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Igor Bykadorov, Andrea Ellero, Stefania Funari,
Sergey Kokovin and Pavel Molchanov

PAINFUL BIRTH OF TRADE UNDER CLASSICAL MONOPOLISTIC COMPETITION

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Igor Bykadorov,[1] *Andrea Ellero,*[2] *Stefania Funari,*[3]
Sergey Kokovin,[4] *and Pavel Molchanov*[5]

Painful Birth of Trade under Classical Monopolistic Competition[6]

Abstract

In the standard Krugman (1979) non-CES trade model, several asymmetric countries typically lose from increasing trade costs. However, all countries transiently benefit from such increase at the moment of closing trade, under almost-prohibitive trade costs (i.e., near autarky, which is possible only under non-CES preferences). In other words, during trade liberalization the first step from autarky to trade is necessarily harmful. Our explanation rests on market distortion and business destruction effects.

Keywords: Trade gains, monopolistic competition, variable elasticity of substitution, free trade, autarky.

JEL Codes: F12, L13, D43.

[1]Sobolev Institute of Mathematics SB RAS, NSU, and NSUEM, bykadorov.igor@mail.ru

[2]Department of Management of University Ca' Foscari Venezia, ellero@unive.it

[3]Department of Management of University Ca' Foscari Venezia, funari@unive.it

[4]National Research University Higher School of Economics, and NSU, skokov7@gmail.com

[5]National Research University Higher School of Economics, PSMolchanov@edu.hse.ru

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1 Introduction

Gains from trade is an evergreen topic. In New Trade theory, this theme again generated vivid discussion (see [Melitz and Redding, 2015], [Behrens et al., 2014]) after [Arkolakis et al., 2012] puzzled theorists with surprisingly low estimated gains amounting to not more than 1.4% of GDP for the US. One of the possible explanations is the constant elasticity of substitution (CES) assumption, which dominates in this discussion. Rare exceptions include [Behrens and Murata, 2007] and [Dhingra, 2013] that find the assumption of variable elasticity of substitution (VES) to be more realistic and discuss related "pro-competitive" effects, promising additional gains. However, under VES, [Arkolakis et al., 2015] finds *lower* estimated trade gains than under comparable CES demand (we will return to this fact in our concluding remarks). Moreover, under VES, free trade may show welfare loss in comparison with autarky. [Bykadorov et al., 2015b] show such harmful trade under general-form preferences and non-linear costs. Necessary and sufficient condition for welfare diminishing free trade equilibrium is "misaligned revenue and utility," originating from [Dhingra and Morrow, 2012]. The explanation is distortion, aggravated by trade. However, misalignment assumption looks too stringent to support belief in real-life harmful *free* trade on these grounds.

By contrast, this paper discovers harmful *costly* trade under very high trade costs, near autarky—under *any* additive VES utilities enabling autarky. As to trade gains, they occur near free trade, at least under realistic preferences. In other words, the gains from gradual trade liberalization are non-monotonic, and are eventually positive, but in the beginning are negative. Thus, the *first step from autarky is harmful.*[7]

A similar effect is known for oligopoly ([Brander and Krugman, 1983]), but our setting is different and standard for the New Trade theory. Our model is a version of Krugman's general one-sector monopolistic competition ([Krugman, 1979]), with unspecified additive utilities, and without outside good. Homogeneous firms use one production factor (labor), having uniform fixed and marginal costs, and consumers are also identical (similar model is studied in [Zhelobodko et al., 2012], [Mrázová and Neary, 2014], [Bykadorov et al., 2015a] but for symmetric countries). For analytical tractability, in the difficult case of asymmetric countries, our model includes only two types of economies: $\mathscr{G}$-countries with great populations, and $\mathscr{L}$-countries, which can be little or equal to $\mathscr{G}$. Labor markets do clear, and trade is balanced. At the equilibrium, any pair of countries can trade or not, depending on the level of iceberg trade cost. Thereby we introduce a new, convenient version of an *asymmetric multi-country* Krugman's trade model (hopefully useful for other studies). However, the assumption of two country-types is mostly expositional, and not crucial for our results. It is not essential for the effect of decreasing welfare near autarky, because it is natural to expect that trade liberalization invites country types into trade one-by-one, not every type simultaneously. Therefore, the "painful birth of trade" effect is general, and allows for the presence of arbitrary number of country types, though not everybody is involved into the first step of liberalization.

In our model, the evolution of trade is driven by the decreasing trade cost coefficient τ and progresses in four steps. At first, liberalization departs from "complete autarky," i.e., from prohibitively high cost τ^{out}. At this stage, "selective trade" arises among certain pairs of countries, afterwards "comprehensive trade" among everybody follows. Finally, the evolution arrives at free trade under zero trade cost ($\tau = 1$). Here, near free trade, welfare locally increases in *each* country with liberalization (under realistic, decreasingly elastic utilities). It is generally not

[7] This effect is firstly found in our discussion paper [Bykadorov et al., 2015a] for two countries and generalized now to multi-country world and incomplete autarky.

surprizing. However, increasing gains for $\mathscr{G}$-countries appear non-evident in view of gradually decreasing and vanishing wage advantage (higher wages in a larger country). Unlike in a CES economy, which has negligible change of variety (mass of firms) under free trade, in our model, welfare increases despite a *decreasing* mass of firms. From the policy viewpoint, this fact means that "business destruction" by trade liberalization is not necessarily harmful.

More subtle and unexpected is the effect at the beginning of liberalization, near complete autarky. Autarky is possible under any additive utility that has a choke-price, which is a finite derivative at zero. This feature ensures locally-decreasing elasticity of utility at zero and increasingly elastic demand near autarky. Under this assumption, we prove that all prices decrease during this initial stage of trade liberalization near autarky, and export increases, but variety (the mass of firms) shrinks due to competition from foreign firms (see Figure 1). The latter effect dominates, so *welfare deteriorates* in each country, i.e., harmful trade takes place. In other words, market distortion is aggravated by the initiation of trade.[8]

The explanation for this phenomenon is "business stealing," which means externalities. Indeed, when all firms and consumers start trading, they correctly anticipate direct mutual benefits, but they do not take into account the spillovers to other firms and consumers in general equilibrium in the form of shrinking domestic variety of goods in response to emerging import. This noticeable reduction in variety is *not compensated* by small arising import. This fact is not intuitive and becomes clear only from algebraic argumentation. The mechanism of welfare reduction in this model is quite different from that of the oligopolistic "painful birth of trade," well-known after [Brander and Krugman, 1983] (see the comparison after Proposition 2). By contrast, in the standard New Trade theory this effect looks new and surprising. Probably, it was overlooked so far, because of dominant CES model, which excludes complete autarky. Additionally, our simulations (Figure 1) show how this paradoxical welfare gain from increasing trade costs is tightly connected with increase in variety, and also ensures that this effect is not negligible in size and zone of presence. The gain in welfare can occur rather far from complete autarky, at the moment when some pair of countries "partially" terminates mutual trade, while each still maintains trade with other countries (third parties). Relaited claim (Conjecture 3) formulates a realistic condition, when a mutual pairwise trade barrier between two countries is beneficial for them, even in the presence of continuing trade with third parties. This finding means that *harmful small-scale trade is not exotic*, and appears robust to any compositions of countries and any additive (non-CES) preferences enabling autarky. Additionally, in our discussion of consumer surplus, we provide a simple explanation for a negative answer to the question posed in [Arkolakis et al., 2015]: "Does the fact that trade liberalization affects firm-level markups, as documented in many micro-level studies, make ... [gains from trade] ... larger or smaller?"

Finally, the interesting effect of harmful trade is found to be robust to extensions, e.g., the assumption of firms' heterogeneity. However, the magnitude of harm becomes smoother than without heterogeneity, hardly noticeable. Therefore the effect is unlikely to be observed empirically in practice. Yet, practical is the idea that *essential gains occur mainly near free trade*, while *a small first step towards trade brings negative or zero welfare gains*.

To summarize: our paper supports [Arkolakis et al., 2015] in the sense that a VES economy may produce smaller trade gains than a CES one. Moreover, we predict that due to VES effects, trade liberalization initially must be harmful. From the policy viewpoint, this claim sounds

[8] This effect happens in all three possible situations at trade termination: either couple $\mathscr{L} - \mathscr{L}$ breaks its trade, or $\mathscr{G} - \mathscr{L}$, or $\mathscr{G} - \mathscr{G}$ couple.

protectionist, but in fact it only suggests *not to liberalize trade gradually*, and that is better to *jump* over initial losses, or just wait until transport costs decrease sufficiently for massive trade. This recipe looks somewhat similar to the "infant industry" argument for postponing trade, but the mechanism is very different, and is not connected with time and learning. It is not young trade but low-volume trade that appears detrimental in this model.

2 Model: Trade among Gullivers and Lilliputians

The world economy consists of $K+1$ great countries—"Gullivers" (e.g., the US, the EU and China) and $k+1$ "little" countries—"Lilliputians" ($K \geq 0$, $k \geq 0$). To rely on the proofs and explanations in our working papers [Bykadorov et al., 2015c], [Bykadorov et al., 2015a], the exposition of the present model follows almost the same notations; all variables for Gullivers (respectively, Lilliputians) are displayed in capital (resp., lowercase) characters, such notation economizes on the use of subscripts. The only production factor is labor, supplied inelastically by L identical consumers/workers in each Gulliver country (resp., l in Lilliputian), and we assume $L \geq l$, not excluding $L = l$. Furthermore, $\Gamma = L \cdot (K+1) + l \cdot (k+1)$ is the total number of consumers in the world. Our single-sector economy exhibits monopolistic competition and involves an endogenous interval $[0, N]$ (resp., $[0, n]$) of identical firms producing varieties of a differentiated good, one variety per firm. Wages can differ, namely, $w^{\mathscr{G}} \equiv w$ denotes the wage in each big country, whereas $w^{\mathscr{L}} \equiv 1$ is the (normalized) wage in each small country.

Each consumer in a Gulliver country maximizes her utility using three kinds of variables: X_i – the domestic consumption of the i-th variety; Y_i – the imported consumption of the i-th variety from another big country; and z_i – the imported consumption of the i-th variety from a Lilliputian country. A representative consumer in a Gulliver country maximizes (subject to the budget constraint) her utility given by

$$\int_0^N u(X_i)\, di + K \cdot \int_0^N u(Y_i)\, di + (k+1) \cdot \int_0^n u(z_i)\, di \to \max_{\{X,Y,z\}},$$

$$\int_0^N P_i^X X_i di + K \cdot \int_0^N P_i^Y Y_i di + (k+1) \cdot \int_0^n p_i^z z_i di \leqslant w,$$

where prices P_i^X, P_i^Y, p_i^z correspond to consumptions X_i, Y_i, and z_i respectively.

Similarly, the problem of a representative consumer in a Lilliputian country is

$$\int_0^n u(x_i)\, di + k \cdot \int_0^n u(y_i)\, di + (K+1) \cdot \int_0^N u(Z_i)\, di \to \max_{\{x,y,Z\}},$$

$$\int_0^n p_i^x x_i di + k \cdot \int_0^n p_i^y y_i di + (K+1) \cdot \int_0^N P_i^Z Z_i di \leqslant 1,$$

where prices p_i^x, p_i^y, P_i^Z correspond to consumptions x_i, y_i, Z_i, respectively (Z_i denoting export from a Gulliver to a Lilliputian, x_i – domestic consumption, y_i – import from similar small country).

The restrictions on elementary utility u (as in [Mrázová and Neary, 2014]), imposed for existence and uniqueness of each consumer's/producer's response to any market situation (and equilibrium), are standard for VES models: u is thrice differentiable, strictly concave, increasing at least on some interval $[0, \breve{z})$, where $\breve{z} \equiv \arg\max_z u(z)$ denotes the satiation point (possibly infinite). Additionally, using the Arrow-Pratt concavity operator $r_g(z) \equiv -\frac{zg''(z)}{g'(z)}$ (defined for any function g), we formulate the restrictions on concavity of u, concavity of u' and behavior of the functions at zero as

$$\{0 < r_u(z) < 1 \ \& \ r_{u'}(z) < 2 \ \forall z \in [0, \breve{z})\}, \ u(0) = 0,$$

$$u'(0) < \infty, \ u''(0) > -\infty, \ u'''(0) \in (-\infty, \infty).$$

In other words, u has finite derivatives at zero, which differs from the CES assumption.

Using these assumptions and the consumer's first order conditions (FOC), we standardly derive six inverse demand functions, one for each kind of variables:

$$P_i^X = \frac{u'(X_i)}{\Lambda}, \quad P_i^Y = \frac{u'(Y_i)}{\Lambda}, \quad p_i^z = \frac{u'(z_i)}{\Lambda}, \tag{1}$$

$$p_i^x = \frac{u'(x_i)}{\lambda}, \quad p_i^y = \frac{u'(y_i)}{\lambda}, \quad P_i^Z = \frac{u'(Z_i)}{\lambda}, \tag{2}$$

Here Λ and λ are the Lagrange multipliers for the Gulliver/Lilliputian budget constraints. The marginal utility of income Λ serves as the main market aggregator in country $\mathscr{G}$, similar to the price index (the role of λ for country $\mathscr{L}$ is analogous).

Producers. The output (size) of the $i-th$ firm in a Gulliver country is given by

$$Q_i(X_i, Y_i, Z_i) = L \cdot X_i + K \cdot \tau \cdot L \cdot Y_i + (k+1) \cdot \tau \cdot l \cdot Z_i$$

while the output (size) of the $i-th$ firm in a Lilliputian country is

$$q_i(x_i, y_i, z_i) = l \cdot x_i + k \cdot \tau \cdot l \cdot y_i + (K+1) \cdot \tau \cdot L \cdot z_i,$$

where $\tau \geq 1$ is the usual regular iceberg trade cost coefficient. Then, total costs take the form

$$C(Q_i) = c \cdot Q_i + F, \ \ C(q_i) = c \cdot q_i + F,$$

where F is the fixed requirement and c is the marginal requirement of labor. It is standard to show the symmetry of producers' behavior, so, it is possible to omit the index i.
Introducing the "normalized revenue" $R(\xi) = u'(\xi) \cdot \xi$ and using the demand functions, the profit maximization program of a firm in a Gulliver country can be written as

$$\max_{(X,Y,Z)} \Pi \equiv L \cdot \frac{R(X)}{\Lambda} + K \cdot L \cdot \frac{R(Y)}{\Lambda} + (k+1) \cdot l \cdot \frac{R(Z)}{\lambda} - w \cdot C(Q(X,Y,Z)). \tag{3}$$

Similarly, the profit maximization program in a Lilliputian country is

$$\max_{(x,y,z)} \pi \equiv l \cdot \frac{R(x)}{\lambda} + k \cdot l \cdot \frac{R(y)}{\lambda} + (K+1) \cdot L \cdot \frac{R(z)}{\Lambda} - C(q(x,y,z)). \tag{4}$$

Standardly, each firm perceives the market aggregates (λ, Λ) parametrically, being "small."

Labor balances mean full employment of labor at the equilibrium. In the two kinds of countries they can be written as

$$N \cdot C(Q) = L, \qquad n \cdot C(q) = l. \tag{5}$$

Trade balances express the idea that the value of all exported goods equals the value of imported goods. In Gulliver and Lilliputian countries the balances are, respectively

$$N \cdot \left(K \cdot L \cdot \frac{R(Y)}{\Lambda} + (k+1) \cdot l \cdot \frac{R(Z)}{\lambda} \right) = N \cdot K \cdot L \cdot \frac{R(Y)}{\Lambda} + n \cdot (k+1) \cdot L \cdot \frac{R(z)}{\Lambda},$$

$$n \cdot \left(k \cdot l \cdot \frac{R(y)}{\lambda} + (K+1) \cdot L \cdot \frac{R(z)}{\Lambda} \right) = n \cdot k \cdot l \cdot \frac{R(y)}{\lambda} + N \cdot (K+1) \cdot l \cdot \frac{R(Z)}{\lambda}.$$

Rearranging the equations above, these two trade balances turn into one equation (which would not be the case under three or more types of countries):

$$N \cdot l \cdot \frac{R(Z)}{\lambda} = n \cdot L \cdot \frac{R(z)}{\Lambda}.$$

Further, substituting N and n from the labor balances (5), we rewrite each trade balance without N and n as

$$\frac{R(Z)}{\lambda \cdot C(Q)} - \frac{R(z)}{\Lambda \cdot C(q)} = 0. \tag{6}$$

Zero-profit conditions (free-entry) at equilibrium are

$$\Pi = 0, \qquad \pi = 0\ , \tag{7}$$

and a producer's first order conditions (FOC) are

$$\frac{\partial \Pi}{\partial X} = 0, \quad \frac{\partial \Pi}{\partial Y} = 0, \quad \frac{\partial \Pi}{\partial Z} = 0, \quad \frac{\partial \pi}{\partial x} = 0, \quad \frac{\partial \pi}{\partial y} = 0, \quad \frac{\partial \pi}{\partial z} = 0. \tag{8}$$

Standardly, the second order condition (SOC) under linear costs can be rewritten in terms of normalized revenue R as

$$R''(\xi) < 0, \quad \xi \in \{X, Y, Z, x, y, z\},$$

which holds true under our assumptions ($r_{u'}(z) < 2$) and guarantees symmetry of producers' behavior.

(Symmetric) **equilibrium** is the bundle of consumptions, wages, prices, masses of firms, and market aggregates

$$\left(X^*, Y^*, Z^*, x^*, y^*, z^*, \Lambda^*, \lambda^*, w^*, N^*, n^*, P^{X^*}, P^{Y^*}, P^{Z^*}, p^{x^*}, p^{y^*}, p^{z^*}\right)$$

that satisfy all the equations imposed, namely:

- utility maximization (1) and (2),
- labor and trade balances (5) and (6),
- free entry (7),
- profit maximization (8).

As is usual, the budget constraints entail the labor balances, which explains too many constraints for five groups of variables.

Welfare. Each Gulliver country's welfare (measured in consumers' total utility) is expressed as

$$W^{\mathscr{G}} = L \cdot (N \cdot (u(X) + K \cdot u(Y)) + n \cdot (k+1) \cdot u(z))$$

and each Lilliputian country's welfare is measured similarly:

$$W^{\mathscr{L}} = l \cdot (n \cdot (u(x) + k \cdot u(y)) + N \cdot (K+1) \cdot u(Z)).$$

Using the labor balance (5), both welfare functions can be reformulated without variables N, n, as

$$W^{\mathscr{G}} = L \cdot \left(L \cdot \frac{u(X) + K \cdot u(Y)}{C(Q)} + l \cdot (k+1) \cdot \frac{u(z)}{C(q)} \right), \tag{9}$$

$$W^{\mathscr{L}} = l \cdot \left(l \cdot \frac{u(x) + k \cdot u(y)}{C(q)} + L \cdot (K+1) \cdot \frac{u(Z)}{C(Q)} \right). \tag{10}$$

Global welfare is not addressed in this analysis.

3 Welfare Consequences of Trade Liberalization

Our goal is to show U-shaped evolution of per-country gains from trade during liberalization, which means decreasing trade costs. Specifically, our analytical proof concerns only initial losses (decreasing welfare) near autarky, and ultimate gains (increasing welfare) near free trade, whereas simulations describe the evolution of welfare and variety in more detail.

3.1 Numerical example

To begin with, let us discuss a numerical example given in Figure 1. Unlike our propositions, it tells the complete story of trade liberalization, not only autarky and free trade stages. The evolution of equilibria is displayed under the following parameters: three big countries ($K = 2$) have populations of $L = 3$ each, and three small countries ($k = 2$) have $l = 1$. Marginal cost is $c = 3.33$ and fixed cost is $F = 1$. The consumer's elementary utility belongs to the Augmented Hyperbolic Absolute Risk Aversion type (which includes CES and almost-linear demand as special cases):

$$u(x, a, m) = \frac{(a+1)^{m+1} \left((a+x)^{1-m} - a^{1-m}\right)}{1-m} - ax$$

with $a = 1$ and $m = 0.25$. In this example, when trade cost τ increases, the countries drop out of trade in three steps. At first, trade stops among the three Gullivers, then it ceases between any Gulliver and any Lilliputian, and finally it disappears among three Lilliputians, who are the most interested in trade (such a sequence of trade termination, with smaller countries halting trade later, appears to be a general rule, although we cannot prove this fact so far). At this final moment τ_3^{out} the world reaches complete autarky. In formal terms, we observe here three trade-breaking points $\tau_1^{out} < \tau_2^{out} < \tau_3^{out}$. For any $\varepsilon > 0$, trade goes below such points, but it stops at the points, i.e., $[Y(\tau_1^{out}) = 0,\ Y(\tau_1^{out} - \varepsilon) > 0]$, $[Z(\tau_2^{out}) = z(\tau_2^{out}) = 0,\ Z(\tau_2^{out} - \varepsilon) > 0,\ z(\tau_2^{out} - \varepsilon) > 0]$ and $[y(\tau_3^{out}) = 0,\ y(\tau_3^{out} - \varepsilon) > 0]$. In our simulation, these points are $\tau_1^{out} \approx 1.43$, $\tau_2^{out} \approx 1.59$ and $\tau_3^{out} \approx 1.79$. The evolution of such

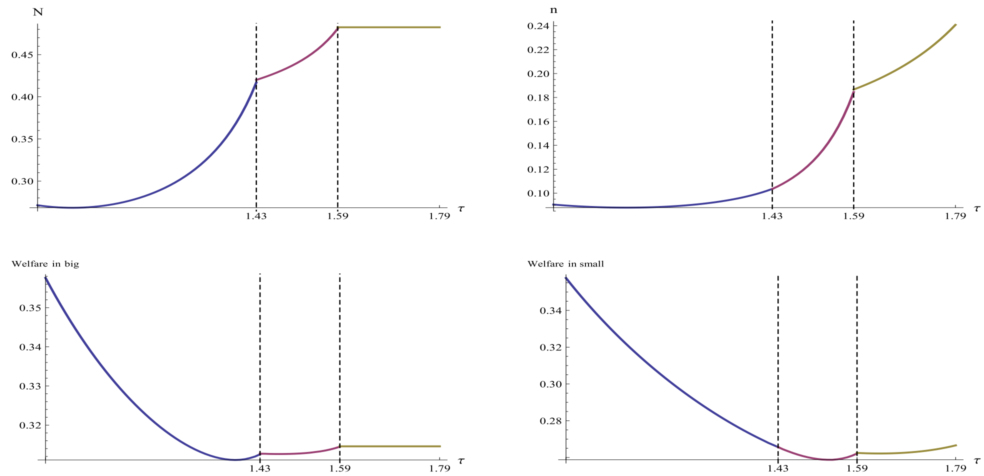

Figure 1: Changing variety and welfare under growing trade cost ($L = 3 = 3l$, $K = 2 = k$, $c = 3.33$, $F = 1$).

economy shown in Figure 1 describes the behavior of two equilibrium variables – variety and welfare under increasing τ.

One can see that welfare is non-monotonic and has a unique global minimum, both for Gullivers and Lilliputians. Gullivers here reach minimum welfare approximately at $\tau \approx 1.38$ within the initial interval $\tau \in (1,\, \tau_1^{out}) = (1,\, 1.43)$, in which all countries do trade. After this, their welfare grows in τ, reaches point τ_1^{out}, at which point the Gulliver-Gulliver trade ceases, and grows further until $\tau_2^{out} \approx 1.59$, at which point Gullivers drop out of any trade and therefore become indifferent to trade cost τ. By contrast, Lilliputians reach minimum welfare at $\tau \approx 1.50$ within another interval $[\tau_1^{out},\, \tau_2^{out})$, and they do not have a local minimum near τ_1^{out}. After minimum at $\tau \approx 1.50$, their welfare grows in τ until the point τ_2^{out}, stabilizes for a moment, then grows again in τ until the complete autarky point $\tau_3^{out} = 1.79$. Thus, in this example, *at each moment when trade ceases, welfare grows for those who cease to trade.* In other words, we always observe the “happy agony of pairwise tradeï¿½ under increasing trade costs, which can be rephrased as “painful birth of pairwise trade” under decreasing trade costs. In the upper part of the graph, two curves depicting the masses of firms $N(\tau)$, $n(\tau)$ show that the increase of trade cost typically *pushes overall variety up* (thereby, consumption of each variety goes down). This fact sheds some light on the mechanism affecting welfare. It is increasing variety that enhances welfare whenever it grows with τ. Another observation is that the interval of harmful trade is rather significant in this example: $\tau \in (1.38,\, 1.79]$, and the amount of harm (the magnitude of the drop in welfare) is also noticeable. This loss amounts to about 14% of the maximal possible gain from free trade. Moreover, gains from trade occur only on the interval [0, 1.38). Comparing it with typical estimates of the contemporary average trade-cost coefficient (e.g., [Melitz and Redding, 2015] reports $\tau \approx 1.8$), one can suggest that contemporary trade gains in a big country indeed can be very small as claimed in [Arkolakis et al., 2012], even negative, or that future gains from further trade liberalization may many times exceed those observed today. Of course, these rough comparisons are far from serious calibration, we cannot insist on them. Instead, our theoretical message is that *essential trade gains should occur near free trade, not in the beginning or middle of liberalization.*

The crude geometric explanation of “the reward at the end of the road” of liberalization can be illustrated through the demand triangle under linear demand. Under trade liberalization

$\tau \to 1$, the marginal cost for export τc decreases from infinity to c, it means decrease of the horizontal line restricting the profit rectangle $(p - \tau c)q$ from below. If not for exit of firms in response, this change would influence both, the maximal profit *quadratically* as $(a - \tau c)^2/4$ and consumer surplus as $(a - \tau c)^2/8$ (where a denotes the choke-price). It means that both these components of "gains" in the beginning ($a \approx \tau c$) should have a zero derivative in τ, then slightly increase, but hike up sharply in the end. However, in general equilibrium, exit of firms somewhat diminishes the consumers' welfare during this process and thus generates the welfare pit observed in the beginning of liberalization in Figure 1, thereby enforcing the difference between the beginning and the end of the process. Generalizing this reasoning to non-linear, more convex demands, would make the welfare increase even steeper than quadratic one. This explains the huge difference between initial and final gains. This example shows, that *during liberalization, half of the way from autarky to free trade may go in vain.* This looks as strange as "almost negligible" trade gains found by [Arkolakis et al., 2012]. Common sense would expect the opposite: that the first gulp of freedom is the most desired one by countries "thirsty for trade," the last step being less important because of "satiation with freedom." Surprisingly, geometry and algebra tell us the opposite: the last gulp contains the greatest rewards.

3.2 General propositions

Now we prove that some effects observed in this example are robust to various modifications of preferences. The main assumption $u'(0) < \infty$ of our analysis excludes only utilities which do not generate autarky like CES functions $u(x) = x^\rho$ (but include functions $u(x) = (x + \varepsilon)^\rho$ arbitrarily close to CES). Under such general conditions, it is not easy to prove the U-shape (quasi-convexity) of whole welfare curve, but we are able to show analytically, that welfare decreases at the beginning and increases at the end of trade liberalization. To demonstrate this, we obtain the total derivative for the equilibrium equations with respect to τ (see Supplementary Appendix) and arrive at the following two propositions about changing equilibria. The first one looks natural: at free trade conditions, a growth of trade cost brings harm to everybody, in other words, liberalization is benefitial near free trade. Small countries react more sensitively than big countries.

Proposition 1. *Under pro-competitive utility ($r'_u > 0$), when trade cost τ increases near free trade ($\tau \approx 1$), domestic consumption, output and all prices also increase, while the mass of firms, imports and welfare $W^{\mathscr{G}}$, $W^{\mathscr{L}}$ decrease in each country; moreover, the speed of these changes obey the following inequalities*

$$\frac{dx}{d\tau} > \frac{dX}{d\tau} > 0, \qquad \frac{dq}{d\tau} > \frac{dQ}{d\tau} > 0, \qquad \frac{dN}{d\tau} < \frac{dn}{d\tau} < 0, \qquad \frac{dn}{d\tau} \cdot \frac{1}{n} < \frac{dN}{d\tau} \cdot \frac{1}{N} < 0 \qquad (\tau \approx 1).$$

Thus, prices react to trade cost pro-competitively under $r'_u > 0$, and the impact of trade costs on firm size is stronger for smaller countries but the impact on the mass of firms is weaker. A decrease in welfare is natural. More surprising is another proposition. It describes the "happy agony of trade" between two countries when τ grows, or the "painful birth of trade" when τ decreases, i.e., trade freeness $\phi \equiv 1/\tau$ increases. In its formulation, to encompass any pair of traders: (Lilliputian-Lilliputian), (Gulliver-Gulliver) or (Gulliver-Lilliputian), we replace (with a little abuse of notation) our upper-case notations X, N with lower-case letters x^j, n^j and denote all imports Y or Z or z or y — by variable y^{ij} where $i, j \in \{\mathscr{G}, \mathscr{L}\}$.

Proposition 2. *Starting from complete autarky ϕ^{aut}, when trade liberalization begins,*[9] *and any two countries of types $i, j \in \{\mathscr{G}, \mathscr{L}\}$ start trading, the local reaction of domestic consumption levels x^i, x^j is negligible ($\left(dx^k/d\phi\right)|_{\phi=\phi^{aut}} = 0,\ \ k \in \{\mathscr{G}, \mathscr{L}\}$); consumption of imports increases ($(dy^{ij}/d\phi)\,|_{\phi=\phi^{aut}} > 0,\ \ i, j \in \{\mathscr{G}, \mathscr{L}\}$), whereas the mass of firms and welfare decrease in both countries:*

$$\frac{dn^k}{d\phi}|_{\phi=\phi^{aut}} < 0,\ \ \frac{dW^k}{d\phi}|_{\phi=\phi^{aut}} < 0,\ \ k \in \{\mathscr{G}, \mathscr{L}\}.$$

So, ϕ^{aut} belongs to a mesurable interval $[\phi^{aut}, \phi^1)$ of freenes ϕ where trade brings harm (in comparison with autarky) to all trading countries.

Proof of happy agony of trade under two symmetric countries. To explain the mechanism of this effect, we prove the proposition in the simple case of two small symmetric countries (see general proof in Appendix). Let $K = 0$, $k = 0$, $l = 1 = L$ and $w = 1$. The equilibrium equations will then contain only two FOCs and one zero-profit condition:

$$c = \frac{R'(x)}{\lambda},\ c\tau = \frac{R'(z)}{\lambda} \tag{11}$$

$$C(Q) = \frac{R(x)}{\lambda} + \frac{R(z)}{\lambda}. \tag{12}$$

To show that $x'_\tau = 0$ and $z'_\tau < 0$ at the autarky point, we totally differentiate these conditions in τ:

$$0 = \frac{R''(x)}{\lambda} x'_\tau - \frac{R'(x)}{\lambda^2} \lambda'_\tau, \tag{13}$$

$$c = \frac{R''(z)}{\lambda} z'_\tau - \frac{R'(z)}{\lambda^2} \lambda'_\tau, \tag{14}$$

$$x'_\tau + z + \tau z'_\tau = \frac{R'(x)}{\lambda} x'_\tau + \frac{R'(z)}{\lambda} z'_\tau - \left(\frac{R(x)}{\lambda^2} + \frac{R(z)}{\lambda^2}\right) \lambda'_\tau. \tag{15}$$

Recalling that $z = 0 = R(0)$ at autarky, then substituting equations (11) into (15) we obtain the derivative of λ:

$$c\,(x'_\tau + \tau z'_\tau) = c\,(x'_\tau + \tau z'_\tau) - \left(\frac{R(x)}{\lambda^2}\right) \lambda'_\tau \ \Rightarrow\ \lambda'_\tau = 0.$$

Plugging this result into the remaining equations (13)-(14) we achieve $x'_\tau = 0$ and the sign of z'_τ as needed:

$$c = \frac{R''(z)}{\lambda} z'_\tau \ \Rightarrow\ z'_\tau < 0.$$

To find welfare changes at autarky, we totally differentiate the country's reduced-form welfare W (9) in trade costs τ:[10]

[9] Trade liberalization means that trade cost τ decreases, ϕ increases.

[10] In the reduced-form welfare (9), the loss from decreasing variety $\Delta N < 0$ is equivalent to the loss from increasing cost of production $\Delta C > 0$.

$$W'_\tau(x,z) = \frac{\left[u'(x)x'_\tau + u'(z)z'_\tau\right] C(Q) - C'(Q)\cdot Q'_\tau \cdot [u(x)+u(z)]}{C^2(Q)} =$$

$$= \frac{\left[u'(x)x'_\tau + u'(z)z'_\tau\right] C(Q) - c\cdot [x'_\tau + z + \tau z'_\tau]\cdot [u(x)+u(z)]}{C^2(Q)}.$$

To simplify this expression, we use conditions (11)-(12) and the properties $z = 0 = u(0) = R(0)$, $x'_\tau = 0$ at the autarky point:

$$W'_\tau(x,z) = \frac{u'(z)z'_\tau \frac{R(x)}{\lambda} - \frac{R'(z)}{\lambda}u(x)z'_\tau}{C^2(Q)} = \tag{16}$$

$$= \frac{z'_\tau}{C^2(Q)\lambda}\left[u'(z)\frac{u'(x)x}{u(x)}u(x) - \left(u'(z)+u''(z)z\right)u(x)\right] = \frac{z'_\tau u'(z)u(x)}{C^2(Q)\lambda}\left[\frac{u'(x)x}{u(x)} - 1\right] > 0.$$

Here, because $\frac{u'(x)x}{u(x)} < 1$ due to the concavity of the utility function u, and $z'_\tau < 0$ at autarky, we conclude that $W'_\tau(x,z) > 0$, i.e., welfare increases in τ. **Q.E.D.**

The formulae above explain why increasing trade cost τ brings benefit near autarky: two effects oppose each other. The harm from diminishing imports z is displayed by the negative first summand in the numerator of marginal welfare in (16), whereas the positive second summand indirectly shows welfare from an increasing mass of firms and variety in each country. Further algebraic transformations show that the latter effect does outweight the first one (no economic intuition can explain why!). Regarding this fact from the opposite perspective, trade liberalization near autarky always brings harm, because its impact on import consumption is not strong enough to outweigh the negative impact of business destruction and consequent reduction of product variety.

This effect (harmful trade near autarky) is new in New Trade literature. Under the CES assumption, harm was never found because CES excludes complete autarky (i.e., excludes any zeros in the export matrix). However, a similar effect, a harmful birth of trade is well known since 1983 in oligopolistic models with homogeneous good ([Brander and Krugman, 1983]). In this paper, the effect stems from strategic behavior and market power in oligopoly, differing from our mechanism of losses, which involves distortion among differentiated varieties within monopolistic competition (a more realistic market structure). In modern literature based on monopolistic competition models, only [Chen and Zeng, 2014] finds similar harm in a two-factor trade model with footloose capital, but does not provide a clear explanation. We suppose that the effect in [Chen and Zeng, 2014] stems from labor/capital substitution and international capital migration. Therefore, our "harm from trade" based on distortion appears to be a phenomenon novel to New Trade. As a cautionary warning, we must mention [Behrens et al., 2014] that criticize any conclusions about welfare under monopolistic competition, because empirically we infer the consumers' utilities only from some demand integration, whereas the unknown constant of integration may influence our welfare predictions.

Partial termination of trade. Let us discuss a more subtle situation than *complete* autarky. Should *any* agony of pairwise trade caused by a growing cost bring benefit to the

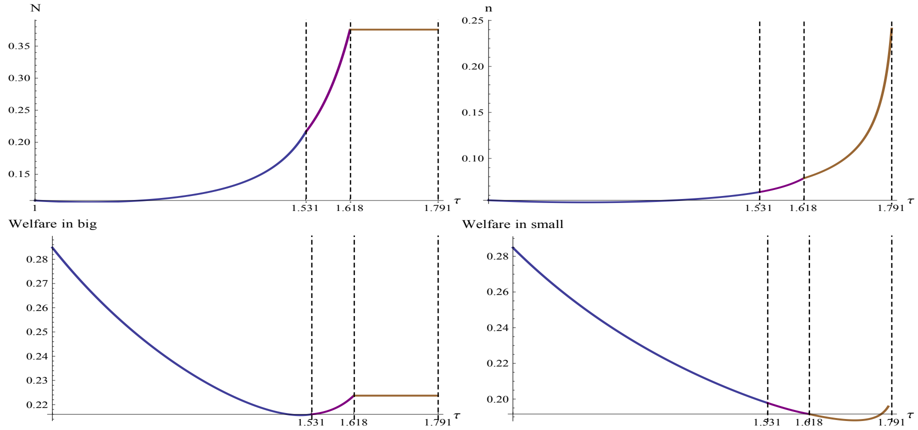

Figure 2: Changing variety and welfare under growing trade cost τ when small countries are numerous ($K = 2,\ k = 30,\ L = 2,\ l = 1$).

traders? This is the case at point τ_2^{out} in our example in Figure 1. Indeed, near the intermediate termination point τ_2^{out} welfare locally increases for everybody. By contrast, near τ_1^{out} welfare increases only for Gullivers, terminating their mutual trade but not for Lilliputians (a third party). Here, any parties stopping trade always win. By contrast, in our second example, when Gullivers stop trade with Lilliputians, the former win but the latter lose.

Example 2 describes a world with numerous Lilliputian countries, and few big countries, the parameters set at $K = 2,\ k = 30,\ L = 2,\ l = 1,\ F = 1,\ c = 3.33,\ a = 1,\ m = 0.25$. Then the impact of increasing trade cost τ is described by the following simulated figures.

To comprehend non-reciprocal gains from $\mathscr{G} - \mathscr{L}$ trade termination here, observe that the large group of Lilliputians is the most important trading partner for any country, $\mathscr{G}$ or $\mathscr{L}$. Consequently, Lilliputians do not care when the two Gullivers terminate trade with $\mathscr{L}$ group, only these Gullivers become happier at this moment (as in Proposition 2) and also when terminating trade with each other. By contrast, during these two dropouts of $\mathscr{G}$ group, the Lilliputians continue bearing losses from increasing cost and decreasing trade with each other. Only their own dropout – the transition to complete autarky, brings a (transitory) benefit to $\mathscr{L}$ group. Thus, we observe that it is not guaranteed that *everbody* always benefit from partially terminating trade (under increasing common trade cost τ). From similar arguments, it looks probable that there exists a situation where *nobody* benefits from increasing τ at a non-final termination of trade. To build such an example, the mass of Lilliputians must be very high to outweigh any small benefits to Gullivers from $\mathscr{G} - \mathscr{G}$ trade termination.

Impact of separately changing trade cost τ_{ij} between a couple of countries. One more similar question is the following: assume that termination of pairwise trade between some countries i, j occurs because *only their bilateral* trade coefficient τ_{ij} (e.g., tariff) increases, while the trade costs τ_{ks} for the rest of the world remain constant and other trade flows remain positive. Without formulating a cumbersome general model of such situation, we are now able to sketch out the scheme of proof that such separate pairwise "agony of trade" should be "happy." These arguments indirectly show why the governments of two trading countries may have reasons to apply some tariff or non-tariff barriers to suppress *negligible* mutual trade, while maintaining substantial trade with third parties (countries).

For example, suppose that world consists of $(k + 1)$ Lilliputians and only two Gullivers numbered #1, #2. What happens, when the mutual trade coefficient $\tau_{12} = \tau_{21}$ between Gullivers decreases, starting from a prohibitive value τ_{21}^{out}, while all other coefficients coefficients τ_{ks} in the world remain constant? One can prove that the level of Gullivers' trade with third parties, Lilliputians, remains almost unaffected by decreasing separate $\mathscr{G} - \mathscr{G}$ coefficients $\tau_{12} = \tau_{21}$.[11] Then estimating the welfare consequences of separate trade liberalization between countries #1, #2 amounts to comparing the benefit of emerging imports $\Delta Y > 0$ to the loss from decreasing variety $\Delta N < 0$. The latter is equivalent, as we know, to the loss from increasing cost of production $\Delta C > 0$. So, we can differentiate the reduced-form welfare function (9) in variable τ_{12} which is the same as differentiating it in Y (at $Y = 0$). We obtain the expression

$$\frac{d}{d\tau} W^{\mathscr{G}} \approx -\frac{\partial}{\partial Y} W^{\mathscr{G}} = -\frac{L^2 K c}{C(Q)} \cdot \left[\frac{u'(0)}{c \cdot \tau_{21}} - \frac{u(X) L + u(z) l \cdot (k+1)}{C(Q)} \right].$$

Applying the same transformations as in the proof of Proposition 2, we can conclude that loss from increasing trade freeness (starting from τ_{21}^{out}) occurs if and only if

$$\frac{u'(0)}{c \cdot \tau_{21}^{out}} < \frac{u(X) L + u(z) l \cdot (k+1)}{f + L \cdot X \cdot c + l \cdot (k+1) \cdot Z \cdot c \cdot \tau}.$$

This condition, after replacing cost by revenue and then simplifying, becomes

$$\frac{u'(0)}{R'(0)} = 1 < \frac{u(X) L + u(z) l \cdot (k+1)}{R(X) L + \frac{R(Z)}{\lambda/\Lambda} l \cdot (k+1)}.$$

Further, using trade balance $\frac{R(Z)}{\lambda/\Lambda} = \frac{R(z) \cdot C(Q)}{C(q)}$ between this Gulliver and remaining trade partners ($k+1$ Lilliputians) we obtain

$$\frac{u'(0)}{R'(0)} = 1 < \frac{u(X) L + u(z) l \cdot (k+1)}{R(X) L + \frac{R(z) \cdot C(Q)}{C(q)} l \cdot (k+1)}.$$

We see, that whenever a firm in Gulliver has (plausibly) equal or bigger total cost than a firm in Lilliputian ($C(Q) \geq C(q)$), exploiting $u(z) > R(z)$, we arrive at the conclusion of "harmful trade." Indeed, each summand in the numerator of the right-hand side is larger than its counterpart in the denominator. Using a reasonable weaker assumption $\frac{R(z) \cdot C(Q)}{u(z) \cdot C(q)} \leq 1$, we summarize these arguments by formulating the following claim (which is incompetely proved, because the model with asymmetric τ_{ij} was not formulated exactly and its trade balances are non-trivial).

Conjecture. *Consider the situation with near-zero trade level $Y \approx 0$ between two Gullivers $i, j \in \{\mathscr{G}, \mathscr{G}\}$, where the relative size of firms in Gullivers and Lilliputians is not too big, in the sense that*

$$\frac{C(Q)}{C(q)} \leq \frac{u(z)}{R(z)},$$

and assume that the separate pairwise trade coefficient τ_{ij} decreases (i.e., freeness ϕ_{ij} increases), and these two countries start trading ($dY^{ij}/d\phi_{ij} > 0, \quad Y^{ij} \approx 0$). Then welfare decreases in both these countries:

$$\frac{dW^k}{d\phi_{ij}} < 0, \quad k \in \{i, j\}.$$

[11] Indeed, repeating the arguments in the proof of Proposition 2, it is possible to prove that trade magnitudes (Z, z) between $\mathscr{G}$ and $\mathscr{L}$ should have a *zero* first derivative in τ_{12} at τ_{21}^{out}, because Z, z (as well as X) depend on τ only indirectly, through variables λ, Λ, whereas these market statistics have zero derivatives in τ at τ^{out}.

Supporting the assumption $\frac{C(Q)}{C(q)} \le \frac{u(z)}{R(z)}$ used here, we must say that we have always observed smaller firms in the bigger country ($\frac{C(Q)}{C(q)} < 1$) in all numerical examples that we have explored. This fact is guaranteed near autarky under the realistic assumption of increasingly-elastic demand function, because in this case production of a variety decreases in population of a closed economy (see [Zhelobodko et al., 2012]); also, Proposition 1 indirectly states $\frac{C(Q)}{C(q)} \le 1$ in situations near free trade.[12] Under another demand function—CES, firm size $Q = q$ does not change, but the assumption holds as well. However, being unable so far to prove inequality $\frac{C(Q)}{C(q)} \le 1$ with arbitrary τ, u, we have used a weaker inequality $\frac{R(z) \cdot C(Q)}{u(z) \cdot C(q)} \le 1$ as an assumption. In particular, the latter inequality always holds true for similar countries having $Q \approx q$, because $\frac{R(z)}{u(z)} < 1$.

Thus, exploring rather realistic situations during trade liberalization, when first small countries start trading, than big ones, we have found that any two big countries (maybe already trading with other partners) *should suppress their arising mutual trade whenever it is small in volume*, for welfare reasons.

Heterogeneous firms. Another modification to the model shows that the harmful trade effect near complete autarky is robust not only to various preferences but also to heterogeneous firms. Following [Bykadorov et al., 2015a], we argue that firms' heterogeneity does not matter at the moment of closing trade. Argumentation for this claim is as follows. When trade cost τ reaches the critical level τ^{out}, the last firms that leave the scene of trade are only the most efficient ones, this group of actors is homogeneous (or approximately homogeneous, if we take a measurable subset of approximately-best firms). Thereby, at the moment of complete autarky the heterogeneous economy reacts to trade costs as a homogeneous one. In other words, Proposition 2 applies here, ensuring that *trade freeness is initially harmful even under heterogeneous firms.*[13]

As to the magnitude of this effect, heterogeneity should smoothen the initial welfare loss during trade liberalization, at least when continuous spectrum of firms is essentially heterogeneous. When freeness increases and a new cohort of firms step into trade bringing related initial losses, the old cohort already trading increases its exports, which may outweigh the losses from new cohort. Moreover, the "cutoff" threshold for the surviving firms may also change. Trade pushes out the inefficient firms, and thereby increases welfare through the selection effect emphasized in [Melitz and Redding, 2015].

In view of these considerations and noticeable firms' heterogeneity in real life, the chances to discover harmful initial trade in the empirical data are slim. Instead of searching for such examples, this paper only argues that *noticeable* gains from trade liberalization are not likely to occur at the onset. In particular, surprisingly small estimates of current trade gains in [Arkolakis et al., 2012] and [Arkolakis et al., 2015] can be explained as follows: the world is now in the middle of globalization, far from free trade. However, our explanation of why the VES assumption in [Arkolakis et al., 2015] brings smaller gains than the CES assumption in [Arkolakis et al., 2012] is different. One can simply compare a demand "triangle" with a CES

[12] Indeed, in free trade, $Q = q$; moreover (see Lemma 2 in Appendix), near free trade

$$\frac{dQ}{d\tau} = (\Gamma - L) \cdot \frac{r_u' \cdot X^2}{r_R \cdot E_R} \le (\Gamma - l) \cdot \frac{r_u' \cdot X^2}{r_R \cdot E_R} = \frac{dq}{d\tau}$$

under $L \ge l$; therefore, $Q \ge q$ when $\tau \approx 1$, $\tau > 1$.

[13] To be more pricise, to prove applicability of Proposition 2, one should formulate a cumbersome general model and exploit zero derivatives of variables λ, Λ, negligibly influencing "unimportant variables," as explained in the proof of Conjecture 3 above.

demand curve to linear demand. Both demands should have the same slope at the "realistic" price level, because their elasticities are similarly estimated at this point only in the papers named. Which model promises a higher trade gains? The linear demand is strictly inside the CES "demand triangle," and has a smaller total area (potential consumer surplus). Such inclusion makes any *estimated* gains from trade under a flat demand smaller than the *estimated* gains under CES hypothesis, and thus explans the driving force of the effect in [Arkolakis et al., 2015].

Returning to the explanation of the surprising "painful birth of trade," we repeat that the reason for such welfare loss is "business stealing" by imports. The mass of domestic firms shrinks too fast in response to emerging imports, *being insufficiently compensated* by the arising insubstantial import. Comparative strength of these opposite effects is demonstrated algebraically in (16) but lacks an intuitive explanation because of interplay of several forces. The harmful distortion results from market spillovers inflicted by each trading firm and its buyers. These indirect spillovers turn out to be stronger than the direct benefit of trade to the actors who voluntarily start it (similarly, increasing freedom sometimes leads to a worse Nash equilibrium in game theory).

In conclusion, we should stress the contrast between the (surprising) harm during the first steps of gradual trade liberalization—and the large welfare gains at the last step of globalization. Only the last stage brings essential benefits. The moral of the story for trade policy is: ***Do not liberalize trade gradually but rather jump over the pit at once, since the main reward is at the end of the road.***

4 Appendix

4.1 Preliminaries

For our proofs we introduce necessary notations.

In what follows, for function $f(\xi)$, we use notations for its elasticity

$$E_f(\xi) = \frac{f'(\xi)\cdot\xi}{f(\xi)}$$

and its Arrow-Pratt measure

$$r_f(\xi) = -\frac{f''(\xi)\cdot\xi}{f'(\xi)} = -E_{f'}(\xi).$$

Note that concavity of utility u restricts its elasticity as $E_u(\xi) < 1\ \forall\xi > 0$.[14]

In comparative statics of equilibria, for any equilibrium variable φ (for consumption, price, etc.) its *total* elasticity w.r.t. trade cost τ is denoted as

$$\mathcal{E}_\varphi = \mathcal{E}_{\varphi/\tau} = \frac{d\varphi}{d\tau}\cdot\frac{\tau}{\varphi}.$$

Also recall the notation for "normalized revenue" $R(\xi) = u'(\xi)\cdot\xi$.

[14]Indeed, for every$\forall\xi > 0, E_u(\xi) < 1 \iff u'(\xi)\cdot\xi - u(\xi) < 0 \quad \forall\xi > 0$. Consider the function $g(\xi) = u'(\xi)\cdot\xi - u(\xi)$. One has $g'(\xi) \equiv u''(\xi)\cdot\xi < 0\ \forall\xi > 0$ due to strictly concavity of u. But $g(0) = u(0) = 0$. Hence $g(\xi) < 0\ \forall\xi > 0$, i.e., $u'(\xi)\cdot\xi - u(\xi) < 0\ \forall\xi > 0$.

Now the Second Order Conditions (SOC) for profit maximization can be written as (cf. [Zhelobodko et al., 2012])

$$r_{u'}(\xi) < 2 \quad \forall \xi \geq 0. \tag{17}$$

Recall that the output (size) of each firm in a Gulliver country is given by

$$Q = Q(X, Y, Z) = L \cdot X + K \cdot \tau \cdot L \cdot Y + (k+1) \cdot \tau \cdot l \cdot Z,$$

while the output (size) of each firm in a Lilliputian country is

$$q = q(x, y, z) = l \cdot x + k \cdot \tau \cdot l \cdot y + (K+1) \cdot \tau \cdot L \cdot z.$$

Further, we express masses N and n from the labor balances

$$N = \frac{L}{C(Q)}, \qquad n = \frac{l}{C(q)}$$

and respectively express the trade balance as

$$TB \equiv \frac{R(Z)}{\lambda \cdot C(Q)} - \frac{R(z)}{\Lambda \cdot C(q)} = 0.$$

Recall the expressions for profits

$$\Pi \equiv L \cdot \frac{R(X)}{\Lambda} + K \cdot L \cdot \frac{R(Y)}{\Lambda} + (k+1) \cdot l \cdot \frac{R(Z)}{\lambda} - w \cdot C(Q(X, Y, Z)),$$

$$\pi \equiv l \cdot \frac{R(x)}{\lambda} + k \cdot l \cdot \frac{R(y)}{\lambda} + (K+1) \cdot L \cdot \frac{R(z)}{\Lambda} - C(q(x, y, z)).$$

Thus, the system of equilibrium equations includes all FOC, two zero-profit conditions and trade balance:

$$\frac{\partial \Pi}{\partial X} = 0, \quad \frac{\partial \Pi}{\partial Y} = 0, \quad \frac{\partial \Pi}{\partial Z} = 0, \quad \frac{\partial \pi}{\partial x} = 0, \quad \frac{\partial \pi}{\partial y} = 0, \quad \frac{\partial \pi}{\partial z} = 0. \tag{18}$$

$$\Pi = 0, \qquad \pi = 0, \tag{19}$$

$$TB = 0. \tag{20}$$

4.2 Comparative statics w.r.t. τ: general case

Totally differentiating the equilibrium conditions in trade cost τ, we get the following basic lemma for comparative statics, describing the total derivatives/elasticities of the equilibrium variables.

Lemma 1. *The total derivatives of the equilibrium variables can be expressed as*

$$\frac{dX}{d\tau} = \frac{1}{\tau} \cdot \frac{\mathcal{E}_\Lambda + \mathcal{E}_w}{\frac{R''(X)}{R'(X)}}, \qquad \frac{dY}{d\tau} = \frac{1}{\tau} \cdot \frac{\mathcal{E}_\Lambda + \mathcal{E}_w + 1}{\frac{R''(Y)}{R'(Y)}}, \qquad \frac{dZ}{d\tau} = \frac{1}{\tau} \cdot \frac{\mathcal{E}_\lambda + \mathcal{E}_w + 1}{\frac{R''(Z)}{R'(Z)}}, \tag{21}$$

$$\frac{dx}{d\tau} = \frac{1}{\tau} \cdot \frac{\mathcal{E}_\lambda}{\frac{R''(x)}{R'(x)}}, \qquad \frac{dy}{d\tau} = \frac{1}{\tau} \cdot \frac{\mathcal{E}_\lambda + 1}{\frac{R''(y)}{R'(y)}}, \qquad \frac{dz}{d\tau} = \frac{1}{\tau} \cdot \frac{\mathcal{E}_\Lambda + 1}{\frac{R''(z)}{R'(z)}}, \tag{22}$$

where $\mathcal{E}_\Lambda$, $\mathcal{E}_\lambda$, $\mathcal{E}_w$ *are the total elasticities of* Λ, λ, w *w.r.t.* τ*, being the solutions to three linear equations*

$$L\cdot\left(\frac{R(X)}{\Lambda}+K\cdot\frac{R(Y)}{\Lambda}\right)\cdot(\mathcal{E}_\Lambda+\mathcal{E}_w)+(k+1)\cdot l\cdot\frac{R(Z)}{\lambda}\cdot(\mathcal{E}_\lambda+\mathcal{E}_w)+\frac{R'(Z)}{\lambda}\cdot(K\cdot L\cdot Y+(k+1)\cdot l\cdot Z)=0, \tag{23}$$

$$(K+1)\cdot L\cdot\frac{R(z)}{\Lambda}\cdot\mathcal{E}_\Lambda+l\cdot\left(\frac{R(x)}{\lambda}+k\cdot\frac{R(y)}{\lambda}\right)\cdot\mathcal{E}_\lambda+\frac{R'(z)}{\Lambda}\cdot(k\cdot l\cdot y+(K+1)\cdot L\cdot z)=0, \tag{24}$$

$$E_C(Q)\cdot\mathcal{E}_Q-E_C(q)\cdot\mathcal{E}_q-E_R(Z)\cdot\mathcal{E}_Z+E_R(z)\cdot\mathcal{E}_z+(\mathcal{E}_\lambda-\mathcal{E}_\Lambda)=0, \tag{25}$$

and

$$\frac{dQ}{d\tau}=L\cdot\frac{dX}{d\tau}+K\cdot L\cdot\left(Y+\tau\cdot\frac{dY}{d\tau}\right)+(k+1)\cdot l\cdot\left(Z+\tau\cdot\frac{dZ}{d\tau}\right), \tag{26}$$

$$\frac{dq}{d\tau}=l\cdot\frac{dx}{d\tau}+k\cdot l\cdot\left(y+\tau\cdot\frac{dy}{d\tau}\right)+(K+1)\cdot L\cdot\left(z+\tau\cdot\frac{dz}{d\tau}\right). \tag{27}$$

Proof of Lemma 1.[15] Totally differentiating the equilibrium conditions (18)–(20) w.r.t. τ, we obtain the system

$$\frac{d}{d\tau}\left(\frac{\partial\Pi}{\partial X}\right)=0,\qquad \frac{d}{d\tau}\left(\frac{\partial\Pi}{\partial Y}\right)=0,\qquad \frac{d}{d\tau}\left(\frac{\partial\Pi}{\partial Z}\right)=0,$$

$$\frac{d}{d\tau}\left(\frac{\partial\pi}{\partial x}\right)=0,\qquad \frac{d}{d\tau}\left(\frac{\partial\pi}{\partial y}\right)=0,\qquad \frac{d}{d\tau}\left(\frac{\partial\pi}{\partial z}\right)=0,$$

$$\frac{d\Pi}{d\tau}=0,\qquad \frac{d\pi}{d\tau}=0,\qquad \frac{d}{d\tau}(TB)=0.$$

Due to additive separability of profits in its variables, the former six equations (total derivatives of FOC (18)) are simplified as

$$\frac{R''(X)}{\Lambda}\cdot\frac{dX}{d\tau}-\frac{R'(X)}{\Lambda^2}\cdot\frac{d\Lambda}{d\tau}-C'\cdot\frac{dw}{d\tau}=0,$$
$$\frac{R''(Y)}{\Lambda}\cdot\frac{dY}{d\tau}-\frac{R'(Y)}{\Lambda^2}\cdot\frac{d\Lambda}{d\tau}-\tau\cdot C'\cdot\frac{dw}{d\tau}-w\cdot C'=0,$$
$$\frac{R''(Z)}{\lambda}\cdot\frac{dZ}{d\tau}-\frac{R'(Z)}{\lambda^2}\cdot\frac{d\lambda}{d\tau}-\tau\cdot C'\cdot\frac{dw}{d\tau}-w\cdot C'=0,$$
$$\frac{R''(x)}{\lambda}\cdot\frac{dx}{d\tau}-\frac{R'(x)}{\lambda^2}\cdot\frac{d\lambda}{d\tau}=0,$$
$$\frac{R''(y)}{\lambda}\cdot\frac{dy}{d\tau}-\frac{R'(y)}{\lambda^2}\cdot\frac{d\lambda}{d\tau}-C'=0,$$
$$\frac{R''(z)}{\Lambda}\cdot\frac{dz}{d\tau}-\frac{R'(z)}{\Lambda^2}\cdot\frac{d\Lambda}{d\tau}-C'=0.$$

Expressing the elasticities $\mathcal{E}_\Lambda, \mathcal{E}_\lambda, \mathcal{E}_w$ through FOC, we obtain the needed equations (18), (22).

Further, plugging (18), (22) into the next two equations (total derivatives of free entry conditions (19)) we obtain the needed equations (23), (24).

[15] More detailed proof of the lemma can be found in [Bykadorov et al., 2015c].

Finally, plugging (18), (22) into the latter equation (the total derivative of trade balance (20)) we obtain the long equation

$$-\frac{R(Z)\cdot C'}{\lambda\cdot(C(Q))^2}\cdot\left(L\cdot\frac{dX}{d\tau}+K\cdot\tau\cdot L\cdot\frac{dY}{d\tau}+(k+1)\cdot\tau\cdot l\cdot\frac{dZ}{d\tau}\right)+\frac{R'(Z)}{\lambda\cdot C(Q)}\cdot\frac{dZ}{d\tau}+$$
$$+\frac{R(z)\cdot C'}{\Lambda\cdot(C(q))^2}\cdot\left(l\cdot\frac{dx}{d\tau}+k\cdot\tau\cdot l\cdot\frac{dy}{d\tau}+(K+1)\cdot\tau\cdot L\cdot\frac{dz}{d\tau}\right)-\frac{R'(z)}{\Lambda\cdot C(q)}\cdot\frac{dz}{d\tau}+$$
$$+\frac{R(z)}{\Lambda\cdot C(q)}\cdot\frac{1}{\tau}\cdot\mathcal{E}_\Lambda-\frac{R(Z)}{\lambda\cdot C(Q)}\cdot\frac{1}{\tau}\cdot\mathcal{E}_\lambda+$$
$$+C'\cdot\left(\frac{R(z)\cdot(k\cdot l\cdot y+(K+1)\cdot L\cdot z)}{\Lambda\cdot(C(q))^2}-\frac{R(Z)\cdot(K\cdot L\cdot Y+(k+1)\cdot l\cdot Z)}{\lambda\cdot(C(Q))^2}\right)=0.$$

Now, using obvious expressions (26), (27) for outputs derivatives $\frac{dQ}{d\tau},\frac{dq}{d\tau}$ through the corresponding derivatives of consumptions, the latter equation becomes (25), that completes the proof.

4.3 Proof of Proposition 1: Free Trade

Recall the formulation of

Proposition 1. *Assume pro-competitive utility ($r'_u>0$). Starting from free trade $\tau\approx 1$, when (negligible) trade cost increases, any country's domestic consumption, output and all prices increase, while the masses of firms, imports and welfare $W^{\mathscr{G}}$, $W^{\mathscr{L}}$ in any country decrease; moreover*

$$\frac{dx}{d\tau}>\frac{dX}{d\tau}>0,\qquad\frac{dq}{d\tau}>\frac{dQ}{d\tau}>0,\qquad\frac{dN}{d\tau}<\frac{dn}{d\tau}<0,\qquad\frac{dn}{d\tau}\cdot\frac{1}{n}<\frac{dN}{d\tau}\cdot\frac{1}{N}<0\qquad(\tau\approx 1).$$

Proof. This proposition considers the situation of free trade, i.e., $\tau=1$. Then, the location of a firm and destination of its output does not matter, that means

$$X=Y=Z=x=y=z,\quad w=1,\qquad\Lambda=\lambda,\ \ Q=q=((K+1)\cdot L+(k+1)\cdot l)\cdot X=\Gamma\cdot X,$$

where Γ denotes the total population of the world. Such symmetry enables simplification of our numerous notations as follows: $R\equiv R(X)=R(Y)$, $R'\equiv R'(X)=R'(Y)$, $C\equiv C(Q)=C(q)$, $E_R\equiv E_R(X)$, $E_C\equiv E_C(Q)$, etc. Similar symmetric simplification of our totally differentiated equations (21)–(25) is also correct *after* the differentiation. However, for expressing the resulting coefficients, we shall need the equilibrium equations expressed for the free trade case as simplified FOC and free entry conditions

$$R'=\Lambda\cdot C',\tag{28}$$

$$\Gamma\cdot R=\Lambda\cdot C,\tag{29}$$

that together entail equality between elasticities of cost and revenue (as in closed economy):

$$E_C=E_R\quad(\equiv 1-r_u).\tag{30}$$

We shall need notations for welfare functions of countries type $\mathscr{G}$ and $\mathscr{L}$, that in free trade situation become[16]

$$W^{\mathscr{G}} = L\cdot(N\cdot(1+K)+n\cdot(k+1))\cdot u = L\cdot(L\cdot(1+K)+l\cdot(k+1))\cdot\frac{u}{C} = L\cdot\Gamma\cdot\frac{u}{C}\,,$$

$$W^{\mathscr{L}} = l\cdot(n\cdot(k+1)+N\cdot(K+1))\cdot u = l\cdot\Gamma\cdot\frac{u}{C}\,.$$

Lemma below is the main part of proof of Proposition 1, containing also interesting additional information about elasticities.

Lemma 2. *In free trade, the masses of firms* $N=\frac{L}{C}\geq\frac{l}{C}=n$ *are proportional to populations; total elasticities of the wage differential and market aggregators* Λ,λ *in* τ *are*

$$\mathcal{E}_w = \frac{L-l}{\Gamma}\cdot E_R\in[0,1]\,, \tag{31}$$

$$\mathcal{E}_\lambda = -\left(-L\cdot(K+1)\cdot\frac{L-l}{\Gamma}\cdot\frac{r_R-1}{E_R+r_R}+\Gamma-l\right)\cdot\frac{E_R}{\Gamma}<0, \tag{32}$$

$$\mathcal{E}_\Lambda = -\left(l\cdot(k+1)\cdot\frac{L-l}{\Gamma}\cdot\frac{r_R-1}{E_R+r_R}+\Gamma-l\right)\cdot\frac{E_R}{\Gamma}\,, \tag{33}$$

and if $r'_u\geq 0$ *then* $\mathcal{E}_\Lambda<\mathcal{E}_\Lambda+\mathcal{E}_w<0$. *Moreover, the total derivatives of consumptions and outputs satisfy conditions*

$$\frac{dX}{d\tau} = -\frac{X}{r_R}\cdot(\mathcal{E}_\Lambda+\mathcal{E}_w)\,,\qquad \frac{dY}{d\tau} = -\frac{X}{r_R}\cdot(\mathcal{E}_\Lambda+\mathcal{E}_w+1)\,,\qquad \frac{dZ}{d\tau} = -\frac{X}{r_R}\cdot(\mathcal{E}_\lambda+\mathcal{E}_w+1)\,, \tag{34}$$

$$\frac{dx}{d\tau} = -\frac{X}{r_R}\cdot\mathcal{E}_\lambda>0,\qquad \frac{dy}{d\tau} = -\frac{X}{r_R}\cdot(\mathcal{E}_\lambda+1)\,,\qquad \frac{dz}{d\tau} = -\frac{X}{r_R}\cdot(\mathcal{E}_\Lambda+1)\,, \tag{35}$$

$$\frac{dQ}{d\tau} = (\Gamma-L)\cdot\frac{r'_u\cdot X^2}{r_R\cdot E_R}\,,\qquad \frac{dq}{d\tau} = (\Gamma-l)\cdot\frac{r'_u\cdot X^2}{r_R\cdot E_R}\,. \tag{36}$$

Further, total derivatives and elasticities of the firms' masses have the sing opposite to r'_u *because*

$$\frac{dN}{d\tau} = -\Gamma\cdot L\cdot(\Gamma-L)\cdot\frac{r'_u\cdot X^3}{C\cdot r_R}\,,\qquad \mathcal{E}_N = -\Gamma\cdot(\Gamma-L)\cdot\frac{r'_u\cdot X^3}{r_R}\,,$$

$$\frac{dn}{d\tau} = -\Gamma\cdot l\cdot(\Gamma-l)\cdot\frac{r'_u\cdot X^3}{C\cdot r_R}\,,\qquad \mathcal{E}_n = -\Gamma\cdot(\Gamma-l)\cdot\frac{r'_u\cdot X^3}{r_R}\,.$$

Finally, the sign of total derivatives of welfare depends on the sign of r'_u *as*

$$\frac{dW^{\mathscr{G}}}{d\tau} = -\frac{L\cdot u}{\Gamma\cdot C\cdot r_R}\cdot\left(A^{\mathscr{G}}+B^{\mathscr{G}}\cdot r'_u\cdot X\right)\,, \tag{37}$$

$$\frac{dW^{\mathscr{L}}}{d\tau} = -\frac{l\cdot u}{\Gamma\cdot C\cdot r_R}\cdot\left(A^{\mathscr{L}}+B^{\mathscr{L}}\cdot r'_u\cdot X\right)\,, \tag{38}$$

[16] Non-importantly for our argument but interestingly, the total world welfare depends on population quadratically: $W=(K+1)\cdot W^{\mathscr{G}}+(k+1)\cdot W^{\mathscr{L}}=\Gamma^2\cdot\frac{u}{C}$.

where three of the four coefficients have definite sign:

$$A^{\mathscr{G}} = (\Gamma \cdot L \cdot K + l \cdot (k+1) \cdot (\Gamma - L + l + (L-l) \cdot r_u)) \cdot \frac{r_R \cdot E_u}{E_R + r_R} > 0, \tag{39}$$

$$B^{\mathscr{G}} = (L \cdot (K+1) \cdot (\Gamma - L) + l \cdot (k+1) \cdot (\Gamma - l)) \cdot \frac{((1-E_u) \cdot E_R + r_u' \cdot X)}{(E_R + r_R) \cdot E_R}, \tag{40}$$

$$A^{\mathscr{L}} = \mathcal{E}_u \cdot \left(L \cdot (K+1) \cdot (L-l) \cdot \frac{r_R \cdot E_R}{E_R + r_R} + \Gamma \cdot (\Gamma - l) \cdot r_u \right) > 0, \tag{41}$$

$$B^{\mathscr{L}} = L \cdot (K+1) \cdot (L-l) \cdot \frac{E_u}{E_R + r_R} + (l \cdot (k+1) \cdot (\Gamma - l) + L \cdot (K+1) \cdot (\Gamma - L)) > 0. \tag{42}$$

Remark. For revealing the signs of the coefficients, note that $r_R > 0$ due to SOC, see (17), and under $r_u' > 0$ the positive sign $B^{\mathscr{G}} > 0$ is guaranteed (using footnote 1), therefore $\frac{dW^{\mathscr{G}}}{d\tau} < 0$, $\frac{dW^{\mathscr{L}}}{d\tau} < 0$.

Proof of Lemma 2.[17] From Lemma 1, substituting $X = Y = Z = x = y = z$ into (21) and (22) we get expressions (34) and (35) for total derivatives of consumptions.

As to equations (23)–(25) from Lemma 1, at free trade they become

$$L \cdot (1+K) \cdot (\mathcal{E}_\Lambda + \mathcal{E}_w) + (k+1) \cdot l \cdot (\mathcal{E}_\lambda + \mathcal{E}_w) + (K \cdot L + (k+1) \cdot l) \cdot E_R = 0,$$

$$L \cdot (1+K) \cdot \mathcal{E}_\Lambda + (k+1) \cdot l \cdot \mathcal{E}_\lambda + (k \cdot l + (K+1) \cdot L) \cdot E_R = 0,$$

$$\left(\frac{C'}{C} \cdot \frac{dQ}{d\tau} + \mathcal{E}_\lambda \right) - \left(\frac{C'}{C} \cdot \frac{dq}{d\tau} + \mathcal{E}_\Lambda \right) - \frac{R'}{R} \cdot \frac{dZ}{d\tau} + \frac{R'}{R} \cdot \frac{dz}{d\tau} = 0,$$

and solving these equations we obtain equalities (31)–(32). The sign $\mathcal{E}_w > 0$ is obvious. As to sign of $\mathcal{E}_\lambda$, we can evaluate it as

$$\mathcal{E}_\lambda = -\left(-L \cdot (K+1) \cdot \frac{L-l}{\Gamma} \cdot \frac{r_R + E_R - E_R - 1}{E_R + r_R} + \Gamma - l \right) \cdot \frac{E_R}{\Gamma} =$$

$$= -\left(-L \cdot (K+1) \cdot \frac{L-l}{\Gamma} + L \cdot (K+1) \cdot \frac{L-l}{\Gamma} \cdot \frac{E_R + 1}{E_R + r_R} + \Gamma - l \right) \cdot \frac{E_R}{\Gamma} =$$

$$= -\left(\frac{\Gamma \cdot (\Gamma - l) - L \cdot (K+1) \cdot (L-l)}{\Gamma} + L \cdot (K+1) \cdot \frac{L-l}{\Gamma} \cdot \frac{E_R + 1}{E_R + r_R} \right) \cdot \frac{E_R}{\Gamma} =$$

$$= -\left(\frac{L \cdot (K+1) \cdot (\Gamma - L) + l \cdot (k+1) \cdot (\Gamma - l)}{\Gamma} + L \cdot (K+1) \cdot \frac{L-l}{\Gamma} \cdot \frac{E_R + 1}{E_R + r_R} \right) \cdot \frac{E_R}{\Gamma} < 0.$$

Similarly, we can evaluate the sign of the sum of elasticities

$$-(\mathcal{E}_\Lambda + \mathcal{E}_w) = \left(l \cdot (k+1) \cdot \frac{L-l}{\Gamma} \cdot \frac{r_R - 1}{E_R + r_R} + \Gamma - L \right) \cdot \frac{E_R}{\Gamma} =$$

$$= \left(l \cdot (k+1) \cdot \frac{L-l}{\Gamma} \cdot \frac{r_R + E_R - 1 - E_R}{E_R + r_R} + \Gamma - L \right) \cdot \frac{E_R}{\Gamma} =$$

$$= \left(l \cdot (k+1) \cdot \frac{L-l}{\Gamma} \cdot \frac{r_R + E_R - 1}{E_R + r_R} - l \cdot (k+1) \cdot \frac{L-l}{\Gamma} \cdot \frac{E_R}{E_R + r_R} + \Gamma - L \right) \cdot \frac{E_R}{\Gamma} =$$

[17] More detailed proof of the lemma can be found in [Bykadorov et al., 2015c].

$$= \left(l \cdot (k+1) \cdot \frac{L-l}{\Gamma} \cdot \frac{r_u' \cdot X}{(E_R + r_R) \cdot E_R} - \frac{l \cdot (k+1) \cdot (L-l) \cdot E_R - \Gamma \cdot (\Gamma - L) \cdot (E_R + r_R)}{\Gamma \cdot (E_R + r_R)} \right) \cdot \frac{E_R}{\Gamma} =$$

$$= \left(\frac{L-l}{\Gamma} \cdot \frac{l \cdot (k+1) \cdot r_u' \cdot X}{(E_R + r_R) \cdot E_R} - \frac{(l \cdot (k+1) \cdot (L-l) + \Gamma \cdot (\Gamma - L)) \cdot E_R - \Gamma \cdot (\Gamma - L) \cdot r_R}{\Gamma \cdot (E_R + r_R)} \right) \cdot \frac{E_R}{\Gamma} =$$

$$= \left(\frac{L-l}{\Gamma} \cdot \frac{l \cdot (k+1) \cdot r_u' \cdot X}{(E_R + r_R) \cdot E_R} + \frac{(l \cdot (k+1) \cdot (\Gamma - L + l) + \Gamma \cdot L \cdot K) \cdot E_R + \Gamma \cdot (\Gamma - L) \cdot r_R}{\Gamma \cdot (E_R + r_R)} \right) \cdot \frac{E_R}{\Gamma}.$$

Hence, if $r_u' \geq 0$ then $\mathcal{E}_\Lambda < \mathcal{E}_\Lambda + \mathcal{E}_w < 0$ and this part of lemma is proven.

Now, we are able to calculate the expressions for total derivatives of outputs

$$\frac{dQ}{d\tau} = L \cdot \frac{dX}{d\tau} + K \cdot L \cdot \frac{dY}{d\tau} + (k+1) \cdot l \cdot \frac{dZ}{d\tau} + Q - L \cdot X =$$

$$= -\frac{X}{r_R} \cdot (L \cdot (\mathcal{E}_\Lambda + \mathcal{E}_w) + K \cdot L \cdot (\mathcal{E}_\Lambda + \mathcal{E}_w + 1) + (k+1) \cdot l \cdot (\mathcal{E}_\lambda + \mathcal{E}_w + 1)) + Q - L \cdot X =$$

$$= -\frac{X}{r_R} \cdot ((K+1) \cdot L \cdot (\mathcal{E}_\Lambda + \mathcal{E}_w) + (k+1) \cdot l \cdot (\mathcal{E}_\lambda + \mathcal{E}_w) + K \cdot L + (k+1) \cdot l) + (\Gamma - L) \cdot X =$$

$$= -\frac{X}{r_R} \cdot (-(K \cdot L + (k+1) \cdot l) \cdot E_R + \Gamma - L) + (\Gamma - L) \cdot X = (\Gamma - L) \cdot \frac{r_R + E_R - 1}{r_R} \cdot X =$$

$$= (\Gamma - L) \cdot \frac{r_u' \cdot X^2}{r_R \cdot E_R},$$

$$\frac{dq}{d\tau} = l \cdot \frac{dx}{d\tau} + k \cdot l \cdot \frac{dy}{d\tau} + (K+1) \cdot L \cdot \frac{dz}{d\tau} + Q - l \cdot X =$$

$$= -\frac{X}{r_R} \cdot (l \cdot \mathcal{E}_\lambda + k \cdot l \cdot (\mathcal{E}_\lambda + 1) + (K+1) \cdot L \cdot (\mathcal{E}_\Lambda + 1)) + Q - l \cdot X =$$

$$= -\frac{X}{r_R} \cdot (-(k \cdot l + (K+1) \cdot L) \cdot E_R + (K+1) \cdot L + k \cdot l) + (\Gamma - l) \cdot X =$$

$$= (\Gamma - l) \cdot \frac{E_R + r_R - 1}{r_R} \cdot X = (\Gamma - l) \cdot \frac{r_u' \cdot X^2}{r_R \cdot E_R}.$$

Thus, we have obtained expressions (36).

Further, using the labor balances and the obtained derivatives (and thereby elasticities) of outputs together with (30), we get the elasticities of N, n:

$$\mathcal{E}_N = -\mathcal{E}_{C(Q)} = -E_C \cdot \mathcal{E}_Q = -E_C \cdot Q \cdot (\Gamma - L) \cdot \frac{r_u' \cdot X^2}{r_R \cdot E_R} =$$

$$= -Q \cdot (\Gamma - L) \cdot \frac{r_u' \cdot X^2}{r_R} = -\Gamma \cdot (\Gamma - L) \cdot \frac{r_u' \cdot X^3}{r_R},$$

$$\mathcal{E}_n = -\mathcal{E}_{C(q)} = -E_C \cdot \mathcal{E}_q = -E_C \cdot Q \cdot (\Gamma - l) \cdot \frac{r_u' \cdot X^2}{r_R \cdot E_R} =$$

$$= -Q \cdot (\Gamma - l) \cdot \frac{r_u' \cdot X^2}{r_R} = -\Gamma \cdot (\Gamma - l) \cdot \frac{r_u' \cdot X^3}{r_R},$$

and related derivatives

$$\frac{dN}{d\tau} = -N \cdot \Gamma \cdot (\Gamma - L) \cdot \frac{r'_u \cdot X^3}{r_R} = -L \cdot \Gamma \cdot (\Gamma - L) \cdot \frac{r'_u \cdot X^3}{C \cdot r_R},$$
$$\frac{dn}{d\tau} = -n \cdot \Gamma \cdot (\Gamma - l) \cdot \frac{r'_u \cdot X^3}{r_R} = -l \cdot \Gamma \cdot (\Gamma - l) \cdot \frac{r'_u \cdot X^3}{C \cdot r_R}.$$

Thus, we have proven every claim of the lemma except for welfare.

As to individual welfare, for a Gulliver country it can be written as

$$\frac{W^{\mathscr{G}}}{L} = L \cdot \frac{u(X) + K \cdot u(Y)}{C(Q)} + l \cdot (k+1) \cdot \frac{u(z)}{C(q)}.$$

Differentiating this expression and substituting the derivatives obtained, we get

$$\frac{1}{L} \cdot \frac{dW^{\mathscr{G}}}{d\tau} = L \cdot \left(\frac{u'(X) \cdot \frac{dX}{d\tau} + K \cdot u'(Y) \cdot \frac{dY}{d\tau}}{C(Q)} - \frac{u(X) + K \cdot u(Y)}{(C(Q))^2} \cdot C'(Q) \cdot \frac{dQ}{d\tau} \right) +$$
$$+ l \cdot (k+1) \cdot \left(\frac{u'(z) \cdot \frac{dz}{d\tau}}{C(q)} - \frac{u(z)}{(C(q))^2} \cdot C'(q) \cdot \frac{dq}{d\tau} \right) =$$
$$= \frac{u'}{C} \cdot \left(L \cdot \frac{dX}{d\tau} + K \cdot L \cdot \frac{dY}{d\tau} + l \cdot (k+1) \cdot \frac{dz}{d\tau} \right) - \frac{u \cdot C'}{C^2} \cdot \left(L \cdot (K+1) \cdot \frac{dQ}{d\tau} + l \cdot (k+1) \cdot \frac{dq}{d\tau} \right) =$$

(substituting (34)–(36))

$$= -\frac{X}{r_R} \cdot \frac{u'}{C} \cdot (L \cdot (\mathcal{E}_\Lambda + \mathcal{E}_w) + K \cdot L \cdot (\mathcal{E}_\Lambda + \mathcal{E}_w + 1) + l \cdot (k+1) \cdot (\mathcal{E}_\Lambda + 1)) -$$
$$- \frac{u \cdot C'}{C^2} \cdot \frac{r'_u \cdot X^2}{r_R \cdot E_R} \cdot (L \cdot (K+1) \cdot (\Gamma - L) + l \cdot (k+1) \cdot (\Gamma - l)).$$

Now, substituting (31) and (33) we get

$$\frac{1}{L} \cdot \frac{dW^{\mathscr{G}}}{d\tau} =$$

$$= -\frac{X}{r_R} \cdot \frac{u'}{C} \cdot \left(\Gamma \cdot \left(-\frac{l \cdot (k+1)}{\Gamma} \cdot \frac{L-l}{\Gamma} \cdot \frac{r_R - 1}{E_R + r_R} \cdot E_R - \frac{\Gamma - l}{\Gamma} \cdot E_R \right) + L \cdot (K+1) \cdot \frac{L-l}{\Gamma} \cdot E_R + \Gamma - L \right)$$
$$- \frac{u \cdot E_C}{\Gamma \cdot C} \cdot \frac{r'_u \cdot X}{r_R \cdot E_R} \cdot (L \cdot (K+1) \cdot (\Gamma - L) + l \cdot (k+1) \cdot (\Gamma - l)) =$$
$$= -\frac{X}{r_R} \cdot \frac{u'}{C} \cdot \left(L \cdot (K+1) \cdot \frac{L-l}{\Gamma} - (\Gamma - l) \right) \cdot E_R -$$
$$- \frac{X}{r_R} \cdot \frac{u'}{C} \cdot \left(-l \cdot (k+1) \cdot \frac{L-l}{\Gamma} \cdot \frac{r_R - 1}{E_R + r_R} \cdot E_R + \Gamma - L \right) -$$
$$- \frac{u}{\Gamma \cdot C} \cdot \frac{r'_u \cdot X}{r_R} \cdot (L \cdot (K+1) \cdot (\Gamma - L) + l \cdot (k+1) \cdot (\Gamma - l)) =$$

$$= -\frac{X}{r_R} \cdot \frac{u'}{C} \cdot \frac{\left(L \cdot (K+1) \cdot \frac{L-l}{\Gamma} - (\Gamma - l)\right) \cdot E_R \cdot (E_R + r_R - 1)}{E_R + r_R} -$$

$$-\frac{X}{r_R} \cdot \frac{u'}{C} \cdot \frac{\left(L \cdot (K+1) \cdot \frac{L-l}{\Gamma} - (\Gamma - l)\right) \cdot E_R}{E_R + r_R} -$$

$$-\frac{X}{r_R} \cdot \frac{u'}{C} \cdot \frac{l \cdot (k+1) \cdot \frac{L-l}{\Gamma} \cdot E_R - l \cdot (k+1) \cdot \frac{L-l}{\Gamma} \cdot E_R \cdot r_R + (\Gamma - L) \cdot (E_R + r_R)}{E_R + r_R} -$$

$$-\frac{u}{\Gamma \cdot C} \cdot \frac{r_u' \cdot X}{r_R} \cdot (L \cdot (K+1) \cdot (\Gamma - L) + l \cdot (k+1) \cdot (\Gamma - l)) =$$

$$= -\frac{X}{r_R} \cdot \frac{u'}{C} \cdot \frac{\left(L \cdot (K+1) \cdot \frac{L-l}{\Gamma} - (\Gamma - l)\right) \cdot (r_u' \cdot X + E_R)}{E_R + r_R} -$$

$$-\frac{X}{r_R} \cdot \frac{u'}{C} \cdot \frac{\left(l \cdot (k+1) \cdot \frac{L-l}{\Gamma} + \Gamma - L\right) \cdot E_R + \left(\Gamma - L - l \cdot (k+1) \cdot \frac{L-l}{\Gamma} \cdot E_R\right) \cdot r_R}{E_R + r_R} -$$

$$-\frac{u}{\Gamma \cdot C} \cdot \frac{r_u' \cdot X}{r_R} \cdot (L \cdot (K+1) \cdot (\Gamma - L) + l \cdot (k+1) \cdot (\Gamma - l)) =$$

(because $L \cdot (K+1) \cdot \frac{L-l}{\Gamma} - (\Gamma - l) = -\left(l \cdot (k+1) \cdot \frac{L-l}{\Gamma} + \Gamma - L\right)$)

$$= -\frac{X}{r_R} \cdot \frac{u'}{\Gamma \cdot C} \cdot \frac{(L \cdot (K+1) \cdot (L-l) - \Gamma \cdot (\Gamma - l)) \cdot r_u' \cdot X}{E_R + r_R} -$$

$$-\frac{X}{r_R} \cdot \frac{u'}{C} \cdot \frac{\left(\Gamma - L - l \cdot (k+1) \cdot \frac{L-l}{\Gamma} + l \cdot (k+1) \cdot \frac{L-l}{\Gamma} \cdot r_u\right)}{E_R + r_R} \cdot r_R -$$

$$-\frac{u}{\Gamma \cdot C} \cdot \frac{r_u' \cdot X}{r_R} \cdot (L \cdot (K+1) \cdot (\Gamma - L) + l \cdot (k+1) \cdot (\Gamma - l)) =$$

(because $\Gamma \cdot (\Gamma - L) - l \cdot (k+1) \cdot (L-l) = \Gamma \cdot L \cdot K + l \cdot (k+1) \cdot (\Gamma - L + l)$)

$$= \frac{X}{r_R} \cdot \frac{u' \cdot r_u' \cdot X}{\Gamma \cdot C} \cdot \frac{(L \cdot (K+1) \cdot (\Gamma - L) + l \cdot (k+1) \cdot (\Gamma - l))}{E_R + r_R} -$$

$$-\frac{X}{r_R} \cdot \frac{u'}{\Gamma \cdot C} \cdot \frac{(\Gamma \cdot L \cdot K + l \cdot (k+1) \cdot (\Gamma - L + l) + l \cdot (k+1) \cdot (L-l) \cdot r_u)}{E_R + r_R} \cdot r_R -$$

$$-\frac{u}{\Gamma \cdot C} \cdot \frac{r_u' \cdot X}{r_R} \cdot (L \cdot (K+1) \cdot (\Gamma - L) + l \cdot (k+1) \cdot (\Gamma - l)) =$$

$$= -\frac{X}{r_R} \cdot \frac{u'}{\Gamma \cdot C} \cdot \frac{\Gamma \cdot L \cdot K + l \cdot (k+1) \cdot (\Gamma - L + l) + l \cdot (k+1) \cdot (L-l) \cdot r_u}{E_R + r_R} \cdot r_R +$$

$$+\frac{(L \cdot (K+1) \cdot (\Gamma - L) + l \cdot (k+1) \cdot (\Gamma - l))}{E_R + r_R} \cdot \frac{r_u' \cdot X \cdot u}{r_R \cdot \Gamma \cdot C} \cdot (E_u - (E_R + r_R)) =$$

$$= -\frac{X}{r_R}\cdot\frac{u'}{\Gamma\cdot C}\cdot\frac{\Gamma\cdot L\cdot K + l\cdot(k+1)\cdot(\Gamma-L+l) + l\cdot(k+1)\cdot(L-l)\cdot r_u}{E_R+r_R}\cdot r_R +$$

$$-\frac{(L\cdot(K+1)\cdot(\Gamma-L)+l\cdot(k+1)\cdot(\Gamma-l))}{E_R+r_R}\cdot\frac{r_u'\cdot X\cdot u}{r_R\cdot\Gamma\cdot C}\cdot\left(1-E_u+\frac{r_u'\cdot X}{E_R}\right) =$$

$$= -\frac{u}{\Gamma\cdot C\cdot r_R}\cdot\left(A^{\mathscr{G}} + B^{\mathscr{G}}\cdot r_u'\cdot X\right),$$

where $A^{\mathscr{G}} > 0$ and $B^{\mathscr{G}}$ are introduced in (39) and (40). Hence if $r_u' \geq 0$ then $\frac{dW^{\mathscr{G}}}{d\tau} > 0$, since $E_u < 1$.

Analogously to Gullivers, we express individual welfare for a Lilliputian as

$$\frac{W^{\mathscr{L}}}{l} = n\cdot(u(x)+k\cdot l\cdot u(y)) + N\cdot(K+1)\,u(Z) = l\cdot\frac{u(x)+k\cdot u(y)}{C(q)} + L\cdot(K+1)\cdot\frac{u(Z)}{C(Q)}.$$

We similarly differentiate and transform it as

$$\frac{1}{l}\cdot\frac{dW^{\mathscr{L}}}{d\tau} = l\cdot\left(\frac{u'(x)\cdot\frac{dx}{d\tau}+k\cdot u'(y)\cdot\frac{dy}{d\tau}}{C(q)} - \frac{u(x)+k\cdot u(y)}{(C(q))^2}\cdot C'(q)\cdot\frac{dq}{d\tau}\right) +$$

$$+L\cdot(K+1)\cdot\left(\frac{u'(Z)\cdot\frac{dZ}{d\tau}}{C(Q)} - \frac{u(Z)}{(C(Q))^2}\cdot C'(Q)\cdot\frac{dQ}{d\tau}\right) =$$

$$= \frac{u'}{C}\cdot\left(l\cdot\left(\frac{dx}{d\tau}+k\cdot\frac{dy}{d\tau}\right)+L\cdot(K+1)\cdot\frac{dZ}{d\tau}\right) - \frac{u\cdot C'}{C^2}\cdot\left(l\cdot(k+1)\cdot\frac{dq}{d\tau}+L\cdot(K+1)\cdot\frac{dQ}{d\tau}\right) =$$

$$= -\frac{X}{r_R}\cdot\frac{u'}{C}\cdot(l\cdot(\mathcal{E}_\lambda+k\cdot(\mathcal{E}_\lambda+1)) + L\cdot(K+1)\cdot(\mathcal{E}_\lambda+\mathcal{E}_w+1)) -$$

$$-\frac{u\cdot C'}{C^2}\cdot\frac{r_u'\cdot X^2}{r_R\cdot E_R}\cdot(l\cdot(k+1)\cdot(\Gamma-l)+L\cdot(K+1)\cdot(\Gamma-L)) =$$

$$= -\frac{X}{r_R}\cdot\frac{u'}{C}\cdot\left(L\cdot(K+1)\cdot\frac{r_R-1}{E_R+r_R}\cdot\mathcal{E}_w-(\Gamma-l)\cdot E_R+L\cdot(K+1)\cdot\mathcal{E}_w+\Gamma-l\right) -$$

$$-\frac{u\cdot C'\cdot Q}{\Gamma\cdot C^2}\cdot\frac{r_u'\cdot X}{r_R\cdot E_R}\cdot(l\cdot(k+1)\cdot(\Gamma-l)+L\cdot(K+1)\cdot(\Gamma-L)) =$$

$$= -\frac{X}{r_R}\cdot\frac{u'}{C}\cdot\left(L\cdot(K+1)\cdot\left(\frac{r_R}{E_R+r_R}+\frac{E_R+r_R-1}{E_R+r_R}\right)\cdot\mathcal{E}_w+(\Gamma-l)\cdot(1-E_R)\right) -$$

$$-\frac{u\cdot r_u'\cdot X}{\Gamma\cdot C\cdot r_R}\cdot(l\cdot(k+1)\cdot(\Gamma-l)+L\cdot(K+1)\cdot(\Gamma-L)) =$$

$$= -\frac{u\cdot E_u}{C\cdot r_R}\cdot\left(L\cdot(K+1)\cdot\frac{L-l}{\Gamma}\cdot\frac{r_R\cdot E_R}{E_R+r_R}+(\Gamma-l)\cdot r_u\right) -$$

$$-\frac{u\cdot E_u}{C\cdot r_R}\cdot\frac{L\cdot(K+1)\cdot(L-l)}{\Gamma\cdot(E_R+r_R)}\cdot r_u'\cdot X -$$

$$-\frac{u \cdot r_u' \cdot X}{\Gamma \cdot C \cdot r_R} \cdot (l \cdot (k+1) \cdot (\Gamma - l) + L \cdot (K+1) \cdot (\Gamma - L)) =$$

$$= -\frac{u \cdot E_u}{\Gamma \cdot C \cdot r_R} \cdot \left(L \cdot (K+1) \cdot (L - l) \cdot \frac{r_R \cdot E_R}{E_R + r_R} + \Gamma \cdot (\Gamma - l) \cdot r_u \right) -$$

$$-\left(L \cdot (K+1) \cdot (L - l) \cdot \frac{E_u}{E_R + r_R} + l \cdot (k+1) \cdot (\Gamma - l) + L \cdot (K+1) \cdot (\Gamma - L) \right) \cdot \frac{u \cdot r_u' \cdot X}{\Gamma \cdot C \cdot r_R} =$$

$$= -\frac{u}{\Gamma \cdot C \cdot r_R} \cdot \left(A^{\mathscr{L}} + B^{\mathscr{L}} \cdot r_u' \cdot X \right),$$

where $A^{\mathscr{L}}$ and $B^{\mathscr{L}}$ are defined in (41) and (42).

Hence, if $r_u' \geq 0$ then $\frac{dW^{\mathscr{L}}}{d\tau} < 0$, that competes the proof of Lemma 2. Using it and related Remark we immediately obtain Proposition 1.

4.4 Proof of Proposition 2: Total autarky

Recall notations for trade freeness $\phi \equiv 1/\tau$. Also, to encompass any pairs of countries trading: (Lilliputian-Lilliputian), (Gulliver-Gulliver) or (Gulliver-Lilliputian), we replace, with a little abuse of notation, our upper-case notations X, N with lower-case letters x^j, n^j and denote all imports (Y or Z or z or y)— by y^{ij} where $i, j \in \{\mathscr{G}, \mathscr{L}\}$.

Recall also formulation of

Proposition 2. *Starting from complete autarky ϕ^{aut}, when trade liberalization begins,*[18] *and some two countries of types $i, j \in \{\mathscr{G}, \mathscr{L}\}$ start trading, the local reaction of domestic consumptions x^i, x^j is negligible ($\left(dx^k/d\phi\right)|_{\phi=\phi^{aut}} = 0,\ \ k \in \{\mathscr{G}, \mathscr{L}\}$); consumption of import increases ($(dy^{ij}/d\phi)\,|_{\phi=\phi^{aut}} > 0,\ i, j \in \{\mathscr{G}, \mathscr{L}\}$), whereas the masses of firms and welfare decrease in both countries:*

$$\frac{dn^k}{d\phi}|_{\phi=\phi^{aut}} < 0,\ \ \frac{dW^k}{d\phi}|_{\phi=\phi^{aut}} < 0,\ \ k \in \{\mathscr{G}, \mathscr{L}\}.$$

So, there is a right neighborhood $[\phi^{aut}, \phi)$ of point ϕ^{aut} where (in comparison with autarky) new-born trade brings harm to both trading countries, and does not affect non-trading countries.

Proof. Partial or complete autarky means that some trade terminates, autarky is "complete" when any international trade stops. In principle, at final point τ_3 of trade, there can be three cases of trade termination, expressed through $\varepsilon \to +0$:

I) At τ_3 trade among countries $\mathscr{L}$ stops: $Y(\tau_3 - \varepsilon) = Z(\tau_3 - \varepsilon) = z(\tau_3 - \varepsilon) = 0, \quad y(\tau_3 - \varepsilon) \to 0$;

II) At τ_3 trade among countries $\mathscr{G}$ and $\mathscr{L}$ stops $Y(\tau_3 - \varepsilon) = y(\tau_3 - \varepsilon) = 0,\ Z(\tau_3 - \varepsilon) \to 0,\ z(\tau_3 - \varepsilon) \to 0$;

III) At τ_3 trade among countries of type $\mathscr{G}$ stops $y(\tau_3 - \varepsilon) = Z(\tau_3 - \varepsilon) = z(\tau_3 - \varepsilon) = 0, \quad Y(\tau_3 - \varepsilon) \to 0$.

Obviously, in Case I), $X, Q, N, W^{\mathscr{G}}$ are constant w.r.t. τ on interval $[\tau_2, \tau_3]$ and these variable play no role here. Analogously, in Case II), $y, Y,$ are constant w.r.t. τ on $[\tau_2, \tau_3]$, and in Case III), $x, q, n, W^{\mathscr{L}}$ are constant w.r.t. τ on $[\tau_2, \tau_3]$.

[18]Trade liberalization means that trade cost τ decreases, ϕ increases.

To prove Proposition 2 we study behavior of equilibrium under trade liberalization, i.e., when ϕ increases, i.e., when $\tau = 1/\phi$ decreases. Using the formula for the derivative of a composite function,

$$\tau = 1/\phi \Longrightarrow \frac{d}{d\phi}\left(g\left(\frac{1}{\phi}\right)\right) = -\frac{dg}{d\tau} \cdot \frac{1}{\phi^2},$$

we conclude that the derivative of an equilibrium variable w.r.t. ϕ has the opposite sign with the derivative of this variable w.r.t. τ. Hence we can directly use the results of comparative statics w.r.t. τ (Lemma 1). So, the following lemma allows us to proof Proposition 2.

Lemma 3. *At final point τ_3 of trade, leading to complete autarky, the total elasticities and derivatives of variables w.r.t. τ have the following values*

- *in case (I) the total elasticities and derivatives are*

$$\mathcal{E}_\lambda = \mathcal{E}_\Lambda = \frac{dx}{d\tau} = 0,$$

$$\frac{dy}{d\tau} = \frac{R'(y)}{R''(y)} \cdot \frac{1}{\tau} < 0, \qquad \mathcal{E}_n = -k \cdot l \cdot \frac{c \cdot \tau^2}{C(q)} \cdot \frac{dy}{d\tau} > 0, \qquad \frac{dq}{d\tau} = k \cdot l \cdot \tau \cdot \frac{dy}{d\tau} < 0,$$

$$\frac{dW^{\mathscr{L}}}{d\tau} = k \cdot l^2 \cdot \tau \cdot \frac{u(x) \cdot E_R(x)}{x} \cdot \frac{E_u(x) - 1}{C(q)} \cdot \frac{dy}{d\tau} > 0;$$

- *in case (II) the total elasticities and derivatives are*

$$\mathcal{E}_\lambda = \mathcal{E}_\Lambda + \mathcal{E}_w = \frac{dX}{d\tau} = \frac{dx}{d\tau} = 0, \qquad \mathcal{E}_w = \frac{\dfrac{R''(Z)}{\lambda \cdot C(q)} - w^2 \cdot \dfrac{R''(z)}{\Lambda \cdot C(Q)}}{\dfrac{R''(Z)}{\lambda \cdot C(q)} + w^2 \cdot \dfrac{R''(z)}{\Lambda \cdot C(Q)}} \in (-1, 1),$$

$$\frac{dZ}{d\tau} = \frac{\dfrac{2}{\tau} \cdot \dfrac{R'(Z)}{\lambda \cdot C(q)}}{\dfrac{R''(Z)}{\lambda \cdot C(q)} + w^2 \cdot \dfrac{R''(z)}{\Lambda \cdot C(Q)}} < 0, \qquad \frac{dz}{d\tau} = w \cdot \frac{C(q)}{C(Q)} \cdot \frac{dZ}{d\tau} < 0,$$

$$\frac{dQ}{d\tau} = l \cdot (k+1) \cdot \tau \cdot \frac{dZ}{d\tau} < 0, \qquad \frac{dq}{d\tau} = L \cdot (K+1) \cdot \tau \cdot \frac{dz}{d\tau} < 0,$$

$$\mathcal{E}_N = -l \cdot (k+1) \cdot \frac{c \cdot \tau^2}{C(Q)} \cdot \frac{dZ}{d\tau} > 0, \qquad \mathcal{E}_n = -L \cdot (K+1) \cdot \frac{c \cdot \tau^2}{C(q)} \cdot \frac{dz}{d\tau} > 0,$$

$$\frac{dW^{\mathscr{G}}}{d\tau} = L \cdot l \cdot (k+1) \cdot w \cdot \frac{R'(z)}{C(Q)} \cdot \frac{E_u(X) - 1}{E_u(X)} \cdot \frac{dZ}{d\tau} > 0,$$

$$\frac{dW^{\mathscr{L}}}{d\tau} = L \cdot l \cdot (K+1) \cdot \frac{1}{w} \cdot \frac{R'(Z)}{C(q)} \cdot \frac{E_u(x) - 1}{E_u(x)} \cdot \frac{dz}{d\tau} > 0;$$

- *in case (III) the total elasticities and derivatives are*

$$\mathcal{E}_\lambda = \mathcal{E}_\Lambda = \frac{dX}{d\tau} = 0,$$

$$\frac{dY}{d\tau}=\frac{1}{\tau}\cdot\frac{R'(Y)}{R''(Y)}<0,\qquad \mathcal{E}_N=-K\cdot L\cdot\frac{c\cdot\tau^2}{C(Q)}\cdot\frac{dY}{d\tau}>0,\qquad \frac{dQ}{d\tau}=K\cdot L\cdot\tau\cdot\frac{dY}{d\tau}<0,$$

$$\frac{dW^{\mathscr{G}}}{d\tau}=L^2\cdot K\cdot\frac{u(X)\cdot E_R(X)}{X}\cdot\tau\cdot\frac{E_u(X)-1}{C(Q)}\cdot\frac{dY}{d\tau}>0.$$

Proof of Lemma 3.[19] Since

$$Q(\tau_3)=L\cdot X(\tau_3),\qquad q(\tau_3)=l\cdot x(\tau_3),$$

our FOC and free entry condition yield

$$E_R(X(\tau_3))=E_C(Q(\tau_3)),\qquad E_R(x(\tau_3))=E_C(q(\tau_3)).$$

Case (I): complete autarky arise in symmetric trade among small countries through $y(\tau)\to 0$ $(\tau\to\tau_3)$. In this case, near $\tau\approx\tau_3$ we have

$$Q=L\cdot X,\qquad q=l\cdot x+k\cdot\tau\cdot l\cdot y,$$

$$\Pi=L\cdot\frac{R(X)}{\Lambda}-w\cdot C(Q),\qquad \pi=l\cdot\frac{R(x)}{\lambda}+k\cdot l\cdot\frac{R(y)}{\lambda}-C(q),$$

where wage w is considered as *constant*, because in countries $\mathscr{G}$ any trade has terminated before, they are out of consideration. This allows to ignore variables X, Y, Λ, z, Z, w in analysis of case I, they become independent of τ in the neighborhood of τ_3.

Equilibrium equations w.r.t. remaining variables (x, y, λ) are $\frac{\partial\pi}{\partial x}=0,\qquad \frac{\partial\pi}{\partial y}=0,\qquad \pi=0,$ i.e.,

$$\frac{R'(x)}{\lambda}=C',\qquad \frac{R'(y)}{\lambda}=\tau\cdot C',$$

$$l\cdot\frac{R(x)}{\lambda}+k\cdot l\cdot\frac{R(y)}{\lambda}=C(q),\quad q=lx+\tau lky.$$

From Lemma 1 (setting $\mathcal{E}_w=0$) we get

$$\frac{dx}{d\tau}=\frac{1}{\tau}\cdot\frac{\mathcal{E}_\lambda}{\frac{R''(x)}{R'(x)}},\qquad \frac{dy}{d\tau}=\frac{1}{\tau}\cdot\frac{\mathcal{E}_\lambda+1}{\frac{R''(y)}{R'(y)}},$$

$$\frac{dq}{d\tau}=l\cdot\frac{dx}{d\tau}+k\cdot l\cdot\left(y+\tau\cdot--\frac{dy}{d\tau}\right),$$

$$l\cdot\left(\frac{R(x)}{\lambda}+k\cdot\frac{R(y)}{\lambda}\right)\cdot\mathcal{E}_\lambda=0.$$

Hence,

$$\mathcal{E}_\Lambda=\mathcal{E}_\lambda=\frac{dX}{d\tau}=\frac{dx}{d\tau}=\frac{dQ}{d\tau}=0,\qquad \frac{dy}{d\tau}=\frac{R'(y)}{R''(y)}\cdot\frac{1}{\tau}<0,\qquad \frac{dq}{d\tau}=k\cdot l\cdot\tau\cdot\frac{dy}{d\tau}<0.$$

[19] More detailed proof of the lemma can be found in [Bykadorov et al., 2015c].

As to mass of firms, from the labor balance we get

$$\mathcal{E}_n = \mathcal{E}_{\left(\frac{l}{C(q)}\right)} = -\mathcal{E}_{C(q)} = -E_C(q) \cdot \mathcal{E}_q = -E_C(q) \cdot \frac{dq}{d\tau} \cdot \frac{\tau}{q} =$$

$$= -E_C(q) \cdot k \cdot l \cdot \tau \cdot \frac{dy}{d\tau} \cdot \frac{\tau}{q} = -k \cdot l \cdot \frac{c \cdot \tau^2}{C(q)} \cdot \frac{dy}{d\tau} > 0.$$

For $\mathscr{G}$-countries, trivially, $\frac{dW^{\mathscr{G}}}{d\tau} = 0$, while for $\mathscr{L}$-countries we differentiate the welfare function

$$W^{\mathscr{L}} = l^2 \cdot \frac{u(x) + k \cdot u(y)}{C(l \cdot x + l \cdot k \cdot y)}$$

and get

$$\frac{1}{k \cdot l^2} \cdot \frac{dW^{\mathscr{L}}}{d\tau} = \frac{1}{k} \cdot \frac{\left(u'(x) \cdot \frac{dx}{d\tau} + k \cdot u'(y) \cdot \frac{dy}{d\tau}\right) \cdot C(q) - (u(x) + k \cdot u(y)) \cdot C' \cdot \frac{dq}{d\tau}}{(C(q))^2} =$$

$$= \frac{1}{k} \cdot \frac{\left(u'(x) \cdot \frac{dx}{d\tau} + k \cdot u'(y) \cdot \frac{dy}{d\tau}\right) \cdot C(q) - (u(x) + k \cdot u(y)) \cdot C' \cdot l \cdot \left(\frac{dx}{d\tau} + k \cdot \left(\tau \cdot \frac{dy}{d\tau} + y\right)\right)}{(C(q))^2} =$$

$$= \frac{u'(y) \cdot C(q) - u(x) \cdot C'(q) \cdot l \cdot \tau}{(C(q))^2} \cdot \frac{dy}{d\tau} = \frac{R'(y) \cdot C(q) - u(x) \cdot C'(q) \cdot l \cdot \tau}{(C(q))^2} \cdot \frac{dy}{d\tau} =$$

$$= \frac{\tau \cdot R'(x) \cdot C(q) - u(x) \cdot C'(q) \cdot l \cdot \tau}{(C(q))^2} \cdot \frac{dy}{d\tau} = \frac{\tau \cdot u'(x) \cdot E_R(x) \cdot C(q) - u(x) \cdot C'(q) \cdot l \cdot \tau}{(C(q))^2} \cdot \frac{dy}{d\tau} =$$

$$= \tau \cdot \frac{u(x)}{x} \cdot \frac{\frac{u'(x) \cdot x}{u(x)} \cdot E_R(x) - \frac{C'(q) \cdot q}{C(q)}}{C(q)} \cdot \frac{dy}{d\tau} = \tau \cdot \frac{u(x)}{x} \cdot \frac{E_u(x) \cdot E_R(x) - E_C(q)}{C(q)} \cdot \frac{dy}{d\tau} =$$

$$= \tau \cdot \frac{u(x)}{x} \cdot \frac{E_u(x) \cdot E_R(x) - E_R(x)}{C(q)} \cdot \frac{dy}{d\tau} = \tau \cdot \frac{u(x)}{x} \cdot E_R(x) \cdot \frac{E_u(x) - 1}{C(q)} \cdot \frac{dy}{d\tau} > 0,$$

which expresses the welfare gain from increasing trade cost, that we are proving.

Case (II): complete autarky arise in asymmetric trade among big and small countries through $z(\tau) \to 0$, $Z(\tau) \to 0$ $(\tau \to \tau_3)$**.** In this case, near $\tau \approx \tau_3$ we have $Y = 0$, $y = 0$,

$$Q = L \cdot X + (k+1) \cdot \tau \cdot l \cdot Z, \qquad q = l \cdot x + (K+1) \cdot \tau \cdot L \cdot z,$$

$$\Pi = L \cdot \frac{R(X)}{\Lambda} + (k+1) \cdot l \cdot \frac{R(Z)}{\lambda} - w \cdot C(Q), \qquad \pi = l \cdot \frac{R(x)}{\lambda} + (K+1) \cdot L \cdot \frac{R(z)}{\Lambda} - C(q),$$

$$W^{\mathscr{G}} = L \cdot \left(L \cdot \frac{u(X)}{C(Q)} + l \cdot (k+1) \cdot \frac{u(z)}{C(q)}\right), \qquad W^{\mathscr{L}} = l \cdot \left(l \cdot \frac{u(x)}{C(q)} + L \cdot (K+1) \cdot \frac{u(Z)}{C(Q)}\right),$$

equilibrium equations w.r.t. $(X, Z, x, z, \Lambda, \lambda, w)$ are

$$\frac{\partial \Pi}{\partial X} = 0, \qquad \frac{\partial \Pi}{\partial Z} = 0, \qquad \frac{\partial \pi}{\partial x} = 0, \qquad \frac{\partial \pi}{\partial z} = 0, \qquad \Pi = 0, \qquad \pi = 0, \qquad TB = 0,$$

i.e.,

$$\frac{R'(X)}{\Lambda} = w \cdot C', \qquad \frac{R'(Z)}{\lambda} = \tau \cdot w \cdot C', \qquad \frac{R'(x)}{\lambda} = C', \qquad \frac{R'(z)}{\Lambda} = \tau \cdot C',$$

$$L \cdot \frac{R(X)}{\Lambda} + (k+1) \cdot l \cdot \frac{R(Z)}{\lambda} = w \cdot C(Q), \qquad l \cdot \frac{R(x)}{\lambda} + (K+1) \cdot L \cdot \frac{R(z)}{\Lambda} = C(q),$$

$$\frac{R(Z)}{\lambda \cdot C(Q)} - \frac{R(z)}{\Lambda \cdot C(q)} = 0.$$

Using comparative statics of Lemma 1, using FOC and linearity of costs we get several equations and signs of derivatives:

$$\mathcal{E}_\Lambda + \mathcal{E}_w = \mathcal{E}_\lambda = 0, \qquad \mathcal{E}_w = \frac{\frac{R''(Z)}{\lambda \cdot C(q)} - w^2 \cdot \frac{R''(z)}{\Lambda \cdot C(Q)}}{\frac{R''(Z)}{\lambda \cdot C(q)} + w^2 \cdot \frac{R''(z)}{\Lambda \cdot C(Q)}}, \qquad \frac{dX}{d\tau} = \frac{dx}{d\tau} = 0,$$

$$\frac{dZ}{d\tau} = \frac{R'(Z)}{R''(Z)} \cdot \frac{1}{\tau} \cdot (1 + \mathcal{E}_w) = \frac{\frac{2}{\tau} \cdot \frac{R'(Z)}{\lambda \cdot C(q)}}{\frac{R''(Z)}{\lambda \cdot C(q)} + w^2 \cdot \frac{R''(z)}{\Lambda \cdot C(Q)}} < 0,$$

$$\frac{dz}{d\tau} = \frac{R'(z)}{R''(z)} \cdot \frac{1}{\tau} \cdot (1 - \mathcal{E}_w) = \frac{\frac{2}{\tau} \cdot w^2 \cdot \frac{R'(z)}{\Lambda \cdot C(Q)}}{\frac{R''(Z)}{\lambda \cdot C(q)} + w^2 \cdot \frac{R''(z)}{\Lambda \cdot C(Q)}} =$$

$$= \frac{\frac{2}{\tau} \cdot \frac{R'(Z)}{\lambda \cdot C(q)}}{\frac{R''(Z)}{\lambda \cdot C(q)} + w^2 \cdot \frac{R''(z)}{\Lambda \cdot C(Q)}} \cdot w^2 \cdot \frac{R'(z)}{R'(Z)} \cdot \frac{\lambda \cdot C(q)}{\Lambda \cdot C(Q)} =$$

$$= \frac{\frac{2}{\tau} \cdot \frac{R'(Z)}{\lambda \cdot C(q)}}{\frac{R''(Z)}{\lambda \cdot C(q)} + w^2 \cdot \frac{R''(z)}{\Lambda \cdot C(Q)}} \cdot w^2 \cdot \frac{1}{w} \cdot \frac{\Lambda}{\lambda} \cdot \frac{\lambda \cdot C(q)}{\Lambda \cdot C(Q)} = w \cdot \frac{C(q)}{C(Q)} \cdot \frac{dZ}{d\tau} < 0.$$

Similarly we estimate the derivatives of outputs:

$$\frac{dQ}{d\tau} = l \cdot (k+1) \cdot \tau \cdot \frac{dZ}{d\tau} = 2 \cdot l \cdot (k+1) \cdot \frac{\frac{R'(Z)}{\lambda \cdot C(q)}}{\frac{R''(Z)}{\lambda \cdot C(q)} + w^2 \cdot \frac{R''(z)}{\Lambda \cdot C(Q)}} < 0,$$

$$\frac{dq}{d\tau} = L \cdot (K+1) \cdot \tau \cdot \frac{dz}{d\tau} = 2 \cdot L \cdot (K+1) \cdot \frac{w^2 \cdot \frac{R'(z)}{\Lambda \cdot C(Q)}}{\frac{R''(Z)}{\lambda \cdot C(q)} + w^2 \cdot \frac{R''(z)}{\Lambda \cdot C(Q)}} < 0.$$

As to mass of firms, from labor balances we get

$$\mathcal{E}_N = -\mathcal{E}_{C(Q)} = -E_C(Q) \cdot \mathcal{E}_Q = -E_C(Q) \cdot \frac{dQ}{d\tau} \cdot \frac{\tau}{Q} =$$

$$= -E_C(Q) \cdot l \cdot (k+1) \cdot \tau \cdot \frac{dZ}{d\tau} \cdot \frac{\tau}{Q} = -l \cdot (k+1) \cdot \frac{c \cdot \tau^2}{C(Q)} \cdot \frac{dZ}{d\tau} > 0,$$

$$\mathcal{E}_n = -\mathcal{E}_{C(q)} = -E_C(q) \cdot \mathcal{E}_q = -E_C(q) \cdot \frac{dq}{d\tau} \cdot \frac{\tau}{q} =$$

$$= -E_C(q) \cdot L \cdot (K+1) \cdot \tau \cdot \frac{dz}{d\tau} \cdot \frac{\tau}{q} = -L \cdot (K+1) \cdot \frac{c \cdot \tau^2}{C(q)} \cdot \frac{dz}{d\tau} > 0.$$

Now we can express the total derivative of welfare in $\mathscr{G}$ as

$$\frac{1}{L} \cdot \frac{dW^{\mathscr{G}}}{d\tau} = L \cdot \left(\frac{u'(X)}{C(Q)} \cdot \frac{dX}{d\tau} - \frac{u(X)}{(C(Q))^2} \cdot C'(Q) \cdot \frac{dQ}{d\tau} \right) +$$

$$+ l \cdot (k+1) \cdot \left(\frac{u'(z)}{C(q)} \cdot \frac{dz}{d\tau} - \frac{u(z)}{(C(q))^2} \cdot C'(q) \cdot \frac{dq}{d\tau} \right) =$$

$$= -L \cdot \frac{u(X)}{(C(Q))^2} \cdot C'(Q) \cdot \frac{dQ}{d\tau} + l \cdot (k+1) \cdot \frac{u'(z)}{C(q)} \cdot \frac{dz}{d\tau} =$$

$$= -L \cdot \frac{u(X)}{(C(Q))^2} \cdot C'(Q) \cdot l \cdot (k+1) \cdot \tau \cdot \frac{dZ}{d\tau} + l \cdot (k+1) \cdot \frac{u'(z)}{C(q)} \cdot \frac{dz}{d\tau} =$$

$$= l \cdot (k+1) \cdot \left(-L \cdot \frac{u(X)}{(C(Q))^2} \cdot C'(Q) \cdot \tau \cdot \frac{dZ}{d\tau} + \frac{u'(z)}{C(q)} \cdot \frac{dz}{d\tau} \right) =$$

$$= l \cdot (k+1) \cdot \left(-L \cdot \frac{u(X)}{(C(Q))^2} \cdot C'(Q) \cdot \tau \cdot \frac{dZ}{d\tau} + \frac{u'(z)}{C(q)} \cdot w \cdot \frac{C(q)}{C(Q)} \cdot \frac{dZ}{d\tau} \right) =$$

$$= l \cdot (k+1) \cdot \frac{w}{C(Q)} \cdot \left(-L \cdot \frac{u(X) \cdot C'(Q)}{C(Q)} \cdot \frac{\tau}{w} + u'(z) \right) \cdot \frac{dZ}{d\tau} =$$

(using the fact that $R(X) \equiv u(X) \cdot E_u(X)$)

$$= l \cdot (k+1) \cdot \frac{w}{C(Q)} \cdot \left(-L \cdot \frac{C'(Q) \cdot R(X)}{C(Q) \cdot E_u(X)} \cdot \frac{\tau}{w} + u'(z) \right) \cdot \frac{dZ}{d\tau} =$$

(simplifying this due to zero-profit condition at autarky: $L \cdot R(X) = \Lambda \cdot w \cdot C(Q)$)

$$= l \cdot (k+1) \cdot \frac{w}{C(Q)} \cdot \left(-\frac{C'(Q)}{E_u(X)} \cdot \tau \cdot \Lambda + u'(z) \right) \cdot \frac{dZ}{d\tau} =$$

(because at autarky $R'(z) = u'(z) \cdot (1 - r_u(z)) = u'(z) \cdot (1 - r_u(0)) = u'(z)$)

$$= l \cdot (k+1) \cdot \frac{w}{C(Q)} \cdot \left(-\frac{C'(Q)}{E_u(X)} \cdot \tau \cdot \Lambda + R'(z) \right) \cdot \frac{dZ}{d\tau} =$$

(due to linearity of costs: $C'(Q) = C'(q) = c$)

$$= l \cdot (k+1) \cdot \frac{w}{C(Q)} \cdot \left(-\frac{C'(q)}{E_u(X)} \cdot \tau \cdot \Lambda + R'(z) \right) \cdot \frac{dZ}{d\tau} =$$

(due to producer's FOC at autarky $C'(q) \cdot \tau \cdot \Lambda = R'(z)$)

$$= l \cdot (k+1) \cdot \frac{w}{C(Q)} \cdot \left(-\frac{R'(z)}{E_u(X)} + R'(z) \right) \cdot \frac{dZ}{d\tau} =$$

$$= l \cdot (k+1) \cdot \frac{R'(z) \cdot w}{C(Q)} \cdot \frac{E_u(X) - 1}{E_u(X)} \cdot \frac{dZ}{d\tau} > 0,$$

since $E_u(X) < 1$. This expresses the welfare gain in country $\mathscr{G}$ from increasing trade cost, that we are proving.

Analogously,

$$\frac{1}{l} \cdot \frac{dW^{\mathscr{L}}}{d\tau} = l \cdot \left(\frac{u'(x)}{C(q)} \cdot \frac{dx}{d\tau} - \frac{u(x)}{(C(q))^2} \cdot C'(q) \cdot \frac{dq}{d\tau} \right) +$$

$$+ L \cdot (K+1) \cdot \left(\frac{u'(Z)}{C(Q)} \cdot \frac{dZ}{d\tau} - \frac{u(Z)}{(C(Q))^2} \cdot C'(Q) \cdot \frac{dQ}{d\tau} \right) =$$

$$= -l \cdot \frac{u(x)}{(C(q))^2} \cdot C'(q) \cdot \frac{dq}{d\tau} + L \cdot (K+1) \cdot \frac{u'(Z)}{C(Q)} \cdot \frac{dZ}{d\tau} =$$

$$= -l \cdot \frac{u(x)}{(C(q))^2} \cdot C'(q) \cdot L \cdot (K+1) \cdot \tau \cdot \frac{dz}{d\tau} + L \cdot (K+1) \cdot \frac{u'(Z)}{C(Q)} \cdot \frac{dZ}{d\tau} =$$

$$= L \cdot (K+1) \cdot \left(-l \cdot \frac{u(x)}{(C(q))^2} \cdot C'(q) \cdot \tau \cdot \frac{dz}{d\tau} + \frac{u'(Z)}{C(Q)} \cdot \frac{dZ}{d\tau} \right) =$$

$$= L \cdot (K+1) \cdot \left(-l \cdot \frac{u(x)}{(C(q))^2} \cdot C'(q) \cdot \tau \cdot \frac{dz}{d\tau} + \frac{u'(Z)}{C(Q)} \cdot \frac{1}{w} \cdot \frac{C(Q)}{C(q)} \cdot \frac{dz}{d\tau} \right) =$$

$$= L \cdot (K+1) \cdot \frac{1}{C(q)} \cdot \left(-l \cdot u(x) \cdot \frac{C'(q)}{C(q)} \cdot \tau + \frac{u'(Z)}{w} \right) \cdot \frac{dz}{d\tau} =$$

$$= L \cdot (K+1) \cdot \frac{1}{C(q)} \cdot \left(-l \cdot \frac{R(x)}{E_u(x)} \cdot \frac{C'(q)}{C(q)} \cdot \tau + \frac{R'(Z)}{w} \right) \cdot \frac{dz}{d\tau} =$$

$$= L \cdot (K+1) \cdot \frac{1}{C(q)} \cdot \left(-\frac{C'(q)}{E_u(x)} \cdot \tau \cdot \lambda + \frac{R'(Z)}{w} \right) \cdot \frac{dz}{d\tau} =$$

$$= L \cdot (K+1) \cdot \frac{1}{C(q)} \cdot \left(-\frac{1}{E_u(x)} \cdot \frac{R'(Z)}{w} + \frac{R'(Z)}{w} \right) \cdot \frac{dz}{d\tau} =$$

$$= L \cdot (K+1) \cdot \frac{1}{C(q)} \cdot \frac{R'(Z)}{w} \cdot \frac{E_u(x) - 1}{E_u(x)} \cdot \frac{dz}{d\tau} > 0,$$

which expresses the welfare gain in country $\mathscr{L}$ from increasing trade cost, that we are proving.

Case (III): complete autarky arise in symmetric trade among big countries through $Y(\tau) \to 0$ $(\tau \to \tau_3)$. This case is analogous to case (I).

Thus, the Lemma is proven, and Proposition 2 follows from it evidently.

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Corresponding author:
Sergey Kokovin,
Leading Research Fellow
E-mail address: skokov7@gmail.com
Phone: 8 (812) 4001347