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RETAILER'S MOTIVATION IN MARKETING: BILEVEL APPROACH [1]

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Summary. We consider a vertical control distribution channel in which a manufacturer sells a single kind of good to a retailer. We assume that wholesale price discounts increase the retailer's sale motivation thus improving sales. We study both the manufacturer's and retailer's profit maximization problems as optimal control models. The controls are the discount on wholesale price (trade discount) which manufacturer present to retailer, and the part of this discount transferred by retailer to the consumer. We consider the cases when one of these control is constant and, moreover, the role of manufacturer and retailer is different.

Introduction

The sales increasing role of advertising and, more in general, of communication has been largely explored by means of dynamic and optimal control models. In particular we recall the model proposed by Nerlove and Arrow [7] where the authors take explicitly into account the role of the *goodwill* of a firm, their paper originated a research stream and a great number of publications (see e.g. the review paper [6]) which continues nowadays (see e.g. [2], [3], [5]).

In this paper we focus (cf. [4]) on the concept of retailer's sale motivation while in former models the concept of goodwill was studied instead. We consider a manufacturer that sells a single kind of good during a limited time period $[t_1, t_2]$ (e.g. it is a seasonal product). The manufacturer acts as a monopolist in a vertical channel (see [8]) selling to the only downstream firm, the retailer. To improve sales the manufacturer can produce a promotional effort by means of a wholesale price discount, and the retailer transfers a percentage of the discount to the consumer.

The model is described as a process in which the state variables are the total sales $x(t)$ during time period $[t_1, t]$ and the retailer's motivation $M(t)$ at time t. The manufacturer's control is the discount $\alpha(t)$ on wholesale price at time t, $\alpha(t) \in [A_1, A_2] \subseteq [0, 1]$. Another control is the part of wholesale price discount $\beta(t)$ transferred to the consumer at time t, $\beta(t) \in [B_1, B_2] \subseteq [0, 1]$.

We assume that the retailer's motivation is summarized by the state variable $M(t)$ whose dynamics is given by

$$\dot{M}(t) = \gamma \dot{x}(t) + \epsilon(\alpha(t) - \overline{\overline{\alpha}}),$$

where $\gamma > 0$ is sales productivity in terms of retailer's sales motivation, $\epsilon > 0$ is price discount productivity in terms of retailer motivation and $\overline{\overline{\alpha}} \in (A_1, A_2)$ is minimum discount expected by the retailer.

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The dynamics of the total amount of sales at time t, $x(t)$, is defined by

$$\dot{x}(t) = -\theta x(t) + \delta M(t) + \eta\beta(t)\alpha(t),$$

where $\delta > 0$ is retailer's motivation productivity in terms of sales, $\eta > 0$ is price discount productivity in terms of sales. We suppose that $x(t_1) = 0$ and $M(t_1) = \overline{M}$, where $\overline{M} > 0$ is the initial motivation of the retailer.

Remark that $\dot{x}(t)$ represents the sales at time t and we suppose that it coincides with the consumer's demand at time t. This means that we assume that the firm sells exactly the produced quantity.

The wholesale price at time t is $p(1 - \alpha(t))$, where p is the wholesale price when no trade discount is applied.

The total profit of the manufacturer is

$$qx(t_2) - p\int_{t_1}^{t_2} \dot{x}(t)\alpha(t)dt,$$

where $q = p - c_0$ and c_0 is unit production cost. Analogously, the total profit of retailer is

$$p\int_{t_1}^{t_2} \dot{x}(t)\alpha(t)(1 - \beta(t))dt.$$

Instead of this "multi-objective" situation, let us consider the cases when either $\alpha(t)$ or $\beta(t)$ is constant and, moreover, the role of manufacturer and retailer is different.

1. β is constant, the retailer is leader

This way for every fixed β the manufacturer's problem is as follows:

$$\begin{aligned} P_1(\beta)\ :\ \text{maximize} \quad & qx(t_2) - p\int_{t_1}^{t_2} \dot{x}(t)\alpha(t)\,\mathrm{d}t, \\ \text{subject to} \quad & \dot{x}(t) = -\theta x(t) + \delta M(t) + \eta\beta\alpha(t), \\ & \dot{M}(t) = \gamma\dot{x}(t) + \epsilon(\alpha(t) - \overline{\overline{\alpha}}), \\ & x(t_1) = 0, \quad M(t_1) = \overline{M}, \quad \alpha(t) \in [A_1, A_2]. \end{aligned}$$

The problem of this type has been considered in [4] for the case when $A_2 = q/p$ and $\theta - \gamma\delta > 0$. We have shown that the kind of optimal control depends on the sign of the parameter

$$(\theta - \gamma\delta)\eta\beta - \delta\epsilon,$$

which is an increasing function with respect to β.

A more deep investigation of the role of β could be interesting. For example, we can consider the bilevel problem in which the retailer chooses β to maximize his profit.

For Problem $P_1(\beta)$ denote the optimal control as $\alpha^*_{1,\beta}(t)$, the optimal state variables as $x^*_{1,\beta}(t)$, $M^*_{1,\beta}(t)$ and let the optimum value of the objective function be

$$F_1(\beta) = qx^*_{1,\beta}(t_2) - p\int_{t_1}^{t_2} \dot{x}^*_{1,\beta}(t)\alpha^*_{1,\beta}(t)\ dt.$$

Now let us consider the problem

$$R_1\ :\ \text{maximize}\ \ G_1(\beta) = p(1-\beta)\int_{t_1}^{t_2} \dot{x}^*_{1,\beta}(t)\alpha^*_{1,\beta}(t)\,\mathrm{d}t \quad \text{subject to} \quad \beta \in [B_1, B_2].$$

Let for Problem R_1 the optimal solution be β_1^* and the optimum value of the objective function be G_1^*, i.e.

$$G_1^* = \max_{\beta \in [B_1, B_2]} G_1(\beta) = G_1(\beta_1^*)$$

is the optimal profit of the retailer. Moreover we can compute $F_1^* = F_1(\beta_1^*)$ as the optimal profit of the manufacturer.

2. α is constant, the retailer is leader

This way for every fixed α the manufacturer's problem is as follows:

$$\begin{aligned} P_2(\alpha) \ : \ \text{maximize} \quad & (q - p\alpha)x(t_2), \\ \text{subject to} \quad & \dot{x}(t) = -\theta x(t) + \delta M(t) + \eta \alpha \beta(t), \\ & \dot{M}(t) = \gamma \dot{x}(t) + \epsilon(\alpha - \overline{\overline{\alpha}}), \\ & x(t_1) = 0, \quad M(t_1) = \overline{M}, \quad \beta(t) \in [B_1, B_2]. \end{aligned}$$

Let for Problem $P_2(\alpha)$ the optimal control be $\beta_{2,\alpha}^*(t)$, the optimal state variables be $x_{2,\alpha}^*(t)$, $M_{2,\alpha}^*(t)$ and the optimum value of the objective function be

$$F_2(\alpha) = (q - p\alpha)x_{2,\alpha}^*(t_2).$$

Now let us consider the problem

$$R_2 \ : \ \text{maximize} \ \ G_2(\alpha) = p\alpha \int_{t_1}^{t_2} \dot{x}_{2,\alpha}^*(t)(1 - \beta_{2,\alpha}^*(t))\,\mathrm{d}t \quad \text{subject to} \quad \alpha \in [A_1, A_2].$$

Let for Problem R_2 the optimal solution be α_2^* and the optimum value of the objective function be G_2^*, i.e.

$$G_2^* = \max_{\alpha \in [A_1, A_2]} G_2(\alpha) = G_2(\alpha_2^*)$$

is the optimal profit of the retailer. Moreover we can compute $F_2^* = F_2(\alpha_2^*)$ as the optimal profit of the manufacturer.

Remark that Problem $P_2(\alpha)$ is linear for every fixed α. Moreover, it is simple to show that the optimal control $\beta_{2,\alpha}^*(t)$ is constant and the objective function $G_2(\alpha)$ of Problem R_2 is quadratic. So in this case we can find explicitly the optimal controls α_2^* and $\beta_{2,\alpha_2^*}^*(t)$, the optimal state variables $x_{2,\alpha_2^*}^*(t)$ and $M_{2,\alpha_2^*}^*(t)$, and also the optimal manufacturer's and retailer's profits $F_2^* = F_2(\alpha_2^*)$ and $G_2^* = G_2(\alpha_2^*)$.

3. β is constant, the manufacturer is leader

This way for every fixed β the retailer's problem is as follows:

$$\begin{aligned} R_3(\beta) \ : \ \text{maximize} \quad & (1 - \beta) \int_{t_1}^{t_2} \dot{x}(t)\alpha(t)\,\mathrm{d}t, \\ \text{subject to} \quad & \dot{x}(t) = -\theta x(t) + \delta M(t) + \eta \beta \alpha(t), \\ & \dot{M}(t) = \gamma \dot{x}(t) + \epsilon(\alpha(t) - \overline{\overline{\alpha}}), \\ & x(t_1) = 0, \quad M(t_1) = \overline{M}, \quad \alpha(t) \in [A_1, A_2]. \end{aligned}$$

Remark that problem $R_3(\beta)$ has the same type as problem $P_1(\beta)$ (see Section 1), i.e is linear with respect to the state variables and is quadratic with respect to control. So to solve it we can use the same technique as in [4].

Let for Problem $R_3(\beta)$ the optimal control be $\alpha^*_{3,\beta}(t)$, the optimal state variables be $x^*_{3,\beta}(t)$, $M^*_{3,\beta}(t)$ and the optimum value of the objective function be

$$G_3(\beta) = (1-\beta)\int_{t_1}^{t_2} \dot{x}^*_{3,\beta}(t)\alpha^*_{3,\beta}(t)\,\mathrm{d}t.$$

Now let us consider the problem

$$P_3 \ : \ \text{maximize} \ \ F_3(\beta) = qx^*_{3,\beta}(t_2) - p\int_{t_1}^{t_2} \dot{x}^*_{3,\beta}(t)\alpha^*_{3,\beta}(t)\,\mathrm{d}t \quad \text{subject to} \quad \beta \in [B_1, B_2].$$

Let for Problem P_3 the optimal solution be β^*_3 and the optimum value be F^*_3, i.e.

$$F^*_3 = \max_{\beta\in[B_1,B_2]} F_3(\beta) = F_3(\beta^*_3)$$

is the optimal profit of the manufacturer. Moreover we can compute $G^*_3 = G_3(\beta^*_3)$ as the optimal profit of the retailer.

4. α is constant, the manufacturer is leader

This way for every fixed α the retailer's problem is as follows:

$$\begin{aligned} R_4(\alpha) \ : \ \text{maximize} \quad & p\alpha\int_{t_1}^{t_2} \dot{x}(t)(1-\beta(t))\,\mathrm{d}t, \\ \text{subject to} \quad & \dot{x}(t) = -\theta x(t) + \delta M(t) + \eta\alpha\beta(t), \\ & \dot{M}(t) = \gamma\dot{x}(t) + \epsilon(\alpha - \overline{\overline{\alpha}}), \\ & x(t_1) = 0, \quad M(t_1) = \overline{M}, \quad \beta(t) \in [B_1, B_2]. \end{aligned}$$

Remark again that Problem $R_4(\alpha)$ has the same type as Problem $P_1(\beta)$ (see Section 1).

Let for Problem $R_4(\alpha)$ the optimal control be $\beta^*_{4,\alpha}(t)$, the optimal state variables be $x^*_{4,\alpha}(t)$, $M^*_{4,\alpha}(t)$ and let the optimum value of the objective function be

$$G_4(\alpha) = p\alpha\int_{t_1}^{t_2} \dot{x}^*_{4,\alpha}(t)(1-\beta^*_{4,\alpha}(t))\,\mathrm{d}t.$$

Now let us consider the problem

$$P_4 \ : \ \text{maximize} \ \ F_4(\alpha) = (q - p\alpha)x^*_{4,\alpha}(t_2) \quad \text{subject to} \quad \alpha \in [A_1, A_2].$$

Let for Problem P_4 the optimal solution be α^*_4 and the optimum value be F^*_4, i.e.

$$F^*_4 = \max_{\alpha\in[A_1,A_2]} F_4(\alpha) = F_4(\alpha^*_4)$$

is the optimal profit of the manufacturer. Moreover we can compute $G^*_4 = G_4(\alpha^*_4)$ as the optimal profit of the retailer.

5. Concluding remarks and Discussions

In this paper we explore bilevel programming approach. In other words, we used (in some sense) the concept of "Stackelberg equilibrium".

The 4 considering cases may not have the same "economical" interest. For example, we can imagine the situation, when the manufacturer "thinks" about two strategy: a strategy of the retailer and his own policy. It may be that if the firm is "large" (or "famous", etc.), then the situation "α is constant" is rather natural. Indeed, it is convenient for the manufacturer to have the constant "game rules" in its relations with the retailer, especially if the time interval is rather short (as in the case of seasonal marketing). Analogously, the situation "β is constant" can be natural if the firm is "small" (or "new", etc.). Further, the question of manufacturer's/retailer's leadership can be considered by the manufacturer in order to maximize his profit taking into account that also the retailer wants to maximize his own profit. From this point of view it can be interesting to compare the values F_i^* for different $i \in I = \{1, \ldots, 4\}$ and/or to compare the values G_i^* for different $i \in I$, and/or to find conditions under which $\exists\, i_0 \in I : \; F_{i_0}^* \geq F_j^* \,,\; G_{i_0}^* \geq G_j^* \; \forall j \in I$.

Finally, it can be interesting to develop this approach, taking into account production period (cf. [3]) and also considering the case of multi-segment marketing (see e.g. [1]).

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