Chapter 1
Entropy and systemic risk measures
Entropia e misure di rischio sistemico

Monica Billio, Roberto Casarin, Michele Costola, Andrea Pasqualini

Abstract The aim of this paper is the construction of an early warning indicator for systemic risk using entropy measures. The analysis is based on the cross-sectional distribution of marginal systemic risk measures such as Marginal Expected Shortfall, Delta CoVaR and network connectedness. These measures are conceived at a single institution for the financial industry in the Euro area. Entropy indicators show forecasting abilities in predicting banking crises revealing to be an effective tool as early warning indicator.
Abstract *Lo scopo di questo lavoro é la costruzione di un indicatore di preallarme per il rischio sistemico attraverso misure di entropia. L'analisi si basa sulla distribuzione cross-sezionale di misure di rischio sistemico come Marginal Expected Shortfall, Delta-CoVaR e misure di network. Queste misure sono ottenute a livello di singola istituzione finanziaria della zona euro. Le misure di entropia mostrano promettenti abilitá di previsione delle crisi bancarie rivelando di essere uno strumento di preallerta efficace.*

Key words: entropy, systemic risk measures, early warning indicators, aggregation

---

Authors' research is supported by funding from the European Union, Seventh Framework Programme FP7/2007-2013 under grant agreement SYRTO-SSH-2012-320270.
Monica Billio
University Ca' Foscari of Venice, Fondamenta San Giobbe 873, 30121 Venezia, e-mail: billio@unive.it

Roberto Casarin
University Ca' Foscari of Venice, Fondamenta San Giobbe 873, 30121 Venezia, e-mail: r.casarin@unive.it

Michele Costola
University Ca' Foscari of Venice, Fondamenta San Giobbe 873, 30121 Venezia, e-mail: michele.costola@unive.it

Andrea Pasqualini
University Ca' Foscari of Venice, Fondamenta San Giobbe 873, 30121 Venezia, e-mail: 828170@stud.unive.it

1.1 Introduction

The relevance of latest financial crises deserved much attention in modeling systemic events which represent a potential threats to financial stability of an interconnected economic and financial system. Given the endogenous nature of systemic risk, its measurement represents a complex task which involves different financial and macroeconomic aspects. In fact, the implications of systemic risk is relevant both in the macro and micro perspectives. At macro level, the aim of policy makers such as European Central Bank (ECB), European Systemic Risk Board (ESRB) and Federal Reserve (FED) is to guarantee the stability of the banking system. Bisias et al. (2012) presents a good survey on systemic risk measures in literature. The motivation of our study relies on the capability to detect and predict likeness of systemic events defined as financial distress condition. We apply here the approach recently developed in Billio et al. (2015). Our aggregation relies on the use of entropy applied to a feature distribution estimated on the market such as the cross-sectional systemic risk measures at a given time. In fact, movements of entropy built on these measures may reveal first signs of changes on systemic risk.

1.2 System Entropy

The novel feature of our application in finance is to apply entropy on systemic risk. Intuitively, in the proximity of a systemic event, the financial institutions that are those that cause this event, because they are the systemic relevant or frail, may be the first to react and thus to provoke a structural change in the cross-sectional distribution. In this regard, entropy can detect structural changes in the (cross-sectional) distribution of these measures. In many applications the object of interest is a function of the probability distribution which summarizes the information content of the distribution. One of the most used probability functional is the entropy.
Let $\pi_t = (\pi_{1t}, \dots, \pi_{mt})$, $t = 1, \dots, T$, be a sequence of vector of probabilities associated to the cross-section distribution of a given feature of the financial assets measured over time t on the market. In this paper we apply entropy to π_t using different proposals for entropy presented in the literature.

The Shannon entropy (Shannon, 1948), also known as Gibbs-Boltzman-Shannon, is defined as

$$H_S(\pi_t) = -\sum_{j=1}^{m} \pi_{jt} \log \pi_{jt} \tag{1.1}$$

where $m < \infty$.

1.3 Features of financial market participants

Our systemic risk variables of interest x_{it} used in the entropy calculation are the Marginal Expected Shortfall (MES), the Δ-CoVar and the network connectedness.
As regards the MES, we follow Acharya et al. (2010) and starting from a series of asset returns r_{it}, $t = 1,\dots,T$, where i denotes the asset, MES_{it} is defined as the expected value of r_{it} when a reference asset (or a reference market) is in its "worst state" and is experiencing losses. This state is identified when the return of the reference asset r_{mt} is below a given quantile q_k. That is, for $k = 0.05$,

$$MES_{it} = \mathbb{E}\{r_{it}|r_{mt} < q_{5\%}\}. \tag{1.2}$$

As it turns out, MES filters data in order to pick loss cascades during market downturns, thus allowing for a specific analysis of tail events.
The Δ-CoVar proposed by Adrian and Brunnermeier (2011) represents the value at risk (VaR) of the financial system conditional on institutions being under distress. Let us define the VaR and CoVar as follows The authors define a contribution of a given institution to systemic risk as the difference between the CoVaR conditional on the institution being under distress and the CoVaR in the median of the institution (Δ-CoVaR), that is

$$\Delta CoVaR_{mit,q} = CoVaR_{mit,q} - CoVaR_{mit,0.5}, \tag{1.3}$$

where r_{it} is the asset return value of the institution i, r_{mt} represents the system and the CoVaR is defined as $\mathbb{P}(r_{jt} \leq CoVaR_{jit,q}|r_{it} = VaR_{it,q}) = q$. $CoVaR_{mit,0.5}$ represents the VaR of the system at time t when returns of asset i are at 50^{th} percentile.

Following Billio et al. (2012), we extract a network from the financial asset returns and then focus on some features of this network.
A network is defined as a set of nodes $V_t = \{1,2,\dots,n_t\}$ and directed arcs (edges) between nodes. The network can be represented through an n_t-dimensional adjacency matrix A_t, with the element $a_{ijt} = 1$ if there is an edge from i directed to j with $i, j \in V_t$ and 0 otherwise. The matrix A_t is estimated by using a pairwise Granger causality approach to detect the direction and propagation of the relationships between the institutions. The in-out degree measure is then defined as

$$IO_{it} = \sum_{j=1}^{n_t} a_{ijt} + \sum_{j=1}^{n_t} a_{jit} \tag{1.4}$$

$t = 1,\dots,T$, where causality implies $a_{jit} = 1$ if j causes i, $a_{ijt} = 1$ if i causes j, and $a_{ijt} = a_{jit} = 1$, if there is a feedback relationship.

As a reference measure we also consider the dynamic causality index (DCI), proposed by (Billio et al., 2012), which is defined as

$$DCI_t = \binom{n_t}{2}^{-1} \sum_{i=1}^{n_t} \sum_{j=1}^{n_t} a_{ijt} \tag{1.5}$$

$t = 1, \dots, T$, when $(DCI_t - DCI_{t-1}) > 0$, there is an increase of system interconnectedness.

1.4 Empirical application

We consider the daily closing price series for the European firms of the financial sector from 1st January 1985 to 12th May 2014. The dataset is composed for a total of 437 financial institutions considering the MSCI Europe index as the proxy for the European market. To estimate the systemic risk measures, we use a rolling window approach (Zivot and J., 2003; Billio et al., 2012; Diebold and Yilmaz, 2014) with a window size of 252 daily observations. Different indicators have been presented in the literature to detect economic and financial crises. In this regard, we use the banking crisis for the European market presented in Alessi and Detken (2014), which represents one of the target variables monitored by European Systemic Risk Board.[1] The variable takes value 1 (0) if there is crisis (not crisis). To study the effectiveness of the entropy-based indicators in detecting conditions of financial distress, we set a logistic model with entropy indicators for MES, $\Delta CoVaR$ and In-Out network degree as covariates. The estimation results from the logit specification are presented in Table 1.1. All entropies are significant at 1% confidence-level. The best explanatory variable is the entropy based on $\Delta CoVaR$.
For sake of brevity, we report in the paper the estimations for Shannon entropy but estimations for Tsallis and Renyi entropy confirm these results.

The percent of correctly predicted indicators for the estimated logit models are 65.32% (MES), 77.51% (ΔCoVaR) and 69.79% (In-Out degree). The entropy based on $\Delta CoVar$ confirms the superior ability in predicting banking crisis.
Concluding, this paper shows promising forecasting abilities of entropy indicators applied to cross-sectional systemic risk measures. In an early warning system perspective, further investigation should be performed using other risk measures and target variables.

References

Acharya, V. V., Pedersen, L. H., Philippon, T., and Richardson, M. P. (2010). Measuring systemic risk. Technical report.

Adrian, T. and Brunnermeier, M. K. (2011). Covar. Technical report, National Bureau of Economic Research.

Alessi, L. and Detken, C. (2014). Identifying excessive credit growth and leverage.

[1] We assume that there is an European banking crisis if more than on country is in (banking) crisis at time t. As Robustness Check we formulate two alternative banking crisis variables by varying the number of countries required to have crisis in all the European area. The results are confirmed.

Table 1.1: Logit specification where the dependent variable is the banking crisis from Alessi and Detken (2014) and the explanatory variables are Shannon entropies.

	Crisis Indicator		
(Intercept)	-5.3340***	-6.3911***	-6.4137***
	(0.1996)	(0.1641)	(0.1851)
$H_S(MES)$	10.9669***		
	(0.4151)		
$H_S(\Delta CoVaR)$		15.5536***	
		(0.4001)	
$H_S(InOut)$			20.0670***
			(0.5817)
R-squared	0.1140	0.2905	0.1952
Adjusted-R-squared	0.1139	0.2905	0.1951
LogLikelihood	-4455.92	-3790.80	-4107.88
LLR	0.0854	0.2219	0.1568
AIC	8915.84	7585.61	8219.77
BIC	8929.56	7599.33	8233.49
Sample	jan-86	jan-86	jan-86
	dec-12	dec-12	dec-12
Obs	7044	7044	7044

Billio, M., Casarin, R., Costola, M., and Pasqualini, A. (2015). A entropy-based early warning indicator for systemic risk. *University Ca'Foscari of Venice, Dept. of Economics Working Paper Series*.

Billio, M., Getmansky, M., Lo, A. W., and Pelizzon, L. (2012). Econometric measures of connectedness and systemic risk in the finance and insurance sectors. *Journal of Financial Economics*, 104(3):535 – 559.

Bisias, D., Flood, M., Lo, A. W., and Valavanis, S. (2012). A survey of systemic risk analytics. *Annu. Rev. Financ. Econ.*, 4(1):255–296.

Diebold, F. and Yilmaz, K. (2014). On the Network Topology of Variance Decompositions: Measuring the Connectedness of Financial Firms. *Journal of Econometrics*, 182(1):119–134.

Shannon, C. E. (1948). A mathematical theory of communication. *The Bell System Technical Journal*, 27(3):379–423.

Zivot, E. and J., W. (2003). *Modelling Financial Time Series with S-Plus®*. Springer.