notice how the variation in surplus affects the incentives to undertake the investment. At the aggregate level, the investment in capacity expansion would be desirable if its cost would be lower than 22, that is, the total welfare gain. However, it may be the case that the expansion of capacity in B-D only requires cooperation between agents B and D, possibly with the contribution of A. In this case, if the investment costs more than 22 but less than 167 (83+84), it would be undertaken, despite the fact that it would not be socially desirable. In other words, there would be a *negative externality* generated by the expansion of capacity in the link B-D.

4 Extensions

4.1 Changing Demand

Consider a case where demand in the smallest market C increases from $Q = 70 - p$ to $Q = 80 - p$. This obviously raises the overall welfare obtainable in the network, from 7170 to 7719. It also changes, however, the bargaining power of all parties, at it is shown in Table 2, comparing the base case with the one with expanded demand in C.

Much of the welfare gain accrues to C. However, it also goes to the nodes which are involved in the supply of C, in proportion to their contribution. As C is typically supplied through the route A-B-C, A and B are also getting significant gains.

4.2 Exogenous Instability

Suppose that one agent in the set, say agent C, is affected by some exogenous factors undermining her "reliability" as a partner in any coalition. For example,

Table 2: Shapley value surplus distributions

agents \ cases	**Base**	**Exp.**	**Diff.**
A	2115	2292	177
B	2222	2375	153
C	530	725	195
D	1488	1500	12
E	815	827	12
Total	***7170***	***7719***	***549***

Table 3: Shapley distributions with instability in C

agents \ cases	**Base**	**Ext.**	**Diff.**
A	2065	2141	+76
B	2172	2266	+94
C	477	382	-95
D	1477	1571	+94
E	849	707	-142
Total	***7040***	***7067***	***+27***

C could refer to a geo-politically unstable country. We assume that there is some probability that the C partner is not available and cooperating. A simple way to capture this exogenous instability is to compute the expected payoffs for all potential coalitions, considering that the coalition could shrink to a smaller one, excluding C. For example, with a 10% probability, the expected payoff of the grand coalition {A,B,C,D,E} would be 0.9*P(ABCDE)+0.1*P(ABDE), where P() is the payoff computed from the NME as in the previous section.

Using this methodology to modify the payoffs for all sub-coalitions including C, new Shapley value distributions can be computed. Table 3 presents the new values, corresponding to the ones in Table 1, under exogenous instability for C.

We see that total expected welfare is lower and, not surprisingly, C is the member which is losing proportionally more (477 instead of 530). Agent E gains from the instability in C (849 instead of 815), because she has more bargaining power now. Indeed, if the path serving market D through C would be disrupted, D could get no more than 60, because of capacity constraints. The price would then go up to 35, making the potential threat by E of not serving D very serious.

If capacity in the link B-D is enlarged, total welfare would increase by 27, which is a bigger increment than before. The value for E falls to 707, because to serve the D market it is not necessary to pass through C if capacity is unbounded in B-D. More importantly, gains for B and D together now sum up to 94*2=188, which is significantly more than the value without instability in C (167), whereas the global gain only increases from 22 to 27. We can therefore deduce that: (a) instability in the node C increases the likelihood that capacity in the arc B-D is enlarged, (b) it is more likely that negative externalities are generated and the network is inefficiently expanded.

4.3 Outside Options

Very often, markets have access to alternative energy sources outside the network. For example, instead of relying (only) on gas or oil, distributed trough pipelines, a country can get energy from renewable sources (e.g. solar). These "outside options" typically have two key characteristics: (1) they are more costly than conventional, network-based goods under normal market conditions; (2) their exploitation does not require cooperation with other agents. In this case, even if an agent may not find it convenient to utilize the outside option when a cooperative network is in place, the mere availability of the outside option affects her bargaining power and the distribution of surplus.

To illustrate the point, consider a market like D in the numerical example discussed above. In the base case, D obtains a good through the network at the price 25. Suppose that D could have produced, autonomously, the same good at a constant cost of 30. Clearly, domestic production would not be economically viable under these conditions. However, to compute the Shapley value we did consider the welfare obtainable by all possible sub-coalitions. The coalition including only D would have got zero surplus in the initial case, but the possibility of autonomous domestic production brings the potential welfare to 2112.5 (price 30, consumed quantity 65). The sub-coalition {A,B,D,E} delivered 60 to market D, bringing the price at 35, which is higher than 30. When the outside option is available, the equilibrium price would instead be 30, and D would be served by both imports through the network and domestic production. This case is depicted in Figure 3.

The price $\bar{p}$ is the constant marginal cost of domestic production, which constitutes an upper bound on the equilibrium price. The network delivers quantity q^N which, without domestic supply, would have brought about a price of p^N. Now the price is kept at $\overline{p}$, the quantity consumed is q, where q^N is delivered by the network and $q - q^N$ is internally produced. The availability of the outside option implies an higher consumer surplus. The gain corresponds to the dark grey shaded area in Figure 3.

To compute the network market equilibrium when outside options, expressed as "backstop technologies" at cost $\bar{p}$, are available, the optimization program (1) has to be modified in the following way:

$$\max_{f_{ij},(i,j)\in A(\Gamma)} W_\Gamma = \sum_{d\in D}\left[\int_0^{f_{id}} P_d(f)df + \int_{f_{id}}^{\overline{P}_d(f)^{-1}} (P_d(f)-\overline{P}_d)df\right] - \\ - \sum_{(i,j)\in A(\Gamma)} C_{ij}(f_{ij}) \tag{3}$$

where $\overline{P}_d$ is the exogenous price of domestic production in market d (possibly very high if no outside option is available), and $\overline{P}_d(f)^{-1}$ is the quantity consumed at this price $\overline{P}_d$.

Results for the case of a backstop technology at cost 30 in market D are shown in Table 4, under the column "Option".

Figure 3: Market equilibrium and welfare with outside option

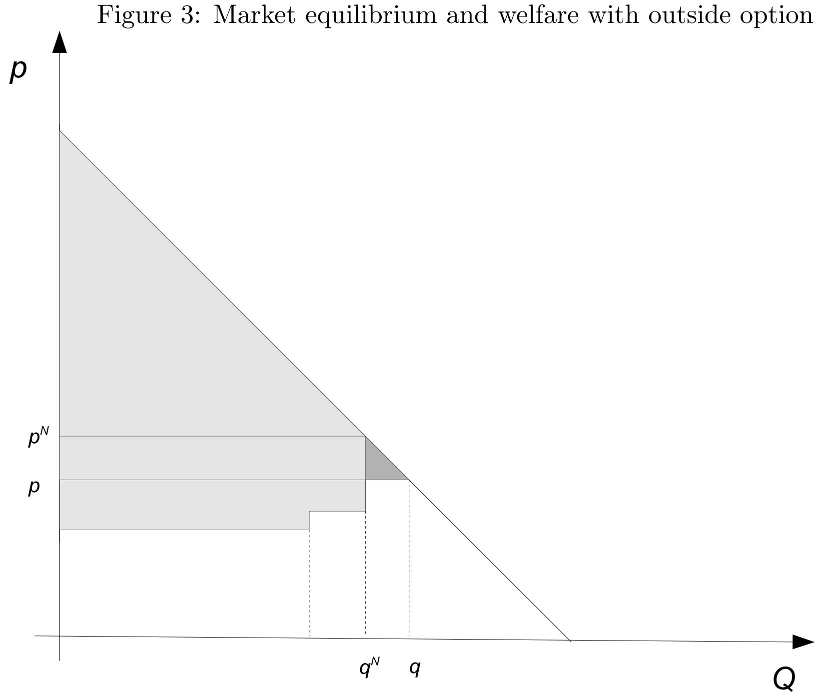


Table 4: Shapley distributions with outside option in node D

agents \ cases	**Base**	**Option**	**Diff.**
A	2115	1656	-459
B	2222	2136	-86
C	530	520	-10
D	1488	2532	+1044
E	815	325	-490
Total	***7170***	***7170***	***0***

We see that total network value is unaffected by the presence of an outside option in D, as it is not economically efficient to use the alternative technology if the network would be run cooperatively by all agents. However, the option significantly improves the bargaining power of agent D, as she makes a much bigger contribution to welfare in all possible coalitions (including the singleton D). In fact, with the possibility of autonomous domestic production, agent D gets the higher share of total surplus, at the expenses of all other agents.

4.4 Market Distortions

The analysis conducted so far assumes that all the potential surplus generated within the network is appropriated by the parties involved in the different coalitions. This hypothesis is consistent with the existence of perfectly competitive markets for network goods or, alternatively, with the presence of a monopolistic supplier, which can perfectly price discriminate among her customers. This second explanation may be defended on the ground that many international networks for oil and gas are based on block pricing schemes, that is, contracts specifying quantity volumes and total prices beforehand.

However, the model described above can be easily modified to accommodate for the existence of distortions in specific markets, like oligopolies or taxes. A common characteristic of market distortions is that the quantity volume exchanged is lower than in the optimum or, equivalently, that market prices are higher than what they would be. Exogenous reductions in consumption volumes can be easily introduced by setting appropriate values for the capacity parameters k_{jd} in distribution links (or, equivalently, by changing the cost functions C_{jd} in the more general formulation). This amounts to assume the existence of import quotas, possibly justified in terms of energy policy[4].

Alternatively, market distortions can imply taxes or profit mark-ups on top of competitive prices. These may also be easily introduced in the model by making the capacity parameters k_{jd} endogenous, dependent on market prices or delivered quantities.

4.5 Exogenous Surplus Factors

Cooperation benefits (or costs) may go beyond the network where cooperation takes place, involving multiple policy dimensions. For example, suppose that nodes in the illustrative example of Figure 1 are countries, and that countries B and D were engaged in a past conflict. A political "peace dividend", associated with cooperation between two former enemies, may then play a role in the distribution of surplus and in the justification of investments in the network infrastructure.

This case could be considered in the example above by raising the payoff of all coalitions including both B and D (e.g, by adding 10 to the surpluses

[4]For example, a government in a country may want to have a portfolio of energy sources, thereby restricting access to the least cost ones.

Table 5: Shapley distributions with exogenous surplus factors

agents \ cases	**Base**	**Ex.S.**	**Diff.**
A	2115	2115	+0.
B	2222	2227	+5
C	530	530	0
D	1488	1493	+5
E	815	815	+0.
Total	***7170***	***7180***	***+10***

obtained in the NME). The Shapley values computed after such modification are displayed in Table 5, where they are compared with the base case.

As could be expected, much of the exogenous extra gain (+10) accrues, symmetrically, to B and D. *However, part of it also goes to A and E.* Why this should be so? In order to grab the additional surplus, B and D must be part of the game, but the network must also be functioning, delivering the goods produced in A and E. If there are no suppliers in a coalition, the coalition would get zero surplus in any case, even if both B and D are into it.

5 Conclusion

In this paper, we introduced a methodology for analyzing cooperation schemes in network markets. The aggregate surplus, to be distributed among agents, was explicitly derived as a market equilibrium, where a set of supply points are connected to a set of demand points through a network structure. Several variants of the base model were discussed, showing how flexible the approach may be and how several complications, possibly arising in real-world cases, may be accounted for.

Perhaps one of the most interesting aspects of the problem of surplus allocation is the difference in incentives between the grand coalition and some sub-set of players. For example, we considered in our numerical example the possibility of expanding capacity in the link B-D. We found that, given an allocation rule like the Shapley value, the increase in surplus jointly obtainable by players B and D may easily exceed the gain for the aggregate of all players. If such a change of network characteristics requires costly investments, with a cost possibly lying in the interval between the two extra surplus values, and if the decision about undertaking the investment only requires agreement of agents B and D, then an inefficient expansion of the network (more capacity or new links) would be the outcome. The reason is that with the modified structure the two players benefit from a stronger bargaining position: individual incentives depend on the bargaining / surplus allocation scheme.

Intuitively, one would expect that the threat of changing the network structure for the benefit of a sub-coalition should increase the *current* bargaining power of the same coalition. To put it differently, if a network change would not be profitable in the aggregate, then the allocation scheme should assign *now* more surplus to those agents who are "tempted" to act on their own, in order to re-align individual and aggregate incentives. Standard equilibrium concepts like the nucleolous or the Shapley value are indeed based on what the coalitions can do "autonomously", but the degree of autonomy does not include the possibility of changing the network structure or, more generally, the determinants of surplus in the game.

This issue is not a purely theoretical construct, but is has important practical implications. Hubert and Ikonnikova (2011) and Hubert and Cobanli (2012) discuss at length the building of the North Stream gas pipeline, directly connecting offshore Russia and Germany. This is an expensive project, whose viability is only due to the fact that some countries (Baltic States, Poland, Belarus, Ukraine) are bypassed, thereby augmenting the bargaining power of Germany and Russia.

The analysis of surplus allocation when the network structure is flexible and modifiable goes beyond the scope of this paper, and it is left for future research. We just mention here that some interesting solutions have been proposed by Jackson (2005)[5], from a theoretical perspective, and by Hubert and Ikonnikova (2011)[6], in terms of applied numerical modeling.

[5]The paper discusses alternative allocation rules for flexible networks. The basic idea is that of considering, alongside the value of each coalition, the maximum value a coalition can achieve, when it is free to alter the network structure. Jackson calls this value *monotonic cover*. He then proposes a modified Shapley value, named "player-based flexible network allocation rule", where monotonic covers simply replace coalition values. The main problem with this formulation is that the sum over all players of allocations gives the value obtainable under the most efficient network, not the value of a given, existing network.

[6]Two cases are considered in this paper: the "status quo", where the network structure cannot be changed, and the "all options" one, where the capacity of all potential new links can be set by those coalitions which can control them. A third case is the so called "general game", which considers the delay associated with implementing new infrastructure projects. Assuming that the current structure of the network will be in place for a given number of periods, necessary to make new projects operational, the value function of the general game can be expressed as a linear combination of values for the status quo and the all options cases. These value functions are used in the paper to compute Shapley values, core and nucleolous allocations.

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Appendix

Using Nucleolus to allocate the cooperative surplus

The Shapley value is not the only equilibrium concept elaborated in cooperative game theory, and other allocation rules could be adopted in the analysis of cooperative networks. The nucleolus, for example, is another concept, appealing due to its relation to the "core" of a cooperative game (Schmeidler, 1969). The core is the set of all possible allocations in which the sum of values assigned to the members of any coalition is at least equal (or greater) than the surplus obtainable by the coalition, acting autonomously. The core may be empty but, when it is not, it often includes many allocations, which limits its applicability. The nucleolus is that allocation which lies in the "centre" of an non-empty core, identified by progressively increasing the payoff of potential sub-coalitions.

Although the nucleolus has a logic and appealing interpretation, its computation for large networks may turn out to be quite complicated and the interpretation not easy. For this reason, we preferred to illustrate the basic ideas in this paper using the Shapley value, which is simpler to compute and possesses a number of desirable properties (e.g., monotonicity and linearity). However, it is not too difficult to compute the corresponding nucleolus values for the simple network discussed in the paper. Here we present our findings, using the nucleolus instead of the Shapley value.

Table 6 corresponds to Table 1. With the nucleolus, demand nodes B, C and D gets relatively more value in the baseline. The effect of expanding capacity in the link B-D is qualitatively similar to the Shapley case, with two differences: (1) node A is harmed, (2) benefits for B and D are not symmetric. Indeed, symmetry is a special property of the Shapley value.

Table 7 corresponds to Table 2, where the case of expanding demand in node C is considered. Again, effects are qualitatively similar, but now the benefits accrue primarily to node C, whereas D and E are completely unaffected by the change in market size.

Table 8 corresponds to Table 3. Here we consider the existence of "exogenous instability" in node C. Of course, allocations change both in the baseline and after the relaxation of capacity constraints. The gains or losses for A, B and C are almost unaffected by the presence of instability, whereas gains for D and losses for E are amplified. Again, the impact of investment in capacity in the link B-D are not symmetric for nodes B and D.

Table 9 corresponds to Table 4. The existence of an outside option in market D strengthen the bargaining power of D and its value share, but much less than in the Shapley case. Actually, most of the gains now accrue to nodes B and C (slightly harmed under Shapley), whereas node E is unaffected (significantly harmed under Shapley).

Table 10 corresponds to Table 5. The incidence of exogenous factors is the same when the nucleolus replaces the Shapley value as an allocation rule.

Table 6: Nucleolus value surplus distributions

agents \ cases	**Base**	**Ext.**	**Diff.**
A	1850	1813	-37
B	2060	2133	+73
C	650	625	-25
D	2485	2546	+61
E	125	708	-50
Total	***7170***	***7192***	***+22***

Table 7: Nucleolus value surplus distributions

agents \ cases	**Base**	**Exp.**	**Diff.**
A	1850	1959	109
B	2060	2169	109
C	650	982	332
D	2485	2485	0
E	125	125	0
Total	***7170***	***7720***	***550***

Table 8: Nucleolus distributions with instability in C

agents \ cases	**Base**	**Ext.**	**Diff.**
A	1818	1782	-36
B	2030	2102	+72
C	585	562	-23
D	2395	2546	+151
E	212	75	-137
Total	***7040***	***7067***	***+27***

Table 9: Nucleolus distributions with outside option in node D

agents \ cases	**Base**	**Option**	**Diff.**
A	1850	1406	-444
B	2060	2174	+114
C	650	924	+274
D	2485	2541	+56
E	125	125	0
Total	***7170***	***7170***	***0***

Table 10: Nucleolus distributions with exogenous surplus factors

agents \ cases	**Base**	**Ex.S.**	**Diff.**
A	1850	1850	+0.
B	2060	2065	+5
C	650	650	0
D	2485	2490	+5
E	125	125	+0.
Total	***7170***	***7180***	***+10***