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Abstract
This paper investigates the timing of wealth transfers between generations. We develop an overlapping generations model in which each generation can borrow against its future income but not against expected bequest. As a result, generations relatively poorer than their parents may end up not smoothing consumption. We prove that if wealth transfers can take place earlier in life, then each generation smooths consumption despite the constraint on borrowing and the first best solution is restored. The model implies that parents transfer resources when the children are credit constrained. This implication is tested using Dutch survey data on households' intentions to make intervivos transfers matched with administrative data that allow to construct a measure of the probability of being in need of a transfer. All in all, the paper highlights the importance of intervivos transfers as a device that households can resort to in order to mitigate inter-generational wealth inequalities.

Keywords
intervivos transfers, credit constraints, overlapping generations

JEL Codes
D12, D13, D91

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When you need it or when I die? Timing of monetary transfers from parents to children*

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Abstract

This paper investigates the timing of wealth transfers between generations. We develop an overlapping generations model in which each generation can borrow against its future income but not against expected bequest. As a result, generations relatively poorer than their parents may end up not smoothing consumption. We prove that if wealth transfers can take place earlier in life, then each generation smooths consumption despite the constraint on borrowing and the first best solution is restored. The model implies that parents transfer resources when the children are credit constrained. This implication is tested using Dutch survey data on households' intentions to make intervivos transfers matched with administrative data that allow to construct a measure of the probability of being in need of a transfer. All in all, the paper highlights the importance of intervivos transfers as a device that households can resort to in order to mitigate inter–generational wealth inequalities.

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1 Introduction

Wealth transmission between generations, and in particular from parents to their offsprings, is an important topic both from an individual and from an aggregate perspective. From a

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microeconomic perspective intended transfers motivated by, e.g., altruism help to extend the standard life cycle model to accommodate for a non–declining saving profile of the elderly. As an example, Hurd (1989) predicts a non–declining or even increasing saving pattern due to a bequest motive plus mortality risk. From an aggregate or macroeconomic point of view, the usual approach to study the effects of fiscal policies on the wealth distribution is to set up an overlapping generations model (OLG), where in each period the government can transfer resources from one generation to another, e.g. using general taxation on workers to finance pensions for the elderly, or financing current public spending by increasing public debt. The extent to which those policies affect the wealth distribution in the population and in particular across generations, depends on the degree of altruism between older and younger individuals. If parents can and are willing to transfer part of their resources to their children in response to a tax–driven income shock on younger generations they may, at least partially, off–set the negative shock (the Ricardian equivalence).

Monetary transfers from parents to children can take place at the moment of parent's death via bequest and during life via intervivos. Therefore, conditional on having decided to transfer money to children, parents face the choice of the timing of such a transfer. If the only driving mechanism behind transfers is altruism and capital markets are perfect, once legal bindings on end-of-life transfers are taken into account, bequests and intervivos should be perfect substitutes. Several authors motivate the existence of intervivos as responses to tax incentives on monetary gifts versus end of life transfers (Page, 2003; Poterba, 2001; McGarry, 2000; Bernheim, Lemke, and Scholz, 2004; Joulfaian, 2004). An alternative approach, followed by Bernheim, Shleifer, and Summers (1985); Cox (1987) and Alessie, Angelini, and Pasini (2014) is to introduce exchange motives: transfers are motivated not only by altruism but also by strategic considerations to incentivize children to take good care of their parents. Empirical evidence does not always support the existence of tax reasons nor bequest motives to anticipate transfers, while there is extensive literature highlighting good reasons to postpone transfers such as a mean of self–insurance against longevity or health risk (Carroll, 1997). Still McGarry (1999) argues that uncertainty about future events does not necessarily imply lower intervivos and higher bequests. Based on a representative sample of American households from the Health and Retirement Study, McGarry observes that intervivos transfers go more often to less well–off children while bequests are divided equally across offsprings. In order to rationalize these observations she develops a two period model that predicts differing behavior towards intervivos transfers and bequests due to uncertainty about future income of the children and due to the fact that children face binding credit constraints.

In this paper we take a standard OLG model as, e.g., in Blanchard and Fischer (1989) as a starting point and without introducing any uncertainty nor market friction we show that intervivos serve as a device to reach an optimal consumption profile if generations have different lifetime income and cannot borrow against bequest. Compared to previous attempts to model intervivos transfers, we introduce only a minimal deviation from a standard life cycle model with altruism: we still assume that each generation can borrow up to their entire lifetime earnings – in other words, financial markets are perfect – but they cannot borrow against future bequest. The effect of relaxing the assumption that individuals can borrow against future bequests is that generations with relatively low lifetime income compared to their predecessors are not able to smooth consumption. Nevertheless if each generation can anticipate part of the transfer by means of intervivos, the first best solution is restored. In other words, each generation transfers resources when the next one is credit constrained. An example are cash transfers (or collateral provision) to children who need a mortgage to buy a house early in their working life.

The main advantage of our choice compared to alternative models is that the assumptions we are making affect the way households reach their first best solution, i.e. consumption smoothing, but not the equilibrium characteristics. This means our life cycle model can be harmlessly integrated into any extension to the standard OLG framework and it is therefore fully compatible with the existing macroeconomic literature on wealth transmission.

One prediction of our model is that inter–generational transfers occur when the young generation needs them to smooth consumption. We test this prediction on the Dutch DNB Household Survey (DHS), that have the unique feature of including very specific questions on intervivos intentions. Survey data are supplemented by administrative data used to construct a measure of the probability of having at least one child in need of the transfer. Results support the prediction of our proposed theoretical model that a transfer is most likely to occur when the next generation is credit constrained and the estimation procedure is robust to a number of different specifications. We therefore provide theoretical and empirical microfoundations to OLG models which allow for bequests and intervivos transfers: if borrowing against bequest is not allowed the timing of transfers is determined by the relative difference in life-time earnings between generations, and each generation smooths consumption.

The paper is organized as follows. In section 2 we develop the theoretical model. Section 3 describes the survey and administrative data. Section 4 provides the details of the empirical analysis and baseline results, while robustness checks are discussed in sections 5. The last section summarizes the contributions of the paper and concludes.

2 An economic model for the timing of transfers

The starting point is an Overlapping Generations (OLG) model with downward altruism *à la* Becker (1974), i.e. each generation's utility depends on next generation's utility. The utility function of a generation born at time t, V_t is

$$V_t = u(c_{1t}) + u(c_{2t+1}) + \alpha V_{t+1},\ t = 1, \ldots, T-1 \tag{1}$$

where an individual born at time t consumes c_{1t} in period t and c_{2t+1} in period $t+1$. Each generation lives for two periods. The per–period utility $u(c_{j\tau})$ is an increasing and strictly concave function of $c_{j\tau}$, α measures the degree of altruism. In order to focus on the essential features of our model we follow Constantinides, Donaldson, and Mehra (2002, 2007): first, we stick to a representative agent model without introducing heterogenous preferences. Second, we abstract from the labour–leisure trade–off, i.e. the wage process is exogenous. Moreover, the model is fully deterministic as e.g. in Auerbach and Kotlikoff (1987): income y_{it} is known to each generation and greater or equal than zero in each period. Finally, we assume that both the interest rate r_t and the rate of time preference θ are equal to zero.[1]

[1]Compared to Blanchard and Fischer (1989) and following their notation notation, $\theta = r_t = 0$. The drawback of this simplifying assumption is that all the comparisons between amounts are amplified due to the absence of

It is worth noticing that our aim is to study the timing of the transfer and not the reason why this transfer takes place. We argue it is altruism to motivate intergenerational transfers as in Becker and Barro (1998), but this is by no means a crucial assumption. If V_{t+1} is interpreted as utility of generation t out of bequest and α as a simple rescaling weight, the maximization problem can be rewritten according to Constantinides, Donaldson, and Mehra (2007) and encompasses the altruistic model we presented, but also egoistic set ups where bequest is motivated e.g. by warm glow considerations *à la* Andreoni (1990) as in Hurd (1989) or Kopczuk and Lupton (2007).

Table 1 provides a description of the timing structure of the model. In period t generation t is in his first period of life. He receives labour income y_{1t} and intervivos R_{t-1} transferred by previous generation $t-1$. During period 1 generation t consumes c_{1t}, saves for the second period A_t and allocates R_t to intervivos transfer for the next generation $t+1$. The intervivos transfer takes place at the end of the first period of life of each generation. In period 2, generation t receives labour income y_{2t+1}, his own savings from period 1 A_t, and the bequest transferred by generation $t-1$, b_t. During period 2 generation t consumes c_{2t+1} and allocates resources to bequest b_{t+1} to be left to generation $t+1$ at the end of period 2 , i.e. at death. We follow the literature on bequests and rule out inter-generational transfers from children to parents, in other words we assume intervivos and bequests cannot be negative: $R_s, b_s \geq 0, \ \forall s$.

2.1 The basic model: no intervivos and no credit constraints

We first solve the model in the case intervivos are all set to zero, $R_s = 0$, $\forall s$ and each generation can borrow against all future resources. If this is the case, generation t maximizes his utility V_t with respect to his choice variables $c_{1t}, c_{2t+1}, A_t, b_{t+1}$:

$$\max_{c_{1t},c_{2t+1},A_t,b_{t+1}} u(c_{1t}) + u(c_{2t+1}) + \alpha V_{t+1} \tag{2a}$$

subject to the following constraints:

$$c_{1t} = y_{1t} - A_t \tag{2b}$$
$$c_{2t+1} = y_{2t+1} + A_t + b_t - b_{t+1} \tag{2c}$$
$$b_{t+1} \geq 0 \tag{2d}$$

With b_1 known and given at time $t = 0$. We rule out corner solutions at zero consumption from the beginning and throughout the paper ($c_{1t} > 0$ and $c_{2t+1} > 0$). This positive net–worth constraint is satisfied if we assume the within period utility functions are of the CRRA type, or more mildly if $\lim_{x \to 0} u'(x) = \infty$. From constraints (2b),(2c), $c_{1t} > 0$ and $c_{2t+1} > 0$ it follows that

$$A_t \in (-y_{2t+1} - b_t, y_{1t}) \tag{3}$$

This means generation t receives the bequest b_t in period 2, but can borrow and consume out of it in period 1 as well.

Since the choices regarding money transfers of generation t affect behavior of the following generation, the model must be solved iteratively. I.e., conditional on the transfer from generation

an inter-temporal discount.

$T-1$ to generation T, b_T, the maximization can be solved for generation $T-1$ and then going backwards for each previous generation. The model is solved if a transversality condition assures b_T is optimal (Kamihigashi, 2008). A natural formulation of this transversality condition that does not entail any loss of generality, is to assume time is finite, i.e. generation $T-1$ is the last one and lives for two periods, $T-1$ and T. As a consequence $b_T = 0$. Maximization of generation $T-1$ does not depend on successive generations and concavity of the per–period utility function $u(c_{jt+i})$ leads to consumption smoothing.

$$c_{1T-1} = c_{2T} = \frac{y_{1T-1} + y_{2T} + b_{T-1}}{2} \tag{4}$$

The terminal condition $b_T = 0$ implies that generation $T-1$ maximizes his utility only with respect to c_{1T-1}, c_{2T}, or equivalently with respect only to A_{T-1}. The value of A_{T-1} at the optimum depends on the bequest generation $T-1$ receives from generation $T-2$, b_{T-1}:

$$A_{T-1}(b_{T-1}) = \frac{y_{1T-1} - y_{2T} - b_{T-1}}{2} \tag{5}$$

Generation $T-2$ anticipates that generation $T-1$ will optimize his own utility taking $T-2$ bequest b_{T-1} as given. A_{T-1} at the optimum is a function only of exogenous income and a choice variable of generation $T-2$, namely b_{T-1}. As a consequence, utility of generation $T-1$, V_{T-1}, is a function only of choice variables of generation $T-2$. Since generation $T-2$ utility depends on V_{T-1}, generation $T-2$ observes the exogenous stream of income of generation $T-1$, and transfers b_{T-1} in order to maximize V_{T-1}. This is the same strategic interaction described in the two generations model of Altonji, Hayashi, and Kotlikoff (1997). Formally, this means generation $T-2$ optimization can be rewritten adding A_{T-1} among the choice variables and adding generation $T-1$ constraints:

$$\max_{A_{T-2}, A_{T-1}, b_{T-1}} u(c_{1T-2}) + u(c_{2T-1}) + \alpha \left[u(c_{1T-1}) + u(c_{2T}) \right] \tag{6a}$$

subject to

$$\begin{aligned}
c_{1T-2} &= y_{1T-2} - A_{T-2} && \text{(6b)}\\
c_{2T-1} &= y_{2T-1} + A_{T-2} + b_{T-2} - b_{T-1} && \text{(6c)}\\
b_{T-1} &\geq 0 && \text{(6d)}\\
c_{1T-1} &= y_{1T-1} - A_{T-1} && \text{(6e)}\\
c_{2T} &= y_{2T} + A_{T-1} + b_{T-1} && \text{(6f)}\\
& && \text{(6g)}
\end{aligned}$$

From the first order conditions follows that

$$A_{T-2} = \frac{y_{1T-2} + b_{T-1} - y_{2T-1} - b_{T-2}}{2} \tag{7}$$

which leads to consumption smoothing:

$$c_{1T-2} = c_{2T-1} = \frac{y_{1T-2} + y_{2T-1} + b_{T-2} - b_{T-1}}{2} \tag{8}$$

Now, V_{T-1} depends on choice variables of generation $T-2$, namely b_{T-1}, but not on choice variables of generation $T-3$. This does not depend on the specific time period: A_t and V_t at the optimum depend on generation t choice variable b_{t+1} and on choice variables of generation $t-1$, namely b_t, but not on choice variables of generation $t-2$:

$$V_t = u(c_{1t}) + u(c_{2t+1}) + \alpha V_{t+1}(b_{t+1}) \tag{9}$$

and

$$A_t(b_t, b_{t+1}) = \frac{y_{1t} - y_{2t+1} + b_{t+1} - b_t}{2}$$

In other words, (generic) generation t decision about bequest will affect directly his own consumption and consumption of generation $t+1$, but not consumption of the successive generations. This is important since it limits the strategic interactions of each generation: generation t moves second in a dynamic game of complete information with generation $t-1$, while moves first in an identical game played with generation $t+1$, and he does not need to account for behavior of generations other than the previous and the following one. Given this setting, each generation smooths income differences between first and second period in order to keep consumption constant and at equilibrium:

$$c_{1t} = c_{2t+1} = \frac{y_{1t} + y_{2t+1} + b_t - b_{t+1}}{2} \tag{10}$$

Moreover, altruism induces each generation to use bequests to smooth income differences with the next generation. This is a well-known result that corresponds to, e.g., the first best solution of the household part of the OLG model in Blanchard and Fischer (1989). Formally, if generation t is willing to bequeath we are at an interior solution for b_{t+1}:

$$MRS_{t,t+1} = \frac{u'(c_{2t+1})}{u'(c_{2t+2})} = \alpha \tag{11}$$

This means the degree of altruism equals the marginal rate of substitution between own and next generation's consumption, thus the closer α is to 1, the more consumption is smoothed between generations.[2] If generation t is not willing to bequeath to generation $t+1$ (non–negativity constraint on bequest is binding), then

$$u'(c_{2t+1}) = \alpha u'(c_{2t+2}) + \mu_{t+1} \Rightarrow MRS_{t,t+1} = \frac{u'(c_{2t+1})}{u'(c_{2t+2})} > \alpha \tag{12}$$

If generation $t+1$ is lifetime richer than generation t, and/or the degree of altruism is low (low α), then it is likely that $MRS_{t,t+1} > \alpha$ and therefore $b_{t+1} = 0$.

2.2 The model with credit constraints but no intervivos

In section 2.1 we assumed that via saving and borrowing, resources can be consumed in the first or second period, independently from the period in which they are effectively at the disposal of

[2] If $\alpha \leq 1$, from (11), $c_{1t} = c_{2t+1} > c_{1t+1} = c_{2t+2}$. If $\alpha > 1$, from (11), $c_{1t} = c_{2t+1} < c_{1t+1} = c_{2t+2}$.

households. This means assuming that households can freely save or borrow against own future income, but also against future bequest. In order for individuals to borrow against bequests, it means banks and financial institutions lend to dynasties, not to single individuals. This may be reasonable in a model with strong altruism, e.g. as in Laitner (1992) where parents treat their descendants as they were themselves. We do not want to restrict the attention to this special case, therefore we adjust equation (3) and assume that each generation can freely borrow against his own future earnings, but not against bequests. Formally, generation t maximizes his utility V_t with respect to his choice variables c_{1t}, c_{2t+1}, A_t and b_{t+1}:

$$\max_{c_{1t},c_{2t+1},A_t,b_{t+1}} u(c_{1t}) + u(c_{2t+1}) + \alpha V_{t+1} \tag{13a}$$

subject to the following constraints:

$$\begin{aligned} c_{1t} &= y_{1t} - A_t && \text{(13b)} \\ c_{2t+1} &= y_{2t+1} + A_t + b_t - b_{t+1} && \text{(13c)} \\ A_t &\geq -y_{2t+1} && \text{(13d)} \\ b_{t+1} &\geq 0 && \text{(13e)} \end{aligned}$$

With b_1 known and given at time $t = 0$. The credit limit, formalized in constraint (13d), states that generation t can borrow up to his whole second period income but not against bequest. It follows that

$$A_t \in [-y_{2t+1}, y_{1t}).$$

As in section 2.1, to solve the model we use backward induction and exploit the fact that each generation solves its problem conditional on the next generation optimal behavior. Generation t, conditional upon the optimal bequest of generation $t+1$ to the following generation denoted by b_{t+2}, faces the following optimization problem:

$$V_t(b_t) = \max_{A_t,A_{t+1},b_{t+1}} u(c_{1t}) + u(c_{2t+1}) + \alpha\left[u(c_{1t+1}) + u(c_{2t+2}) + \alpha V_{t+2}(b_{t+2})\right] \tag{14a}$$

subject to the following constraints

$$\begin{aligned} c_{1t} &= y_{1t} - A_t && \text{(14b)} \\ c_{2t+1} &= y_{2t+1} + A_t + b_t - b_{t+1} && \text{(14c)} \\ c_{1t+1} &= y_{1t+1} - A_{t+1} && \text{(14d)} \\ c_{2t+2} &= y_{2t+2} + A_{t+1} + b_{t+1} - b_{t+2} && \text{(14e)} \\ b_{t+1} &\geq 0 && \text{(14f)} \\ A_\tau &\geq -y_{2\tau+1},\ \tau = t, t+1 && \text{(14g)} \end{aligned}$$

The credit constraint (14g) implies

$$A_t = \frac{y_{1t} + b_{t+1} - y_{2t+1} - b_t}{2} \geq -y_{2t+1} \tag{15}$$

Equation (15) can be rewritten as follows:

$$b_t \leq y_{1t} + y_{2t+1} + b_{t+1} \tag{16}$$

Generation t smooths consumption if the bequest received b_t is relatively small. This means that if generation t does not receive any bequest ($b_t = 0$), then he will certainly smooth. Moreover, simple comparative statics highlights that, other things being equal, the larger is the bequest generation t is willing to leave to generation $t+1$ (b_{t+1}), the more likely it is that generation t smooths consumption. This is due to the fact that each generation leaves a bequest to the next generation at the end of the second period. The larger the bequest, the less likely it is that such a generation will need to borrow in his first period of life in order to smooth consumption and, therefore, also the less likely that he will be credit constrained. Equation (16) also implies that generation t is credit constrained and does not smooth consumption if the difference between received and given bequests $b_t - b_{t+1}$ is larger than lifetime earnings $y_{1t} + y_{2t+1}$. The optimal consumption path in this case is

$$c_{1t} = y_{1t} + y_{2t+1} < c_{2t+1} = b_t - b_{t+1}$$

Since each generation has the same degree of altruism and the same per period utility function, what determines whether a specific generation is credit constrained or not is heterogeneity in income across generations. Compared to the existing literature, e.g. McGarry (1999, 2000); Altonji, Hayashi, and Kotlikoff (1997), there is no need to invoke heterogeneous preferences nor unexpected income shocks to observe a suboptimal solution for a given generation. For a generation to be credit constrained in this model it suffices that his permanent income is relatively low compared to previous generation's permanent income. Equation (16) implies that the difference in permanent income between generation $t-1$ and t must be very large in order to observe t not smoothing. Note anyway this is simply an artifact of two combined simplifying assumptions: first, we set the intertemporal discount factor and interest rates equal to zero. Second, we assumed there are no market frictions and therefore each generation can entirely borrow his own second period income. If we assume financial markets are not perfect and individual can borrow only a $(1-\delta)$ fraction of their second period income where $\delta \in (0,1)$, then (14g) becomes

$$A_\tau \geq -(1-\delta)y_{2\tau+1}, \ \tau = t, t+1$$

and (16) becomes

$$b_t \leq y_{1t} + (1-2\delta)y_{2t+1} + b_{t+1}$$

Now, if we assume agents can borrow only $\delta = 0.5$ of their future income, it is enough that the difference between received and given bequests $b_t - b_{t+1}$ is larger than first period income y_{1t} and generation t will not smooth.[3] The crucial point is that if borrowing against bequest is

[3]Note that if $\delta > 0.5$ we should add additional assumptions on the relative magnitude of $y_{1\tau}$ and $y_{2\tau+1}$, and/or a positive interest rate r_τ in order to close the model. As we already mentioned, this would complicate the analysis without providing further insight regarding the key results of the paper.

not possible, without invoking any further deviation from the standard, textbook OLG model presented in section 2.1, we can observe suboptimal solutions to the model, i.e. some generations do not smooth consumption and choose second–best solutions.

2.3 The full model with bequests and intervivos transfers

Now suppose that generation t may also receive an intervivos transfer R_{t-1} at the beginning of period t from the previous generation, and can transfer via intervivos to the next generation $t+1$ at the end of period t (R_t). The full maximization problem is the following:

$$\max_{c_{1t}, c_{2t+1}, R_t, b_{t+1}} u(c_{1t}) + u(c_{2t+1}) + \alpha V_{t+1} \tag{17a}$$

subject to the following constraints

$$\begin{aligned} c_{1t} &= y_{1t} - A_t + R_{t-1} - R_t && (17b) \\ c_{2t+1} &= y_{2t+1} + A_t + b_t - b_{t+1} && (17c) \\ b_{t+1} &\geq 0 && (17d) \\ R_t &\geq 0 && (17e) \\ A_t &\geq -y_{2t+1} && (17f) \end{aligned}$$

With b_1, R_0 known and given at time $t = 0$. It is important to observe that we are not introducing any fiscal incentive to prefer intervivos to bequest as in Poterba (2001) or in Nishiyama (2002). This can be easily done, but our aim is to highlight the fact it is enough to drop the assumption that households can borrow against bequest to observe intervivos transfers.

From constraints (17b),(17c) and (17f), as well as from the maintained assumption that $c_{1t} > 0$ and $c_{2t+1} > 0$, it follows that $A_t \in [-y_{2t+1}, y_{1t} + R_{t-1} - R_t)$. The credit constraint still limits the span of generation t choice about A_t. There are two possibilities to transfer money between generations, intervivos and bequests, and each generations decision about transfers affects directly only the maximization problem of the next generation and not of the generations thereafter. It is then possible to rewrite the maximization problem of generation t conditional on the choice variables R_{t+1} and b_{t+2} of generation $t+1$ evaluated at their optimum, and to include saving A_{t+1} of generation $t+1$ among the choice variables of generation t, as well as all the relevant constraints in the maximization problem, and solve the maximization problem of generic generation t.

$$V_t(R_{t-1}, b_t) = \max_{A_t, A_{t+1}, R_t, b_{t+1}} u(c_{1t}) + u(c_{2t+1}) + \alpha \left[u(c_{1t+1}) + u(c_{2t+2}) + \alpha V_{t+2}(R_{t+1}, b_{t+2}) \right] \tag{18a}$$

subject to the following constraints

$$
\begin{aligned}
c_{1t} &= y_{1t} - A_t + R_{t-1} - R_t && \text{(18b)}\\
c_{2t+1} &= y_{2t+1} + A_t + b_t - b_{t+1} && \text{(18c)}\\
c_{1t+1} &= y_{1t+1} - A_{t+1} + R_t - R_{t+1} && \text{(18d)}\\
c_{2t+2} &= y_{2t+2} + A_{t+1} + b_{t+1} - b_{t+2} && \text{(18e)}\\
b_{t+1} &\geq 0 && \text{(18f)}\\
R_t &\geq 0 && \text{(18g)}\\
A_\tau &\geq -y_{2\tau+1},\ \tau = t, t+1 && \text{(18h)}
\end{aligned}
$$

The solution follows exactly the same steps as in section 2.2: we solve the model backwards assuming transfers to the last generation (R_{T-1} and b_T in this case) are optimal, or, without loss of generality, time is finite and therefore $R_{T-1} = b_T = 0$. If generation t is not credit constrained, then

$$c_{1t} = c_{2t+1} = \frac{y_{1t} + R_{t-1} - R_t + y_{2t+1} + b_t - b_{t+1}}{2}$$

And from (18h)

$$b_t - R_{t-1} \leq y_{1t} + y_{2t+1} + b_{t+1} - R_t$$

That is the transfer received by generation t in his second period in excess to what he received in the first period, $b_t - R_{t-1}$ must be relatively small. If also generation $t+1$ is not credit constrained and smooths consumption, then it also holds that

$$b_{t+1} - R_t \leq y_{1t+1} + y_{2t+2} + b_{t+2} - R_{t+1} \tag{19}$$

Again from simple comparative statics, and holding total transfers from generation t to generation $t+1$ ($R_t + b_{t+1}$) constant, the timing of transfers affects the chances that generations t and $t+1$ are both credit constrained. If generation t postpones the transfers, i.e. b_{t+1} is relatively big compared to R_t, then it is more likely generation t will not be credit constrained while generation $t+1$ will be credit constrained. The reason being that a higher transfer late in life of generation t reduces the borrowing needs for who is making the transfer, while it increases the need to borrow of generation $t+1$. We are now at the key theoretical result of the paper: we can prove that at equilibrium each generation will set R_t in such a way the credit constraint is not binding for generation $t+1$, that is intervivos R_t allow to restore the first best solution.

Theorem. (Intervivos transfers restore the first best solution) *Suppose each generation maximizes (17a) under constraints (17b) to (17f), i.e. agents cannot borrow against bequests, but can transfer money both via bequests and via intervivos. Moreover, suppose generation 1 does not receive any bequest ($b_1 = 0$), and $R_0 \geq 0$ is known. Then the credit constraint is never binding and all generations smooth consumption.*

Proof. The optimization problem (17) can be rewritten as (18) and the corresponding Lagrangian function is:

$$\begin{aligned}L &= u(y_{1t} - A_t + R_{t-1} - R_t) + u(y_{2t+1} + A_t + b_t - b_{t+1}) \\ &+ \alpha \left[u(y_{1t+1} - A_{t+1} + R_t - R^*_{t+1}) + u(y_{2t+2} + A_{t+1} + b_{t+1} - b^*_{t+2}) + \alpha V_{t+2}(R^*_{t+1}, b^*_{t+2})\right] \\ &+ \mu_{t+1}(b_{t+1}) + \nu_t(R_t) + \lambda_t(A_t + y_{2t+1}) + \lambda_{t+1}(A_{t+1} + y_{2t+2})\end{aligned}$$

From the first order conditions it follows that:

$$u'(c_{1t}) = u'(c_{2t+1}) + \lambda_t \quad \Leftrightarrow \quad u'(c_{1t}) = u'(c_{2t+1})(1 + \lambda^*_t) \tag{20a}$$

$$\alpha u'(c_{1t+1}) = \alpha u'(c_{2t+2}) + \lambda_{t+1} \quad \Leftrightarrow \quad \alpha u'(c_{1t+1}) = u'(c_{2t+2})(\alpha + \lambda^*_{t+1}) \tag{20b}$$

$$u'(c_{2t+1}) = \alpha u'(c_{2t+2}) + \mu_{t+1} \quad \Leftrightarrow \quad u'(c_{2t+1}) = u'(c_{2t+2})(\alpha + \mu^*_{t+1}) \tag{20c}$$

$$u'(c_{1t}) = \alpha u'(c_{1t+1}) + \nu_t \quad \Leftrightarrow \quad u'(c_{1t}) = u'(c_{1t+1})(\alpha + \nu^*_t) \tag{20d}$$

where $\lambda^*_\tau = \lambda_\tau / u'(c_{2\tau+1}),\ \tau = t, t+1$ denote rescaled Kuhn-Tucker multipliers corresponding to the two liquidity constraints (18h). $\mu^*_{t+1} = \mu_{t+1}/u'(c_{2t+2})$and $\nu^*_t = \nu_t/u'(c_{1t+1})$ are the rescaled Kuhn-Tucker multipliers corresponding to the non-negativity constraints (18f) and (18g). Equations (20) imply the following relationship between these four Kuhn-Tucker multipliers:

$$\frac{(\alpha + \lambda^*_{t+1})}{\alpha(1 + \lambda^*_t)} = \frac{(\alpha + \mu^*_{t+1})}{(\alpha + \nu^*_t)} \tag{21}$$

Equality (21) rules out the possibility that only one of the four inequality constraints (18h),(18f) and (18g) is binding (in that case only one of the four Kuhn-Tucker multipliers is positive while the other ones are equal to zero).[4] Equality (21) also rules out the following cases:

$$\lambda^*_t > 0;\ \lambda^*_{t+1} = 0;\ \mu^*_{t+1} > 0;\ \nu^*_t = 0$$

$$\lambda^*_t = 0;\ \lambda^*_{t+1} > 0;\ \mu^*_{t+1} = 0;\ \nu^*_t > 0$$

Moreover, generation $t+1$ cannot be at the same credit constrained ($A_{t+1} = -y_{2t+2},\ \lambda^*_{t+1} > 0$) and a non-recipent of a bequest ($b_{t+1} = 0,\ \mu^*_{t+1} > 0$ because in that case period 2 consumption of generation $t+1$ would not be positive according to equation (18e) ($c_{2t+2} = -b_{t+2} \leq 0$). This means that there are no admissible solutions where generation t smooths consumption while generation $t+1$ is credit constrained. By using dynamic programming argument, one can easily show that in that case the successive generations $t+2, \ldots$ are also not liquidity constrained.

Finally, if b_1 and R_0, which are known at time t $= 0$, are such that $R_0 \geq b_1 = 0$, then equation (19) hold for period 1 and therefore generation 1 is not liquidity constrained and smooths consumption. By using the previous argument, successive generations are also not liquidity constrained. After ruling out all the not admissible cases, we conclude that at the optimum it holds that:

1. $c_{1t} = c_{2t+1}$ ($\lambda^*_t = 0$); $c^*_{1t+1} = c^*_{2t+2}$ ($\lambda^*_{t+1} = 0$); $b_{t+1} > 0$ ($\mu^*_{t+1} = 0$); $R_t > 0$ ($\nu^*_t = 0$)
2. $c_{1t} = c_{2t+1}$ ($\lambda^*_t = 0$); $c^*_{1t+1} = c^*_{2t+2}$ ($\lambda^*_{t+1} = 0$); $b_{t+1} = 0$ ($\mu^*_{t+1} > 0$); $R_t = 0$ ($\nu^*_t > 0$)

[4] If $\alpha = 0$, then it is possible to have $\lambda^*_t > 0$ and the other 3 Kuhn-Tucker multipliers different from zero, but in this case any transfer to the next generation reduces utility . Each generation smooths his own consumption

That is, all generations smooth consumtion and the first best solution is restored. □

A natural choice of initial values R_0 and b_1 is $R_0 = b_1 = 0$: in period 1 there is no previous generation to receive transfers from. Note that anyway the condition $R_0 \geq b_1 = 0$ can be further relaxed: as long as equation (19) holds for period 1, then generation 1 and all the following ones smooth consumption.

In the benchmark model presented in section 2.1, the smoothing result comes from the fact that each generation anticipates the amount he will receive from the previous generation at the end of period 2, and borrows against it. In the extended model we just presented, still each generation anticipates the amount the parent wants to transfer. The parent (generation t) knows the income the child will earn in each period and sets $b_{t+1} - R_t$ in such a way the child will not be credit constrained and will smooth consumption. Note that this does not lead to anticipate all the transfer from the end of life as bequest to the first period as intervivos: the only admissible solutions are such that either generation t does not transfer any resource to generation $t+1$, or he transfers both via intervivos but also via bequest. This result follows from the fact that generation t wants to smooth as well, therefore anticipates only what is strictly necessary for generation $t+1$ to smooth his own consumption: as we explained, reducing b_{t+1} and increasing R_t makes it more likely that generation t becomes credit constrained. In other words, we highlight the existence of a trade-off each generation faces, a trade-off that it is not possible to appreciate if not in our extended life cycle model: generation t wants $(b_{t+1} - R_t)$ to be big in order to smooth his own consumption, but wants also $(b_{t+1} - R_t)$ to be small in order to let generation $t+1$ smooths as well.

Our result has a clear empirical implication: in equilibrium generation t sets intervivos and bequest in order to relieve the credit constraint of the next generation $t+1$. The key contribution to the existing literature on inter–generational transfers is that there is no need to invoke unexpected shocks in order to justify intervivos transfers: if the parent knows the child will have low income during his first period, e.g. because while young the child has children himself or buys a house, the parent can use intervivos to let the child smooth. In the next sections we empirically test this implication.

3 Data: the DNB Household Survey and the Income Panel Study of the Netherlands

The DNB Household Survey (DHS) is a unique data set that allows to study both psychological and economic aspects of financial behavior. This panel survey was launched in 1993 and comprises information on work, pensions, housing, mortgages, income, assets, loans, health, economic and psychological concepts, and personal characteristics. We refer to Teppa, Vis et al. (2012) for more detailed information about the DHS. The sample consists of, on average, 2,000 Dutch households per year and it is a representative sample of the Dutch-speaking population. For our main analysis we only use data for the period 2001–2008. We start from the 2001 wave because in order to keep the maintain representativeness in 2001 there was a major resampling. In addition, we only include data for the years up to and including 2008 in order to avoid the economic crisis. As we will see in section 5 replicating the analysis on the full sample results do not change.

Important for our paper is the questionnaire section on "economic and psychological concepts" in which all household members are asked about their intentions about bequests and intervivos transfers. As regards intervivos transfers, the key question (PLAN) reads as follows:

> Do you give substantial amounts of money to your children in order to transfer part of your capital to them, or are you planning to do so in the future, e.g. every year?
>
> 1. no
> 2. yes, I already give substantial amounts now
> 3. yes, I am planning to give substantial amounts in the future
> 4. don't know

Each respondent could give at most one answer as no multiple answers were allowed. This question is asked only to respondents with children. Figure 1 reports age profiles of each answer. Values on cohort/age cells with less than 5 observations are set to missing in order to avoid peaks due to low frequency of answers. For the same reason, we consider in this figure only individuals aged 20–80.[5] Age profiles follow the intuition: as people age, they first start planning to transfer in the future and gradually reduce such a planning and start to actually transfer. There are no clear cohort effects.

The DHS contains also questions about monetary transfers within the family, in particular respondents are asked whether they gave money to any member of their family in the year prior to the interview, and which amount was transferred. These information can be used to proxy intervivos, but differently from PLAN they are meant to capture transfers to any family members and not only to children, in particular respondents may transfer money to their parents, rather than their children. Moreover, compared to PLAN they refer only to the year prior to the interview. We, therefore, do not use this information to replace PLAN, but only to restrict to large intervivos for a sensitivity analysis in section 5.

The dataset covering the period 2001-208 contains 7152 individuals and 26198 year-person observations. The DHS questionnaire is administered to all household members older than 16. We restrict the attention to household heads and their spouses or cohabiting partners, and given that we are looking at intervivos transfers we focus on respondents with at least one child. Once observations with missing values in key variables are deleted, the estimation sample consists of 1892 individuals for 6648 year-person observations. Lastly, we are interested in parents trade-off between relieving the credit constraint of (one of) the children and postponing a transfer to smooth his or her own consumption. As we saw in section 2.3, a parent who is poorer than his child can optimally decide not to make an intervivos transfer despite the child is in need of money. If this is the case, the data would not allow to validate the model. Therefore we restrict the analysis to homeowners. Our final sample consists of 1465 individuals and 5053 year-person observations.

[5] Cohorts are always defined over a ten years period in order to have a reasonable number of observations in each of them.

The available information about respondents' children in the DHS is limited to age and gender, there is not a measure of children income nor of children wealth. For this reason, we resort to the Income Panel Study of the Netherlands (IPO) to compute a proxy of the presence of credit constraints. IPO is an administrative database of individual incomes collected by Statistics Netherlands from official records such as tax records, population registry, institutions that pay out (insurance) benefits and the department of housing (because of rent subsidies). The IPO is a representative sample of the Dutch population of, on average, about 95,000 individuals per year from 1995 onward. Most important for our study is that IPO contains data on the demographic compositions of the households the individuals belong to and whether he or she (or his or her partner) is a homeowner. Individuals remain in the panel for as long as they are alive and residing in the Netherlands. We compute the hazard rate by age of having a child in the subsequent year (i.e. the probability of a child being born conditional on yet having children), the hazard rate of being a homeowner (i.e. the probability of buying a house conditional on not yet owning one) and the hazard rate of the joint event. We use these hazards to compute proxies of the likelihood of facing a reduction of future disposable income. Figure 2 reports the hazard rates by age based on the IPO years 1995-2010.[6] The hazard rate of being a homeowner peaks right before age 30, while the hazard rate of having a child rises until age 33 and then declines faster than the housing hazard rate. The hazard rate of the joint event is relatively low at each age.

We merge the three hazard rates to the DHS data based on the age of each child in the household. For each child we can compute the hazard rate for the union of the two events: having the first child in the subsequent year or buying the first home in the subsequent year. The final step is to compute the variable *haz-union* for each respondent as the probability that at least one of the children has a child for the first time in the subsequent year, or at least one of the children buys a house for the first time. In order to compute such a variable we have to assume that the choices of each child are independent of the choices of his or her brothers and sisters. We will use this variable as our preferred proxy for the likelihood that at least one child is in need of a transfer: the identifying assumption is that parents with higher *haz-union* probability are more likely to have at least one child with a binding credit constraint. The bottom left panel of figure 3 reports the distribution of *haz-union* by age and cohort of the respondent. The distribution is right–skewed, and cohort differences are clear: older cohorts tend to have higher probabilities of having children who are facing a reduction in life–time disposable income at older ages. Looking at the top panels of the same figure, it is evident that the cohort effect is due both to the hazard rate of having a grandchild, and to the hazard rate of at least one child buying his or her first house.

4 Empirical results

The key empirical implication of our theoretical model is that if individuals cannot borrow against bequest, the presence of credit constraints on the child increases the probability of observing intervivos transfers. The empirical strategy is to define a discrete indicator for the

[6] Hazard rates cannot be computed by year due to low frequencies. Computing them over sub-periods, it is possible to observe only limited and gradual changes in hazard rates over time.

presence or intention to transfer based on the variable PLAN and to regress it on *haz-union*. The estimation model is the following:

$$dplan_{it} = \beta_0 + \beta_1 haz\text{-}union_{it} + \boldsymbol{x}'_{it}\boldsymbol{\beta} + \theta_i + \mu_t + u_{it} \tag{22}$$

where θ_i are individual specific effects that capture time-invariant differences across households such as degree of altruism, μ_t are year specific effects that capture macro shocks or cohort differences, and u_{it} is the idiosyncratic error term. The dependent variable $dplan_{it}$ takes value 0 if PLAN is equal to 1 (the respondent is not transferring, nor he is planning to transfer in the future), or to 4 (the respondent is still undecided whether to transfer or not in the future), while it takes value 1 if PLAN is equal to 2 or 3 (the respondent is transferring, or he is planning to transfer in the future). This assumption will be validated as part of a large set of robustness checks in section 5. Equation (22) is estimated by least squares when using a linear probability model specification and by maximum likelihood for probit model specification. Standard errors are always clustered at household level to account for common unobservables.

Income and consumption streams of each child are unobserved, and so it is the likelihood a child is in need of a transfer to smooth consumption. The identifying assumption is that parents with higher *haz-union* probabilities are more likely to have at least one child credit constrained. Under this assumption a positive effect of *haz-union* on $dplan_{it}$ might be considered empirical evidence in favor of our theoretical model of section 2.3 and would highlight the importance of intervivos transfers as a device that households can resort to in order to relieve the credit constraints of their children.

The theoretical model we proposed in section 2.3 deals only with intended transfers and bequests, and does not model explicitly other motives behind transfers. The advantage for the theoretical outline was that it resulted in clear predictions. However, in order to bring the model to data we need to accommodate for other determinants of the presence and timing of intervivos transfers. As we mentioned in the introduction, tax differentials between intervivos and bequest can induce the parent to anticipate/postpone the transfer (Page, 2003; Poterba, 2001). Moreover, postponing the transfer allows the parent to use it as a buffer saving to cover unforeseen income/wealth shocks, as for example due to unemployment, deteriorating health conditions, or longevity (see e.g. Carroll, 1997). Precautionary savings therefore are likely to lead to unintended bequest. Performing our empirical analysis on Dutch data turns out to be helpful in this respect: Alessie and Kapteyn (2001) find that the pervasive welfare system in the Netherlands is likely to reduce the incentives to postpone transfers. Similarly Hochguertel (2003), which also uses the DHS, finds a limited role for precautionary savings in portfolio decisions. Still, we have a number of variables to control for other determinants of timing and amount of transfers. We use individual income, household financial and real wealth, and a subjective question about the propensity to save in order to cover unforseen expenses[7]; self reported health to control specifically for health risk, and dummies for being employed, self employed or out of labour force to control for unemployment and income risk. All specifications include also regional dummmies, an indicator for being living in Amsterdam, Rotterdam or Den Haag (the

[7]The specific question, SPAARM10 asks respondents to rate how important is "to save to have some savings to cover unforeseen expenses" on a scale ranging from 1 to 7, where 1 means "very unimportant" and 7 "very important"

three biggest cities in the Netherlands) and a detailed set of controls for household composition: a dummy on whether a partner is living in the household, a quadratic polynomial in the number of children, the age of the oldest child, the total number of grandchildren. Fixed effects specifications include a quadratic term in age to account for nonlinear age effects (De Ree and Alessie, 2011), while specifications that allow for time invariant regressors also include education dummies and gender as well as a quadratic in age. Concerning the latter, we experimented with higher order polynomials but that left the results virtually unchanged. Finally, as we already mentioned, the baseline specifications is estimated on a sample of homeowners.

Table 2 reports fixed effects linear probability model (LPM) estimates of equation (22). Fixed effects allows to control for individual specific unobservables affecting both the decision to transfer and right hand side variables. The marginal effect of *haz-union* is positive and statistically significant at the 10 percent level. If the probability that at least one of the children has a child himself or buys a house in the following year increases by one percentage point, then the chances a transfer is planned or performed increase by 0.4 percentage points. This marginal effect increases to 0.5 percentage points and becomes significant at the 5 percent level if we augment the specification with some control variables (see columns 2 and 3 of Table 2). As regards the other right hand side variables, the variable 'poor health' has a positive impact on the probability of transferring money indicating that the exchange motive might be important (see e.g. Cox, 1987; Alessie, Angelini, and Pasini, 2014). An alternative explanation is that poor health indicate a relatively short residual lifespan and thus the significant marginal effect may point to an important role for tax incentives. The relation between the probability of intervivos transfers and the number of children seems hump shaped: parents with three children transfer most frequently. Economic resources do not play a statistically significant role[8], while being unemployed reduces the probability of transferring with respect to the omitted category, which is being retired or out of labour force.

Fixed effects estimation does not allow to estimate the effects of time invariant variables. To obtain insights into effects of gender and education column (1) of Table 3 reports random effects estimates. These results are qualitatively in line with what we found with fixed effect. The estimated magnitude of the marginal effect of the proxy for credit constraints is smaller: a one percentage point increase in the probability that one of the children faces a vredit constraint increases the chances of transferring or planning a transfer by 0.31 percentage points. As regards the time invariant regressors, education level does not seem to have an impact on the transfer via intervivos, while women are less likely to perform such a transfer. As regards the other regressors, in this specification financial wealth is significantly positively related with the probability to perform an intervivos transfer. As it is well-known, the random effect estimator requires the individual specific, time invariant unobservable characteristic to be (partially) uncorrelated with the observables. This is strong assumption and in column (2) of Table 3 we relax it and estimate a model that allows for correlation between the error term and time varying right hand side variables by including their individual specific means as suggested by Mundlak (1978). Individual specific averages are jointly significant indicating that the RE estimates in column (1) are inconsistent. Interestingly, gender and the education dummies become insignificant by

[8]We applied the following inverse hyperbolic sine transformation to income and wealth variables: $IHS(x) = \ln(x + \sqrt{x^2 + 1})$. Notably, $IHS(x) \approx \ln(2x)$ if $x >> 0$

the inclusion of individual specific averages.[9]

Column (3) of Table 3 reports the average partial effects based on the RE probit model using the baseline specification outlined in equation (22). The marginal effect of *haz-union* is significant and similar to the point estimate obtained with a comparable RE LPM (cf. column (1) of table 3). Column (4) reports marginal effects obtained from a correlated random effects probit model (Wooldridge, 1995). Again, these results are similar to the estimates obtained when using the Mundlak model (column (2); table 3).

The overall conclusion is that our empirical results are in support of the prediction of the theoretical model of section 2.3: a higher probability that the younger generation faces a credit constraint increases the probability that the older generation transfers or plans to transfer part of his wealth (an intervivos transfer) before death to relieve such a constraint.

5 Robustness checks

Results presented in section 4 may depend on sample selection and variables definition. The first potential mis–specification may regard the dependent variable. A planned transfer may take place a long time after the respondent declares he is willing to transfer in the future. If this is the case, the coefficient of *haz-union* may capture a spurious correlation. In column (1) of table 4 the dependent variable takes value 1 only if the respondent declare that he is actually transferring, and it is set to zero also for planned transfers. All regressions in table 4 are fixed effects estimates of a linear probability model, and are therefore comparable with our baseline estimates in column (3) of table 2. The estimated marginal effect of *haz-union* obtained with this more stringent definition of the dependent variable is very similar to the baseline estimate. In column (2) of table 4 the dependent variable takes value 1 if the respondents report to be currently transferring, and the respondents transfers to family members (not limited to children) at least 10,000 euros per year. The idea is that we want to focus on substantial transfers in order to rule out small gifts. The marginal effect of interest is still positive and statistically significant, the magnitude is somewhat smaller than the baseline estimate.

As explained in the previous section, we restricted the sample for estimation to households where the older generation has wealth to bequeath. In column (3) we extend the baseline specification by including an interaction term between *haz-union* and the hyperbolic sine of net financial wealth. The coefficient corresponding to this interaction is positive and significant at the 10 percent level. This result suggests that richer parents are more likely to perform intervivos transfers to their credit constrained children than poorer ones.

The baseline sample includes both partners within a household. If transferring is a joint household decision, then clustering standard errors may not be sufficient to properly account for correlations across cross–sectional observations. Column (4) reports estimates when we include only one individual per household namely the primary respondent. The magnitude of the marginal effect of interest again is very close to the baseline case, but only significant at the 10 percent level. The smaller precision in the estimates may be due to the fact that the sample is

[9]The Mundlak estimates of the time–varying right hand side variables differ slightly from the fixed effect ones because we have not included the individual specific averages of the year dummies.

reduced from 5053 observations and 1465 individuals to 3191 observations relative to less than a thousand individuals, or it may be the case that the selected household member is not in charge of financial transfers. The last column of table 4 reports estimates computed on the full panel rather than on the 2001-2008 period. Results are again very close to the baseline results, as regards the marginal effect of the parameter of interest.

In table 5 we consider alternative proxies for the likelihood that at least one child is in need of a transfer to smooth consumption. In column (2) we use the hazard rate that at least one of the children of the respondent has a first child *and* at least one child buys a house in the following year. This is clearly a tighter requirement than the one used in the baseline specification: it is reasonable to expect higher chances of being in financial distress when the two events take place in the same year. The point estimate is in line with this expectation, but the standard error is considerably larger than in the baseline specification due to the fact the joint event is extremely rare. Nonetheless the point estimate is statistically significant. In column (3) we use as proxy the probability that at least one child buys a house in the following year and in column (4) the probability of having a grandchild in the following year. Both marginal effects have a positive and significant effect and both the point estimate and the standard errors are bigger than in the baseline case.

6 Conclusion

Overlapping generations models "hide" an assumption: young generations can borrow against bequest. This is a very strong and, arguably, undesirable assumption as it means that banks lend money to dynasties rather than to individuals. In this paper we first relaxed this assumption and imposed a credit constraint to each generation equal to its own life cycle income. We showed that if financial markets are perfect but it is not possible to borrow against previous generation's resources, relatively poor generations end up not smoothing consumption. We then introduced intervivos transfers and showed that by allowing a generation to decide about the timing of the transfer to the next generation the first best solution is restored, i.e. the usual consumption smoothing result of standard OLG models.

Compared to existing literature on intervivos transfers, we did not need to invoke heterogeneous preferences, market frictions or uncertainty to justify the existence of intervivos transfers. Moreover, the characteristics of the equilibrium we described are exactly the same as in a standard OLG model. Our life cycle model can, therefore, harmlessly be integrated into any extension of a standard Overlapping Generations framework, i.e. it is fully compatible with the existing macroeconomic literature on wealth transmission.

The empirical implication of the model is clear-cut: looking at parents and children, if the latter face a credit constraint, the former are more likely to perform an intervivos transfer. The analysis on Dutch data provided empirical evidence in support of the theoretical model: the more likely it is that children become parents themselves or buy a house for the first time, the more likely it is that their parents perform an intervivos transfer. This result was shown to be robust to a wide range of different specifications.

All in all, we provided theoretical and empirical support for the existence and importance of intervivos transfers as a transmission mechanism within the family which can alleviate wealth inequalities between generations.

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A The basic model: no intervivos and no credit constraints

Generation t maximizes his utility V_t with respect to his choice variables $c_{1t}, c_{2t+1}, A_t, b_{t+1}$:

$$\max_{c_{1t},c_{2t+1},A_t,b_{t+1}} \quad u(c_{1t}) + u(c_{2t+1}) + \alpha V_{t+1} \tag{23a}$$

subject to the following constraints.

$$c_{1t} = y_{1t} - A_t \tag{23b}$$
$$c_{2t+1} = y_{2t+1} + A_t + b_t - b_{t+1} \tag{23c}$$
$$b_{t+1} \geq 0 \tag{23d}$$

With b_1 known and given at time $t = 0$. From constraints (23b),(23c), $c_{1t} > 0$ and $c_{2t+1} > 0$ it follows that

$$A_t \in (-y_{2t+1} - b_t, y_{1t})$$

This means individuals can borrow against future resources, including bequests. Now we have a money transfer between generations, therefore it will be necessary to solve it starting from last period. The transversality condition imposes b_t to be optimal, we impose wlg $b_T = 0$.

A.1 Generation $T - 1$

Generation $T - 1$ solves the following optimization problem:

$$\max_{c_{1T-1},c_{2T},A_{T-1}} \quad u(c_{1T-1}) + u(c_{2T}) \tag{24a}$$
$$\text{s.t. } c_{1T-1} = y_{1T-1} - A_{T-1} \tag{24b}$$
$$c_{2T} = y_{2T} + A_{T-1} + b_{T-1} \tag{24c}$$

The received bequest b_{T-1} acts a an additional exogenous source of income in period 2. We can simplify the optimization problem by substituting constraints (24b) and (24c) into the utility function (24a) and then solving the maximization with respect to A_{T-1}:

$$\max_{A_{T-1}} \quad u(y_{1T-1} - A_{T-1}) + u(y_{2T} + A_{T-1} + b_{T-1})$$

from which the first order condition follows:

$$u'(y_{1T-1} - A_{T-1}) = u'(y_{2T} + A_{T-1} + b_{T-1})$$

which implies

$$A_{T-1} = \frac{y_{1T-1} - y_{2T} - b_{T-1}}{2}$$

and

$$c_{1T-1} = c_{2T} = \frac{y_{1T-1} + y_{2T} + b_{T-1}}{2}$$

Generation $T - 1$ smooths consumption.

A.1.1 Generation $T-2$

Generation $T-2$ maximization is the following:

$$\begin{aligned} \max_{c_{1T-2}, c_{2T-1}, b_{T-1}, A_{T-2}} \quad & u(c_{1T-2}) + u(c_{2T-1}) + \alpha V_{T-1} \\ \text{s.t. } c_{1T-2} &= y_{1T-2} - A_{T-2} \\ c_{2T-1} &= y_{2T-1} + A_{T-2} + b_{T-2} - b_{T-1} \\ b_{T-1} &\geq 0 \end{aligned}$$

Generation $T-2$ anticipates that generation $T-1$ will optimize his own utility: V_{T-1} is generation $T-1$ utility function evaluated at its maximum. Due to the terminal condition $b_T = 0$ generation $T-1$ maximizes his utility only with respect to c_{1T-1}, c_{2T}, or equivalently with respect only to A_{T-1}. The value of A_{T-1} at the optimum depends on the bequest generation $T-1$ receives from generation $T-2$, b_{T-1}:

$$A^*_{T-1}(b_{T-1}) = \frac{y_{1T-1} - y_{2T} - b_{T-1}}{2}$$

Since A^*_{T-1} at the optimum is determined by a choice variable of generation $T-2$, namely b_{T-1}, V_{T-1} is a function only of choice variables of generation $T-2$. This means generation $T-2$ optimization can be rewritten as a maximization with respect to A_{T-1} as well, replacing V_{T-1} and adding generation $T-1$ constraints:

$$\max_{A_{T-2}, A_{T-1}, b_{T-1}} u(c_{1T-2}) + u(c_{2T-1}) + \alpha \left[u(c_{1T-1}) + u(c_{2T}) \right]$$

subject to the following constraints

$$\begin{aligned} c_{1T-2} &= y_{1T-2} - A_{T-2} \\ c_{2T-1} &= y_{2T-1} + A_{T-2} + b_{T-2} - b_{T-1} \\ b_{T-1} &\geq 0 \\ c_{1T-1} &= y_{1T-1} - A_{T-1} \\ c_{2T} &= y_{2T} + A_{T-1} + b_{T-1} \end{aligned}$$

The Lagrangian is

$$\begin{aligned} L = & \, u(y_{1T-2} - A_{T-2}) + u(y_{2T-1} + A_{T-2} + b_{T-2} - b_{T-1}) + \\ & + \alpha u(y_{1T-1} - A_{T-1}) + \alpha u(y_{2T} + A_{T-1} + b_{T-1}) + \mu_{T-1} b_{T-1} \end{aligned}$$

from which the Kuhn-Tucker conditions follow:

$$\frac{\partial L}{\partial A_{T-2}} \; : \; -u'(y_{1T-2} - A_{T-2}) + u'(y_{2T-1} + A_{T-2} + b_{T-2} - b_{T-1}) = 0 \tag{25a}$$

$$\frac{\partial L}{\partial A_{T-1}} \; : \; -\alpha u'(y_{1T-1} - A_{T-1}) + \alpha u'(y_{2T} + A_{T-1} + b_{T-1}) = 0 \tag{25b}$$

$$\frac{\partial L}{\partial b_{T-1}} \; : \; -u'(y_{2T-1} + A_{T-2} + b_{T-2} - b_{T-1}) + \alpha u'(y_{2T} + A_{T-1} + b_{T-1}) + \mu_{T-1} = 0 \tag{25c}$$

$$\mu_{T-1} \geq 0 \tag{25d}$$

$$b_{T-1} \geq 0 \tag{25e}$$

$$\mu_{T-1} b_{T-1} = 0 \tag{25f}$$

$$\tag{25g}$$

CASE 1: $\mu_{T-1} = 0;\; b_{T-1} \geq 0$

- FOC (25a) implies that

$$A_{T-2} = \frac{y_{1T-2} + b_{T-1} - y_{2T-1} - b_{T-2}}{2}$$

  and

$$c_{1T-2} = c_{2T-1} = \frac{y_{1T-2} + y_{2T-1} + b_{T-2} - b_{T-1}}{2}$$

- FOC (25b) implies that

$$A_{T-1} = \frac{y_{1T-1} - y_{2T} - b_{T-1}}{2}$$

  which leads to

$$c_{1T-1} = c_{2T} = \frac{y_{1T-1} + y_{2T} + b_{T-1}}{2}$$

  Generation $T-1$ smooths his consumption as well

- From FOC (25c) it follows that

$$MRS_{c_{2T-1},c_{2T}} = \frac{u'(c_{2T-1})}{u'(c_{2T})} = \alpha$$

  The degree of altruism determines which generation consumes more.

CASE 2: $\mu_{T-1} > 0;\; b_{T-1} = 0$

- As for CASE 1, FOC (25a) implies that

$$A_{T-2} = \frac{y_{1T-2} - y_{2T-1} - b_{T-2}}{2}$$

  which leads to

$$c_{1T-2} = c_{2T-1} = \frac{y_{1T-2} + y_{2T-1} + b_{T-2}}{2}$$

- Generation $T-1$ does not receive any bequest. Therefore,

$$c_{1T-1} = c_{2T} = \frac{y_{1T-1} + y_{2T}}{2}$$

- from FOC (34d) it holds that

$$u'\left(\frac{y_{1T-2} + y_{2T-1} + b_{T-2}}{2}\right) = \alpha u'\left(\frac{y_{1T-1} + y_{2T}}{2}\right) + \mu_{T-1}$$

  which leads to

$$MRS_{c_{2T-1},c_{2T}} = \frac{u'(c_{2T-1})}{u'(c_{2T})} > \alpha$$

  Given the concavity of the utility function $u(.)$, this condition more likely holds if c_{2T-1} is relatively low in comparison with c_{2T}. If generation $T-1$ is lifetime richer than generation $T-2$, and the degree of altruism is low (low α), it is likely that $b_{T-1} = 0$. In this case generation $T-1$ anticipates that he will not receive a bequest and smooth his consumption over his lifecycle.

A.1.2 Generation $T-3$

Generation $T-3$ maximization is the following:

$$\begin{aligned} \max_{c_{1T-3},c_{2T-2},b_{T-2},A_{T-3}} \quad & u(c_{1T-3}) + u(c_{2T-2}) + \alpha V_{T-2} \\ \text{s.t. } c_{1T-3} &= y_{1T-3} - A_{T-3} \\ c_{2T-2} &= y_{2T-2} + A_{T-3} + b_{T-3} - b_{T-2} \\ b_{T-2} &\geq 0 \end{aligned}$$

Generation $T-3$ anticipates that generation $T-2$ will optimize his own utility: V_{T-2} is generation $T-2$ utility function evaluatated at its maximum. Generation $T-2$ maximizes his utility with respect to c_{1T-1}, c_{2T} and b_{T-1}, or equivalently with respect only to A_{T-2} and b_{T-1}. As we already noticed at the beginning of the previous section, generation $T-2$ sets his optimal choice of $b_{T-1} = b^*_{T-1}$ taking into account that the optimal saving choice of generation $T-1$, A^*_{T-1}, is completely determined once generation $T-2$ sets b_{T-1}. Optimal saving of generation $T-2$, i.e. the value of A_{T-2} at the optimum, is a function of the bequest generation $T-2$ receives from generation $T-3$, b_{T-2}, and of the (optimal) bequest left to generation $T-1$:

$$A^*_{T-2}(b_{T-2}, b^*_{T-1}) = \frac{y_{1T-2} + b^*_{T-1} - y_{2T-1} - b_{T-2}}{2}$$

Generation $T-3$ anticipates $T-2$ will set $b_{T-1} = b^*_{T-1}$ in such a way b^*_{T-1} maximizes generation $T-1$ as well as generation T utility, according to what we saw in previous sections.

Conditional on $b_{T-1} = b^*_{T-1}$, A_{T-2} and therefore V_{T-2} are functions only of choice variables of generation $T-3$. This means generation $T-3$ optimization can be rewritten as a maximization with respect to A_{T-2} as well, replacing V_{T-2} and adding generation $T-2$ constraints:

$$\max_{A_{T-3},A_{T-2},b_{T-2}} u(c_{1T-3}) + u(c_{2T-2}) + \alpha\left[u(c_{1T-2}) + u(c_{2T-1}) + \alpha V_{T-1}(b^*_{T-1})\right]$$

subject to the following constraints

$$\begin{aligned}
c_{1T-3} &= y_{1T-3} - A_{T-3} \\
c_{2T-2} &= y_{2T-2} + A_{T-3} + b_{T-3} - b_{T-2} \\
b_{T-2} &\geq 0 \\
c_{1T-2} &= y_{1T-2} - A_{T-2} \\
c^*_{2T-1} &= y_{2T-1} + A_{T-2} + b_{T-2} - b^*_{T-1}
\end{aligned}$$

As we discussed in previous section, V_{T-1} depends on choice variables of generation $T-2$, namely b^*_{T-1}, but not on choice variables of generation $T-3$. This does not depend on the specific time period: A_t and V_t at the optimum depend on generation t choice variable b_{t+1} and on choice variables of generation $t-1$, namely b_t, but not on choice variables of generation $t-2$:

$$V_t = u(c_{1t}) + u(c_{2t+1}) + \alpha V_{t+1}(b_{t+1})$$

and

$$A_t(b_t, b_{t+1}) = \frac{y_{1t} - y_{2t+1} + b_{t+1} - b_t}{2}$$

This means we can now solve generation $T-3$ as well as all previous generations solving the general maximization of generation t conditional upon the optimal choice of the next generation (generation $t+1$ bequest to generation $t+2$, namely b^*_{t+2}).

A.2 The general maximization problem: Generation t

The optimization of generation t, conditional upon the optimal choice of generation $t+1$ bequest to the following generation b_{t+2}, denoted by b^*_{t+2} is the following:

$$V_t(b_t) = \max_{A_t, A_{t+1}, b_{t+1}} u(c_{1t}) + u(c_{2t+1}) + \alpha\left[u(c_{1t+1}) + u(c_{2t+2}) + \alpha V_{t+2}(b^*_{t+2})\right]$$

subject to the following constraints

$$\begin{aligned}
c_{1t} &= y_{1t} - A_t && \text{(26a)} \\
c_{2t+1} &= y_{2t+1} + A_t + b_t - b_{t+1} && \text{(26b)} \\
c_{1t+1} &= y_{1t+1} - A_{t+1} && \text{(26c)} \\
c^*_{2t+2} &= y_{2t+2} + A_{t+1} + b_{t+1} - b^*_{t+2} && \text{(26d)} \\
b_{t+1} &\geq 0 && \text{(26e)}
\end{aligned}$$

The Kuhn-Tucker conditions are as follows:

$$\begin{aligned}
\frac{\partial L}{\partial A_t} &: \quad -u'(y_{1t} - A_t) + u'(y_{2t+1} + A_t + b_t - b_{t+1}) = 0 && \text{(27a)} \\
\frac{\partial L}{\partial A_{t+1}} &: \quad -\alpha u'(y_{1t+1} - A_{t+1}) + \alpha u'(y_{2t+2} + A_{t+1} + b_{t+1} - b^*_{t+2}) = 0 && \text{(27b)} \\
\frac{\partial L}{\partial b_{t+1}} &: \quad -u'(y_{2t+1} + A_t + b_t - b_{t+1}) + \alpha u'(y_{2t+2} + A_{t+1} + b_{t+1} - b^*_{t+2}) + \mu_{t+1} = 0 && \text{(27c)} \\
& \quad \mu_{t+1} \geq 0;\ b_{t+1} \geq 0;\ \mu_{t+1} b_{t+1} = 0 && \text{(27d)}
\end{aligned}$$

FOC (27a) implies that

$$A_t = \frac{y_{1t} + b_{t+1} - y_{2t+1} - b_t}{2}$$

which leads to

$$c_{1t} = c_{2t+1} = \frac{y_{1t} + y_{2t+1} + b_t - b_{t+1}}{2}$$

Generation t smooths his consumption. Since this result does not depend on the specific period t, also generation $t+1$ alsways smooths consumption:

$$c^*_{1t+1} = c^*_{2t+2} = \frac{y_{1t+1} + y_{2t+2} + b_{t+1} - b^*_{t+2}}{2}$$

Now suppose generation t is willing to bequeath to generation $t+1$. That is, we are at an interior solution for b_{t+1}, i.e. constraint (26e) is not binding, i.e. $\mu_{t+1} = 0$ and $b_{t+1} \geq 0$. We can rewrite (27c) as

$$MRS_{t,t+1} = \frac{u'(c_{2t+1})}{u'(c^*_{2t+2})} = \alpha$$

Differences in the level of life–time consumption between generations depend on α.[10] If generation t is not willing to bequeath to generation $t+1$ ($b_{t+1} = 0,\ \mu_{t+1} > 0$), then

$$MRS_{c_{2t+1},c^*_{2t+2}} = \frac{u'\left(\frac{y_{1t}+y_{2t}+b_t}{2}\right)}{u'\left(\frac{y_{1t+1}+y_{2t+2}-b^*_{t+2}}{2}\right)} > \alpha$$

Again, consumption smoothing across generation depends on the specific value of the parameter α and on exogenous income.[11]

B The model with credit constraints but no intervivos

Generation t maximizes his utility V_t with respect to his choice variables $c_{1t}, c_{2t+1}, b_{t+1}$:

$$\max_{c_{1t},c_{2t+1},A_t,b_{t+1}} u(c_{1t}) + u(c_{2t+1}) + \alpha V_{t+1} \tag{28a}$$

subject to the following constraints:

$$c_{1t} = y_{1t} - A_t \tag{28b}$$
$$c_{2t+1} = y_{2t+1} + A_t + b_t - b_{t+1} \tag{28c}$$
$$A_t \geq -y_{2t+1} \tag{28d}$$
$$b_{t+1} \geq 0 \tag{28e}$$

[10]If $\alpha \leq 1$, from (A.2), $c_{1t} = c_{2t+1} > c^*_{1t+1} = c^*_{2t+2}$. If $\alpha > 1$, from (A.2), $c_{1t} = c_{2t+1} < c^*_{1t+1} = c^*_{2t+2}$.

[11]If $\alpha \geq 1$ then $c_{1t} = c_{2t+1} \leq c^*_{2t+2} = c^*_{1t+1}$. If $\alpha < 1$ we can't say anything about consumption smoothing across generations. Given the concavity of the utility function $u(.)$, this condition more likely holds if if c_{2t+1} is relatively low in comparison with c^*_{2t+2}. If generation $t+1$ is lifetime richer than generation t, and/or the degree of altruism is low (low α), it is likely that $b_{t+1} = 0$. In this case generation $t+1$ anticipates that he will not receive a bequest and smooth his consumption over his life cycle.

From constraints (28b),(28c), $c_{1t} > 0$ and $c_{2t+1} > 0$ it follows that $A_t \in (-y_{2t+1} - b_{t+1}, y_{1t})$. This is the standard assumptions about savings in OLG models: each generation can borrow up to his whole future resources. Constraint (28d) says generation t can borrow up to his whole second period income, but not against bequest. It follows that in our case

$$A_t \in [-y_{2t+1}, y_{1t})$$

As in section A there's a money transfer between generations, therefore it will be necessary to solve it starting from the last period.

B.1 Generation T-1 maximization

Generation $T-1$ will not bequeath any money, but he will receive at the beginning of period T an inheritance b_{T-1} from generation $T-2$. Therefore, generation $T-1$ solves the following optimization problem:

$$\begin{aligned} \max_{c_{1T-1}, c_{2T}} & \quad u(c_{1T-1}) + u(c_{2T}) && (29a) \\ \text{s.t. } c_{1T-1} &= y_{1T-1} - A_{T-1} && (29b) \\ c_{2T} &= y_{2T} + A_{T-1} + b_{T-1} && (29c) \\ A_{T-1} &\geq -y_{2T} && (29d) \end{aligned}$$

We can simplify the optimization problem by substituting constraints (29b) and (29c) into the utility function (29a) and then solving the maximization with respect to A_{T-1}:

$$\begin{aligned} \max_{A_{T-1}} & \quad u(y_{1T-1} - A_{T-1}) + u(y_{2T} + A_{T-1} + b_{T-1}) && (30) \\ \text{s.t. } A_{T-1} &\geq -y_{2T} \end{aligned}$$

The Lagrangian is

$$L = u(y_{1T-1} - A_{T-1}) + u(y_{2T} + A_{T-1} + b_{T-1}) + \lambda_{T-1}(A_{T-1} + y_{2T})$$

from which the Kuhn-Tucker conditions follow:

$$\begin{aligned} \frac{\partial L}{\partial A_{T-1}} \; : \; & -u'(y_{1T-1} - A_{T-1}) + u'(y_{2T} + A_{T-1} + b_{T-1}) + \lambda_{T-1} = 0 && (31a) \\ & \lambda_{T-1} \geq 0 && (31b) \\ & A_{T-1} + y_{2T} \geq 0 && (31c) \\ & \lambda_{T-1}(A_{T-1} + y_{2T}) = 0 && (31d) \end{aligned}$$

CASE 1: $\lambda_{T-1} = 0$, $A_{T-1} > -y_{2T}$
Constraint (28d) (i.e., Kuhn-Tucker condition (31c)) is not binding. From (31a) it follows that

$$A_{T-1} = \frac{y_{1T-1} - y_{2T} - b_{T-1}}{2}$$

which leads to

$$c_{1T-1} = c_{2T} = \frac{y_{1T-1} + y_{2T} + b_{T-1}}{2} \tag{32}$$

Substituting (32) into (31c) it follows

$$\begin{aligned} y_{1T-1} - y_{2T} - b_{T-1} &\geq -2y_{2T} \\ b_{T-1} &\leq y_{1T-1} + y_{2T} \end{aligned} \tag{33}$$

From (32) and (33): if for generation $T-1$ the received bequest is not larger than lifetime resources, then he is able to smooth consumption, i.e. the constraint (28d) in borrowing is not binding. Note that this seems to be satisfied in most reasonable applications. Still, magnitudes should not be taken at face value: comparisons between values of income and beqests of different generations lead to somewhat unrealistic, but simplified expressions due to the assumptions of no intetemporal discounting.

CASE 2: $\lambda_{T-1} > 0$**,** $A_{T-1} = -y_{2T}$
Constraint (28d) is binding. From (31a) it follows that

$$y_{1T-1} + y_{2T} \quad < \quad b_{T-1}$$

and

$$\begin{aligned} c_{1T-1} &= y_{1T-1} + y_{2T} \\ c_{2T} &= b_{T-1} \end{aligned}$$

If for generation $T-1$ the received bequest is larger than lifetime resources, then he does not smooth because he cannot borrow against bequest and is forced to borrow up to the maximum ($A_{T-1} = -y_{2T}$), consume all his lifetime income in period $T-1$ and consume out of bequest in period T. As we showed in section A, given the total amount of resources $y_{1T-1} + y_{2T} + b_{T-1}$ this consumption pattern is suboptimal since the first best would be to smooth and consume also part of the bequest in period $T-1$.

B.1.1 Generation $T-2$

Generation $T-2$ maximization is the following:

$$\begin{aligned} \max_{c_{1T-2}, c_{2T-1}, b_{T-1}} \quad & u(c_{1T-2}) + u(c_{2T-1}) + \alpha V_{T-1} \\ \text{s.t. } c_{1T-2} &= y_{1T-2} - A_{T-2} \\ c_{2T-1} &= y_{2T-1} + A_{T-2} + b_{T-2} - b_{T-1} \\ A_{T-2} &\geq -y_{2T-1} \\ b_{T-1} &\geq 0 \end{aligned}$$

Generation $T-2$ anticipates that generation $T-1$ will optimize his own utility: V_{T-1} is generation $T-1$ utility function evaluatated at its maximum. Due to the terminal condition

$b_T = 0$ generation $T-1$ maximizes his utility only with respect to c_{1T-1}, c_{2T}, or equivalently with respct only to A_{T-1}. The value of A_{T-1} at the optimum depends on the bequest generation $T-1$ receives from generation $T-2$, b_{T-1}:

$$A_{T-1}(b_{T-1}) = \begin{cases} \frac{y_{1T-1}-y_{2T}-b_{T-1}}{2} & \text{if} \quad b_{T-1} \leq y_{1T-1} + y_{2T} \\ -y_{2T} & \text{if} \quad b_{T-1} > y_{1T-1} + y_{2T} \end{cases}$$

Since A_{T-1} at the optimum is determined by a choice variable of generation $T-2$, namely b_{T-1}, V_{T-1} is a function only of choice variables of generation $T-2$. This means generation $T-2$ optimization can be rewritten as a maximization with respect to A_{T-1} as well, replacing V_{T-1} and adding generation $T-1$ constraints:

$$\max_{A_{T-2}, A_{T-1}, b_{T-1}} \quad u(c_{1T-2}) + u(c_{2T-1}) + \alpha \left[u(c_{1T-1}) + u(c_{2T}) + \alpha V_T \right]$$

subject to the following constraints

$$\begin{aligned} c_{1T-2} &= y_{1T-2} - A_{T-2} \\ c_{2T-1} &= y_{2T-1} + A_{T-2} + b_{T-2} - b_{T-1} \\ b_{T-1} &\geq 0 \\ A_{T-2} &\geq -y_{2T-1} \\ c_{1T-1} &= y_{1T-1} - A_{T-1} \\ c_{2T} &= y_{2T} + A_{T-1} + b_{T-1} \\ A_{T-1} &\geq -y_{2T} \end{aligned}$$

V_T is equal to $u(y_{1T})$ and therefore does not depend on any choice variable of generation $T-2$. The Lagrangian is

$$\begin{aligned} L = & \; u(y_{1T-2} - A_{T-2}) + u(y_{2T-1} + A_{T-2} + b_{T-2} - b_{T-1}) + \\ & + \alpha u(y_{1T-1} - A_T) + \alpha u(y_{2T} + A_{T-1} + b_{T-1}) \\ & + \mu_{T-1} b_{T-1} + \lambda_{T-2}(A_{T-2} + y_{2T-1}) + \lambda_{T-1}(A_{T-1} + y_{2T}) \end{aligned}$$

from which the Kuhn-Tucker conditions follow:

$$\frac{\partial L}{\partial A_{T-2}} \; : \; -u'(y_{1T-2} - A_{T-2}) + u'(y_{2T-1} + A_{T-2} + b_{T-2} - b_{T-1}) + \lambda_{T-2} = 0 \tag{34a}$$

$$\frac{\partial L}{\partial A_{T-1}} \; : \; -\alpha u'(y_{1T-1} - A_{T-1}) + \alpha u'(y_{2T} + A_{T-1} + b_{T-1}) + \lambda_{T-1} = 0 \tag{34b}$$

$$\frac{\partial L}{\partial b_{T-1}} \; : \; -u'(y_{2T-1} + A_{T-2} + b_{T-2} - b_{T-1}) + \alpha u'(y_{2T} + A_{T-1} + b_{T-1}) + \mu_{T-1} = 0 \tag{34c}$$

$$\mu_{T-1} \geq 0, \; b_{T-1} \geq 0, \; \mu_{T-1} b_{T-1} = 0 \tag{34d}$$

$$\lambda_{T-2} \geq 0, \; A_{T-2} + y_{2T-1} \geq 0, \; \lambda_{T-2}(A_{T-2} + y_{2T-1}) = 0 \tag{34e}$$

$$\lambda_{T-1} \geq 0, \; A_{T-1} + y_{2T} \geq 0, \; \lambda_{T-1}(A_{T-1} + y_{2T}) = 0 \tag{34f}$$

CASE 1

$\lambda_{T-2} = 0$ $A_{T-2} \geq -y_{2T-1}$	$\lambda_{T-1} = 0$ $A_{T-1} \geq -y_{2T}$	$\mu_{T-1} = 0$ $b_{T-1} \geq 0$

- FOC (34a) implies that

$$A_{T-2} = \frac{y_{1T-2} + b_{T-1} - y_{2T-1} - b_{T-2}}{2}$$

which leads to

$$c_{1T-2} = c_{2T-1} = \frac{y_{1T-2} + y_{2T-1} + b_{T-2} - b_{T-1}}{2}$$

Moreover,

$$b_{T-2} \leq y_{1T-2} + y_{2T-1} + b_{T-1}$$

Generation $T-2$ smooths his consumption if the received bequest b_{T-2} is not larger than his lifetime income plus what wants to transfer to generation $T-1$.

- FOC (34b) implies that

$$A_{T-1} = \frac{y_{1T-1} - y_{2T} - b_{T-1}}{2}$$

which leads to

$$c_{1T-1} = c_{2T} = \frac{y_{1T-1} + y_{2T} + b_{T-1}}{2}$$

Moreover,

$$A_{T-1} = \frac{y_{1T-1} - y_{2T} - b_{T-1}}{2} \geq -y_{2T} \Rightarrow b_{T-1} \leq y_{1T-1} + y_{2T}$$

Generation $T-1$ smooths his consumption if the received bequest b_{T-1} is not larger than his lifetime income.

- From FOC (34d)

$$u'\left(\frac{y_{1T-2} + y_{2T-1} + b_{T-2} - b_{T-1}}{2}\right) = \alpha u'\left(\frac{y_{1T-1} + y_{2T} + b_{T-1}}{2}\right) \tag{35}$$

which can be rewritten as

$$MRS_{c_{2T-1},c_{2T}} = \frac{u'(c_{2T-1})}{u'(c_{2T})} = \alpha$$

The degree of altruism determines which generation consumes more.[12]

[12] If $\alpha < 1 \Rightarrow c_{1T-2} = c_{2T-1} > c_{1T-1} = c_{2T}$. This means that generation $T-2$ consumption level is higher than generation $T-1$ consumption level despite the bequest b_{T-1} which generation $T-2$ to generation $T-1$. If $\alpha \geq 1 \Rightarrow c_{1T-2} = c_{2T-1} \leq c_{1T-1} = c_{2T}$. When does it happen?

$$\frac{y_{1T-2} + y_{2T-1} + b_{T-2} - b_{T-1}}{2} < \frac{y_{1T-1} + y_{2T} + b_{T-1}}{2}$$

or

$$b_{T-1} > \underbrace{(y_{1T-2} + y_{2T-1} + b_{T-2})}_{\text{lifetime resources gen } T-2} - \underbrace{(y_{1T-1} + y_{2T} + b_{T-1})}_{\text{lifetime resources gen } T-1}$$

Now, using condition (35) this can be rewritten as

$$y_{1T-2} + y_{2T-1} + b_{T-2} \leq 3(y_{1T-1} + y_{2T})$$

Consumption of gen $T-1$ is higher than consumption of gen $T-2$ only if $\alpha > 1$ and generation $T-2$ is not much richer than generation $T-1$.

CASE 2

$\lambda_{T-2} = 0$ $A_{T-2} \geq -y_{2T-1}$	$\lambda_{T-1} > 0$ $A_{T-1} = -y_{2T}$	$\mu_{T-1} = 0$ $b_{T-1} \geq 0$

- As for CASE 1, FOC (34a) implies that

$$A_{T-2} = \frac{y_{1T-2} - y_{2T-1} - b_{T-2} + b_{T-1}}{2}$$

which leads to

$$c_{1T-2} = c_{2T-1} = \frac{y_{1T-2} + y_{2T-1} + b_{T-2} - b_{T-1}}{2}$$

Moreover, generation $T-2$ smooth his consumption if the received bequest b_{T-2} is no larger than his lifetime income plus what wants to transfer to generation $T-1$, i.e.

$$b_{T-2} \leq y_{1T-2} + y_{2T-1} + b_{T-1}$$

- Since $A_{T-1} = -y_{2T}$ it follows that

$$c_{1T-1} = y_{1T-1} + y_{2T}; \qquad c_{2T} = b_{T-1}$$

Moreover, since $\lambda_{T-1} > 0$ from FOC (34b)

$$y_{1T-1} + y_{2T} < b_{T-1}$$

i.e., generation $T-1$ does not smooth since received bequest is larger than lifetime income, and he cannot borrow against bequest hitting the credit limit constraint.

- From FOC (34d)

$$MRS_{c_{2T-1},c_{2T}} = \frac{u'(c_{2T-1})}{u'(c_{2T})} = \alpha \tag{36}$$

and the degree of altruism determines which generation consumes more exactly as in CASE 1.[13]

[13] From equation (36), $u'\left(\frac{y_{1T-2}+y_{2T-1}+b_{T-2}-b_{T-1}}{2}\right) = \alpha u'\left(b_{T-1}\right)$. If $\alpha < 1$

$$u'\left(\frac{y_{1T-2} + y_{2T-1} + b_{T-2} - b_{T-1}}{2}\right) < u'(b_{T-1}) \Leftrightarrow$$

$$\frac{y_{1T-2} + y_{2T-1} + b_{T-2} - b_{T-1}}{2} > b_{T-1} \Leftrightarrow$$

$$b_{T-1} < \frac{1}{3}(y_{1T-2} + y_{2T-1} + b_{T-2})$$

Now using the liquidity constraint condition we get

$$3(y_{1T-1} + y_{2T}) < y_{1T-2} + y_{2T-1} + b_{T-2}$$

If generation $T-2$ is much more lifetime richer than generation $T-1$, generation $T-2$ smooths and generation $T-1$ doesn't even if altruism is low. If $\alpha \geq 1$ then

$$b_{T-1} \geq \frac{1}{3}(y_{1T-2} + y_{2T-1} + b_{T-2})$$

If altruism is high ($\alpha \geq 1$) it is enough that gen $T-2$ bequeaths at least a third of his lifetime resources to have gen $T-1$ not smoothing. If you consider real estate wealth, this is not necessarily a big amount.

CASE 3

$\lambda_{T-2} = 0$ $A_{T-2} \geq -y_{2T-1}$	$\lambda_{T-1} = 0$ $A_{T-1} \geq -y_{2T}$	$\mu_{T-1} > 0$ $b_{T-1} = 0$

- As for CASE 1, FOC (34a) implies that

$$A_{T-2} = \frac{y_{1T-2} - y_{2T-1} - b_{T-2}}{2}$$

which leads to

$$c_{1T-2} = c_{2T-1} = \frac{y_{1T-2} + y_{2T-1} + b_{T-2}}{2}$$

Moreover, generation $T-2$ smooth his consumption if the received bequest b_{T-2} is no larger than his lifetime income plus what wants to transfer to generation $T-1$, i.e.

$$b_{T-2} \leq y_{1T-2} + y_{2T-1} + b_{T-1}$$

- again, from FOC (34b)

$$A_{T-1} = \frac{y_{1T-1} - y_{2T}}{2}$$

which leads to

$$c_{1T-1} = c_{2T} = \frac{y_{1T-1} + y_{2T}}{2}$$

Moreover,

$$A_{T-1} = \frac{y_{1T-1} - y_{2T}}{2} \geq -y_{2T} \Leftrightarrow y_{1T-1} + y_{2T} \geq 0$$

Condition (B.1.1) is always satisfied since $\forall i, t,\ y_{it} \geq 0$.

- from FOC (34d) it holds that

$$u'(\frac{y_{1T-2} + y_{2T-1} + b_{T-2}}{2}) = \alpha u' \left(\frac{y_{1T-1} + y_{2T}}{2} \right) + \mu_{T-1}$$

which leads to

$$MRS_{c_{2T-1}, c_{2T}} = \frac{u'(c_{2T-1})}{u'(c_{2T})} > \alpha$$

Given the concavity of the utility function $u(.)$, this condition more likely holds if c_{2T-1} is relatively low in comparison with c_{2T}. If generation $T-1$ is lifetime richer than generation $T-2$, and the degree of altruism is low (low α), it is likely that $b_{T-1} = 0$. In this case generation $T-1$ anticipates that he will not receive a bequest and smooth his consumption over his lifecycle.

CASE 4

$\lambda_{T-2} = 0$ $A_{T-2} \geq -y_{2T-1}$	$\lambda_{T-1} > 0$ $A_{T-1} = -y_{2T}$	$\mu_{T-1} > 0$ $b_{T-1} = 0$

This is not an admissable case. FOC (34b) and $\lambda_{T-1} >$ imply that

$$b_{T-1} > y_{1T-1} + y_{2T} > 0 \tag{37}$$

At the same, it should hold that $\mu_{T-1} > 0$ and $b_{T-1} = 0$ which contradicts (37): If $b_{T-1} = 0$ then generation $T-1$ smooths consumption.

CASE 5

$\lambda_{T-2} > 0$ $A_{T-2} = -y_{2T-1}$	$\lambda_{T-1} = 0$ $A_{T-1} \geq -y_{2T}$	$\mu_{T-1} = 0$ $b_{T-1} \geq 0$

- Since $A_{T-2} = -y_{2T-1}$ it follows that

  $$c_{1T-2} = y_{1T-2} + y_{2T-1}; \qquad c^*_{2T} = b_{T-2} - b_{T-1}$$

  Moreover, since $\lambda_{T-2} > 0$ from FOC (34a)

  $$y_{1T-2} + y_{2T-1} < b_{T-2} - b_{T-1}$$

  Generation $T-2$ cannot smooth his consumption since the received bequest b_{T-2} is larger than his lifetime income plus what wants to transfer to generation $T-1$ and therefore he hits the credit constraint.

- From FOC (34b)

  $$A_{T-1} = \frac{y_{1T-1} - y_{2T} - b_{T-1}}{2}$$

  which leads to

  $$c_{1T-1} = c_{2T} = \frac{y_{1T-1} + y_{2T} + b_{T-1}}{2}$$

  Moreover, since $A_{T-1} \geq -y_{2T}$, it follows

  $$b_{T-1} \leq y_{1T-1} + y_{2T}$$

- From FOC (34d)

  $$MRS_{c_{2T-1},c_{2T}} = \frac{u'(c_{2T-1})}{u'(c_{2T})} = \alpha \tag{38}$$

  and the degree of altruism determines which generation consumes more exactly as in CASE 1.[14]

[14] Replacing optimal second period consumption for both generations in (38), it follows that

$$u'(b_{T-2} - b_{T-1}) = \alpha u' \left(\frac{y_{1T-1} + y_{2T} + b_{T-1}}{2} \right)$$

If $\alpha < 1$ (low altruism), $u'(b_{T-2} - b_{T-1}) < u' \left(\frac{y_{1T-1}+y_{2T}+b_{T-1}}{2} \right)$, which implies

$$c_{2T-1} = b_{T-2} - b_{T-1} < \frac{y_{1T-1} + y_{2T} + b_{T-1}}{2} = c^*_{1T-1} = c_{2T}$$

CASE 6

$\lambda_{T-2} > 0$ $A_{T-2} = -y_{2T-1}$	$\lambda_{T-1} > 0$ $A_{T-1} = -y_{2T}$	$\mu_{T-1} = 0$ $b_{T-1} \geq 0$

- As in CASE 5, from $A_{T-2} = -y_{2T-1}$ it follows that

$$c_{1T-2} = y_{1T-2} + y_{2T-1}; \qquad c^*_{2T} = b_{T-2} - b_{T-1}$$

Moreover, since $\lambda_{T-2} > 0$ from FOC (34a)

$$b_{T-2} > y_{1T-2} + y_{2T-1} + b_{T-1} \tag{39}$$

Generation $T-2$ cannot smooth his consumption since the received bequest b_{T-2} is larger than his lifetime income plus what wants to transfer to generation $T-1$ and therefore he hits the credit constraint.

- Since $A_{T-1} = -y_{2T}$ it follows that

$$c_{1T-1} = y_{1T-1} + y_{2T}; \qquad c_{2T} = b_{T-1}$$

Moreover, since $\lambda_{T-1} > 0$ from FOC (34b)

$$y_{1T-1} + y_{2T} < b_{T-1} \tag{40}$$

i.e., generation $T-1$ does not smooth since received bequest is larger than lifetime income, and he cannot borrow against bequest hitting the credit limit constraint.

- From FOC (34d)

$$MRS_{c_{2T-1},c_{2T}} = \frac{u'(c_{2T-1})}{u'(c_{2T})} = \alpha \tag{41}$$

and the degree of altruism determines which generation consumes more exactly as in CASE 1.[15]

Generation $T-2$ consumes more than generation $T-1$ at least in period 2 (remember that due to the binding credit constraint $c_{1T-2} < c_{2T-1}$). If $\alpha \geq 1$ then $b_{T-2} - b_{T-1} \leq \frac{y_{1T-1}+y_{2T}+b_{T-1}}{2}$, which implies

$$c_{1T-2} < c_{2T-1} = b_{T-2} - b_{T-1} \leq \frac{y_{1T-1} + y_{2T} + b_{T-1}}{2} = c_{1T-1} = c_{2T}$$

If altruism is high, generation $T-2$ consumption is always below generation $T-1$ consumption.

[15] Replacing optimal consumption for both generations in (41), it follows that $u'(b_{T-2} - b_{T-1}) = \alpha u'(b_{T-1})$. If $\alpha < 1$, then $b_{T-2} - b_{T-1} > b_{T-1}$, which implies $c_{2T-1} < c_{1T-1}$. Given equation (40) it also holds that

$$b_{T-2} > 2(y_{1T-1} + y_{2T})$$

Again if generation $T-2$ is much lifetime richer than generation $T-1$, the latter receives a large bequest that does not allow to smooth consumption, even with low altruism. In this case this is due to a very large bequest b_{T-2} so that generation $T-2$ also doesn't smooth consumption over his lifecycle.

If $\alpha \geq 1 \Rightarrow b_{T-2} - b_{T-1} \leq b_{T-1}$. Given equation (39) it holds that $b_{T-1} > y_{1T-2} + y_{2T-1}$. So, if altruism is high AND generation $T-2$ is not smoothing, b_{T-1} is bigger that total lifetime earnings of generation $T-2$. Moreover, b_{T-2} received by generation $T-2$ was huge. Again, magnitudes are somewhat unrealistic due to the simplifying assumption of no intertemporal discounting.

CASE 7

$\lambda_{T-2} > 0$ $A_{T-2} = -y_{2T-1}$	$\lambda_{T-1} = 0$ $A_{T-1} \geq -y_{2T}$	$\mu_{T-1} > 0$ $b_{T-1} = 0$

- Since $A_{T-2} = -y_{2T-1}$ it follows that

  $$c_{1T-2} = y_{1T-2} + y_{2T-1}; \qquad c_{2T} = b_{T-2}$$

  Moreover, since $\lambda_{T-2} > 0$ from FOC (34a)

  $$y_{1T-2} + y_{2T-1} < b_{T-2}$$

  Generation $T-2$ cannot smooth his consumption since the received bequest b_{T-2} is larger than his lifetime income (he does not want to transfer anything to the next generation), and therefore he hits the credit constraint

- From FOC (34b)

  $$A_{T-1} = \frac{y_{1T-1} - y_{2T}}{2}$$

  which leads to

  $$c_{1T-1} = c_{2T} = \frac{y_{1T-1} + y_{2T}}{2}$$

  Moreover,

  $$A_{T-1} = \frac{y_{1T-1} - y_{2T}}{2} \geq -y_{2T} \Leftrightarrow y_{1T-1} + y_{2T} \geq 0$$

  which is always satisfied since $\forall i, t,\ y_{it} \geq 0$.

- From the FOC (34a) and (34c) it follows that

  $$u'(y_{1T-2} + y_{2T-1}) = u'(b_{T-2}) + \lambda_{T-2} \tag{42}$$

  $$u'(b_{T-2}) = \alpha u'(\frac{y_{1T-1} + y_{2T}}{2}) + \mu_{T-1} \tag{43}$$

  Generation $T-1$ is smoothing. Substitution of equation (43) into (42)

  $$u'(y_{1T-2} + y_{2T-1}) = \alpha u'(\frac{y_{1T-1} + y_{2T}}{2}) + \mu_{T-1} + \lambda_{T-2} \Rightarrow$$

  $$MRS_{c_{2T-1},c_{2T}} = \frac{u'(y_{1T-2} + y_{2T-1})}{u'(\frac{y_{1T-1}+y_{2T}}{2})} > \alpha$$

  This scenario is more likely if lifetime earnings of generation $T-2$ is relatively low compared with lifetime earnings of generation $T-1$ and/or the degree of altruism is low.

CASE 8

$\lambda_{T-2} > 0$ $A_{T-2} = -y_{2T-1}$	$\lambda_{T-1} > 0$ $A_{T-1} = -y_{2T}$	$\mu_{T-1} > 0$ $b_{T-1} = 0$

This case is not admissable for reasons mentioned under CASE 4 (see above).

ALL IN ALL

Regardless of whether generation $T-2$ smooths or not,

- if $b_{T-1} = 0$ then generation $T-1$ smooths consumption;
- generation $T-1$ is more likely to be liquidity constrained and not able to smooth if generation $T-2$ is (much) lifetime richer than generation $T-1$, regardless on whether these resources come from earnings or bequest from generation $T-3$.

B.1.2 Generation $T-3$

Generation $T-3$ maximization is the following:

$$\begin{aligned}
\max_{c_{1T-3}, c_{2T-2}, b_{T-2}} \quad & u(c_{1T-3}) + u(c_{2T-2}) + \alpha V_{T-2} \\
\text{s.t. } c_{1T-3} &= y_{1T-3} - A_{T-3} \\
c_{2T-2} &= y_{2T-2} + A_{T-3} + b_{T-3} - b_{T-2} \\
A_{T-3} &\geq -y_{2T-2}
\end{aligned}$$

Generation $T-3$ anticipates that generation $T-2$ will optimize his own utility: V_{T-2} is generation $T-2$ utility function evaluatated at its maximum. Generation $T-2$ maximizes his utility with respect to c_{1T-1}, c_{2T} and b_{T-1}, or equivalently with respect only to A_{T-2} and b_{T-1}. As we already noticed at the beginnning of the previous section, generation $T-2$ sets his optimal choice of $b_{T-1} = b^*_{T-1}$ taking into account that the optimal saving choice of generation $T-1$, A^*_{T-1}, is completely determined once generation $T-2$ sets b_{T-1}.

Optimal saving of generation $T-2$, i.e. the value of A_{T-2} at the optimum, depends on the bequest generation $T-2$ receives from generation $T-3$, b_{T-2}, and on the (optimal) bequest left to generation $T-1$:

$$A^*_{T-2}(b_{T-2}, b^*_{T-1}) = \begin{cases} \frac{y_{1T-2} + b^*_{T-1} - y_{2T-1} - b_{T-2}}{2} & \text{if} \quad b_{T-2} \leq y_{1T-2} + y_{2T-1} + b^*_{T-1} \\ -y_{2T-1} & \text{if} \quad b_{T-2} > y_{1T-2} + y_{2T-1} + b^*_{T-1} \end{cases}$$

Generation $T-3$ anticipates $T-2$ will set $b_{T-1} = b^*_{T-1}$ according to what we saw in previous section.

Conditional on $b_{T-1} = b^*_{T-1}$, A_{T-2} and therefore V_{T-2} are functions only of choice variables of generation $T-3$. This means generation $T-3$ optimization can be rewritten as a maximization with respect to A_{T-2} as well, replacing V_{T-2} and adding generation $T-2$ constraints:

$$\max_{A_{T-3}, A_{T-2}, b_{T-2}} u(c_{1T-3}) + u(c_{2T-2}) + \alpha \left[u(c_{1T-2}) + u(c_{2T-1}) + \alpha V_{T-1}(b^*_{T-1}) \right]$$

subject to the following constraints

$$\begin{aligned}
c_{1T-3} &= y_{1T-3} - A_{T-3} \\
c_{2T-2} &= y_{2T-2} + A_{T-3} + b_{T-3} - b_{T-2} \\
b_{T-2} &\geq 0 \\
A_{T-3} &\geq -y_{2T-2} \\
c_{1T-2} &= y_{1T-2} - A_{T-2} \\
c_{2T-1} &= y_{2T-1} + A_{T-2} + b_{T-2} - b^*_{T-1} \\
A_{T-2} &\geq -y_{2T-1}
\end{aligned}$$

As we discussed in previous section, V_{T-1} does depend on choice variables of generation $T-2$, namely b_{T-1}, but not on choice variables of generation $T-3$. This does not depend on the specific time period: the specific values of A_t and V_t at the optimum depend on generation t choice variable b_{t+1} and on choice variables of generation $t-1$, namely b_t, but not on choice variables of generation $t-2$:

$$V_t = u(c^*_{1t}) + u(c^*_{2t+1}) + \alpha V_{t+1}(b^*_{t+1})$$

and

$$A^*_t(b_t, b^*_{t+1}) = \begin{cases} \frac{y_{1t}+b*_{t+1}-y_{2t+1}-b_t}{2} & \text{if} \quad b_t \leq y_{1t} + y_{2t+1} + b^*_{t+1} \\ -y_{2t+1} & \text{if} \quad b_t > y_{1t} + y_{2t+1} + b^*_{t+1} \end{cases}$$

This means we can now solve generation $T-3$ as well as all previous generations solving the general maximization of generation t conditional upon the optimal choice of the next generation (generation $t+1$ bequest to generation $t+2$, namely b^*_{t+2}).

B.2 The general maximization problem: Generation t

The optimization of generation t, conditional upon the optimal choice of generation $t+1$ bequest to the following generation b_{t+2}, denoted by b^*_{t+2} is the following:

$$V_t(b_t) = \max_{A_t, A_{t+1}, b_{t+1}} u(c_{1t}) + u(c_{2t+1}) + \alpha \left[u(c_{1t+1}) + u(c_{2t+2}) + \alpha V_{t+2}(b^*_{t+2}) \right] \tag{45a}$$

subject to the following constraints

$$c_{1t} = y_{1t} - A_t \tag{45b}$$
$$c_{2t+1} = y_{2t+1} + A_t + b_t - b_{t+1} \tag{45c}$$
$$c_{1t+1} = y_{1t+1} - A_{t+1} \tag{45d}$$
$$c_{2t+2} = y_{2t+2} + A_{t+1} + b_{t+1} - b^*_{t+2} \tag{45e}$$
$$b_{t+1} \geq 0 \tag{45f}$$
$$A_\tau \geq -y_{2\tau+1}, \ \tau = t, t+1 \tag{45g}$$

The Lagrangian is

$$\begin{aligned} L = {} & u(y_{1t} - A_t) + u(y_{2t+1} + A_t + b_t - b_{t+1}) \\ & + \alpha \left[u(y_{1t+1} - A_{t+1}) + u(y_{2t+2} + A_{t+1} + b_{t+1} - b^*_{t+2}) + \alpha V_{t+2}(b^*_{t+2}) \right] \\ & + \mu_{t+1}(b_{t+1}) + \lambda_t(A_t + y_{2t+1}) + \lambda_{t+1}(A_{t+1} + y_{2t+2}) \end{aligned}$$

from which the Kuhn-Tucker conditions follow:

$$\frac{\partial L}{\partial A_t} \quad : \quad -u'(y_{1t} - A_t) + u'(y_{2t+1} + A_t + b_t - b_{t+1}) + \lambda_t = 0 \tag{46a}$$

$$\frac{\partial L}{\partial A_{t+1}} \quad : \quad -\alpha u'(y_{1t+1} - A_{t+1}) + \alpha u'(y_{2t+2} + A_{t+1} + b_{t+1} - b^*_{t+2}) + \lambda_{t+1} = 0 \tag{46b}$$

$$\frac{\partial L}{\partial b_{t+1}} \quad : \quad -u'(y_{2t+1} + A_t + b_t - b_{t+1}) + \alpha u'(y_{2t+2} + A_{t+1} + b_{t+1} - b^*_{t+2}) + \mu_{t+1} = 0 \tag{46c}$$

$$\mu_{t+1} \geq 0 \tag{46d}$$

$$b_{t+1} \geq 0 \tag{46e}$$

$$\mu_{t+1} b_{t+1} = 0 \tag{46f}$$

$$\lambda_\tau \geq 0, \tau = t, t+1 \tag{46g}$$

$$A_t + y_{2t+1} \geq 0 \tag{46h}$$

$$A_{t+1} + y_{2t+2} \geq 0 \tag{46i}$$

$$\lambda_\tau (A_\tau + y_{2\tau}) = 0, \;\; \tau = t, t+1 \tag{46j}$$

If generation t is not liquidity constrained in period t, i.e constraint (45g) is not binding for period t and $\lambda_t = 0$, FOC (46a) implies that

$$A_t = \frac{y_{1t} + b_{t+1} - y_{2t+1} - b_t}{2} \tag{47}$$

which leads to

$$c_{1t} = c_{2t+1} = \frac{y_{1t} + y_{2t+1} + b_t - b_{t+1}}{2} \tag{48}$$

Moreover, since

$$A_t = \frac{y_{1t+1} + b_{t+1} - y_{2t+1} - b_t}{2} \geq -y_{2t+1} \tag{49}$$

it follows that generation t smooth his consumption if the bequest received b_t is not larger than his lifetime income plus what he want to leave to the next generation, i.e.

$$b_t \leq y_{1t} + y_{2t+1} + b_{t+1}$$

If also generation $t+1$ is not liquidity constrained ($\lambda_{t+1} = 0$),

$$c^*_{1t+1} = c^*_{2t+2} = \frac{y_{1t+1} + y_{2t+2} + b_{t+1} - b^*_{t+2}}{2}$$

Vice versa, if generation $t+1$ is liquidity constrained ($\lambda_{t+1} > 0$),

$$c^*_{1t+1} = y_{1t+1} + y_{2t+2} < c^*_{2t+2} = b_{t+1} - b^*_{t+2}$$

Now suppose generation t is willing to bequeath to generation $t+1$. That is, we are at an interior solution for b_{t+1}, i.e. constraint (45f) is not binding, i.e. $\mu_{t+1} = 0$ and $b_{t+1} \geq 0$. We can rewrite (46c) as

$$MRS_{t,t+1} = \frac{u'(c_{2t+1})}{u'(c^*_{2t+2})} = \alpha \tag{50}$$

Cross-generation differences in the level of life–time consumption depend on α and on exogenous income.[16] If generation t is not liquidity constrained ($\lambda_t = 0$) and is not willing to bequeath to generation $t+1$ ($b_{t+1} = 0,\ \mu_{t+1} > 0$), then generation $t+1$ is not liquidity constrained as well (i.e. $b_{t+1} = 0 \leq y_{1t+1} + y_{2t+2} + b^*_{t+2}$). Then it holds that

$$u'(\frac{y_{1t} + y_{2t+1} + b_t}{2}) = \alpha u'\left(\frac{y_{1t+1} + y_{2t+2} - b^*_{t+2}}{2}\right) + \mu_{t+1} \ \Rightarrow$$

$$MRS_{c_{2t+1}, c^*_{2t+2}} = \frac{u'(\frac{y_{1t}+y_{2t}+b_t}{2})}{u'\left(\frac{y_{1t+1}+y_{2t+2}-b^*_{t+2}}{2}\right)} > \alpha$$

Again, consumption smoothing across generation depends on the specific value of the parameter α and on exogenous income.[17]

If generation t is liquidity constrained and consequently does not smooth consumption, i.e. $\lambda_t > 0$ and $A_t = -y_{2t+1}$. Then $c^*_{1t} = y_{1t} + y_{2t+1} < c^*_{2t+1} = b_t - b_{t+1}$ and from Kuhn-Tucker condition (46a) it follows that

$$u'(y_{1t} + y_{2t+1}) > u'(b_t - b_{t+1}) \Leftrightarrow y_{1t} + y_{2t+1} < b_t - b_{t+1}$$

or

$$b_t > y_{1t} + y_{2t+1} + b_{t+1} \tag{51}$$

At an internal solution on bequests, i.e. $\mu_{t+1} = 0$ from (46a) and (46c) we get again

$$MRS_{c_{2t+1}, c^*_{2t+2}} = \frac{u'(c_{2t+1})}{u'\left(c^*_{2t+2}\right)} = \alpha$$

and comparative statics on consumption smoothing across generations can be easily obtained.[18] If generation t is liquidity constrained and not willing to bequeath, i.e.

$$\lambda_t > 0 \Rightarrow A_t = -y_{2T-1}$$

[16] If $b_t \leq y_{1t} + y_{2t+1} + b_{t+1}$ and $b_{t+1} \leq y_{1t+1} + y_{2t+2} + b^*_{t+2}$ (gen t and $t+1$ not liq constrained)

1. If $\alpha < 1$, from (50), $c_{1t} = c_{2t+1} > c^*_{1t+1} = c^*_{2t+2}$
2. If $\alpha = 1$, from (50), $c_{1t} = c_{2t+1} = c^*_{1t+1} = c^*_{2t+2}$
3. If $\alpha > 1$, from (50), $c_{1t} = c_{2t+1} < c^*_{1t+1} = c^*_{2t+2}$

If $b_t \leq y_{1t} + y_{2t+1} + b_{t+1}$ and $b_{t+1} > y_{1t+1} + y_{2t+2} + b^*_{t+2}$ (gen $t+1$ not smoothing)

1. If $\alpha < 1$, from (50), $c_{1t} = c_{2t+1} > c^*_{1t+1} > c^*_{2t+2}$
2. If $\alpha = 1$, from (50), $c_{1t} = c_{2t+1} = c^*_{2t+2} > c^*_{1t+1}$
3. If $\alpha > 1$, from (50), $c_{1t} = c_{2t+1} < c^*_{2t+2}$ Here we cannot say much on the difference between c^*_{1t+1} and c_{2t+1}

[17] If $\alpha \geq 1$ then $c_{1t} = c_{2t+1} \leq c^*_{2t+2} = c^*_{1t+1}$. If $\alpha < 1$ we can't say anything about consumption smoothing across generations. Given the concavity of the utility function $u(.)$, this condition more likely holds if if c_{2t+1} is relatively low in comparison with c^*_{2t+2}. If generation $t+1$ is lifetime richer than generation t, and/or the degree of altruism is low (low α), it is likely that $b_{t+1} = 0$. In this case generation $t+1$ anticipates that he will not receive a bequest and smooth his consumption over his life cycle.

[18] If $b_t > y_{1t} + y_{2t+1} + b_{t+1}$ and $b_{t+1} \leq y_{1t+1} + y_{2t+2} + b^*_{t+2}$ (gen. $t+1$ not liq constrained)

1. If $\alpha < 1$, $c_{1t} < c_{2t+1}$ and $c_{2t+1} > c^*_{2t+2} = c^*_{1t+1}$
2. If $\alpha = 1$, $c_{1t} < c_{2t+1} = c^*_{1t+1} = c^*_{2t+2}$

$$\mu_{t+1} > 0 \Rightarrow b_{t+1} = 0$$

From the FOC (46a) and (46c) it follows that $c_{1t} = y_{1t} + y_{2t+1}$, $c_{2t+1} = b_t$, $c_{2t+1} > c_{1t}$ and generation $t+1$ always smooths. As regards intergenerational consumption smoothing, we obtain again

$$MRS_{c_{2t+1},c^*_{2t+2}} = \frac{u'(b_t)}{u'\left(\frac{y_{1t+1}+y_{2t+2}-b^*_{t+2}}{2}\right)} > \alpha$$

from which it is easy to compute comparative statics.[19]

GENERAL COMMENTS ON GENERATION t SOLUTION

If individuals cannot borrow against bequests, solutions may change compared to the standard case:

1. For any level of altruism α, it is possible to observe solutions in which some generations hit their liquidity constraint and do not smooth consumption.
2. This happens if $b_t > y_{1t} + y_{2t+1} + b_{t+1}$. That is if received bequests from previous generations is larger than lifetime earnings plus what generation t wants to transfer to the next generation $t+1$.
3. Since the within period utility functions are the same across generations given resources, the marginal propensities to consume is the same across generations.
4. The same for altruism
5. This means that it is enough to have heterogeneity in income across generations to observe people hitting the liquidity constraint. No need for heterogenous preferences.
6. Whatever shocks to generation t permanent income, even expected which brings down generation t resources with respect to the previous generation, lead to a suboptimal solution for generation t: generation t would like to smooth but cannot consume out of bequests in their first period.

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3. If $\alpha > 1$, $c_{1t} < c_{2t+1} < c^*_{1t+1} = c^*_{2t+2}$

If $b_t > y_{1t} + y_{2t+1} + b_{t+1}$ and $b_{t+1} > y_{1t+1} + y_{2t+2} + b^*_{t+2}$ (both generations liquidity constrained liq constrained)

1. If $\alpha < 1$, $c_{1t} < c_{2t+1}$ and $c_{2t+1} > c^*_{2t+2} > c^*_{1t+1}$
2. If $\alpha = 1$, $c_{1t} < c_{2t+1} = c^*_{2t+2}$
3. If $\alpha > 1$, $c_{1t} < c_{2t+1} < c^*_{2t+2}$

[19] If $\alpha \geq 1$, $c_{1t} < c_{2t+1} < c^*_{2t+2} = c^*_{1t+1}$. If $\alpha < 1$, nothing can be said on smoothing across generations. This scenario is more likely if lifetime earnings of generation t is relatively low compared with lifetime earnings of generation $t+1$ and/or the degree of altruism is low.

C The full model with bequests and intervivos transfers

Suppose now that generation t now also receives an intervivos transfer R_{t-1} at the beginning of period t from the previous generation and can give an intervivos transfer the the next generation $t+1$ at the end of period t (R_t).

$$\max_{c_{1t}, c_{2t+1}, R_t, b_{t+1}} u(c_{1t}) + u(c_{2t+1}) + \alpha V_{t+1} \tag{52a}$$

subject to the following constraints

$$c_{1t} = y_{1t} - A_t + R_{t-1} - R_t \tag{52b}$$
$$c_{2t+1} = y_{2t+1} + A_t + b_t - b_{t+1} \tag{52c}$$
$$b_{t+1} \geq 0 \tag{52d}$$
$$R_t \geq 0 \tag{52e}$$
$$A_t \geq -y_{2t+1} \tag{52f}$$

With b_1, R_0 known and given at time $t = 0$. From constraint (52b),(52c), $c_{1t} > 0$ and $c_{2t+1} > 0$ it follows that $A_t \in (-y_{2t+1} - b_t + b_{t+1}, y_{1t} + R_{t-1} - R_t)$. A generation which does not want to transfer any money, can save or borrow within $A_t \in (-y_{2t+1} - b_t, y_{1t} + R_{t-1})$. This is the standard assumptions about savings in OLG models: each generation can borrow up to his whole future resources. Constraint (52f) says generation t can borrow up to his whole second period income, but not against bequest. It follows that in our case, if $b_t > b_{t+1}$

$$A_t \in [-y_{2t+1}, y_{1t} + R_{t-1} - R_t)$$

As in sections B and A there are money transfers between generations (two in this case, intervivos and bequests), therefore it will be necessary to solve it starting from the last period. Generation T consumes all his income and the money received from previous generation as an inter-vivos $c^*_{1T} = y_{1T} + R_{T-1}$; $V_T = u(y_{1T} + R_{T-1})$.

C.1 Generation $T-1$

Generation $T-1$ leave an inter-vivos transfer but not a bequest, due to the terminal conditions $b_T = 0$ and $R_{T-1} = 0$. The optimization problem is the following:

$$\begin{aligned} \max_{c_{1T-1}, c_{2T}, A_{T-1}} \quad & u(c_{1T-1}) + u(c_{2T}) \\ \text{s.t. } c_{1T-1} &= y_{1T-1} - A_{T-1} + R_{T-2} \\ c_{2T} &= y_{2T} + A_{T-1} + b_{T-1} \\ A_{T-1} &\geq -y_{2T} \end{aligned}$$

So the Lagrangian becomes

$$L = u(y_{1T-1} - A_{T-1} + R_{T-2}) + u(y_{2T} + A_{T-1} + b_{T-1}) + \lambda_{T-1}(A_{T-1} + y_{2T})$$

FOC

$$\frac{\partial L}{\partial A_{T-1}}: \ -u'(y_{1T-1} - A_{T-1} + R_{T-2}) + u'(y_{2T} + A_{T-1} + b_{T-1}) + \lambda_{T-1} = 0 \tag{53}$$

We can distinguish two cases:

1. $\lambda_{T-1} = 0$; $A_{T-1} + y_{2T} \geq 0$ (liquidity constraint not binding)
   From FOC (53): Generation $T-1$ smooths his consumption

$$c^*_{1T-1} = c^*_{2T} = \frac{y_{1T-1} + R_{T-2} + y_{2T} + b_{T-1}}{2}$$

$$b_{T-1} - R_{T-2} \leq y_{1T-1} + y_{2T}$$

2. $\lambda_{T-1} > 0$; $A_{T-1} = -y_{2T}$ (liquidity constraint binding)
   From FOC (53): No consumption smoothing

$$c^*_{1T-1} = y_{1T-1} + y_{2T} + R_{T-2}; \quad c^*_{2T} = b_{T-1}$$

   Moreover,

$$b_{T-1} - R_{T-2} > y_{1T-1} + y_{2T}$$

C.2 Generation t

$$\max_{c_{1t}, c_{2t+1}, R_t, b_{t+1}} u(c_{1t}) + u(c_{2t+1}) + \alpha V_{t+1}$$

subject to the following constraints

$$\begin{aligned} c_{1t} &= y_{1t} - A_t + R_{t-1} - R_t \\ c_{2t+1} &= y_{2t+1} + A_t + b_t - b_{t+1} \\ b_{t+1} &\geq 0 \\ R_t &\geq 0 \\ A_t &\geq -y_{2t} \end{aligned}$$

We can rewrite generation t maximization conditional on generation $t+1$ choice variables evaluated at their optimum, namely R^*_{t+1} and b^*_{t+2}, including generation $t+1$ saving A_{t+1} among the choice variables of generation t, as well as all the relevant constraints in the maximization problem.

$$V_t(R_{t-1}, b_t) = \max_{A_t, A_{t+1}, R_t, b_{t+1}} u(c_{1t}) + u(c_{2t+1}) + \alpha \left[u(c_{1t+1}) + u(c_{2t+2}) + \alpha V_{t+2}(R^*_{t+1}, b^*_{t+2}) \right]$$

subject to the following constraints

$$\begin{aligned}
c_{1t} &= y_{1t} - A_t + R_{t-1} - R_t \\
c_{2t+1} &= y_{2t+1} + A_t + b_t - b_{t+1} \\
c_{1t+1} &= y_{1t+1} - A_{t+1} + R_t - R^*_{t+1} \\
c_{2t+2} &= y_{2t+2} + A_{t+1} + b_{t+1} - b^*_{t+2} \\
b_{t+1} &\geq 0 \\
R_t &\geq 0 \\
A_\tau &\geq -y_{2\tau+1}, \;\; \tau = t, t+1
\end{aligned}$$

The Lagrangian is

$$\begin{aligned}
L = & \, u(y_{1t} - A_t + R_{t-1} - R_t) + u(y_{2t+1} + A_t + b_t - b_{t+1}) \\
& + \alpha \left[u(y_{1t+1} - A_{t+1} + R_t - R^*_{t+1}) + u(y_{2t+2} + A_{t+1} + b_{t+1} - b^*_{t+2}) + \alpha V_{t+2}(R^*_{t+1}, b^*_{t+2}) \right] \\
& + \mu_{t+1}(b_{t+1}) + \nu_t(R_t) + \lambda_t(A_t + y_{2t+1}) + \lambda_{t+1}(A_{t+1} + y_{2t+2})
\end{aligned}$$

from which the Kuhn-Tucker conditions follow:

$$\frac{\partial L}{\partial A_t} : \; -u'(y_{1t} - A_t + R_{t-1} - R_t) + u'(y_{2t+1} + A_t + b_t - b_{t+1}) + \lambda_t = 0 \tag{54a}$$

$$\frac{\partial L}{\partial A_{t+1}} : \; -\alpha u'(y_{1t+1} - A_{t+1} + R_t - R^*_{t+1}) + \alpha u'(y_{2t+2} + A_{t+1} + b_{t+1} - b^*_{t+2}) + \lambda_{t+1} = 0 \tag{54b}$$

$$\frac{\partial L}{\partial b_{t+1}} : \; -u'(y_{2t+1} + A_t + b_t - b_{t+1}) + \alpha u'(y_{2t+2} + A_{t+1} + b_{t+1} - b^*_{t+2}) + \mu_{t+1} = 0 \tag{54c}$$

$$\frac{\partial L}{\partial R_t} : \; -u'(y_{1t} - A_t + R_{t-1} - R_t) + \alpha u'(y_{1t+1} - A_{t+1} + R_t - R^*_{t+1}) + \nu_t = 0 \tag{54d}$$

$$\mu_{t+1} \geq 0 \tag{54e}$$

$$b_{t+1} \geq 0 \tag{54f}$$

$$\mu_{t+1} b_{t+1} = 0 \tag{54g}$$

$$\nu_t \geq 0 \tag{54h}$$

$$R_t \geq 0 \tag{54i}$$

$$\nu_t R_t = 0 \tag{54j}$$

$$\lambda_\tau \geq 0, \tau = t, t+1 \tag{54k}$$

$$A_t + y_{2t+1} \geq 0 \tag{54l}$$

$$A_{t+1} + y_{2t+2} \geq 0 \tag{54m}$$

$$\lambda_\tau(A_\tau + y_{2\tau}) = 0, \;\; \tau = t, t+1 \tag{54n}$$

If b_1 and R_0, which are known at time $t = 0$, are such that $R_0 \geq b_1 = 0$, by the Theorem proved in main body of the article, the only admissible cases are the following

1. $c_{1t} = c_{2t+1}$; $c^*_{1t+1} = c^*_{2t+2}$; $b_{t+1} > 0$; $R_t > 0$
2. $c_{1t} = c_{2t+1}$; $c^*_{1t+1} = c^*_{2t+2}$; $b_{t+1} = 0$; $R_t = 0$

CASE 1 $\lambda_t = \lambda_{t+1} = \mu_{t+1} = \nu_t = 0$: (liquidity constraints not binding (consumption smoothing); positive intervivos and bequests)

From (54a): Generation t smooths his consumption

$$c_{1t} = c_{2t+1} = \frac{y_{1t} + R_{t-1} - R_t + y_{2t+1} + b_t - b_{t+1}}{2}$$

and

$$A_t = \frac{y_{1t} + R_{t-1} - R_t - y_{2t+1} - b_t + b_{t+1}}{2}$$

Since $A_t \geq -y_{2t+1}$

$$b_t - R_{t-1} \leq y_{1t} + y_{2t+1} - R_t + b_{t+1}$$

$b_t - R_{t-1}$ denotes the *'excess' transfer received* by gen. t in his second period of life wrt first period.

$b_{t+1} - R_t$ denotes the *'excess' transfer given* by gen. t in his second period of life wrt first period.

Nothing changes in comparison the model without intervivos transfers as long as we look at the optimization problem in terms of $b_{t+1} - R_t$ which is 'equivalent' to b_{t+1} in model without intervivos.

From (54b)

$$c^*_{1t+1} = c^*_{2t+2} = \frac{y_{1t+1} + R_t - R^*_{t+1} + y_{2t+2} + b_{t+1} - b^*_{t+2}}{2}$$

and

$$A^*_{t+1} = \frac{y_{1t+1} + R_t - R^*_{t+1} - y_{2t+1} - b_{t+1} + b^*_{t+2}}{2}$$

Since $A^*_{t+1} > -y_{2t+2}$

$$b_{t+1} - R_t \leq y_{1t+1} + y_{2t+2} + b^*_{t+2} - R^*_{t+1}$$

From FOC (54c) and (54d):

$$\frac{u'(c_{2t+1})}{u'(c^*_{2t+2})} = \frac{u'(c_{1t})}{u'(c^*_{1t+1})} = \alpha$$

Consumption smoothing across generations, i.e. whether generation t or generation $t+1$ per-period consumption is higher depends on whether $\alpha \lesseqgtr 0$.

CASE 2 $\lambda_t = \lambda_{t+1} = 0$; $\mu_{t+1} > 0$; $\nu_t > 0$: (liq constraint not binding for gen $t+1$ and t, $b_{t+1} = 0$, $R_t = 0$)

Again,

$$c_{1t} = c_{2t+1}$$

$$c^*_{1t+1} = c^*_{2t+2}$$

Together with FOC (54c) and (54d), the equations above imply the following

$$\frac{u'(c_{2t+1})}{u'(c^*_{2t+2})} = \frac{u'(c_{1t})}{u'(c^*_{1t+1})} > \alpha$$

This is an admissable solution. If $\alpha > 1$, consumption of generation $t+1$ is higher than that of t.

Table 1: The structure of the model

		time t		time $t+1$		time $t+2$		...
		beginning	end	beginning	end	beginning	end	
Generation t	money IN	y_{1t}, R_{t-1}		$A_t,\ y_{2t+1}, b_t$				
	money OUT	c_{1t}	A_t, R_t	c_{2t+1}	b_{t+1}			
Generation $t+1$	money IN			y_{1t+1}, R_t		$A_{t+1},\ y_{2t+2}$,b_{t+1}		
	money OUT			c_{1t+1}	$A_{t+1},, R_{t+1}$	c_{2t+2}	b_{t+2}	
$\vdots$								

		...	time $T-2$		time $T-1$		time T	
			beginning	end	beginning	end	beginning	end
$\vdots$								
Generation $T-2$	money IN		$y_{1T-2},\ R_{T-3}$		A_{T-2},$b_{T-2},\ y_{2T-1}$			
	money OUT		c_{1T-2}	$A_{T-2},\ R_{T-2}$	c_{2T-1}	b_{T-1}		
Generation $T-1$	money IN				$y_{1T-1},\ R_{T-2}$		A_{T-1},b_{T-1},y_{2T}	
	money OUT				c_{1T-1}	$A_{T-1},\ R_{T-1}=0$	c_{2T}	$b_T=0$

Table 2: Baseline: Linear probability model

VARIABLES	(1) FE	(2) FE	(3) FE
prob child has first child OR buys house	0.403*	0.505**	0.504**
	(0.231)	(0.242)	(0.241)
number of children		0.091**	0.092**
		(0.045)	(0.045)
number of children squared		-0.015**	-0.015***
		(0.006)	(0.006)
how many grandchildren		0.006	0.006
		(0.004)	(0.004)
partner present in the household		-0.051	-0.048
		(0.079)	(0.079)
age of the oldest child		-0.004	-0.003
		(0.005)	(0.005)
poorhealth			0.036*
			(0.019)
IHS total earnings			0.002
			(0.002)
IHS net financial wealth			-0.001
			(0.002)
IHS net real wealth			0.001
			(0.001)
employed or self employed			-0.014
			(0.028)
unemployed and looking for work			-0.089**
			(0.044)
save to cover unforeseen expenses			0.001
			(0.006)
Observations	5,053	5,053	5,053
Number of personid	1,465	1,465	1,465

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Standard errors clustered at household level reported in parenthesis. All specifications include a full set of year dummies and a quadratic term in age to account for non linear age effects (De Ree and Alessie, 2011). 'IHS' stands for Inverse Hyperbolic Sine' transformation. Columns (2) and (3) include also a full set of regional dummies.

Table 3: Alternative estimation methods: RE and probit

VARIABLES	(1) RE	(2) mundlak	(3) re probit	(4) cor re probit
prob child has first child OR buys house	0.307**	0.503**	0.245*	0.378*
	(0.144)	(0.241)	(0.131)	(0.205)
number of children	0.041*	0.093**	0.043	0.102**
	(0.024)	(0.045)	(0.028)	(0.051)
number of children squared	-0.009**	-0.015***	-0.009**	-0.016**
	(0.003)	(0.006)	(0.004)	(0.007)
how many grandchildren	0.003	0.006	0.002	0.004
	(0.004)	(0.004)	(0.003)	(0.002)
partner present in the household	0.021	-0.047	0.014	-0.049
	(0.035)	(0.079)	(0.034)	(0.069)
age of the oldest child	-0.006***	-0.003	-0.006***	-0.003
	(0.002)	(0.005)	(0.002)	(0.005)
poorhealth	0.018	0.036*	0.017	0.030*
	(0.016)	(0.019)	(0.015)	(0.018)
IHS total earnings	0.002	0.002	0.002	0.002
	(0.002)	(0.002)	(0.002)	(0.002)
IHS net financial wealth	0.003**	-0.001	0.004**	-0.001
	(0.001)	(0.002)	(0.002)	(0.002)
IHS net real wealth	0.003**	0.001	0.004**	0.001
	(0.001)	(0.001)	(0.002)	(0.002)
employed or self employed	-0.007	-0.014	-0.005	-0.017
	(0.019)	(0.028)	(0.019)	(0.028)
unemployed and looking for work	-0.079**	-0.087**	-0.107*	-0.127**
	(0.037)	(0.044)	(0.055)	(0.059)
save to cover unforeseen expenses	-0.001	0.001	-0.000	0.000
	(0.005)	(0.006)	(0.005)	(0.006)
college education	0.033	0.025	0.029	0.023
	(0.020)	(0.020)	(0.019)	(0.019)
Low or no education	-0.043	-0.039	-0.049	-0.045
	(0.032)	(0.032)	(0.036)	(0.035)
female	-0.032*	-0.028	-0.035**	-0.032*
	(0.017)	(0.017)	(0.017)	(0.018)
Observations	5,053	5,053	5,053	5,053
Number of personid	1,465	1,465	1,465	1,465
Significance mundlak terms (p–val)		0.0001		0.0003

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Standard errors clustered at household level reported in parenthesis. Table reports marginal effects at the mean. All specifications include a full set of year dummies, a set of regional dummies and a quadratic polynomial in age. 'IHS' stands for Inverse Hyperbolic Sine' transformation.

Table 4: Alternative specifications

VARIABLES	(1) curr transf	(2) subst transf	(3) interacted	(4) One obs/hh	(5) full panel
prob child has first child OR buys house	0.517**	0.395**	0.299	0.483*	0.519***
	(0.206)	(0.186)	(0.264)	(0.285)	(0.130)
IHSnfinw*haz_union			0.022*		
			(0.012)		
number of children	0.029	0.024	0.087**	0.146***	0.042*
	(0.030)	(0.028)	(0.044)	(0.057)	(0.023)
number of children squared	-0.004	-0.003	-0.014**	-0.021***	-0.007**
	(0.005)	(0.004)	(0.006)	(0.007)	(0.003)
how many grandchildren	0.001	0.001	0.006	0.011***	0.007*
	(0.002)	(0.002)	(0.004)	(0.003)	(0.004)
partner present in the household	-0.062	-0.053	-0.048	-0.050	-0.015
	(0.052)	(0.061)	(0.079)	(0.085)	(0.044)
age of the oldest child	-0.004	-0.004	-0.003	-0.005	-0.005
	(0.004)	(0.004)	(0.005)	(0.007)	(0.004)
poorhealth	0.016	0.009	0.036*	0.016	0.031**
	(0.015)	(0.014)	(0.019)	(0.027)	(0.015)
IHS total earnings	-0.001	-0.001	0.001	0.005**	0.001
	(0.001)	(0.001)	(0.002)	(0.002)	(0.001)
IHS net financial wealth	0.001	0.001	-0.004**	-0.002	0.000
	(0.001)	(0.001)	(0.002)	(0.002)	(0.001)
IHS net real wealth	0.001**	0.001*	0.001	0.001	0.001
	(0.001)	(0.001)	(0.001)	(0.002)	(0.001)
employed or self employed	-0.010	0.001	-0.016	0.006	-0.001
	(0.020)	(0.019)	(0.028)	(0.041)	(0.023)
unemployed and looking for work	0.003	0.061	-0.091**	-0.115*	-0.084**
	(0.020)	(0.039)	(0.044)	(0.059)	(0.042)
save to cover unforeseen expenses	0.003	0.003	0.001	0.007	0.000
	(0.004)	(0.003)	(0.006)	(0.008)	(0.004)
Observations	5,053	5,053	5,053	3,191	10,389
Number of personid	1,465	1,465	1,465	928	3,043

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Standard errors clustered at household level reported in parenthesis. All specifications include a full set of year dummies, a set of regional dummies anda quadratic term in age to account for non linear age effects (De Ree and Alessie, 2011). 'IHS' stands for Inverse Hyperbolic Sine' transformation. In column 1 the dependent variable takes value 1 only if the parent is transferring at the monent of the interview. In column 2 the dependent is 1 if the parent is transferring and transfers to family members are above 10000 euros per year. The last column includes an interaction with total wealth

Table 5: Alternative proxy for credit constraints

VARIABLES	(1) baseline	(2) inters	(3) house	(4) gchild
number of children	0.092**	0.092**	0.096**	0.089**
	(0.045)	(0.046)	(0.045)	(0.045)
number of children squared	-0.015***	-0.014**	-0.016***	-0.014**
	(0.006)	(0.006)	(0.006)	(0.006)
how many grandchildren	0.006	0.005	0.006	0.006
	(0.004)	(0.004)	(0.004)	(0.004)
partner present in the household	-0.048	-0.046	-0.047	-0.048
	(0.079)	(0.079)	(0.079)	(0.079)
age of the oldest child	-0.003	-0.003	-0.003	-0.003
	(0.005)	(0.005)	(0.005)	(0.005)
poorhealth	0.036*	0.036*	0.036*	0.036*
	(0.019)	(0.019)	(0.019)	(0.019)
IHS total earnings	0.002	0.002	0.002	0.002
	(0.002)	(0.002)	(0.002)	(0.002)
IHS net financial wealth	-0.001	-0.001	-0.001	-0.001
	(0.002)	(0.002)	(0.002)	(0.002)
IHS net real wealth	0.001	0.001	0.001	0.001
	(0.001)	(0.001)	(0.001)	(0.001)
employed or self employed	-0.014	-0.008	-0.018	-0.011
	(0.028)	(0.028)	(0.028)	(0.028)
unemployed and looking for work	-0.089**	-0.082*	-0.090**	-0.086**
	(0.044)	(0.044)	(0.044)	(0.044)
save to cover unforeseen expenses	0.001	0.001	0.001	0.001
	(0.006)	(0.006)	(0.006)	(0.006)
prob child has first child OR buys house	0.504**			
	(0.241)			
prob child has first child AND buys house		9.557**		
		(4.434)		
prob a child buys a house			0.812**	
			(0.379)	
prob a child has a first child				0.726*
				(0.433)
Observations	5,053	5,053	5,053	5,053
Number of personid	1,465	1,465	1,465	1,465

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Standard errors clustered at household level reported in parenthesis. All specifications include a full set of year dummies, a set of regional dummies and a quadratic term in age to account for non linear age effects (De Ree and Alessie, 2011). 'IHS' stands for Inverse Hyperbolic Sine' transformation.

Figure 1: PLAN, age profiles of each answer category

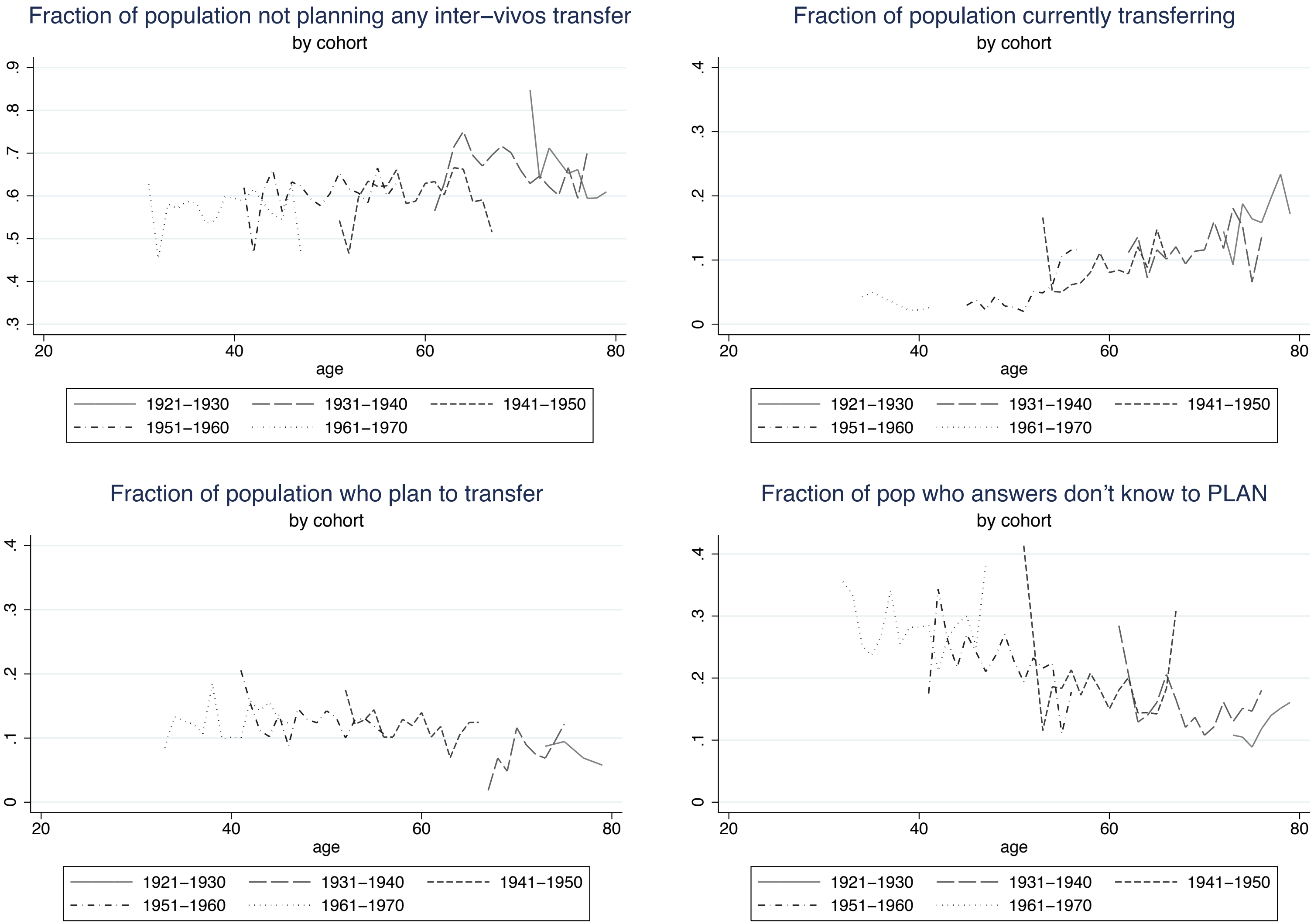

Figure 2: Hazard rates by age

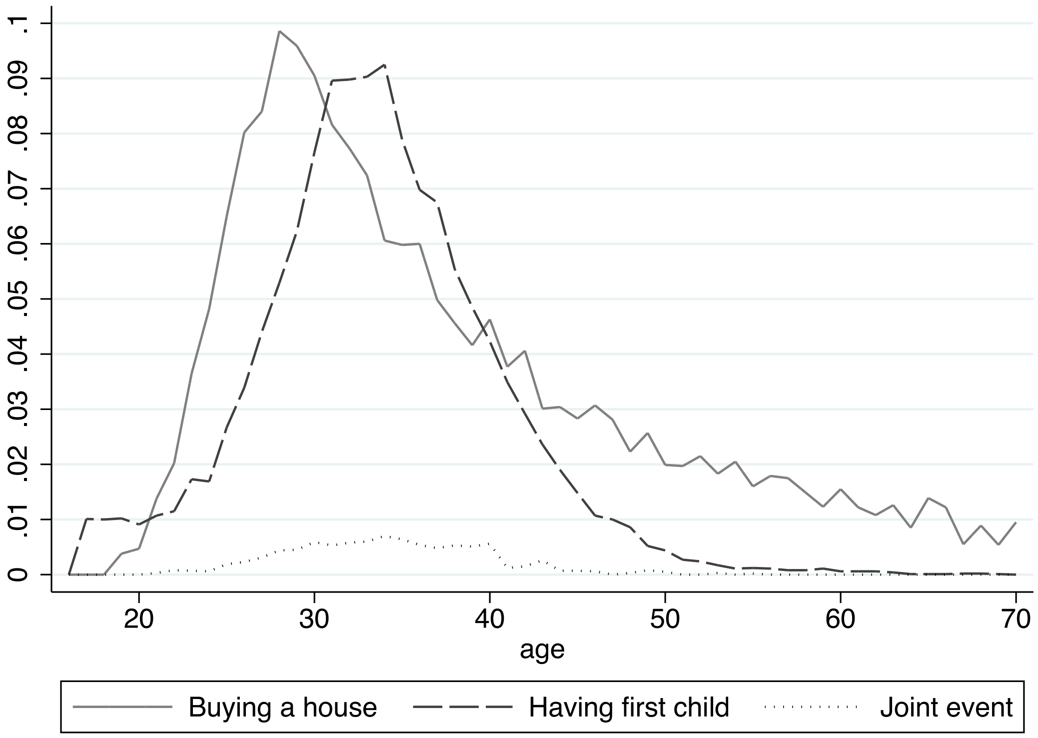

Figure 3: Credit limit proxies, by cohort