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SOME REMARKS ON THE CAUSALITY DEFINITIONS IN THE NON-LINEAR THEORY OF STOCHASTIC PROCESSES

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1. INTRODUCTION

The Granger causality concept is extensively used in econometrics and several works have applied the Granger causality definition to test hypotheses about economic structures. The applications have shown controversial results depending on the test structures, the time series length, the lag orders and the filtering. In particular different test structures were originated by different definitions on causality derived from Sims [1972] and Pierce-Haugh [1979].

The knowledge of theoretical equivalences between the definitions avoids the attribution of the discrepancies of the results, using the same data but different tests, to the different definitions. In the following the discussion about the causality concepts is made from a theoretical point of view, all the definitions are stochastic, outside the framework of the philosophical discussions on causality concepts, to which the last debate in econometrics is referred (see the papers of Granger [1980], Zellner [1979] and of Gambetta and Sartore [1983]).

The aim of this paper is to propose an extension of Pierce-Haugh causality definition in the non-linear case, showing that this extended definition is implied by the non-linear definition of Granger causality.

2. ALTERNATIVE CONCEPTS OF LINEAR NON-CAUSALITY

Let $[(x_t, y_t),\ t = \ldots, -1, 0, 1, \ldots]$ be a collection of random variables defined overs a common probability space, that is a bivariate stochastic process.

"y non-causes x" means:

– following Granger [1969], that the linear predictor (in mean square error sense) of x_{t+1} based on $x_t, x_{t-1}, \ldots y_t, y_{t-1}, \ldots$ equals the linear predictor

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based on $x_t, x_{t-1}, \ldots$

– following Sims [1972], that the linear predictor of y_t based on $\ldots, x_{t-1}, x_t, x_{t+1}, \ldots$, equals the linear predictor based on $x_t, x_{t-1}, \ldots$

– following Pierce e Haugh [1977], that the corss-correlation function between the innovations of the stochastic process y_t, that is η_t, $-\infty < t < q$, and those of the stochastic process x_t, that is ϵ_t, $-\infty < t < p$, is equal to zero if $q < p$.

These definitions can be formalized in terms of Hilbert space geometry.

Let L^2 the Hilbert space of $\mathscr{F}$-measurable functions (generally complex valued) defined on the probability space $(\Omega, \mathscr{F}, P)$, such that $\int_\Omega |f(\omega)|^2 P(d\omega) < \infty$, $\omega \in \Omega$. $L^2 = L^2(\Omega, \mathscr{F}, P)$ is an Hilbert space where the inner product and the norm are defined as, respectively, $<f, g> = \int_\Omega f(\omega) g(\omega) P(d\omega)$, $\|f\| = \sqrt{<f, f>}$. One point of the space $L^2(\Omega, \mathscr{F}, P)$ represents one random variable of the stochastic process z_t. Here we consider the bivariate stochastic process, $z_t = (x_t, y_t)$, such that $\int_\Omega x_t(\omega) P(d\omega) = \int_\Omega y_t(\omega) P(d\omega) = 0$, $\forall t$, where t varies, for simplicity, over a discrete set.

Linear prediction is simply a projection referring to Hilbert spaces. If $\mathscr{P}$ is the projection operator and x_n^m, $n < m$, the linear subspace of L^2 generated by the sequence of non collinear random variables $x_n, \ldots, x_m$, then:

$$\mathscr{P}(x_t / x_n^m) = \sum_{j=n}^{m} a_j x_{t-j} = \hat{x}_{t,m,n} \qquad n < m < t$$

in general:

$$\mathscr{P}(x_t / x_{-\infty}^m) = \sum_{j=-\infty}^{m} a_j x_{t-j} = \hat{x}_{t,m} \qquad m < t$$

Now $\hat{x}_{t,m;n}$ is the only element of x_n^m such that:

$$\|x_t - \hat{x}_{t,m,n}\| = \inf_{x_{t,m,n} \in x_n^m} \|x_t - x_{t,m,n}\|$$

were $x_{t,m,n} \in x_n^m$.

Similarly for $\hat{x}_{t,m}$ we have $\lim_{n \to \infty} \|\hat{x}_{t,m,n} - \hat{x}_{t,m}\| = 0$, so that the prediction based on the finite past is an approximation of the prediction based on the infinite past.

If $z_t = (x_t, y_t)$, then:

(*G*) y does not cause x in Granger sense if:

$$\mathscr{P}(x_{t+1} / z_{-\infty}^t) = \mathscr{P}(x_{t+1} / x_{-\infty}^t)$$

where $z^t_{-\infty} = x^t_{-\infty} + y^t_{-\infty}$ is the linear subspace generated by $x^t_{-\infty}$ and $y^t_{-\infty}$.

(*S*) y does not cause x in Sims sense if:

$$\mathcal{P}(y_t / x^{\infty}_{-\infty}) = \mathcal{P}(y_t / x^t_{-\infty})$$

(*PH*) y does not cause x in Pierce-Haugh sense if:

$$\epsilon_{t+1} = x_{t+1} - \mathcal{P}(x_{t+1} / x^t_{-\infty})$$

$$\eta_{s+1} = y_{s+1} - \mathcal{P}(y_{s+1} / y^s_{-\infty})$$

$$< \epsilon_{t+1}, \eta_{s+1} > = 0, \quad \text{per} \quad \forall\, s < t.$$

The equivalence between definition (*G*) and (*S*) was given by Sims [1972]. He considered z_t as a jointly stationary stochastic process in covariances without singular component, in vector autoregressive form, dual to the decomposition used by Wold-Zasuhin. Later Hosoya [1977] gave a more general demonstration expressing the process z_t as a curve in the Hilbert space.

Florens e Mouchart [1985] showed that (*PH*) can be derived from (*G*), but the equivalence between the two, (*PH*) and (*G*), derives from initial conditions, that is by imposing $y_{-\infty} \in x^t_{-\infty}$, where $y_{-\infty} = \bigcap_{j \geq 1} y^{t-j}_{-\infty}$.

To test the hypothesis "y does not cause x", the infinite sequence of random variables of the stochastic process must be truncated in accordance with the sample data available.

Now we ridefine *G, S, PH*, respectively as follows:

(*G'*) $\mathcal{P}(x_{t+1} / y^t_{t-q} + x^t_{t-p}) = \mathcal{P}(x_{t+1} / x^t_{t-p}) \qquad t > \max(p, q)$

for fixed p and q.

(*S'*) $\mathcal{P}(y_t / x^{t+p}_{t-q}) = \mathcal{P}(y_t / x^t_{t-q}) \qquad t > q$

for fixed p and q.

(*PH'*) $\epsilon_{t+1} = x_{t+1} - \mathcal{P}(x_{t+1} / x^t_{t-p}) \qquad t > p$

$\eta_{s+1} = y_{s+1} - \mathcal{P}(y_{s+1} / y^s_{s-q}) \qquad s > q$

$< \epsilon_{t+1}, \eta_{s+1} > = 0$, per $\forall\, s < t$ and for fixed values of p and q.

Now, (*G'*), (*S'*) and (*PH'*) are not equivalent and also (*G*) and (*G'*), (*S*) and (*S'*), (*PH*) and (*PH'*) are not equivalent. In the latter three cases the equivalence can be demonstrated by introducing other hypothesis on the relations between the processes x_t e y_t (1).

3. GRANGER'S AND SIMS' NON-CAUSALITY IN NON-LINEAR STOCHASTIC PROCESSES

The Hilbert space L^2 gives a simplified mathematical representation of the information set; in fact the conditional independence between processes has to be expressed in terms of orthogonality between vectors, hence in terms of linear conditional independence.

It is possible to apply a more strict concept of conditional independence, by making use of probability theory.

If X is a $\mathscr{F}$-measurable function, defined on the probability space (Ω, $\mathscr{F}$, P), P-integrable, and if $\mathscr{G}$ is a σ-algebra, such that $\mathscr{G} \subset \mathscr{F}$, then there exists (Radon-Nikodym theorem) a $\mathscr{G}$-measurable function, the conditional expected value of X given $\mathscr{G}$, that is $E(X/\mathscr{G})$, such that $\int_A X\, dP(\omega) = \int_A E[X/\mathscr{G}]\, dP(\omega)$, $\forall A \in \mathscr{G}$.

The σ-algebra gives the general mathematical representation of the information set.

Now we can state the following:

Theorem: Let $\mathscr{G}_i$, $(i = 1, 2, \ldots)$ be a sub-σ-algebra of $\mathscr{F}$. $\mathscr{G}_1$ and $\mathscr{G}_2$ are conditional independent from $\mathscr{G}_3$, and we write $\mathscr{G}_1 \perp\!\!\!\perp \mathscr{G}_2 / \mathscr{G}_3$, if one of the following equivalent conditions (2) is satisfied:

i) for all $A_1 \in \mathscr{G}_1$,

$$P\{A_1/\sigma(\mathscr{G}_2 \cup \mathscr{G}_3)\} = P\{A_1/\mathscr{G}_3\} \qquad \text{a.s.;}$$

ii) for every $\mathscr{G}_1$-measurable and integrable function X,

$$E\{X/\sigma(\mathscr{G}_2 \cup \mathscr{G}_3)\} = E\{X/\mathscr{G}_3\}. \qquad \text{a.s..}$$

Proof: Chow-Teicher [1978], Theorem 1, pp. 217-218.

As far as the various definitions of non-causality are concerned, it is sufficient here to employ the more general definition of $z_t = (x_t, y_t)$ as stochastic process following the axiomatic theory of probability of Kolmogorov [1933] (3).

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(1) These hypotheses are satisfied for stationary stochastic processes of second order (see Florens and Mouchart [1985]).

(2) Other equivalent conditions can be mentioned (see Florens and Mouchart [1982, Appendix]).

(3) See Doob [1953, p. 46].

Later it will be necessary, due to operational problems, to introduce a more restrictive hypothesis on the process z_t.

Without loss of generality, let us assume $E(x_t) = E(y_t) = 0$, $\forall t$. If Z_n^m, where m and n can be infinite, is the sub-σ-algebra generated by the sequence of random variables $z_n, z_{n+1}, \ldots, z_{m-1}, z_m$, then:

(G'') y does not causes x in Granger sense if:

$$x_{t+1} \perp\!\!\!\perp Y_{-\infty}^{t} / X_{-\infty}^{t} \qquad \forall\, t>0$$

(S'') y does not causes x in Sims sense if:

$$X_{t+1} \perp\!\!\!\perp y_t / X_{-\infty}^{t} \qquad \forall\, t>0$$

Chamberlain [1982], Florens e Mouchart [1982], stated that (G'') implies (S''), but not viceversa, as in linear case. Anyway they give a new definition of (S'') and thus they state the difference between (G'') and the new (S''), that is:

$(S''\text{-bis})$ $$X_{t+1}^{\infty} \perp\!\!\!\perp y_t / \sigma(X_{-\infty}^{t} \cup Y_{-\infty}^{t-1}) \qquad \forall\, t>0$$

with the new non-causality condition:

$$X_{-\infty} \perp\!\!\!\perp y_{-\infty} / x_{-\infty}$$

satisfied if $y_{-\infty} \subset x_{-\infty}$, where $y_{-\infty} = \bigcap_{p>1} y_{-\infty}^{t-p}$ and $x_{-\infty} = \bigcap_{p \geq 1} X_{-\infty}^{t-p}$.

4. NON-LINEAR STOCHASTIC PROCESSES AND PIERCE AND HAUGH'S NON-CAUSALITY

The conditions on the information set are too general that is there is not a definition (PH'') corresponding to the definition (PH) in the linear case. In this latter case since the information set is an Hilbert space $L^2(\Omega, \mathcal{F}, P)$ the inner product and the norm can be used and therefore a stochastic process has first and second moments.

A question arises: is it possible to give a definition (PH'') corresponding to (PH)? But another question arises that is better answered first: is it possible to interpret definitions (G'') and (S'') in terms of prediction theory? The answer is yes if a particular class of stochastic processes is considered.

In fact, given the stochastic process $z_t = (x_t, y_t)$, the definition (G'') can be interpreted as:

$(G''\text{-bis})$ $$E[x_{t+1} / \sigma(X_{-\infty}^{t} \cup Y_{-\infty}^{t})] = E[x_{t+1} / X_{-\infty}^{t}], \qquad \forall\, t>0.$$

for every x_{t+1} is $X_{-\infty}^{t+1}$-measurable in $L^2(X_{-\infty}^{t+1})$.

Now if:

$$\hat{x}_{t+k} = E\,[x_{t+k}/X^t_{-\infty}]\,, \qquad k = 1, 2, \ldots$$

and if $E\,|\,x_t\,|^2 < \infty$, $\forall t$, using the rule of iterated conditional expectation and Jensen's inequality, we have:

$$E\,[\,|\,\hat{x}_{t+k}\,|^2\,] < E\{E\,[\,|\,x_{t+k}\,|^2/X^t_{-\infty}]\} = E\,[\,|\,x_{t+k}\,|^2\,], \qquad k = 1, 2, \ldots$$

Moreover, if f is a measurable function with respect to $X^t_{-\infty}$, such that f^2 is integrable, we have (4):

$$E\,[(x_{t+k} - \hat{x}_{t+k})\,f\,] = 0 \qquad k = 1, 2, \ldots$$

Then we have:

$$E\,[\,|\,x_{t+k} - f\,|^2\,] = E\,[\,|\,(x_{t+k} - \hat{x}_{t+k}) + (\hat{x}_{t+k} - f)\,|^2\,]$$
$$= E\,[\,|\,x_{t+k} - \hat{x}_{t+k}\,|^2\,] + E\,[\,|\,\hat{x}_{t+k} - f\,|^2\,],$$
$$k = 1, 2, \ldots$$

which is minimum only if $f = \hat{x}_{t+k}$.

Thus $\hat{x}_{t+k}$ can be defined as the best predictor of x_{t+k} because it minimizes the expected values of the square of the prediction error $\epsilon_{t+k} = x_{t+k} - \hat{x}_{t+k}$, where ϵ_{t+k} is orthogonal to every function f, $X^t_{-\infty}$-measurable and squared integrable.

Now the problem is to determine if there is any sense in the use of a non linear prediction based only on a part of the past described by the information set. Obviously this is the case if the prediction is an approximation of the prediction based on the infinite past.

Again the answer to the question is affermative because (5):

$$\lim_{s \to \infty} E\,[x_{t+k}/X^t_s\,] = E\,[x_{t+k}/X^t_{-\infty}]$$

with probability 1, as $E\,|\,x_{t+k}\,| < \infty$ and because, being $x^t_{s-1} \supset x^t_s$, for all $s < t$, x^t_s is a monotone decreasing sequence in s for which there exists:

$$\lim_{s \to \infty} X^t_s = X^t_{-\infty} \supset \bigcup_{s=-\infty}^{t} X^t_s\,.$$

It can be observed that, as in the linear case, there is not a stationary condition on the stochastic process and therefore no need to refer to the Masani-Wiener [1959] theoretical approach. In fact, all the conditions given for the

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(4) The result follows from theorem 8.3, I, Dobb [1953].

(5) See Dobb [1953], VII, Theorem 4.3.

definition of the non linear predictor are satisfied by the class of stochastic process with finite second moments. In this class (G''-bis) can be thought as an equality between non linear predictors.

When the stochastic process is gaussian it can be shown that the best predictor is linear and as before (6) the Hilbert space geometry can be used to develop the theory.

Now the question is if it makes any sense to define (PH'') for non linear theory as (PH) for the linear theory.

A definition implied by (G'') is the following:

(PH'') y does not causes x in Pierce-Haugh sense if, being:

$$\epsilon_{t+1} = x_{t+1} - \hat{x}_{t+1}, \quad \forall\, t, \text{ and}$$

$$\eta_{s+1} = y_{s+1} - \hat{y}_{s+1}, \quad \forall\, s < t$$

it is:

$$E\,[\epsilon_{t+1}\,\eta_{s+1}/\sigma\,(X^t_{-\infty} \cup Y^s_{-\infty})] =$$

$$E\,[\epsilon_{t+1}/\sigma\,(X^t_{-\infty} \cup Y^s_{-\infty})]\; E\,[\eta_{s+1}/\sigma\,(X^t_{-\infty} \cup Y^s_{-\infty})]\,.$$

Now the following can be shown:

Theorem: (G'') implies (PH'').

Proof:

i) y does not cause x in Granger sense implies:

$$E\,[\epsilon_{t+1}\,\eta_{s+1}/\sigma\,(X^t_{-\infty} \cup Y^s_{-\infty})] = 0.$$

In fact:

$$\begin{aligned} E\,[\epsilon_{t+1}\,\eta_{s+1}/\sigma\,(X^t_{-\infty} \cup Y^s_{-\infty})] &= E\,[x_{t+1}\,y_{s+1}/\sigma\,(X^t_{-\infty} \cup Y^s_{-\infty})] - \\ &E\,[y_{s+1}\,E\,[x_{t+1}/X^t_{-\infty}]/\sigma\,(X^t_{-\infty} \cup Y^s_{-\infty})] - \\ &E\,[x_{t+1}\,E\,[y_{s+1}/Y^s_{-\infty}]/\sigma\,(X^t_{-\infty} \cup Y^s_{-\infty})] + \\ &E\,[E\,[x_{t+1}/X^t_{-\infty}]\,E\,[y_{s+1}/Y^s_{-\infty}]/\sigma\,(X^t_{-\infty} \cup Y^s_{-\infty})]\,. \end{aligned}$$

From (G''-bis) being y_{s+1} ($s < t$) $\sigma\,(X^t_{-\infty} \cup Y^t_{-\infty})$-measurable the second term

(6) See Cramer-Leadbetter [1967].

of the right hand side can be rewritten:

$$E\,[E\,[x_{t+1}\,y_{s+1}/\sigma\,(X^t_{-\infty} \cup Y^t_{-\infty})]/\sigma\,(X^t_{-\infty} \cup Y^s_{-\infty})] =$$

$$E\,[x_{t+1}\,y_{s+1}/\sigma\,(X^t_{-\infty} \cup Y^s_{-\infty})]$$

as $\sigma\,(X^t_{-\infty} \cup Y^s_{-\infty}) \subset \sigma\,(X^t_{-\infty} \cup Y^t_{-\infty})$. Therefore the second term and the first sum equal to zero.

Then the third term, as $E\,[y_{s+1}/y^s_{-\infty}]$ is $\sigma\,(X^t_{-\infty} \cup Y^s_{-\infty})$-measurable, can be rewritten:

$$E\,[y_{s+1}/Y^s_{-\infty}]\,E\,[x_{t+1}/\sigma\,(X^t_{-\infty} \cup Y^s_{-\infty})]$$

Finally, the fourth term can be rewritten:

$$E\,[y_{s+1}/Y^s_{-\infty}]\,E\,[E\,[x_{t+1}/X^t_{-\infty}]/\sigma\,(X^t_{-\infty} \cup Y^s_{-\infty})] =$$

$$E\,[y_{s+1}/Y^s_{-\infty}]\,E\,[x_{t+1}/\sigma\,(X^t_{-\infty} \cup Y^s_{-\infty})]$$

this last equality follows from (G''-bis). Then the third and the fourth terms have sum equal to zero so i) is true.

ii) y does not cause x in Granger sense implies:

$$E\,[\epsilon_{t+1}/\sigma\,(X^t_{-\infty} \cup Y^s_{-\infty})] = 0\,.$$

It is:

$$E\,[\epsilon_{t+1}/\sigma\,(X^t_{-\infty} \cup Y^s_{-\infty})] = E\,[x_{t+1} - E\,[x_{t+1}/X^t_{-\infty}]/\sigma\,(X^t_{-\infty} \cup Y^s_{-\infty})]$$

From (G''-bis), as $\sigma\,(X^t_{-\infty} \cup Y^s_{-\infty}) \subset \sigma\,(X^t_{-\infty} \cup Y^t_{-\infty})$:

$$E\,[x_{t+1} - E\,[x_{t+1}/X^t_{-\infty}]/\sigma\,(X^t_{-\infty} \cup Y^s_{-\infty})] =$$

$$E\,(x_{t+1} - E\,[x_{t+1}/\sigma\,(X^t_{-\infty} \cup Y^t_{-\infty})]/\sigma\,(X^t_{-\infty} \cup Y^s_{-\infty})] =$$

$$E\,[x_{t+1}/\sigma\,(X^t_{-\infty} \cup Y^s_{-\infty})] - E\,[x_{t+1}/\sigma\,(X^t_{-\infty} \cup Y^s_{-\infty})] = 0$$

Q.E.D.

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RIASSUNTO

Alcune osservazioni sulle definizioni di causalità nella teoria non lineare dei processi stocastici

Il concetto di causalità di Granger è usato estensivamente in econometria e si assiste ad una notevole mole di lavori in cui tale concetto viene utilizzato nella fase della verifica empirica per una più corretta specificazione della struttura dei modelli.

Queste applicazioni hanno mostrato risultati anche contrastanti tra loro, poiché la verifica empirica dipende dalla diversa struttura dei tests, dalla numerosità campionaria della serie storica, dall'ordine dei ritardi temporali utilizzati nel modello per una corretta specificazione dinamica e dalle procedure di "filtering" delle serie al fine di ottenere la condizione di stazionarità. In particolare, le diverse definizioni di causalità di Sims e di Pierce-Haugh hanno dato origine a differenti procedure per la verifica dell'ipotesi di causalità.

Risulta quindi rilevante analizzare le relazioni di equivalenza tra le diverse definizioni esistenti in letteratura affinché le discrepanze tra verifiche empiriche non risultino erroneamente imputabili all'utilizzazione di una diversa definizione del concetto di causalità. Nell'articolo, la discussione sui concetti di causalità è condotta all'interno della teoria dei processi stocastici, al di fuori quindi del dibattito recentemente sviluppatosi in ambito filosofico.

Scopo della ricerca è quello di proporre un'estensione del concetto di causalità proprio di Pierce-Haugh al caso non lineare, dimostrando che questa definizione estesa è implicata dalla definizione non lineare di causalità nel senso di Granger.

RÉSUMÉ

Remarques sur les définitions de la causalité dans la théorie non linéaire des processus stochastiques

Le concept de causalité de Granger est massivement utilisé en économétrie, et un très grand nombre de travaux l'emploie lors de la phase de vérification empirique afin de parvenir à une spécification plus correcte de la structure des modèles.

Ces applications ont présenté parfois des résultats contradictoires dans la mesure où la véritification empirique dépend des différentes structures des tests, de la longueur de la série historique, de l'ordre des retards temporels utilisés dans le modèle pour une spécification dynamique convenable, et des processus de filtrage des séries visant à obtenir la condition de stationnarité. En particulier, les diverses définitions de la causalité de Sims et de Pierce-Haugh ont donné lieu à différents processus de vérification de l'hypothèse de causalité.

Il apparaît donc intéressant d'analyser les relations d'équivalence entre les différentes définitions existantes dans la littérature, afin que les divergences relevées entre les vérifications empiriques ne soient pas imputables, à tort, à l'utilisation d'une autre définition du concept de causalité.

La discussion sur les concepts de causalité sera conduite, dans cet article, en se situant à l'intérieur de la théorie des processus stochastiques, et par conséquent, en dehors du récent débat d'ordre philosophique sur la question.

Le but de cette recherche est de proposer une extension du concept de causalité de Pierce-Haugh au cas non-linéaire, en démontrant que cette définition plus large est impliquée par la définition non-linéaire de la causalitè au sens de Granger.