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Talking About Uncertainty

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Declaration of Authorship

I, Carlo Romano Marcello Alessandro Santagiustina, declare that this thesis titled, Talking About Uncertainty and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
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- Where I have consulted the published work of others, this is always clearly attributed.
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- I have acknowledged all main sources of help.
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Signed: Carlo R. M. A. Santagiustina
Date: September 11, 2018

With the introduction of uncertainty - the fact of ignorance and necessity of acting upon opinion rather than knowledge - into this Eden-like situation, its character is completely changed. With uncertainty absent, man's energies are devoted altogether to doing things; it is doubtful whether intelligence itself would exist in such a situation; in a world so built that perfect knowledge was theoretically possible, it seems likely that all organic readjustments would become mechanical, all organisms automata.[...] Consciousness would never have developed if the environment of living organisms were perfectly uniform and monotonous, conformable to mechanical laws. [...] There is a manifest tendency to economize consciousness, to make all possible adaptations by unconscious reflex response. [...] The true uncertainty in organized life is the uncertainty in an estimate of human capacity, which is always a capacity to meet uncertainty.

Frank Knight

Ca'Foscari University of Venice

Abstract

Department of Economics

Doctor of Philosophy

Talking About Uncertainty

by Carlo Romano Marcello Alessandro Santagiustina

In the first article we review existing theories of uncertainty. We devote particular attention to the relation between metacognition, uncertainty and probabilistic expectations. We also analyse the role of natural language and communication for the emergence and resolution of states of uncertainty. We hypothesize that agents feel uncertainty in relation to their levels of expected surprise, which depends on probabilistic expectations-gaps elicited during communication processes. Under this framework above tolerance levels of expected surprise can be considered informative signals. These signals can be used to coordinate, at the group and social level, processes of revision of probabilistic expectations. When above tolerance levels of uncertainty are explicated by agents through natural language, in communication networks and public information arenas, uncertainty acquires a systemic role of coordinating device for the revision of probabilistic expectations.

The second article of this research seeks to empirically demonstrate that we can crowd source and aggregate decentralized signals of uncertainty, i.e. expected surprise, coming from market agents and civil society by using the web and more specifically Twitter as an information source that contains the wisdom of the crowds concerning the degree of uncertainty of targeted communities/groups of agents at a given moment in time. We extract and aggregate these signals to construct a set of *civil society uncertainty* proxies by country. We model the dependence among our *civil society uncertainty* indexes and existing policy and market uncertainty proxies, highlighting contagion channels and differences in their reactiveness to real-world events that occurred in the year 2016, like the EU-referendum vote and the US presidential elections.

Finally, in the third article we propose a new instrument, called **Worldwide Uncertainty Network**, to analyse the uncertainty contagion dynamics across time and areas of the world. Such an instrument can be used to identify the systemic importance of countries in terms of their *civil society uncertainty* social percolation role. Our results show that *civil society uncertainty* signals coming from the web may be fruitfully used to improve our understanding of uncertainty contagion and amplification mechanisms among countries and between markets, civil society and political systems;

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I would like to express all my gratitude to all of those that during these four years dedicated their time, actions and thoughts to me and to this work.

Without their help, presence and support, the path I have chosen to undertake and pursue would have been burdensome and certainly less enjoyable and stimulating.

The completion of this study, my maturation as person and researcher, and the fact that my energy, spirit and creativity didn't fade in these four long years is mostly due to them...

To Giulia,

To my family,

To my supervisors, Massimo Warglien and Michele Bernasconi,

To Agar Brugiavini, Roberto Casarin and Giacomo Pasini,

To all the reviewers of this work,

To Lisa Negrello,

To my friends, colleagues and professors

and to all of those that helped me along this journey.

To them I dedicate this work, as well as my most sincere feelings of friendship, affection and esteem.

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Chapter 1

Uncertainty: reviewing the unknown

By Carlo R. M. A. Santagiustina

Abstract

This article reviews existing theories of uncertainty. Through a comparative approach, we highlight the distinctive attributes associated to uncertainty, at the agent, group and social level. Starting from mainstream characterizations of uncertainty in economics, information theory, as well as social and cognitive sciences, we move towards uncertainty modelling and measurement research frontiers. We devote particular attention to the relation between metacognition, uncertainty and probabilistic expectations. We describe the relation between higher order beliefs and uncertainty. We analyse the role of natural language and communication for uncertainty phenomena emergence, persistence, contagion, reduction and eventual resolution. By so doing, we reconstruct a robust uncertainty phenomena-concept reference relation, where uncertainty characterizes metacognitive processes. The roots of uncertainty are shown to reside in a de facto epistemic situation that characterizes all human agents and their systems: *having to learn, while learning to learn*. Following cues from recent applications of information and belief theory to economics, we hypothesize that agents feel uncertainty in relation to their levels of expected surprise, which depends on probabilistic expectations gaps elicited during communication processes. Expected surprise will be measured through relative entropy, as formalized by Kullback and Leibler. Under this framework above tolerance levels of expected surprise can be considered informative signals. These signals can be used to coordinate, at the group and social level, processes of revision of probabilistic expectations. When above tolerance levels of uncertainty are explicated by agents through natural language, in communication networks and public information arenas, uncertainty acquires a -new- systemic role of coordinating device for the revision of probabilistic expectations and the anticipation of expected utility.

1.1 Introduction

"The fundamental principle underlying organized activity is the reduction of the uncertainty in judgments"[1]

This article is a review and a synthesis of modern theories used to represent uncertainty phenomena and to identify, measure, analyse and model its occurrence and effects in - and beyond- economic affairs. In the following subsections, we introduce the topics treated in uncertainty literature and formulate a set of research questions and hypotheses, concerning the emergence and role of uncertainty.

A starting point: distinguishing uncertainty from risk representations

Uncertainty phenomena is often confused with the metaheuristics[2] used to represent, project and reduce uncertainty; among which, probability theory [3–5], as formalized in Kolmogorov's axioms [6], stands out as the formal system used to quantitatively represent uncertainty. As we will show, the contemporary risk framework[7–9], which is based on, but not limited to, probability theory, is used both in scientific and business domains to jointly represent and reduce *"quantifiable uncertainties"*. For those who know its axioms and methods, probability theory is a powerful tool for mental representation[10, 11] and convergent thinking[12, 13], in relation to repetitive decisions under imperfect information. In particular, probability theory appears to be useful for formalizing coherent systems of expectations, to undertake and rationally justify decisions on the basis of the latter. However, when ones' uncertainties, in relation to observed phenomena, are quantified and analysed in a probabilistic framework, the resulting probabilistic representation is not necessarily an exhaustive or unbiased representation of former uncertainties. The methodological constraints of probability theory, as well as the chosen frame of discernment, (re)determine uncertainties. Probability theory re-projects uncertainties through its use. For example, non-exhaustivity may emerge in relation to measures that are finitely and non-countably additive, which are not admitted in probability-space based representation in the classical probabilistic framework: A hypothetical *"uniform distribution"* over the set of natural numbers does not satisfy the three Kolmogorov axioms[6]. Whereas, biasedness may emerge in relation to the distinguishability requirement for events in the probability-space: The granularity and dimensionality of a probability-space often depends on the characteristics of sensory instruments used for observation, which by determining distinguishable outcomes, project real world phenomena in a outcome space.

Uncertainty as a characterization of metacognition and communication processes

In the following sections, we will propose a re-examination of uncertainty phenomena and theories from a metacognition and communication perspective[14, 15]. If we trivialize the concept of metacognition, it can be described as a process through which an aware agent elicitates and affects his belief system. Such a process has the structure of an iterated circular process through which belief system reflexion and reviewal are alternated[16, 17]. Communication is here defined in its more extensive meaning, as described by Shannon and Weaver in their Mathematical Theory of

Communication[18]: *"All of the procedures by which one mind may affect another"*

This definition of communication englobes any process through which meaning is constructed, codified, transferred, decodified and used to review beliefs and expectations [19–28]. As we will see, metacognition itself implies communication among different cognitive levels. Communication characterizes also those situations where the communication-related tasks are seen as instrumental or subsidiary to the ends of the agent(s) that undertake(s) them. Any communication results in a change of beliefs and uncertainties across coupled systems. In our review, we will illustrate evidence in favour of the hypothesis that human agents knowingly use their metacognitive and communication capacities to try to jointly reduce their beliefs related uncertainties. In particular, in relation to the foreseeing of future states of the world and the commensuration of the likelihood of future events. This activity is undertaken by mentally speculating on, and anticipating the effects of, future events. By so doing agents can represent the dynamics of the systems in which they operate and evaluate what to do to render those systems, in actual or prospect terms, more favourable or closer to ideal states. In neoclassical economics, the aforementioned mechanism corresponds to the possibility of formalizing a probability space and maximizing expected utility conditional on probabilistic expectations. In our framework, changes in uncertainty can, for example, be undertaken through communication occurring in:

- **Market transaction processes:** through the enacting of each transaction and determination of a price, a buyer (seller) comes to know that, under a given *"state of the world"*, there is at least one seller (buyer) that values a good/service less (more) than the transaction price. Where the price is identified with reference to fiat currency, a numeraire, or, in case of a barter, of the ratio of the cardinality of exchanged goods/services. If the transaction does not take the form of a barter among goods/services that are to be consumed instantly after the transaction, it is always conditional on beliefs of the transacting parties concerning the foreseen utility and price of the exchanged goods/services. Therefore, a result of market transactions is the redistribution across agents of uncertainties and of their effects, in relation to foreseen utility and prices of exchanged goods/services. The latter effects are implicit to any transaction of goods/services that are not instantaneously consumed.

- **Empirical and experimental evidence collection:** each time an individual observes (samples) the state of a system in a natural (empirical) or controlled (experimental) setting. We can consider observation a type of pull communication.

- **Deliberation and communication about coordination devices:** all those processes used to generate or modify reference systems, common knowledge beliefs, expectations, conventions and metaheuristics, in particular, in relation to the reduction of strategic uncertainty. Which are generally used for decision-making, coordination, sense-making, foreseeing or anticipation purposes under imperfect information;

Towards a cognitivist turning of uncertainty paradigms?

In our review we will explore the dominant paradigms proposed by economic, social, cognitive and information sciences' literature. Studies in these research fields appear to be edified on imperfectly overlapping assumptions on what uncertainty is, and consequently, how to measure and model it. Their findings are comparable only with respect to limited aspects, which we call the core of phenomenological human uncertainty. We explore recent evidence and theorizations from the cognitive sciences, which link human uncertainties to metacognitive and communication processes[14–29].

Metacognition, uncertainty, beliefs and communication

As we will see, metacognitive processes can be considered particularly important for economists in relation to those beliefs that are used to formulate and review probabilistic expectations and to evaluate the degree of confidence attributed to them. At a systemic level, expectation revision interdependencies emerge when agents elicit and try to reduce uncertainty, at the group and social level, through communication. As we will explain, uncertainties at the group and social level can be elicited, reduced and eventually resolved, through the communication of probabilistic expectations. In this framework, a necessary condition for agent to exhibit uncertainty is to be aware of the (non-null) divergence between alternative systems of probabilistic expectations[30], for example the divergence between the prior expectations of an agent of those of the agents with whom the former communicates. Such a divergence can be considered a measure of the pressure for reviewing expectations, for reducing expected surprise. Metacognition itself, can be considered a process of iterative and reflexive reconstruction of beliefs and expectations, based on controlled communication between:

- **Cognition:** Lower level cognition implements the use of a belief system and monitors its outcomes in terms of decision-making and sense-making;
- **Metacognition:** Higher (meta) level cognition controls and reviews the belief system on the basis of information provided by lower level cognition;

Internal communication, between different hierarchical levels of cognition, uses as support one's belief system and retrieved memory based on the latter[31, 32]. Whereas, external communication uses as support a portion of a shared environment, which can be physical or virtual. The shared environment is transformed in a communication medium, i.e. a channel. This medium becomes object of the joint attention of communicating parties. By joint attention we mean that parties simultaneously or one after the other send probing impulses to the medium. The communication medium's state is used as a signal transmitting device. Therefore, the controllability of the medium, in terms of costs for probing it and for switching among states; the cardinality of the set of states; the saliency and distinguishability of states; the maximum frequencies at which the state can be switched and the vulnerability/isolabilty of the medium from noise are all key elements for the success of the communication processes.

Higher order beliefs that frame the understanding and role of uncertainty

As we will illustrate in the second and third section of our review, a large amount of studies by psychologists [33–37] suggest that metacognitive processes are used for the controlled revision of higher order beliefs. Through this review we will explain why, beliefs of commensurability and ergodicity, are key concepts in the economic debate about uncertainty, which profoundly differentiate the Neoclassical and the (new)Keynesian uncertainty analysis frameworks. By commensurability, we do not simply refer to meaning-invariance in relation to the observation of systems' states, dimensions and processes. Which can be defined in terms existence and identification of coherent and invariant reference systems (vocabularies), measures and measurands [38]. In our framework, rather that viewing the commensurability from an epistemic perspective we evaluate it in relation to individual-specific belief requirements, which are the conditions considered jointly necessary for the quantitative integration and/or comparability of specific sets of beliefs associated to sensory experiences.

Commensurability can be seen as a composite belief attribute of sets of beliefs, which represents the faculty of comparing, in quantitative terms, the perceptions of sensory experiences considered distinct, phenomenologically speaking, but integrated in the same belief system. Beliefs are considered commensurable when they are themselves believed to be comparable in terms of, one or more, shared (higher-order) quantitative belief attributes. Where attributes represent original or projected perceptual dimensions of sensory experiences. For example, under this approach, markets and observed (monetary) prices of goods/services, can be used by individuals to reduce the uncertainties elicited for goods/services procuration. Where by procuration we mean the process through which one may get hold of goods/services, and, to dispose of them. Individuals become consumers precisely when they have the possibility to simplify goods/services procuration through the use of a market system that projects the material and immaterial costs to be sustained for the acquisition of goods/services to a unidimensional proxy measure, making the costs of goods/services mentally commensurable through a unidimensional common measure: prices. Without markets and observable prices for goods/services one would be in the situation of having to mentally represent all alternative procuration technologies, i.e. multidimensional combinations of material and immaterial resources, usable to obtain desired services/goods, with ensuing problems of commensurability and uncertainty associated to the elicitation, comparation, choice and utilization of these technologies. Similarly, beliefs of ergodicity concerning those systems in relation to which one wants to formalize expectations, are also particularly important in economics. Where by ergodic, we mean systems whose processes have *"identical time averages and ensemble averages"*[39]. Ergodicity makes sampled observations representative of the system's actual state and phase-space densities, making possible state learning and foreseeing. If one beliefs that some observed human systems are incommensurable and/or non-ergodic, then, observational data concerning these systems' states cannot be, respectively, measured or used to forecast the states of these systems. As we can see, higher order beliefs, especially those concerning commensurability and ergodicity, may deeply affect the meaning and feeling of uncertainty of human agents. These higher-order beliefs are particularly important for economic research, because they make possible the construction of probabilistic expectations, which are used in neoclassical economic modelling of rational decision making under imperfect information.

Information-gap and expectations-gap related uncertainty

Through a review of recent literature we will identify two distinguishable but intertwined types of uncertainty that are particularly important in terms of their economic implications. The first can be seen as the outcome of self-metacognition, whereas the second one is the outcome of group and social metacognition. As we will see, feelings of uncertainty that emerge in relation to expectation-gaps evidenced during group or social metacognition processes, signal the degree of expected surprise conditional on one's expectations and those communicated to the latter by other members of a group, or, by his social network neighbours. Whereas, feelings of uncertainty that emerge in relation to information-gaps during self metacognition processes, signal the degree of surprise caused by new evidence conditional on one's prior beliefs. These two types of uncertainty are both elicited, reduced and eventually resolved within metacognitive processes. As we will document, communication plays a crucial role in such processes. It is through metacognitive processes that beliefs, like probabilistic expectations, are knowingly revised. We hypothesize that uncertainty reduction is a fundamental characterization of human metacognition. Metacognition can be seen in this framework as a way to reduce uncertainty by reviewing beliefs. Our vision is based on recent works by Golman and Lowenstein[40–42]. The axiomatic microfoundations proposed by Golman and Lowenstein can be considered a cognitivist reframing of the concept of utility to englobe the effects of information-gaps and related uncertainty. Inspired by their framework, in the last section we will explored how expectations-gaps related uncertainties may be integrated in EU maximization models. These uncertainties are not considered in the classical EU framework. They can be viewed as a way of integrating many of the ideas behind the Keynesian concept of conventional expectations[43–47] in the Neoclassical EU framework.

Relative-entropy based measures of expectations related uncertainty

We show that, the above illustrated expectations-gaps related uncertainties, can be measured through relative entropy, as formalized by Kullback and Leibler[48, 49]. Kullback-Leibler's relative entropy is a generic measure of the divergence among distributions that can be interpreted as expected surprise when applied to divergence among pairs of probabilistic expectations of future events. Under this framework social-metacognition can be seen as a coordination mechanism through probabilistic expectations: by communicating and reviewing expectations, agents are able to elicit and try to locally reduce, group or social uncertainties related to expectations-gaps, while jointly maximizing their expected utility.

Expectations-gaps related uncertainty as a belief reviewing coordination device

Under this perspective states of uncertainty related to expectations-gaps can be considered individually and collectively informative signals, which, if explicated, can be respectively used to coordinate, at the individual and group/collective level metacognitive processes. When uncertainty states are explicated by agents through language, in communication and deliberation systems, uncertainty acquires a systemic role of coordination device for belief revision. By being able to explicit and communicate (extreme or above tolerance) degrees of uncertainty to their peers, agents can request additional time, evidence and more or less intense and extensive communication to better coordinate their revision of expectations and reduce expected surprise. By so doing they can facilitate the process of reduction of expected surpise, i.e. uncertainty.

An economic re-reading of the role and effects of uncertainty aversion

To conclude, we will illustrate how, in the previously mentioned framework, by maximizing a modified expected utility function, agents can jointly and optimally modify through a unique mechanism:

- Their degree of expectation-gaps related uncertainty levels, or expected surprise;
- Their probabilistic expectations of future events;

Such a mechanism implicitly contains the twofold identity represented by the sentence **having to learn, while learning to learn**. Which in economic terms may be summarized by the following concepts:

- **Expected utility maximizers:** Where, in complement to the standard characterization of expected utility, the more agents are averse to expected-surprise, the stronger will be their convergence towards the group/social probabilistic expectations distribution. This, to reduce the disutility of expected-surprise generated by the awareness of expectations-gaps, in relation to subjective probabilities of events, which is a linguistic partition of the future, communicated by other agents. Where communicated expectations are sets of at least two subjective probabilities of events belonging to a representation/partitioning of the phase/state space of a system;
- **Simil bayesian social-learners:** Human agents, by being able to integrate others' beliefs/expectations in their own, can improve their (expected) prediction accuracy of future events, by minimizing expected surprise conditional on communicated beliefs (priors) concerning the probabilities of future events. In situations where percepts (observational evidence) is distributed across a large-world environment and locally observable, i.e. signals have limited duration and/or intensity, agents, which can be seen as local sensors (subject to noise), will perceive, process and integrate these local percepts in their beliefs. Agents can also (indirectly) learn about events in a large-world through percepts that are not directly accessible to them, by means of a iterated communication of beliefs, in particular expectations, among neighbouring agents. In this framework expectations, which are discrete probability distributions over future events considered possible, are informative representations of locally pre-processed percepts and prior communications of expectations among neighbouring agents;

As we will see, our expectations-gaps uncertainty measurement approach appears to be particularly well suited for empirical studies in those domains of knowledge that have to do with multi agent network systems. In particular, situations in which the pay-off of agents, or their expected utility, may -also- depend on the degree of convergence among agents' beliefs (expectations) and preference relations for hypothesized future events. For example, in those economic and financial empirical applications in which uncertainty concerning expectations, and their distribution across agents cannot be observed directly, but where it plays a relevant role in determining observed outcomes and/or prices, as well as their volatility.

Why have we evolved to feel and communicate uncertainty: on the informative and coordination value of uncertainty

To conclude, we will review main findings and give an intuitive explanation of why, under our perspective, in which uncertainty is related to the degree of expected surprise due to expectations-gaps, the public communication of uncertainty states can be considered a coordination strategy for group or social-network beliefs revision. Especially during those events that jointly change, for a large share of the agents in a group or social network, the foreseen conditional utilities and probability distribution for a large subset of possible outcomes in the event-space. For example, when the United Kingdom's EU referendum results, or, the victory of Trump in the 2016 United States' presidential elections were communicated to the worldwide public.

1.1.1 Summary of objectives and research questions

Our main objective is to outline, through a review of existing works, the characteristics and effects of uncertainty phenomena in relation to markets, expert judgements and communicative/deliberative arenas. In addition, by reviewing literature on the issue, we want to highlight the pros and cons of the methods that have been identified and used to model and measure uncertainty and uncertainty aversion. Finally, we wish to expose and enrich the metacognition-communication uncertainty framework, by integrating it to literature about information-gaps and expectations-gaps, which, under specific conditions and assumptions, allows us to model and measure individual, group and social uncertainty. Here follow our main research questions:

1. How uncertainty has been conceptualized in modern times?
2. What is the relation between choice difficulties, indifference and uncertainty?
3. What is the relation between uncertainty and risk?
4. How uncertainty has been conceptualized in contemporary economics, information sciences and cognitive sciences?
5. Which relation exists between uncertainty and information?
6. What do those formal representations have in common and in which terms do they differ?
7. Can we represent human uncertainty in terms of belief related entropy and relative entropy?
8. How is our uncertainty reshaped through metacognitive processes?
9. Can public communications of states of uncertainty expressed through natural language be used as a signals of expectations-gap related uncertainty?
10. What is the relation between expectations communication, expectations-gaps, group/social metacognition and uncertainty?
11. In which terms individuals' uncertainty is linked to the updating of their beliefs and maximization of their expected utility?

1.2 Conceptualizations of uncertainty in literature

In the following subsections we will highlight the emergence, drifts, transformations and crisis of uncertainty paradigms[50] in economics and neighbouring disciplines, which study the role and effects of uncertainty in human cognition and behaviour. We would like to acquire knowledge about uncertainty states, by identifying a stable reference relation between representations of uncertainty and uncertainty phenomena in the human world. However, the condition for the very existence of -intertemporal- knowledge concerning human uncertainty phenomena, in a factive sense, is that there must exist an objective truth to discover that mustn't be influenced by the observer: some general relation between observed states of uncertainty of agents in isolation, in groups, in networks, or in societies non conditional on the observer, as if the former where closed and isolated systems. The aforementioned feature of knowledge is very rarely observed in practice, in particular with reference to uncertainty phenomena. Because, theories and models of uncertainty do not simply measure states of uncertainty, they try to reduce uncertainty concerning these states by imputing uncertainty to specific causes, for example the existence of noise or random perturbations. In the sections that follow, we will progressively try to explore and illustrate alternative methods and sources, that we can use to extract local knowledge concerning uncertainty states of human agents, that can then be aggregated at the desired level and frequency. Methods which, differently from the EU theory and risk aversion measures, do not seek to extract intertemporal "truths" concerning agents' behaviour when facing uncertainty, but, which can be used to observe contingent states of uncertainty through (public) communication systems. The latter may be used to characterize and describe individual and aggregate uncertainty, when analysed or modelled in the short or very short term, with data aggregated at high (day) or very high (infraday) frequency.

We will start our review by illustrating modern works on uncertainty. Then we illustrate contemporary research on the issue starting from economic research fields, where we highlight differences between Neoclassical and Keynesian theorizations of uncertainties. Finally, we expose studies about uncertainty from the cognitive and information sciences. This second section is an essential building block for identifying the frontiers of existing frameworks for the analysis and measurement of uncertainty in economic studies. Through our review we will outline the common findings and gaps of existing research.
Given the space constraints of this article, the works presented in the following sections are not exhaustive of their fields of research. The amount of work produced in the aforesaid areas is so large that some relevant works may have been omitted, we apologize with authors for eventual omissions. Our aim is to build a review, as much as possible, representative of open or unresolved research questions in relation to market agents' uncertainties, their measurement, and, on the relation between the latter and observed aggregate market phenomena.

1.2.1 On the modern understanding of uncertainty

The reflections on classical and modern economic thought that will follow are only a small window on the economic world and theoretical paradigms of the last four centuries. Despite their non-exhaustiveness, these reflections allow us to show to which extent uncertainty has always been a fundamental aspect of economic affairs, and, associated theoretical representations. We have voluntarily omitted Keynes from this subsection, because, even if he was a contemporaneous of Knight, we consider his thoughts, as well as raised questions related to uncertainty phenomena closely related to very recent economic debates on human uncertainty, and, to the (re)emerging of uncertainty analysis paradigms alternative to the neoclassical risk framework.

Dubium existentiae

The importance of the state of uncertainty for the revision of higher order beliefs and for the reconstruction of knowledge and belief systems was highlighted since the seventeenth century by Descartes. In his *Meditations On First Philosophy*[51] he stated that *"Although the utility of a Doubt which is so general does not at first appear, it is at the same time very great, inasmuch as it delivers us from every kind of prejudice [...] if I am able to find one some reason to doubt, this will suffice to justify my rejecting the whole"*. Descartes had a very stylized vision of his own beliefs, probably due to his radical *"confidence in rationality"*[52]: he considered himself able to control his own propositional attitudes, through his -at will- capability of doubting about them and applying reasoned thought to test propositions and by so update his belief system. However, even though he knew that some of his beliefs were based on *"opinions, in some measure doubtful [i.e. prepositions possessing a propositional attitude belief attribute that is neither true nor false]"*, if these were *"at the same time highly probable, so that there is much more reason to belief in than to deny them" he considered them appropriate to be eventually used as "masters of [his] beliefs"*, and claimed that he would never lose *"the habit of deferring to them or of placing [his] confidence in them"*[51]. Descartes was certainly one of the first to grasp and express the idea that one's propositional attitudes, may admit higher order belief attributes, which not necessarily must be binary. Higher order beliefs trough which one can, for example, represent the truthfulness, the degree of confidence, the plausibility or the probability of a preposition, belonging to one's belief system. Descartes also understood that, by voluntarily doubting, i.e. activating metacognition, one could put under the lens of reason propositions and beliefs, and by so doing, update his own belief system. Under the aforementioned perspective belief systems are used in all cognitive processes. Uncertainty, signals that the belief system is object of potential revision during a metacognitive process. Uncertainty is for this reason described as a state in which an agent is reluctant or unwilling to use his belief system for formulating expectations and taking relevant decisions.

Paradoxon cognitionis et exspectationis humanae

Seventy years later, in 1713, the mathematician Nicolas Bernoulli explicated his uncertainty in relation to the St. Petersburg paradox through a letter[53] written to his friend Pierre Rémond de Montmort. The St. Petersburg paradox is a lottery with an infinite mathematical expectation, valued a limited amount of money, i.e. a finite certain equivalent, by market agents while facing decision-making under risk.
Such a lottery made Bernoulli doubt of the reasonableness of the usage of mathematical expectation for the valuation of price lower-bounds for monetary lottery tickets. When knowledge generates anomalies in relation to evidence and intuitive

thinking, paradoxes emerge together with metacognition. In such situations, humans start doubting, collecting additional evidence, and, if evidence is insufficient to collapse to a coherent and justified representation from which derive an optimal choice, by speculating and simulating on possible representations and models of reality one may find a representation, which fulfils better than the others, choice justification requirements and criteria of the thinker. Bernoulli was certainly a clever mathematician, but, despite his doubting and thinking, he was never able to formalize a solution to the St. Petersburg paradox. However, this paradox revealed to be the cognitive fuel used by the spark of human reason, embodied by Bernoulli's cousin, to bring to light the foundations of the neoclassical theory of decision making under risk: the expected utility hypothesis. In 1738, Daniel Bernoulli[54], cousin of Nicolas, exposed for the first time a mathematical function that discounted, in utility terms, the expected value of lotteries over monetary pay-offs, to account for, and represent, some stylized facts concerning observed bets and human preferences among alternative lotteries, later conceptualized as the risk aversion of individuals. The die of the paradigm of expected utility was cast.

Aporia et indifferentiae oeconomicas

One century later, William Stanley Jevons summarized, in his *Brief Account of a General Mathematical Theory of Political Economy*[55], an extremely refined and modern view of the role of uncertainty in economic affairs. Not only Jevons was a precursor of the quantitative and cognitive turn of economics, he affirmed that feelings are *"quantities capable of scientific treatment"*, but also, he claimed that *"every expected future pleasure or pain affects us with similar feelings in the present time, but with an intensity diminished in some proportion to its uncertainty and its remoteness in time"*. Through his words Jevons grounded the intuition behind the hypothesis that people discount utility on the basis of the granularity and saliency of their (beliefs of) knowledge concerning future situations and their foreseen utility:

- **(I) foreseeing horizon distance:** The more the foreseen horizon is remote, in terms of time distance and prototypicality;
- **(II) complexity and non-ergodicity:** The more a situation appears to be indeterminate, non-ergodic, incommensurable or complex to be mentally represented in terms of utility and probability of events;
- **F(I,II) representativeness and coherence of the belief system:** The harder will result the process of mental speculation, modelling, simulation and accounting of all possible combinations of actions and effects;
- **G(F(I,II)) foreseen utility intensity:** The higher will be the "discounting rate" of the foreseen utilities at a given time/situation horizon;

Jevons had also understood that *"all the critical points of the theory [of economics] will depend on that nice estimation of the opposing motives which we make when these are nearly equal, and we hesitate between them"*. We may call the latter, situations of rational doubt or uncertainty related to the incapacity to consider one alternative strictly superior to all others. According to Jevons, the incapacity to discriminate prospects in terms of utility was seen as a critical point, because it was a possible source of cognitive difficulties in real life, and possibly, random behaviour.
Today, it is considered socially acceptable that, if one is indifferent among a series

of choice alternatives, he may choose by undertaking a random choice among alternatives considered equally good and superior to all others; however, at the time of Jevons, random choice, even between a small subgroup of choice alternatives, still appeared to lack the requirement of justifiability, to oneself and to others, both in moral, volitional and rational grounds. From a rational perspective, when two or more alternatives, superior to all others, are considered equally good, if time constraints are not binding, there is an alternative mechanism to random choice that is strictly superior to the latter from a global efficiency perspective.
The superior solution consists in exploiting the full information extraction potential of the choice, by decomposing it, and, delegating or selling the sub-choice among alternatives considered equally good and superior to all others to someone that is willing to bear it, or even better, to pay for it. If there exists an individual that values positively the possibility of undertaking the sub-choice and will buy it, or accept to undertake it, it means that the latter will be able to extract some additional utility through the same choice. Therefore, by decomposing choices when one encounters a situation of indifference, allows the extraction of additional utility from the initial choice. Utility that would otherwise be lost in case of a random choice between indifferent alternatives superior to all others. This until the full information and utility extraction potential of the choice is "consumed". As a result, if time constraints are non binding, random choices are effectively a-priori non-optimal choices.

At the time of Jevons, if one found himself in such a situation of indifference, choice was generally postponed until one of the alternatives would reveal to be strictly preferred with respect to others. The latter perspective results particularly interesting if we understand that under this conception, indifference was seen as relative/-conditional to beliefs of imperfect information and a too weak or absent volition. Therefore, by undertaking a random sub-choice, the utility, volitional and information extraction potential of the choice is inefficiently used. Such inefficiency may appear small to us, but was considered a very serious waste in a society in which information, as well as resources were extremely scarce; situation that, excluding the last century, has always been the norm in human societies. For this reason indifference, which was considered caused-by unresolved uncertainties, themselves due to weak (choice) volition, lack of information and/or poor cognitive and metacognitive capacities, shouldn't be settled by random choice. Under this perspective, indifference characterizes situations where utility differences among alternatives do not exist in observable terms, however, they may exist in the latent information and utility space that hasn't yet been commensurated by the decision-maker. In such circumstances situations of indifference may be metaphorically assimilated to Pandora boxes, potentially full of unexpected externalities, in terms of (dis)utility and sub-efficient information extraction. The latter situations, can be also resolved through communication or additional information retrieval, to explore the latent information and utility space of oneself and others hence involved in the decision, refining the grain of human representations and knowledge of each other. In general, through communication and information retrieval, preference relations that may initially appear to be indifferences become, sooner or later, strong. Indifference can hence be a signal of information and utility extraction global sub-efficiency, which generally emerges in relation to situations of decision-making under uncertainty: when one undertakes a choice among alternatives temporally or cognitively very remote compared to contingent, observed or experienced states of the world, his representation of the state-space will likely exhibit poor granularity. If one uses a thick grained representation to commensurate differences in the foreseen utility among alternatives,

alternatives may appear as equally good/preferred only because the information set used to compare them is too little to distinguish their actual differences in terms of utility. Knight, whose thoughts on uncertainty will be mentioned in detail further in this section, also discussed this issue and considered it one of the channels through which the economic effects of uncertainty phenomena were amplified and propagated through markets[1].

Last but not least, Jevons anticipates, through a written intuition, some findings from a field of economic research, related to uncertainty and expectations, that will emerge more than a century later, that of anticipated utility[56–60]. He claimed that: *"we must carefully distinguish actual utility in present use from estimated future utility, which yet, by allowing for the imperfect force of anticipation, and for the uncertainty of future events, gives a certain present utility"*. In his view, utility could be anticipated by individuals precisely through the foreseeing of future utility. We will return to this issue later on when we will discuss the advantages and disadvantages of foreseeing the future and formulating probabilistic expectations of future utility levels.

Adaequatio intellectus nostri cum re

Close to the turn from the nineteenth to the twentieth century, William James, a radical empiricist that laid the foundations of modern psychology claimed[61] that the concept of consciousness was *"on the point of disappearing altogether"*. According to his view, consciousness was about to be rendered, by modern science, epistemically obsolete through direct empirical observation, measurement and analysis of perceptual phenomena. James questioned the very need to continue to assume the existance of consciousness, as a necessary tool to explain doubt and uncertainty, in line with this thought, he considered consciousness a non-entity[62].

William James had identified a more practical role for doubt and uncertainty, closely related to theoretic rationalism. According to James, consciousness was a casing layer for world-views based on theoretic rationalism, which he described as the *"the passion for parsimony, for economy of means in thought"* and the ensuing *"habit of explaining parts by wholes"*[63]. Under this perspective, rational world-views are used to give relief to the overwhelming process of empirical contemplation of the *"richness of the concrete world"*, through which humans' percepts incessantly inspire a multiplicity of inconsistent views of the world, with *"little pictorial nobility"*. According to James, rational world-views, emerge and are revised for practical inter-individual and intertemporal coherence purposes, linked to people's preferences to jointly reduce actual and expected surprise, i.e. uncertainty.
James related uncertainty to the feeling of unrest, surprise and uncontrollability, and claimed that, through rational world-views, humans can temporally *"banish uncertainty from the future"*, and by so doing, their *"feeling of strangeness disappears"*. James represented rational sense-making as a distributed and modularized form of theoretical substitute for perceptual reality[64], which could be used to limit and reduce uncertainty created by empirical analysis and reflections on new percepts. Rational sense-making alleviates extreme surprise because individuals *"come back into the concrete from [rational] journey into these abstractions, with an increase both of vision and power"*[63]. According to James single perceptual *"experience in its immediacy seems perfectly fluent"*, it is in relation other percepts and their joint explanation that uncertainty emerges. Therefore, rational sense-making can be considered a process of ex-post imputation of percepts, or empirical evidence, to modularized theoretical

representations, related to the objectification of percepts, in beliefs and knowledge systems, through a rational sense-making framework[65]. Its difficulties are represented by regret and residual uncertainties, which are themselves attributed by rationality to exogenous factors and noise.

If we transpose James' view of rational sense-making to contemporaneous economic, cultural and scientific environments and institutions[66] and their specialization by functional fields of knowledge, theoretic rationalism can be considered an instrument used to reduce human uncertainties by minimizing the duration, extension and diffusion of rational world-views reviewing processes when new empirical evidence is collected by agents and groups. The former process is undertaken by attributing actual and expected uncertainty conditional on new evidence, to existing theoretic representation modules, eventually reviewed, when considered rationally convenient, for the reduction of actual and prospected surprise, i.e. uncertainty. The task of maintaining, communicating and eventually reviewing theoretic representation modules is entrusted to individuals and groups experienced and functionally specialized in the reduction of uncertainty generated by percepts in a specific area of the perceptual-space; and eventually, if uncertainty resolution is impossible, to exogenous factors and noise.

Cogito incertum et opus incertum

In 1921, Frank Knight[1] dedicated the third part of **Risk Uncertainty and Profit** to the analysis of the *"conditions of existence"* of uncertainty in economic affairs. According to Knight, uncertainty in economic affairs encompasses the problem of estimation of the expected utility of alternative money usages/allocations. He is one of the first to expose a systemic and epistemological representation of human uncertainty phenomena. In his writings he claimed that such a state is intrinsic to the application of the *"dogma of science"* to the understanding and anticipation of cause-effect relations in a indeterministic and ever changing universe, which enables the possibility of incurring in a *"knowledge paradox"*, i.e. a state of radical uncertainty:

> *"Change of some kind is prerequisite to the existence of uncertainty [.. and] change in some sense is a condition of the existence of any problem whatever in connection with life or conduct, and is the actual condition of most of the problems of pure thought[...] The existence of a problem of knowledge depends on the future being different from the past, while the possibility of the solution of the problem depends on the future being like the past.[...] The point for us here is that change according to known law does not give rise to uncertainty.[...] But the process of formulating change in terms of unchanging "laws" cannot be carried to completeness, and here our minds invent a second refuge to which to flee from an unknowable world, in the form of the law of permutations and combinations. A law of change means given behavior under given conditions. But the given conditions of the behavior of any object are the momentary states and changes of other objects. Hence the dogma of science, that the world is "really" made up of units which not only do not change, but whose laws of behavior are simple and comprehensible. But it is contended that there are so many of these units that the simple changes which they undergo give rise to a variety of combinations which our minds are unable to grasp in detail. We have examined this dogma and been forced to the conclusion that whatever we find it pleasant to assume for philosophic purposes, the logic of our conduct assumes real indeterminateness[...] Real indeterminateness, however, gives mind a new means of prediction, through grouping phenomena into classes and applying probability reasoning. This device enables us to*

predict what will happen in groups of instances[...] this method also has its limits. Both methods in fact, prediction by law in individual cases and by probability reasoning in groups of cases, have rather narrow limitations in everyday life in consequence of the organic costs of applying them and the time required to get the necessary data; both outlay and time are commonly much greater than circumstances will allow us to consume in deciding upon a course of action"[1]

The indeterminateness mentioned by Knight is not necessarily intrinsic to the *physical world* but results from beliefs that are the outcome of interactions and coordination among agents, and between the latter and their local information environments. In addition, Knight claims that certain factors like the *"inflexibility of prices, due to habit, indifference, rounding off of figures"* may *"aggravate the effect of uncertainty"* and disturb the adjustments towards theoretical market equilibrium conditions, as well as the functioning of market clearing mechanisms themselves.

1.2.2 Uncertainty in contemporary economic literature

Since the formalization of the Von Neumann-Morgenstern (VN-M) expected-utility (EU) hypothesis[67], the economic study of uncertainty has been superseded by that of risk. In the VM-N EU theory, risk characterizes environments with imperfect information concerning the states of world. It is a problem of optimal inference in a resource and information constrained environment. Through our review we will briefly describe the EU theory in its objective and subjective probability version, as well as their violations, like the paradoxes identified by Allais and Ellsberg, together with some extensions and revisions, like cumulative prospect theory[68]. As we will point out, the VN-M EU theory has contributed to the advancement of almost all disciplines that study uncertainty from the risk perspective, ranging from finance to engineering. However, uncertainty phenomena and its analysis is not confined to the risk framework. Therefore, to enrich our understanding of uncertainty, we will also review alternative theories and representations of uncertainty in decision and sense making. We summarize the main differences in the hypothesis of these theories of uncertainty, with particular attention to those proposed in recent times in Neoclassical and (new)Keynesian economic works. As we will see, the search lines that emerged from the Knightian/Keynesian and the Von Neumann Morgenstern-Savage paradigms are extremely different, also within each framework some relevant differences exist and will be highlighted in the following sections. Since the former theoretic frameworks are based on non perfectly translatable assumptions and reference-systems, they are not necessarily comparable and integrable.

The expected-utility framework and its normative effects on uncertainties

The Von Neumann-Morgenstern expected-utility (EU) hypothesis[67] is a formal theoretical framework used to describe, compare, aggregate and compose risky prospects, and to evaluate agents' preferences among them. It is based on Kolmogorov's first order probability axioms[5, 6, 69] and on an axiomatic representation of market agents as resource constrained objective function maximizers[70]. In this framework uncertainty is seen as a consequence of randomness or chance. Randomness is definable as a indeterministic perturbation or noise affecting a system or a process. However, behind random noises and stochastic perturbations may be hidden some

non-linearities or chaos in the dynamics of deterministic systems[71, 72], which, by being open, or, computationally too complex or too costly to commensurate, are represented, for convenience or necessity, as random noises or stochastic processes. One may be unaware, or, deny for convenience or epistemic strategy[73], that the *"fog of randomness"*[74] in his perceptual experience may (also) be due to random noise or stochastic perturbations in his sensory and cognitive states, while probing a system through observation.
Therefore, not necessarily randomness in a representation is isomorphic to the phenomenon it refers-to. For example, one may consider randomness:

1. As a characteristic of a system/process observed in isolation, under all possible conditions, or, under specific conditions, of the observed system/process;
2. As a characteristic of an observed (non isolable) system/process, in relation to its, conditional or unconditional, coupling with other systems/processes;
3. As a characteristic of the observed system/process in relation to its observer(s), under specific conditions, or, under all possible conditions, of the observer(s), of the observed system/process, or, of the observation process;
4. As a characteristic of the observer(s) in relation to the observed system/process, under specific conditions, or, under all possible conditions, of the observer, of the observed system/process, of the observation process;
5. As a characteristic of a set of observations in relation to the observer(s), under specific conditions, or, under all possible conditions, of the observer, of the observed system/process, of the observation process;
6. As a characteristic of a set of observations in relation the observed system/process, under specific conditions, or, under all possible conditions, of the observer, of the observed system/process, of the observation process;
7. As a characteristic of a set of observations in relation to, the whole reference population, or, the complement of that set;

These distinctions are not trivial from the point of view of possible effects resulting from the imputation of the belief attribute of randomness to a specific representation of a phenomenon. In addition, one may also mentally represent randomness as a particular combination or composition of the above characterizations.
For example, if one accepts, by default, the hypothesis *(1)* that noise or stochastic perturbations characterize an observed system/process, independently from the observer(s) and from the observation processes, a belief-mantle of objective external randomness shapes the representations of that system/process. This belief-mantle, can push one to consider the dynamics of such system/process a-priori indeterministic. Under the acceptance of any (non degenerate) randomness hypothesis, future states of the world cannot be forecast with certainty. Agents who attribute a randomness belief to their representations of a system/process, may however have the necessity or preference to foresee possible states/outcomes of a system/process, and, through the latter, make a-priori rational decisions under randomness-related uncertainties. The EU hypothesis is a normative framework, which formally describes a way of representing and dealing with the aforementioned decision-making situations.

In works belonging to the objective EU framework, uncertainty is assimilated to a particular type of randomness, called chance and linked to the frequentist conceptualization of probability[75, 76]. In this framework the facing of risky prospects is considered an objective fact. Risky prospects are lotteries/gambles over known monetary outcomes, with known probabilities. Where objective probabilities are limiting relative frequencies of the outcomes of an idealized infinite repetition of the same statistical experiment, i.e. the lottery. The only information that is subjectively commensurated in the objective EU framework is one's utility function, all other things being seen as external, objective characterizations of a system/process whose state/outcome is uncertain only to the extent that it is assimilated to a random draw from a known probability distribution, over the space of possible events/outcomes. In the objective EU framework, agents try to behave a-priori optimally, from a normative rationality perspective, by allocating their resources among available lotteries in such a way that, the resulting composite lottery, is the preferred probability distribution among all possible ones, over one's future earnings, i.e. the distribution that maximizes expected utility.

In situations where:

- One faces a choice among known lotteries, or, a money allocation choice among known lotteries;
- Probabilities of monetary outcomes for each lottery are objectively known, or, believed to be objectively known in terms of physical propensity of a phenomenon;
- One is able to commensurate the utility of any possible monetary outcome of original lotteries and all possible composite lotteries;
- The same choice among lotteries is repeated a very large or infinite number of times, to "activate" the law of large numbers;
- There is no path dependency from a choice to another, i.e. money and utility is not transferable across lottery choices;

Choosing how to allocate money among lotteries on the basis of the axioms of rationality is, in objective terms, an optimal rational strategy to pursue if all the above conditions are met. Unfortunately very few real-world situations of uncertainty comply jointly to these conditions. To extend its applicability, if physical theory justifies it, one may switch from a frequentist view to a physical propensity view of probability[77], which renders, in rational terms, EU maximization an a-priori optimal strategy even if the same choice is undertaken once: when the physical propensities of the considered phenomena are (hypothesized to be) known, objective EU maximization becomes rationally optimal also if applied to single events. To make the EU theory even more "adherent to reality", in terms of its representativeness of a larger number of real-life situations of uncertainty due to imperfect information, assumptions concerning the objectivity of probabilities can be further soften through the, so called, subjective probability framework.
In works belonging to the subjective EU framework[78, 79], pioneered by Leonard J. Savage[80] in 1954, the probabilities of occurrence of possible outcomes/events are subjectively commensurated and represent the belief attitude vis-a-vis a given phenomenon[78]. Probabilities in such setting must still be comparable, numeric, exhaustive and comply to Kolmogorov's axioms of probability. As pointed out by

Suppes[81], in the subjective EU framework *"probabilities are measures of degree of belief"*. Uncertainties can be therefore transformed in risks by associating probabilities to foreseeable events. In the subjective EU framework, uncertainties are jointly commensurated and projected to a consistent representation through a probability space, and then, collapsed to an optimal choice through the maximization of an objective function. Objective function that represents the expected utility of an agent, which is given by the sum of the utility of each possible event/outcome multiplied by its subjectively commensurated probability.
In the years that followed the axiomatization of the EU hypothesis, the risk framework was object of a huge number of accrual contributions, in particular after its subjectivist turn. The subjective EU framework rendered risk phenomena something that extends from the physical world to the cognitive and metacognitive domain of mental representation and commensuration of probabilities. As a result of the academic attention devoted to this framework, a great amount of empirical and experimental evidence was collected to test the EU hypothesis, systematic deviations from the theory and violations were observed and explained:

- **The so called EU paradoxes[82, 83]-** ***for example:*** The Allais paradox, which revealed inconsistencies of lottery choices in the vicinity of certainty that violated the independence principle of Savage; the Ellsberg paradox, which showed that not all uncertainties are representable in the EU framework, because, regardless of one's utility function and ensuing risk aversion, all individuals appear to prefer lotteries with precisely known odds. They are averse to lotteries with partially-specified or ambiguous probabilities. Through these paradoxes, the existence of more radical and higher order uncertainties, which cannot be assimilated to risk, were revealed;

- **The so called EU fallacies[84–88]-** ***for example:*** Samuelson's fallacy of large numbers, as well as its extensions and revisions, demonstrate that, unless there is no path dependency and the same choices among lotteries are repeated a large/infinite number of times, normative rational choices are not a rational decision-making instrument: maximizing the geometric mean of utility outcomes in long sequences of investing or gambling is not an optimal choice for maximizing one's utility in expected terms;

- **Evidence against the EU hypothesis assumptions and implications[89–96]-** ***for example:*** There is evidence that expected-utility theory makes incompatible predictions about the relationship between risk aversion over modest stakes and that over large stakes, and that therefore, it doesn't provide a plausible and coherent account of risk aversion over all scales of stakes; a large number of experimentally observed inconsistencies with the EU hypothesis, in particular choice reversals, have been identified by psychologists and mapped to a series of possible heuristics used to choose among lotteries, some of which will be described later, and biases, explainable in terms of non-observable cognitive and computation costs, indifference, misunderstandings and misrepresentation of incentives and probabilities;

Despite critiques to the EU framework, both as explanatory and normative theory of human decision-making under uncertainty, its axiomatic and probabilistic foundation became the most common formal-language used for risk-analysis and decision-making under imperfect information. Nowadays, a great number of highly-qualified

professional categories systematically employ it, as a formal representation and belief frame, in decision-making and sensemaking processes under uncertainty; making our human world look like an ever-growing risk society[97–107]. As a result, on a daily or infra-day basis real-world uncertainties are quantified by risk experts, analysts and automatized algorithms, who, on the basis real-world real-time data, commensurate subjective probabilities of events, and, represent real-world situations as lotteries over an (imagined or inferred) outcome/event space. Outcome/event space that can also be multidimensional, and, which can range from monetary pay-offs to number of civil casualties per square mile. Multidimensional outcomes are then collapsed through a multi-criteria objective function, to identify the system governance/control strategy whose outcome is preferred by the decision maker, given his elicited choice criteria. The subjective EU framework has become, thanks to its elasticity, the dominant building-block of representations and justifications of the behaviour of human agents not only under risk, but also under uncertainty.
Expected utility, together with its extensions, generalizations, concepts and measures of risk aversion, has reached, in the past half century, such a degree of diffusion among human agents and organizations that it has become the conventional uncertainty representation paradigm[10]. Subjective EU theory has also been employed and extended, sometimes inappropriately, to fit to practical purposes that go far beyond the field of economic research and theorizations: by banks[108], brokers[109], insurance companies[110], investors[111], central banks[112], financial market vigilance authorities[113], managers[114] and judges[115]. The usage of risk-aversion related concepts in a court ruling case[115] is probably the most revealing example of the diffusion of the aforementioned framework and emergence of an ideological risk-frame used to confront with uncertainty and its effects: in 2013, the Court of Appeal of Milan ruled that a bank, operating in Italy, had to reimburse one of its clients for an investment gone bad, because the investment, a swap contract, accepted by the client wasn't compatible with his degree of risk aversion, even though the latter had read and signed the contract. The bank's fault consisted in the omission of a formal process for the elicitation of the degree of risk aversion of the client before proposing such a *high-risk propensity preference* investment. As the above example illustrates, what was at first only a hypothesis has progressively become a normative theory[79, 116–119] and a modeling convention that can be used to ground and justify actions and decisions with uncertain outcomes on basis of the axiomatic foundations of neoclassical economic theory, the so-called rationality axioms, and, resulting claims of rational optimality, or, of deviations from the latter.

We hypothesize that the EU framework, as well as other risk theories based or inspired by the EU hypothesis, became so widely acknowledged and employed by analysts, market agents and judges, not only because they are believed to be an adherent-to-reality formal representation of the behaviour of human agents while facing risks; but also because such a framework can be used as a metaheuristic to actually face uncertainty, and, face stakeholders and people that bear externalities, when having to justify a choice under metacognitive uncertainty. We hypothesize that the subjective EU framework, its extensions, as well as other normative theories of optimal decision making under risk or uncertainty, can be used as a metaheuristic for:

- **Information extraction, elicitation and signalling conditional on beliefs of rationality and common knowledge of probability spaces.** ***For example:*** to infer the degree of (absolute, relative) risk aversion from observed choices under (known) risky prospects, conditional on the EU hypothesis being true and knowing the probability space used by an agent; to measure the probability of observing a specific set of evidence (choices) made by an agent conditional on the EU hypothesis being true; to signal rationality through choices under risky prospect uncertainty;

- **Reducing the costs of decision-making and justification, under any type of uncertainty, by bringing a decision into the subjective EU framework.** ***For example:*** by simulating or randomly generating missing information that is required to use the EU framework, like a subjective probability space, and/or, a choice space, and/or, an objective function. Once an agent disposes of this information he can choose, as a EU maximizer would do, and simultaneously, has all the information necessary to explain, to himself and to others, the rational grounding arguments in favour of his action;

- **Create ex-post rationality illusions, or, narratives of rationality for convenience and justification purposes.** ***For example:*** to justify a decision, or, to hide information concerning the probability-space, utility function or criteria an agent actually used to choose among alternatives, by identifying all the possible combinations of utility functions and subjective probability spaces that can justify a posteriori a decision on the grounds of its compatibility with the EU hypothesis; to infer, a posteriori, which combinations of probability spaces and utility functions maximize the probability of EU hypothesis being true given an observed (set of) choice(s) that have been previously undertaken;

Rabin[94] pointed out that theories of risk attitudes can reveal to be useful procedures, or metaheuristics, for reducing or neutralizing risk aversion. In addition, as remarked by Painter[120], the risk framework shifts the frame and attention of decision makers and stakeholders away from the belief that *"decisions should be delayed until conclusive proof or absolute certainty is obtained (a criterion that may never be satisfied), towards timely action informed by an analysis of the comparative [expected] costs and risks of different choices and options"*, which is somehow related to the point made by Javons, and described in the previous subsection, in relation to what indifference, in economic choices under imperfect information, signals and represents.

Beyond expected-utility: On the commensuration of uncertainties through first order probability frameworks

To our knowledge, the fact of being used as a metaheuristics in real-world affairs, concerns -almost- all contemporary theories of decision-making under risk or uncertainty developed in economics, finance and game-theory. This because, the paradigm of rational decision-making represents uncertainty as a fact, caused by imperfect information or randomness, which generates some extra (utility) costs to bare, measured, in the subjective or objective probability framework, in terms of the distance between the mathematical expectation and the certainty equivalent of a bet for an agent, in case one is risk averse and all available prospects/strategies are risky. As a result, risk studies that have a rationally-optimal decision-making frame, try to go beyond the identification of elicitation mechanisms or measures to quantify agents' uncertainties, and, seek to explain the processes through which, despite the existence of such uncertainties, their consequences, in terms of the psychological and physical costs associated to the foresight/anticipation of expected utility by risk averse agents, can be limited, as much as possible, through the identification of a-priori rationally optimal strategies, in relation to available choices and information. This can be done in a multiplicity of ways, for example: by diversifying resource allocation among risky prospects so that in all *states of the world* the pay-off is (foresight to be) the same; by insuring against the occurrence of specific events with extreme pay-offs by transferring these risks to other agents for which the certainty equivalent of the gamble/lottery is higher.

Under the above illustrated perspective, the commensuration and reduction of uncertainty manifests itself also through the use of other types of decision-making heuristics[95, 121–127] considered alternative or complementary, in explanatory and normative terms, to the EU framework. The latter decision-making rules and methods, like the rule of thumb[128] or max-min[129], do not necessarily give as output the first best solution to a normative expected utility maximization problem and are not necessarily immune to systematic biases, but, may be used to abbreviate and simplify complex decision-making processes under uncertainty through simpler or more intuitive inference procedures. Heuristics are evolution, imitation or experience sourced techniques, well suited to rapidly choose an action/strategy in dynamic environments in which windows for action under imperfect information may appear at unforeseeable moments in time and have a limited duration, or, when the possibility of successfully implementing an action/strategy negatively depends on the amount of time required to infer that action/strategy.
Before going further in our analysis of economic literature concerning economic theories of uncertainty and their implications in terms of optimal behaviour, some further clarifications on the concepts of objectivity and subjectivity, in relation to evidence, probability and uncertainty commensuration must be done. According to Knight[1], to be able to associate the attribute of objectivity to numeric probability distributions of future states/outcomes of a system/process, one should have an a-priori perfect knowledge concerning the unknowability of the factors and not simply the facts of ignorance, in relation to the system or process being observed. Where explanatory factors, if jointly known and not-ignored, can be used to deterministically infer, and causally explain, facts. To consider a numeric probability objective, the unknowability of explanatory factors should prescind from observers and from their processes of observation, being an external property of a system or process. In the aforesaid epistemic circumstances, where unknowable factors are known, and hence, isolable from knowable factors, limiting distributions of frequentist probabilities[76], i.e. relative frequencies, are perfectly informative isomorphisms of the indeterminate states/outcomes of such systems/processes. These should be considered human mind independent characterizations of the world. In such circumstances, one could talk not only about objective randomness, in terms of its independence from the observer, but also, about physical probabilities, given their hypothesized independence from observers and observation processes.

If the latter epistemic condition is verified, once the objective probabilities of states/outcomes of a system/process are known or assumed to be known, probability reasoning becomes a tautological activity, because an objective probability distribution must be by definition unconditionally true[130], therefore from an epistemic perspective the objectivization of probability knowledge renders the latter self-referential. When objective probability distributions, which describe the joint effects of all known unknowable explanatory factors on the state/outcome of a given physical system/process, are assumed to exist and to be known, they become signal-noise isolation and commensuration devices, used for the measurement[131, 132] of the influence of knowable and commensurable explanatory factors on the state/outcome of a system/process. The aggregate effects of known unknowables on the state/outcome of a system/process are implicit to the shape of the objective probability distribution. Objective probabilities are assumed to represent, completely and perfectly, potential knowledge concerning the joint effects of disturbances due to all known-unknowables and their interactions, on the state/outcome of the system/process. By considering probabilities as objective and objectively known, new collected evidence concerning the state/outcome of a system/process and its knowable explanatory factors, is by assumption considered irrelevant for the updating of probabilistic knowledge concerning the effects of unknowable factors. In a specular way, deviations from expected states/outcomes of a system/process conditional on knowable explanatory factors, are considered random noise or stochastic perturbations. As if, the residual distance between what is expected on the basis of all knowable explanatory factors and what is observed should be, by construction, attributed to random draws from the objective probability distribution, from which no additional information or knowledge concerning knowable explanatory factors can be extracted. For this same reason, Knight claimed that *"if the real [objective] probability reasoning is followed out to its conclusion, it seems that there is "really" no probability, but certainty"*, because objective *"knowledge [of the unknowability of explanatory factors] is already complete"*[1] and perfectly represents known-unknowns in terms of their joint effects on the state/outcome of a system/process. Because objective probabilities are an isomorphism of the state/outcome indeterminacies of a physical system/process. In such situations where probabilities are, or are assumed to be, objective, the principle of cogent reason overshadows that of insufficient reason, and, objective probabilistic knowledge represents completely and perfectly potential knowledge concerning the effects of known-uknowables on the states/outcomes of a system/process. Therefore, what one may learn through observation of a system/process in such circumstances, is conditional on a prior separation between known-knowables and known-unknowables, whose effects are separated through, and thanks to, objective probabilities and their distributions.

Despite the epistemic value of the search for objective probabilities we should always remember that even if we, the humans, are embedded in a common physical universe of which we ideally seek to acquire common and objective knowledge, our uncertainties, in relation to the former, emerge in our mental and conceptual spaces, which are not necessarily overlapping among agents: we become aware of, and feel, uncertainties in relation to both internal and external phenomena, through the emergence of aware thoughts during metacognitive processes. Even though some physical characterizations of human uncertainty have been experimentally identified and isolated in neurological studies, to our knowledge no evidence has been yet found against the hypothesis that: uncertainty feelings emerge, are commensurated and eventually reduced or resolved, through, and conditional on, metacongition.

Since uncertainty is, jointly, a phenomenon characterizing the mind and the brain, the objective probability approach, which is constructed to prescind from the observer and his subjective beliefs, is not necessarily the optimal road-map to the observation, commensuration, measurement, analysis and understanding of human uncertainty phenomena.

On the other hand, subjective probability spaces can be considered explicitable and implicitable elastic cognitive and metacognitive instruments. Human mind can generate these instruments to respond to the preference for coherently and formally representing one's own and others' beliefs and quantifying and comparing uncertainties in relation to the former. Subjective probabilistic quantifications of uncertainties are beliefs representation or projection systems, which, being inspired by probability theory, should have as conditions the beliefs of commensurability, comparability and exhaustivity of the uncertainties concerning propositions and beliefs represented through this framework. Several theories of subjective probability exist, among which the bayesian approach to subjective probability[133–135] is certainly the more rigorously oriented to learning. The bayesian framework can be used for the rational updating of prior beliefs when new observational evidence is available[136, 137]. The conceptualization of subjective probability that we will adopt in the sections that follow, is somehow halfway between the bayesian view and that used in psychological experiments[138], where subjective probability is assimilated to the *mental expectancy*[139–141] of a phenomenon. In general, outside the bayesian framework, only first order subjective probabilities are considered to be mentally commensurable by individuals in real life: events' subjective probabilities are elicited as if they were real numbers between 0 and 1. If the aforesaid hypothesis is true, two options are possible, either first order probabilistic mental representations of uncertainties prescind from higher-order and radical uncertainties, if any, or they are low dimensional uncertainties projection systems, in which first order subjective probabilities are eventually distorted by higher order or radical uncertainties. In support of this second view, it has been found that when individuals are asked to elicit first order subjective probabilities and attribute equal probability values to all possible states of the world, the so called " fifty-fifty" probabilistic expectation when there are only two possible states, these probabilities do not represent anymore (simply) the numerical subjective expected relative-frequency or expectancy of the occurrence of these states, but a different type of higher order uncertainty attribute associated to the whole probability space, called epistemic uncertainty[142]. These higher order and radical uncertainties and their effects will be further discussed in the next sections.

Subjective probability spaces can hence be used to commensurate and elicit uncertainties in relation to specific sets of beliefs concerning the states/outcomes of a target system/process, and eventually, to decide under which circumstances and through which mechanisms beliefs concerning the former system/process should be revised. Moreover, differently from risk aversion, uncertainty feelings associated to probabilistic representations of systems/processes are dynamic. Uncertainties may change in terms of their degree and identified sources during metacongitive processes. As we will explain later in this work, belief revision during metacognitive processes largely depends on new evidence collected through communication with other agents and with the environment. The higher is the compatibility between new evidence and prior beliefs, the smaller is the actual surprise generated by the communication of such evidence, and, the lower will likely be the uncertainty in relation to the latter, and hence, the contingent pressure to review beliefs. Similarly, the higher is the compatibility between expectations communicated by others and prior beliefs on the issue of the recipient of the communication the smaller will be the surprise generated by the communication of these expectations, and, the lower will likely be the uncertainty (expected surprise) in relation to the latter, and hence the pressure to review probabilistic expectation related beliefs. Shackle[143–145] was probably the first economist to link the feeling of uncertainty to surprise, conditional on prior beliefs. According to Shackle[143], actual surprise is *"what we feel* *when an expectation has gone wrong"*, whereas potential surprise is *"the degree of difficulty which an individual has in banishing [an hypothesis concerning the future] from his mind"*. Shackle's intuition on the link between uncertainty and surprise will be further discussed in the information theory section, in relation to information entropy and relative entropy measures.

Given the simplicity, elasticity and coherence of first order subjective probability theory, this framework has been used in several theories of rational decision-making under uncertainty, many of which prescind from the expected utility hypothesis[57, 59, 146–151], or, relax and change some of its assumptions[152–155]. One of the most promising alternative frameworks for the representation of uncertainty in rational decision-making is called regret theory[156–160]. Regret theory, in its original formalization[158], is also based on subjective probabilities and expected utility maximization, however expected utility is conceptually and axiomatically formalized differently. In this framework, also inspired by Bernoulli's work on psychological anticipation, agents foresee and actualize possible future rejoicing or regret, due to the consequences of the undertaking of an action conditional on the consequences that could have occurred under alternative actions, for all possible states of the world. Differently from the standard EU framework actions are represented by n-tuples of consequences, where n is the cardinality of the set of states of the world. A modified utility function allows to represent the negative utility impact of anticipated potential regret, and, the positive utility impact of anticipated potential rejoicing. Similarly to the classical EU framework, the optimal action is the one that maximizes the sum of the product of modified utility and subjective probability for all states of the world.

From imprecise probabilities to radical uncertainties

It could seem a twist of fate, but, the notion of imprecise risk and probabilities entered public debate in the sector, that of nuclear energy, and the country, the United States, where one would have hoped that statisticians, together with engineers and physicists, would have been able to infer, through their "hard science" knowledge and methods, non-disputable first-order probabilities, and error bounds, of nuclear facility failures, and, their possible consequences in terms of probabilistic distributions of fatalities, per facility, per unit of time. In 1953, the statistics director of *General Electrics* submitted a memorandum, titled *The Evaluation of Probability of Disaster*, which proposed a methodology for the commensuration of probabilities of chain events that could culminate in a nuclear disaster. Twenty years later, in 1974, the *US Nuclear Regulatory Commission* published its *Reactor Safety Study*, called *WASH-1400*, which estimated, in probabilistic terms, the risk of an early human fatality due to a hundred nuclear power plants in the United States to be $2 * 10^{-10}$ per year[161]. *WASH-1400* was acclaimed as one of the best risk assessments ever accomplished[162]. Through this statistical work, probabilities of fatalities due to Nuclear disasters where inferred and compared, in terms of expected fatalities, to other risks, more familiar to the US public. Risk benchmarks ranged from fires, to air crashes and hurricanes.

A few years later, the Lewis Committee was commissioned by the US government to review the study's conclusions by analyzing the error bounds for the estimated probabilities reported in the latter, the report of the committee states as follows: *"we* *are unable to determine whether the absolute probabilities of accident sequences in WASH-1400 are high or low, but we belief that the error bounds on those estimates are, in general, greatly understated"*[163]. The question of if, when and how to commensurate and represent these higher-order uncertainties was raised, and submitted to the evaluation of US's civil society and its scientific community.

The Lewis Committee claimed that *"the spectrum represented by that team [the RSS that authored WASH-1400] was not broad enough to encompass the full range of scholarly opinion on the subject. This led the RSS team to make estimates with a narrower range of stated 'uncertainty' that would otherwise have been the case"*[164]. The RSS team was criticized for not being able to speculate sufficiently on the effects of scholarly known unknowns, concerning possible causes of nuclear accidents and their effects, and by so doing, having failed to elicit, acknowledge and communicate the full range and degree of uncertainties concerning the outcomes of their nuclear risk analysis, in particular in relation to the degree of imprecision of inferred probabilities of fatalities.
The degree of imprecision of these probabilities had been understated because of the objective difficulty and impossibility, for the RSS team, to elicit and commensurate all knowable unknowns related to nuclear disasters[165]. In particular, those higher order uncertainties that would have emerged in relation to the speculation on possible causal chains that may produce as outcome a system failure resulting in a nuclear accident. But also, those uncertainties concerning the effects of nuclear accidents, in terms of probabilities of human fatalities. This because these estimates depend on behavioural hypotheses on the reactions of human agents, inside and outside the nuclear facility, once they become aware of the nuclear accident[166]; as well as, on estimates of the health impact of nuclear radiation on human bodies. The aforementioned higher order uncertainties should have been propagated through the inference process, and, should have affected the error bounds of inferred probabilities of fatality, per nuclear plant, per year; but, were ignored and hence rendered invisible in final estimates of first-order probabilities of fatalities and their error bounds. Numerical probabilities are salient information[167, 168], which can overshadow higher-order uncertainties and non-commensurated risk factors[169]. First-order probability spaces grant to commensurated risk factors and elicited unknowns the perceived quality of internal consistency, coherence, completeness and precision[170]. For the aforementioned reasons, inferred probability-spaces can result extremely useful to oust from people's mind residual uncertainties and worries concerning risk factors that may be difficult to commensurate. This because residual unknowns are masked by the apparent completeness, precision and hence reliability of these inferred numerical probabilities[171, 172]. Furthermore, by assessing and representing risks through first-order probabilities over a predefined outcome-space, known knowns and known unknowns are froze, and, clearly separated from residual unknowns. Residual unknowns which may be either unknown unknowns, or, non-considered knowable unknowns, or, non-elicited knowable unknowns.

The fact of considering first-order numeric probabilities as lower-level projections, or expected values, of their latent higher-order counterparts, becomes particularly controversial when the shape of the higher-order distributions of the latter are not known, or, cannot be safely assumed or inferred. For example, a second-order probability with a continuous uniform distribution on the interval zero-one would have the same expected value of a second-order (degenerate) probability with all its mass on the value 0.5, clearly these two second-order probabilities describe very distinct states of epistemic uncertainty which, if possible, should be clearly distinguished.

Moreover, if these latent higher-order probabilities are assimilable to degenerate random variables only conditionally on commensurated risk factors, but not also, on non-commensurated risk factors, then, inferred first-order probability values and error bounds, can be biased, because only commensurated unknowns are allowed to affect the inferred first-order probabilities and their error bounds, whereas, possible effects on the latter of incommensurable uncertainties and risk factors are jointly ignored.

In addition, the more one becomes familiar and accustomed to the use of probability measures in decision and sense-making, the more probabilistic information on commensurated uncertainties will likely become salient to him[173], up to the point of creating an illusion of risk control[174], and, of objective knowledge about the effects of unknowable explanatory factors on the state/outcome the considered system/process[172]. The latter phenomena has been annoverated among the causes of overconfidence[175–177].

Another important point concerning the practical limits of the use of a probabilistic framework for uncertainty representation and commensuration, is linked to epistemic priors concerning the probability-space to which probability mass may be attributed. Such a space should:

- be preliminarily defined and remain unchanged during the whole process of evidence collection and measurement, i.e. a statistical experiment;
- be an exhaustive representation of all possible and distinguishable states of the world;

Hence, a probability-space, once formalized and used to represent the randomness of a system/process, generates some extra costs imputable to the activities of speculation -in the philosophical sense- on residual unknowns and commensuration of previously non-considered risk factors. This because, probability-space based representations, by being systems of interdependent and self-consistent knowledge, imply some sunk costs and switching barriers, related to the eventual necessity of re-defining the whole structure of the probability-space, and, re-attributing the weight of evidence (probability mass) to distinguishable outcomes, in such a way that all probability axioms are respected. The latter necessity of redefining the probability space generally emerges in relation to the outcomes of the aforementioned commensuration and speculation processes, on residual unknowns. Residual unknowns which once elicited could result incompatible with the prior probabilistic representation of a system/process.

Moreover, the concept of probability measure, which is instrumental to that of probability space, used in real-world applications to represent risks, implies that events that are indiscernible are considered identical, this means that any probability space depends on a frame of discernment used by the observer. Among many other factors, the ability to discern states of an observed system depends on the process through which a sensory device, i.e. a measurement system or/and the observer, is able to probe and mimic (isomorphically or approximately represent) the states/outcomes of the target system/process or its transitions. The maintenance of frames of discernment requires agents to face specific costs, for example: for the maintenance/calibration of probing sensors, for rounding measurements if memory is not unlimited, and, for the memorization of the frame of discernment and collected evidence. The willingness to support these costs should not be given for granted, especially with reference to probabilistic representations of complex systems. Therefore, if individuals are rational, and, if they do not value per-se frames of discernment, it must be that the latter contribute to their utility through some other mechanism. We hypothesize that frames of discernment contribute to agents' utility by being used to elicit, commensurate, aggregate and represent commensurable known unknowns, in a coherent and formal system, and through the latter stabilize, as much as possible, their expectations under imperfect information. Conditionally on those "stabilized" probabilistic expectations agents can anticipate expected utility and expected surprise, also called uncertainty. We will come back on this point in the last section of this work.

Many years before the *WASH-1400* study was published, during the first half of the XX^{th} century, issues related to imprecise and higher-order probabilities and beliefs were at the fulcrum of the economic debate among incompatible schools of economic thought. Alternative conceptual orientations on these topics were highlighted by the contraposition between the Neoclassical and the Keynesian views of uncertainty phenomena. These two uncertainty paradigms could be distinguished precisely in relation to the concept, role and usage of probabilities in the two frameworks. Since its origin the Keynesian approach was *"characterized by the deep conviction that the economic, and social, environment is dominated by uncertainty that cannot be reduced to risk and treated with the traditional tools of [first order] probability theory"*[178].

Such a conviction of Keynesian economists, may be seen as related to the existence of epistemic uncertainties that cannot be commensurated, represented or resolved through probability reasoning, for example, those uncertainties emerging from beliefs of incommensurability and non-ergodicity. Under beliefs of non-ergodicity the information separation axiom[179] of information theory, also called Shannon-Khinchin's 4th axiom, is violated. The 4th axiom allows us to consider information atomistically: if the axiom holds, statistical dependencies among single information units can be considered negligeable in terms of their entropy effects. When the latter axiom holds we can simplify entropy with its Gibbs-Boltzmann version, and, give to observed densities in the phase-space of dynamical systems a probabilistic interpretation, which is required to formulate probabilistic expectations[180]. Whereas, under beliefs of incommensurability, the very references of cognition appear to be reversed, absolute uncertainty becomes the de-facto equilibrium mental state for those that belief that real-world systems and phenomena are incommensurable. That said, beliefs of commensurability and ergodicity, even if subjective, may reveal to be rather stable and useful to be held, and will generally last until a new anomaly, indeterminacy or paradox, in relation to one's belief and knowledge systems emerges. These higher-order beliefs are not necessarily the results of a process of inference of the commensurability and ergodicity properties of the actual target system that one may want to represent, they could be simply never updated priors. Moreover, agents that may have correctly inferred, speculated or guessed that a system is non-ergodic or incommensurable, would derive no concrete advantages from these beliefs, both in terms of forecasting and control capacities in relation to the latter system, and, in terms of mental stance towards the future. In addition, if it is true that expected utility can be mentally foreseen by human agents, and, produces a present anticipated-utility effect at the moment in which expected-utility is elicited[58, 59, 149, 181, 182], if one beliefs that a system is non-ergodic or incommensurable he will not be able to forecast and hence anticipate any utility that may derive from his interaction with that system. The latter is certainly an evolutionary disadvantage for those that may, even rightly, belief that some real-world systems are non-ergodic and/or incommensurable.

The aforementioned epistemic position, frequently assumed by Keynes, was implicit to many of his early economic works[183]. According to Keynes, in many real-world situations *"there is no scientific basis on which to form any calculable probability whatever. We simply do not know. Nevertheless, the necessity for action and for decision compels us as practical men to do our best to overlook this awkward fact and to behave exactly as we should if we had behind us a good Benthamite calculation of a series of prospective advantages and disadvantages, each multiplied by its appropriate probability waiting to be summed"*[46]. This view also emerged clearly in his *Treatise on Probability*, in which probability magnitudes, were not considered necessarily commensurable or comparable:

"No exercise of the practical judgment is possible, by which a numerical value can actually be given to the probability of every argument. So far from our being able to measure them, it is not even clear that we are always able to place them in an order of magnitude. Nor has any theoretical rule for their evaluation ever been suggested. The doubt, in view of these facts, whether any two probabilities are in every case even theoretically capable of comparison in terms of numbers, has not, however, received serious consideration. There seems to me to be exceedingly strong reasons for entertaining the doubt. [...] There are some pairs of probabilities between the members of which no comparison of magnitude is possible; that we can say, nevertheless, of some pairs of relations of probability that the one is greater and the other less, although it is not possible to measure the difference between them; and that in a very special type of case a meaning can be given to a numerical comparison of magnitude."[184]

Keynes viewed probability judgments as commensurations on the level of partial entanglement between an argument's rational expectancy and its epistemic uncertainty: *"Unlike the relative frequency theory of probability, in which probability is interpreted as a property of the physical world, Keynes treats probability as a property of the way individuals think about the world. As a degree of belief, this property is subjective to the extent that information and reasoning powers vary between persons. But it is not subjective, according to Keynes, in the sense that the probability bestowed on a proposition given the evidence may be subject to human caprice. The probability of a conclusion given the evidence is objective and corresponds to the degree of belief it is rational to hold."*[185] The objectiveness of probabilities is seen by Keynes as an emerging property of rational reasoning on an argument in situations of epistemic uncertainty.

A very interesting -not merely statistical- innovation introduced by Keynes's *Treatise on Probability* is that the probability of any argument is a bi-dimensional entity[184]:

1. the first dimension, called probability *magnitude*, is similar to a classical first-order probability value that is based on both evidence in favour or against an argument: *"as the relevant evidence at our disposal increases, the magnitude of the probability of the argument may either decrease or increase, according as the new knowledge strengthens the unfavourable or the favourable evidence*;

2. the second dimension, called probability *weight*, represents the amount of retrieved and elicited relevant evidence (informative known knowables), with respect to all possible relevant evidence (informative knowables). Probability weight therefore represents the epistemic support used for a judgement, on which the probability *magnitude* of an argument is built-upon: *"As the relevant evidence at our disposal increases [...] we have a more substantial basis upon which to rest our [probability magnitude] conclusion. I express this by saying that an accession of new evidence increases the weight of an argument. New evidence will sometimes decrease the probability [magnitude] of an argument, but it will always*

increase its weight". Keynes added to the concept of weight the following remark, through which we can associate uncertainty represented by probability weights to the inverse of entropy in information theory: *"we may say that the weight is increased when the number of alternatives is reduced, although the ratio of the number of favourable to the number of unfavourable alternatives may not have been disturbed [... And] we may say that the weight of the probability is increased, as the field of possibility is contracted"*;

Keynes's view of probability is for many conceptual aspects similar to theories of imprecise[186–191] and higher-order probabilities[192–195] which started emerging, and were applied in economics[196–200] and psychology[201, 202] studies, almost sixty years after the publication of *A Treatise on Probability*. For an analysis of the relation between Keynes' probability and epistemic uncertainty we refere to the works by Runde[203, 204], Weatherson[205] and Dow[45, 206].

In his *General Theory*, Keynes highlighted the connection between uncertainty and low probability weights[205], and, clearly dissociated the concept of uncertainty from probability magnitudes, which represent risks. According to Keynes, lotteries, like those described in the EU framework, are not situations of uncertainty but only of risk[207]. Uncertainties are represented by Keynes as low probability weights. Weights therefore represent the degree to which rational probabilities are an epistemically reliable guide to rational decision, given the potential surprise which may ensue from their usage. Accordingly, Keynes claimed[184] that the main limit of the classical probability framework is that it imposes a random view of uncertainty that is debatable. This point is precisely the argument used by Dempster and Shaffer[208] to explain why a more flexible and less randomness-oriented theory of evidence under uncertainty, is necessary to correctly treat epistemic uncertainties that are ignored or overshadowed by randomness in classical probability theory. The DempsterShafer theory of belief functions[209–212], is a generalization of the Bayesian theory of subjective probability, which, despite its versatility and capacity to represent epistemic uncertainty, has received until now little consideration by economists, or maybe, by economic journals; with some welcomed exceptions[213–217].

As we have explained throughout this subsection transition between radical uncertainties to probabilistic risks, of various orders, are epistemic belief driven. One is confronted to a risk when facing a situation considered, subjectively or objectively, indeterminate, commensurable and ergodic, with a stable and consistent probabilistic frame of discernment, and, a stable and tollerable degree of epistemic uncertainty, with reference to a process that exhibits stochastic dynamics. On the other side, one is confronted to radical uncertainty when facing a situation considered, subjectively or objectively, indeterminate with a belief system that, given the instability or the intolerably high degree of epistemic uncertainty, doesn't allow one to make, or rationally rely upon, inferences and forecasts conditional on contingent beliefs and evidence. This can happen for different reasons[218–220] For example: because the belief system is temporally inconsistent or incomplete, because beliefs are being reviewed; because probabilities/utilities/states are believed to be incommensurable; because even though probabilities/utilities/states are believed to be commensurable their commensuration would require too much time or computational capacity, in relation to one's constraints, preferences and rationally optimal behaviour.

Under this perspective, uncertainty and risk theories, may be both viewed as sense-making frameworks to avoid the occurrence or reduce the duration of states of radical uncertainty. Indeterminate situations are transformed in elicited uncertainties and measurable risks, obtained by grouping similar phenomenological instances in event categories, in such a way observed phenomena become recognizable and countable occurrences of events in a measure-space. States of radical uncertainty are therefore more likely to occur when agents are confronted to phenomena, agents or environments with volatile, or, rarely observed characteristics/attributes, which make them difficult to be categorized and represented in a formal sense-making and decision-making framework. We can transpose this concept to the aggregate level by saying that aggregate radical uncertainty in economic systems may be viewed as the average frequency at which agents incur in a state extreme or intolerable expected surprise, when having to make expectations and take decisions while facing indeterminate situations. At the aggregate level this value will likely depend on the degree of complexity, openness, speed of structural change of the system, and, the information, belief and resource endowments and constraints faced by the agents during communication, decision-making and sense-making in a multi-agent system.

As we will see in the next sections, in situations where epistemic uncertainty is intolerably high, communication and the formation of conventional expectations can be used as shared beliefs systems to coordinate action and reduce the frequency and duration of states radical uncertainty.

Elicitation of beliefs, markets and uncertainty

Here follows a brief subsection in relation to markets, uncertainty and the elicitation of beliefs, of various order. An important characteristic of contemporary economies stands on their capacity of eleciting market agents' preferences[221–226] and beliefs[227–231]. Elicited information about beliefs can be used to represent/map the agents-beliefs space in relation to the diffusion of known unknowns, and possibly, exploit agents' information-gaps as a market opportunity[40–42]. In relation to the elicitation of beliefs, Karni has recently shown[232, 233] that agents' subjective information structures under Knightian uncertainty, intended as second-order beliefs over a set of different priors, can be inferred through a revealed-preference procedure. If one can elicit subjective information structures of others, and determine if and when the set of priors changes, new entrepreneurial opportunities in markets for beliefs become available[234]. Elicited known unknowns become commercially exploitable: by offering to an agent the information that would allow him to eliminate the information-gaps related to his belief of ignorance and ensuing state of epistemic uncertainty, at a price that is inferior or equal to the expected-utility gains of such a change in his beliefs and epistemic uncertainty.

In this rather dystopian vision of the world, agents' subjective information structures and epistemic beliefs attributes can be endogenized through markets, which offer services to redefine beliefs of various order, through beliefs transforming technologies, based on controlled communication treatments[235]. All communicated messages would be tailored, on the basis of beliefs elicited by the targeted agent, to reduce his state of epistemic uncertainty, in relation to his known unknowns. Through communication, agents would be facilitated in the process of converge to epistemic belief attributes of faith and agnosis, in the doxastic philosophical meaning, which are states in which epistemic uncertainty is absent and therefore expected surprise, which is generally considered an economic bad, is absent/null. Faith, agnosis[236] and ignorance-of-ignorance[237, 238], represent together the epistemic (set-theoretic) complement of states of uncertainty.

It is interesting to note that, market agents endowed with technologies that extend agnosis, the realm of unquestioned or unquestionable known unknowables, or faith, the realm of unquestioned or unquestionable known knowns, to priorly known unknowns, can provide through markets a service that reduces uncertainty. Demand fluctuations of these markets for beliefs would hence depend on the amount of epistemic transitions from unknown unknowns to known unknowns, of uncertainty averse agents. Preferences for the reduction of uncertainty would push the latter to pay to reduce their information and expectations gaps and ensuing uncertainties. In such a way, uncertainty itself becomes a green-field for moral hazard[239], because belief-markets can continue to exist only if a sufficient number of individuals are in a state of uncertainty, and, keep on being averse to such a state. This market situation is very similar to that of the so-called "arbitrageurs", described by Miyazaki[240].

1.2.3 Uncertainty in information and communication theories *from an economist's perspective*

In the following subsection we briefly review the concept of uncertainty as defined and measured in information theory, pioneered by Shannon[18] and Weaver[241]. Throughout the subsection we will try to relate uncertainty measures from information theory, to interpretations and conceptualization of uncertainty in economics, as described in the previous subsections.

Communication, messages, signals and noise

As described by Shannon[242] the problem of communication is that of reproducing messages from an environment or agent to another. *"Messages have meaning; that is they refer to or are correlated according to some system with certain physical or conceptual entities. [...] The significant aspect is that the actual message is one selected from a set of possible messages. The [communication] system must be designed to operate for each possible selection, not just the one which will actually be chosen since this is unknown at the time of design"*. The problem of communication is therefore associated to the openness to external influence of real-world systems and human agents, and, the ensuing dynamics of state entanglements among them. Communication can be seen as a device for (mental/physical) state coordination among loosely coupled systems and agents, which exhibit non null degrees of freedom in their communication process. Degrees of freedom emerge in relation to the maximum/limiting *"number of independent signals that can be exchanged between the [message] transmitter and the receiver"*[243]. This implies that the two systems that can communicate do not deterministically determine each others' state prior to, and unconditionally from, the communication process. They are both in a latent state of readiness to communication-driven change.

Uncertainty exists precisely because agents, through their communications:

- elicit these degrees of freedom and their latent readiness to change;
- disturb their own state, by determining/choosing the message to be sent;
- disturb the state of the communication medium, in a instrumental way;

- disturb the state of the receiver, by stimulating a state shift through the signal, the transformation of the medium, which must be interpreted by the receiver to be able to reduce his uncertainty in relation to the meaning of the signal. Signal which can hence be considered a conditioning impulse with potentially indeterministic effects;

By so doing agents can jointly collapse their degree of expected surprise and their degree of communication freedom, reducing available signalling capacity, in terms of time, space, memory and cognitive resources available for further communication. If the mental/physical state shift of the receiver, in relation to the received signal, corresponds to that desired by the sending agent, the message has been successfully transferred. Otherwise, there exists some noise or perturbation in the communication process. Noisy message transfers can occur in relation to:

- differences between the representation and reference systems used by the two agents to synthesize/interpret signals;
- interferences of other signals transferred through the same medium by others;
- perturbations and noise that characterize the medium/channel;

However, since the sender may have access to the receiver's mental/physical state only through feedback signals of the state of the receiver, sender's uncertainty concerning the degree of communication success may depend on additional iterative communication steps. Through iterated communication, the sender that had initiated the communication process, can try to change his own entropy level, in addition to that of the receiving agent(s) and of the communication medium or environment. From a rational perspective, a message should always determine a collapse or shift of the senders' expectations of receiver's possible actions/states, to a posterior distribution, conditional on the sent message, which is strictly preferred, by the message sender, to the original one. Moreover, the message sender, by receiving feedback signals concerning the response of the receiver to his impulse, can try to infer the success rate/degree of the prior communicating process. All signals elicited and memorized during communication processes can be used as evidence, to infer the capacity of signals to affect, in the desired or undesired way, each others' states. These inferred relations can both increase or decrease agents' uncertainty, however if the system is ergodic, in the long run uncertainties should be lowered through this evidence collection mechanism. Communication must hence be seen as a process that can be used to reduce uncertainties concerning non-deterministic dependency and coordination relations among agents and systems.

Entropy

To keep this subsection as intuitive and simple as possible we will present and describe the discrete versions of entropy and relative entropy measures, in classical probability and in the Dempster-Shaffer framework.

Entropy represents the average amount of surprise produced by a stochastic source of data. In terms of communication, entropy can be seen as the expected number of atomistic/independent informative units contained in a message. Message that may concern a system/process, in relation to which, the receiver of the message, is aware of being in a state of imperfect knowledge and hence of potential surprise, concept that will be explained further on in this subsection. States of awareness of imperfect knowledge may generate in one's mind these feelings of uncertaint(y)/(ies), which in information theory are measured in terms of surprise. Changes in entropy are related to these changes in the set of known unknowns of an agent. Entropy is related to the variety and distribution of possible answers to a question. Imagine a draw from a known degenerate random variable distribution, receiving a message concerning the realized state of such a draw doesn't change the entropy of the agent because that message doesn't dissipate/resolve any prior uncertainty. This because the question had little surprisal potential given the prior shape. The agent was almost certain of the result of the draw, the information content of the message was therefore ininfluent to him. Whereas, imagine to receive a message that tells you that an event previously considered very improbable occurred, the surprisal potential of the transmitted information is very great precisely because the event wasn't expected, given prior beliefs.

Entropy and normalized entropy in the probabilistic framework

In classical probability theory, given a discrete probability distribution, called P, with n mutually exclusive and collectively exhaustive events $j \in \{1, \dots, n\}$. The entropy of this probability distribution can be computed as follows:

$$H(P) = -\sum_{j=1}^{n} P(j) \ln(P(j)) \tag{1.2.1}$$

In the discrete case, the value of $H(P)$ is at the maximum entropy value H_{max}, if the probability of each possible event is the same, i.e. the distribution P is uniform. Which is equivalent to say that for all j $P(j) = \frac{1}{n}$. The ratio between the entropy and the maximum entropy of a discrete distribution P with n events, called normalize entropy or efficiency, can be computed as follows:

$$\begin{aligned} \frac{H(P)}{H_{\text{max}}} &= \frac{-\sum_{j=1}^{n} P(j) \ln(P(j))}{-\sum_{j=1}^{n} \frac{1}{n} \ln(\frac{1}{n})} \\ &= -\sum_{j=1}^{n} \frac{P(j) \ln(P(j))}{\ln(n)} \end{aligned} \tag{1.2.2}$$

The value of $H(P)$ is at its minimum entropy value H_{min}, when the probability of one event is equal to one (1), all others being identical and equal to zero (0), i.e. the distribution P is degenerate. Shannon entropy can be generalized by the Rényi entropy[244], which can be used to represent situations in which there is non-null entropy entanglement between atomistic information particles, i.e. when the chain rule of conditional probability doesn't hold.

Entropy, specificity and other aggregate uncertainty measures in the Dempster-Shaffer framework

In the more general framework of Dempster-Shafer[209], which allows us to represent a belief structure under imprecise information, we can compute Shannon entropy as follows:

Assume that $X = \{x_1, \dots, x_n\}$ is a finite set of elements of cardinality n, called the frame of discernment or universe of discourse, which represent the support of a belief structure m. A belief structure m allows us to assign a mass $m(A) = a$ to any subset of the frame of discernment $A \subseteq X$. In the Dempster-Shafer framework we

can impute shared mass among a subset of multiple elements from X, without indicating how it is shared among them.
Any $A \subseteq X$ such that $m(A) = a$ is called a focal element. If each focal element consists of only one element from the frame of discernment, i.e. focal points are singletons, the Dempster-Shafer belief structure corresponds to a Bayesian belief structure, and, Shannon entropy measures are computed as in the classical probability case. Otherwise, we have to proceed as follows:

Call belief of A, i.e. $Bel(A)$, the sum of all the masses of all possible subsets $B \subseteq X$ of the set A, such that:

$$Bel(A) = \sum_{B|B\subseteq A} m(B) \tag{1.2.3}$$

Call plausibility of A, i.e. $Pl(A)$, the sum of all the masses of all sets $B \subseteq X$ that intersect A, such that:

$$Pl(A) = \sum_{B|B\cap A\neq\emptyset} m(B) \tag{1.2.4}$$

As pointed out by Yager[245]the connection between probabilistic information and the Dempster-Shafer framework is based upon the fact that belief and plausibility are respectively lower and higher boud for the underlying probability of an event A. We can hence compute the Shannon entropy of m as follows:

$$H(m) = -\sum_{B\subseteq X} m(A)\ln(Pl(A)) \tag{1.2.5}$$

In addition to Shannon entropy, in the Dempster-Shafer framework there is a specificity $S(m)$ measure that represents the non-random epistemic vacuousness component of uncertainty, which can be computed as follows:

$$H(m) = -\sum_{B\subseteq X,\, A\neq\emptyset} \frac{m(A)}{n_A},\ n_A = cardA \tag{1.2.6}$$

When $S(m) = 1$ m is a Bayesian belief structure, and $H(m)$ is equivalent to its classical probability version.
Many other measures of doxastic uncertainty have been developed in the DempsterShafer, framework[246–250]. Like that of Harmanec and Klir[251], which have developed a symmetric, continuous, additive and subadditive aggregate uncertainty measure, which gives the maximum value of the set of Shannon entropies, of each possible probability distribution that is consistent with the lower DempsterShafer belief bound.

An extension of the Dempster-Shaffer belief framework, called Transferable Belief Model (TBM)[252–254], allows one to attribute mass also to the empty set, non-null mass on the empty set allows one to represent beliefs related to the possibility of occurrence of gray and black swan events[255], events that an agent a-priori knows he cannot distinguish or foresee, but which could be possibly imagined if the agent had infinite time to speculate on possible, and at the moment indiscerned, states of the future world; but also, on the infinitely many almost-impossible events that would push one to represent the outcomes of a process/system through a continuous space.

Relative entropy as surprise

Kullback and Leibler[49] developed a measure of the divergence among distributions, also called relative entropy, which is very useful to measure surprise in relation to the comparing of belief structures. For example, when used to measure the divergence between prior and posterior beliefs, it can be considered as measure of the (extra) surprise implied by the "switch" between the two distributions. Let P and Q denote two discrete distributions on the same support: both P and Q have the same partition with n mutually exclusive and collectively exhaustive events $j \in \{1, \ldots, n\}$. Then the Kullback-Leibler (KL) divergence, also called relative entropy, between P and Q is defined as:

$$\begin{aligned} \mathrm{KL}(P \parallel Q) &= \sum_{j=1}^{n} \ln\left(\frac{P(j)}{Q(j)}\right) P(j) \\ &= \sum_{j=1}^{n} \ln\left(P(j)\right) P(j) - \sum_{j=1}^{n} \ln\left(Q(j)\right) P(j) \\ &= H(P,Q) - H(P) \end{aligned} \tag{1.2.7}$$

Where $H(P,Q)$ is the cross entropy of P and Q, and $H(P)$ is the entropy of P. $\mathrm{KL}(P \parallel Q)$ divergence can be also interpreted as a measure of communication efficiency losses (in cross entropy terms), when encoding a message using a distribution (Q) other than the real one (P). We remind our readers that communication efficiency consists in the rate of infomativeness of a message, which is equivalent to its average surprise (entropy) per signal element. Under the aforementioned perspective $\mathrm{KL}(P \parallel Q)$ can be seen as the (extra) surprise which derives from believing in a prior Q and then coming to know that the true distribution is P.

Since $\mathrm{KL}(P \parallel Q)$ represents the degree of information inefficiency due to use of an approximation (Q) in place of a hypothetical true distribution (P), we can imagine a situation in which **agent A** has a set of prior beliefs Q which he knows to be a subjective and potentially biased probabilistic representation, which he uses to approximate and anticipate the true unobservable "randomness" of a real-world process. Let us now imagine that, through communication, **agent A** comes to know that the belief structure of another agent, called **agent B**, concerning the same process, is distributed as P, $\mathrm{KL}(P \parallel Q)$ would hence represent the uncertainty, or better the (extra) expected surprise, derived from the following self-questioning by **agent A**:
*What if **agent B** is right, and, his beliefs (P) correspond to the truth, while my own beliefs (Q) are only an approximate and therefore biased representation of reality that will further limit my capacity to anticipate/foresee we world?* $\longrightarrow \mathrm{KL}(P \parallel Q)$ ***(the latter is a measure of the (extra) expected surprise that agent A will feel when experiencing such a state of doubt and skepticism concering his own beliefs, in relation to those of agent B, which are temporally evaluated and hypothesized to represent the truth.)***

One of the most interesting properties of the KL divergence resides on its non-symmetry:

$$\begin{aligned} \mathrm{KL}(P \parallel Q) - \mathrm{KL}(Q \parallel P) &= \sum_{j=1}^{n} \ln\left(\frac{P(j)}{Q(j)}\right) (P(j) + Q(j)) \neq 0 \\ &\Rightarrow \mathrm{KL}(P \parallel Q) \neq \mathrm{KL}(Q \parallel P) \end{aligned} \tag{1.2.8}$$

This property allows us to represent changes in expected surprise in a non symmetric way. Which is a very likely psychological hypothesis in relation to beliefs communicated and then compared by agents. In the above describe situation, if the communication of beliefs is bilateral, **agent B**, which comes to now beliefs Q of **agent A**, could hypothesize that the inverse truth-approximation relation among beliefs exists: *What if **agent A** is right [...]?*
In such circumstances **agent B** will experience an (extra) expected surprise, different from that of **agent A**, and, equivalent to $\mathrm{KL}(Q \parallel P)$. If eventual zeroes of P and Q are associated to the same events, $\mathrm{KL}(P \parallel Q)$ divergence is finite and contained in the zero one interval. $\mathrm{KL}(P \parallel Q)$ tends to infinity when the distribution Q, our so-called approximation, imputes 0 probability mass to some events that have non null probabilities in the distribution hypothesized to be true P. This situation can be assimilated to a state of extreme expected surprise, or radical uncertainty. It is equivalent to a situation in which the aforesaid **agent A**, self-questioning his beliefs in relation to those of another agent (hypothesized to be true), imagines the surprise-effect of the potential realization of an event which he considered almost impossible, given his prior subjective beliefs Q, but which, according to the communicated beliefs of **agent A**, which are hypothesized to be true, could happen with non-null probability.

1.2.4 Uncertainty in cognitive sciences *from an economist's perspective*

In the following section we will briefly review how uncertainty is conceptualized and studied in the cognitive sciences. We will try to highlight shared paradigms, in relation to previously described uncertainty frameworks.

The neurological characterizations of uncertainty

To identify the neurological characterizations of states of (self-declared) uncertainty, Harris et al.[256] employed functional neuroimaging. In their experimental setting, people were asked to *"judge written statements to be true (belief), false (disbelief), or undecidable (uncertainty)"*. The objective of the study was to *"characterize belief, disbelief, and uncertainty in a content-independent manner"*, by including statements from a *"wide range of categories: autobiographical, mathematical, geographical, religious, ethical, semantic, and factual"*. The results of the study clearly evidenced that, subjects which declared that the belief attribute of a statement was undecidable *"differentially activated distinct regions of the prefrontal and parietal cortices, as well as the basal ganglia"*, with respect to when the state of belief or disbelief were declared. Therefore, from a neurological point of view, the 2nd order belief characterization called "uncertainty" is observable and clearly distinguishable from that of conscious belief and disbelief, such a meta-cognitive process has a physical counterpart, i.e. associated phenomenon, therefore it is not only a conceptual human construct.
Additional research from the neural sciences[257–259], has confirmed this view and evidenced that responses of the human brain to (higher-order) uncertainty, in particular ambiguity, are clearly distinguishable from those caused by risk, i.e. choices among predefined gambles in a formal and explicit probability-space. In particular, neurologists[260–262] have found that activity in the inferior frontal gyrus and

posterior parietal cortex is significantly higher when confronted to uncertainty (ambiguity or ignorance) compared to risk, they conclude that these regions may be involved in searches for hidden/simulated evidence during expectations formation or outcome anticipation tasks under (higher-order) uncertainty. Neural responses to situations of higher-order uncertainties, with reference to the activation of the posterior parietal cortex, posterior dorsolateral prefrontal cortex and anterior insula, suggest that higher-order and strategic uncertainties, are neurologically similar phenomena, clearly distinguishable from risk, i.e. the first-order uncertainties emerging during economic gambles, when gambles are known. Uncertainty and risk, are not only different from a mental and epistemic perspective, they are also distinct neurological phenomena.

The psychological origins and implications of uncertainties

After World War II, psychologists, through their experimental analysis approach, started filling the gap of uncertainty theories and frameworks in relation to other fields of knowledge[68, 92, 93, 95, 127, 263–273]. In particular with reference to theories of risk and expected utility from economics, and, measures of entropy from communication and information sciences.

In 1957, the experimental psychologist and philosopher Daniel Berlyne, in a work titled *Uncertainty And Conflict*[274], illustrated almost perfectly the process of increasing dependencies and contagion between the *humanae scientiae*, occurring at that time. In particular, in relation to the analysis and representation of uncertainty phenomena. Berlyne claimed that emerging psychological information theory was *"a type of theory in the scientific sense: it applies information-theory measures to phenomena within the purview of psychology and uses information-theory language to formulate laws or hypotheses with testable implications about behavior. [...] The phenomena that concern behavior theory consist, in fact, of two sets that can be partitioned into subsets with associated probabilities, namely stimuli and responses. The language of information theory is therefore, in principle, applicable to everything within the competence of behavior theory. [...] measures as "amount of information," "uncertainty," and "relative uncertainty" can be applied. [...] Reaction time, retention of verbal material, and accuracy of psychophysical judgment, to cite examples, appear to be functions of "uncertainty" and "amount of transmitted information." [...] An observer can compute information-theory measures from data not accessible to the individuals he is observing. But there is not likely to be much connection between these measures and variables of psychological importance [, like uncertainty], unless there is some isomorphism between the situation as viewed by the observer and the situation as it impinges on the observed organism [... these] observed response tendencies [to stimuli, can be considered] "reaction potentials". Cognitive behavior theories would describe them as "expectations" of the consequent stimuli, and the "expectation" resembles the "reaction potential" insofar as both imply the occurrence of a particular response, if certain additional conditions are met".* Berlyne also linked psychological conflict and competing tendencies to entropy and utility as follows :*" if we examine the information theorist's formula for "uncertainty" or "entropy", we find that it satisfies the first five of our requirements for a degree-of-conflict function, but not the sixth. It increases with the number of alternative responses and is at a maximum when their strengths are equal. But it does not vary with their absolute strengths [... Entropy] can be regarded as an indication of the "complexity" of a conflict, or of the difficulty that an observer would have in predicting which of the conflicting responses will be the first to occur. It does not reflect the "scale" of the conflict,* *which depends on the energy invested in the competing response tendencies. There may be a temptation to relate these two components to the utility and probability of-outcome factors that must be taken into account in decision theory."* Finally, Berlyne related uncertainty to states of metacognitive activation, like doubt, perplexity and ambiguity as follows: *"Other words that seem apposite to situations that call for investigatory behavior are "doubt", "perplexity", and "ambiguity." These words likewise imply some degree of behavioral conflict; they indicate that different aspects of a situation evoke discordant reactions or else that a particular reaction is called forth by one aspect and inhibited by another. They are opposite in meaning to words like "clear" and "distinct", which generally imply that certain response tendencies have come, through discriminatory learning, to predominate over their competitors. "Doubtful", "perplexing", or "ambiguous" stimulus situations are usually also cases of high "uncertainty" in the information-theory sense, both because the subject cannot predict very successfully what the future behavior or the hidden properties of the entities will be, and because observers will not be able to predict very successfully how he will react to them."*

In the years that followed, the representation of uncertainty as a source of psychological conflict emerging from communication and metacognition[14, 15, 275] was further explored, and implemented to multi-agent frameworks and experiments[276–278, 278, 279, 279, 280]. These works showed that uncertainty emerges in relation to metacognitive processes undertook to evaluate, and eventually correct, belief and knowledge systems used to represent the world, in relation to signals communicated by other agents or coming directly from the environment. Uncertainty in this framework appears as a psychological conflict at the metacognitive level in relation to learning and coordination problems. This view of uncertainty is a cognitivist transposition of the notion of strategic uncertainty, which in the cognitive framework acquires not only a subjective perspective, but also, a social one. These works show that eliciting and measuring higher-order uncertainty, during individual and social metacongition processes, is the key to explain observed deviations from the EU hypothesis framework at the individual and aggregate level.

A very recent stream of psychological literature, linked to the latter, has focused on the cognitive relation between uncertainty, variance/bias tradeoffs and learning in open systems[281–286]. This area of research, pioneered by the German psychologist Gerd Gigerenzer[10, 287–289], clearly distinguishes optimal action in *small worlds*, with respect to optimal action in *large worlds*. *Large worlds* that, given their complexity and openness, are intrinsically more "uncertain" than the former. Where by uncertain we do not refer to physical indeterminacies, but to the frequency and intensity of states of metacognitive uncertainty, i.e. extreme or untollerable expected surprise. This stream of literature clearly evidenced that, rationally optimal action in the the "two worlds" rarely coincide. Therefore, rationally optimal decision-making heuristics and metaheuristics to be used in *small worlds* should be different from that used for rational decision-making in *large worlds*. For example, De Miguel et al. [290] have shown that Markowitzs Nobel prize-winning mean-variance portfolio allocation model[291], performed worse than the "naive" **1/N** risk diversification heuristic when applied to real financial asset price time-series.

1.3 Communication, metacognition, uncertainty and beliefs revision

In the following section, we will use very recent literature, from the cognitive and social sciences, to show in which terms uncertainty states are associated to the activation of higher levels of cognition, through which agents try to solve decision-making and sense-making problems, at the epistemic level, under imperfect information. As we will show these processes are undertaken through communication in groups and/or social networks. We are particularly interested in the social embeddedness of these metacognitive processes, and, on the relation between inter-agent doxastic communication and epistemic uncertainty, but also, on the role of the latter in belief revision. We will show why self and social metacognition processes are necessary conditions for the commensuration, mitigation and resolution of states of uncertainty in aware intelligent systems, which in our case are human agents and their societies. We will show how, aware intelligent systems which are endowed with epistemic beliefs (priors) concerning their environment, are able to review the latter after communication, to locally reduce epistemic uncertainty, conditionally on their priors and preferences, and, on beliefs communicated by others; without "loosing", through falsification, all prior knowledge that is not perfectly compatible with "evidence" received while communicating with other agents. We will put in relation our findings to the social embeddedness of expectations revision, and, we will link the latter to the Keynesian notion of conventional beliefs and expectations, and, their role as knowledge compression device used for coordination.

Searching for a mental mechanism for epistemic signal-noise separation

In the last decades, human knowledge and belief systems have been extensively studied both in relation to cognition[292–294] and metacognition[17, 37, 295–297]. The study of metacognition concerns cognition about cognition, including, but not limited to, normative and sensemaking matters, like truth value judgements, justifications and updating of epistemic beliefs[298–301]. In the cognitivist framework that we will illustrate, uncertainty can be seen as a -latent- property emerging from metacognition[275, 302, 303], undertook by human agents in relation to communication processes[36] and the epistemic surprise they generate[14, 34, 304]. This because, to be epistemically valued, non-metacognitively-denoised (doxastic) messages received by agents through communication, also called perceptions of beliefs[305], are, in epistemic terms, at first and by default assumed to be true, hence they immediately become the meter of judgement of one's prior epistemic beliefs[301], i.e. knowledge. Because, a-priori to metacognition, the latent claims that are under investigation are our epistemic beliefs, and not, aware percepts of the outer world, which are considered evidence. However, a-posteriori, once epistemic beliefs are updated through metacognitive processes, new evidence is separated in a metacognitively cleaned epistemic-signal, the a-posteriori believed-to-be "true" content of (doxastic) messages, and, a residual epistemic-noise, the a-posteriori believed-to-be "false" content of (doxastic) messages. Where the epistemic-signal is that part of evidence which is a-posteriori non-dissonant to, and integrated into, reviewed beliefs, whose surprise effects were reduced through metacognition. Whereas, the epistemic-noise is that part of evidence which is a-posteriori dissonant and orthogonal with respect to reviewed beliefs; whose surprise effects were not reduced through metacognition. Epistemic-noise can be considered the information waste given reviewed epistemic beliefs. Epistemic-noise represents quasi-information, that is, evidence which produces a irreducible surprise effect, but not also, an epistemic effect, in terms of changes in epistemic beliefs and hence of optimal behaviour inferred from those beliefs. Therefore, it can be considered as the degree of communicating and learning inefficiency. Epistemic-noise represents latent information that is not a-priori false, but which is a-posteriori considered unreliable for necessity of reducing as much as possible expected surprise through belief revision, conditionally on all available evidence, priors and preferences. The epistemic signal-to-noise ratio, conditional on reviewed beliefs, represents the expected surprise, or the expectation of epistemic disappointment[306], which we consider assimilable to the Knightian and Keynesian views of uncertainty.

Agents' epistemic beliefs can be therefore seen as a dynamic compressed version of past evidence that one has been sensible to, and which hasn't been knowingly and instrumentally considered false/unreliable [307–310]. If individuals were not averse to "feelings" of surprise, which is an attribute of the relation between evidence and beliefs, they would have no pressure to learn by changing their beliefs in a subjectively optimal way. The epistemic notion of truth and falsity of information can therefore be (also) considered instrumental to the reduction of metacognitive uncertainty[311].

Metacognition

The concept of metacognition includes, among others, processes of knowledge falsification and updating and their ensuing effects on beliefs and expectations reviewal. As claimed by Nagel[312], in *The View From Nowhere*, through cognition *"we can add to our knowledge of the world by accumulating information at a given levelby extensive observation from one standpoint"*, however, *"we can raise our understanding to a new level only if we examine that relation between the world and ourselves which is responsible for our prior understanding, and form a new conception that includes a more detached understanding of ourselves, of the world, and of the interaction between them"*, the latter metacognitive processes are generally referred to as *epistemic cognition*[313].

Metacognition is considered the system of control of cognitive processes at various hierarchical levels[314]. Through metacognition, cognitive processes are horizontally aggregated and recursivily represented and iterated at higher levels of abstraction[315]. Lower level of cognition provide the information that are processed by higher levels. Through metacognitive control mechanism lower levels of cognition are hence steered. Phenomena of neural hierarchical aggregation, reflexivity and recursivity, somehow isomorphic to the concept of metacognition, have been extensively identified in neurological studies, as an organizational principle of human cortical networks and functions[316–319]. Experimental results have shown that, under situations of increasing perceptual discrimination difficulty quantified through objective measures, the descriptions of undertaken tasks by agents, before knowing their performance, revealed *"reflexive self-awareness in the sense that humans are aware of themselves as cognitive monitors [...] responses were prompted by feelings of uncertainty and doubt about the correct answer on the trial"*[320], which were significantly increasing with the perceptual discrimination difficulty of the task. Metacognition appears to be a critical factor in the determination of the outcomes of lower cognitive processes under situations of objective environment complexity[321, 322] or information overload[36, 323–325]. Metacognitive uncertainty monitoring ability, elicited both before and throughout the execution of experiment tasks, was identified as a one of the most relevant and statistically significant predictors of accuracy improvements in learning[326], and also, of the precision of the self judgement of one's expected and actual performance, respectively, after the description of the task, and, after its execution but before knowing the objective performance measurements.

Given the above stated findings related to metacognition, it has been argued that rationality cannot simply be reduced to the use of logic but requires agency: *"Agency is intrinsically and unavoidably subjective in its nature but reflection on and coordination of ones reasons and reasoning can enhance rationality and objectivity. This enables the progress of rational agents through qualitatively distinct levels of rationality. These are [...] largely levels of epistemic cognition. Logic is important in this view, but rationality is fundamentally metacognitive rather than logical. Our knowledge and control of our inferential processes is not limited to logical inferences. Even in the domain of logic, what makes us rational is our metalogical understanding about the epistemic nature and role of logic and our corresponding ability to distinguish, coordinate, and interpret logical inferences, not just make them mindlessly along with inferences of all sorts. More generally, epistemic cognition supports better inferences but it is the epistemic cognition itself that is central to our rationality, not the correctness of the resulting inferences as determined by an external expert or standard."* [327]

During metacognitive processes, rationality may be therefore seen as a form of *metasubjective objectivity*[328–331]: *"subjectivity need not be construed as a realm of idiosyncratic ideas and feelings. Rather, it may be seen as a property of cognitive actions (reasoning, remembering, perceiving, etc.) that take place, as they must, from some point of view [priors, preferences and evidence]. Objectivity, on this view, is not a realm of absolute truth and rigorous logic distinct from the realm of subjectivity. Rather, subjectivity and objectivity are complementary poles of the relationship of knowing. Given that knowing always takes place from some point of view, ones knowledge is always a function of ones viewpoint and thus unavoidably subjective. To the extent that knowledge is constrained by a reality distinct from the knower, however, it is also a function of that reality and thus, to that extent, objective. [...] continuing self-reflections [...] never transcend subjectivity but nevertheless may allow increasing objectivity. If we define the reflective analysis and reconstruction of ones subjectivity as metasubjectivity, we can then define rationality as metasubjective objectivity. It is important to emphasize that psychological reflection takes place in the course of transactions[communications] with ones environment. From an external point of view, the object of reflection is not pure subjectivity but a subject-object (or subject-subject) relationship. The construction of that external (metasubjective) point of view enables explicit understanding and reconstruction of the previously implicit subject-object relationship. [...] "*[332]

In cognitive psychology, epistemic beliefs are not simply subjective beliefs to which agents commit, they are dynamic mental constructs reviewed and justified, to oneself and to others, by spontaneously or voluntarily probing the environment and other agents in search of evidence. If evidence is perfectly coherent with epistemic priors agents experience no surprise, they are metacognitively certain about the truthfulness of their beliefs. Metacognitive certainty refers to *"the extent to which a person is convinced of a belief and views the belief as valid. Applied to the self, two people might each belief that they are outgoing (primary thought). However, one of these people might be convinced that this belief is correct, whereas the other person might hold some* *reservations about the validity of this belief (both secondary thoughts). When a person holds a self-view with high rather than low certainty, the selfview tends to be more predictive of behavior and information processing, more stable over time, and more resistant to change. [...] Furthermore, when [agents] interact with someone whose expectations [...] countered their self-beliefs, those low (but not high) in certainty changed their behavior to align with their partners expectations."*[333]

The more prior beliefs of agents are improbable given the evidence that they observe, the more they experience surprise. If an agent is uncertainty averse, surprise acts as a "epistemological pressure" for beliefs revision, to render beliefs more probably "true" with respect to observed evidence, and by so doing reduce a-posteriori uncertainty, i.e. expected surprise, conditional on beliefs and collected evidence.

Interruptions to lower level cognitive processes are the result of highly discrepant events with respect to schema or prior expectations: *"these events triggered not only feeling of difficulty but surprise as well. This is an important finding because it reveals the close relation between metacognition, in the form of feeling of difficulty, and emotions, such as surprise. Surprise serves the relocation of attention from the prevalent schema to the discrepant event. Feeling of difficulty along with surprise provide the input for better appraisal of the demands of the situation as well as for better control decisions."*[326] In the psycho-evolutionary surprise framawork[334], it has claimed that *"the most important functional property of conscious states is widely thought to be their system-wide accessibility and their being (thereby) poised for exerting global control. The information that the surprise feeling reliably provides concerns the occurrence and intensity of mental interruption and/or the occurrence and degree of a schema-discrepancy. Note that, on both counts, the formation provided by the surprise feeling can be said to be metacognitive in character that is, it is information about, respectively, the person's cognitive processes or the status of his or her belief system. Hence, on both counts, surprise can be called a "metacognitive" or a "metarepresentational" feeling. Taken together, these points suggest that the function of the surprise experience is to make this information globally available [...] to exercise global control specifically, to influence goal-directed actions such as epistemic search. Surprise elicits curiosity because it informs the conscious self about the occurrence of schema-discrepancies or of mental interrupts. [...] Subjective experience of surprise [...] differs in crucial respects from that of other emotions because, in contrast to the latter, it is hedonically neutral, and the information that it provides is uniquely metarepresentational."*

Metacognitive certainty and metacognitive uncertainty are non necessarily symmetrical concepts: if on one side, metacognitive certainty refers to the extent to which an agent considers his beliefs under evaluation to be "true", from a higher-order belief perspective. On the other side, metacognitive uncertainty can refer to the lack of the aforementioned (higher order) epistemic characterizations of evaluated (lower-order) beliefs, or, their truth value of being "false", or, their truth value being "true" while contemporaneously acknowledging/believing that there are some known unknowns that, once known, could imply a revision of present beliefs. Under the latter perspective, agents can be considered sceptical towards their own epistemic beliefs, and, the latter are seen as instrumental and therefore precarious forms of knowledge, used to reduce as much as possible uncertainty, by limiting one's own freedom and volatility of representation, given observed evidence. Therefore, we can see metacognitive procedures for epistemic beliefs revision as metaheuristics, used to, dynamically, keep expected-surprise as low as possible conditionally on preferences, priors and evidence. Where by preferences we mean: aversion to uncertainty, aversion to risk and preferences for states of the world, in terms of foreseen utility as a function of events or states of the world. As new evidence becomes available, epistemic beliefs and uncertainty change in relation to preferences in an optimal way.

Social metacognition as a mechanism for uncertainty reduction

The analysis of the interdependencies between social metacognition and uncertainty is no new stream of literature. More than half a century ago, the social psychologist Leon Festinger claimed that *"individuals understandings of the world are held as true to the extent that they can be affirmed by some social group"*[335].

Two decades before, Muzafer Sherif, illustrated his view on the effects of the embeddedness of beliefs in social structures and their communication networks. In *Group Norms and Conformity*[336], which soon after became the founding pillar of modern social psychology, Sherif claimed that: *"an opinion, a belief, an attitude is perceived as correct, valid, and proper to the extent that it is anchored in a group of people with similar beliefs, opinions, and attitudes.[... Once it] is standardized and becomes common property of the group [... in which it is considered] objective reality.* Sherif also explained the social conditions and processes under which beliefs and norms are reviewed. He did so by linking conventional beliefs reviewal to metacognitive uncertainty: *"when there are [cognitive] stresses and tensions in the lives of many people in the community, the equilibrium of life ceases to be stable, and the air is pregnant with possibilities. [...] Such a delicate, unstable situation is the fertile soil for the rise of doubts [...] The doubt and the challenge which no one would listen to before, now become effective. These are times of transition from one state to another [...] The transition is not simply from the orderliness of one set of norms to chaos, but from one set of norms to a new set of norms through a stage of uncertainty"*.

In relation to the aforementioned claims by Sherif, it has been recently found that signals of states of surprise and uncertainty, elicited by agents in groups and social networks through natural language, *"support event analysis by communicating to others the mental state of the sender and in this way solicit their help with explaining the event"*[337]. Agents participate to group or social metacognition processes, precisely because these processes offer to their participants rich information environments, through which beliefs elicited or communicated by others can be used as elastic doxastic supports to review or stabilize epistemic beliefs in a locally optimal way: *"Peoples judgments regarding the meaning of their metacognitive experiences can impact other, downstream judgments. What is more, peoples judgments regarding the meaning of their metacognitive experiences are malleable, indicating that people who are having similar metacognitive experiences may show very different ultimate judgments as a function of their lay theories linking these experiences with meaning"*[333]

According to uncertainty-identity theory[338], social groups that are very homogeneous and polarized from a belief[339, 340] and preference[341, 342] homophily perspective, can provide a stabilizing support for the beliefs of their members. Through doxastic communication, beliefs are attracted towards the belief barycenter of the group[343]. Therefore, an agent that is able to elicit the distribution of beliefs within groups to which he is connected through social ties, can target his communication processes towards those groups and agents that exhibit the greatest doxastic affinity with him; with respect to the concentration of probability mass of expectations of group members on the states of the world that are preferred by the agent[343, 344]. Agents will therefore be "socially" and hence doxastically attracted by the largest groups whose norms and expectations are closer to their "ideal" ones. Hogg and

Blaylock[345] have found that the more a group is large and doxastically polarized, the more it *"provide[s] a sense of shared reality to their members [... these groups] serve the function of reducing these persons uncertainty. Accordingly, the greater members need for certain knowledge about the world, the greater should be their attraction to groups with a firm sense of shared reality. Such epistemic need for firm knowledge has been termed the need for cognitive closure. One may expect, therefore, that when individuals need for cognitive closure is high, groups that are able to provide coherence, consistency, order, and predictability to belief systems acquire particular appeal [... Where] the need for [cognitive] closure is defined as the desire for a quick and firm answer to a question and the aversion toward ambiguity or uncertainty. Ample evidence exists that a heightened need for closure leads to a seizing and freezing on available information and on judgments that such information implies"*.
Moreover, those events that generate high levels of surprise because considered impossible or almost impossible, have been shown to *"elevate peoples need for cognitive closure"* because through closure agents try to reduce actual and expected surprise, i.e. uncertainty. Finally, *"there is much support for the notion that a heightened need for closure leads to a syndrome of group centrism, including pressures toward uniformity, rejection of opinion deviates, in-group favoritism, out- group derogation, and the endorsement of autocratic leadership.*[345] Therefore, when agents are very much averse to surprise, the occurrence of black swan events[206, 255, 346] at first destabilizes, in doxastic reviewal pressure terms, the epistemic beliefs of agents which experience extreme surprise. Hence, it produces an even more deleterious and long-lasting effect on the structure of social networks and on the intensity of communications. Agents being put under pressure by uncertainty exhibit an increasing need for cognitive closure. However, such a need for closure, while temporally mitigating expected surprise by reducing average information flows, can further polarize society, from the point of view of its degree of doxastic group segregation. Doxastic segregation which clearly doesn't favour inter-group communication and limits the spreading and dissemination of locally emerging evidence across the social system. Limiting the forecasting accuracy of agents, and therefore, increasing the (unseen) surprise potential that the future holds for them.

According to J. G. March and H. A. Simon[347] communications across groups and social networks act as uncertainty absorption mechanisms: *"Uncertainty absorption takes place when inferences are drawn from a body of evidence and the inferences, instead of the evidence itself, are then communicated."* Therefore, as suggested by Baecker[348], communication can be seen as the process of *"determination of the indeterminate but determinable"*, i.e. the epistemic beliefs, with the aim of *"understanding the determinate"*, i.e. received doxastic signals.

The philosopher Donald Davidson, claimed[349, 350] that human rationality can be better understood as the (a-priori) fitting of beliefs to evidence, and, the (a-posteriori) judgement of observed evidence, in terms of signals and noises, on the basis of (posterior) beliefs. Where beliefs represent and characterize the patternization and causal justification of evidence. Resulting beliefs become beliefs of knowledge, and are elevated to the status of epistemic "truth", until new evidence contradicts, reveals to be unprobable, or, rationally non-justifiable, with respect to expected patterns of new evidence. According to this view through rational sense-making beliefs become justifiable and hence transferable to the mind of another human, who can hence judge his own (prior) beliefs in relation to recieved doxastic signals, and, eventually converge towards them. By so doing, the process of patternization and causal justification of evidence, including communicated doxastic evidence, is carried further in terms of epistemic completeness and signal-noise ratio[351]. Davidson also suggested that the same sort of relation occurs in a single mind through metacognition[352]. Luhman[353–357] also considered social interactions and ensuing uncertainty as a composite mechanism through which conventional beliefs shifts occur in groups and social systems. Tensions between conventional beliefs and communicated information/beliefs determines the re-negotiation of epistemic signal-noise separation mechanisms, i.e. the new shared truths and doxastic conventions necessary to coordinate the representations and forecasts of real-world phenomena. If on one hand, uncertainty is necessary for the autopoietic reorganization of societies and for their adaptation to changing information environments, on the other hand, society members cannot tolerate excessive levels of uncertainty and hence try to invisibilize sources of uncertainty by reviewing their doxastic endowments for the minimization of contingent surprise, given observed evidence. Under this perspective double contingency[358] may be viewed as a mechanism to reduce differences in epistemic beliefs, while contemporaneusly surprise emerges and is hence contracted through iterated communication and beliefs reviewal processes. Society itself can therefore be seen as an *"operative oscilation of uncertainty and organization"*[359].

Natural Vs formal language, and, the granularity of communicated uncertain(ty) signals

"Are information and uncertainty part of each other?"[360]

Uncertainty reporting and elicitation schema[361–366] have been extensively and increasingly used in the last decades, for aiding NGOs, international organizations, governments and corporates, to include experts' judgements in formal decision-making framework under "imperfect information". As pointed out by Parker and Risbey[170], there are two basic requirements that uncertainty reports should meet, generally referred-to as faithfulness and completeness:

- **faithfulness:** *"an uncertainty report should accurately describe what the agent believes the extent of current uncertainty [i.e. expected surprise] to be; it should not imply that uncertainty [i.e. expected surprise] is greater than, less than or otherwise different from what the agent actually believes it to be"*;
- **completeness:** *"an uncertainty report should take account of all significant sources of uncertainty [i.e. expected surprise], and should consider all available (relevant) information when doing so"*;

In formal uncertainty reporting[367], it is often assumed that the representation of uncertainty should take the form of a standardized schema, regardless of the extent of available information. Often the assumption is that uncertainty should be represented using precise first-order probabilities. Outcomes of interest are generally presented through probability distributions, specified over the values of a parameter or variable considered possible. In these formal uncertainty representation settings, natural language terms are also codified to avoid possible meaning ambiguities, derived from the common interpretation of a language, which not necessarily corresponds to its use in a "specialized" field. In addition, confidence bounds are added to estimated probabilities to reveal the imprecision or volatility of these inferred numbers. If, on one hand, these formal uncertainty representation schemes are powerful instruments to commensurate and elicit uncertainty; on the other hand, *"metrics to* *assess information may engender confusion when low confidence levels are matched with very high/low likelihoods that have implicit high confidence"*.[368].

Evidence shows that real-world experts, especially those that study human systems and their outcomes, like central bank governors or military councilors of governments, publicly describe their degree of confidence in their judgements with coarse-grained probabilistic/possibilistic expressions, often using natural language[369–374]. It must be remarked, that experts are not considered experts on the basis of their above-average degree of confidence in their judgments, but, on the basis of their capacity to estimate with high precision their degree of uncertainty in relation to the latter, i.e. their expected surprise conditional on their epistemic beliefs. Hence, if those who are called experts voluntarily choose to use coarse-grained judgements, then, we can hypothesize that, conditional on their beliefs that also concern the receiver of the message and noise sources of the medium, such vague/coarse-grained signals must a-priori have been considered the optimal choice to vehicle/convey a specific message, and its actual information, to targeted receiver(s). By codifying the desired message in such a way that it has, according to its sender, the highest signal-noise ratio for the reciever in expected-terms. Moreover, when experts are forced to formalize probability judgments interesting biases emerge[168, 176, 375–379]: sometimes they spontaneously use numeric intervals for probabilities, in other cases, they use numeric probability values in such a way that the number of digits of elicited probabilities is proportional to their confidence on the information set used infer that probability judgement. Both, the number of digits and the probability interval, are implicit representations of higher-order uncertainties, which are evidenced when one tries to represent formally the sources of his expected-surprise, in numeric first-order probabilistic terms. Higher-order uncertainties can therefore be "hidden" within first-order probability judgement. So maybe, granularity is not always a crude approximation of information, as we often assume, but reflects the granularity of epistemic beliefs used in the judgement, and therefore, it could be the optimal information encoding scheme for describing the actual degree of uncertainty of agents, not only experts, given their beliefs.

If we think to information from Shannon's perspective[242], which viewed the latter as a measure of surprise. Then, when an agent sends, to a target group of agents, a message of "uncertainty" concerning the state/outcome of a (named) system/process; for example: by using the noun "uncertainty" in relation to a forthcoming decision of monetary policy, like the fixing of the official lending interest rate by a central bank; he elicits and signals his epistemic beliefs on the issue. The noun "uncertainty" must be clearly distinguished from the adjective "uncertain", because the former conveys a message on epistemic beliefs that is not exclusively personal, but collective, whereas the latter doesn't. "Uncertainty" relates the beliefs of all parties involved in the communication. When an agent sends a message of radical "uncertainty", he jointly elicits and signals his epistemic beliefs concerning:

1. his own and others' epistemic beliefs. Where "others" represents the agent(s) to which the message is addressed;
2. the fact that he considers the latter doxastic endowments *(1)* totally unfit to rationally infer/anticipate the state/outcome of a (named) system/process;
3. the fact that, given *(1,2)*, he anticipates his own and others' extreme/infinite surprise, to be expected in relation to the (actual or expected) state/outcome of the aforementioned system/outcome;

Therefore when one communicates, to a target group of individuals, the occurrence of a state of, so called, radical "uncertainty", in relation to a real-world system/process and its state/outcome; we must think of it as a message with infinite entropy potential. Because in such circumstances, the sender wants to convey to the receiver(s) an idea similar to the following:
On the basis of my known knowns and known unknowns concerning the [actual or expected] state/outcome of a system/process [the grammatical complement of the word "uncertainty"] and about your epistemic beliefs concerning it; the message that i want to give you, to help you reduce the surprise that awaits you in relation to the observation of the latter state/outcome, is that neither you nor I, given our epistemic beliefs, are, at the moment, able to imagine, identify or attribute expectancy/probability mass, to the state/outcome that is more likely to occur, therefore, you as I should expect the unexpectable and prepare to it, by speculating as much as possible on the latter system/process and collecting evidence that at the moment neither you nor I have evaluated or had access to.

Such a message is by construction instrumental to social metacognition, because it refers to the senders' epistemic state, but also to the receiver(s)' epistemic state(s), as represented by the sender. The interesting point is that the message is elastic/malleable from an surprisal effect point of view, even if it has infinite surprise potential. Because:

- if the receiver is uncertainty averse, he will prefer to avoid as much as possible surprise in relation to such message. He can obtain this effect by considering the latter a declaration of total ignorance of the sender, with respect to a (named) system/process, and/or, his beliefs concerning the beliefs of the receivers. The receiver can therefore interpret the message simply as a request for help/information, which has nothing to do with his own capacity to infer the state/outcome of the (named) system/process;
- on the other hand, if the receiver is uncertainty seeking, he will prefer to think that, the sender has "complete" knowledge of knowables and unknowables, of both, himself (the sender) and the receiver, and therefore, his suggestion should be totally embraced. The receiver would experience extreme/infinite surprise, and, should consider all states/outcomes to which he previously attributed probability mass as almost impossible, and, identify states/outcomes that were considered almost impossible, or not even considered in his outcome/state space, and attribute to the latter the whole probability mass;
- a third path is also possible, if the receiver is uncertainty neutral, he will prefer to improve, as much as possible, his epistemic situation, his feeling of knowing, conditionally on his priors (those of the receiver), concerning the degree of knowledge that the sender has about himself (the receiver) and about the (named) system/process. Information/surprise extracted from the message will in this case totally depend on the reciever's priors;

We can imagine a situation in which a uncertainty averse human agent, objectively incapable of projecting colors in the hue, saturation, and brightness continuous-space, is obliged to play a game in which he has to extract every minute a ball from an urn, which he is told to contain coloured balls, of an unknown number and of unknown colors. Probably, to reduce his expected surprise, before extracting any ball he will attribute, through his beliefs, non-null probability/expectancy mass to colors he already knows, maybe through an uninformative discrete prior. However, imagine that the agent has extracted 100 balls from the urn, and at each extraction, the extracted ball is of a (objectively) different color with respect to those previously extracted, at a given moment the better epistemic strategy to reduce surprise, is to attribute all the expectancy/probability mass, in equal parts, to those colors that the agent can imagine, which haven't yet occurred, and, attributing mass 0 to those that have already occurred. Now imagine that all colors that the agent can imagine have occurred, to try to anticipate through his beliefs the extraction, and, reduce ensuing surprise, he will have to attribute mass not to imaginable colors, but to their complement, the set of unimaginable colors, which are only unimaginable and not also indiscernible. Therefore, having no other alternative he will attribute probability mass to the empty set, that will represent perceptually discernible but unimaginable events. The only valuable knowledge for him in such situation, is his awareness of the existence of these unimaginable unknowns, this is precisely what radical uncertainty represents. Now imagine that another agent enters the room and asks to the player what is the best forecasting strategy to play the game, if the two agents share the same imagination capacity, the most informative message he can give him will probably be:

prepare yourself for radical uncertainty!

As we have seen in this section, communication, in particular the communication of states of uncertainty and radical uncertainty, plays an essential role in human social metacognition, feelings of surprise and beliefs reviewal: if contingent communication activity is ignored, it will be impossible to understand and predict the degree of uncertainty of socially embedded agents, because posterior beliefs and uncertainty are mutually defined through social metacognition and its underlying communication and surprise[380]. Society is therefore the structure that creates the*"common context of reference that render shared knowledge and consensus a possibility"*. Therefore, crowd sourced measures of civil society uncertainty, based on real world communication among agents, should be developed to capture the magnitude and effects of the above mentioned phenomena, and estimate their possible economic effects.

1.4 A brief discussion on, and proposal for, endogenizing uncertainty and doxastic communication in economics

On the basis of previously reviewed works, we propose a small contribution to the existing Neoclassical EU framework. Through a change to the expected utility function, which allows to endogenize expectation communication and social metacongition processes, and to show how, on the basis of the latter, agents can jointly update their beliefs, maximize their expected utility and reduce their expected surprise, i.e. uncertainty. Our vision has been inspired by recent works by Golman and Lowenstein[40–42]. In their works, information-gaps related uncertainty aversion is integrated through the specification of a new utility function; where information utility corresponds to the preference for clarity over possible answers to questions of interest, related to the awareness of information-gaps. Information-gaps which are represented by a function of entropy weighted by attention. Through our own reformulation of expected-utility, uncertainty can be clearly distinguished from risk, without requiring to abandon the probabilistic framework for risk commensuration. This contribution, should be seen as inferred from works described and analysed in this review. Its implications will be only sketched here, and possibly, more rigorously formalized and explored in future works by the author.

Expectations communication, EU maximization and uncertainty reduction

We can imagine that, human agents may exploit communicated expectations, i.e. priors concerning the probabilities of future events, to contemporaneously maximize their expected utility and reduce their expected surprise by updating their beliefs conditional on evidence. Including doxastic evidence concerning others' expectations. By so doing agents would indirectly coordinate their expectations with others, with whom they communicate through markets and social networks. Communication of expectations among agents generates surprise and surprise is the activator of metacognition, which determines the emergence of the *third dimension of the problem of choice*, as defined by Ellsberg[83], which implies the evaluation of the nature of *"one's information concerning the relative likelihood of events. What is at issue [during metacongitive processes] might be called the ambiguity of this information, a quality depending on the amount, type, reliability and unanimity of information"*; but also, the revision of beliefs to reduce as much as possible uncertainty, conditionally on one's preferences.

Under this perspective, agents would choose individually optimal inter-subjective expectations, on the basis of their preferred trade-offs between:

- preferences for attributing probability mass to events that are more favourable to them, from which the can (mentally) anticipate the greatest utility;
- preferences for reducing expected surprise, measured as a function of relative entropy, i.e. Kullback-Leibler divergence, between (optimally chosen) posteriors and priors of their neighbours, with whom they coordinate through expectations communication, and optionally, their own;

This view may be seen as a mathematical transposition of what Allais[82] called the instrumental deformation of subjective probabilities:*"certaines personnes qui ont confiance en leur etoile sous-estiment la probabilité des évènements qui leur sont défavorables et surestiment la probabilité des évènements qui leur sont favorables"*; jointly with the idea that *"too much choice [concerning probabilities of future events] can produce a paralyzing uncertainty"*[381].

In a simplified version of what should be developed as a multi-agent network model, agents, expected utility (EU) maximizers, would endogenize probabilistic expectations, i.e. sets of probabilities of possible events at a given time horizon, using them as EU maximization arguments. These probabilities should be subject to min-max constraints determined, for each possible event, by the range of observed probability values in the probabilistic expectations set containing the prior of the agent and the priors communicated by other agents with whom there has been communication. Uncertainty averse agents, would take into account the cost opportunity of -self and social- metacongitive uncertainty by having, as an additional component in their expected utility function, a penalty equal to a weighted sum of pairwise Kullback-Leibler (KL) divergence, between optimal posterior expectations and the priors of agents with whom they have communicated, and optionally, their own prior. This utility penalty represents the cognitive difficulty of holding and relying on a set of beliefs, concerning the future, which diverge with respect to those held by other (neighbouring) agents operating in the same environment.

Similar theoretic settings have been previously used to model coordination games in presence of imperfect information or strategic uncertainty. For example, Golub and Jackson[382–384] model the lack of consensus in a dynamic network coordination game, through a distance of beliefs measure, that has various points in common, but which is not identical to, the concept and measure of uncertainty that we have proposed. In their framework, consensus -belief/information coherence in the network- is reached through an iterated process of beliefs contagion among neighbours. Agents start with different belief sets concerning the distribution of possible state of the world. They are imperfect sensors, because they receive signals subject to group-specific random-noise structures. Agents know that they do not perfectly observe the state of the world, and, that also others do not perfectly observe such a state. By being aware of the possible imperfection of knowledge derived from local signals, they are individually and collectively uncertain/doubtful about their understanding of the world and of events occurring within it. By so doing they exhibit the individual and social metacognitive capacity that determines a state that we may label as uncertainty, as described in the previous sections of this work.

Accordingly, they coordinate on the unknown true state of the world by communicating with their neighbors. To disentagle signals -of the true state of the world- from noise, and by so doing, they reduce uncertainty. They synthesize posteriors by averaging the prior beliefs of their neighbors. Interesting enough, the authors find that the dynamics of belief convergence in such games can be inferred through a reduced "representative agent" network model, in which there is only one node for each type of agent, which allows to approximate consensus distance dynamics through the simplified framework. The authors claim that an average-based belief updating mechanism has a number of properties that render it a technically appealing and empirically plausible assumption[384]:

- it leads distributed network systems of agents to converge to an efficient coordination equilibrium using, iteratively, computationally cost efficient heuristics.
- under some non stringent conditions, concerning size, the group distribution, the density and the degree of omophily in the network, such an updating scheme converges -in probability- to fully rational limiting beliefs, i.e. Bayesian posterior conditional on all agents' initial information sets;
- it is similar to what the majority of individuals appear to do when placed interacting an experimental social learning setting and in empirical studies;

Relative entropy of communicated expectations-gaps as expected surprise

In our framework, communicated expectations are considered local signals about the event-space already elaborated by network neighbours and hence integrated in their probabilistic expectations.

Imagine that there are m agents, jointly communicating to each other their priors (probabilistic expectations) in a clique, concerning n mutually exclusive and collectively exhaustive events at a given horizon, where the partitioning and horizon is common knowledge. Since in our setting agents are communicating in a network through natural language we can imagine that each event is defined by a unique sequence of words and that, in such circumstances, one event should always be the complement of the union of all others, therefore the latter event will be definable only in relation all others, for example through the words *"none of the aforesaid events will occur"*. Call $\mathbf{P} = \{P_1, \dots, P_m\}$ the set of priors, i.e. expectations before communication; where P_k represent the discrete distribution of probabilistic expectations of the k^{th} **agent**. The prior probability of occurrence of the event j for agent k is given by $P_k(j)$. In relation to what we have explained in section 1.2.3, we can imagine that, for the k^{th} **agent**, the sum of all Kullback-Leibler belief divergences with the priors communicated by agents, including his own: the sum of $\mathrm{KL}(P_x \parallel P_k)$ for $x \in \{1, \dots, m\}$ that we call Aggregate Relative Entropy (ARE), will represent the aggregate (extra) expected surprise of agent k, in relation to a specific partitioning of the outcome space, emerging through the process of communication and comparing of his prior with probabilistic expectations communicated by his neighbours:

$$\begin{aligned} \mathrm{ARE}(k) &= \mathrm{KL}(\mathbf{P} \parallel P_k) \\ &= \sum_{i \in \{1,\dots,m\}} \mathrm{KL}(P_i \parallel P_k) \\ &= \sum_{i \in \{1,\dots,m\}} \left(\sum_{j=1}^{n} \ln \left(\frac{P_k(j)}{P_i(j)} \right) P_i(j) \right) \end{aligned} \tag{1.4.9}$$

Clearly, it will be sufficient that the k^{th} **agent** considers one event j almost impossible ($P_k(j) = 0$) when at least one of the other $k - 1$ agents considers it possible with non-null probability ($\exists i \neq k \; P_i(j) \neq 0$), to experience a state of extreme expected surprise: $\mathrm{ARE}(k) = +\infty$ also called radical uncertainty.

If we divide aggregate relative entropy 1.4.9 by the number of agents with whom k^{th} **agent** interacts, we obtain the Mean Relative Entropy (MRE):

$$\begin{aligned} \mathrm{MRE}(k) &= \frac{1}{m-1} \mathrm{KL}(\mathbf{P} \parallel P_k) \\ &= \frac{1}{m-1} \sum_{i \in \{1,\dots,m\}} \mathrm{KL}(P_i \parallel P_k) \\ &= \frac{1}{m-1} \sum_{i \in \{1,\dots,m\}} \left(\sum_{j=1}^{n} \ln \left(\frac{P_k(j)}{P_i(j)} \right) P_i(j) \right) \end{aligned} \tag{1.4.10}$$

Mean relative entropy represents the mean (extra) expected surprise emerging through the process of communication and comparing of probabilistic expectations with a neighbouring agent in a communication/social network.

If the k^{th} **agent** changes his probabilistic expectations after the communication process, by choosing a new set of probabilities $P_k^{post} \neq P_k$, the sum of all Kullback-Leibler divergences between communicated priors, including the prior of the agent k, and the posterior P_k^{post}, called Posterior Aggregate Relative Entropy of (PARE), will represent the posterior expected surprise of agent k, given his new beliefs P_k^{post}, in relation to previously communicated expectations and his own prior:

$$\begin{aligned}\mathrm{PARE}(k) &= \mathrm{KL}(\mathbf{P} \parallel P_k^{post}) \\ &= \sum_{i\in\{1,\ldots,m\}} \mathrm{KL}(P_i \parallel P_k^{post}) \\ &= \sum_{i\in\{1,\ldots,m\}} \left(\sum_{j=1}^{n} \ln\left(\frac{P_k(j)^{post}}{P_i(j)} \right) P_i(j) \right)\end{aligned} \tag{1.4.11}$$

If $\mathrm{PARE}(k) < \mathrm{ARE}(k)$ by changing his probabilistic expectations from P_k to P_k^{post} the k^{th} **agent** was able to reduce aggregate relative entropy by $(1 - \frac{\mathrm{PARE}(k)}{\mathrm{ARE}(k)}) * 100$ percent, by reviewing his beliefs.

A modified expected-utility function with expected surpise

If agents are uncertainty averse, to reduce expectations-gaps related uncertainty, i.e. ARE, resulting from the communication of expectations among agents, they can update their priors to reduce expected surprise. In such a framework posterior expectations would represent individually optimal inter-subjective probabilities of events, conditionally on preferences and partitioning used in the expectations communication process. Posterior expectations would take into account the expected surprise that can emerge in relation to coordination failure, with reference to neighbouring agents in the social/communication network. Neighbouring agents which, from a causal point of view, will more likely determine the perceptual/information/doxastic evidence and environment to which an agent will be subject to, and its ensuing surprise potential. In such a framework the KL penalty, which represents the subjective costs of uncertainty, in terms of aggregate expected surprise, will curve the expected-utility indifference curves of the agent, in the event-probability space, producing individually optimal inter-subjective probability indifference curves. However, choosable combinations of probabilities will have to respect the probability laws. In addition, for each event j, with $j \in \{1, ..., n\}$, probabilities will have to be contained in the min-max range of "observed" probabilities $[\min(\mathbf{P}(\mathbf{j})), \max(\mathbf{P}(\mathbf{j}))]$ for that event, i.e. in the expectations-set of the agent, containing his own and others' communicated priors. Under some conditions on the distribution of communicated beliefs/expectations, these indifference curves will be convex, as the classical utility indifference curves when represented in the goods-quantity space.

We can imagine a round of probabilistic expectations revision to be organized as follows:

1. **Individual observation and expectations updating phase:** the agent observes his environment, collects evidence and reviews his beliefs on the basis of non-doxastic evidence, using his preferred updating mechanism, for example: bayesian updating; *We do not explain or refer to this phase through our model*

2. **Social doxastic communication phase:** the agent turns to the social world to communicate his expectations for a specific common knowledge partitioning of the outcome-space at a given horizon. By coming to know the probabilistic expectations of others, each agent has the possibility to metacognitively project himself and his probabilistic expectations in the support delimited by "observed" upper and lower probability bounds; for each event these are the

extremum (min and max) values of the set containing probabilities communicated by others and one's own prior;

3. **Social expectations updating phase:** The agent maximizes his expected utility as a function of probabilistic expectations, which, if the agent has an optimistic future orientation, are individually optimal inter-subjective beliefs used to anticipate future utility. By so doing, he also optimally reduces his uncertainty, in terms of expected surprise, which perturbs/damages his capacity to anticipate utility from probabilistic expectations, *as explained here below*;

The optimal posterior probabilistic expectations P_k^{post*}, which is the vector of posterior probabilities P_k^{post} which maximizes the (modified) expected utility $EU_k(P_k^{post})$ function of the k^{th} **agent**, are given by:

$$\begin{aligned}
&P_k^{post*} = \arg\max_{P_k^{post}} EU_k(P_k^{post}) \\
\text{Where:}\quad & EU_k(P_k^{post}) := \beta_k \sum_{j=1}^{n} u_k(j) P_k(j)^{post} - \alpha_k \mathrm{KL}(\mathbf{P} \parallel P_k^{post}) \\
&\equiv \beta_k \sum_{j=1}^{n} u_k(j) P_k(j)^{post} - \alpha_k \sum_{i=1}^{m} \mathrm{KL}(P_i \parallel P_k^{post}) \\
&\equiv \beta_k \sum_{j=1}^{n} u_k(j) P_k(j)^{post} - \alpha_k \sum_{i=1}^{m} \left(\sum_{j=1}^{n} \ln\left(\frac{P_i(j)}{P_k(j)^{post}} \right) P_i(j) \right) \\
\text{Subject to:}\quad & \forall j \in \{1,\ldots,n\} : 0 \le \min(\mathbf{P}(j)) \le P_k(j)^{post} \le \max(\mathbf{P}(j)) \le 1 \\
&\sum_{j=1}^{n} P_k(j)^{post} = 1 \quad , \quad \beta_k \in \{-1,0,1\} \quad , \quad \alpha_k \in \mathbb{R}
\end{aligned} \tag{1.4.12}$$

Where α_k is a scaling factor parameter,
which represents the agent's degree of aversion to expected surprise;
if $\alpha_k > 0$ the agent k is uncertainty (expected surprise) averse;
if $\alpha_k < 0$ the agent k is uncertainty (expected surprise) seeking;
if $\alpha_k = 0$ the agent k is uncertainty (expected surprise) neutral;
Where $\beta_k \in \{-1,0,1\}$ is a parameter that represents the future orientation psychology of the agent;
if $\beta_k = 1$ the agent k is optimistic;
if $\beta_k = 0$ the agent has no future orientation
he is indifferent in relation to the anticipation of future utility and disappointment;
if $\beta_k = -1$the agent is pessimistic;

The utility $u_k(j)$ represents the actual anticipated utility that the k^{th} **agent** would experience if all agents, including himself, would be sure that the event j will occur almost certainly, with probability one, all other events being almost impossible.

The posterior probability $P_k(j)^{post*}$ represents the rationally optimal *mental expectancy*[139–141] of event j for agent k.

In general we will have that $\beta_k = 1$ and $\alpha_k > 0$, which means that the k^{th} **agent** is optimistic and uncertainty averse, other parameter values are related to very special cases. It must be remarked that the utility penalty due to the divergence from prior beliefs is independent from how these beliefs were derived.

The future orientation represents an asymmetry in the treatment of doxastic evidence concerning the future. If agent k is optimistic ($\beta_k = 1$) he will attribute relatively higher probability weights to those events that generate the greatest anticipated utility in his mind, than if he would have done if he had no future orientation ($\beta_k = 0$). An optimistic agent, by attributing greater probability weights to favourable events, should in average be more surprised by the ex-post occurrence of events in the left side of the distribution for which relatively more doxastic evidence has been underweighted/ignored, i.e. the events that were a-priori considered "unfavorable" in terms of maximum anticipable utility ($u_k(j)$). Such an agent should be "by evolutionary nature" more resilient to "bad" news, or in more precise terms, less sensible (in terms of expectations change and disappointment) to the communication of expectations that attribute high probability to those events relatively more unfavourable to him in terms of maximum anticipable utility.
If agent k is pessimistic ($\beta_k = -1$) he will attribute relatively greater probability weights to unfavorable events and will be in average ex-post more surprised by the occurence of events in the right side of the distribution, for which relatively more doxastic evidence has been underweighted/ignored. Such an agent is more vulnerable to "bad" news, or in more precise terms, more sensible to the communication of expectations that attribute high probability to those events relatively more unfavourable to him in terms of maximum anticipable utility. From a psychological and neurological perspective pessimistic agents are agents for which the right hemisphere of the brain predominates during speculations about the future; which avoid sources of possible (ex-post) disappointment by underweighting the probabilities of events that are more favourable to them. As pointed out by Hecht[385] *"the optimistic schema is scaffolded and assimilated into neural structures and systems within the LH [left hemisphere] , while the pessimistic schema is primarily associated with, and integrated into, neural circuits and networks in the RH [right hemisphere]*. On one side stubborn optimism (when $\beta_k = 1$ and $\alpha_k = 0$) *"may lead to negligent and reckless behaviors - e.g. not taking the necessary precautions to prevent common hazards - which may result in a catastrophe"*; on the other side, a socially exasperated pessimism (when $\beta_k = -1$ and α_k very large) may push one to worry *"too much about potential dangers [mentioned by social network neighbours and in the news] and focusing on what might go wrong [, which] leads to avoidance behavior, passivity, exacerbation of low mood and an increase in the vulnerability to depression"*. As we can see from our model, agents that have a future orientation ($\beta_k \neq 0$) are vulnerable to preference-driven expectation confirmation bias, on the right side (optimism) or on the left side (pessimism) of the distribution of the anticipable utility of events. The parameter α determines how reactive an agent is to communicated expectations, i.e. his belief reactivity, or probabilistic expectations elasticity, in relation to the psychological pressure for the minimization of expected surprise after expectations communication.

Residual beliefs divergence $\mathrm{KL}(\mathbf{P} \parallel P_k^{post*})$ of the k^{th} **agent** is considered the epistemic-noise, the residual incoherence of expectations, which we generally call uncertainty. Which represents residual expected surprise at the end of the process of beliefs revision, when optimal posterior beliefs P_k^{post*} have been determined. It is residual not because impossible to reduce, but because considered inconvenient to be further reduced, in terms of deviation from the optimal trade-offs between expected surprise $\alpha_k \mathrm{KL}(\mathbf{P} \parallel P_k^{post})$ and anticipated utility $\sum_{j=1}^{n} u_k(j) P_k(j)^{post}$. This epistemic-noise doesn't represent, in objective terms false or wrong doxastic information, concerning future events. But, more simply, it represents that part of doxastic information which is not compatible with an agents' optimal probabilistic expectations, which however still produces for him an expected surprise.

We can imagine real-life situations in which an agent exhibits distinct preferences for priors not in terms of their contents but in terms of their source, i.e. the agent from which they come, including oneself. In this case our (modified) expected utility function would be rewritten as follows:

$$EU_k(P_k^{post}) := \beta_k \sum_{j=1}^{n} u_k(x_j) P_k(j)^{post} - \alpha_k \sum_{i \in \{1,\dots,m\}} \gamma_k^i \mathrm{KL}(P_i \parallel P_k^{post}) \qquad (1.4.13)$$

The parameter γ_k^i can be used to represent several social factors, like authority, legitimacy, affection or reputation, recognized by the k^{th} **agent** to the i^{th} **agent**, in terms of weight/relevance given to his communicated expectations. The γ_ks should be rescaled in such a way that $\sum_{i\in\{1,\dots,m\}} \gamma_k^i = 1$
When expectations are discriminated by source, the γ_k^i are different, we expect that for the k^{th} **agent** $\gamma_k^k > \gamma_k^i$ for $i \neq k$. Because γ_k^k represents the weight given to his own prior expectations. Our model is almost an isomorphism of Keynes's view of *conventional expectations*:

"Notwithstanding uncertainty, individuals have to make decisions and act. They do so by pretending that they have behind them a good Benthamite calculation of a series of prospective advantages and disadvantages, each multiplied by its appropriate probability, waiting to be summed. In order to behave in such a way, some techniques are devised. They are essentially conventions like assuming that the present is a reliable guide to the future despite the past evidence to the contrary or trying to conform to the behavior of the majority. This means that individual decisions and actions cannot be regarded as independent of one another." [46]

We extract four key points in relation to the role of *conventional expectations* that unite our framework to Keynes's view:

- uncertainty is a distributed state concerning expectations, and more generally epistemic beliefs, which is related to the local incoherence of expectations communicated in multiple agent network systems. Agents coordinate their actions through the communication/contagion of beliefs concerning the future, i.e. expectations;
- under given conditions, related to the structure of the network and the distribution of preferences, the system may slowly converge (in probability) to coherence, i.e. the global optimum of the system;
- if all agents have the same preferences, risk aversion and uncertainty aversion, the system benefits as a whole if all agents are represented through a unique (common knowledge) set of conventional expectations. Because the convention reduces the need of continuously communicating with all neighbouring agents to elicit their updated beliefs, and also, because it is memory efficient. Agents' doxastic identity becomes insignificant because they all coordinate through the same set of impersonal conventional expectations. In information terms, all agents' beliefs are efficiently approximated by conventional expectations of a representative agent;

In our framework beliefs are Markovian. Once a node (agent) communicates with his neighbouring nodes and reviews his expectations on the basis of evidence he forgets his old priors. However, we hypothesize that the memory of past uncertainty levels may be stored, and, used later on to compare the present uncertainty situation to the past, and by so doing, evaluate if it is a good moment for communication or for closure. On the basis of value, sign and/or slope of the surprise and uncertainty outcomes of communication. For example: if the residual expected surprise $\mathrm{KL}(\mathbf{P} \parallel P_k^{post*})$ is very high respect to its average value, in past expectations revision processes, it may seem convenient for agents, in expected terms, to communicate such a state of uncertainty to immediately require/trigger another round of communication. In such a way agents can elicit posterior expectations of neighbouring agents $\mathbf{P}^{post*}$ and their potential information/surprise effects $\mathrm{KL}(\mathbf{P}^{post*} \parallel P_k^{post*})$, and iterate the process of revision of beliefs again, to try to converge to states of lower metacognitive uncertainty. Therefore, real-world public communications of uncertainty should be considered precious signals to evaluate locally (for specific agents) and at the aggregate (social) level expected surprise in human systems, which may also influence the behaviour of agents in markets.

In our model, metacognitive processes and their doxastic outcomes are not independent from preferences. Preferences for events, produce indirectly the effect of preferences for probabilistic expectations, but only in a instrumental way, because mediated by doxastic evidence and its expected surprise effects. If agents are indifferent to events, the process of learning is in epistemic terms optimal, the posterior maximizes the feeling of knowing, because the signal/noise ratio of doxastic evidence communication is maximized when the sum of Kullback-Leibler divergences is minimized, i.e. when agents feel the smallest possible expected surprise given their beliefs and observed evidence. If agents are not indifferent to events, i.e. they derive more utility from some events respect to others, the process of learning is still optimal in rational terms, i.e. the posterior maximizes the modified expected utility function, but not in terms of the minimization of expected surprise, agents are biased in their usage of doxastic evidence by their preferences. However, posteriors are always objectively optimal conditionally on the agents' preferences. Residual uncertainty represents the expected-surprise effect due to the awareness of the existence doxastic evidence that is ignored because incompatible with optimal beliefs. Voluntarily ignored doxastic evidence consists of information that one hasn't yet been (sufficiently) incentivized to integrate in his rationally optimal belief system, given his will to maximize the modified expected utility function, which represents the psychological benefits/well-being that an agent feels by anticipating utility through his rationally optimal inter-subjective probabilistic expectations. In a more sophisticated version of the model, we could imagine that ignored doxastic evidence is ex-post not simply treated as noise, because the agent who ignored it could in a second moment regret his "disbelief choice", if, doxastic evidence first considered noise reveals a-posterior to be useful to reduce actual surprise and disappointment, when the time horizon of the probabilistic expectations is reached and one of the possible events occurs.

For seek of realism, we could also hypothesize that the partitioning process itself determines the future orientation (pessimism and optimism) of agents, when having to update their beliefs after a process of communication. We could imagine that when an agent is confronted with a partition that describes in detail and with a thin (event) grain negative scenarios, i.e. those with the smallest anticipable utilities, and jointly uses a thick grain or omits to differentiate and describe positive scenarios by leaving them in the residual event, i.e. the event defined as the complement of the union of the all the other events, the future orientation of this agent will very likely be pessimistic; this because, through the aforementioned partition, the agent's thoughts are oriented and focused towards more adverse scenarios, which are rendered salient and explicit. Whereas, when an agent is confronted with a partition that describes in detail and with thin (event) grain positive scenarios, i.e. those with the large anticipable utilities, and omits to differentiate and describe in detail negative ones by leaving them in the residual event, i.e. the complement of the union of all others, his future orientation will very likely be optimistic. The aforesaid mechanism could be a way of endogenizing the future orientation in our model. We could finally imagine that the negotiation of the partition used by agents taking part to the process of communication of probabilistic expectations is strategic, agents could manipulate their own and others' future orientations in the preferred way by strategically partitioning the outcome space through natural language.

1.5 Discussion

As we have seen in this article, in its more radical interpretation, metacongitive uncertainty represents situations in which the only valuable knowledge one possesses is his awareness of the existence of unimaginable unknowns. Under this perspective, since ancient times, uncertainty has been seen in philosophic literature as a epistemic state of extreme scepticism towards forecasts of future events, which can push one up to the point of considering specific real-world systems/processes and their states/outcomes unimaginable or incommensurable.

However, as we have seen throughout this review, in particular in relation to cognitivist theorizations of uncertainty, if an agent is averse to this epistemic state of surprise and expected surprise, he will try to individually and collectively reduce his uncertainty trough self and social metacognition and metaheuristics: he will search for evidence, including doxastic evidence communicated by other agents, which can be used, together with metaheuristics, like bayesian probability, to review his beliefs in such a way that the ensuing expected surprise, conditional on evidence and his own preferences, is reduced as much as possible. If an agent values positively the epistemic state of uncertainty, in relation to a specific or all aspects of perceived reality, he may simply contemplate his expectation of infinite surprise in relation to the latter without updating his epistemic beliefs.

Uncertainty can therefore be seen as the state of awareness of knowledge failure in expected terms. We metacognitively consume surprise/information, to optimally learn and collapse to more stable and justifiable epistemic beliefs, in relation to observed evidence and one's preferences, and by so doing, we reduce expected surprise. It is through the pressure of surprise and expected surprise that we are pushed to synthesize new beliefs instrumental to the survival of any intertemporal feeling of knowing. To release ourselves from the ineluctable tendency to aware *epistemic horizon* failure that uncertainty brings. Aware failure of our knowledge of the world, of other beings and of their beliefs, and, aware failure of the preservation and survival of our own awareness and knowledge, as individuals, groups, societies and species. For uncertainty averse agents, economic and social life can be transformed in a search for an escape way from uncertainty. In belief revision terms it can be seen as the use of rationality in a environment that is not deterministically imposed to human agents, but, which is exploitable and transformable by the latter through their choices, expectations and communications.

Orientations towards the future, probabilistic expectations and their communication shape the doxastic evidence environment that will be later observed. The escape direction from uncertainty that one chooses, if he is uncertainty averse, represents precisely his capacity in creating mental attractors for his awareness, through self and social metacognition and ensuing sense-making and decision-making. Mental attractors which may well be preference-driven. The more one is able to "create" (live in) a stable environment, in terms of controllable probing signals and predictability of ensuing evidence, the easier it will be for him to stabilize his epistemic beliefs. From this point of view, preferences, which favour the repetitive and voluntary seeking of particular evidence, can be considered, at the individual and collective level, epistemic belief stabilizing forces.

However, the farther we get from uncertainty, through the artificial creation of evidence in our environment, the greatest is the implicit sacrifice, in terms of individual, group and social fixation of our beliefs, and, the greater will be the surprise, if our perceptual/information/doxastic evidence environments are perturbed in an unforeseen way. Therefore, the alienation of uncertainty is jointly the measure of the power of individually and collectively using our metacongitive capacities to regenerate and protect our feeling of knowing, and, the measure of our will to undertake the path of denial of the acknowledgement of our ignorance of the future, as individuals, groups, societies and species. Uncertainty can be considered as one of the outcomes of a metacognitive mechanism that allows one to evaluate the signal importance of epistemic "noise", a process of trembling cognition, which can serve to abandon local optima and explore alternative belief and knowledge systems. Our civilizations, languages, concepts and institutions, represent such a millinery struggle against uncertainty, through the construction and fall, in our imagination and human worlds, of temples of conventions. For the teaching of metaheuristics and for the use of metacognition; necessary to coordinate our epistemic beliefs and to protect aware life from unreachable explanations and unexplainable understandings.

Chapter 2

Twitter uncertainty indexes and uncertainty contagion during the unfolding of the Brexit-Trump Era

By Carlo R. M. A. Santagiustina

Abstract

We develop a set of *non-market uncertainty* measures, called Twitter uncertainty (TU) indexes, for the United Kingdom (UK) and the United States (US), by aggregating decentralized signals of uncertainty elicited through a real-world online communication medium, the Twitter news and social networking platform. We use our new measures to infer *market non-market uncertainty* contagions, within and among these two countries, in a VAR modelling setting. To compute our TU indexes we use messages containing the word *uncertainty*, published publicly on Twitter during a nine months time interval that covers UK's EU-referendum and the 2016 US's presidential election. We exploit Twitter as an information source that contains the *wisdom of the crowds* concerning the degree of *civil-society uncertainty*, self-reported on Twitter by the worldwide english-speaking community, in relation to the United Kingdom (UK-TU) and the United States (US-TU), across the 2^{nd}, 3^{rd} and 4^{th} quarter of the year 2016. We hence estimate a structural VAR model to infer the dependencies among: *civil society uncertainty*, measured with our US-TU and UK-TU indexes; *policy uncertainty*, measured with the US-EPU and UK-EPU; *market uncertainty*, measured by VIX and VFTSE option-implied volatility indexes. Results show that, at the country level, there is a relevant bidirectional Granger causation relationship between *civil society uncertainty* and *market uncertainty*, and only for UK, between *policy uncertainty* and *market uncertainty*. In addition, *civil society uncertainty* shocks in the US had positive and significant spillover effects on UK's *market uncertainty* during the year 2016.

2.1 Introduction

Objectives and delineation of the research structure and boundaries

In this article, we propose a new set of uncertainty indexes that allow us to aggregate and measure decentralized signals uncertainty coming from the Twitter social network and its community, called Twittersphere. We use these signals to model processes of inter-source and inter-area uncertainty contagion during the year 2016. This study is of particular interest because of the occurrence of two major events, widely associated to the concept and phenomenon of uncertainty, that unfolded during the aforementioned year in the two countries analysed in this work: the vote in favour of Brexit in the United Kingdom and the election of Trump in the United States. These occurrences are a natural experiment setting, which we will exploit, to infer processes of uncertainty contagion among market and non-market systems in the UK and the US, both within and between these two countries, in the year 2016. In addition, this study contributes to existing literature because it allows us to differentiate the ripple effects on market systems of *civil society uncertainty* from those of *policy uncertainty*, in terms of impulse-responses of option-implied volatility in these two countries.

In the first section we describe the methodology used to build Twitter Uncertainty (TU) indexes from a database of more than one million Twitter posts, also called tweets, written in english and containing the term *"uncertainty"*. Tweets are textual messages published publicly by users, with a maximum lenght of 140 characters. Tweets are generally used for microblogging, social deliberations and real-time news publishing, commenting and retweeting. We explain in which terms our TU indexes are proxy measures for *civil society uncertainty* in relation to the two geographic-areas of interest for this study. By *civil society* we mean a public information and deliberation space between States, NGOs, firms, financial markets, medias and households, in which agents undertake processes of group or social metacognition[14, 15] and communicate their ensuing uncertainties. By *uncertainty* we mean actual and expected surprise in relation to information-gaps and expectations-gaps of agents participating to processes of communication and social metacognition. For an in-depth explanation of our approach to *uncertainty*in general and to *civil society uncertainty* in particular, we refer to the first article in this collection, titled *Uncertainty: reviewing the unknown*.

In the second section we present *market uncertainty* and *policy uncertainty* proxy meausures that will be used in the third section to model inter-source inter-area uncertainty contagions in the United Kingdom and United States. *Market uncertainty* in UK and US is respectively proxied through the VFTSE and VIX option implied volatility indexes. Whereas *policy uncertainty* in UK and US is proxied through the daily Economic Policy Uncertainty indexes by country, developed by Baker, Bloom and Davis[386]. We explain their construction methodology and compare their time series to our *civil society uncertainty* TU indexes' time series.

In the third section we use our Twitter Uncertainty indexes, jointly with other *market and non-market uncertainty* measures, to infer if, and in which terms, the proxied degree of *civil society uncertainty*, allows us to anticipate and explain, in a structural VAR modelling setting with constant parameters, the observed degree of *market uncertainty* and *policy uncertainty* in these two countries, across the 2nd, 3rd and 4th quarter of the year 2016. We comment the results of our inference results in terms of lagged Granger causation and instantaneous Wold causality. We show in which terms the fluctuations of these different uncertainty sources may be anticipated through the use of our VAR model for forecasting. We finally undertake an historical decomposition of observed uncertainty variables' fluctuations, to impute changes in each uncertainty measure, to endogenous shocks and fluctuations of other uncertainty measures endogenized through our VAR model.

To put a limit to the length of this article, and to try to avoid omitting details potentially of interest for the readers, we included a rich appendix with an in-depth analysis of the content of UK-TU and US-TU indexes, which may be considered a manual validation process to further empirically legitimate the use of our TU indexes as proxies of *civil society uncertainty*. Always in the appendix we describe the details of our model specification choices, like the information criteria used for the choice of the lag order and the stationarity tests. We include orthogonalized cumulative impulse-response functions and the forecast error variance decomposition resulting from the estimated structural VAR model. As well as the robustness checks, together with the estimation of our uncertainty contagion model under alternative specifications, and, constraints on contagion channels.

In the introductory subsections, which follow, we delineate the phenomenological context and research field in which this work is empirically and theoretically ubicated. We link its content to real-world events that occurred during the period under investigation and to existing studies that analysed, measured or modelled the effects of uncertainty on market systems during the unfolding of the Brexit-Trump Era. We briefly cover and review the topics, questions and hypothesis formulated in other works concerning the inference of *market non-market uncertainty* dependency relations. We briefly survey existing literature on text-based proxies for latent variables, in particular, with reference to risk and uncertainty measurement. We illustrate the uses in literature of these uncertainty proxies, in particular, for the modelling of the interdependences among *market and non-market uncertainty* variables. Finally, in relation to reviewed works and existing literature gaps, we illustrate our own research questions and hypotheses, which will be empirically tested and discussed in the next sections of this work.

Context and field of enquiry

"How Brexit Uncertainty Could Produce a British Boom" (June the 29th, 2016)

is the title of a post[387] published on *The Wall Street Journal*'s web-page, few days after the EU-referendum vote in the United Kingdom.

"An Age of uncertainty is upon us" (November the 19th, 2016)

is the first sentence of an article[388] published on the of *The Economist*, few days after the United States presidential election.

"Finance, it's the Age of uncertainty"(December the 15th, 2016)

is the heading of an article[389] published on the weekly magazine *l'Espresso*, few days after the Italian Constitutional Referendum. *"Measuring uncertainty is no game for economists"*, states the article among its conclusions, in bold.

Will 2016 be remembered as the year of the global apogee of the concept and feeling of uncertainty? Or maybe, it's just an illusory perception of contemporaries, which will soon be refuted. Have we, contemporary men and women, sailed too long in dead calm seas? Forgetting the fear and the tumult caused by the arrival of the umpteenth storm: the storm of (expected but still indeterminate) change. Is uncertainty only a *"wipping boy"*, created to justify our limited foreseeing capacity, and, the lack of consensus among policy experts', economists' and market operators' expectations, as supposed[390] by the Bank of England's external member of the Monetary Policy Committee Kristin Forbes. The same Kristin Forbes which, a few months after Brexit, also affirmed that *"uncertainty is not as bad for the economy as feared"*[391]. Probably, this fear of a speculative bubble on the word uncertainty, stems from the fact that the Bank of England itself used the word "uncertainty" 123 times in official communications during the year 2016[392]. In the light of that, what is was the use of the word uncertainty signalling?

In the world of finance and business, it was shared opinion that *"events in Britain, Italy and the U.S. created turmoil in the markets, while decisions by the Federal Reserve, the European Central Bank and the Bank of Japan further stoked it"*[393]. As pointed out by the president of UBS investment bank, Andrea Orcel: during the year 2016 financial industry and operators *"have moved from trying to manage risk where you prepare for it, you have historic series, you have hedges, you make decisions, you debate them, you plan for different scenarios to managing uncertainty. Uncertainty is very different. Uncertainty is, 'OK, there's going to be an election. Is Brexit going to happen?' How do you judge that? If it happens, what's the consequence? How do you hedge? How do you price for actions as a result of a tweet or a throwaway comment without getting it wrong?"*[394].

This generalized state of uncertainty that followed the Brexit vote and the election of Trump was not limited to local and isolable political, financial and economic affairs[395]. It rapidly extended globally, across borders and systems, like an underground river in full spate, which shook the foundations of human expectations, making people jointly feel in precarious balance in all spheres of life, as if their expected surprise related to the foreseeing of the future had dramatically and suddenly grown, especially in relation to economic decision and sense making. While uncertainty eroded the epistemic pillars of knowledge and belief systems, human capacity to formulate and justify probabilistic expectations, and, to reach, through the communication of the latter, social and market consensus on the more likely economic and political scenarios to be expected, was temporally lost. The process of coordination of expectations failed. People started looking to their future with a sense of systemic uncontrollability, helplessness, concern and anxiety, awaiting the opening of the next economic, financial or political chasm. Risks became systemic, difficult to commensurate and isolate, because political, economic and social phenomena appeared to such an extent intertwined that they were almost-impossible to compartmentalize and represent, through rational sense-making, as isolable-systems, uncertainty became once again the one master of peoples' mind, giving the go to a social metacognition mechanism capable of forcing people to confront with intolerable levels of expected surprise, resolvable only through further communication and revision of beliefs, that can eventually ultimate in the construction of revised conventional inter-subjective belief systems, usable to coordinate human expectations and actions in market and social systems.

Given the complexity of the aforementioned global "uncertainty puzzle", very few macro-economic studies tried to model the dependency relations among different

-specialized- uncertainty variables that represent distinct sources or types of uncertainty, to which a market system may be subject to. As Jurado et al. pointed out[396]: "*the conditions under which common [uncertainty] proxies are likely to be tightly linked to the typical theoretical notion of uncertainty may be quite special. Stock market volatility can change over time even if there is no change in uncertainty about economic fundamentals, if leverage changes, or if movements in risk aversion or sentiment are important drivers of asset market fluctuations. Cross-sectional dispersion in individual stock returns can fluctuate without any change in uncertainty if there is heterogeneity in the loadings on common risk factors. Similarly, cross-sectional dispersion in firm-level profits, sales, and productivity can fluctuate over the business cycle merely because there is heterogeneity in the cyclicality of firms' business activity*".

However, in most studies concerning the period of the Brexit vote and election of Trump, uncertainty was proxied by only one of these measures, or, by using the first principal component extracted from two or more among them[390]. Models that use only one uncertainty measure or one principal component are potentially subject to omitted variables and aggregation biases. These are typical problems in the area of macro-economic studies that seek to commensurate the effects of "generic" uncertainty, which produce the following consequences:

- **No uncertainty heterogeneity:** By representing uncertainty through a unique aggregate measure it results impossible to identify eventual significant differences among the responses of aggregate economic variables to uncertainty impulses/shocks that are different "by nature";
- **No dependencies among specialized uncertainty variables:** By representing uncertainty through principal components is equivalent to assume that variables' observed values are independent in time, therefore by using PCA we cannot appraise the lagged dependency structure -and hence reinforcing feedback mechanisms- among different specialized uncertainty variables. By doing so one may obtain autocorrelated residuals without understanding the source of the observed autocorrelation;

A recent study, published in the ECB's Financial Stability Review of May 2017[397], started to fill the overstated literature gap by estimating the dependencies between *non-market uncertainty*, proxied with the EPU indexes, *market uncertainty*, proxied with the option implied volatility indexes, and other macroeconomic variables, for US and UK during the year 2016, using a Bayesian VAR model. The authors find that:

- "*Policy uncertainty had a notable tightening effect on US and UK financial conditions [and co-determined an increase in option implied volatility (see chart A.3)], in particular around the respective political events*";
- "*All else being equal, the surge in US economic policy uncertainty since November [2016] would have had a tightening impact on US financial conditions. This effect was, however, outweighed by a positive demand shock*";

Despite the aforementioned study is the first -to us known- tentative of modeling the interactions among *market and non-market uncertainty* measures, it may potentially suffer from variable aggregation problems that render its findings questionable, due to the fact of using time series aggregated at a monthly frequency. Monthly frequency that is certainly too low, and therefore inappropriate, to identify and disentangle causal dependency relations between *market and non-market uncertainty* variables and associated phenomena, to which these measures may be differently sensible, in terms of responsiveness, latency and sensitivity. The Governor of the Bank of Italy, Ignazio Visco, has recently tried to attract the attention of the public towards this issues, by claiming that despite in 2016 "*there has been a sharp rise in global policy uncertainty. [And] this is a cause for concern [... because] there is ample empirical support for the claim that economic policy uncertainty if persistent dampens economic activity and trade as well. We should be aware of the fact that economic policy uncertainty measures with all the caveats that apply to news-based approaches may capture longer-term concerns only partly correlated with perceptions of the short-term macroeconomic outlook on which financial markets tend to focus*"[398], therefore these two measures are not and should not be considered substitutes. The Governor of the Bank of Italy added that, after the 2016 EU-referendum and US elections, there has been a "*divergence between economic policy uncertainty measures [EPU indexes] and financial market volatility [option-implied expected volatilities]*"[398]. The divergent pattern of *market and non-market uncertainty* proxies, has raised additional questions[399] relative to the dependencies among different uncertainty sources and their measures, certainly highly correlated, but differently sensible to real world phenomena. Reason for which in this article we have developed our crowd-sourced text-based *civil society uncertainty* proxy, to try to disentangle direct uncertainty dependencies between policy/political and market outcomes. In particular, to disentangle the effects of *policy uncertainty* on *market uncertainty*, proxied through option-implied volatility, from *market uncertainty* fluctuations mediated, amplified or originated by *civil society uncertainty*, proxied by Twitter communications and deliberations on uncertainty related issues. *Civil society uncertainty*, which, as we will see, generally emerges in relation to patterns of rapid and diffused divergence from/of conventional expectations, within a society or social group, producing above tolerance levels of actual and expected surprise, signalled through the use of the word uncertainty, during processes of group or social metacognition, directly observable on online social networks like Twitter.

Text-based measures have already been developed and used in economic and financial studies, to obviate the problem of proxying unobservable risk and uncertainty variables. For example Romer and Romer[400]have measured monetary policy shocks using -among others- textual information sources published by the FED, like the *Minutes of Federal Open Market Committee*. They develop an ad-hoc manual procedure to deduce Federal Reserve interest rate change intentions from published FOMC narratives. Cavallo and Wu[401] used information from specialized journals: *Oil Daily* and *Oil & Gas Journal* to create a binary daily index which distinguishes dates in which the price of oil was driven by arguably exogenous events from those in which it was driven by developments related to the state of the oil market. Casarin and Squazzoni[402] have constructed three -daily frequency- bad news indexes for the 2008-2009 financial crisis, based on the content of the front page banner headlines of financial newspapers. The index values depend on the daily number of negative banner headlines concerning the crisis, on the number of columns where such news were reported and on the number of negative words therein contained. Caldara and Iacovelli[403] have constructed a worldwide index of Geopolitical Risk, called GPR Index and two specialized indexes, called Geopolitical Threats (GPT) and Geopolitical Acts (GPA) indexes. They used an automated token-dictionary based text-mining methodology, based on rescaled monthly counts of the occurrence of words related

to geopolitical tensions in articles from international newspapers in English.
Finally, Baker, Bloom and Davis[386] have developed a set of country specific Economic Policy Uncertainty (EPU) indexes based on newspapers articles, which we will use in this work as proxies for *policy uncertainty*. These indexes are based on a rescaled counting of the number of daily articles, coming from a set of country specific newspapers, in which specific tokens, used as identifiers of economic policy uncertainty, occur. The aforesaid work inspired our Twitter Uncertainty measures, which, as we will see, have very different qualities and potential drawbacks respect to existing text-based uncertainty measures employed in economic and financial studies.

The main difference between these measures and the one that we propose is that existing measures are based on narratives and data produced by relatively small groups of experts and analysts, i.e. figures considered technically capable of evaluating objectively and with scientific method specific instances of events and risks, which they have the role of monitoring and reporting about. Like rating agencies, analysts, journalists and other professional political, financial and economic commentators. Whereas, the uncertainty measures that we develop and use in this work, are based on textual uncertainty signals produced and diffused by hundred of thousand agents, that publicly communicate and deliberate about socially relevant uncertainty issues, through an online distributed information network system, called Twitter.

Hypotheses

As previously anticipated, one of the key hypothesis behind our interest for the measurement of *civil society uncertainty*, trough new Twitter based proxy indexes, is the possible existence of causal relations between *civil society uncertainty* and *market uncertainty*. In our study *market uncertainty* will be proxied through measures of option-implied volatility: VIX and VFTSE.
Relationships between *market and non-market uncertaint* have been a constant subject of debate among market operators during the last years[404–407]. In particular, precautionary savings and risk appetite[408–412] have been one acknowledged as the main contagion channels of *non-market uncertainty* on economic decisions and outcomes. When widely diffused, *wait and see*[413] behaviors may lead to a stagnation or contraction of transactions, making markets become thinner and their prices more volatile. This mechanism has also been identified as a possible explanation for cyclical burst-boom patterns in aggregate investment, where the trough of the cycle corresponds to major elections and is therefore associated to *political/policy uncertainty*[414]. The existance of contagion channels among policy/political uncertainty and market volatility have been object of numerous investigations in finance and economic disciplines, especially in the past decade. These market non-market contagion channels appear to be particularly relevant in periods of crisis and structural change[415, 416]. It has been found that political uncertainty was among the main sources of stock market volatility in Germany between 1880 and 1940 [417]. Uncertainty shocks, proxied by the innovations in the VIX index and in the EPU[386], contributed to the fall in the US GDP during the recent 2007-2009 recession[418]. In addition, it has been hypothesized and tested[419], that, during the last financial crisis, rises in the volatility of *policy uncertainty* have dampen stock market returns and

increased stock market volatility, and, increases in stock market volatility have reduced stock market returns and increased *policy uncertainty*. Volatility in basic commodity price levels has been idetified as one of the main causes of increasing political turmoil and hence uncertainty before the Arab Spring[420]. Moreover, political uncertainty, caused by the Arab Spring itself, led to significant increases Middle East's and North Africa's stock market volatility indexes[421]. Finally, downward stock market volatility jumps have been causally linked to the resolution of *policy uncertainty*[422].

We must remark that it is sufficient that "*individuals who do not invest in the stock market are likely to use its ups and downs as a guide to the state of the economy*"[423] to formulate their expectations, to create a contagion channel *between stock market volatility* and *non stock market uncertainty*. On the other side, through the public debt channel, *political and policy uncertainty*, at the country level, has clearly an effect on the dynamics of international capital markets and hence on the volatility of prices of assets traded in these markets. As we can see from the above stated studies in some circumstances political uncertainty is among the causes of market volatility, in others, political uncertainty is a by product -or externality- of market volatility. Given the complexity and the -suggested- existence of loop mechanisms, it often results very difficult to disentangle causes from effects, especially using aggregated low frequency data. This is also a good reason for developing a new set of uncertainty proxies which can aggregate *civil society uncertainty* signals by target area, at the desired frequency.

In this study we will hypothesize, and test if, the TU indexes and the EPU indexes, which are respectively used to proxy *civil society uncertainty* and *policy uncertainty*, can be considered complementary aggregate uncertainty measures, in relation to their contributions in explaining observed fluctuations in *market uncertainty*, in a linear VAR modelling setting. We will also try to infer the eventual existence, sign and type of dependencies among the aforementioned *non-market uncertainty* measures, within and across the two countries considered in this study, the United States and the United Kingdom. In addition, we will hypothesize, and test if, these two sources of uncertainty affect in distinct ways and with different latencies, both in terms of granger causation and estimated impulse-response functions, *market uncertainty*, here measured through option-implied volatility indexes.

Here follow some additional hypotheses on the dependencies among *market and non-market uncertainty* in the United States and United Kingdom, that will be tested through the estimation of our VAR uncertainty contagion model in the third section of this work:

- *Hypothesis 1: non-market uncertainty* (TU and/or EPU) **Granger causes** *market uncertainty* (VIX or VFTSE) in the corresponding geographic-area;

Remark: if Strong Efficient Market Hypothesis (Strong EMH) were true hypothesis 1 should be rejected;
We are interested in testing *hypothesis 1* because we wish to see if information provided by the lagged values of *civil society uncertainty* and *policy uncertainty* uncertainty proxies have explanatory power for *market uncertainty*, i.e. have statistically significant coefficients in the VIX and VFTSE equations.

- *Hypothesis 2: non-market uncertainty* (TU and/or EPU) in one of the two geographic-areas, **Granger causes** *market uncertainty* (VIX or VFTSE) in the other geographic-area;

We are interested in testing *hypothesis 2* because we wish to see if information provided by the lagged values of *civil society uncertainty* and *policy uncertainty* uncertainty proxies in one country have explanatory power for *market uncertainty* variable of another country. This hypothesis is linked to the existence of international uncertainty spillover effects.

- *Hypothesis 3:* **The lagged dependence relation** between *civil society uncertainty* or *policy uncertainty*, and, *market uncertainty* in the corresponding geographic-area **is positive and bidirectional, i.e. reinforcing feedback mechanisms exist**;

Remark: hypothesis 3 is compatible with *market uncertainty* embeddedness in social and political systems an their processes;
Hypothesis 3 may be viewed as a transposition of Granovetter's[424] concept of embeddedness to *market uncertainty* phenomena. Under this perspective *market uncertainty*, like option-implied volatility, is embedded in social and political processes. *Market uncertainty* jointly affects and is affected by *non-market uncertainty* phenomena. In particular we hypothesize that processes of political and social metacognition, through public communications and deliberations about expectations, affect the perception of expectations-gaps and hence the uncertainties of market agents, in terms of their actual and expected surprise. Changes in the degree of expected surprise which can in turn influence economic behaviour and choices of market agents, in particular in relation to options which can be seen as conditional insurances against specific events or states of the world, on which probability of occurrence there is no convergence of views among market agents. Similarly we hypothesize that publicly communicated information about *market volatility*, may affect individuals' *non-market uncertainty*and expectations, for example the volatility of public debt may affect policy and political expectations. Through such a mechanism *market uncertainty* may affect the non-market uncertainties of agents, in terms of actual and expected surprise associated to political and social issues. This hypothesis should be evaluated in relation to social-metacognition[15, 425, 426] theories, which have been exposed in the first article of this collection. If *market uncertainty* is not independent from other types of uncertainty, by considering option implied volatility a proxy measure of *market uncertainty* and considering *market uncertainty* independent from other type of uncertainty to which a socio-economic system may be subject to, we could underestimate the impact of non-economic uncertainty shocks on *market uncertainty* and consequently on risk premia.

- *Hypothesis 4:* The **cumulative response** of *market uncertainty* to a *civil society uncertainty* shock is significantly **different from zero only in the short term**, and **not significantly different from zero in the medium-long term**;

Remark: hypothesis 4 is compatible with Semi-Strong EMH;

We wish to test *Hypothesis 4* because it could be that *civil society uncertainty* measures (US-TU and UK-TU) are neutral in the medium and long term even if they may Granger-cause *market uncertainty* proxies (VIX and VFTSE) in the short run. Neutrality is measured in terms of the cumulative responses of VIX and VFTSE to UK-TU and US-TU impulses: *civil society uncertainty* proxies are neutral in the medium and long term if cumulative responses are non-statistically significantly different from 0, or exhibit confidence intervals very close to zero, two months (40 observations) after the impulses;

- *Hypothesis 5:* Through our VAR modelling setting, it is easier to correctly predict the sign of next-day *market uncertainty* variations, i.e. changes in option-implied volatility, with respect to those of *civil society uncertainty* and *policy uncertainty*;

Remark: hypothesis 5 is compatible with the hypothesis that extreme market volatility phenomena, in the timespan under study, were more easily and precisely foreseeable compared to social and political uncertainty phenomena;

We wish to test *Hypothesis 5* because we believe that *civil society uncertainty* and *policy uncertainty* where the main drivers of *market uncertainty* in the US and UK during the year 2016.

- *Hypothesis 6:* Through our uncertainty contagion modelling setting, *market uncertainty* spillover effects are the main channel of contagion of *civil society uncertainty* and *policy uncertainty* shocks to other countries;

We wish to test *Hypothesis 6* because we believe that the main uncertainty contagion channel is the international interdependence among market systems, which allows local *civil society uncertainty* and *policy uncertainty* (extreme) events capable of affecting local markets, to rapidly propagate worldwide and produce international *market and non-market uncertainty* ripple effects.

Research questions

Here follow the main research questions that we will try to answer through this study:

1. Can we measure *civil society uncertainty* through twitter data?
2. Which are the pros and cons of our Twitter-based proxy measures of *civil society uncertainty*?
3. In which terms our *civil society uncertainty* proxies represent different phenomena with respect to existing *policy uncertainty* and *market uncertainty* measures?
4. Which were the dependecies between *civil society uncertainty* , *policy uncertainty* and *market uncertainty* in the US and UK during the year 2016?
5. Which were the spillover effects of *civil society uncertainty* , *policy uncertainty* and *market uncertainty* between the US and UK during the year 2016?
6. Is the usage of non-normal gamma-like stochastic disturbances, still a necessary *market uncertainty* modeling assumption when *civil society uncertainty* and *policy uncertainty* are used as explanatory variables?
7. Can we use of *civil society uncertainty* and *policy uncertainty* proxies to forecast next-day *market uncertainty*? Which is the out of sample next-day forecasting performance?
8. To which extent can we consider *market uncertainty* observed in the US and UK during the year 2016 impredictable?

9. In which terms unexplained *civil society uncertainty* and *policy uncertainty* can be associated to the concept of radical uncertainty and *black swan*[255, 346, 427] events?

10. Is it more difficult to predict next-day positive variations, i.e. uncertainty booming patterns, with respect to negative variations, i.e. uncertainty resolution patterns? for which variables and in which terms our forecasting performance is asymmetric?

11. What do differences in our *civil society uncertainty* and *policy uncertainty* forecasting performance tell us about the events that occurred in the year 2016?

2.2 From Twitter Uncertainty data to Twitter Uncertainty indexes

2.2.1 Twitter Uncertainty data

In the last decade "*Twitter has evolved from a phatic and ambient intimacy machine [...] to an event-following and news machine. [...] Twitter increasingly has come to be studied as an emergency communication channel in times of disasters and other major events as well as an event-following and aid machine*"[428]. Among open source repositories of crowd communication data, some, like Twitter, are not specialized by topic or by professional category of users. Twitter with its more than 330 million monthly active users, mostly English speakers, is at the same time a communication medium and a deliberative platform for its international open community.
Therefore, Twitter can be considered an instrument for collective elicitation and interpretation of global events and expectations, and hence, of the cognitive states explicitly associated to these events and expectations. By being an extremely large multi-national open and free access online community, in which posted information is freely and publicly observable through a unique platform, Twitter performs the role of a virtual public information and deliberation space. Accordingly, as we will document in this study, its information flows may be captured and used to proxy (online) civil society's "states of nature", especially for what concerns English speaking countries. Accordingly we exploit this characteristic of Twitter to track and measure at hight frequency, potentially in real time and at worldwide scale, uncertainty phenomenon through a single system: Twitter.

Twitter as a data source for empirical studies

In the last decade, a great number of academic studies have used Twitter and its freely accessible data to analyze the opinion dynamics related to specific global events or phenomenon[429–433], like epidemics[434–439], natural disasters[440–450], revolutions[451–459] and elections[460–474]. Twitter has also been used to predict stock market prices[475–481], to extract consumer preferences[482], to analyze consumer satisfaction[483, 484] and brand engagement[485–489].

The advantage of using online data sources is that they can *"leverage distributed human knowledge to obtain information that does not exist in conventional databases"*[490]. The Internet can therefore be viewed as a system for collective attention and interpretation[491–493], where people virtually gather to discuss and make sense of

what is happening, what others believe is happening and what could happen in a specific environment, at a particular moment in time[494–497]. It has been shown that "*partially ignorant actors in a distributed system can accurately interpret complex situations when they interact appropriately*[498]. Since, when individuals deal with "*uncertain situations highlighted by potential danger [...] they will seek information from a variety of sources [... and] one channel that provides many opportunities for this purpose is the Internet*"[499], tweets about uncertainty appear to be a potentially good proxy for analysing *civil society uncertainty* in relation to a given area o the world. Because Twitter will be at the same time source of information and expression tool for its users, which renders it an idle instrument for collective interpretation of global events.
The Twitter corporation itself is investing huge efforts in developing new instruments for businesses to access and process this data. Recently Twitter affirmed through its official blog that their "*data is being used by a variety of financial market participants*"[500]. In September 2015, Bloomberg publicly announced[501] "*it has signed a long-term data agreement with Twitter that will further enhance financially relevant information found on the social media platform for users of the Bloomberg Professional service*". Other more practical arguments for using a crowd-driven web data sourcing approach include "*reduced cost, increased data sizes, and [information] environments closer to those in the real world with respect to an experimental setting. These characteristics may ultimately enable research not possible via traditional methods*"[502].

Data collection

Our dataset comprehends Twitter posts (tweets) in English, containing the term *"uncertainty"*, published on the social network Twitter from the first of April 2016 to the first of January 2017. The data has been collected progressively, by repetitively querying the API of Twitter during the above mentioned period of study. We used the Search API and not the Stream API because our downloading infrastructure, a small scale home server, could not be considered beforehand sufficiently robust to guarantee, at the moment we started the downloading process, operativeness without interruptions during the entire period of our investigation. Therefore the Search API was for us the best solution, because in case of interruption of the downloading process, we could go back up to seven days to eventually get the missed tweets, like if we had a buffer in case of inoperativeness of our server. Luckily our server never had problems during the period under study.

Our querying algorithm, coded in RapidMiner Server, a Java server program, repeated up to 180 times per fifteen minute period a get query to the Twitter API. Besides the keyword filter *(q="uncertainty")* and the language filter *(lang="en")* parameters, each get query contained a different value in the *since_id* parameter, the value of this parameter was set equal to the tweet identifier number of the most recent tweet that had been downloaded in the previous query iteration, this to have a time sliding downloading window that allows us to catch as much posts as possible, and, to avoid downloading multiple times the same tweet. Since the rate limit of the Rest API is 180 queries per 15 minute period, and each query can get up to 100 observations, the maximum capacity of our data downloading infrastructure is 10 800 tweets per hour, i.e. 259 200 tweets per day. Even in peaking hours we didn't approach this rate limit, therefore we can consider to have, if not the whole population, a very large sample of the tweeting activity in English about uncertainty, both in terms of size of the population and content representativeness for each day.

Data cleaning and processing

Raw dataset contains 1 439 686 observations. We filter the dataset by Source Name, removing all observations for which the Source Name contains the regular expression "*[Bb][oO][tT]*": 33 566 tweets are removed from T_{raw}. We do so because, since we consider the use of the word uncertainty a signaling device used among humans to express their perception of uncertainty, i.e. expected surprise, we want to omit content produced by profiles that explicitly state to be programs in their Source Name. In addition we remove all tweets that are uploaded through an online horoscope service called "Twittascope": 116 228 tweets are removed from T_{raw}. When one retweets a post of another user, the following structure is added to the message "*RT @[username]:*", this structure lengthens, in terms of number of characters, the original tweet. Retweets that, given the length of the retweet structure, would be longer than 140 characters are cut. A special UTF-8 character (U+2026) is added automatically at their end, to indicate that the message is not identical to the original tweet, and that at least one character is missing. We therefore use this special character as an identifier of cut retweets. To attenuate this message truncation issue, when a match is found we replace cut retweets with their original version. We have repeated this process for all observations in T_{raw} that are retweets, we call the resulting set T_{clean}, which is our cleaned observations dataset.

Descriptive statistics

Our cleaned database contains 1 289 892 tweets in English containing the term "*uncertainty*". 495 777 of which are unique, i.e. have a post content distinct from all other tweets in our database. Among non-unique tweets 619 340 are retweets. The remainder are copies of other posts in our database. Almost 10% of total tweets (105 107) are direct messages to other users, these messages start with the user-name (@tag) of the user to whom they are intended. Tweets have been posted by 742 924 different users, in a period of 275 days, from the first of April 2016 to the first of January 2017. As we can see from table 2.1, June and November have been particularly intense months from the point of view of twitter posting activity about uncertainty, with more than two hundred thousand tweets per month. These two months contain, respectively, the UK EU-referendum and the US presidential elections.

Table 2.1. Number of observations per month

Month	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
N.Obs.	97 462	101 798	212 834	161 789	112 220	110 843	145 265	213 224	133 323

January (2017) has been omitted from the above table because it contained only one day -with 1134 obs.-

All observations respect the 140 character twitter post length limit. The number of tokens per tweet ranges from one to thirty-five. The mode is eighteen tokens per tweet. The marginal distribution density is rather flat and higher than 0.05 (5%) for values between ten and twenty, the majority of our observations fall in this range. Tweets with one token should contain solely the term *"uncertainty"* or an URL which contains this term. Three observations do not fulfil this requirement, we remove them.

More than half of the total number of tweets have been uploaded through mobile twitter applications or browsers. Whereas only about 20% have been uploaded through Twitter's web-client interface. Apple portable devices are the primary uploading source for tweets in our database, followed by android and windows devices. Only a small fraction of the total population has been uploaded through other social networks, like Facebook or LinkedIn, or blog platforms, like WordPress. The remainder have been uploaded through third party services, like IFTTT and Hootsuite or through user specific uploading services.

2.2.2 Twitter Uncertainty Indexes

The key difference between the Twitter Uncertainty (TU) indexes, which we will construct in this subsection, and pre-existing aggregate uncertainty measures based on textual data from newspapers[386, 403], is that our TU indexes, being based on Twitter posting activity, exploit the characteristics of this online social network and decentralized news provider, which is an open access, open content, distributed information communication and deliberation system, with millions of active users per day, to recognize and measure the degree of, area specific, *civil society uncertainty* events. We use Twitter and the Twittersphere as a natural sensor for uncertainty phenomena. A receiving, amplifying and archiving system for human signals of uncertainty, which are elicitations of mental states associated to actual and expected surprise. These signals generally emerge and are communicated during social metacognition processes, when agents communicate their expectations and observe their degree of divergence. Given our index computation methodology, each published textual observation containing the word *uncertainty* in relation to a given area counts alike. Our uncertainty measures are only apparently naive, because the weighting of uncertainty signals is implicitly delegated to the Twitter network and its decentralized community of users, which through their online interacting behaviours, generate the global social metacognition patterns that we observe in relation to uncertainty, which we aggregate by day and by target area. Twitter users are jointly the communicating parties and the sensors of such multi-agent network system for global social metacognition, called the Twittersphere. By reporting, disseminating, transforming, citing and commenting information about uncertainty events, the Twittersphere becomes itself a model and gauge of the degree of civil society uncertainty in relation to specific events, or, areas of the world in which the latter have, or may, occur.

In the following section we will illustrate the methodology used to construct our uncertainty indexes from our T_{clean} database of tweets containing the term "*uncertainty*". This subsection starts with a brief review of the criteria used to filter observations and construct our dataset of Twitter observations about uncertainty. We then highlight the main differences between our indexes and the most similar pre-existing uncertainty indexes based on textual data, i.e. the Economic Policy Uncertainty (EPU) indexes by Baker, Bloom and Davis[386].

Index construction technique and differences from existing uncertainty measures

Similarly to the Economic Policy Uncertainty (EPU) index by Bloom, Baker and Davis [386], the presence of the word *uncertainty*" in a textual observation (a tweet) is a necessary condition for that observation to be potentially considered and counted as a signal for *civil society uncertainty*. For this reason the term "*uncertainty*" was one among the query conditions used to download our Twitter Uncertainty dataset.

A major difference between our TU indexes and the EPU indexes is that for the construction of the EPU indexes, besides the term "*uncertainty*", the term "*uncertain*" is also used as an identification signal for uncertainty. We voluntarily choose to avoid using this second term -*"uncertain"*- in the construction of our indexes for the following reasons: tweets are not exclusively news containers, they are also used as narration and story-telling devices for private life events and personal thoughts. The adjective "*uncertain*" is generally used to qualify one's individual mental state in relation an actual or expected phenomenon, like the weather or the outcome of a job-interview, under such circumstances it is only a quality of a noun, in a sentence. In tweets in which it appears, the adjective "*uncertain*" is generally used instrumentally, as a non-necessarily essential qualification of a entity. Whereas, the term "*uncertainty*", being a noun, is the main or only element of subjects of verbs, or, of objects of verbs or prepositions in a sentence.
While the noun "*uncertainty*" is frequently used in sentences that describe external and collective situations, using the third person singular or a plural form, "*uncertain*" is frequently used in statement in the first person singular, to describe ones' personal thoughts.
Especially in tweets, the word "*uncertainty*" is very rarely used to elicit private-life uncertainties, but, it is frequently used with reference to states of diffused uncertainty in reference to a group, community or society, to which the writer of the post belongs-to or observes. In such situations "*uncertainty*" signals that the target group, community or society is experiencing, collectively, above tolerance levels of surprise or expected surprise, in relation to the degree of divergence of opinions or expectations elicited through processes of communication and social metacongition. Therefore *civil society uncertainty*, which is the phenomenon that we wish to identify and measure in this study, is therefore more precisely and robustly identified in tweets using only the noun "*uncertainty*".

Twitter Uncertainty indexes are constructed by counting the number of messages (tweets), per time interval (days), contained in subsets of the T_{clean} dataset. The dataset is subsetted using token dictionaries as logical boolean functions conditions, and using tokens (sequences of characters) contained in observations as boolean inputs to evaluate the membership of a given observation to a given subset of the T_{clean} dataset. If the conditions of a dictionary are matched, i.e. the dictionary function applied to a given observation gives TRUE, then the observation belongs-to the data subset represented by the dictionary. Tokens in the geographic-area dictionaries are assumed to represent the necessary conditions for the imputation of a state of *civil society uncertainty*, publicly declared through a tweet, to a specific geographic-area. Dictionaries may contain three different types of textual tokens:

- ***REGEX exact matching tokens** -*for example: "United States" and "Great Britain";*
- **** REGEX exact matching after tokenization tokens** -*for example: "US" and "UK";*

- ***** REGEX exact matching, preceded by white-space condition tokens -** *for example: "U.K" and "U.S"; E.U;*

To minimize the probability of including false positives in our dataset we voluntarily choose to exclude the use of quasi-synonyms of the word uncertainty (like "*doubt*" or "*confusion*") or other words that frequently co-occur or are associated with uncertainty (like "*fear*" or "*anxiety*") as signals for *civil society uncertainty*.

Index Validation by Upstreaming Information Cascades

To identify the information cascade processes that determined the peaks in our uncertainty indexes we have upstreamed the information sources contained in our observations by searching for signals of the original source of the information mentioned by twitter users. We manually analyse the most-frequently observed contents in messages that where posted in *civil society uncertainty* peaking days. We have upstreamed these information cascades as much as possible, using the following signals:

- *URL links:* these links often point to online articles from web-newspapers, press agencies and blogs; as well as from official press releases or speeches by Central Bank governors/personnel, national government cabinet officials and executives of listed companies, NGOs, intergovernmental or international organizations;
- *Quotations:* direct quotations -in brackets- of statements and fragments of speeches, which predominantly come from members of the above-stated categories; as well as from influential political, cultural, artistic and scientific figures;
- *Mentions:* Explicit reference to a named entity *(person, organization and place, etc.)* in a observation;
- *Reference Citations:* Explicit reference to the documentary sources of the content/ideas expressed in a observation -without linking by URL to it-. This technique is generally used to refer to information sources that are not available on the Internet, that have been deleted or that the user is unable to recover;

By so doing we wish to exploit the web as a digital historical archive of information about uncertainty. Through which we can identify, retrieve and analyse the primary sources to which our observations refer-to. We do so with a historical method and approach. By exploring who relies on the different sources of information available on the web, we can recognize the uncertainty signalling online authority and reputation dynamics, and, disentangle the distribution of influence among information providers, at a given moment in time or in relation to a given event or segment of the population.

For the two TU indexes that we present and use in this work, the US-TU and the UK-TU, we have included in the section 2.0.2 of the Appendix at the end of the article the outcomes of the aforesaid index validation process.

Twitter Uncertainty indexes: US-TU and UK-TU

As previously explained, Twitter Uncertainty (TU) indexes specialized by geographic-area are derived by subsetting T_{clean} with a area-specific token dictionary, and then, counting the number of observations per day. Here follow our TU by geographic-area for the and the United Kingdom (UK-TU) and the United Stated (US-TU).

United Kingdom Twitter Uncertainty (UK-TU) index

The UK-TU index has been constructed using jointly two dictionaries of geographic-area tokens, juxtaposed with an OR filter. Those of the United Kingdom and Great Britain:

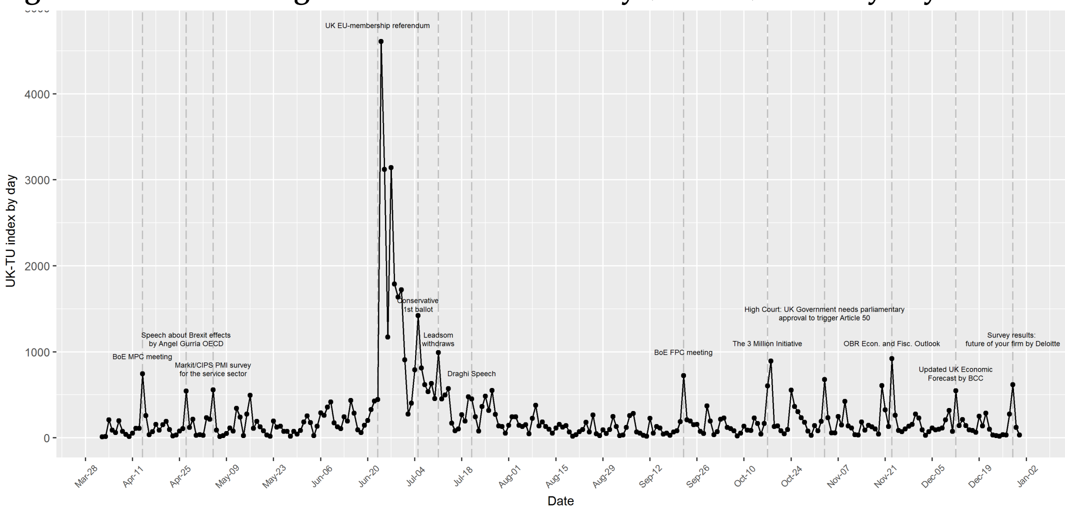
Figure 2.1. UK-TU geographic-area tokens dictionary

UNITED KINGDOM		GREAT BRITAIN
"United Kingdom"* OR "UNITED KINGDOM"* OR "united kingdom"* OR "UK"** OR "uk"** OR "U.K"*** OR "u.k"*** OR "U.K."*** OR "u.k."***	OR	"Britain"** OR "BRITAIN"** OR "britain"** OR "gb"** OR "GB"** OR "g.b"*** OR "G.B"*** OR "g.b."*** OR "G.B."***

REGEX are case sensitive

We did so because from a semantic point of view these terms are very close to each other. Even though strictly speaking Great Britain's geo-political area corresponds to the UK without Northern Ireland, they are often used in twitter posts and news as alternative identifiers (labels) for the same geographic-area.

Figure 2.2. United Kingdom Twitter Uncertainty (UK-TU) index by day



The UK-TU index counts in total, from the first of April to the end of December, 69 372 observations. The index's time series, represented in figure 2.2, evidences the extraordinary uncertainty impact of the British EU-referendum outcome. The uncertainty effects of this event are the largest in the period covered by this study by several orders of magnitude, with more that 4 000 observations in the day of the referendum results announcement. Brexit is clearly also the more persistent uncertainty shock we observe. Till July the 14th the index stays above 400 observations per day.

United States Twitter Uncertainty (US-TU) index

Our uncertainty index for the United States of America, named US-TU index has been computed using the following terms as geographic-area identification tokens for filtering the ATU database:

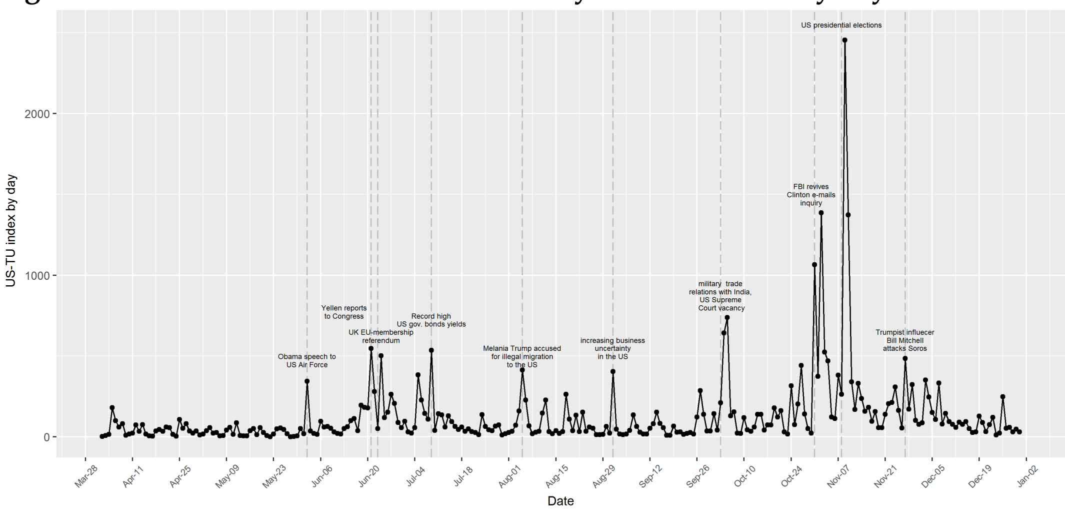
Figure 2.3. US-TU geographic-area tokens dictionary

UNITED STATES

"United States"* OR "UNITED STATES"* OR "united states"* OR "US" OR "USA"** OR "usa"** OR "U.S"*** OR "u.s"*** OR "U.S."*** OR "u.s."*** OR "U.S.A"*** OR "U.S.A."*** OR "u.s.a"*** OR "u.s.a."*****

REGEX are case sensitive

Figure 2.4. United States Twitter Uncertainty (US-TU) index by day



The US-TU has a total of 34 024 observations, which is about one half of UK's number of observations. At a first glance, the US-TU index's time series, represented in figure 2.4, evidences recurrent small and mild uncertainty shocks in June and July, especially in dates near to UK's EU-referendum. Then a relatively long low uncertainty period, from mid July to the beginning of October, with few isolated events. Followed by increasingly recurrent and more persistent uncertainty events, some with exceptionally high peaks. Major shock events start during the last two weeks of the US presidential elections campaign, when the FBI enquiry on Clinton's email server is revived. US-TU reaches the maximum observed value the day after the US elections, day in which the victory of Trump becomes known. This date represents the positive extremum of US-TU and counts more than two thousand observations in a single day. From this moment on Twitter users start signalling above average uncertainty in the US, in relation to their political expectation, in particular in relation to US's foreign and climate policy under Trump.

As we can see from the commented graphs of the time series of the UK-TU and US-TU indexes, as well as, from the detailed analysis in section 2.0.2 of the Appendix: our indexes are highly reactive and highly sensitive quantitative indicators of the degree and dynamics of aggregate *civil society uncertainty* across time. Not only our indexes respond almost instantaneously to major political, economic, financial information impulses that may affect the degree of uncertainty of civil society in these two countries, but also, they appear to be short memory processes. By having a relatively short memory, our indexes can reveal the existence and magnitude of sequential/iterated (in the time dimension) *civil society uncertainty* impulses, also in the period that follows major events. This latter characteristic is rare if not unique among uncertainty proxy measures, like newspaper-based uncertainty indexes or option implied volatility indexes, which generally exhibit higher hysteresis.

2.3 (Other) endogenous uncertainty variables

In relation to the model on uncertainty contagion, between and within the UK and US, which will be estimated and analyzed in the next sections, besides the US-TU and UK-TU indexes, which have been already described, four other endogenous variables will be used. Two Economic Policy Uncertainty[386] indexes, called US-EPU and UK-EPU, and, two option-implied volatility indexes, called VFTSE and VIX.

Economic Policy Uncertainty indexes: US-EPU and UK-EPU

- **Daily United States economic policy uncertainty index (US-EPU)**[503]: the daily value of the US-EPU index is obtained by using the Newsbank's newspaper archive service, called Access World News[504], to count the number of article published by UK newspapers covered by this archive, containing at least one token from each of the following topic dictionaries:

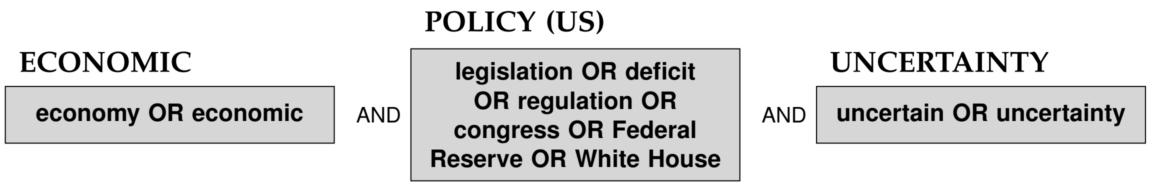


Figure 2.5. Token dictionaries used to construct the daily US-EPU index, by Baker, Bloom and Davis

  the values obtained through the daily count of articles matching the above-stated filtering criteria are then rescaled, by dividing each daily value by the count of the total number of articles from US newspapers published in the corresponding day. This to take into account the fact that the number of newspapers covered by the Newsbank archive increased across time. But also, because the number of articles per newspaper may also change from one day to another. Finally, the index value is frequently renormalized to match the value of the monthly EPU index in specific dates.

- **Daily United Kingdom economic policy uncertainty (UK-EPU) index**[505]: the daily value of the UK-EPU index is also obtained by using Access World News[504], to count the number of article published by UK newspapers covered by the archive, containing at least one token from each of the following topic dictionaries:

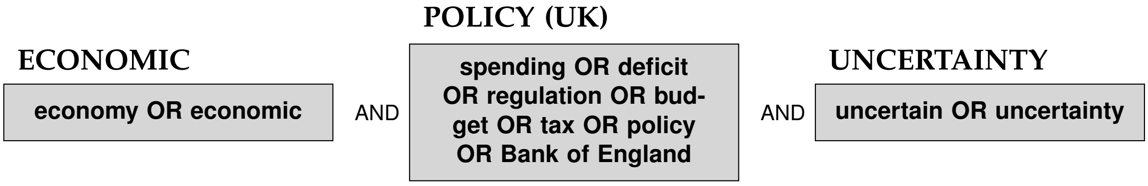


Figure 2.6. Token dictionaries used to construct the daily UK-EPU index, by Baker, Bloom and Davis

As for the daily US-EPU, the values of the UK-EPU are also rescaled by dividing each daily count by the total number of articles published by UK newspapers in the corresponding day.

For what concerns the EPU indexes by Baker, Bloom and Davis, it has been found that an upward 90 points innovation of the EPU index causes a "*drop in industrial production of about 1 percent and a rise in the unemployment rate of about 25 basis points*"[386]. The authors of the EPU also find that *economic policy uncertainty* has "*sizable [and significant estimated] effects on the cross-sectional structure of stock-price volatilities, investment rates and employment growth*"[386]. Even though formally the EPU indexes are *economic policy uncertainty* indexes, they have been used as generic uncertainty proxies in a great variety of studies:

- **Asymmetric changes in risk premia:** it has been empirically observed that, when political uncertainty increases, i.e. when the EPU index of a country experiences a positive shock, investors ask for an extra political uncertainty premia to invest in the country that experienced the shock[506]. This extra premia is increasing in the magnitude of the shock and in the riskiness of the investment -i.e. decreasing in the credit rating-, and decreasing in the GDP growth rate of the country that experienced the shock. As a result, the bigger is the uncertainty shock and the worse is the economic situation of a country that experienced it, the more investors will disinvest/flee from high-risk high expected profitability investments in that country. Investors "*will not reallocate efficiently until market perceive uncertainty as completely normalized [EPU goes below its average value]*"[507]. Therefore, unexpected political uncertainty shocks, proxied by the innovations in the EPU index, may cause investors' hoarding of safe havens -like gold and triple A sovereign among others- during periods of political turmoil and uncertainty.

- **Changes in the volatility and systemic risks of financial markets:** it has been found that when *policy uncertainty* increases in a country, prices in that country become more volatile and more correlated. Pastor and Veronesi[506], using the EPU as proxy of *policy uncertainty* in a General Equilibrium model, find that "*volatility is more than 50% higher in bad conditions (21% versus 13.4%) and the correlation [among equity prices] is 80% higher (78% versus 43%). The reason is that political uncertainty is higher in bad economic conditions[... And] this uncertainty affects all firms, so it cannot be diversified away*".

Option-implied volatility indexes: VIX and VFTSE

- **Daily -opening and closing values- of the S&P500 option-implied volatility index (CBOE VIX)**[508]: the VIX index is calculated with a model free methodology, based on the work of Demeterfi et al.[509] on pricing variance swaps. S&P500 put and call options are subdivided by expiring date and then ordered/indexed by increasing strike price (index *i*). Only options with non-zero bid price, with more than 23 days and less than 37 days to expiration are considered. In addition, once two consecutive put options with zero bid prices are found, no put options with lower strike price are considered. Similarly, once two consecutive call options with zero bid prices are found, all calls with higher strikes are excluded. VIX is constructed as a weighted average of the aforesaid subset of options on a 14 days long rolling interval, centered on 30 days. Therefore ,VIX is a measure of volatility expectations of the S&P500 Index for the next month. For more details on the methodology used to compute the VIX we refer to the CBOE VIX White Paper[508].
- **Daily -opening and closing values- of the FTSE100 option-implied volatility index (NYSE-Euronext VFTSE)**[510]: the VFTSE index can be considered the FTSE100 equivalent of the VIX index, as the VIX it is a proxy measure of expected volatility of stock prices, for publicly traded firms. The VFTSE index shares the same methodology of the VIX index. It is computed by using the market pricing data of FTSE100 options. For a detailed overview of the calculation methodology and information content of the VFTSE index see the paper by Siriopoulos & Fassan[510].

The above stated volatility indexes have been extensively employed, in macro-economics and finance literature, as uncertainty proxy measures. Prior studies have shown that option-implied volatility measures, like the VIX and VFTSE, include "*information about future volatility beyond that contained in past volatility*"[510].It has been found that volatility indexes fall the day after monetary policy announcements. More specifically, the "*VIX index (and hence the S&P 500 options market) reacts in a systematic manner surrounding US monetary policy announcements.[...] The index falls on average by 2% on the day of Federal Open Market Committee meetings. No significant movements on days prior to or after the meetings have been identified*[511].

In section 2.0.4 of the Appendix the time series of US and UK twitter uncertainty (TU) indexes have been compared to the Economic Policy Uncertainty (US-EPU and UK-EPU) and volatility (VIX and VFTSE) indexes of the corresponding country.

2.4 A structural VAR Model for the inference of uncertainty contagion channels in the UK and the US

2.4.1 Model concept

Since the the EPU indexes and option-implied volatility measures are constructed using different information sources compared to TU indexes, respectively news articles and stock option prices, they are ideal candidates, to be used in a VAR modelling setting, to estimate eventual uncertainty dependencies, feedback mechanisms and granger causation structures between the following sources of uncertainty, within and between countries:

1. **Degree of *market uncertainty* in the US and UK reflected by option prices:** proxied through option-implied volatility indexes (VIX and VFTSE);
2. **Degree of *policy uncertainty* in the US and UK assessed by professional journalists and experts:** proxied through the daily Economic Policy Uncertainty indexes developed by Baker, Bloom and Davis (US-EPU and UK-EPU);
3. **Degree of *civil society uncertainty* in the US and in UK perceived by the online community:** proxied through the daily Twitter Uncertainty indexes by geographic-area (US-TU and UK-TU);

Here follows a diagrammatic representation and description of known and expected relations among our endogenous variables:

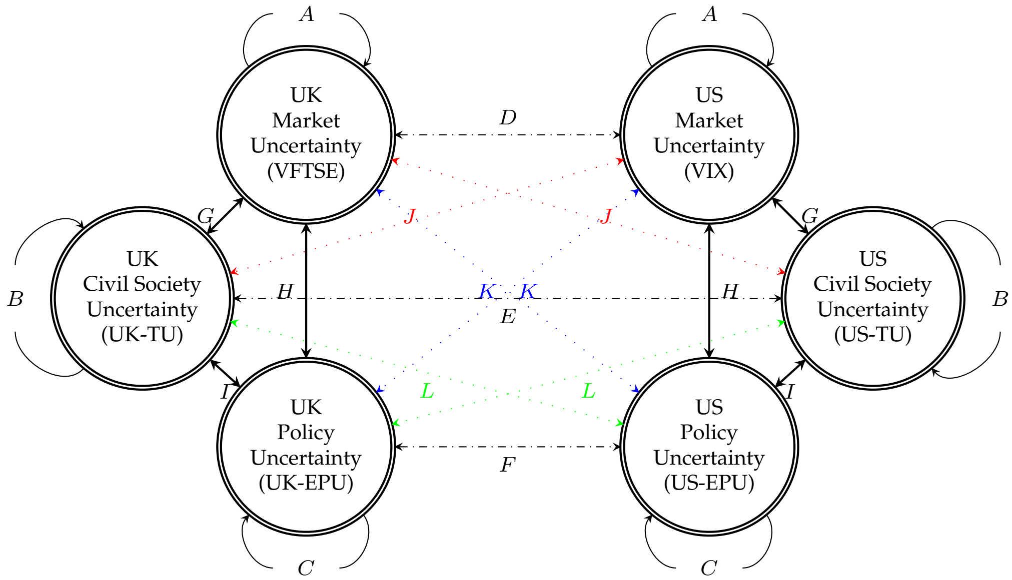


Figure 2.7. Interactions among uncertainty variables

A- Financial Uncertainty feedback mechanisms: option-implied volatility processes are often considered (medium term) mean or median reverting processes, subject to gamma-like distributed shocks. Therefore, we expect that after a impulse option implied volatility (if no additional relevant perturbation occurs) slowly converges back to its prior average/median long-term stationary level. The empirical distributions of option-implied volatilities indexes are both leptokurtic and right skewed (see Fig. 2.25 and Fig. 2.26);

B- *Civil Society Uncertainty* feedback mechanisms: uncertainty information diffusion processes in social networks, like online reactions to uncertainty related events, are generally assumed to be short memory processes. Our TU index time series appear, to exhibit this kind of behaviour (see Fig. 2.23 and Fig. 2.24). The empirical distributions of TU indexes are very leptokurtic and right skewed. As we can see from section 2.0.2 of the Appendix, our indexes are very reactive to information about uncertainty coming from an great variety of sources. As a result, they exhibit short term peaking and decaying patterns, and eventually, rapidly dissipating positive feedback effects, responsible for inter-day information cascades, clearly visible during major uncertainty shocks, like the Brexit vote in the UK and the election of Trump in the US;

C- ***Policy Uncertainty* feedback mechanisms:** we expect *policy uncertainty*, measured through the EPU indexes, to have a larger hysteresis, and more persistent peaking patterns than our twitter uncertainty measures. This because the content of published articles is generally richer -in terms of quantity and quality of the information therein contained- compared to messages about uncertainty posted by people on online social networks, therefore news articles (EPU observations) require more time to be written, understood, and eventually re-elaborated before being diffused by other newspapers, which rarely "copy and past" *policy uncertainty* signals delivered by other agents, like institutions, politicians and other media. Therefore, published *policy uncertainty* news cascades should be smaller in scale -relative to the average values of the index- and produce more durable effects than their social network based counterparts. The fact that the empirical distribution of our the EPU indexes (see Fig. 2.21, and Fig. 2.22) is also right skewed and leptokurtic, but less leptokurtic and right skewed compared to our TU indexes is compatible with the aforementioned hypothesis;

D- **Inter-country interaction between *market uncertainty* variables:** empirical studies have shown that, uncertainty spillover effects, i.e. the relation between option-implied volatility indexes for stock markets of different countries, depends on the level of integration among the capital markets of those countries, the more capital markets are integrated the higher is the correlation coefficient between volatility indexes. The magnitude of this relation may vary across time, it often increases during periods of economic and political uncertainty[512]. According to Rahmaniani et al.[513] US and UK investors' expectations about future uncertainty are highly integrated: from 2004 to 2014 the value of the time varying conditional correlation coefficient between VIX and VFTSE has oscillated within the $[0.4, 0.6]$ interval.

E- **Inter-country interaction between *civil society uncertainty* variables:** in normal times we expect our US and UK *civil society uncertainty* variables -proxied with TU indexes- to exhibit weak or statistically null lagged interdependence coefficients. In general, excluding exceptional events, there is no evident reason to hypothesize that an increase in *civil society uncertainty* in a country should cause an increase in *civil society uncertainty* in another country. However, as we have seen in the first section, during extreme shocks with potentially global repercussions, like the EU-referendum and the US presidential elections, *civil society uncertainty* did propagate from one country to the other becoming a relevant uncertainty contagion channel among countries. This channel may become particularly important when the source of uncertainty are supranational or not geographically bounded or circumscribable, like for example:

- uncertainty concerning climate change mitigation policies of the US after the election of Trump;
- uncertainty related to the outcome Syrian conflict;
- uncertainty related to possible development and use of nuclear weapons by North Korea;

- uncertainty generated by terrorism, which recurrently propagates across borders from the civil society in which the last attack took place to -at least- all allied countries and their civil societies;

F- Inter-country interaction between *policy uncertainty* variables: since the proxy used for *policy uncertainty* are based on newspapers articles filtered by source (geographic location of the newspapers' headquarters) the US *policy uncertainty* index reacts not only to uncertainty news concerning the US, but to all news articles published by US newspapers containing at least a token from each filtering dictionary. Therefore articles counted by the US-EPU may therefore report about uncertainty in other countries, like the UK. The same reasoning applies to UK *policy uncertainty*, the UK-EPU may well contain signals of uncertainty concerning the US. As a result we expect the two EPU indexes to have a high positive instantaneous correlation coefficient. Since the share of articles concerning foreign countries in each EPU index is unknown and may change across time, becoming larger when a foreign country experiences a major *policy uncertainty* shock while the situation in the home country is calm. We cannot distinguish the effects of these different *policy uncertainty* sources that are contained in each EPU index;

G- Intra-country interaction between *market and civil society uncertainty* variables: if option prices instantaneously reflect, in all moments in time, all available information, including information concerning *civil society uncertainty*, then the lagged values of US/UK *civil society uncertainty* shouldn't be informative predictors for US/UK *market uncertainty*. Whether the aforementioned efficient markets hypothesis is true or false, it is probable that the lagged values of US/UK *market uncertainty* are informative predictors for US/UK *civil society uncertainty*. The reason behind this hypothesis is very simple, the TU indexes used as proxies for *civil society uncertainty* contain discussions, remarks and comments concerning uncertainty in markets, like changes in option-implied volatility, and since people may mention some of these happenings in the days that follow the market event, *market uncertainty* may well positively influence, from a causal point of view, *civil society uncertainty* in the corresponding country;

H- Intra-country interaction between *market and policy uncertainty* variables: the same remarks made for the interaction between *market uncertainty* and *civil society uncertainty* apply to the relation between *market uncertainty* and *policy uncertainty*. If markets are efficient, lagged values of US/UK *policy uncertainty* shouldn't be informative predictors for US/UK *market uncertainty*. Whereas lagged values of US/UK *market uncertainty* are expected to be informative predictors for US/UK *policy uncertainty*.

I- Intra-country interaction between Policy and *civil society uncertainty* variables: as we have seen in the first section, our TU index often mention or cite online news articles about uncertainty. Therefore we may expect that lagged values of US/UK *policy uncertainty* are informative predictors for US/UK *civil society uncertainty*. However, since TU indexes are potentially more reactive to real world events compared to EPU indexes, it is also possible that lagged values of US/UK *civil society uncertainty* are informative predictors for US/UK *policy uncertainty*. Therefore, we hypothesize the existence of a bilateral granger causation relation (with feedback mechanisms) between policy and *civil society uncertainty* variables of the same country.

J- Inter-country interaction between *market and civil society uncertainty* variables: we have no a priori knowledge about the relation between market and *civil society uncertainty* of two different countries;

K- Inter-country interaction between *market and policy uncertainty* variables: we have no a priori knowledge about the relation between market and *policy uncertainty* of two different countries;

L- Inter-country interaction between *policy and civil society uncertainty* variables: we have no a priori knowledge about the relation between civil society and *policy uncertainty* of two different countries;

Given the numerous possible interaction channels between our variables, we believe the use of a VAR modelling setting is the appropriate methodological choice to estimate the relevance of the aforementioned uncertainty contagion and amplification channels. We model the interactions between our variables, at a daily frequency, including all trading days common to the US (VIX) and the UK (VFTSE) markets. All our time series have been rescaled to have a mean of 0 and standard deviation of 1, i.e. they have been standardized. Since all our variables have rather similar -gamma like- empirical distribution shapes, once standardized it will be easier for us to compare the time series and interpret the values of estimated coefficients from our VAR models.

2.4.2 VAR model specification

VAR process models[514–516] have been widely adopted for the analysis of multivariate time series in which the present value of the endogenous variables is determined in large part by their own history, apart from deterministic regressors, like a constant and trend, and, if necessary, exogenous variables, like seasonal dummies.

A $VAR(p)$ process for a set of K endogenous variables and N exogenous dummy variables can be defined as:

$$\mathbf{y}_t = \mathbf{A}_0 + \sum_{i=1}^{p} \mathbf{A}_i \mathbf{y}_{t-i} + \sum_{j=1}^{N} \mathbf{B}_j e_{j,t} + \mathbf{u}_t \tag{2.4.1}$$

Where:

y_t is a K-dimensional vector with endogenous variables at time t;

$\mathbf{A}_i$ are $K * K$ coefficients matrices, with $i \in \{1, ..., p\}$;

$\mathbf{u}_t$ is a white noise K-dimensional process s.t. $E(\mathbf{u}_t) = \mathbf{0}$ and $E(\mathbf{u}_t \mathbf{u}_t^\top) = \Sigma_\mathbf{u}$ is a time invariant positive definite covariance matrix;

$\mathbf{A}_0$ is a K-dimensional vector with constant terms;

$\mathbf{B}_j$ (with $j \in \{1, ..., N\}$) is a K-dimensional vector containing the coefficients of the j'th exogenous dummy variable;

$e_{j,t}$ (with $j \in \{1, ..., N\}$) is a one-dimensional vector containing the value of the j'th exogenous dummy variable at time t;

In our model we have six endogenous standardized variables:

$\mathbf{y_t} = (UK\text{-}TU_t, US\text{-}TU_t, VFTSE_t, VIX_t, UK\text{-}EPU_t, US\text{-}EPU_t)$

For stationarity and unit root tests on endogenous variables' time series see section 2.0.4 of the Appendix.

In addition to endogenous variables we have a set of 4 exogenous day of the week dummy variables:

- *Tuesday* $(e_{1,t})$: a binary one-dimensional vector, whose element is equal to one if day t is Tuesday and equal to zero otherwise;
- *Wednesday* $(e_{2,t})$: a binary one-dimensional vector, whose element is equal to one if day t is Wednesday and equal to zero otherwise;
- *Thursday* $(e_{3,t})$: a binary one-dimensional vector, whose element is equal to one if day t is Thursday and equal to zero otherwise;
- *Friday* $(e_{4,t})$: a binary one-dimensional vector, whose element is equal to one if day t is Friday and equal to zero otherwise;

Since three out of four information criteria tell us that the optimal lag order is two (see section 2.0.5 of the Appendix) we estimate a *VAR(2)* model which can be written as follows:

$$\mathbf{y}_t = \mathbf{A}_0 + \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2\mathbf{y}_{t-2} + \sum_{j=1}^{4}\mathbf{B}_j e_{j,t} + \mathbf{u}_t \tag{2.4.2}$$

Given the relatively small sample size (182 observations), our lack of prior knowledge about the structure of error variance/covariance matrix and the fact that we do not impose restrictions on our equations -they all have the same explanatory variables-, we choose to estimate the model using OLS.

2.4.3 VAR(2) Estimates and residuals analysis

Since all roots of the characteristic polynomials of our model lie inside the unit circle, our VAR equation system is stable/stationary. As we can see from the Table 2.2, more than half of the estimated coefficients of endogenous variables (37 out of 72) are not statistically significant at the 0.1 confidence level. About one third of the estimated coefficients of endogenous variables (29 out of 72) are statistically significant at the 0.05 confidence level. Finally, only sixteen estimated coefficients of endogenous variables are statistically significant at the 0.01 confidence level.

Table 2.2. VAR(2) model estimates

VFTSE at close prices (4:30PM UTC); VIX at open prices (2:30PM UTC); all endogenous variables have been standardized

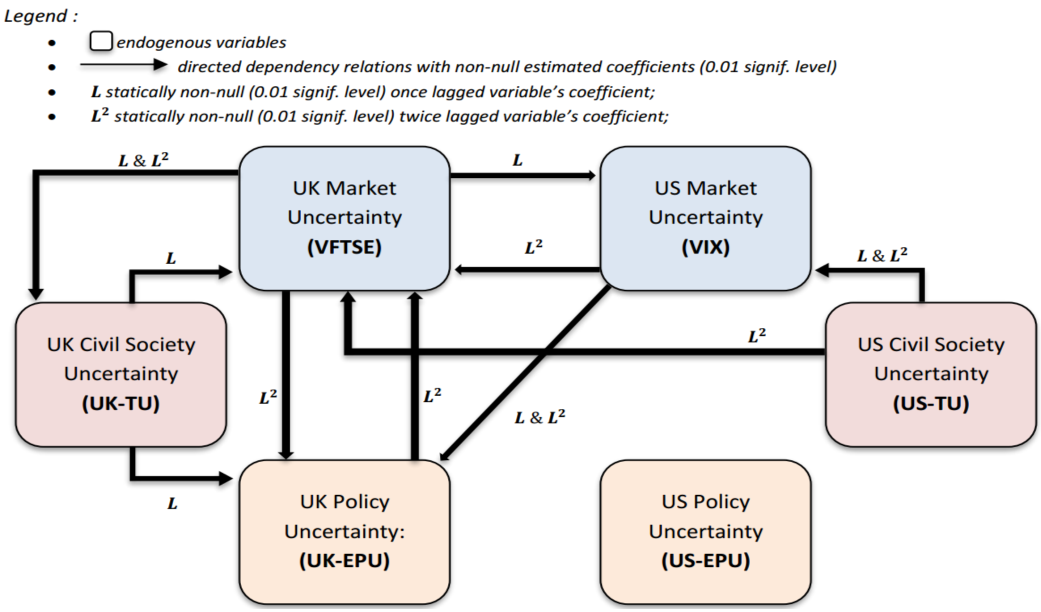
L means once lagged variable, L^2 means twice lagged variable

	Dependent variables:					
	UK-TU	US-TU	VFTSE	VIX	UK-EPU	US-EPU
L UK-TU	0.45***	−0.13	0.10***	0.07	0.37***	0.15
	(0.09)	(0.11)	(0.04)	(0.06)	(0.08)	(0.10)
L^2 UK-TU	0.09	−0.03	−0.02	−0.02	−0.03	−0.15
	(0.10)	(0.12)	(0.04)	(0.06)	(0.09)	(0.11)
L US-TU	−0.12	0.32***	0.03	−0.12***	0.05	0.12
	(0.07)	(0.08)	(0.03)	(0.04)	(0.06)	(0.07)
L^2 US-TU	0.08	0.15*	0.06**	0.17***	0.07	0.06
	(0.07)	(0.09)	(0.03)	(0.04)	(0.06)	(0.08)
L VFTSE	−0.73***	−0.39	0.86***	0.27**	−0.45**	−0.31
	(0.20)	(0.24)	(0.08)	(0.12)	(0.17)	(0.21)
L^2 VFTSE	0.94***	0.26	0.18**	−0.04	0.65***	0.44**
	(0.21)	(0.25)	(0.08)	(0.13)	(0.18)	(0.21)
L VIX	0.33**	0.42**	0.04	0.77***	0.33***	0.25*
	(0.14)	(0.17)	(0.05)	(0.09)	(0.12)	(0.14)
L^2 VIX	−0.33**	−0.08	−0.15***	−0.05	−0.44***	−0.28*
	(0.14)	(0.17)	(0.05)	(0.09)	(0.12)	(0.15)
L UK-EPU	−0.06	0.01	0.01	−0.07	0.22**	−0.03
	(0.10)	(0.12)	(0.04)	(0.06)	(0.09)	(0.11)
L^2 UK-EPU	0.05	0.03	−0.08**	−0.06	0.15*	0.07
	(0.09)	(0.11)	(0.04)	(0.06)	(0.08)	(0.10)
L US-EPU	0.07	−0.02	−0.03	−0.01	0.04	0.22***
	(0.07)	(0.09)	(0.03)	(0.05)	(0.06)	(0.08)
L^2 US-EPU	0.02	−0.01	−0.06**	−0.06	0.05	0.37***
	(0.07)	(0.09)	(0.03)	(0.05)	(0.06)	(0.08)
Const	0.05	−0.06	0.06	0.05	0.01	0.24*
	(0.13)	(0.16)	(0.05)	(0.08)	(0.11)	(0.14)
Tuesday	−0.15	0.03	−0.04	−0.22**	0.14	−0.13
	(0.18)	(0.21)	(0.07)	(0.11)	(0.15)	(0.19)
Wednesday	0.07	0.37*	−0.03	0.02	0.15	−0.32*
	(0.18)	(0.22)	(0.07)	(0.11)	(0.16)	(0.19)
Thursday	0.03	0.01	−0.16**	−0.07	−0.21	−0.39**
	(0.18)	(0.22)	(0.07)	(0.11)	(0.16)	(0.19)
Friday	−0.19	−0.11	−0.11	0.02	−0.11	−0.27
	(0.18)	(0.22)	(0.07)	(0.11)	(0.16)	(0.19)
Observations	182	182	182	182	182	182
R^2	0.51	0.30	0.93	0.81	0.63	0.47
Adjusted R^2	0.46	0.24	0.92	0.79	0.60	0.42
Resid. SE (df=165)	0.74	0.88	0.28	0.46	0.64	0.77
F Stat. (df=16;165)	10.69***	4.51***	134.05***	43.99***	17.86***	9.04***

Note: *p<0.1; **p<0.05; ***p<0.01

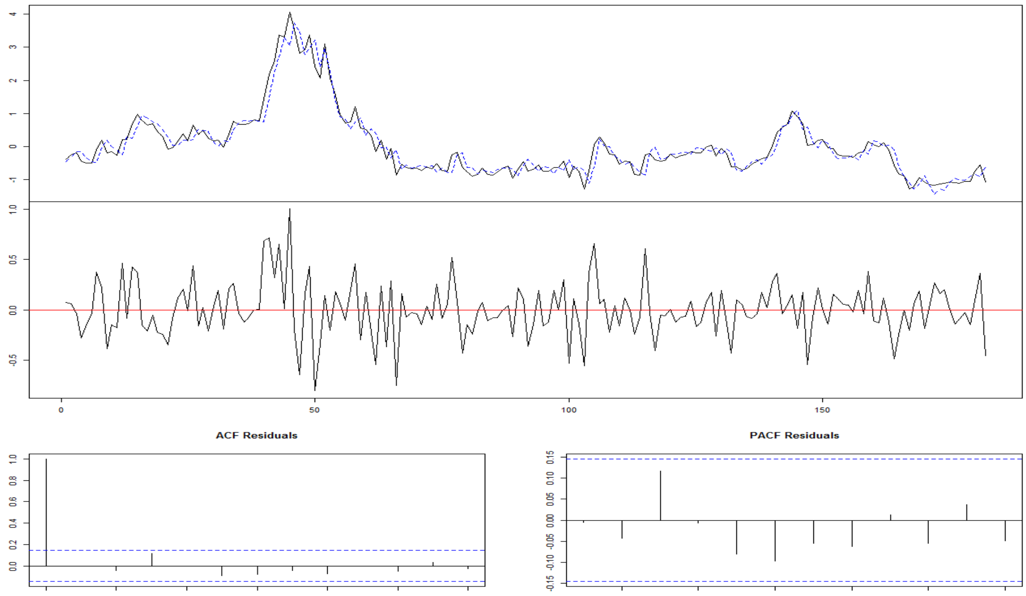
As we can see from the VAR(2) regression table (Table 2.2) and its summary diagram (Fig. 2.8), while Adjusted R^2 of volatility indexes are very high (0.92 for the VFSTE and 0.79 for the VIX) the ones of twitter uncertainty indexes rather low (0.42 for the UK-TU index and 0.22 for the US-TU index). The high adjusted R^2 of both option implied volatility indexes, which are used as proxies of UK and US *market uncertainty*, show that, in the timespan of our study, which contains both the EU-referendum and US presidential elections, *market uncertainty* variability in the UK and the US appears to be in large part explainable through *non-market uncertainty* proxies' variations. Whereas, through our model, it is more difficult to explain observed oscillations in *non-market uncertainty* variables through the swings of *market uncertainty* proxies. Furthermore, we observe that a relatively larger share of the variance of US *market uncertainty* (VIX) remains unexplained by our model compared to its UK counterpart (VFTSE). This finding may be partially explained by the higher degree of interdependence between market and *non-market uncertainty* proxies in UK (UK-TU, UK-EPU, VFTSE) compared to US (UK-TU, UK-EPU, VIX). The three UK endogenous variable have higher R^2 compared to their US counterparts, this could be due to a higher interdependence between the three uncertainty proxies in UK compared to US, a higher signal/noise ratio for UK uncertainty variables with respect to US variables, or, to the higher relevance of omitted variables for US compared to UK. This finding may be linked to the peculiar situation that UK experienced after the EU-referendum, when, given the intensity and rapid succession of several disruptive events/shocks, uncertainty in UK may have been reinforced through positive inter-source feedback effects and propagation mechanisms among *market and non-market uncertainty* variables that are endogenized in our model. In addition, only for the VFTSE (column 3), VIX (column 4) and US-EPU (column 6) indexes at least one day of the week dummy variable is statistically significant. This means that only these three time series exhibit constant day of the week cyclical trends. Finally, for what concerns uncertainty spillover effects, the coefficients that describe US variables' lagged dependency on UK variables are in general terms less significant, from a statistical point of view, with respect to UK (Market and Policy) uncertainty lagged dependency on US uncertainty variables.

Figure 2.8. Statistically significant inter-day dependency relations among endogenous variables



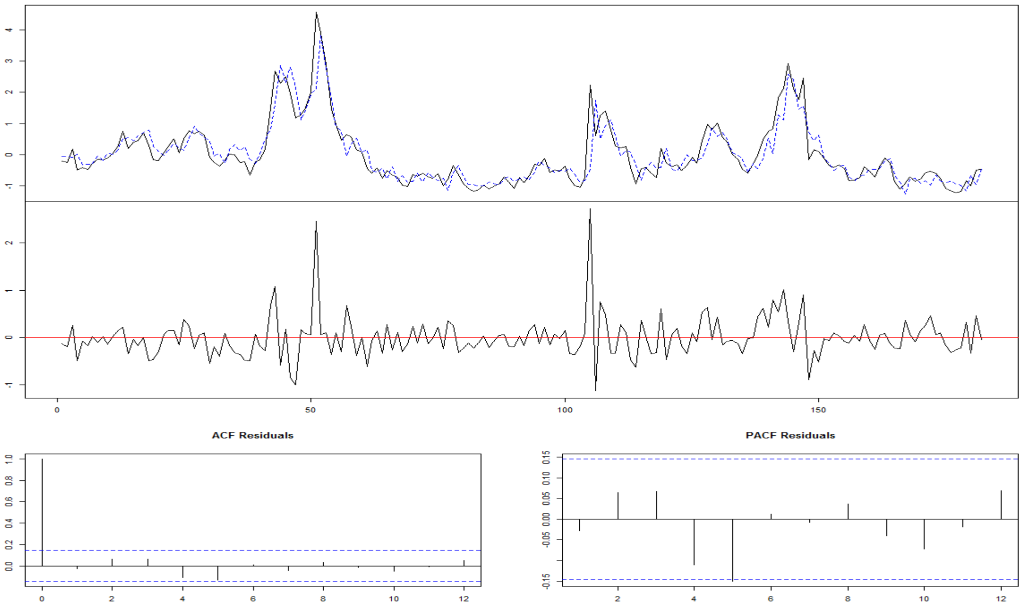
The residuals of the UK-TU index fluctuate close to zero until a week before the EU-referendum, after this point they appear to be more autocorrelated and distributed in a non-normal way for at least two weeks. Successively UK-TU residuals fluctuate again very close to 0, until a week before the US-elections. The gamma like distributed residuals observable after the EU-referendum may represent unexpected *civil society uncertainty* in UK, caused by the vote result in favour of Brexit. Similarly, residuals of the US-TU fluctuate close to zero until a week before the US-elections, after this point residuals appear to be autocorrelated and right skewed for almost three weeks. Successively they fluctuate again very close to 0. Like for the UK-TU index, gamma like distributed residuals observable around the US elections represent unexpected *civil society uncertainty* in US, which was caused by higher election uncertainty just before the vote and by "Trump related" uncertainty after the vote. The two *civil society uncertainty* variables -proxied with TU indexes- appear to be less dependent on other lagged uncertainty measures compared to the EPU and volatility indexes. Partial autocorrelation among the residuals of UK-TU and US-TU is probably due to the aforementioned periods of exceptional unexpected *civil society uncertainty*.

Figure 2.9. Standardized VFTSE fit, residuals and residuals' autocorrelation



As we can see from Fig. 2.9, the fitted regression curve of VFTSE remains very close to the observed value of VFTSE in almost the whole period under study. Residuals' autocorrelation and partial autocorrelation values never come out from the confidence interval, therefore we do not refuse the null hypothesis of absence of autocorrelation and partial-autocorrelation. The residuals of the VIX index (Fig. 2.10) also oscillate rather close to 0 during the entire period of study, with two exceptions around observations number 50 and 105. Around these two points residuals appear to have a higher variance and are more right skewed.

Figure 2.10. Standardized VIX fit, residuals and residuals' autocorrelation



Finally, the residuals of the UK-EPU index and those of the US-EPU indexes are halfway between those of the volatility indexes and the ones of the TU indexes. With very few exceptions they appear to have a constant variance across time. The residuals' autocorrelation and partial autocorrelation coefficients are, all but one (PACF for the UK-EPU residuals, for lag 5), in the confidence intervals.

The distributions of the residuals of the VIX, VFTSE, US-EPU and UK-EPU appear to be almost normal when plotted against a normal distribution. It was rather surprising for the author to see to which degree, by using four extremely leptokurtic and right skewed, gamma-like distributed variables (the TU and EPU indexes) as regressors, we were able to obtain almost normally distributed residuals, especially for what pertains to the residuals of the volatility indexes (VIX and VFTSE).

For an in-depth analysis of the stability of estimated coefficients and other robustness checks we refer to subsection 2.0.8.1 of the Appendix.

2.4.4 Structural impulse-response functions

In the following subsection we compute the impulse response (IR) functions of our estimated system of equations. As Tsay[515] points out, in VAR models, error terms can "*be correlated, that is, $\Sigma_{\mathbf{u}}$ is not a diagonal matrix*". Since *error terms consist of all the influences and variables that are not directly included in the endogenous variables vector [...] correlation of the error terms [-estimated through the residuals variance-covariance matrix-] may indicate that a shock in one variable is likely to be accompanied by a shock in another variable. In that case, setting all other residuals to zero [while computing impulse-response functions] may provide a misleading picture of the actual dynamic relationships between the variables [and] obscure the actual relation between the [endogenous] variables*"[517]

By looking to the residuals variance-covariance matrix $\Sigma_{\hat{\mathbf{u}}}$ we can see that our estimated VAR(2) model appears to have non-null covariance relations among error terms.

Table 2.3. $\Sigma_{\hat{\mathbf{u}}}$ Residuals Variance Covariance Matrix

	UK-EPU	US-EPU	VFTSE	VIX	UK-TU	US-TU
UK-EPU	0.406	0.076	-0.019	0.078	0.245	0.113
US-EPU	0.076	0.587	-0.018	0.057	0.123	0.195
VFTSE	-0.019	-0.018	0.079	0.036	-0.009	-0.027
VIX	0.078	0.057	0.036	0.210	0.144	0.085
UK-TU	0.245	0.123	-0.009	0.144	0.544	0.057
US-TU	0.113	0.195	-0.027	0.085	0.057	0.770

The covariance among endogenous variables' residuals appears to be particularly high and positive for the EPU indexes. We could expect shocks to the two EPU variables to be highly and positively interdependent -from an instantaneous point of view- because both variables are based on Newspapers' content, and, even though this content comes from information sources of two different countries, the US for the US-EPU and the UK for the UK-EPU, it may -and certainly will- occur that newspapers from the two countries of the same day talk about the same subjects or worldwide relevant events, like UK's EU-referendum and the US presidential elections. To a lesser extent, we observe that for both geographic-areas there is a positive covariance of residuals of the EPU and TU indexes considered by country. This instantaneous relation was also expected since Civil Society instantaneously reacts to information stimuli about *policy uncertainty* concerning the same geographic-area.

To embed in the impulse-response functions the non-null instantaneous dependency relations evidenced by the residuals' variance-covariance matrix. We must perform our impulse-response analysis in terms of orthogonalized MA (Moving Average) representation of our VAR system:

$$\mathbf{y_t} = \sum_{\mathbf{i=0}}^{\infty} \mathbf{\Theta_i} \omega_{\mathbf{t-i}}$$

Where $\omega_{\mathbf{t}} = (\omega_{1,t}, \omega_{2,t}, ..., \omega_{6,t})$ are orthogonalized innovation terms with unit variance: $\Sigma_{\omega} = I_K$ and the $\mathbf{\Theta_i} = (\Theta_{1,i}, \Theta_{2,i}, ..., \Theta_{6,i})$ coefficients are the dynamic MA responses to orthogonalized innovations. In practice, we can orthogonalize the innovations through a Choleski decomposition of the residuals' variance covariance matrix $\Sigma_{\hat{\mathbf{u}}}$.

The results of the Choleski decomposition depend upon the ordering of our endogenous variables, also called Wold causal ordering. The Wold causal ordering co-determines the orthogonalized IR functions and is therefore critical for the interpretation of our estimated system. It cannot be determined with statistical methods, and must be specified -by the authors- through a recursive technique based on a-priori beliefs and theoretical knowledge about which endogenous variables may have an instantaneous effect on other endogenous variables. Such that, innovations of the first variable y_1 have a potential immediate impact on all other variables $(y_2, y_3, y_4, y_5, y_6)$; the ones of the second variable have a potential immediate impact on all other variables except the first one (y_3, y_4, y_5, y_6), and so on.

We choose to place the EPU indexes before all other endogenous variables in our Wold causal ordering for the following reasons:

- By being based on printed newspapers their daily content is determined -once and far all- at the beginning of the day.
- Markets and twitter users "consume" newspapers as information sources during the day, hence daily agents' activity on markets and twitter is likely influenced by newspapers contents whereas the effect of volatility and twitter uncertainty innovations may have an impact only on next-day printed newspapers, and therefore the EPU indexes are by construction not sensible to intraday twitter and *market uncertainty*.

We chose to place option-implied volatility indexes after the EPU indexes and before the TU indexes -in our Wold causal ordering-. This because, from a chronological point of view, daily TU activity concerning markets pursues after the closing of markets and may have as object daily market events, like volatility shocks. Whereas markets cannot instantaneously react to daily TU information published after their closing.

In addition, for each group of variables -EPU indexes, TU indexes and volatilities- we choose to put the United Kingdom variable before the United States one, for the following reasons:

- **EPU indexes:** given that days are identified through UTC time, we will have that UK daily newspapers are generally printed/distributed before US newspapers. It is therefore very unlikely that the content of daily US newspapers may influence the content of UK ones -since when the latter are published the first may have still to be written-, while the inverse relation may be true;
- **Option-implied volatilities:** when US's S&P 500 market opens UK's FTSE100 market has already undergone six hours of trading; therefore it is more likely for US markets' option-implied volatility at opening to be influenced by UK's option implied-volatility near closing compared to the possibility of the reverse relationship being true;
- **TU indexes:** TU observation concerning UK are more frequently written during UK daylight/working hours whereas TU observation about US are more frequently written during US daylight/working hours. Therefore the distribution (by hours - UTC time) of TU activity concerning UK is centered at the left of TU activity concerning US; it is therefore more probable that daily innovations in UK-TU activity may influence innovations in US-TU rather than the reverse relationship being true;

The chosen Wold causal ordering used to orthogonalize the innovations -through Cholesky decomposition/factorization of $\Sigma_{\hat{\mathbf{u}}}$- is the following:

$$(\mathbf{UK-EPU}, \mathbf{US-EPU}, \mathbf{VFTSE}, \mathbf{VIX}, \mathbf{UK-TU}, \mathbf{US-TU})$$

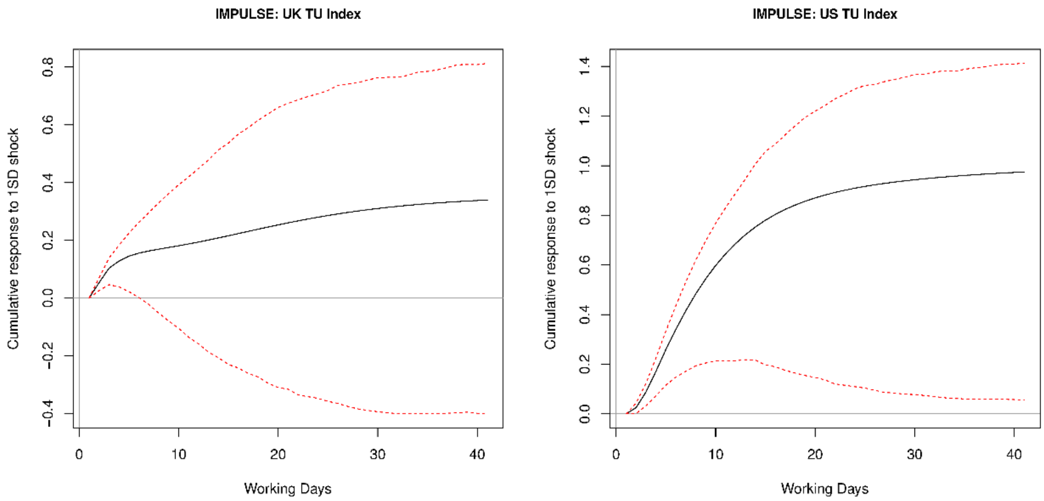
We obtain the following lower triangular matrix T, s.t. $\Sigma_{\hat{\mathbf{u}}} = T * T^t$:

Table 2.4. Choleski factorization of the Residuals Variance Covariance Matrix (T)

	UK-EPU	US-EPU	VFTSE	VIX	UK-TU	US-TU
UK-EPU	0.637	0	0	0	0	0
US-EPU	0.120	0.756	0	0	0	0
VFTSE	-0.029	-0.019	0.279	0	0	0
VIX	0.122	0.056	0.144	0.414	0	0
UK-TU	0.385	0.102	0.017	0.216	0.581	0
US-TU	0.177	0.230	-0.061	0.142	-0.111	0.806

By multiplying both sides of the equation of our VAR(2) estimates by T we obtain a structural form VAR model, with orthogonalized error terms. Orthogonalized IR and cumulative IR functions are simulated up to 40 steps ahead (2 months) using the structural estimated model. Bootstrapped 68% confidence bands (CBs) -based on 1000 bootstrap replications- are obtained through the use of Hall's percentile method[517, 518]. For reasons of space constraints we include only the plots of the cumulative impulse-response functions of *market uncertainty* variables, to impulses of *civil society uncertainty variables*. The cumulative IR functions plots and comments for all other variables have been included in section 2.0.6 of the Appendix. Cumulative IR (y-axis) are functions of time (x-axis) in working-week days. They should be interpreted as the sum of the responses, from time 0 to t, of the variable j to a unitary shock (one standard deviation) of the impulse variable i that has occurred t working days ago.

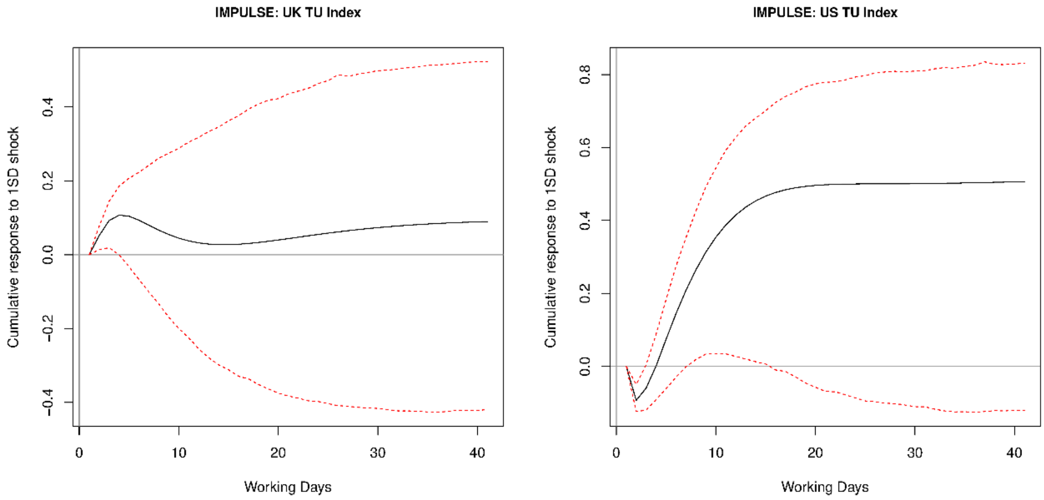
Figure 2.11. Cumulative responses of UK *market uncertainty* (VFTSE) to one standard deviation (SD) impulses of *civil society uncertainty* variables



Bootstrapped 68% confidence bands (in red)

As we can see from Fig. 2.11, an impulse to the UK-TU causes a positive bell shaped cumulative response of the VFTSE index in the first week after the shock. The cumulative response then becomes statistically equivalent to 0. An impulse to the US-TU causes a positive and increasing cumulative response of the VFTSE index. The cumulative response tends to a close to unitary value about one month after the impulse. The lower confidence band approaches 0 as the time-distance from the impulse increases.

Figure 2.12. Cumulative responses of US *market uncertainty* (VIX) to one standard deviation (SD) impulses of *civil society uncertainty* variables



Bootstrapped 68% confidence bands (in red)

For what concerns the VIX index, as we can see from Fig. 2.12, an impulse to the US-TU index, causes a small and negative cumulative response of the VIX the first day after the impulse. The cumulative response then becomes positive and statistically different from 0 between eight and fourteen days after the US-TU impulse. Whereas, an impulse to the UK-TU index causes a bell shaped positive cumulative response of the VIX index in the short run (first five days), which then becomes statistically equivalent 0.

2.4.5 Next-day forecasting of uncertainty variables

In the following subsection we sequentially compute and analyze the one step ahead (next-day) forecasts for each endogenous variable in our estimated VAR(2) model. The scope of this subsection is not to compare the forecasting performance of our VAR(2) specification relative to forecasts obtained under alternative functional forms, but, more simply, to appraise in which terms and to which degree *market uncertainty*, *policy uncertainty* and *civil society uncertainty* fluctuations may be anticipated -the day before- thanks to our -rather simple- daily re-estimated and unrestricted VAR(2) model. Moreover, we wish to verify if, as we expect, for all types of uncertainty it is more difficult to anticipate positive -compared to negative- daily variations, i.e. to predict the occurrence/intensification of uncertainty events compared to their dissipation/attenuation. We verify this hypothesis by looking to our daily variation sign prediction performance, as well as other forecasting performance indicators. One of the advantages of using a vector autoregressive model, is that -by construction- once estimated this type of model is well-suited to sequentially forecast next day values of the endogenous variables. Imagine to be at time *t-1*, we could use the estimates of our VAR(2) model, estimated using all observations available until time *t-1*, to forecast -with great precision- the values at time *t* of the endogenous variables. This because, in our VAR(2) model, the values at *t* are based on the observed values of endogenous variables at *t-1* and *t-2*, and, on the present value of exogenous variables. Since at *t-1* we have already observed the values of the endogenous variables at time *t-1* and *t-2*, and since we used as exogenous variables only day of the week dummies, whose values are known in advance: at time *t-1* we have all the information required to forecast -using the model estimates obtained at time *t-1*- the value of our endogenous variables at time *t*. By construction, the only thing that we do not and cannot observe in at time *t-1* are the innovations at *t*.

Following the above-stated process, we sequentially forecast the one step ahead (next working day) values of our endogenous variables: the first VAR(2) model is estimated using the first five weeks of data (25 observations) and with the estimated model we forecast the value of the 26th observation. Then, we estimate again our VAR(2) model using 26 observations and use it to forecast the 27th observation, and so on. As a result our one-step ahead forecast interval, called *I*, goes from May the 13th to December the 31st 2016.

Since our VAR(2) model is estimated using standardized variables, to obtain meaningful and comparable forecasts from our re-estimated VAR(2) model, for each *t* belonging to *I*, we must:

1. Subset the time-series of the endogenous variables, keeping only the first *t-1* observations for each time-series;
2. Save two vectors, one containing the means:

   $$\mathbf{m}_{t-1} = (m_{1,t-1}, ..., m_{6,t-1})$$

   the other containing the standard deviations:

   $$\mathbf{sd}_{t-1} = (sd_{1,t-1}, ..., sd_{6,t-1})$$

   of the subsetted time-series;
3. Standardize the values of the subsetted time-series;
4. Estimate the $VAR(2)$ model using the $(t-1)*6$ matrix containing standardized subsetted time-series:

   $$\mathbf{std}(\mathbf{y}) = (\mathbf{std}(\mathbf{y})_1, ..., \mathbf{std}(\mathbf{y})_6)$$

5. Use the estimated model $\widehat{VAR(2)}_{t-1}$ to forecast the standardized values of the endogenous variables at time (observation) *t*:

   $$\widehat{\mathbf{std}(\mathbf{y})}_{t|t-1} = (\widehat{std(y)}_{1,t|t-1}, ..., \widehat{std(y)}_{6,t|t-1})$$

6. Transform the forecast vector $\widehat{\mathbf{std}(\mathbf{y})}_{t|t-1}$ back to level values $\widehat{\mathbf{y}}_{t|t-1}$, by multiplying it by the transposed vector of standard deviations $\mathbf{sd}^t_{t-1}$ and by adding to it the transposed vector of means $\mathbf{m}^t_{t-1}$ of the time-series used to estimate the model $\widehat{VAR(2)}_{t-1}$;

By doing so, we obtain the next-day (one step ahead) level forecasts' time-series for each endogenous variable, which can be compared to observed (level) values to evaluate the forecasting performance of our VAR(2) model. Table 2.5 summarizes the forecasting performance of our VAR(2) unrestricted model.

Table 2.5. One Step Ahead Sequential Forecasting - Summary Statistics

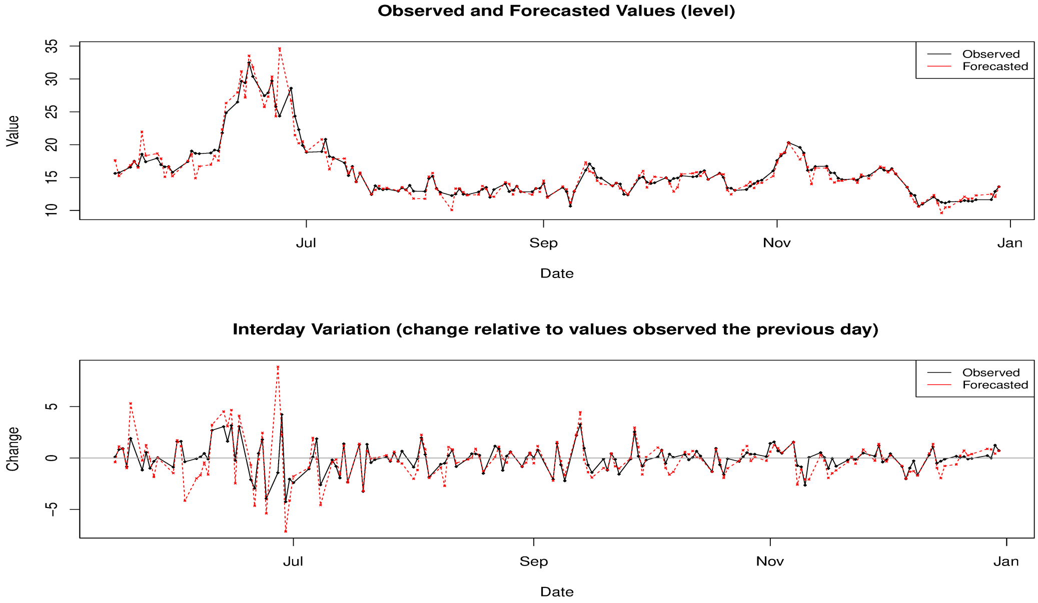
	Correctly predicted sign of variations (% total)	Correctly predicted positive ($\Delta > 0$) variations (% N positive)	Correctly predicted negative ($\Delta < 0$) variations (% of N negative)
UK-TU	63%	72%	56%
US-TU	73%	68%	78%
VFTSE	84%	77%	92%
VIX	79%	77%	81%
UK-EPU	73%	66%	79%
US-EPU	72%	68%	76%
	Mean Error	**Mean Absolute Error**	**Pseudo R^2**
UK-TU	34.305	239.784	0.532
US-TU	-47.343	91.278	0.540
VFTSE	-0.128	0.730	0.914
VIX	-0.097	0.715	0.821
UK-EPU	-48.801	176.505	0.350
US-EPU	-7.981	29.821	0.381

As we can see from Table 2.5, given the simplicity of our model, the frequency of observations and the limited lag order our *civil society uncertainty* forecasting performance are reasonable satisfying, especially for US-TU, for which in 73% of cases we correctly predict the sign of the daily variations (differenced observation values). Especially in dates in which the fluctuations are small we tend to overestimate the magnitude of *civil society uncertainty* fluctuations. We tend to overestimate UK-TU and underestimate US-TU, this is probably due to the fact that the EU-referendum takes place at the beginning of our observed sample and the US-elections close to the end, between the two events the determinants of the fluctuations of US-TU (the coefficients of the US-TU equation) are probably underestimated compared to the ones of UK-TU, which reacted to (integrated the information of) extreme uncertainty during EU-referendum event in the coefficients' value.

The performance of our VAR(2) model in forecasting (one step ahead) *market uncertainty* is particularly good, especially for VFTSE (Fig.2.14):

- In 84% of cases our model correctly forecasts the sign of the daily variations;
- The performance is asymmetric, the model predicts better negative variations (92% of observed cases are correctly predicted) with respect to positive ones (77% of observed cases are correctly predicted)
- The mean error is very small and negative, in average we slightly underestimate UK *market uncertainty*; however the values of VFTSE in the days just after the EU-referendum are overestimated by our model, VFTSE under-reacted in those dates compared to its explanatory variables, i.e. other *non-market uncertainty* proxies;
- The mean absolute error (0.730) is smaller than the mean variation of the VFTSE time series (0.922);

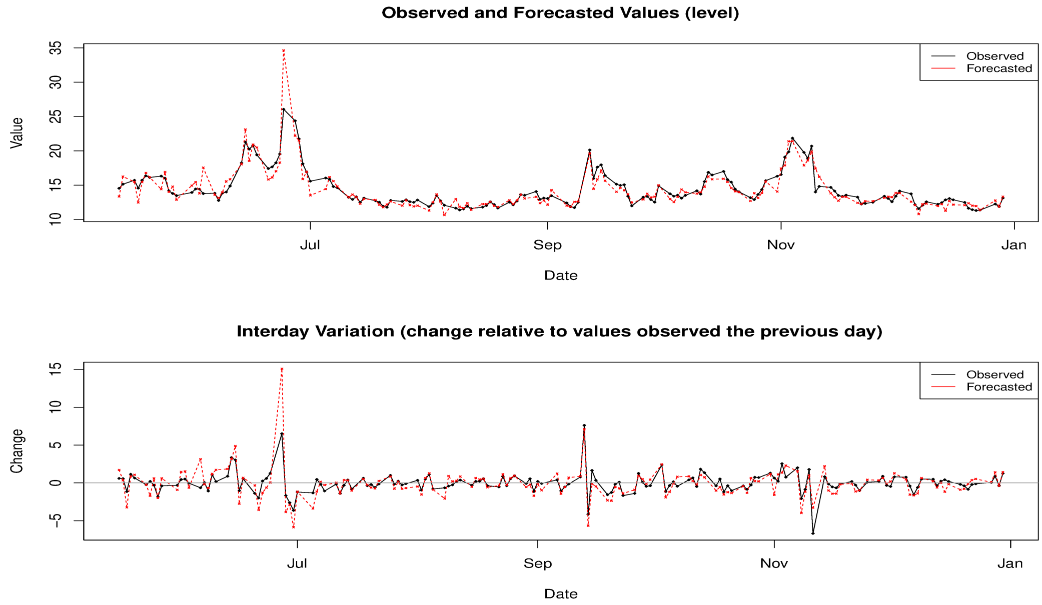
Figure 2.13. VFTSE Index next day out-of-sample forecasts



The forecasting performance for VIX (Fig.2.13) is slightly inferior to that of VFTSE, but still very good:

- In 79% of cases our model correctly forecasts the sign of the daily variations of VIX;
- The performance is still asymmetric, we more often correctly predict negative sign variations (81% VS 77%), but the performance gap is smaller for VIX compared to VFTSE.
- The mean error is very small and negative, in average we slightly underestimate UK *market uncertainty*; however the values of VIX in the days just after the EU-referendum are overestimated by our model, in dates close to the US elections the model performs well in forecasting next day values of VIX;
- The mean absolute error (0.730) is smaller than the mean variation of the VFTSE time series (0.922);

Figure 2.14. VIX Index next day out-of-sample forecasts



The forecasting performance of our model for *policy uncertainty* variables is inferior to that of *market uncertainty* and similar to that of US *civil society uncertainty*. For UK-EPU we correctly predict in 73% of cases the sign of the variation, the for US-EPU the performance is very similar (72% of correctly predicted variation signs). Both *policy uncertainty* variables are in average underestimated. It looks like *policy uncertainty* variables exhibit, after extreme uncertainty events a very large hysteresis: the values of US-EPU and UK-EPU decline very slowly to their pre-event steady state values after major shocks. It looks like *market uncertainty* and *civil society uncertainty* processes have a shorter memory and are less "noisy" after these events, therefore no linear combination of these variables' values appears to be appropriate to anticipate the recurrently high post-event values and variations exhibited by *policy uncertainty* indexes in the time-frame of this study. It is also possible that the autoregressive coefficients of *policy uncertainty* may vary -increasing- during extreme uncertainty events, and then slowly re-converge to their log-term pre-event values.

In this subsection we have shown that our VAR(2) model is able to correctly anticipate in at least 63% of cases the sign of the daily variations of our six endogenous uncertainty measures. Excluding UK-TU, our model correctly predicts more frequently the sign of negative variations compared to positive ones, this performance gap is particularly large for US and UK *policy uncertainty* and for US *civil society uncertainty*.

Market uncertainty appears to be more easily predictable compared to *policy uncertainty* and *civil society uncertainty*, like if the key determinants of *market uncertainty* during the year 2016 were included in the model and more stable across time compared to those of *policy uncertainty*and *civil society uncertainty*; or more simply, because *market uncertainty* processes are less "noisy" and volatile with respect to political and societal ones. In addition, it looks like the forecasting performance of our model was better around the US elections with respect to the EU-referendum. This could be due to various reasons:

- More observations are used to predict the values of uncertainty variables close to the US elections compared to the EU-referendum. Given that the latter event is rather close to the starting date of our observation sample, the estimated coefficients could be highly volatile or imprecise (they haven't yet stabilized) at this point (EU-referendum), whereas when the US-elections occur we are almost at 2/3 of our sample;

- Uncertainty innovations during the EU-referendum were more difficult to anticipate (more erratic or larger in magnitude) compared to those innovations that occurred close to the US-elections;
- Some unobserved or omitted variable, like monetary policy response, may be particularly important close to the EU-referendum, and, the fact that we do not include such a variable in our model may cause our forecasting performance to drop in the time interval in which this variable is probably more relevant, during and just after an extreme uncertainty event;

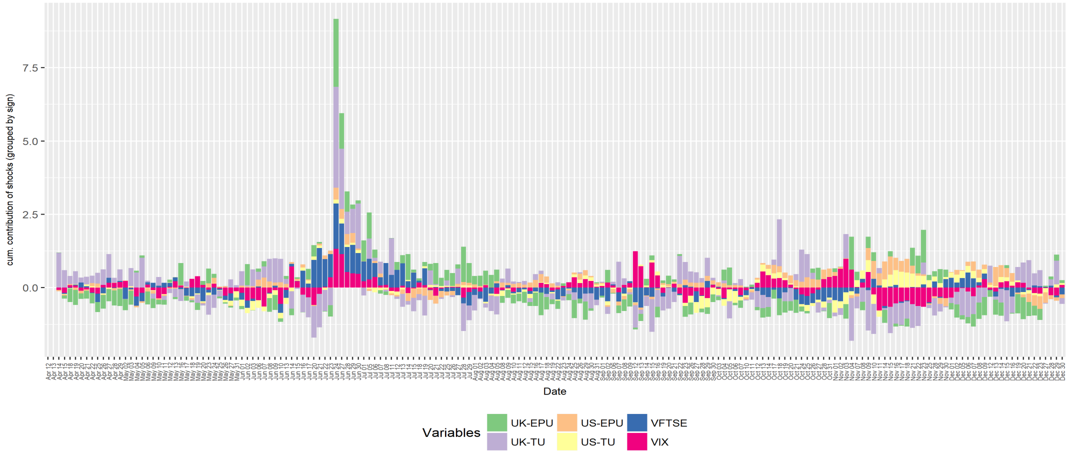
Or, more simply, uncertainty dynamics close to the EU-referendum are more difficult to anticipate. If this was true, close to the EU referendum we may have been facing a situation of irreducible Knightian uncertainty.

2.4.6 Historical decomposition of uncertainty by source

In this subsection we will comment the historical Historical Decomposition (HD) of endogenous *civil society uncertainty*, *policy uncertainty* and *market uncertainty* variables. Historical decomposition allows us to impute observed endogenous variables' fluctuations to the joint cumulative effects across time, of the sequence of -estimated- disturbances to which the endogenous variables have been subject to, i.e. the lagged and contemporaneous effects of innovations, represented in our structural VAR(2) estimated model by the lagged and recursive (infraday) propagation of residuals to the various endogenous variables in the system. Therefore HD can be used to appraise the contribution of -model derived- sequences of innovations, to each endogenous variable, on the other endogenous variables' structural equations. Showing us which were the drivers of fluctuations of *civil society uncertainty*, *policy uncertainty* and *market uncertainty* in US and UK across the year 2016. In particular, it allows us to look to specific intervals of dates, in which major events occurred, to gauge the contribution each variable's prior innovations to the observed values of a given variable of interest in that particular day.

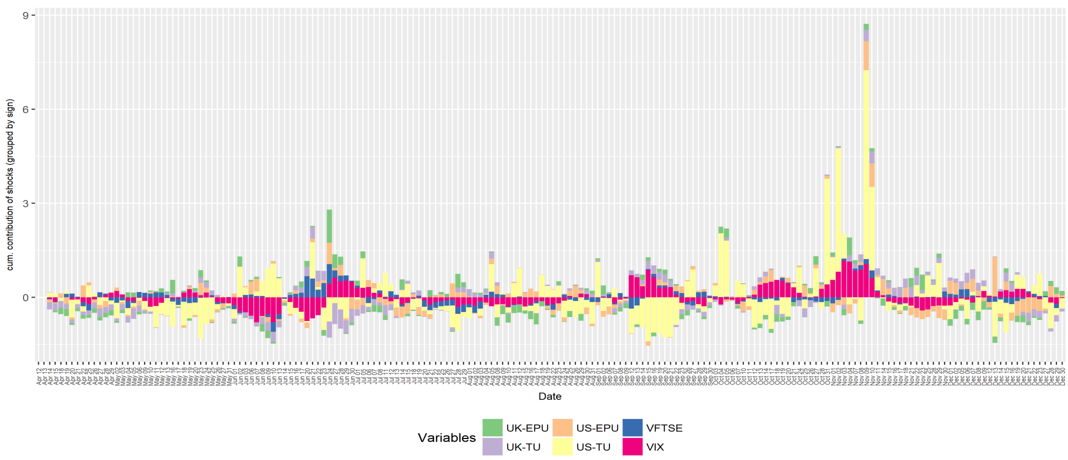
HDs have been obtained by using the innovation-accounting technique first proposed by Burbidge & Harrison[519], which exploits the MA representation of the structural VAR estimated model, to compute the estimated contribution of prior -sequences of- innovations to each endogenous variable, on observed values of the selected endogenous variable of the model. We compute the contribution of innovations to each standardized endogenous variable, for each day from April the 12th to December the 30th 2016, for each standardized endogenous variable in our orthogonalized VAR(2) model. As for impulse-response functions, results are dependent on the selected Wold causal ordering (UK-EPU, US-EPU, VFTSE, VIX, UK-TU, US-TU).

Figure 2.15. UK-TU Historical Decomposition



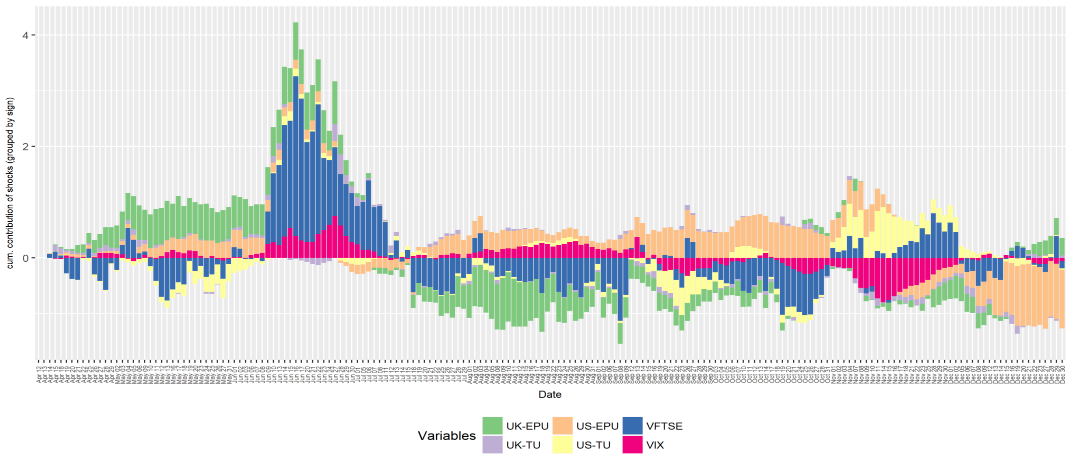
As we can see from Fig. 2.15 the contribution to UK-TU of innovations to other endogenous variables varied substantially across the time interval of this study. Innovations to VFTSE (UK *market uncertainty*) have been relatively more relevant for UK-TU (UK Civil-Society Uncertainty) from the beginning of the week that preceded the EU-referendum to the second ballot of the Conservative Party leadership elections on July the 7th. Their contribution was particularly large and positive in this interval of time and almost negligible out of it. Whereas the role of innovations to the UK-EPU index (*policy uncertainty*) have been particularly relevant for the UK-TU in the two months that followed the EU-referendum. Until the beginning of august their effect on UK-TU has been positive and very large, then, until the end of august it became negative. VIX (US *market uncertainty*) played a positive -peak reinforcing-role on UK-TU during the week after the EU-referendum, from the 12th to the 16th of September and during the week before the US elections. Elsewhere US *market uncertainty* innovations had either a negative or a almost null effect on UK-TU. US-TU (US *civil society uncertainty*) had a negative effect on UK *civil society uncertainty* from late September to mid October; whereas, its contribution to UK-TU has been positive during the whole month of November, i.e. during the week that preceded the US presidential elections and during the three weeks that followed this event.

Figure 2.16. US-TU Historical Decomposition



US Civil-Society Uncertainty exhibits country-inverted endogenous variable shock dependence dynamics compared to UK-TU. As we can see from Fig. 2.16 the innovations played the major role for US Civil-Society Uncertainty -besides innovations to US-TU itself- were those of VIX (US *market uncertainty*). The contribution of US *market uncertainty* innovations to US *civil society uncertainty* has been negative or close to null during the three months that preceded the EU-referendum. It then became positive -and rather large in magnitude- during the two weeks that followed the EU-referendum. VIX contribution turned again negative or close to zero from mid July to the 9th of September. Then, until the election of Trump, it became again positive. VIX innovations have been particularly relevant, in terms of magnitude of their effects, for US *civil society uncertainty* during the two weeks that preceded the US-elections. Similarly, innovations to VFTSE (UK *market uncertainty*) had a particularly relevant positive effect on US *civil society uncertainty* from the week before to the week after the EU-referendum. For what pertains to *policy uncertainty*, the contribution of innovations to US-EPU on US-TU has been positive during the two days that followed the US elections. Whereas UK-EPU played a similar role on US *civil society uncertainty* the three days after the EU-referendum. Finally, UK-TU (UK *civil society uncertainty*) innovations had a positive effect on US *civil society uncertainty* during the week before the EU-referendum, from 12th to the 23rd of September and from the beginning of November to mid December. Elsewhere, the contribution of UK *civil society uncertainty* on US *civil society uncertainty* has been either negative or close to zero.

Figure 2.17. VFTSE Historical Decomposition



As we can see from Fig. 2.17 fluctuations of UK *market uncertainty* were predominantly driven by the innovations to the variable itself. The contribution of innovations to VIX (US *market uncertainty*) on VFTSE has been -almost every day- positive from the beginning of April to the beginning of September, and has then turned negative until the end of the year 2016. UK-EPU (UK *policy uncertainty*) innovations had a large positive effect on UK *market uncertainty* during the three months before the EU-referendum and also during the two weeks that followed this event. The contribution of UK-EPU innovations then became negative for almost all the following days of the year 2016. With the exception of observations in the two weeks that followed the EU-referendum and those from the second half of November to the end of the year 2016, US-EPU (US *policy uncertainty*) innovations had a positive effect on UK *market uncertainty*. The relevance of US-EPU contribution increased closer to the date of the US presidential elections. Innovations to UK-TU (UK *civil society uncertainty*) had a non negligible positive effect on UK *market uncertainty* only during the three weeks after the EU-referendum, effect which declined progressively from the

27th of June to the 13th of July. Whereas US-TU (US *civil society uncertainty*) impulses had a rather large positive effect on UK *market uncertainty* during the whole month of November and during the first decade of December.

Figure 2.18. VIX Historical Decomposition

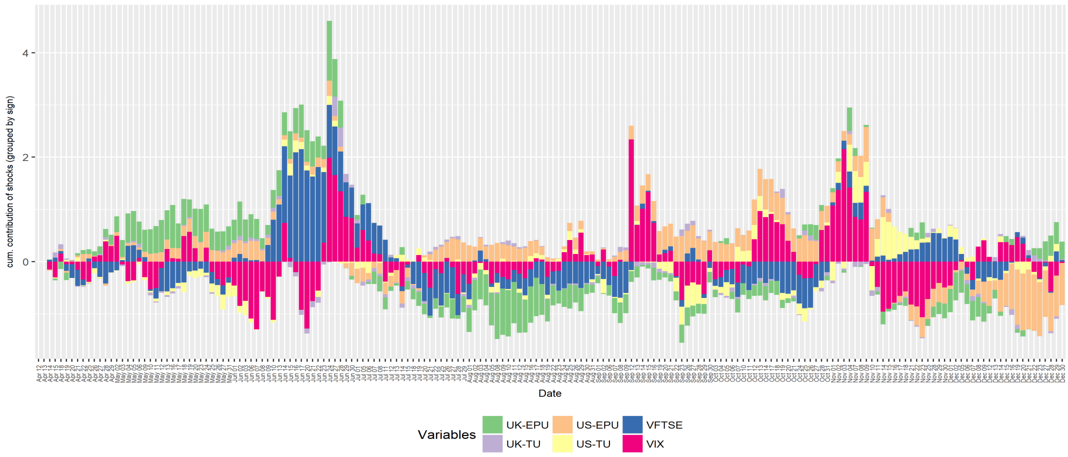
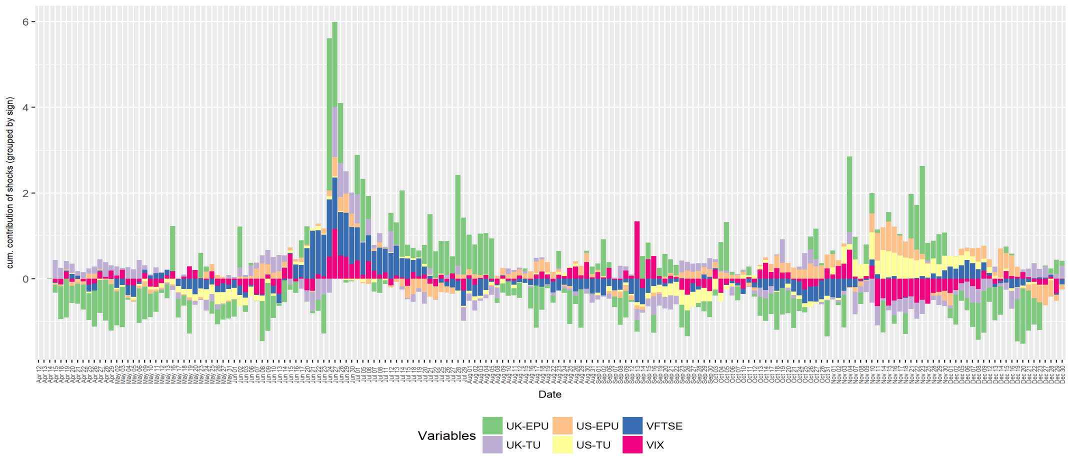


Fig. 2.18 reveals that fluctuations of US *market uncertainty* were predominantly driven by *market uncertainty* innovations. However the contribution of VFTSE innovations to VIX during the EU-referendum was larger in magnitude and opposite in sign compared to that of VIX innovations on VFTSE during the US elections. More specifically, the contribution of innovations to VFTSE (UK *market uncertainty*) on VIX has been positive and particularly large in dates from June the 9th to July the 11th, and also positive but smaller in magnitude during the month of November. UK-EPU (UK *policy uncertainty*) innovations had a large positive effect on US *market uncertainty* during the three months before the EU-referendum, until the 5th of July. The contribution of UK-EPU innovations then became negative for almost all the following days of the year 2016; with the exception of the two weeks before the US elections and the second half of December. As for VFSTE, innovations to US-EPU had positive effects on VIX from the second half of July until the week after the US presidential elections. US-EPU innovations' contribution increased in magnitude from the beginning September to the end of October, and reached its positive extremum during the two days that followed the US presidential elections (9-10 November). If on one side innovations to UK-TU (UK *civil society uncertainty*) had a non negligible positive effect on US *market uncertainty* only during the week after the EU-referendum, innovations to US-TU (US *civil society uncertainty*) had a more pronounced and enduring effect on US *market uncertainty*: during the whole month of November, especially in the first decade, the positive contribution of US *civil society uncertainty* has been one of the main drivers of US *market uncertainty*.

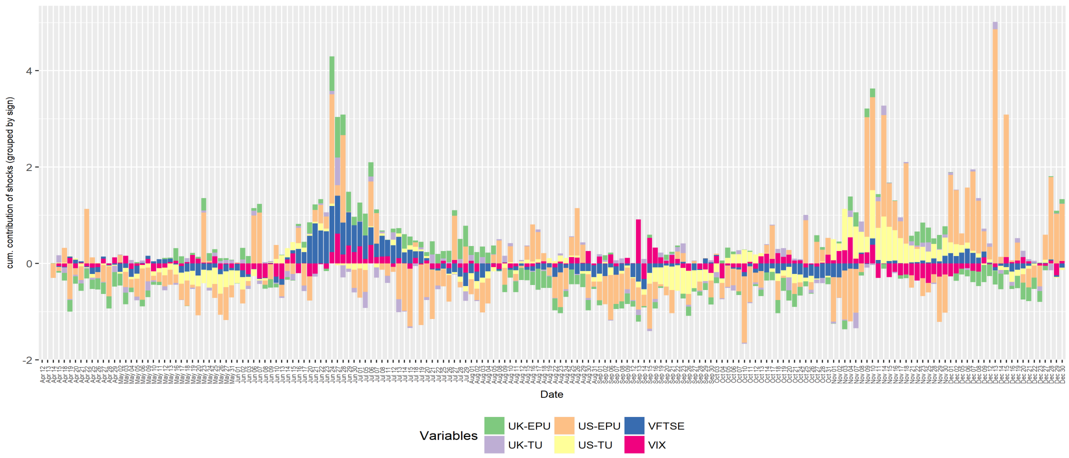
BRIEF EXCURSUS: the observed asymmetries of inter-market uncertainty innovations' contributions is -also- determined by the selected Wold causal ordering of the endogenous variables. Although we have justified this ordering with logical and practical motivations, and it appears to be the most appropriate one. The instantaneous dependency relations are constrained by such an ordering, which determines how innovations are recursively propagated across the system's equations, allowing a one-way instantaneous infraday propagation of disturbances from UK uncertainty variables to the corresponding US ones. Creating a infraday dependency asymmetry between UK and US uncertainty variables.

Figure 2.19. UK-EPU Historical Decomposition



For what pertains to the historical decomposition of *policy uncertainty* in the United Kingdom (Fig.2.19) and United States (Fig.2.20), both variables had been influenced in a similar way by innovations to *market uncertainty* variables. Innovations to VFTSE (UK *market uncertainty*) had a large positive contribution on *policy uncertainty* in both countries from mid June to mid July. Whereas innovations to VIX (US *market uncertainty*) produced, smaller in magnitude but still relevant, positive effects on *policy uncertainty* in both countries during the two weeks that follow the EU-referendum, during the week that preceded the US elections, and between the 12th and 16th of September, when US stocks' prices experienced their biggest decline since the UK's referendum. UK-EPU (UK *policy uncertainty*) innovations had a particularly large positive effect on US *policy uncertainty* during the six weeks after the EU-referendum, and, from the 21st to the 29th of November, when the OBR published its Economic and Fiscal Outlook for UK. Whereas, US-EPU (US *policy uncertainty*) innovations contributed positively to UK *policy uncertainty* from late September until mid December. Innovations to UK-TU and US-TU (*civil society uncertainty*) had rather different contributions, both in terms of sign and magnitude, to US and UK *policy uncertainty*. US-TU innovations had large positive effects on both US-EPU and UK-EPU from the beginning of November to the first decade of December 2016. Whereas US-TU played a minor role, with a relevant positive contribution only after the EU-referendum. Positive contributions of UK *civil society uncertainty* innovations to *policy uncertainty* lasted for a longer period for UK (two weeks) with respect to US (two days).

Figure 2.20. US-EPU Historical Decomposition



In this subsection we have identified a lower bound for imputable contributions of our TU indexes to the observed values of other uncertainty variables. We found that *civil society uncertainty* innovations played a relevant role in post EU-referendum/US-elections events, and have co-determined *market uncertainty* and *policy uncertainty* peaks and slow recovery observed patterns. The cumulative effect of UK *civil society uncertainty* innovations on UK *market uncertainty* (VFTSE) reaches its maximum value (0.35SD of VFTSE) three days after the EU-referendum (June the 27th). Similarly, the cumulative contribution of US *civil society uncertainty* innovations to US *market uncertainty* touched its maximum value (0.73 SD of VIX) six days after the US-elections. Given the chosen Wold causal ordering, these contribution values -of US-TU and UK-TU - on other uncertainty variables are lower bounds. These effects are therefore worthy of interest, they legitimize our effort to construct our TU measures and are motivations for further and deeper analysis and investigation on the topic of uncertainty contagion among *market and non-market uncertainty* variables. For what pertains to the dependencies between *civil society uncertainty* and *policy uncertainty*, we find that the contributions and effects of the first (UK-TU and US-TU) on the latter (UK-EPU and US-EPU) are long lasting. Hence our TU indexes allow us to improve our forecasting capacity and enrich our understanding of *market uncertainty* dynamics in the short run, and, of the dynamics of *policy uncertainty* both in the short but also in the medium-long term.

2.5 Conclusion

In this article we have used Twitter Uncertainty data to develop some new *civil society uncertainty* proxy measures for the United Kingdom and the United States. Our two *civil society uncertainty* indexes, called UK-TU and US-TU, have been used in a structural VAR modelling setting to infer the dependencies among *civil society uncertainty*, *policy uncertainty* and *market uncertainty*, in the United Kingdom and United States, during the unfolding of the Brexit-Trump Era, i.e. the 2^{nd}, 3^{rd} and 4^{th} quarter of the year 2016. Our unrestricted VAR(2) model estimates show that these three distinct sources of geographic-area specific uncertainty exhibit statistically significant interdependencies, within and across these two countries. Both in terms of lagged dependencies -Granger causality- and from a recursive infra-day -Wold causality- point of view.

As we have seen in the impulse-response subsection, the cumulative responses of *market uncertainty* variables (VIX and VFTSE) to *civil society uncertainty* (US-TU and UK-TU) impulses are -almost always- positive and exhibit bell shaped lower confidence bands. As a result, in the medium term (more than two months) the cumulative effects of *civil society uncertainty* impulses on *market uncertainty* variables tend to dissipate: they are either statistically equivalent to zero or their lower confidence band is close to zero. The observed dependency between *civil society uncertainty* and *market uncertainty* is somehow comparable to the relation between monetary policy and aggregate output, effect which is often hypothesized to be positive in the short term and neutral in the long run. Our estimated model clearly evidences that these dependencies are bidirectional and that during the year 2016 there have been intra-area inter-source loop effects among the three sources of uncertainty considered in this study. Especially in the short term, these feedback effects are relevant -from an economic point of view- and statistically significant. It appears that UK *market uncertainty* depends both on US's and UK's *policy uncertainty* and *civil society uncertainty*; whereas, United States' *market uncertainty* depends relatively more on internal (intra-area) *civil society uncertainty* and *policy uncertainty*.

The dependency of *market uncertainty* (VIX and VFSTE) on *non-market uncertainty* variables (UK-TU, US-TU, UK-EPU, US-EPU) appears to be both stronger and more significant for *civil society uncertainty* variables (UK-TU, US-TU) with respect to *policy uncertainty* variables (UK-EPU, US-EPU), for which the intra-area Granger causality relation with *market uncertainty* is significant -at the 0.05 level- and bidirectional only for the United Kingdom.

When considering also infra-day dependencies, given the Wold causality ordering, we observe through the Historical Decomposition of the time series of option-implied volatility indexes (*market uncertainty* proxies) that the contribution of *policy uncertainty* variables on *market uncertainty* variables is -in general- more persistent and -in average- larger through the time span of our study, compared to that of *civil society uncertainty* innovations, which, nevertheless are relevant in the week(s) that follow major uncertainty events, like the EU-referendum and the US-elections. Elsewhere the contribution of *civil society uncertainty* innovations on *market uncertainty* (VIX and VFTSE) appears to be small.

In both unrestricted and restricted versions of our VAR model, whose estimates are contained in subsection 2.0.8.2 of the Appendix, it appears that the strongest spillover effects between the two studied areas, the United Kingdom and the United States, are market driven. Markets are inter-area contagion and propagation channels not only for *market uncertainty* phenomena, but also for *civil society uncertainty* and *policy uncertainty*. Only to a lesser extent, given estimated coefficients and impulse-response functions, inter-area contagions of uncertainty between the US and UK also occurred, in an assymmetric way, through *non market-to-market uncertainty* dependencies. From our structural model estimates it appears that UK *market uncertainty* was rather vulnerable to US *civil society uncertainty* impulses close to the US presidential elections and during the three weeks that followed the victory of Trump; whereas, close to UK's EU-referendum and after the vote in favour of Brexit the estimated contribution to US *market uncertainty* of UK *civil society uncertainty* impulses was more limited.

Appendix

Descriptive statistics

Figure 2.21. United Kingdom Economic Policy Uncertainty (UK EPU) by day

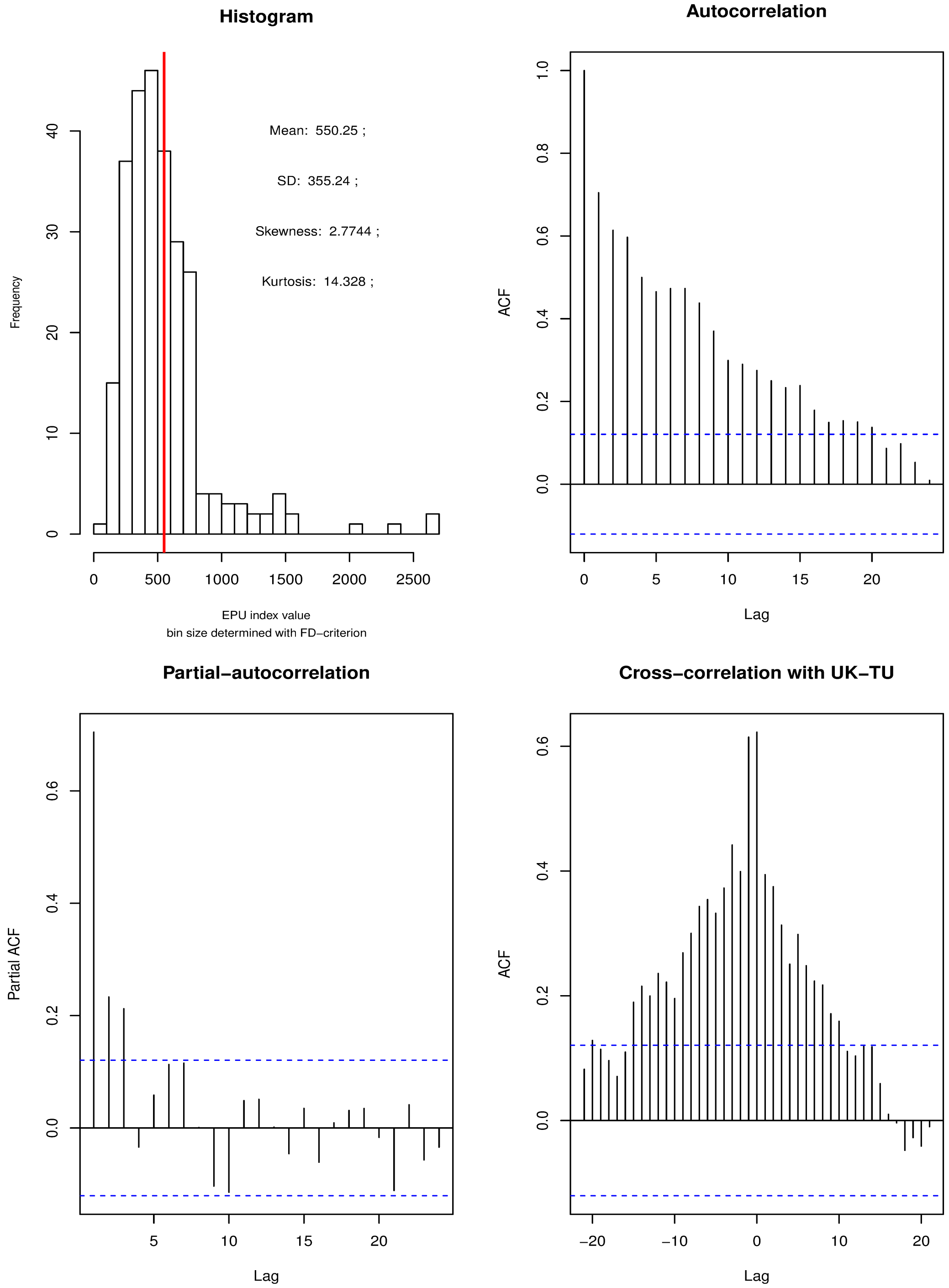


Figure 2.22. United States Economic Policy Uncertainty (US EPU) by day

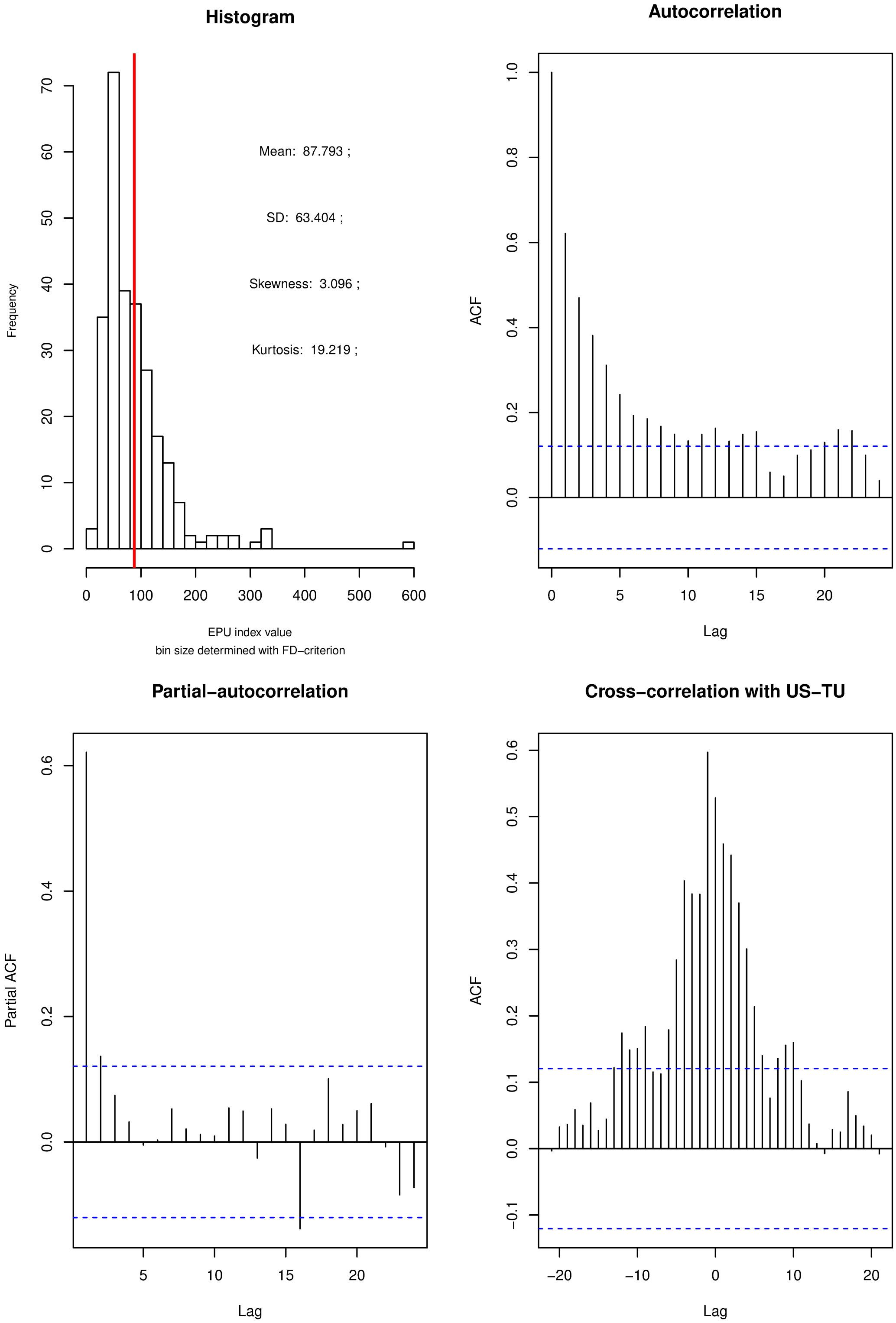


Figure 2.23. United Kingdom Twitter Uncertainty (UK-TU) by day

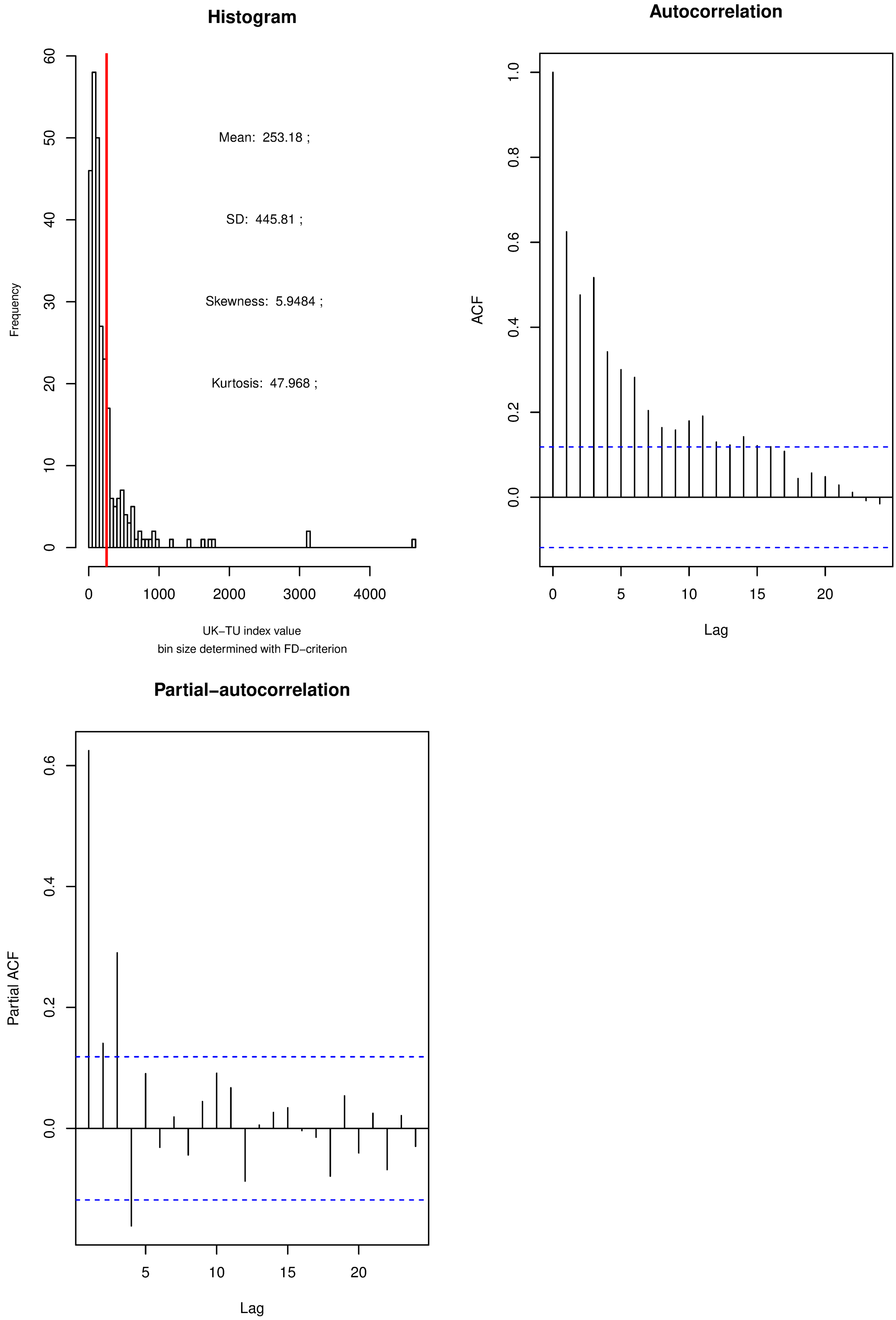


Figure 2.24. United States Twitter Uncertainty (US-TU) by day

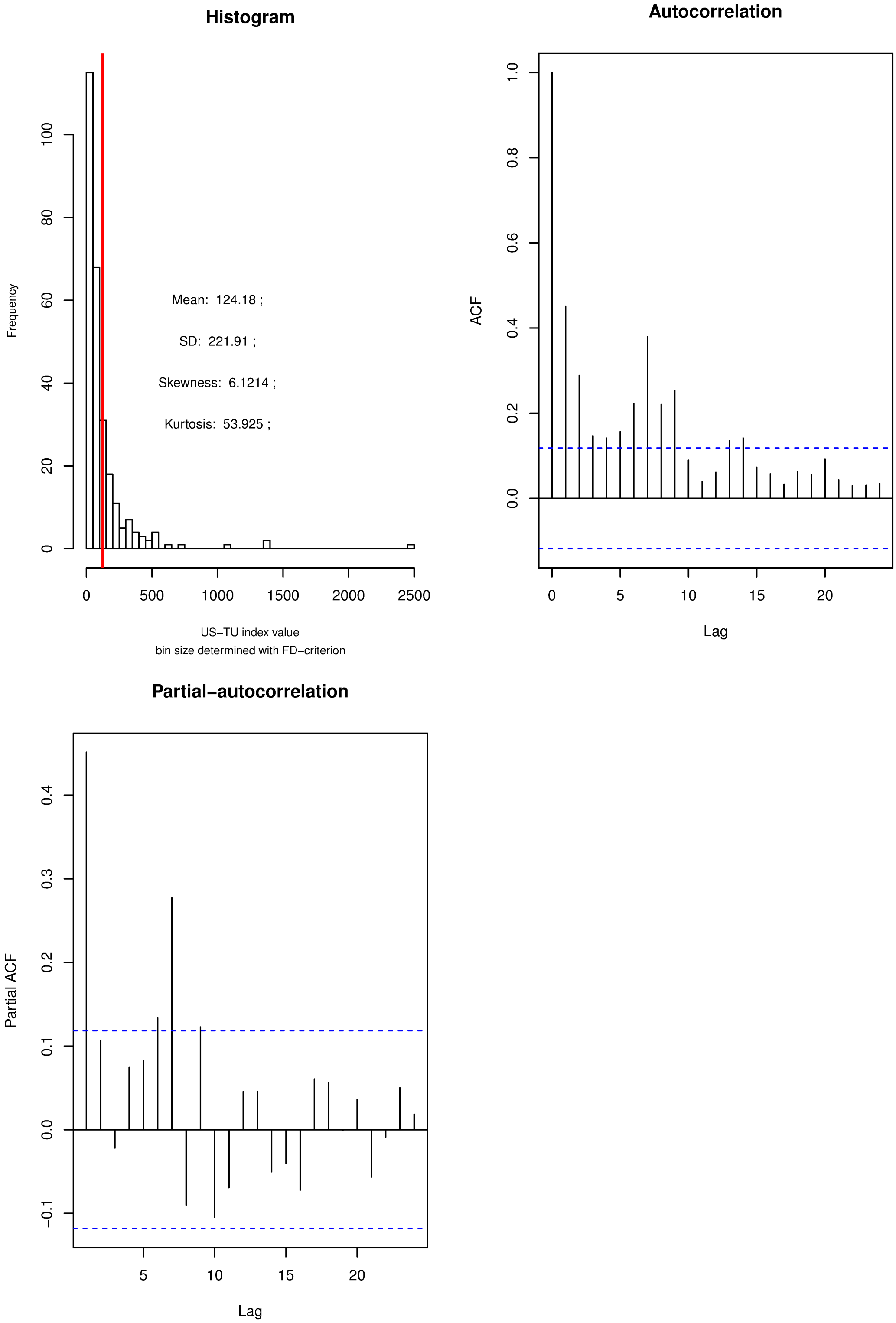


Figure 2.25. CBOE Volatility Index (VIX) by day

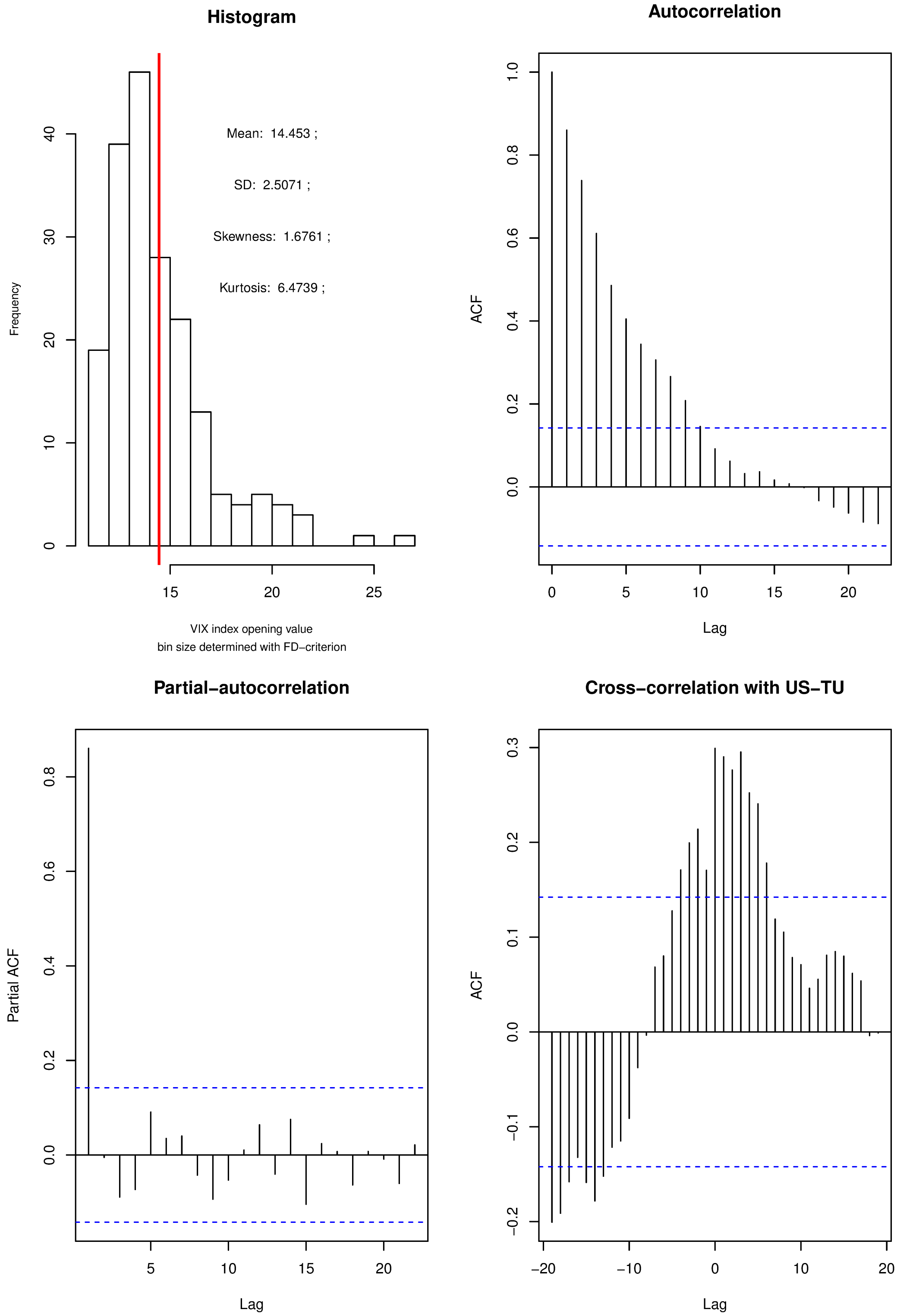
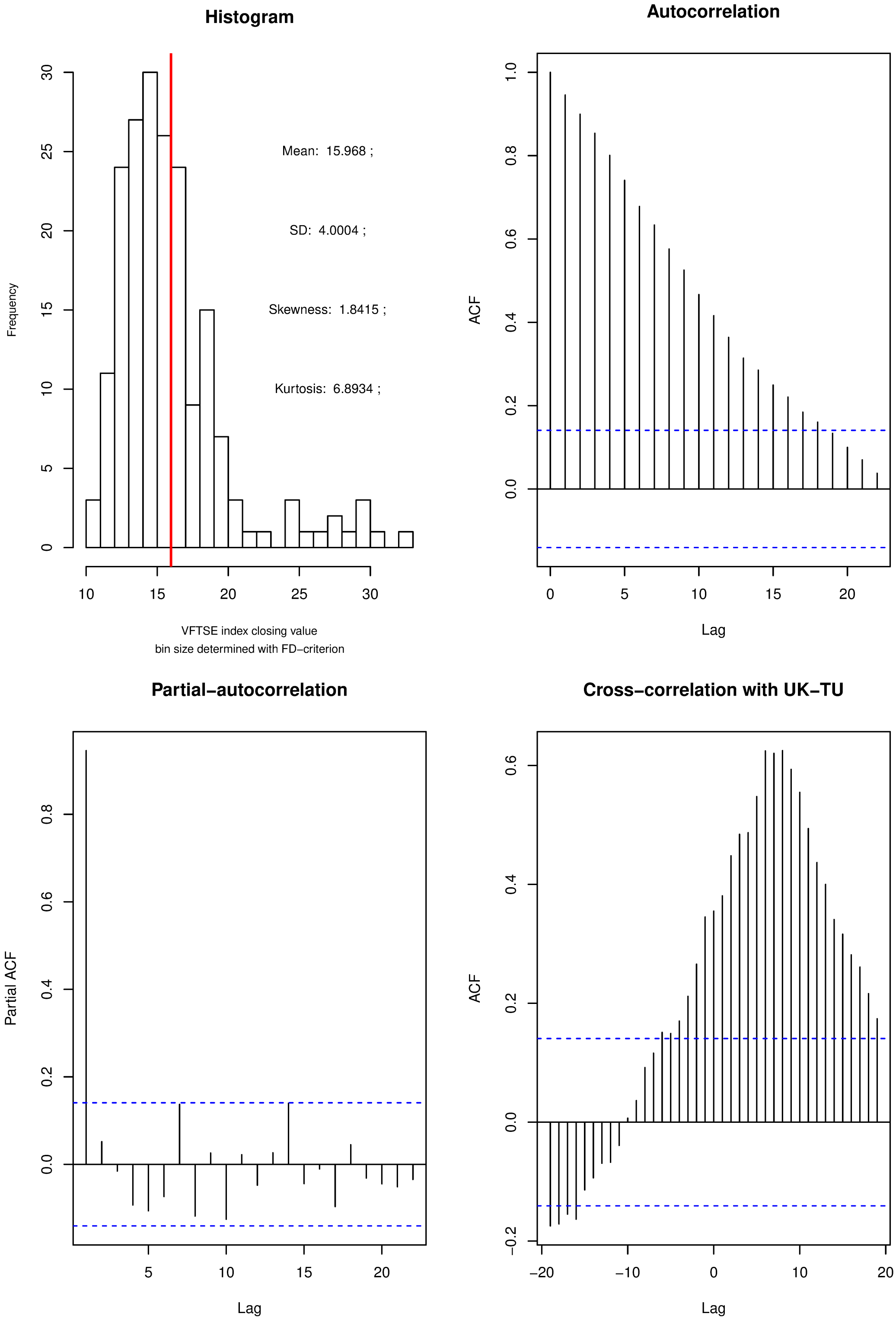


Figure 2.26. FTSE100 Volatility Index (VFTSE) by day



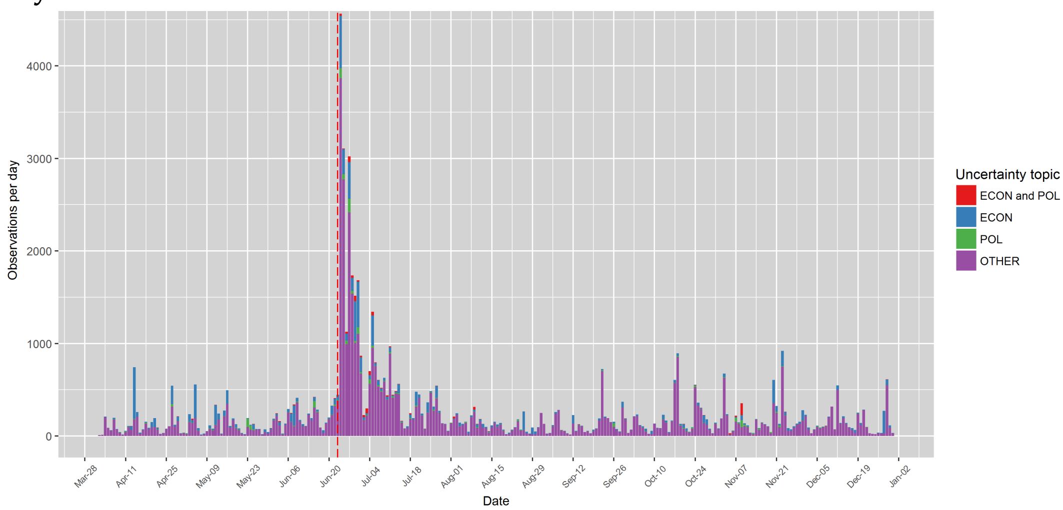
Index validation and decomposition by topic

In addition to the validation of the index by upstreaming information cascades. We have here disaggregated the UK-TU and US-TU according to an additional **Topics** dimension, a polytomous variable with four possible outcomes:

- **"ECON and POL"** - *observations that belong to this category must have:* one or more tokens that belong to the "economics" dictionary AND one or more tokens that belong to the "politics" dictionary;
- **"POL"** - *observations that belong to this category must have:* one or more tokens that belong to the "politics" dictionary AND NO tokens that belong to the "economics" dictionary;
- **"ECON"**- *observations that belong to this category must have:* one or more tokens that belong to the "economics" dictionary AND NO tokens that belong to the "politics" dictionary;
- **"OTHER"** - *observations that belong to this category must have:* Neither tokens from the "economics" dictionary NOR tokens from the "politics" dictionary

Unites Kingdom Twitter Uncertainty (UK-TU)

Figure 2.27. United Kingdom Twitter Uncertainty (UK-TU) index by topic and by day



- **April 14 (744):** Day after the Bank of England's Monetary Policy Committee (MPC) meeting. The MPC's monetary policy summary [520], published the 14th of April, states that "*uncertainty relating to the EU referendum has begun to weigh on certain areas of activity, as some decisions, including on capital expenditure and commercial property transactions, are being postponed pending the outcome of the vote*", the reporting of this information is the subject of the majority of twitter posts about UK uncertainty that day.
- **April 27 (545):** The OECD's Secretary-General Angel Gurría, delivers a speech at the London School of Economics titled "*To Brexit or not to Brexit: A Taxing Decision*". She predicts that "*from the moment of a Brexit vote until the arrangements for divorce are definitively settled -years later- there would be heightened economic uncertainty, with*

damaging consequences. [...] Taking into account the effects of heightened uncertainty and the less favourable trading environment while new arrangements are negotiated, we put the Brexit tax at some 2200 pounds per household by 2020"[521]. The same day UK's Office for National Statistics delivers a statistical bulletin with the new projections of the GDP for the first quarter 2016: "*GDP is estimated to have increased by 0.4%*"[522] which is equivalent to a growth slowdown of 0.2% compared to the previous quarter. These news are quickly rebounded and commented by the medias[523, 524] and extensively mentioned throughout twitter posts regarding uncertainty in the UK.

- **May 5 (559):** UK economic growth is "near stalling", partly due to uncertainty over the EU referendum, according to a closely-watched survey.

As we approach the EU-referendum vote, changes in the UK-TU index appear to be both, highly correlated to, and caused by, community-wide reactions to news and consequent updates of expectations, following the disclosure to the public and medias of new technical assessments and economic forecasts regarding both the economic effects of the referendum per se, and the implications of an eventual exit of Britain from the EU. More specifically, the more it is claimed, by British *-Bank of England, Chancellor of the Exchequer, Office for National Statistics, among others-* and international *-OECD, ECB, World Economic Forum , IMF, World Bank, among others-* economic and financial institutions that the referendum or the prospects of victory of the exit front increase uncertainty and that uncertainty can hamper UK's growth, investments and employment, the more the UK-TU index rises as a result of the sharing and discussions of these news through Twitter. In addition, the UK-TU appears to grow in dates in which the exit front seems to be ahead in polls[525] and therefore when the prospect of an exit from the EU becomes more concrete, in particular in working weekdays from June the 6th to June the 16th, day in which the MP Joe Cox was murdered. After that date and up to the voting day, the polls' voting intention shares reversed and the pro-European front appeared to be again slightly ahead. However, despite the claimed advantage of the Europeanists in polls, during the five days that preceded the EU-referendum the UK-TU index raised slowly but constantly from 38 observations per day to slightly more than 400 in June the 23rd, the voting day.

The uncertainty shock due to the EU-referendum outcome begins in June the 24th, the day after the vote. Throughout the morning it becomes known that the British voted (52% against 48%) in favour of leaving the European Union, reason why that very day D. Cameron announced his intention to resign from the role of Prime Minister. The uncertainty effects of this event are the largest in the period covered by this study by several orders of magnitude, with more that 4 000 observations in the day of the referendum results announcement. Brexit is clearly also the more persistent uncertainty shock we observe throughout our area-specific indexes: The UK-TU index stays above its average value (220) for more than three weeks, until mid July. During the week that follows the EU-referendum vote we count more than 17 000 observations referring about uncertainty in the UK. After a first apparent recovery during the first weekend of July, with 274 and 404 observation respectively the 2nd and 3rd of July, the value of the UK-TU index increases again to 1421 the 5th of July, day of the First ballot of the Conservative Party leadership elections. Till July the 14th the index stays above 400 observations per day. After a few "calm" days, the UK-TU index value approaches again 500 observations in July the 20th and 21st:

- **July 20 (478):** The rating agency Moody's publishes a Banking System Outlook for the UK, titled "*United Kingdom: Uncertainty Following Brexit Vote Will Weaken Credit*

Fundamentals"[526]. Carlos S. Duarte, Senior Vice President at Moody's, affirms that *"increased uncertainty about the UK's future trade relationship with the EU will likely lead to reduced confidence and lower investment and consumer spending in the UK*"[527], reason why Moody's decided to leave unchanged its negative outlook for UK's banking system. Outlook that was modified to negative after the vote. Moody's twitter username is mentioned in 79 observations that day. Other posts mention information contained in the update by the IMF of the World Economic Outlook, titled: "*Uncertainty in the Aftermath of the U.K. Referendum*"[528], published the day before (July the 19th).

- **July 21 (451):** Mario Draghi has a press conference at the ECB. During the conference[529] he affirms that: "*Following the UK referendum on EU membership, our assessment is that euro area financial markets have weathered the spike in uncertainty and volatility with encouraging resilience. [...] Given prevailing uncertainties, the Governing Council will continue to monitor economic and financial market developments very closely and to safeguard the pass-through of its accommodative monetary policy to the real economy. [...] Large uncertainties prevail, because first of all these figures will in the end depend on how long is going to be the stretch of time for these negotiations to be completed and therefore to give a certain outlook, which we don't have with us today [...] it's very difficult to understand how these big geopolitical uncertainties would affect the recovery, because the channels are not obvious. [...] The message that will probably come out -well, the message that will come out, the message that I can foresee or expect to come out, will be a message of stability, and the message that will come out specifically from the eurozone will be a message of a recovery that continues, though at a slower pace, in the midst of great uncertainties. Uncertainties that are not only or not especially, actually coming from the eurozone, but they come from various parts of the world. And in this climate of general uncertainty, not necessarily economic uncertainty but probably mostly geopolitical uncertainty, it's very important that a message of stability comes out of the G20. Policies are everywhere very accommodative. The financial system and the banking system are stronger than they were before, and so it's very important that a message like that comes out.*" That day a great number of observations refer to the above mentioned press conference, some of these observations are retweets of posts by the ECB twitter profile, which refer to a particular part of the speech of their President, as follows: *"Draghi: After UK referendum euro area financial markets have weathered the spike in uncertainty and volatility with encouraging resilience*"[530]. The previous content has been posted (Id: 756104446464159745) by the ECB twitter account (@ecb). Given the content of the tweet we hypothesize that the ECB choose this part of the speech by Draghi to lower the general feeling of uncertainty due to Brexit. Since Draghi speaks about a "weathering of uncertainty" we could think that our, rather naive, UK-TU index does not correctly reflect the uncertainty in UK that specific day. However, the peaking behavior of our TU index in this situation reveals to be helpful, because it allows us to reveal the successful communication tentative, given the number of retweets and the diffusion of Draghi's citation in the medias, of the European monetary governance system that desires to mitigate the perception of uncertainty also through its communication policy, which mentions uncertainty. Speeches and conferences may be more effective economic and monetary policy levers than it has been hypothesized by contemporary economic theory. These communication policy levers can be used at relatively low costs, if not at the expenses of the reputation and authority of the declaimer in the event that the expectations and forecasts he expressed turn out to be too far from perceived reality, and, if people are able to remember the author of the latter.

After that, **from the July the 27nd to September the 21st, the UK-TU index stays**

below 400 observations per day. We cannot say if this is due to the "natural" mitigation and previous discounting of the uncertainty effects of Brexit metioned by Draghi during his press conference, or to any other factor. Nevertheless the mitigation of uncertainty after his speech would certainly deserve a separate in-depth analysis.

Subsequently, from the end of September on, the UK-TU peaks again, with values above 500 (observations per day) in the following dates:

- **September 22 (724):** The Bank of England (BoE) publishes on its website the statements of the Financial Policy Committee (FPC) meeting that took place in September the 20th[531], which states that:"*The financial system has demonstrated resilience to spikes in uncertainty and risk aversion. [...] Although financial stability has been maintained in the United Kingdom through a period of volatility, and a number of economic indicators have picked up from their post-referendum low points, the United Kingdom faces a challenging period of uncertainty and adjustment. [...] Heightened uncertainty about the near-term macroeconomic outlook and the United Kingdom's future relationship with the EU is reinforcing domestic risks. [...] There remains a high degree of political and policy uncertainty in many advanced economies*". That day, the high number of posts quoting the content or retweeting news regarding the above statements by the BoE's FPC determined the high value of the UK-TU index.

- **October 17-18 (605 and 893):** "*The 3 million*" initiative[532], "*a grassroot organisation for EU citizens living in the UK offering practical guidance, lobbying the government to protect the rights of EU citizens in the UK*"[533] is the dominating topic discussed in the observations counted by our UK-TU index in these two days. The following words: "*I'm one of 3 million EU citizens in the UK. This uncertainty is unbearable*", said by one of the founders of the initiative, Anne-Laure Donskoy, are quoted, mentioned, or referred to, in more than six hundred tweets, posted during the 48 hours. The initiative rapidly attracts the attention of national and international television networks and newspapers who devote articles and twitter posts to this subject[534–536]. Other contents among the most shared in these two days include:

  1. An article[537] titled "*UK car industry fears effects of Brexit tariffs on supply chain*" published by the Financial Times and menitioned by the FT's twitter profile (@FT) through a tweet (Id: 787670212007067649), which states that "*suppliers to UK car industry say uncertainty over trade agreements may force them to relocate overseas*", this content is retweeted by almost a hundred users between October the 17th and 18th.

  2. An article of the Foreign Policy (FP) magazine[538] titled: "*Britain Is Becoming an Emerging Market*", which is directly mentioned through FP's twitter account (@ForeignPolicy). The text content of the tweet (Id: 788131737163423745) is the following: "*With a plunging pound and economic uncertainty, the UK is now looking a lot like the developing world*".

- **October 24 (554):** The British broadcaster ITV announces it will have to cut more than a hundred jobs as Brexit uncertainty damages UK's TV ad market. Major newspapers and news agencies publish articles on the issue[539–543]. In a sigle day, on twitter the ITV topic is mentioned in almost four hundred twitter observations considered in our UK-TU index. The most retweeted post on the ITV subject (Id: 790555236737875968):"*ITV cuts 120 jobs as uncertainty over Brexit slows UK TV ad* *market*", is posted by the user @Scientists4EU, a campaigning profile by UK scientists to keep the UK in the EU, this tweet also contains a link to an article from *The Guardian*[540].

- **November 3 (676):** After the decision of the British High Court that "*has ruled that the Government does not have power to trigger Article 50 without parliamentary approval and a vote from MPs*[544]. Mark McFadden, a notorious broadcaster and journalist at ITV, expresses all his doubts and uncertainty through the following tweet:""*After the Brexit vote, UK was left with uncertainty. Now we don't have the certainty of uncertainty. Even our uncertainty is now uncertain*", referring to the possible implications of the above mentioned High Court's decision. The post (Id: 794198371644899330) is retweeted by about four hundred observations.

  The 8th and 9th of November, which correspond to the voting day of the US presidential elections and the day in which the results are revealed to the public, the UK-TU index remains below 500 observations per day. We infer that the election of Donald Trump was not considered at that date, by English speaking twitter users, as a relevant source of uncertainty for the United Kingdom, under the specific circumstances generated by the Brexit vote in UK. Brexiteers and pro-brexit newspapers and tabloids, especially far-right ones, welcomed the Trump's presidency[545, 546], believing it will benefit the UK, both in its negotiations strategy with the EU and by reinforcing the historical cultural, trade and political relation among the two countries in the post-brexit era. An article in The Sun claims that: "*Trump's win could also mean victory for Brexit Britain when it comes to trade deals with the United States*"[547]. While according to the Daily Express the benefit will derive from the possibility of imitating the "non green" energy policy envisaged by Trump: "*Trump presidency could benefit the world at large and could be very good news for the UK in particular [...] This could help reset the UK Government's attitude to climate change [...] The UK could perhaps follow suit on this front and will benefit from exploiting the mountain of carbon fuel we sit on. More jobs and cheaper energy could give our economy an enormous boost.*" [548]. Among specialized commentators and analysts, there are widely varied opinions regarding the uncertainty effects for the UK due to the election of Trump. The Economist weekly magazine considered the election of Trump an obstacle for UK's negotiations with the EU: "*Meanwhile the British economy was already in a fragile state before last night's result, with the pound weakened, business uncertainty mounting and some evidence of slowing investment. The economic shock of a Trump presidency may exacerbate these trends. It will also harden politics in the mainland European countries with which Britain will shortly start negotiating, where populists emboldened by his win will reduce mainstream leaders' freedom to approve a pragmatic deal with Britain*" [549]. While more recently, Benjamin Broadbent, Deputy Governor for Monetary Policy at the Bank of England, affirmed that the election of Trump had benefited UK's economy. The FT reports Broadbent's words as follows: "*the new White House administration had already led to improving business and financial market conditions, boosting the UK's economic prospects on the margin*"[550]. Between the second half of November and the end of the year 2016 the UK-TU peaks four times, at the following dates:

- **November 20 (606):** Three days before the official release of the November Economic and Fiscal Outlook[551] by the UK Office for Budget Responsibility (OBR), Peter Hammond is guest at the Andrew Marr Show on BBC One[552]. During the talk Marr and the Chancellor of the Exchequer discuss about OBR's "difficulties" in modeling and forecasting UK's economic situation given the uncertainty around the UK government negotiating strategy:

- **Marr:** "*Have you had a conversation with the OBR? because it is a very strange situation, they have to forecast what happens to the British economy over five years, which means after Brexit, so on what basis can they possibly make a forecast given that the government doesn't know how you're going to deal with that?*"
- **Hammond:** "*Clearly that's why there is a large degree of uncertainty than usual about the economy over the next couple of years. It will be for the OBR themselves to explain in their report how they dealt with that higher degree of uncertainty. But of course we don't know exactly where we are going to end up, at the end of that period of negotiation...*" [overlapping voices]
- **Marr:** "*It's an almost meaningless process if you haven't given them specific private information about what you intend to do, then they are just making the same kind of guess that anybody watching this program could make, and therefore there is no particular reason to think that their 100 billion pound black-hole -if that's what it is- is real, or anything else they say*"
- **Hammond:** "*It isn't about us giving them information. The fact is we're going to enter into a negotiation with our EU partners, about the terms of our future relationship with them. It's not about the Government's view of the outcome, it's about where those negotiations end up over a period of a couple of years of hard fought negotiations. We will get the very best possible deal that we can for Britain, but the OBR will have to make its own judgment about where we are likely to end up and how that's likely to affect the economy.*"

Hammond successively suggests an alternative interpretation of uncertainty concerning the negotiation strategy of the UK government. Hammond claims that UK's Brexit plans must be kept confidential, because their secrecy is a strategic advantage for UK in its negotiations with EU countries: "*Those who are urging us to reveal our tactics are inviting us to undermine our own negotiations*". UK-TU observations prevalently refer to Hammond's warnings about UK uncertainty caused by negotiations over the next two years.

- **November 23 (921):** Kristin Forbes, external member of the BoE Monetary Policy Committee, gives a presentation[390] at a conference titled "*Uncertainty about Uncertainty*", at J.P. Morgan Cazenove, in London. During her speech Forbes explains the many difficulties in capturing signals of uncertainty through existing proxy measures and why, in her opinion, the effects of uncertainty have been overestimated. She proposes the joint use of multiple proxy measures of uncertainty, trough a principal component analysis, as a best practice to proxy unobservable uncertainty. She also illustrates the interaction mechanisms between uncertainty and aggregate economic variables, highlighting the confounding risks deriving from the use of single uncertainty proxies. Especially when proxies are used to estimate empirical models, she claims it is dangerous to assume that a single uncertainty proxy can mimic the latent variable uncertainty. Therefore, the proxy's estimated coefficients should not be straightforward interpreted as if they were those of the latent variable uncertainty. Forbes also hypothesizes the existence of an inflating "uncertainty" bubble, where the term "uncertainty" is misused by economists, governance institutions, medias and policy experts, which label as "uncertainty" all phenomena with negative socio-economic effects that is strategically claimed to be uncontrollable or unforeseeable. Forbes name is mentioned by more than fifty UK-TU observations. The same day the OBR publishes its November Economic and Fiscal Outlook[551], which contains forecasts of UK's public finances and economic situation for the upcoming 5 years.

The OBR's outlook mentions the word uncertainty more than one hundred and fifty times, about once every two pages. In the executive summary of the OBR's report it's written that[551]:

– "*Given the uncertainty surrounding the choices and trade-offs that the Government may have to make, and the consequences of different outcomes, we have not attempted to predict the precise end result of the negotiations. Instead we have made a judgement consistent with most external studies that over the time horizon of our forecast any likely Brexit outcome would lead to lower trade flows, lower investment and lower net inward migration than we would otherwise have seen, and hence lower potential output.[...]*"

– "*In the near term, as the negotiations get under way, we assume that GDP growth will continue to slow into next year as uncertainty leads firms to delay investment and as consumers are squeezed by higher import prices, thanks to the fall in the pound. [...] Our forecasts are currently somewhat less pessimistic than those in the Bank of England's November Inflation Report and the Treasury's published pre-referendum analysis, but in current circumstances the uncertainty around them is even greater than it would be in normal times.[...]*"

– "*The referendum result and forthcoming post-Article 50 negotiations have generated uncertainty for firms that will lead to some investment being postponed or cancelled. We have revised business investment down relative to our March forecast in all years, which also reduces trend productivity growth due to slower capital deepening.[...]*"

– "*The monetary policy easing announced by the Bank of England in August is likely to have reduced the impact of post-referendum uncertainty on GDP growth. This implies a faster effect on the economy than is typical in economic models, but is consistent with the Bank having acted to head off a drop in activity before signs of it appeared in actual data.[...]*"

– *We expect the economy to be running 0.7 per cent below full capacity by the end of 2017 (compared to 0.2 per cent in the third quarter this year), with above-trend growth then closing this output gap by mid-2021. At this stage we have not assumed any further uncertainty-related hit to growth in 2019 when the UK's exit from the EU is assumed to be completed.*

– *We do not, at this stage, forecast that Brexit-related uncertainty will prompt more aggressive job-shedding.[...]*"

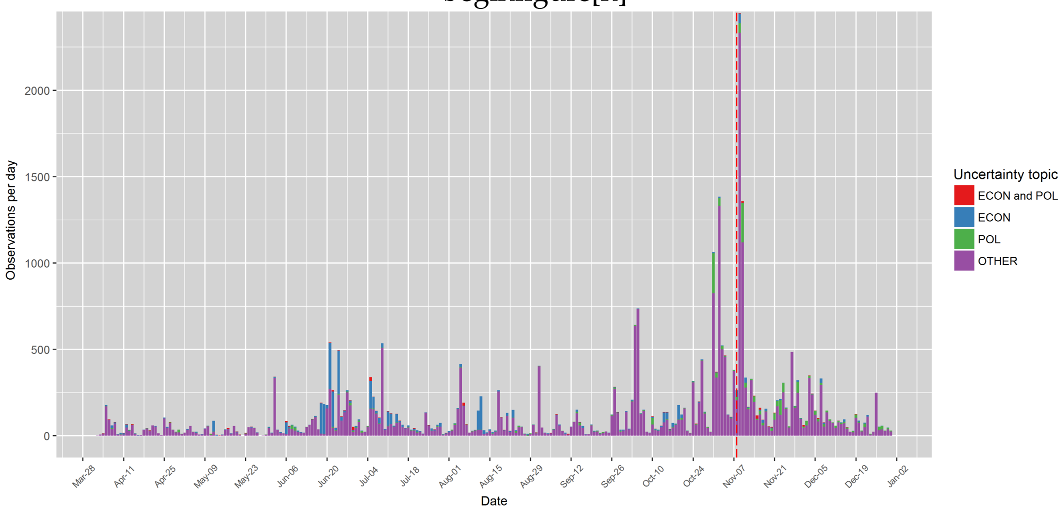
The summary of the OBR outlook finally points out that: "*For this and subsequent forecasts, there are numerous risks and uncertainties associated with the period leading up to and following the UK's exit from the EU, related to policy setting and the response of households and firms, with little by way of precedent to guide the assumptions in this forecast.[...] The uncertainties around our central forecast reflect those regarding the outlook for the economy and those regarding the performance of revenues and spending in any given state of the economy.*" About one hundred UK-TU observations mention or quote OBR's outlook and more than fifty other observations tell about Hammond's Autumn Statement at the House of Commons[553]. During his speech to the Parliament Chancellor Hammond summarizes OBR's previsions and briefly illustrates his economic and fiscal policy plan[554, 555].

- **December 12 (547):** The British Chambers of Commerce (BCC) publishes an update of the UK's Economic Forecast (EF). The online version of the EF update[556] states that: "*the current level of economic momentum is set to slow over the next two years, as* *continued uncertainty around the UK's future relationship with the EU and higher inflation are expected to dampen growth in the medium term.[...] While some firms see significant opportunities over the coming months, many others now see increasing uncertainty, which is weighing on their investment expectations and forward confidence.[...] As the Brexit negotiations commence, steps will need to be taken to help ambitious firms overcome the risks, real and perceived, borne out of political uncertainty.[...] Higher inflation and continued uncertainty over Brexit will weigh on the UK's growth prospects, with consumer spending and business investment likely to be hardest hit.[...] Uncertainty remains over the longer-term outlook, but the UK's structural imbalances, including the over reliance on services and household spending as drivers of growth, continues to leave the UK vulnerable to rapid changes in economic conditions*". In addition, many tweets about UK refer to a news article from The Guardian[557]. The article, titled "*End Brexit uncertainty for EU citizens in the UK, report urges*", explains why, according to a study financed by the British Future thinktank[**?**], "*the government should end uncertainty for EU nationals living in Britain by promising that those based in the UK when article 50 is triggered can stay*".

- **December 29 (616):** Like every end of the year, newspapers formulate their expectations for what will await UK in the year 2017. According to the FT[558] "*Uncertainty will shape the UK economy in 2017*". Following this trend, among the most mentioned twitter contents related to UK uncertainty for this day there is a BBC news article titled "*UK firms' finance bosses say uncertainty is new normal*". The article[559] is based on a survey by Deloitte, in which 119 finance directors working in UK where questioned about the future of their firm. The survey[560] shows that "*despite the rebound in business confidence, most finance bosses expect more economic and financial uncertainty following Brexit.[...] half of those surveyed plan to cut spending [...] 48% also expect hiring and mergers and acquisition activity to slow down*".

Unites States Twitter Uncertainty (US-TU)

beginfigure[h]



Here follows a chronological analysis of events occurring in the days in which the US-TU peaks, and most frequently mentioned contents in twitter posts about US uncertainty for each peak. In the first four months, between April and July the US-TU index daily value exceeds two hundred and fifty observations, in the following dates (in brackets the daily value of the index):

- **June 2 (344):** A great majority of the posts about US uncertainty refer to a speech by B. Obama to US Air Force Academy graduates[561]. The speech took place in Colorado Springs at 10:20AM MDT, and was rapidly covered by newspapers and TV broadcasters[562–564]. During his talk President Obama referred to the role of the US in international conflicts as follows:"*As we navigate this complex world, America cannot shirk the mantle of leadership. We can't be isolationist. It's not possible in this globalized, interconnected world. In these uncertain times, it's tempting sometimes to pull back and try to wash our hands of conflicts that seem intractable, let other countries fend for themselves. But history teaches us, from Pearl Harbor to 9/11, that oceans alone cannot protect us.[...] Weak public health systems on the other side of the world allow diseases to develop that end up reaching our shores. So we cannot turn inward. We cannot give in to isolationism. That's a false comfort. Allowing problems to fester over there makes us less secure here*"[561]. The most frequently observed sentence referring to this speech, considering only posts from our US-TU database, states:"*Obama to address Air Force grads amid uncertainty on US role*" followed by links to the news.

- **June 21-22 (546, 280):** The FED chair Janet Yellen is asked to report to both chambers of the US Congress about the monetary policy and US's economic situation and outlooks, during the Federal Reserve's Semiannual Monetary Policy Report to the Congress[565]: to the Senate the 21st[566] and to the House of Representatives the 22nd[567] of June. During these two days US medias extensively mention in their economic news updates the FED's report and Yellen's statement to the Senate Banking Committee[568–571]. In her prepared oral comments[572] Yellen refers that: "*the economic outlook is uncertain, so monetary policy is by no means on a preset course and FOMC participants' projections for the federal funds rate are not a predetermined plan for future policy. The actual path of the federal funds rate will depend on economic and financial developments and their implications for the outlook and associated risks [...] Considerable uncertainty about the economic outlook remains. The latest readings on the labour market and the weak pace of investment illustrate one downside risk - that domestic demand might falter.*" About three hundred and fifty twitter posts considered by the US-TU index explicitly mention or cite the part of the speech by Yellen that refers to uncertainty.

  In the day in which the results of UK's EU-referendum are known, medias[573–575] and twitter users discuss the implications of Brexit for the US. Several twitter users post messages saying that "*US dollar is soaring*" as Brexit induced uncertainty grows worldwide. Some expect that with a stronger dollar the US commercial balance could become more negative. On the subject, an article on Blomberg[576], titled "*For U.S. Exporters Who Profited From One Europe, New Uncertainty*", explains potential new difficulties for US firms to export their goods in a fragmenting European Union; the article is mentioned in several posts. If on one side many twitter users focus on the present affirming that "*US stock market posed for massive losses this AM after UK vote to leave EU*", other users claim that there will soon be a rebound in US stock prices as "*US stocks [are the] only place to go amid Brexit uncertainty*". The following summary of the statement on Brexit vote by Hillary Clinton[577]:"*Hillary Clinton on #Brexit: 'First task' is to ensure 'uncertainty created by these events does not hurt' US economy*", posted (Id: 746348779566145537) by ABC News's twitter profile (@ABCPolitics), is among the most frequently retweeted posts about US uncertainty that day.

- **July 9 (535):** US government bonds yields, at that time of the year cosidered a safe-heaven by worldwide investors, touch record low as financial forecasters expect "*more British Pound downside*"[578] against the dollar. The Financial Times dedicates to the subject an article[579] titled:"*US stocks and bond yields in record territory due to uncertainty*". The title of the article is used as text content in twitter posts and retweeted by several hundred users that day. In particular, the Dominican entrepreneur and businessman Luis de Jesus Rodriguez (@LuisRodriguezRD) is among the first to tweet the content of the above mentioned FT article, his post (Id: 751558280393007104) is the daily most retweeted among observations about US uncertainty.

Then US uncertainty progressively raises as we approach the presidential elections voting day. The month of October starts with a major uncertainty event, followed by two minor events:

- **October 4-5 (642, 738):** During these two days, the most discussed topic related to US uncertainty, is military trade relations between India and the US. In particular, India's government desires to strengthen military trade agreements and to hasten the negotiations regarding their request for 22 US Predator Guardian drones to settle a binding agreement before the US elections[580, 581]. More than two hundred observations mention an article published by Reuters[582] on this subject. A great number of tweets referring to Reuters's article use as textual content the title of the article: *"India tries to hasten US defence deals amid election uncertainty"*. Several other posts refer to recent developments in the turbulent diplomatic relations between the Philippines and the US[583]. After having threatened to expel US soldiers from the Philippines[584, 585] the President R. Duterte finally gives the go-ahead to joint military drills with the US army[586, 587]. Many posts refer to this news as follows: "*US, Philippines launch war games amid uncertainty over ties*".

Then, from October the 31st, several major uncertainty shocks closely follow each other during the last two weeks of the election campaign:

- **October 31 - November 4 (1064, 374, 1385, 524, 470):** During the weekend (29th and 30th of October) the Democrat candidate Clinton sees her edge in polls over Trump narrowed as the FBI "*reveals it has revived an inquiry*"[588] into Hillary Clinton's email server, after having discovered the presence of Clinton's emails, containing potentially classified information, on a computer belonging to the former congressman A. D. Weiner[589]. As a result, uncertainty grows in the following trading days and impacts stock markets prices worldwide[590]. **On Monday** our US-TU index peaks to 859 observations per day, due to a great number of posts referring to uncertainty about US elections outcome as the cause of the stock markets "negative" closings. This post(Id: 793055632668655616) by Yahoo Finance (@YahooFinance):"*World shares mostly lower as FBI probe raises US uncertainty*", represents very well the general feeling expressed in US-TU observations that day. **On Tuesday**, negative effects of election uncertainty on financial markets persist[591]. In particular, it is claimed by several news articles[592–595] mentioned by twitter users, that election uncertainty is causing the depreciation of US dollar against the Japanese Yen and the Euro among others. **On Wednesday**, the FED's Board of Governors "*left unchanged the interest rate paid on required and excess reserve balances at 0.50 percent ... [and] took no action to change the discount rate (the primary credit rate), which remains at 1.00 percent*"[596]. However, several twitter posts that refer to financial news articles[597–599] claim that decisions by the FED will not invert or slowdown previous days' worldwide market trends caused by increasing uncertainty regarding US elections result. **On**

Thursday twitter users focus on news about the recent effects of US elections uncertainty on gold and safe heavens prices[600, 601], which increased or maintained their value during this turbulent week. **On Friday** numerous posts focus on the recent rise of the US's "fear index": *The CBOE's VIX is at a two-month high. The index, which tracks the price of futures contracts for the S&P 500 stock index, is used as a shorthand for volatility in the markets.[...] While the VIX has risen sharply in recent weeks, the increase comes after a relatively quiet summer after the EU referendum*"[602] says an article cited in many US-TU observations. Observations which frequently quote the following text: "*Clinton or Trump? Uncertainty the only safe bet for markets ahead of US elections*", which is the title of the article.

- **November 7-8 (382,264):** Discussions about the effects of US elections uncertainty on stock markets, which are well summarized by the following twitter post:"*#Finance week ahead: US election uncertainty continues to dominate markets*" are still the prevalent topic among US-TU observations in these two days. Another frequently mentioned topic is uncertainty emerging in climate change policy discussions during the United Nations' Marrakech Climate Change Framework Convention Conference[603]. The following post content:"*US election uncertainty overshadows climate talks in Morocco*", which is a citation of a news article by France24[604], is among the most frequently observed posts on the subject.

The day in which the electoral victory of Trump becomes known we observe the positive extremum of our US-TU index, with more than two thousand observations in a single day:

- **November 9-11 (2453, 1373, 340):** During the first hours of November the 9th, while US votes are still being scrutinized, twitter users start expressing their worries regarding US's future and talk about an uncertainty without precedent in recent US History. A Wall Street Journal's article[605], cited in numerous tweets, goes even further, by claiming that: "*The presidential race has increased small-business uncertainty to a 42-year high*". While The Boston Globe writes through its Twitter profile (@BostonGlobe):"*Welcome to the United States of Uncertainty. And division. And recrimination [URL LINK TO: Boston Globe][606]*", referring to foreseen consequences for US in case of Trump's win. Once it became clear that Donald Trump would be the 45th President of the United States of America twitter users start expressing great anxiety and fears about US's future through a record high number of tweets about US uncertainty, almost two thousand in a single day. A great number of first moment reaction posts are personal thoughts, they do not mention news or cite other users' posts. The following observations can serve as examples to illustrate the kind of worries expressed in US-TU observations at first, when people come to know about Trump's election:
  - "*Its not the end of the world but uncertainty is a bad thing. The US electorate has tied our horse to an unknown.*"
    (From: @SteevSimmonds, Post Id: 796244636993929217)
  - "*Trump election shock & associated rise in uncertainty just might put the US into recession... with no policy options except yet more debt!*"
    (From: @jandehn, Post Id: 796254603478237184)
  - "*The US has the ability of making anyone that is remotely different become terrified of their own uncertainty.*"
    (From: @CataGuimaraes, Post Id: 796241728210632705)

Then came the moment for the "rational" evaluation of the implications of Trump's election. The moment in which people looked to media to interpret reality and form a coherent judgement about likely consequences of this event. Here follow two among the most frequently retweeted posts, which illustrate this situation:

– "*Paris climate deal thrown into uncertainty by US election result [URL LINK TO: The Guardian][607]*"
(From: @Guardian, Post Id: 796329601916080128, 253 retweets)
(From: @GuardianEco, Post Id: 796328164616065024, 107 retweets)
(From: @ClimateDesk, Post Id: 796389493171359744, 27 retweets)
(From: @GuardianWorld, Post Id: 796331285870362624, 26 retweets)
(From: @GuardianAus, Post Id: 796329899103490049, 21 retweets)
(From: @NatureClimate, Post Id: 796357060359233536, 20 retweets)

– "*Trump victory heralds uncertainty and market volatility. Longer-term, US remains on steady growth path. #USElections*"
(From: @CreditSuisse, Post Id: 796267615228481536, 39 retweets)

The most discussed topic associated to US uncertainty is the "Trump effect" on US's foreign policy, energetic policy and environmental policy.
In the following two days the US-TU index is still high but progressively decreases from 2453 to 340 posts per day. The most retweeted post among our US-TU observations is about foreign leaders reaction to Trump's victory.:

– "*World leaders have mixed reactions to Trump's election as US president, from mentioning a "period of uncertainty" to calling it "great news"*
(From: @AFP, Post Id: 796515420689879040, 387 retweets)

Despite the numerous and somehow unexpected post-election endorsements of Donald Trump by worldwide leaders, the above-mentioned tweet contains the term "uncertainty" that was mentioned by the French President François Hollande[608] when asked to give his opinion about likely consequences of Trump's victory.

During the days that follow the victory of Trump demonstrations and protests against policy reforms envisaged by the neo-elected President rapidly surge across the US[609–614]. The business magnate and philanthropist George Soros, a major donor to Democrat's 2016 elections campaign[615], whose name is mentioned in more than 604 TU observations, is blamed by Trump supporters[616, 617] for financing and organizing many of these protests through his NGOs and media partners. November the 10th, Trump publicly affirms in a tweet (Id: 796900183955095552) that:"*Now professional protesters, incited by the media, are protesting. Very unfair!*"

During an interview for CBS News[618] he reaffirms this belief. At fist, claims regarding Soros's financing of protesters are labeled as "fake news" by major US and international newspapers[619, 620]. Only few US medias[621, 622], Russian news agency Sputnik[623, 624] and RT[625] support Trump's claims about protesters being paid by political opponents. Nonetheless, at the end of November Trump supporters move to action "in defense" of the new presidency: "*billionaire investor and philanthropist George Soros has become a target of Donald Trump supporters, who began organizing protests against the prominent Democratic donor they accuse of contributing to civil unrest in the wake of the elections*" says an article of USA Today[626] referring to Trumpist's move against Soros.

- **November 27 (485):** One of Trump's most influential twitter supporters, Bill Mitchell (@mitchellvii), considered by the MIT Media Lab[627] among the top 30 US elections influencers, enters the "anti-Soros" media campaign arena tweeting the following message (Id: 802862614820823041): "*Rumor was Soros heavily shorted the US markets prior to this election. Perhaps he is funding this to add uncertainty and slow the rally?*" The afore-cited post is retweeted by several hundred users that day.

  It appears that the hypothesis, according to which during the election campaign and after the victory of Trump, uncertainty in US markets was "voluntarily fabricated" by Soros through financial and media operations, with the purpose of generating a crisis in the US, first to hinder the election campaign of Trump and once elected to create governance difficulties to the new presidency, exerted and still exerts great appeal on pro-Trump twitter users and is intensively debated in the medias[628–645], fomenting a post-truth "info-war" among opposing US political factions. Since organizations and individuals taking part to the uncertainty debate on Twitter may use the term "uncertainty" strategically, our uncertainty indexes, as people, can be influenced by all forms of "information stimuli", also strategically biased news containing the term "uncertainty".

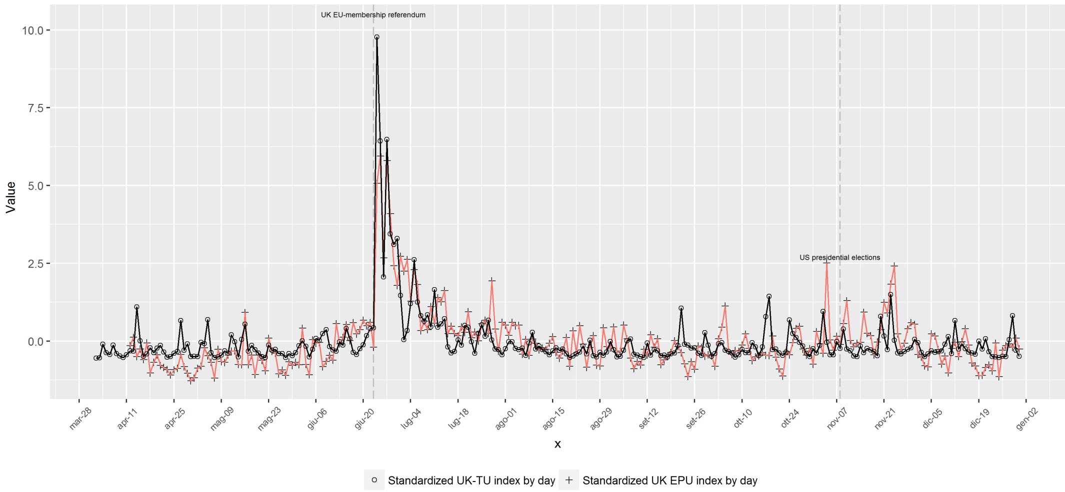
Comparative analysis

In the following subsection we compare the time series of US and UK twitter uncertainty (TU) indexes with the Economic Policy Uncertainty (US-EPU and UK-EPU) and volatility (VIX and VFTSE) indexes of the corresponding country.

Comovement analysis of civil society and policy uncertainty: UK-TU Vs UK-EPU and US-TU Vs US-EPU

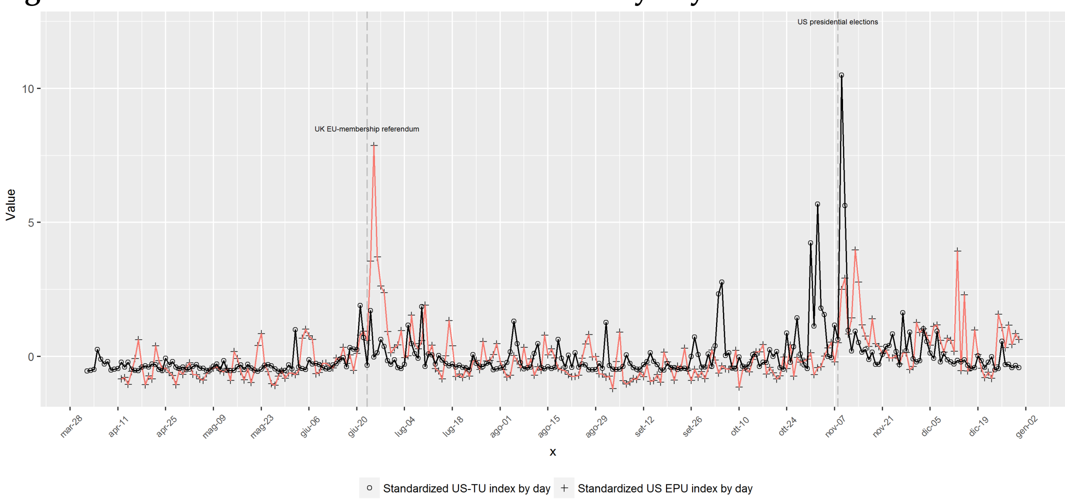
As we can see from the joint plot of the UK-EPU (in red) and UK-TU (in black) standardized time series (Fig. 2.28) the two UK uncertainty measures exhibit at a first look very similar dynamics in the time interval considered by this study. We observe that while the UK-TU index exhibits extreme shocks as large as ten standard deviations, the UK-EPU index never peaks above six standard deviations. After large shocks the standardized UK-TU index returns to values close to zero faster than the standardized UK-EPU index. The UK-EPU exhibits lower positive peaks and a lower average value compared to the UK-TU index during the three months before the EU-referendum. Whereas, during the three months after the EU-referendum the opposite relation is true, the UK-EPU exhibits higher volatility, more numerous and higher positive peaks and a higher average value than the UK-TU index. In the last week of the US presidential election campaign and in the month that follows the US elections the UK-EPU index exhibits higher volatility, higher positive peaks and a larger average value than the UK-TU index. In this range of dates the two UK uncertainty indexes peak on the same dates. However, the UK-EPU index appears to be more sensible to post Trump election uncertainty shocks.

Figure 2.28. Standardized UK-EPU and UK-TU by day



Let us now compare the US-EPU (in red) and US-TU (in black) standardized time series (Fig. 2.29). The two US uncertainty measures remarkably differ in the magnitude and timing of their peaking dynamics. We observe that the US-EPU index exhibits a seven standard deviations peak two days after the EU-referendum, this peak dissipates only six days later during the first weak of July, whereas the US-TU index exhibits a relatively small -two standard deviations- peak a few days before the EU-referendum (July the 21st), this peak dissipates after only two days and is followed by another -one standard deviation- peak the day after the EU-referendum.

Figure 2.29. Standardized US-EPU and US-TU by day



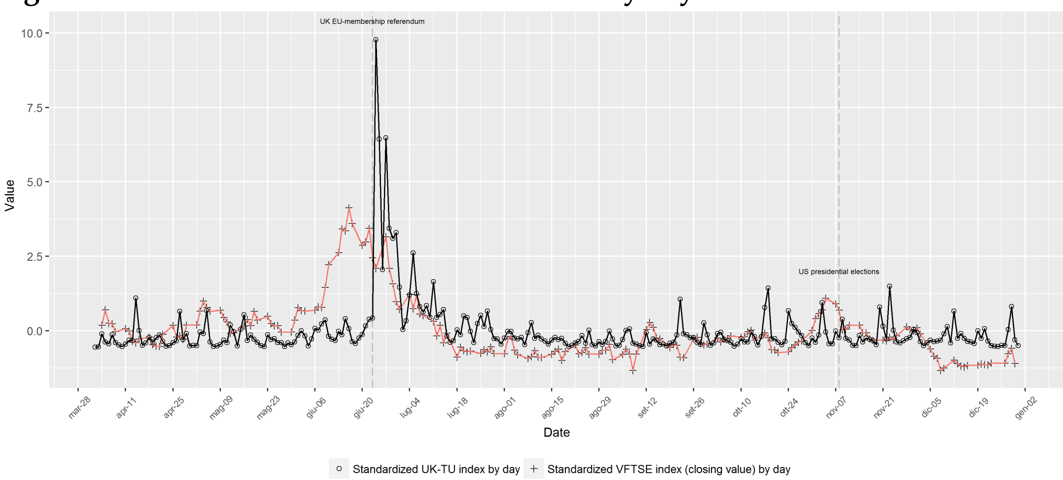
While the US-TU index exhibits two extreme shocks as large as four standard deviations during the last two weeks of the US election campaign the US-EPU index never peaks during the last weeks of the US-election campaign. The day after the US elections, when the victory of Trump is announced, the standardized US-TU index experiences a eleven standard deviations shock. The same day the US-EPU has a value close to 2.5 standard deviations. The US-EPU touches its post-election extremum value one day later (November the 9th), and while the US-TU index returns close to zero four days after the elections (November the 12th) the US-EPU index experiences two other positive peaks, which dissipate as far as November the 21st.

Finally, in December, the US-EPU exhibits more frequent and higher positive peaks and a much larger average value compared to the US-TU index.

Comovement analysis of civil society and market uncertainty: US-TU Vs VIX and UK-TU Vs VFTSE

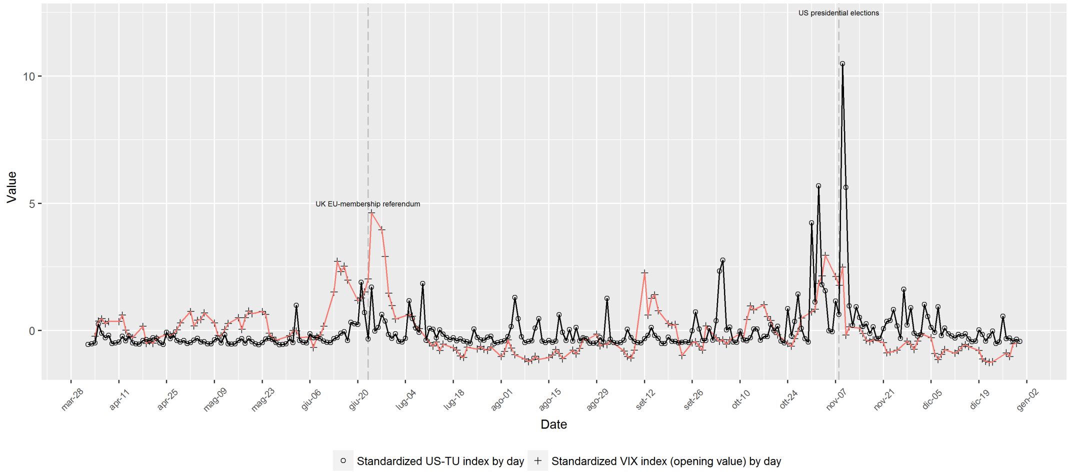
By looking to the standardized time series of the VFTSE and UK-TU (Fig. 2.30) we remark that in April and from mid-September to the end of October there is no apparent correspondence between peaks in the VFTSE index and peaks in the UK-TU index. From the beginning of May to mid-June, UK-TU peaks appear to anticipate slightly or co-occur with VFTSE peaks, which dissipate more slowly than UK-TU peaks. Three weeks before the EU-referendum the VFTSE index starts growing and touches its maximum value five days before the voting day. Whereas, the UK-TU index value slowly grows until the voting day and rapidly peaks in the following day to its maximum observed value. Both indexes are close to zero or negative from mid-July to mid-September. Finally, during the last week of the US election campaign both indexes exhibit a -one standard deviation- peak, but the UK-TU shock dissipates faster.

Figure 2.30. Standardized VFTSE and UK-TU by day



As we can see from Fig. 2.31, standardized VIX and US-TU exhibit rather different dynamics. From April to mid-June the VIX index is very volatile and has a much larger average value than the US-TU index. Then the VIX index peaks both one week before the EU-referendum and the day that follows the referendum. Whereas, the US-TU index peaks only two days before the referendum in UK. Both indexes exhibit low -almost only negative- values during July and August. Then, from September to the end of October, the VIX and US-TU peaks almost never co-occur, the two time series exhibit very different patterns in this range of dates. Finally, both variables peak the weak before and the day after the US presidential elections. However the US-TU extremum is much higher than the VIX value in the corresponding date (11 VS 3 standard deviations);

Figure 2.31. Standardized VIX and US-TU by day



Time series stationarity tests

In the following section we test for the stationarity of the time series used as endogenous variables in our VAR models. We remark that in our framework stationarity is important because applying a (least squares) regression on non-stationary variables can give misleading parameter estimates[646, 647]: unit root processes may have a non-finite variance and an ever changing mean. We suspect that, given the short timespan of this study and the daily frequency of our time series, the hypothesis that volatility indexes (VIX and VFTSE) have a unit root will be refused only for high significance levels ($\alpha > 0.3$). However, this doesn't imply that option-implied volatility indexes are not stationary mean reverting processes in a longer/different time interval or at a lower frequency, as literature suggests[648]. Despite there are no evident reasons for our twitter uncertainty indexes and for the EPU and TU indexes to be non-stationary, it could be possible that, if we considered a longer term model, these indexes could also depend on some omitted variables that may vary in the long run. For example, the number of active users in Twitter or the share of BOTs active in the social network, which are assumed to be constant in this study. Here follow (2.6) the results of the following unit root and stationarity tests[649] on the levels and on and first differences of all our time series:

- Augmented Dickey-Fuller (ADF)[650, 651];
- Phillips-Perron (PP)[652];
- Kwiatkowski-Phillips-Schmidt-Shinb (KPSS)[646];

Table 2.6. Stationarity tests

all endogenous variables have been standardized; Dif stands for once differenced

	ADF-stat	lags	pvalue	PP-stat	lags	pvalue	KPSS-stat	lags	pvalue
UK-TU	-3.507	5	0.044	-7.107	13	<0.01	0.199	9	>0.1
Dif UK-TU	-7.226	5	<0.01	-24.327	13	<0.01	0.035	9	>0.1
US-TU	-3.388	5	0.059	-9.521	13	<0.01	0.416	9	0.070
Dif US-TU	-8.757	5	<0.01	-31.363	13	<0.01	0.035	9	>0.1
VFTSE	-2.787	5	0.247	-2.675	13	0.294	0.513	9	0.037
Dif VFTSE	-5.465	5	<0.01	-14.020	13	<0.01	0.050	9	>0.1
VIX	-3.186	5	0.092	-3.781	13	0.0214	0.209	9	>0.1
Dif VIX	-7.037	5	<0.01	-15.213	13	<0.01	0.031	9	>0.1
UK-EPU	-2.915	5	0.194	-6.615	13	<0.01	0.144	9	>0.1
Dif UK-EPU	-7.204	5	<0.01	-23.001	13	<0.01	0.036	9	>0.1
US-EPU	-3.755	5	0.023	-9.000	13	<0.01	0.348	9	0.100
Dif US-EPU	-8.076	5	<0.01	-35.634	13	<0.01	0.035	9	>0.1

The augmented DickeyFuller test (ADF) stat has a unit root under the null hypothesis H_0
Lags of the ADF test: $lags = (N-1)^{1/3}$
The Philips Perron (PP) test stat has a unit root under the null hypothesis H_0
Lags of the PP test: $lags = 12 * (N/100)^{1/4}$
The KPSS test stat is "level" stationary under the null hypothesis H_0
Lags of the KPSS test: $lags = 10 * N^{1/2}/14$
For a significance level of 0.1 we reject the ADF test null hypothesis -presence of unit root- for the following level variables:

- UK-TU, US-TU, VIX, US-EPU;

For a significance level of 0.1 we reject the PP test null hypothesis -presence of unit root- for the following level variables:

- UK-TU, US-TU, VIX, UK-EPU, US-EPU;

For a significance level of 0.1 we reject the KPSS test null hypothesis -stationarity- for the following level variables:

- VFTSE

Despite VFTSE appears to be non-stationary at a 0.1 significance level, the results of the stationarity/cointegration tests are likely due to the changing trends before and after the EU-referendum, and depend on the magnitude of this event. For similar reasons, due to pre and post US election trends, stationarity test statistics suggest that VIX and UK-EPU could also be non-stationary in the particular time interval considered in this study. But, if we use the whole historical time series of the UK-EPU, VIX and VFTSE to test the stationarity of these three processes (through ADF and PP tests), we refuse the null hypothesis of unit root at the 0.05 significance level for the three time series. Moreover, literature[648] suggests that volatility is a stationary mean-reverting processes in the medium term. Therefore, in this study we assume that all our time series are stationary *I(0)* processes.

Criteria for the choice of the number of lags of the VAR model

To select the lag order of our VAR model, we have searched for the number of lags that minimized the majority of the following four information criteria, given our endogenous time series and conditional on exogenous day-of-the-week explanatory dummy variables:

- *Akaike information criterion (AIC) is minimized for L=2*
- *Hannan-Quinn information criterion (HQC) is minimized for L=2*
- *Schwarz Bayesian information criterion (BIC) is minimized for L=1*
- *Akaike final prediction error (FPE) is minimized for L=2*

Max lag order = 5, which is equivalent to one week becuase we use only working days' observations;

Since three out of four criteria give us that the optima lag order is two we choose to model our system of endogenous variables with a *VAR(2)* model which can be written as follows:

$$\mathbf{y}_t = \mathbf{A}_0 + \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2\mathbf{y}_{t-2} + \sum_{j=1}^{4}\mathbf{B}_j e_{j,t} + \mathbf{u}_t \tag{2.0.3}$$

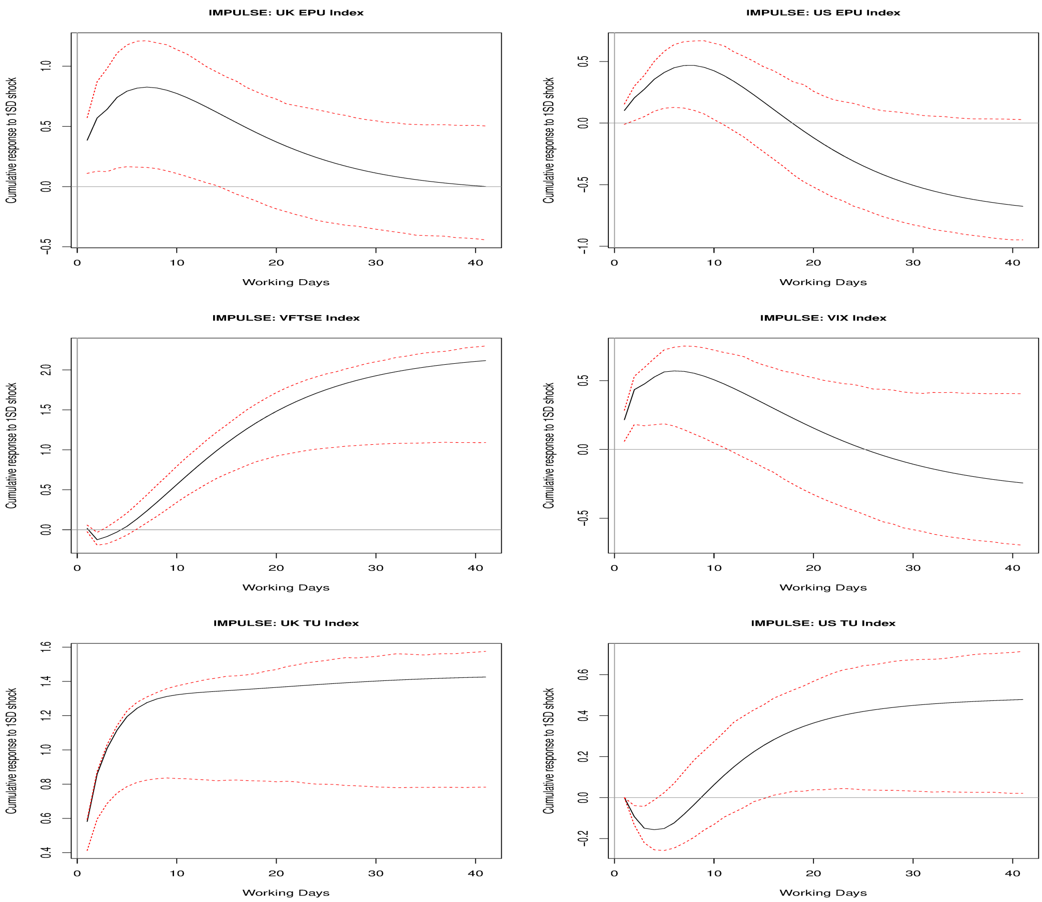
Orthogonalized Cumulative Impulse-Response functions

Here follow a detailed description of orthogonalized impulse response functions.

[As we can see from Fig. 2.36 the UK-TU index reacts as follows to unitary impulses:

- An impulse to the UK-TU index causes a rapidly increasing cumulative response in the UK-TU index from 0 to 10 days from the impulse, which then stabilizes near to a unitary response/impulse ratio;
- An impulse to the VFTSE causes an almost null cumulative response in the UK-TU index at first. But then the response/impulse ratio increases logarithmically across time;
- An impulse to the VIX causes a small positive cumulative response in the UK-TU index in the first two weeks (10 days) and then becomes statistically equivalent to 0;
- An impulse to the US-EPU, or to the UK-EPU, causes a small positive cumulative response in the UK-TU index in the two weeks (10-12 days) after the impulse, the cumulative response then becomes statistically null in the long run;
- An impulse to the US-TU causes a small negative cumulative response in the UK-TU index in the first three days and then increases logarithmically and becomes positive from 15 to 40 days after the impulse;

Figure 2.32. Response of Standardized UK-TU to one SD impulses

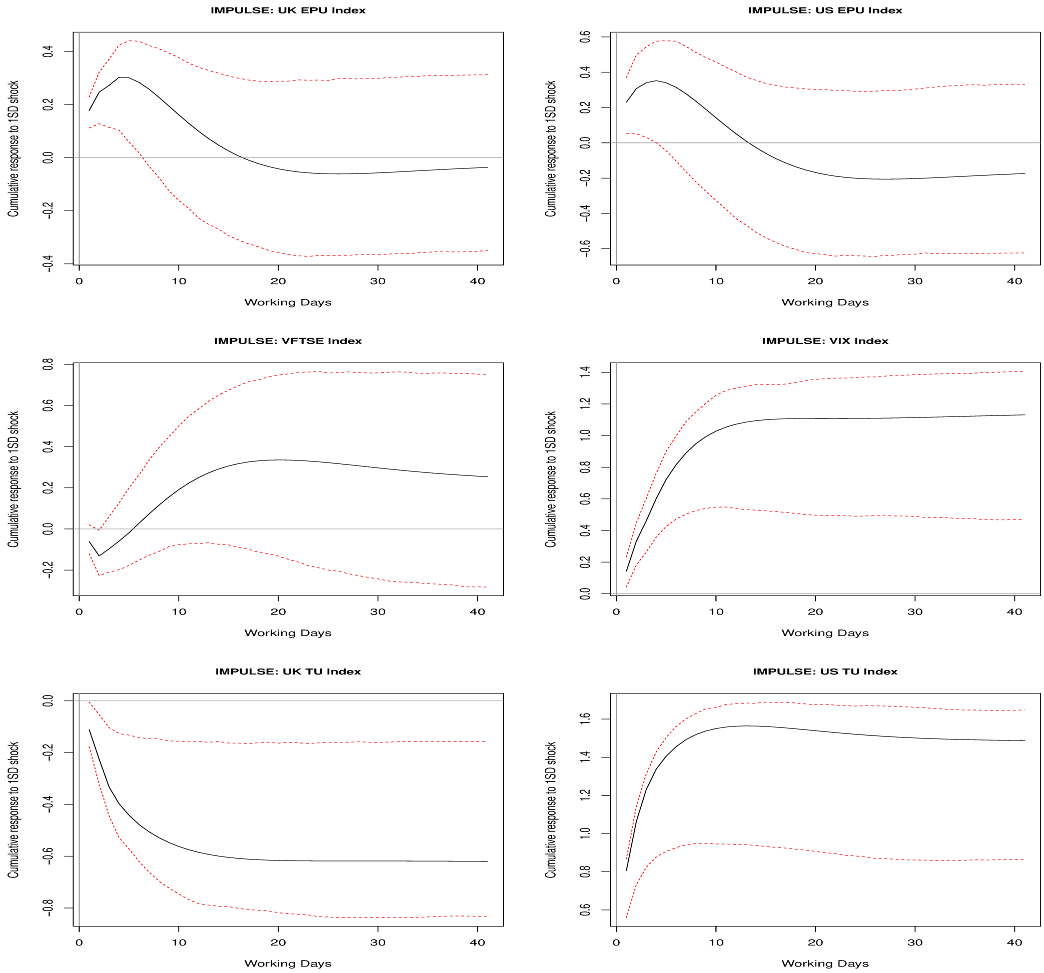


Bootstrapped 68% confidence bands (in red)

As we can see from Fig. 2.33 the US-TU index reacts as follows to unitary impulses:

- An impulse to the US-TU index causes an positive and cumulative response in the US-TU index that tends to a value close to one in the long run;
- An impulse to the UK-TU index causes an increasingly negative cumulative response in the US-TU index in the short run, which tends to a value close to 0.6;
- An impulse to the US-EPU, or to the UK-EPU, causes a small and positive cumulative response in the US-TU index at first, which then becomes statistically equivalent to 0 about one week after the impulse;
- An impulse to the VIX causes a positive and increasing cumulative response in the US-TU index that stabilizes close to a unitary response value after two weeks;
- An impulse to the VFTSE causes a statistically null cumulative response in the US-TU index;

Figure 2.33. Response of Standardized US-TU to one SD impulses

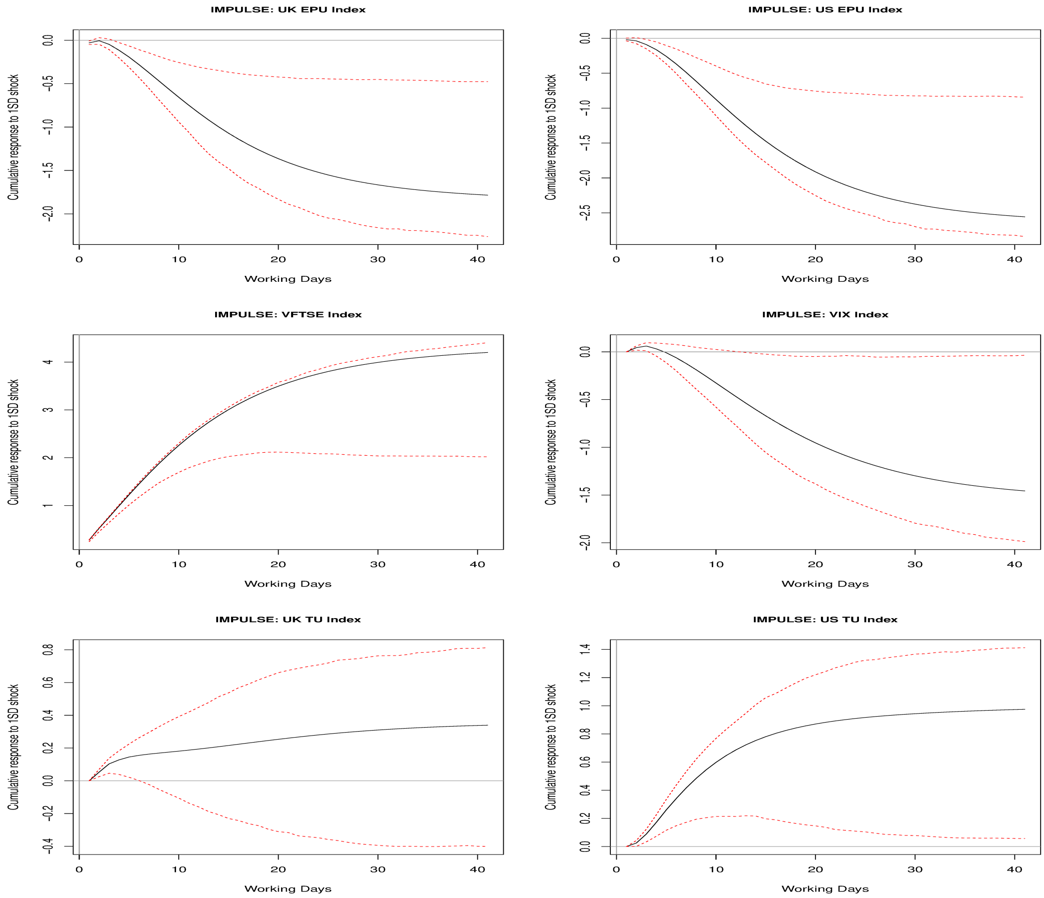


Bootstrapped 68% confidence bands (in red)

As we can see from Fig. 2.34 the VFTSE index reacts as follows to unitary impulses:

- An impulse to the UK-EPU index, or to the US-EPU index, causes a increasingly negative cumulative response of the VFTSE, which tends to values close to 2.5 about two months after the impulse;
- An impulse to the VFTSE index causes a positive and increasing cumulative response of the VFTSE index, that tends to a value close to 4 two months after the shock;
- An impulse to the VIX causes a small positive cumulative response of the VFTSE in the short run. The cumulative response then becomes negative and tends to a value close to -1.5 about two months after the impulse;
- An impulse to the UK-TU causes a positive, bell shaped like, cumulative response of the VFTSE index in the first week after the shock. The cumulative response then becomes statistically equivalent to 0;
- An impulse to the US-TU causes a positive and increasing cumulative response of the VFTSE index. The cumulative response tends to a close to unitary value about one month after the impulse. The lower confidence band approaches 0 as the time from the impulse increases;

Figure 2.34. Response of Standardized VFTSE to one SD impulses

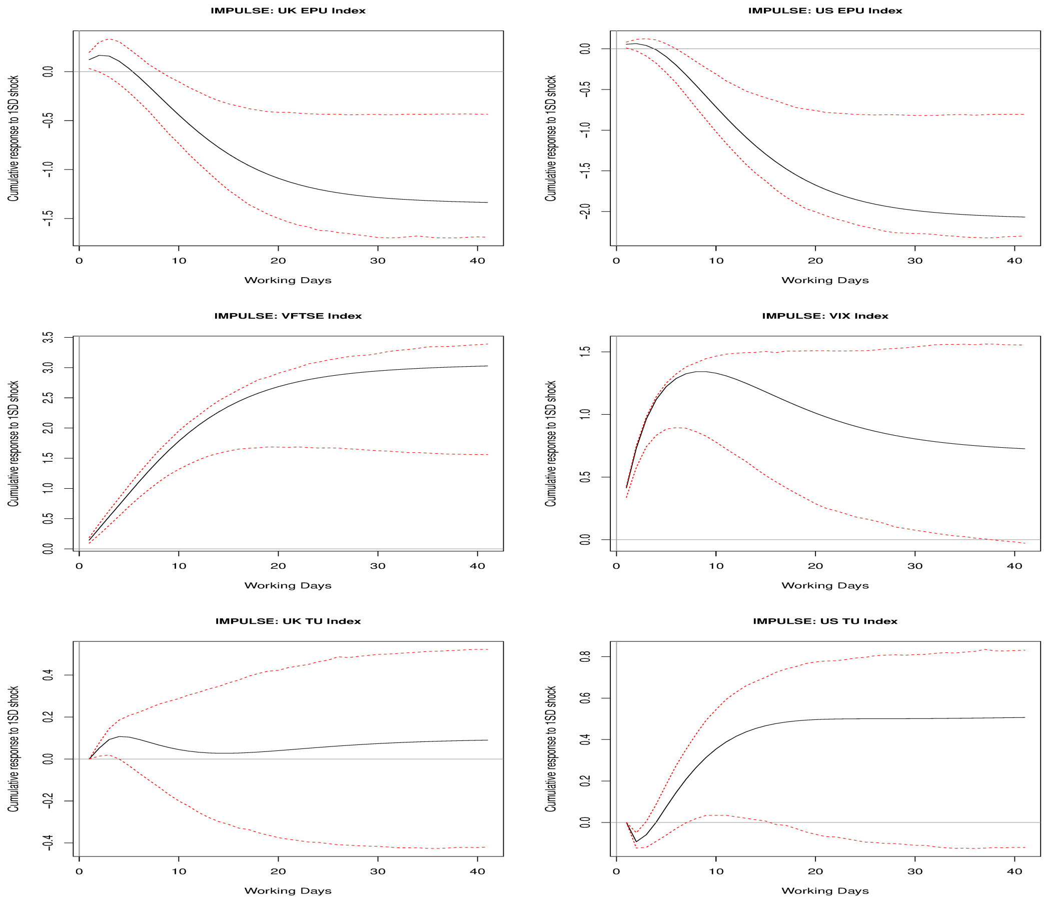


Bootstrapped 68% confidence bands (in red)

As we can see from Fig. 2.35 the VIX index reacts as follows to unitary impulses:

- An impulse to the UK-EPU index, or to the US-EPU index, causes a positive and very small cumulative response of the VFTSE in the very short run. The cumulative response then becomes increasingly negative and approaches -1.5 about one month after the impulse;
- An impulse to the UK-TU index, causes a -bell shaped- positive cumulative response of the VIX index in the short run (first five days), which then becomes statistically equivalent 0;
- An impulse to the US-TU index causes a small and negative cumulative response of the VIX index the first day after the impulse. The cumulative response then becomes positive and statistically different from 0 between eight and fourteen days after the impulse;
- An impulse to the VIX index, causes a -bell shaped- positive cumulative response of the VIX index in the first thirty five days, which then becomes statistically equivalent to 0;
- An impulse to the VFTSE index, causes a positive logarithmically increasing cumulative response of the VIX index that tends to a response/impulse ratio of 3;

Figure 2.35. Response of Standardized VIX to one SD impulses

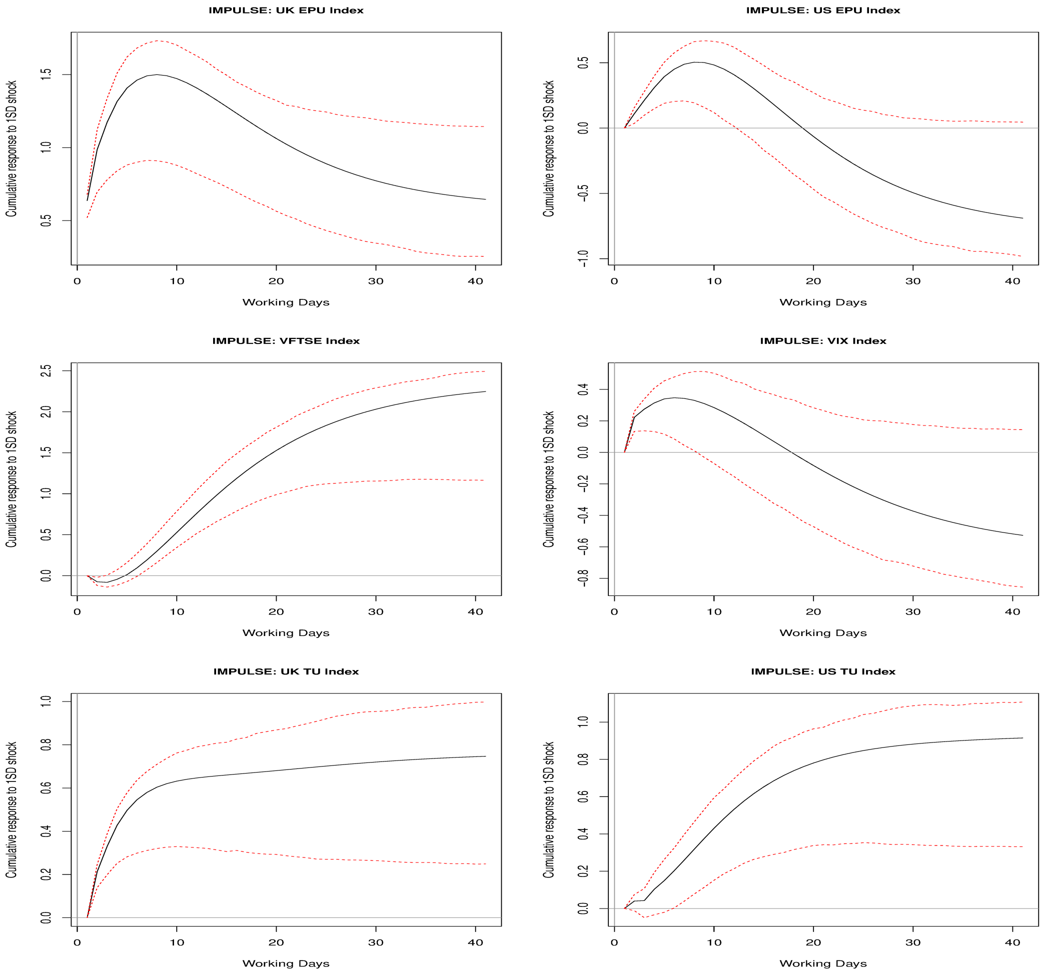


Bootstrapped 68% confidence bands (in red)

As we can see from Fig. 2.36 the UK-EPU index reacts as follows to unitary impulses:

- An impulse to the UK-EPU causes a positive, bell shaped like, cumulative response of the UK-EPU index. The highest cumulative effect is touched around two weeks after the impulse. Then the cumulative response slowly decreases, but remains positive;
- An impulse to the VFTSE causes an almost null cumulative response of the UK-EPU index the first week after the impulse. Then the cumulative response slowly increases towards a value close to two, two months from the impulse;
- An impulse to the UK-TU, or to the US-TU, causes a positive and increasing cumulative response of the UK-EPU index at first. But then the response/impulse ratio slowly increases towards a value close to 0.8;
- An impulse to the VIX, or to the US-EPU, causes a positive, bell shaped like, cumulative response of the UK-EPU index. After about ten days the cumulative response becomes statistically equivalent to 0;

Figure 2.36. Response of Standardized UK-EPU to one SD impulses



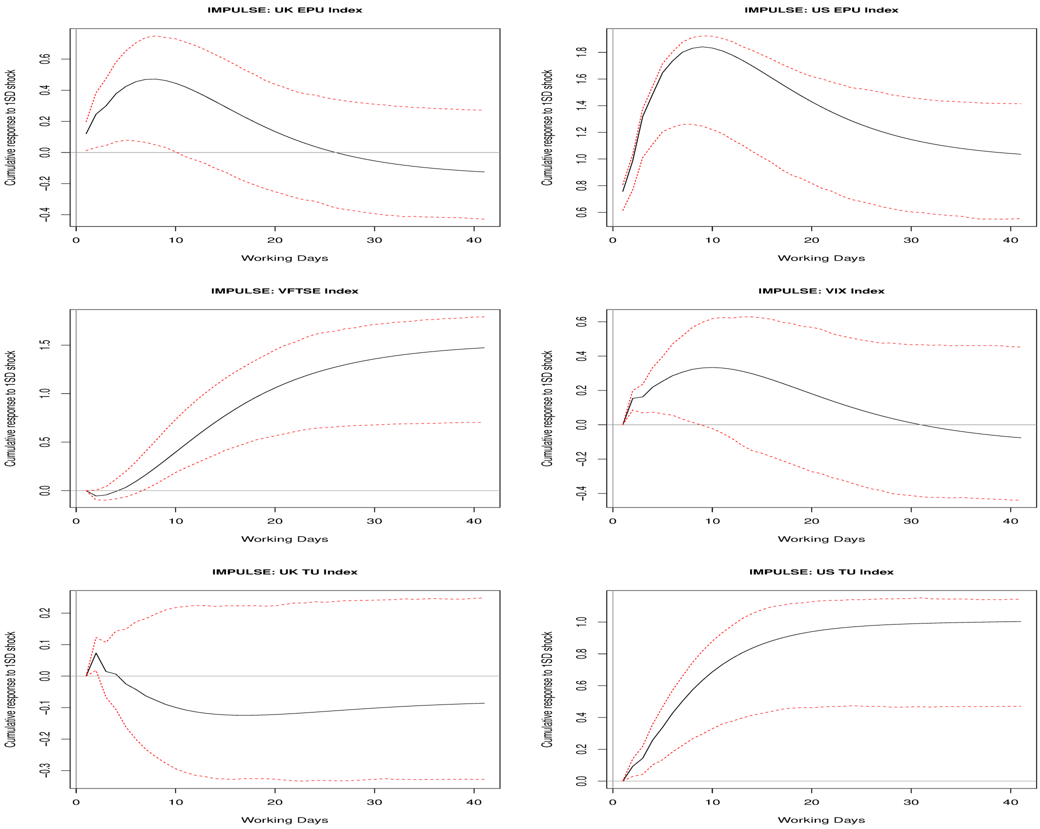
Bootstrapped 68% confidence bands (in red)

As we can see from Fig. 2.37 the US-EPU index reacts as follows to unitary impulses:

- An impulse to the UK-EPU causes a positive, bell shaped like, cumulative response of the US-EPU index. The highest cumulative effect is touched around one week after the impulse. Then the cumulative response slowly decreases and becomes statistically null after two weeks;
- An impulse to the US-EPU causes a positive bell shaped like- cumulative response of the US-EPU index. The highest cumulative effect is touched around ten days after the impulse. Then the cumulative effect slowly decreases to a close to unitary value;
- An impulse to the VFTSE causes a positive and increasing response of the US-EPU index. That tends to a cumulative response value close to 1.5;
- An impulse to the VIX causes a positive, bell shaped like, cumulative response of the US-EPU index in the first two weeks. The cumulative response then becomes statistically equivalent to 0;
- An impulse to the UK-TU, causes a positive bell shaped like, cumulative response of the US-EPU index in the very short run (first two days). The cumulative response then becomes statistically equal to 0;

- An impulse to the US-TU, causes a positive and increasing cumulative response of the US-EPU index, that tends to a close to unitary value in the long run;

Figure 2.37. Response of Standardized US-EPU to one SD impulses



Bootstrapped 68% confidence bands (in red)

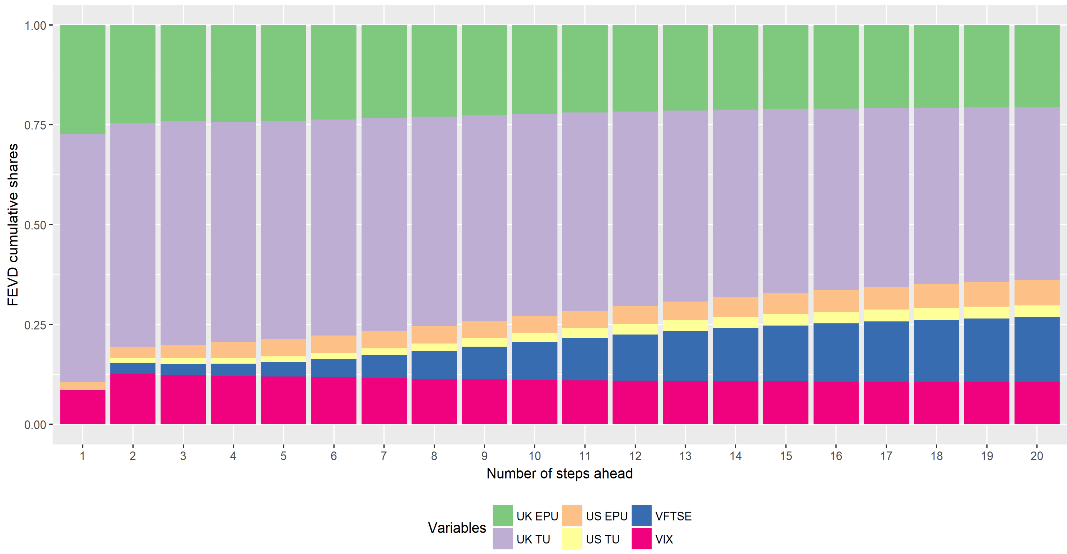
Forecast error variance decomposition

In the following appendix section we analyze and comment on the generalized forecast error variance decomposition (GFEVD)[517, 653] of our estimated unconstrained structural VAR(2) model equations. GFEVD allows us to appraise the proportion of the *n*-steps ahead forecast error variance of an endogenous variable, accounted for by innovations to the various equations (endogenous variables) in our orthogonalized VAR system. Since GFEVD values are computed from the orthogonalized impulse-response functions, they are not unique. The Wold causal ordering -motivated and used in the previous sections- determines the outcome of GFEVDs.

By being the first couple of variables in the chosen Wold causal ordering, innovations to *policy uncertainty* (EPU indexes) are allowed to instantaneously affect -propagate to- other endogenous variables. Therefore, the contribution of the EPU indexes on other variables' GFEVDs could potentially be overestimated if the selected Wold causal ordering doesn't correspond to reality. Whereas, by being the last couple of variables in the Wold causal ordering, the contribution of our TU indexes could potentially be underestimated if the Wold causal ordering is wrong. However, we prefer to run the risk of underestimating the contribution of innovations to *civil society uncertainty* (proxied by TU indexes) on market and political uncertainty variables' forecast error variance, rather than risking to overestimate them. Similarly, since -for each couple of endogenous variables- the UK variable precedes the US one in the Wold causal ordering, we risk to overestimate the intra-type contribution of innovations to UK variables on the corresponding US variable, and to underestimate the reverse relation. Since we are interested in identifying a lower bound for the contribution of *civil society uncertainty* on *market and policy uncertainty* variables our approach results directly from our identification objectives. Under/over estimation risks are intrinsic to any GFEVD analysis and cannot be mitigated if not by analyzing the outcomes under -plausible- alternative Wold causal ordering specifications.

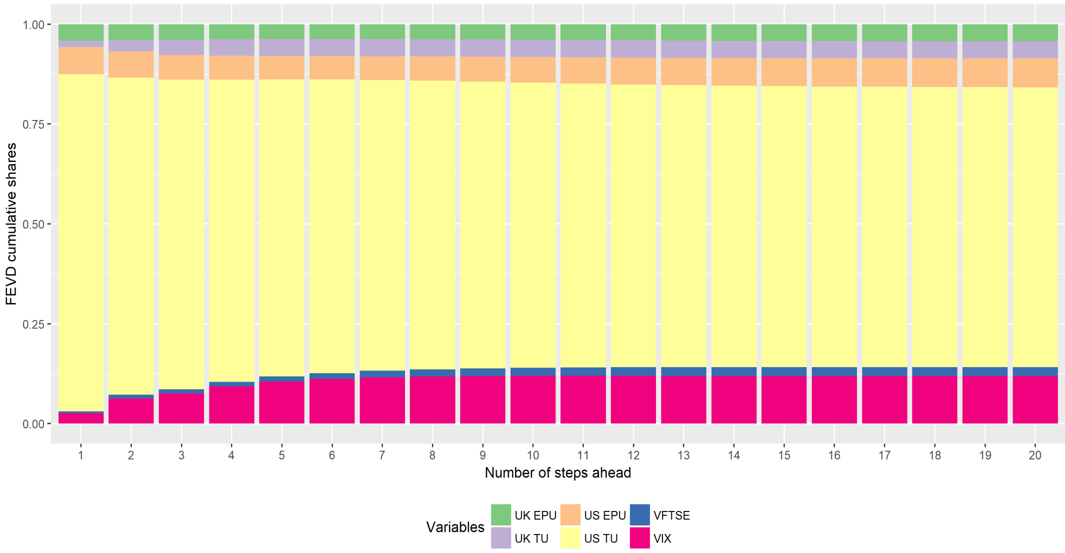
Here follow, for each endogenous variable the GFEVD stacked bar-plots up to 20 steps ahead forecasts (4 weeks):

Figure 2.38. UK-TU Generalized Forecast Error Variance Decomposition



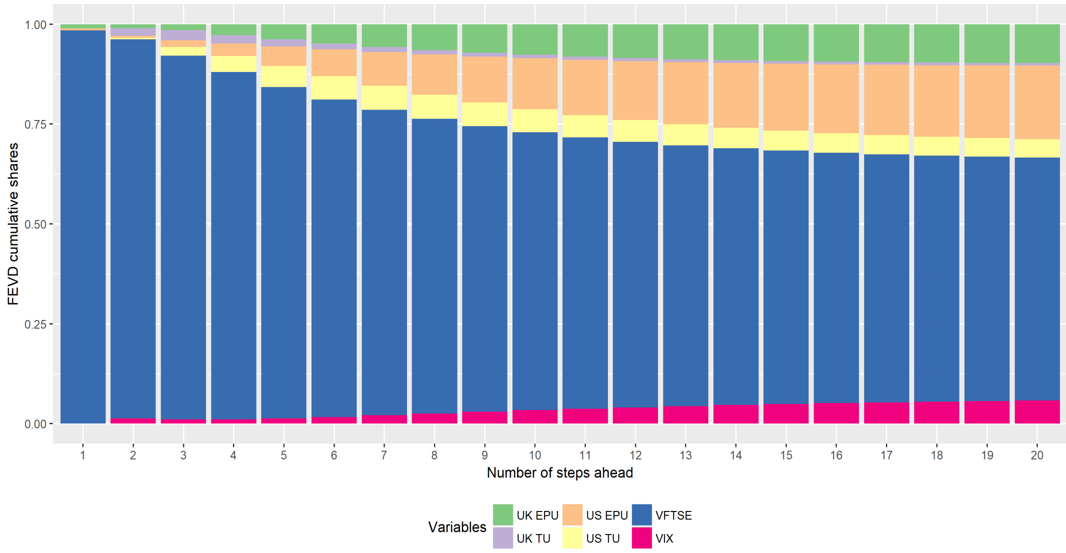
As we can see from Fig.2.38, about 50% of the forecast error variance (FEV) of UK *civil society uncertainty* (UK-TU) is explained by innovations to UK-TU itself. Innovations to UK *policy uncertainty* (UK-EPU) also play a relevant role -about 25%- in explaining UK-TU FEV. For what pertains to uncertainty innovations from the other side of the Atlantic, innovations to US *market uncertainty* (VIX) have an almost constant weight on UK-TU FEV, close to 10%. Whereas the contribution to UK-TU FEV of innovations to UK *market uncertainty* (VFTSE) increases with the number of steps, from about 2% to about 17% in four weeks ahead forecasts. US Policy (US-EPU) and Civil Society (US-TU) uncertainty have relatively minor roles in explaining UK-TU FEV, however their weight appears to increase from a forecast step to the next and then stabilizes. In four weeks ahead forecasts, the contribution of UK-TU to the FEV of UK-TU is about 3%, whereas that of US-EPU is a little above 6%.

Figure 2.39. US-TU Forecast Error Variance Decomposition



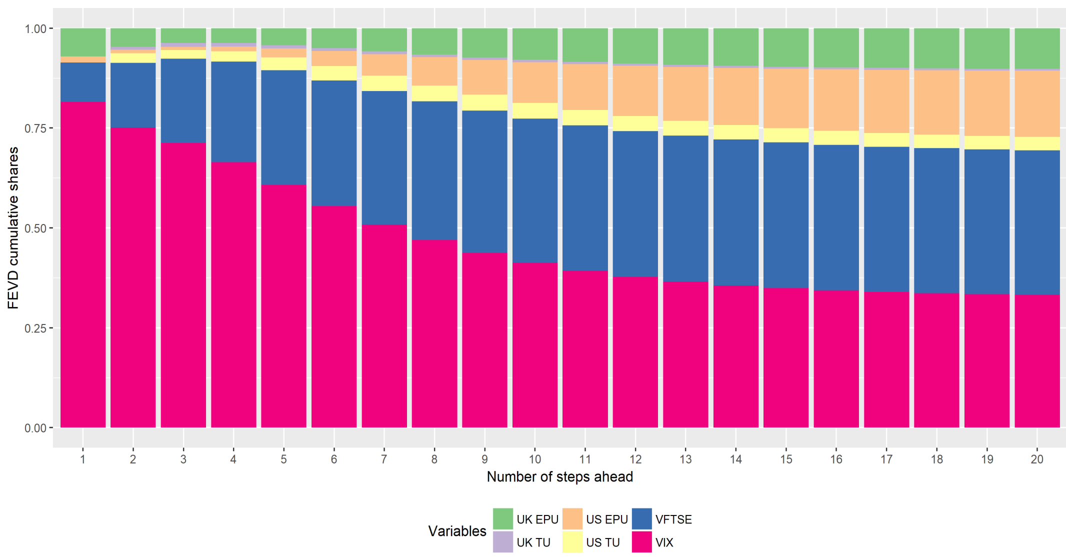
The FEV of US *civil society uncertainty* (Fig.2.39) is in large part explained by innovations to US-TU, its contribution ranges from 85% to about 70% of US-TU's FEV and stabilizes around the latter value. Innovations to *policy uncertainty* play a minor role for US -*civil society uncertainty* FEV- with respect to UK. US-EPU is accounted for only about 7% of US-TU's FEV, and, UK-EPU for about 4%. UK-TU plays a similar role to UK-EPU, with a contribution to US-TU's FEV close to 4%. *Market uncertainty* (VFTSE and VIX) have both a minor contributions US-TU's FEV at first, but their weight increases with the number of steps ahead of the forecast. Touching respectively 2% -for VFTSE- and about 12% -for VIX- in four weeks ahead forecasts.

Figure 2.40. VFTSE Generalized Forecast Error Variance Decomposition



UK *market uncertainty* FEV (Fig.2.40) is in large part explained by innovations to itself, its contribution ranges from 98% to about 60% of VFTSE's FEV and stabilizes around the latter value. Innovations to *policy uncertainty* play a relevant contribution to VFTSE's FEV, contribution which increases as a function of the number of steps ahead of the forecast. UK-EPU and US-EPU tend to -FEV contribution share-values respectively close to 10% and 18%. The contribution shares of *civil society uncertainty* innovations on VFTSE's FEV exhibit bell shaped dynamics. US-TU's FEV contribution touches its maximum (6%) in 7-days ahead forecasts of VFTSE, it then decreases to values close to 5%. Whereas the FEV contribution of UK-TU touches its maximum (2.5%) in the 3-days ahead forecasts of VFTSE and then decreases to values below 1%. US *market uncertainty* (VIX) innovations play a minor contribution to VFTSE's FEV in short term forecasts, but their weight increases with the number of steps ahead of the forecasts towards FEV contribution share values close to 6%.

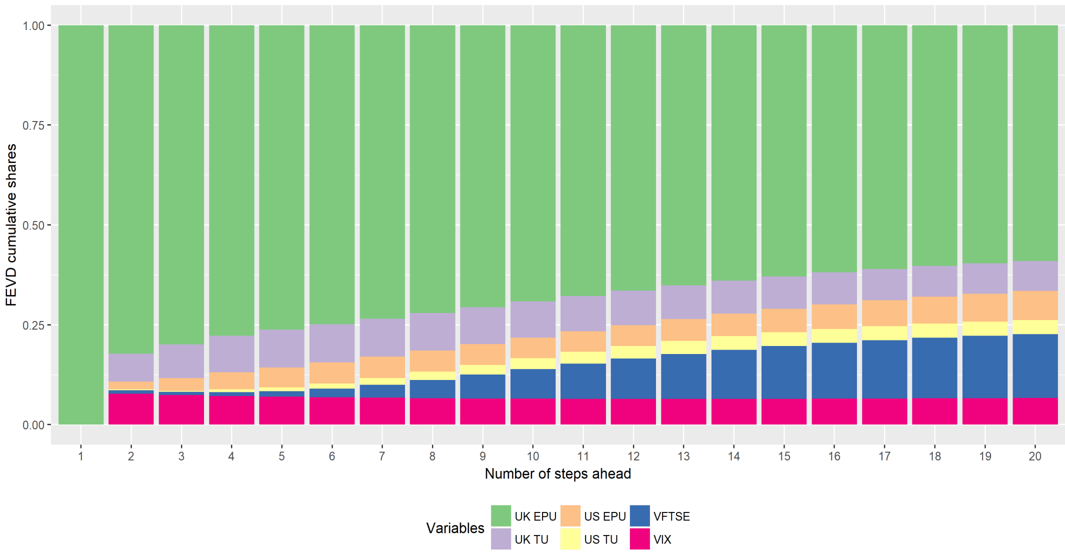
Figure 2.41. VIX Generalized Forecast Error Variance Decomposition



The FEV of US *market uncertainty* (Fig.2.41) is -as for the other variables of the system-in large part explained by innovations to itself. FEV contribution that ranges from 81% to 33%. Innovations to *policy uncertainty* -as for VFTSE- also play a relevant contribution to VIX's FEV that increases with the time horizon of the forecasts. US-EPU and UK-EPU tend to -FEV contribution shares- respectively close to 17% and 10%. The contributions of *civil society uncertainty* on VFTSE's FEV exhibits a bell shape dynamic. US-TU's FEV contribution touches its maximum (4%) in 9-days ahead forecasts of VFTSE, it then decreases to values close to 5%. Whereas UK-TU reaches its maximum (1%) to VIX's FEV in the 3-days ahead forecasts and then decreases to values close to 0%. UK *market uncertainty* (VFTSE) innovations play an important contribution to VIX's FEV. VFTSE's weight increases with the number of steps ahead of the forecasts towards a FEV contribution share close to 36% in twenty observations ahead forecasts.

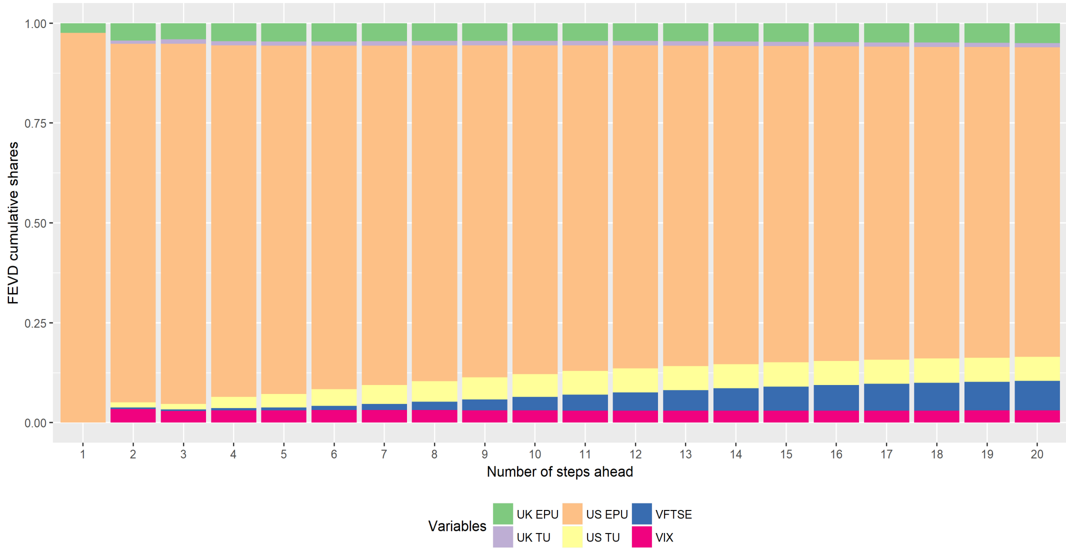
The FEV of *policy uncertainty* in UK (UK-EPU) principally depends on disturbances to UK-EPU itself. This contribution declines from the 100% (the full FEV depends only on UK-EPU) for one-step ahead UK-EPU forecasts, to 59% in twenty-steps ahead forecasts. The contribution of US-EPU to UK-EPU FEV is relatively minor and increase as a function of the forecasts' number of steps ahead from 2% to 7%. Excluding the one-step ahead forecast, the share of the contribution -to UK-EPU'FEV- of US *market uncertainty* (VIX) ranges from 6% to 8%. Whereas the contribution of VFTSE increases logarithmically from 0% to about 16%. *civil society uncertainty* plays an important role on UK *policy uncertainty*. UK-TU contributes for about 8% of the UK-EPU's FEV and US-TU for about 3%.

Figure 2.42. UK-EPU Generalized Forecast Error Variance Decomposition



The contribution of US *policy uncertainty* on its FEV varies from 98% -in one step ahead forecasts- to 77% -in twenty steps ahead forecasts- . The contribution of innovations to UK-EPU for UK-EPU's FEV is relatively stable across forecasting horizons and close 5%. The share of the contribution -to US-EPU'FEV- of US *market uncertainty* (VIX) ranges is 0% for the one-step forecasts, and close to 3% for other forecast horizons. Whereas the contribution of VFTSE increases logarithmically across horizons from 0% to about 7%. US *civil society uncertainty* (US-TU) plays an rather important role for US *policy uncertainty*, it contributes up to 6% of the UK-EPU's FEV from ten-steps horizons onwards. Whereas the role of US-TU is limited, close to 1%.

Figure 2.43. US-EPU Forecast Error Variance Decomposition



In this section we have identified a lower bound for the contribution of our TU indexes to the forecast error variance of other uncertainty variables. As we have seen in this section, by using TU indexes as proxies for *civil society uncertainty*, we find that *civil society uncertainty* plays a minor but still relevant contribution in explaining *market uncertainty* FEV, almost exclusively in the very short run. This because the cumulative effects of *civil society uncertainty* innovations on *market uncertainty* variables either rapidly declines or totally dissipates across time, after about a month their contribution is either very low or almost insignificant. On the side of the dependencies between *civil society uncertainty* and *policy uncertainty*, we find that the contributions and effects of the first (UK-TU and US-TU) on the latter (UK-EPU and US-EPU) are long lasting. Hence our TU indexes allow us to improve our forecasting capacity and enrich our understanding of *market uncertainty* dynamics in the short run, and, of the dynamics of *policy uncertainty* both in the short but also in the long term.

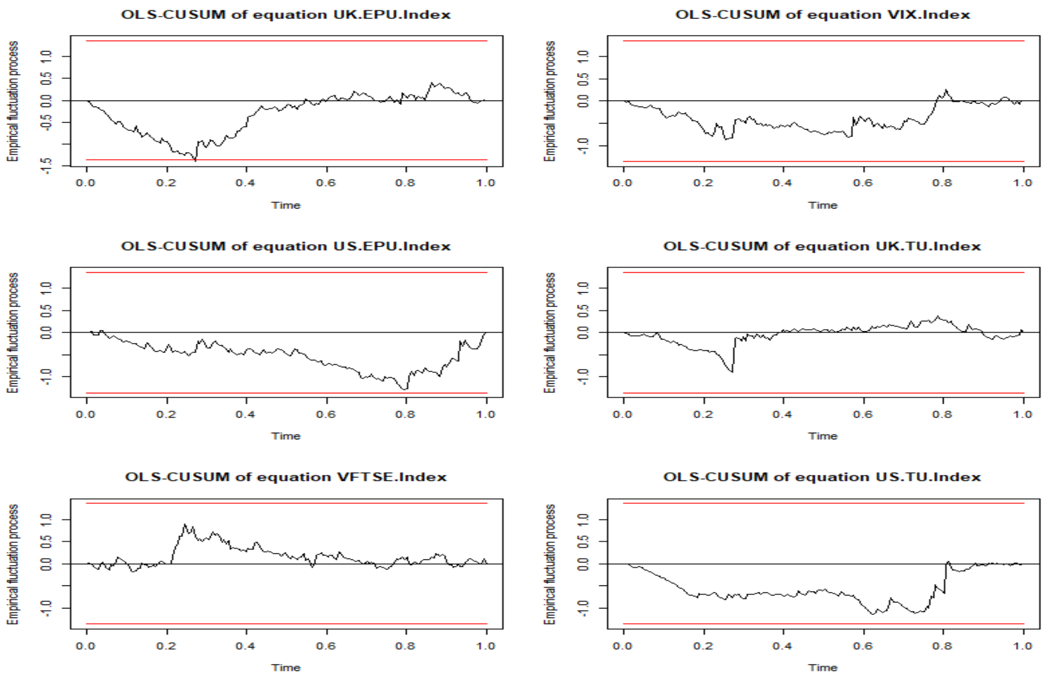
Robustness checks and alternative VAR specifications

In this appendix section, we look if the estimated coefficients of the endogenous variables are stable across the time interval of this study by using the CUSUM and MOSUM tests (2.5). We then estimate three distinct restricted versions of our VAR(2) model. To evaluate which are the losses, in terms of fitness and eventual violations of the models' assumptions, if we constrain some endogenous variables' coefficients to zero on the basis of theory based hypothesis and previous findings. By analyzing the estimates and residuals of our restricted models we can also appraise if our coefficients' estimates are sensible, in terms of sign, size and variance, to the specifications of our VAR(2) model.

Structural changes and coefficients stability

To detect if eventual structural changes took place in the time span covered by our sample of observations, as suggested by Krämer et al.[654], we analyze the least-squares moving (MOSUM) and cumulative (COSUM) sums of residuals for the six equations of our VAR(2) model, to test if our null hypothesis: **absence of structural change**, must be rejected given the observed values of the COSUM/MOSUM empirical fluctuation processes (*efp(t)*). The null hypothesis is rejected when the fluctuation of the empirical process *efp(t)* gets improbably large compared to a boundaries of the limiting process, which is a Brownian Bridge process[655]. If one of the MOSUM/COSUM *efp(t)* crosses one of the two boundaries at any t, then the null hypothesis is rejected at the -previously- chosen confidence level α (here $\alpha = 0.05$). Here follow the plots of the least-square COSUM (2.44) and MOSUM (2.45) *efp*s.

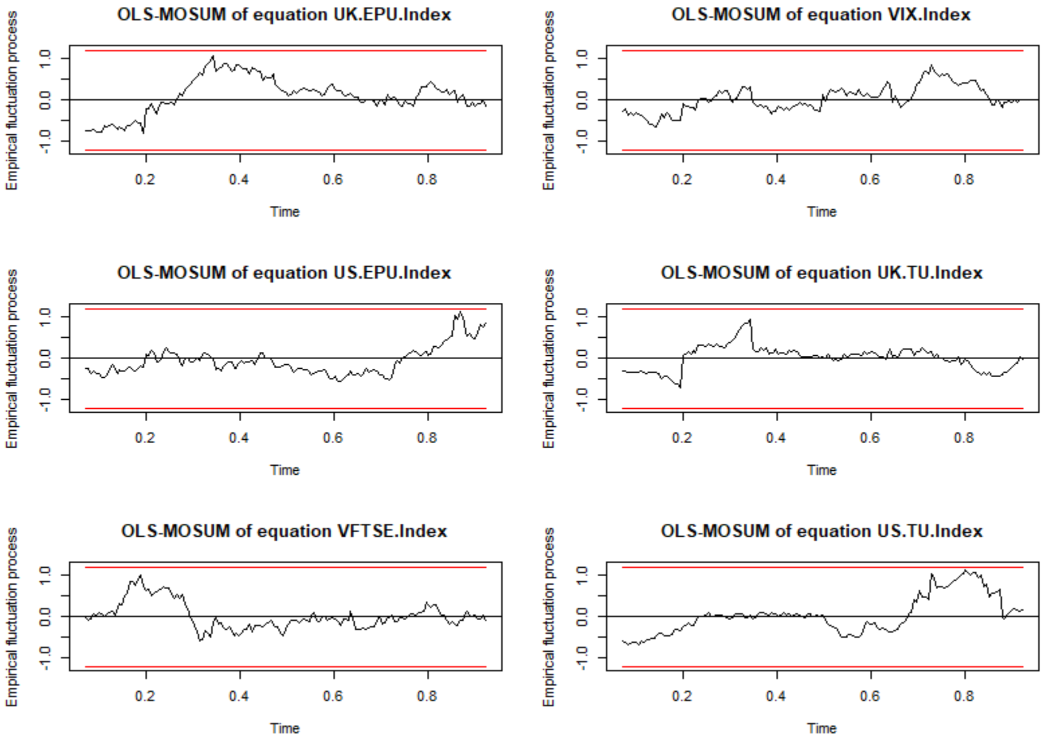
Figure 2.44. least-square COSUM



in red process Boundaries (at the $\alpha = 0.05$ *significance level)*

The COSUM stability tests indicate moderate deviations from parameter constancy during extreme uncertainty events. Only the OLS-COSUM of the UK-EPU crosses (once) the -$\alpha = 0.05$ significance level- lower boundary, the day after the EU-referendum (June 24). Given the u-shape of the UK-EPU COSUM *efp* in proximity to the EU-referendum it appears that UK-EPU is underestimated by our VAR(2) model in dates around this event. A similar, but not statistically significant, pattern is observable for the US-EPU in dates around the US presidential elections (November 8).

Figure 2.45. least-squares MOSUM



Moving Window Bandwidth=$\frac{1}{15}N \approx 12$
in red CI ($\alpha = 0.05$ *significance level*)

MOSUM *efp*s never cross the -$\alpha = 0.05$ significance level- boundaries. However, moderate (negative) deviations from parameters stability are clearly visible during periods of extreme uncertainty. Non-market US uncertainty MOSUM *efp*s (US-TU, US-EPU) exhibit moderate negative deviations around the US presidential elections, whereas Non-market UK uncertainty MOSUM *efp*s (US-TU, US-EPU) exhibit similar deviations in dates close to the UK-referendum.

Given the shapes of the COSUM and MOSUM *efp*s for the VIX and VFTSE, which are never too close to the boundaries, we can -rather safely- accept the null hypothesis of no structural change in the coefficient parameters of the VIX and VFTSE equations of our VAR(2) system in the observations' time-span here considered. Considering that our major modeling interest consists in explaining *market uncertainty* through non-*market uncertainty* proxies, our unrestricted VAR(2) model with time-unvarying parameters appears to be a satisfactory representation of the dynamics of *market uncertainty* (proxied through option-implied volatilities) during the 2^{nd}, 3^{rd} and 4^{th} quarter of the year 2016.

Alternative VAR model specifications

No inter-area dependencies

If UK and US uncertainty variables were independent, by restricting our VAR(2) model in such a way that there is (by construction) no inter-area dependencies among endogenous uncertainty variables (see Fig.2.46), we should obtain a fit as good as our previously presented unrestricted VAR(2): Both from the point of view of the *Adjusted* R^2 of the various fitted equations, and, for what pertains to the non refusal of the hypothesis on the residuals of the model, like the absence of autocorrelation and heteroskedasticity. In the following subsection we estimate such a VAR(2) restricted model and compare it to its unrestricted version. As we can see from Table

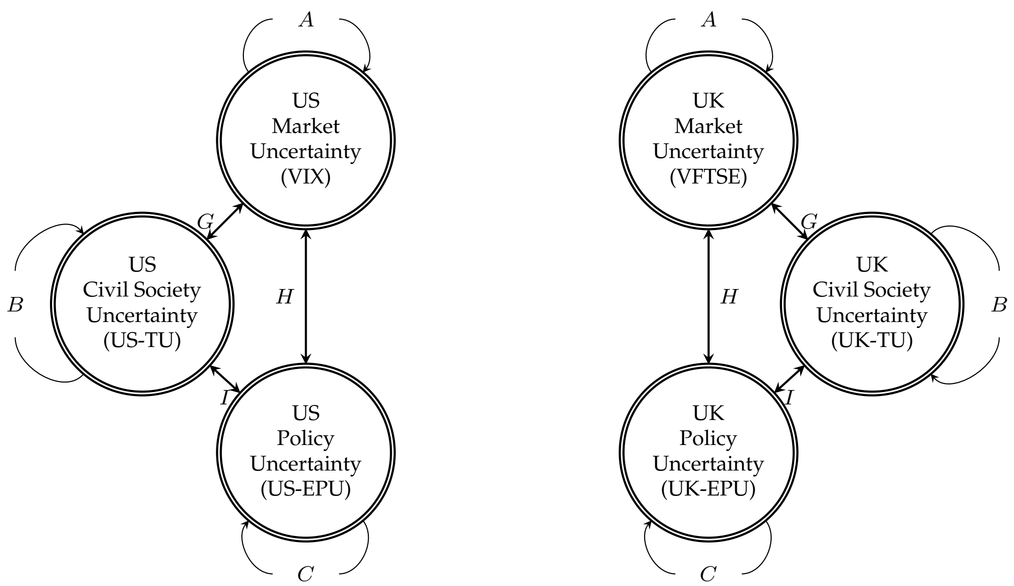


Figure 2.46. Reduced Model: No inter-area dependencies

2.7 that contains the estimates of our VAR(2) restricted model without inter-area dependencies, compared to the unrestricted model, the fit of all the equations in the model worsens, the Adjusted R^2 diminishes for all fitted equations. The magnitude of the drop is larger the US-EPU indexes and for the VIX, this is an indication that US *market uncertainty* and *policy uncertainty* variables are more interdependent on inter-area uncertainty events compared to *civil society uncertainty* variables. It is interesting to note that although some of the coefficients change value, they all maintain their sign.The residuals of the restricted model are more heteroskedastic, especially EPU indexes during the EU-referendum and US-elections. In addition, US-TU and UK-EPU exhibit signs of non-null residuals' autocorrelation that were already present -but smaller- in the unrestricted model. By looking to residuals' distributions we remark that these lasts have more bumps in right tail and are more right skewed. Given the aforementioned findings we refuse the hypothesis of absence of inter-area dependencies among US and UK uncertainty variables.

Table 2.7. Reduced VAR(2) model estimates;
Reductions: No inter-area dependencies

VFTSE at close prices; VIX at open prices; All endogenous variables have been standardized
L means once lagged variable, L^2 means twice lagged variable

	Dependent variables:					
	UK-TU	US-TU	VFTSE	VIX	UK-EPU	US-EPU
L UK-TU	0.53***		0.08**		0.43***	
	(0.09)		(0.03)		(0.08)	
L^2 UK-TU	0.04		−0.08**		−0.13	
	(0.09)		(0.04)		(0.08)	
L VFSTE	−0.55***		0.93***		−0.28*	
	(0.18)		(0.07)		(0.16)	
L^2 VFSTE	0.77***		0.05		0.43***	
	(0.19)		(0.07)		(0.17)	
L UK-EPU	−0.06		0.02		0.28***	
	(0.10)		(0.04)		(0.09)	
L^2 UK-EPU	0.05		−0.09**		0.16*	
	(0.09)		(0.04)		(0.08)	
L US-TU		0.37***		−0.16***		0.12*
		(0.08)		(0.04)		(0.07)
L^2 US-TU		0.16*		0.17***		0.01
		(0.08)		(0.04)		(0.07)
L VIX		0.23*		0.95***		0.31**
		(0.14)		(0.07)		(0.12)
L^2 VIX		−0.06		−0.05		−0.21*
		(0.14)		(0.08)		(0.12)
L US-EPU		−0.05		−0.02		0.26***
		(0.08)		(0.05)		(0.07)
L^2 US-EPU		−0.03		−0.09**		0.38***
		(0.08)		(0.04)		(0.07)
const	0.11	−0.04	0.07	0.03	0.06	0.21
	(0.13)	(0.15)	(0.05)	(0.08)	(0.12)	(0.14)
Tuesday	−0.20	0.02	−0.07	−0.21*	0.07	−0.15
	(0.18)	(0.21)	(0.07)	(0.12)	(0.16)	(0.19)
Wednesday	−0.04	0.35	−0.05	0.04	0.03	−0.33*
	(0.18)	(0.22)	(0.07)	(0.12)	(0.16)	(0.19)
Thursday	−0.06	−0.01	−0.17**	−0.04	−0.24	−0.34*
	(0.18)	(0.21)	(0.07)	(0.12)	(0.16)	(0.19)
Friday	−0.25	−0.13	−0.09	0.04	−0.16	−0.16
	(0.18)	(0.21)	(0.07)	(0.12)	(0.16)	(0.19)
Observations	182	182	182	182	182	182
R^2	0.48	0.28	0.92	0.78	0.59	0.43
Adjusted R^2	0.45	0.23	0.91	0.77	0.57	0.40
Resid. SE (df = 171)	0.74	0.88	0.29	0.48	0.66	0.78
F Stat.(df = 11; 171)	14.41***	6.05***	180.43***	56.65***	22.77***	11.91***

Note: *p<0.1; **p<0.05; ***p<0.01

No inter-area inter-source dependencies

It could be possible that even though UK and US uncertainty are not totally independent, nevertheless there is no contagion between uncertainty variables of both different sources (Market, Policy, Civil Society) and different geographic-areas (UK and TU) . To verify this hypothesis we restrict our VAR(2) model in such a way that inter-area dependencies among different uncertainty sources are not permitted (See Fig.**??**), i.e. their coefficients are null by construction. As before, if this hypothesis was true we should obtain a goodness of fit comparable to that of our unrestricted VAR(2) model. In the following subsection we estimate such a restricted model and compare it to the unrestricted one.

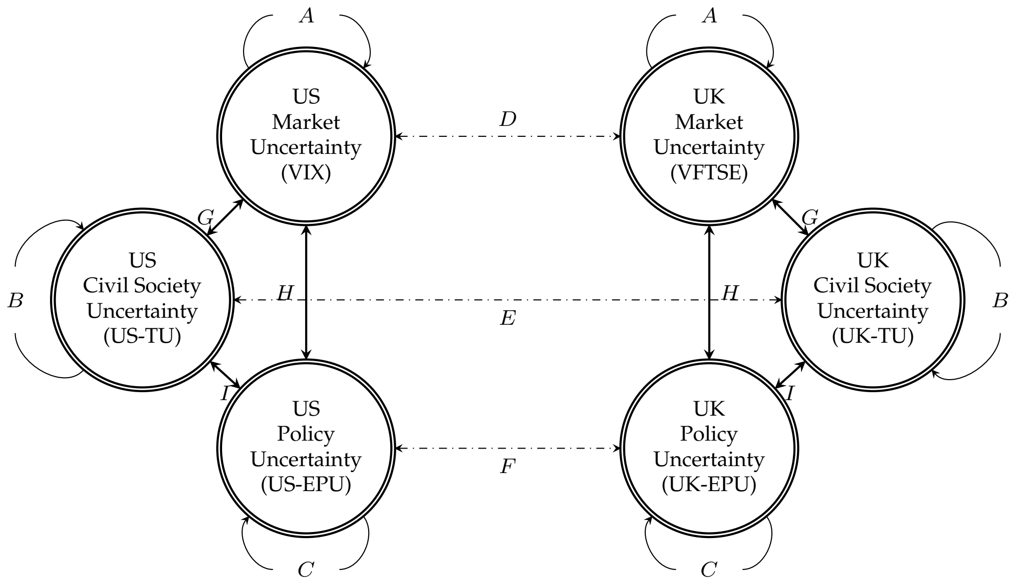


Figure 2.47. Reduced Model: No inter-area inter-source dependencies

As we can see from Table 2.8 that contains the estimates of our VAR(2) restricted model with no inter-area inter-source dependencies, the Adjusted R^2 of *market uncertainty* equations (VFTSE and VIX) are - at two digits precision- identical. For the other endogenous variables the fit worsens compared to the unrestricted model. As for the previous restricted model, the magnitude of the drop is larger for the EPU indexes compared to TU indexes: *policy uncertainty* depends more on inter-area Market and *civil society uncertainty* than *civil society uncertainty* depends on inter-area *market and policy uncertainty*. Coefficients all maintain their sign. The autocorrelation and heteroskedasticity structure of residuals doesn't appear to change in a remarkable way compared to the baseline unrestricted VAR(2) model. The distributions of the residuals of the EPU and TU indexes are slightly more right skewed. It is interesting to remark that -in the EPU and TU equations-, the coefficients of uncertainty variables of the same source but of the other geographic area are not statistically significant. Given the above findings, it looks like for both geographic-areas (US or UK) *market uncertainty* variables do not depend on the lagged values of non *market uncertainty* variables of the other geographic-area. Whereas for non *market uncertainty* variables (TU and EPU indexes) inter-area inter-source uncertainty dependencies (especially with *market uncertainty* variables) may still play a relevant role.

Table 2.8. Reduced VAR(2) model estimates;
Reductions: No inter-area inter-source dependencies

VFTSE at close prices; VIX at open prices; All endogenous variables have been standardized
L means once lagged variable, L^2 means twice lagged variable

	Dependent variables:					
	UK-TU	US-TU	VFTSE	VIX	UK-EPU	US-EPU
L UK-TU	0.53***	−0.11	0.07*		0.41***	
	(0.09)	(0.10)	(0.04)		(0.08)	
L^2 UK-TU	0.03	−0.03	−0.06		−0.14	
	(0.10)	(0.10)	(0.04)		(0.08)	
L VFSTE	−0.57***		0.90***	0.26**	−0.25	
	(0.19)		(0.08)	(0.12)	(0.16)	
L^2 VFSTE	0.80***		0.12	−0.05	0.41**	
	(0.19)		(0.08)	(0.12)	(0.16)	
L UK-EPU	−0.05		0.02		0.25***	0.02
	(0.10)		(0.04)		(0.09)	(0.09)
L^2 UK-EPU	0.04		−0.09**		0.14*	0.02
	(0.09)		(0.04)		(0.08)	(0.09)
L US-TU	−0.08	0.35***		−0.12***		0.12*
	(0.07)	(0.08)		(0.04)		(0.07)
L^2 US-TU	0.04	0.14		0.17***		0.02
	(0.07)	(0.08)		(0.04)		(0.07)
L VIX		0.27*	0.05	0.82***		0.31**
		(0.15)	(0.05)	(0.08)		(0.12)
L^2 VIX		−0.04	−0.11**	−0.10		−0.22*
		(0.15)	(0.05)	(0.08)		(0.13)
L US-EPU		−0.02		−0.02	0.07	0.25***
		(0.09)		(0.04)	(0.06)	(0.08)
L^2 US-EPU		0.003		−0.09**	0.05	0.37***
		(0.08)		(0.04)	(0.06)	(0.08)
const	0.10	−0.05	0.07	0.05	0.05	0.26*
	(0.13)	(0.16)	(0.05)	(0.08)	(0.12)	(0.14)
Tuesday	−0.19	0.02	−0.05	−0.22*	0.06	−0.18
	(0.18)	(0.21)	(0.07)	(0.11)	(0.16)	(0.19)
Wednesday	−0.03	0.34	−0.02	−0.004	0.03	−0.44**
	(0.18)	(0.22)	(0.07)	(0.12)	(0.16)	(0.19)
Thursday	−0.03	−0.01	−0.18**	−0.09	−0.22	−0.39**
	(0.18)	(0.21)	(0.07)	(0.11)	(0.16)	(0.19)
Friday	−0.25	−0.10	−0.09	0.05	−0.11	−0.22
	(0.18)	(0.21)	(0.07)	(0.11)	(0.16)	(0.19)
Observations	182	182	182	182	182	182
R^2	0.49	0.29	0.92	0.80	0.60	0.43
Adjusted R^2	0.45	0.23	0.92	0.79	0.57	0.39
Resid. SE (df = 169)	0.75	0.88	0.29	0.46	0.66	0.78
F Stat. (df = 13; 169)	12.26***	5.28***	155.96***	53.15***	19.68***	9.99***
Note:						*p<0.1; **p<0.05; ***p<0.01

Inter-area dependencies only for *market uncertainty*

Given that, in the previously estimated restricted model (Table 2.8), the coefficients of intra-source inter-area dependencies in the *policy uncertainty* and *civil society uncertainty* equations were not significantly different from zero. More specifically:

- **In the UK-TU estimated equation:** Both coefficients of lagged US-TU were not statistically significant;
- **In the US-TU estimated equation:** Both coefficients of lagged UK-TU were not statistically significant;
- **In the UK-EPU estimated equation:** Both coefficients of lagged US-EPU were not statistically significant;
- **In the US-EPU estimated equation:** Both coefficients of lagged UK-EPU were not statistically significant;

We choose to further restrict the aforementioned model by constraining to zero TU and EPU inter-area intra-source dependency relations' coefficients. In such away we obtain a model in which uncertainty may flow from a geographic-area to the other only through (financial) markets' uncertainty (VIX and VFTSE) that are the only endogenous variables that have a non constrained to zero inter-area dependency coefficient (see Fig.2.48). In the following subsection we estimate and comment this final restricted VAR(2) model, which results the best among our three restricted models.

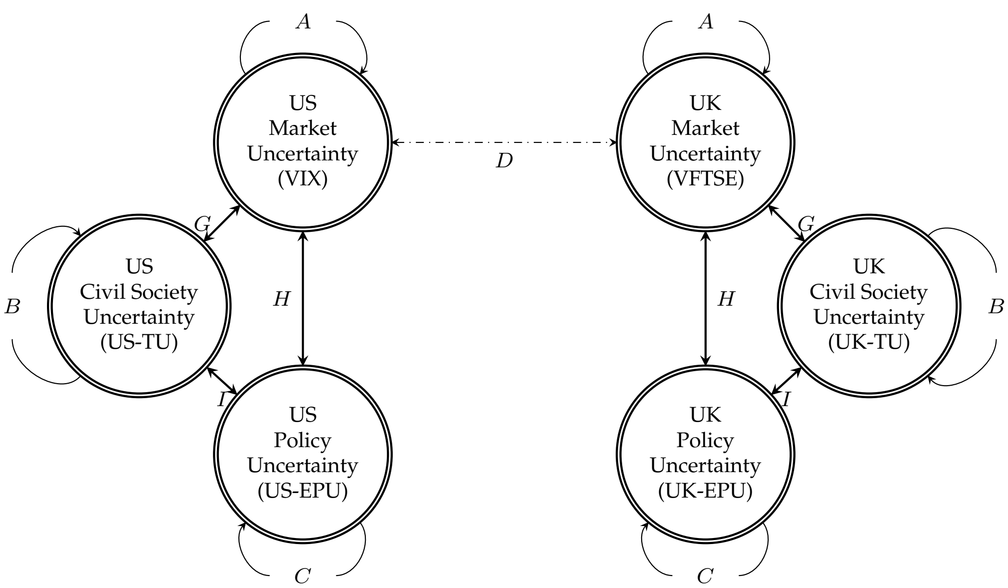


Figure 2.48. Reduced Model: Inter-area dependencies only for *market uncertainty*

As we can see from Table 2.9 that contains the estimates of our VAR(2) restricted model with inter-area uncertainty dependency only through *market uncertainty*, the Adjusted R^2 of all the endogenous variables' equations are the highest among the restricted models' ones. Nevertheless the unrestricted model still appears to be slightly superior from the point of view of the fitness and residuals' non violations of the assumptions. Also in this case coefficients of lagged endogenous variables maintain their sign. The autocorrelation and heteroskedasticity structure of residuals is rather similar to that of the baseline unrestricted VAR(2) model, but the residuals' distributions of EPU and TU indexes are slightly more right skewed. Some extreme uncertainty events remain -relatively more- unexplained compared to the baseline model, this is probably due to impossibility of inter-area inter-source contagion among Policy and *civil society uncertainty* in the restricted model. It is possible that, even though in normal times this channel of inter-area contagion between policy and *civil society uncertainty* has a relatively minor role with respect to other contagion channels, its importance could grow during extreme uncertainty events and in their trails. However -by being linear and with constant parameters- our model is unable to integrate these -certainly plausible- nonlinear components of inter-source inter-area uncertainty contagion.

We also remark that, in the estimated equations of US and UK *civil society uncertainty* (US-TU and UK-TU), the coefficients of lagged *policy uncertainty* variables (UK-EPU and US-EPU) are not statistically significant. However for both US and UK the residuals of Policy and *civil society uncertainty* exhibit relatively high instantaneous inter-area inter-source correlation coefficients, which range from 0.2 to 0.4. Therefore, when it occurs, the contagion of *policy uncertainty* innovations to *civil society uncertainty* must occur at a infra-day time-scale, this means that the twitter community -that we exploit to build TU indexes- is very reactive to *policy uncertainty* news reporting -also coming from foreign countries- but then rapidly forgets foreign *policy uncertainty* events -it is a short memory contagion process-. Whereas the impulse-response of *policy uncertainty* to *civil society uncertainty* innovations is more persistent and delayed, as the n-shape of the cumulative-impulse response functions confirms.

Finally, given that this last restricted model appears to be the best among the three, we can rather safely claim that Markets are the primary inter-area contagion channel for uncertainty: VIX-VFSTE dependencies are the main uncertainty link between uncertainty in UK and US. This link binds together the two systems in a unique integrated trans-national macro-uncertainty system, through which the effects of Policy and *civil society uncertainty* may flow, "exploiting" inter-area *market uncertainty* dependencies as a vehicle through which non-*market uncertainty* impulses may be propagated to foreign countries. Therefore Policy and *civil society uncertainty* impulses, whose effects would otherwise be geographically limited, may become international through their chain effect on dependent uncertainty variables: First, by affecting local markets' uncertainty; then from local markets to foreign countries' markets, and finally -if the magnitude of the initial shock is sufficiently large-, to foreign countries' Policy and *civil society uncertainty*.

Table 2.9. Reduced VAR(2) model estimates;
Reductions:Inter-area dependencies only for *market uncertainty*

VFTSE at close prices; VIX at open prices; All endogenous variables have been standardized
L means once lagged variable, L^2 means twice lagged variable

	Dependent variables:					
	UK-TU	US-TU	VFTSE	VIX	UK-EPU	US-EPU
L UK-TU	0.53***		0.07*		0.43***	
	(0.09)		(0.04)		(0.08)	
L^2 UK-TU	0.04		−0.06		−0.13	
	(0.09)		(0.04)		(0.08)	
L VFTSE	−0.55***		0.90***	0.26**	−0.28*	
	(0.18)		(0.08)	(0.12)	(0.16)	
L^2 VFTSE	0.77***		0.12	−0.05	0.43***	
	(0.19)		(0.08)	(0.12)	(0.17)	
L UK-EPU	−0.06		0.02		0.28***	
	(0.10)		(0.04)		(0.09)	
L^2 UK-EPU	0.05		−0.09**		0.16*	
	(0.09)		(0.04)		(0.08)	
L US-TU		0.37***		−0.12***		0.12*
		(0.08)		(0.04)		(0.07)
L^2 US-TU		0.16*		0.17***		0.01
		(0.08)		(0.04)		(0.07)
L VIX		0.23*	0.05	0.82***		0.31**
		(0.14)	(0.05)	(0.08)		(0.12)
L^2 VIX		−0.06	−0.11**	−0.10		−0.21*
		(0.14)	(0.05)	(0.08)		(0.12)
L US-EPU		−0.05		−0.02		0.26***
		(0.08)		(0.04)		(0.07)
L^2 US-EPU		−0.03		−0.09**		0.38***
		(0.08)		(0.04)		(0.07)
const	0.11	−0.04	0.06	0.05	0.06	0.21
	(0.13)	(0.15)	(0.05)	(0.08)	(0.12)	(0.14)
Tuesday	−0.20	0.02	−0.06	−0.21*	0.07	−0.15
	(0.18)	(0.21)	(0.07)	(0.11)	(0.16)	(0.19)
Wednesday	−0.04	0.35	−0.03	0.02	0.03	−0.33*
	(0.18)	(0.22)	(0.07)	(0.11)	(0.16)	(0.19)
Thursday	−0.06	−0.01	−0.16**	−0.08	−0.24	−0.34*
	(0.18)	(0.21)	(0.07)	(0.11)	(0.16)	(0.19)
Friday	−0.25	−0.13	−0.08	0.03	−0.16	−0.16
	(0.18)	(0.21)	(0.07)	(0.11)	(0.16)	(0.19)
Observations	182	182	182	182	182	182
R^2	0.48	0.28	0.92	0.80	0.59	0.43
Adjusted R^2	0.45	0.23	0.92	0.79	0.57	0.40
Resid. SE (df)	0.74 (171)	0.88 (171)	0.29 (169)	0.46 (169)	0.66 (171)	0.78 (171)
F Stat. (df)	14.41*** (11)	6.05*** (11)	155.96*** (13)	53.15*** (13)	22.77*** (11)	11.91*** (11)

Note: *p<0.1; **p<0.05; ***p<0.01

VFTSE at close prices; VIX at open prices; All endogenous variables have been standardized
L means once lagged variable, L^2 means twice lagged variable

Chapter 3

The Worldwide Uncertainty Network: mapping the global dynamics and contagion channels of *civil society uncertainty*

By Carlo R. M. A. Santagiustina

Abstract

In this article will develop an instrument to measure, visualize and analyse worldwide dynamics of *civil society uncertainty* by sovereign-country. Our tool exploits decentralized signals of *uncertainty* elicited through Twitter, a global news and social networking platform. By filtering Twitter posts in English containing the term *"uncertainty"* and attributing them to single countries or relations among countries on the basis of their textual content we are able to reconstruct a network which proxies global *civil society uncertainty*, we call it **Worldwide Uncertainty Network** (**WUN**). Such a network can be constructed and visualized at the desired frequency. In this article we will illustrate the process of construction and analysis of the outcomes of a static (**S-WUN**) and a rescaled dynamic (**RD-WUN**) projection of worldwide *civil society uncertainty* discussions through Twitter, using our original methodology. Both versions are based on the same dataset, which contains Twitter posts (textual observations) signalling states of *civil society uncertainty*, published across the 2^{nd}, 3^{rd} and 4^{th} quarter of the year 2016. In the static version (**S-WUN**) observations are aggregated for the entire period of study; whereas, in the rescaled dynamic version (**RD-WUN**) observations are aggregated, re-weighted and analysed at a daily frequency. As we will see, the static version (**S-WUN**) allows us to appraise which countries where subject to major *civil society uncertainty* events that year and which where the most relevant contagion channels for uncertainty among them, allowing us to represent and highlight through a time invariant framework the backbone of national and international *civil society uncertainty* phenomena during unfolding of the Brexit-Trump Era. Whereas, the rescaled dynamic version (**RD-WUN**) will allow us to appraise the time-varying structure of contagion channels among countries as events unfold, as well as country level dynamics of *civil society uncertainty* emerging in relation to both local and international events, like elections, referendums, civil-wars and international political or military conflicts. For example, to evaluate the local and international *civil society uncertainty* impact dynamics of: UK's EU-referendum, the US presidential elections, the failed coup in Turkey, the Italian constitutional referendum and the Syrian civil-war, among many other major and minor events of interest for our readers that occurred during the year 2016 and which have been captured by our **Worldwide Uncertainty Network**.

3.1 Introduction

A great number of events that occurred in the year 2016, like the EU-referendum vote in favour of Brexit, the election of Trump, the Syrian civil-war, the failed coup in Turkey, the failed Italian constitutional reform and ensuing resignation of the Prime Minister Renzi, have shown that the *civil society uncertainty* impact of geographically localized events may encompass the boarders of the country in which a given event has physically occurred, and, influence the degree of *civil society uncertainty* in neighbouring countries and beyond.

Differently from the method developed and used in this paper, in the second article of this collection the magnitude of contagion channels among *civil society uncertainty* variables by country was not inferred from the textual content of agents' online uncertainty signaling behaviour, through Twitter posts containing the term *"uncertainty"*, but was based on the lagged linear dependency relations among time series of uncertainty variables for different countries. **Twitter Uncertainty** (**TU**) indexes as well as other existing text-based uncertainty measures, like the **Economic Policy Uncertainty** (**EPU**) indexes constructed by Baker, Bloom and Davis[386], do not exploit the possibility of using co-occurrences of country labels in textual observations containing the term *"uncertainty"* as signals of the occurrence of *civil society uncertainty* events that may concern several countries at the same time. Events which encompass national boarders and which signal the existence of *civil society uncertainty* contagion channels between them.

However, it is well documented fact[656–663] that *civil society uncertainty* generated by economic, political and social events, even if geographically localized, may ripple in other countries and areas of the world for a multiplicity of reasons. The simplest one is that economic, social or political phenomena that generate *civil society uncertainty* can be transnational or international by nature, for example: the UE referendum in the United Kingdom appeared as soon as its date was fixed by Cameron, as a transnational event. With expected disruptive political and economic effects -in case of a vote in favor of Brexit- not only within the UK but also for other countries in the European Union [664–667]. The EU-referendum was transnational to such an extent that several analysts and commentators described it as a unilateral vote -by United Kingdom's people- on the survival of the *"European dream"*[668], i.e. the pursuit of the European Union project of economic, political and socio-cultural integration.

Some specific post EU-referendum inter-area *civil society uncertainty* contagion channels have been already identified by literature, using a qualitative event study approach. In a recent paper Belke et al.[669] have claimed that "*apart from direct economic linkages, Brexit might also generate political and institutional uncertainty about the EU. [... for example] the success of the Brexit movements might generate momentum for similar movements in other countries increasing the probability of more countries leaving the EU. [...] Political uncertainty may therefore spread across Europe, especially affecting countries whose sovereign solvency is closely linked to the existence of the EU and the euro area namely Spain, Portugal, Italy and Greece."* More generally, all withdrawals, or threats of withdrawals from multilateral or international agreements and treatises, like the Paris climate agreement or the NAFTA, are by "nature" transnational/international events, which may cause pikes in *civil society uncertainty* across several countries concerned directly or through side-effects by the agreement/treatise. Therefore, it is often sufficient that one of the parties menaces to withdraw from such agreements, to produce an international uncertainty shock related to the degree of divergence of expectations concerning the possible effects of the withdrawal. A second category of events, for which there can be inter-area uncertainty contagion, are collective belief driven contagions of *civil society uncertainty* among countries. The latter situation occurs when a great number of agents living in a given country/area, which we call target, believe that, given the perceived similarities and interdependencies between their own -economic, political, social, etc.- conditions and those of another country, events that occur in the area object of the comparison, which we call source, could "cross the borders" and produce -similar or identical- consequences in the target area.

Civil society uncertainty contagion processes are, as we will see, amplified by cultural relatedness[670, 671], shared identification[672, 673] and social emulation[659, 674, 675]. The perceived degree of similarity between cultural, political, social and economic scenarios occuring in different countries, as well as beliefs of cultural, political, social and economic interdependence act as catalysers for *civil society uncertainty* contagion processes. Through the diffusion across boarders of news and beliefs concerning cause-consequence relations, civil societies of distinct areas that exhibit similar conditions become interdependent in terms of expectations and ensuing *civil society uncertainty*. Local uncertainty phenomena may hence "ripple" outside the country from where they first emerged. For example the Arab Spring, which began in Tunisia and then, through the reporting -in Arabic and through Social medias- and emulation -of protesting and demonstration techniques- progressively extended to other countries in North Africa and Middle-East, causing a transnational *civil society uncertainty* phenomenon[676–679]. Another group of instances belonging to the latter category are general public's reactions to terrorist attacks, especially within the European Union. These -rather isolated- local violences give rise to Europe wide situations of *civil society uncertainty* and security alert. Such uncertainty phenomena are certainly accentuated by the worldwide integration of information systems and media: by storytelling and reporting in great detail about terrorist attacks occurring in foreign countries and dedicating huge spaces to these happenings, the media -willingly or unwillingly- propagate signals capable of causing or stimulating feelings of uncertainty and unsafety across country borders. The above stated information diffusion processes lead people to further identify themselves with the local communities that actually suffered terrorist attacks and renders foreign public empathetic, from an "uncertainty feeling" point of view, to the vicissitudes and suffering of the local communities that experienced in first person these dreadful experiences.

The existence of the aforesaid channels of contagion renders sovereign country systems vulnerable to the contagion of *civil society uncertainty* from other areas of the world. Given the lack of adequate indicators for measuring and monitoring international *civil society uncertainty* phenomena and transnational contagions of uncertainty among geographic areas, in this final article we choose to address this issue by developing a tool for systemic uncertainty analysis and visualization, which is constructed in such a way that it can be updated in real time. This to appraise and visualize the links among *civil society uncertainty* in the different countries of the world. We consider this instrument the building block of a real-time *civil society uncertainty* observatory that the author will try to further develop and improve in future works.

More specifically, in this final article we seek to measure, describe and analyse *civil society uncertainty* dependencies among sovereign countries, using the subset of observations in the TU dataset that contain at least one token used to identify countries from the list of dictionaries available in section 3.0.1 of the Appendix at the end of this article. In the next section, we start by analysing the counts by country of the number of observations (tweets) in which country related labels occur or co-occur. We use the term *country occurrence* to refer to the situations where only labels associated to a single country (tokens belonging to the same country dictionary) are contained in a textual observation (tweet), and we use the term *country co-occurrence* to refer to the situation where labels of two or more areas (tokens from multiple country dictionaries) are contained in the same observation (tweet). Considering the entire period of our investigation, which ranges from April the 4th to December the 31st 2016, we build the **Static Worldwide Uncertainty Network** (**S-WUN**), by counting the number of occurrence for each country and co-occurrences of each pair of countries, the latter values are computed through a weighted count that allows us to ensure that every observation (tweet) has the same contribution on the edges/node weight of the inferred network whatever is number of countries mentioned in a given observation. Successively, we apply the same technique to Twitter data aggregated by day, to construct the **Dynamic Worldwide Uncertainty Network** (**D-WUN**), i.e. a three dimensional tensor representing the *civil society uncertainty* dynamics and dependency relations, among countries, across time. Since all observations contained in our dataset are written in English and Twitter users capable of writing in English are not uniformly distributed across the areas of the world, our counts and weighted counts of node/edge specific daily observation sets that determine the weights of the nodes and edges are -very likely- distorted measures of the magnitude of *civil society uncertainty* events actually occurring worldwide, therefore they are not necessarily reliable for doing inter-area (among nodes' weights), or, inter contagion channel (among edges' weights) comparisons of uncertainty levels. In **D-WUN**, the relevance of events in countries in which only a small share of the population speaks in English or have access to Internet/Twitter will likely be underestimated, especially when compared to countries in which a majority of the population are native English speakers, like the UK, the US, Canada and Australia among others. To mitigate this data representativeness problem we rescale the content of our **D-WUN** tensor, through three distinct transformations of the edge/nodes weights contained in the original tensor. We obtain three distinct tensors that represent the following distinct features of *civil society uncertainty* in a given moment in time, for a given node/edge:

1. The redundancy of the *civil society uncertainty* signal, which represents the variety of the sources that created, in a given day, *civil society uncertainty* signals concerning a specific edge/node. To show how reliable are the Twitter information sets on which a daily node/edge *civil society uncertainty* measurements is based;
2. The intensity of present *civil society uncertainty* with respect to past *civil society uncertainty* in that node/edge;
3. The intensity of present *civil society uncertainty* in that node/edge compared to present *civil society uncertainty* in all other nodes/edges;

By computing the Hadamard product of the three tensors we obtain a unique dynamic network representation of worldwide *civil society uncertainty*, for which the weights are now more reliable for doing comparisons of *civil society uncertainty* levels between countries and across time. We call it **Rescaled Dynamic Worldwide Uncertainty Network** (**RD-WUN**).

We conclude the article by analysing main results and illustrating the advantages and limits of our **Worldwide Uncertainty Network** (**WUN**) tools for the measurement of worldwide *civil society uncertainty* by country. We explain why the decomposition of **D-WUN** in three distinct *civil society uncertainty* dynamic feature tensors can facilitate and enrich the interpretation of crowd-sourced uncertainty signals, by helping us to identify: the systemic importance of countries in terms of their *civil society uncertainty* social percolation[680–686] role and major **WUN** changes across time. Despite the fact that our **WUN** is still in an early development phase, the preliminary methods and findings here exposed may help us understand which could be the potential contribution, to contemporary economic and sociological literature, of studies based on textual data extracted from the web. In particular, for the measurement, analysis and modelling of *civil society uncertainty* dynamics, at a worldwide scale, for which, at the moment, there is no comparable alternative instrument.

3.2 From occurrences and co-occurrences of country labels in tweets to a Static Worldwide Uncertainty Network (S-WUN)

In this section we will describe the procedure that allows us to use Tweets in English about uncertainty states, associated to one or more countries of the world, to appraise and visualize -as a network- *civil society uncertainty* associated to one or more countries; and, even more important, to differentiate among activity concerning *civil society uncertainty* in a specific country from activity concerning international *civil society uncertainty* events and *civil society uncertainty* contagion channels among different countries of the world.

To obtain the **Static Worldwide Uncertainty Network**, we start by subsetting our population of tweets belonging to the T_{clean} dataset (see section 2.2.1 of the second article) by keeping only those observations (tweets) that contain at least one token that matches the conditions of (at least) one of the country dictionaries presented in section 3.0.1 of the Appendix. We use only token dictionaries that identify sovereign countries. *For example, the European Union, Antarctica, Greenland as well as many other geographic areas that are not sovereign countries are not considered in the following article.* We refer to this list of country dictionaries as *DICT*. *DICT* contains 163 different dictionaries, representing 163 sovereign countries of the world. Each dictionary contains a set of textual tokens used to identify a specific country in observations. Some very small sovereign countries, in terms of population, geo-political and economic importance at the global scale, haven't been included in *DICT*, this to avoid having a too large network, and, because the volume of Twitter activity concerning uncertainty in these very small countries is either very close to zero or null. *For example, the Republic of San Marino, The Principality of Monaco, Liechtenstein, the Principality of Andorra, as well as other very small autonomous islands, like the Republic of Kiribati and Nauru, do not belong to DICT and are therefore not considered in this study.* In addition, as we did in the second article of this collection, we group together the United Kingdom and Great Britain, and hence their dictionaries. We refer to this area through the label *GB/UK*. The resulting list of countries whose country dictionaries have been included in *DICT*, is shown in section 3.0.1 of the Appendix.

Country dictionaries contained in $DICT$ are boolean functions which can be applied to textual observations. As explained in section 2.2.1 of the second article, dictionaries' conditions are sequences of characters or regular expressions used to mention

-and hence identify- countries to which tweets about uncertainty refer to. For example, "*U.K*" is one of the ways used to mention the United Kingdom in tweets, for this reason it has been included in the United Kingdoms' token dictionary. As a result of our subsetting, the dataset of tweets that we will use in the remainder of this article contains 170 806 observations -from T_{clean}-, which match the boolean conditions of at least one of the country dictionaries in *DICT*, all other observations have been dropped. This subset of T_{clean} is called $T_{country}$. Observations in our $T_{country}$ dataset have been published between April (the 4th) and December (the 31st) 2016. We then categorize $T_{country}$ observations in two groups on the basis of their textual content:

- In the first group we have those observations in which a single country is mentioned, in isolation. We call this group of observations country **occurrences**, they will jointly determine the weights of the country nodes in the Static Twitter Uncertainty Network.
- In the second group we have all observations in which several (two or more) countries are mentioned together. We call this group of observations country **co-occurrences**, they will jointly determine the weights of the country edges in the Static Twitter Uncertainty Network.

Table 3.1. Geographic Areas (countries) whose country dictionaries have been included in *DICT*

A-C	D-L	L-P	Q-Z
afghanistan	denmark	latvia	qatar
albania	ecuador	lebanon	romania
algeria	egypt	lesotho	russian federation
angola	el salvador	liberia	rwanda
arab emirates	eritrea	libya	saudi arabia
argentina	estonia	lithuania	senegal
armenia	ethiopia	luxembourg	serbia
australia	finland	macedonia	singapore
austria	france	madagascar	slovakia
azerbaijan	gabon	malawi	slovenia
bahrain	gambia	malaysia	somalia
bangladesh	GB/UK	mali	south africa
belarus	georgia	malta	south korea
belgium	germany	mauritania	south sudan
belize	ghana	mexico	spain
benin	greece	moldova	sri lanka
bhutan	guatemala	mongolia	suriname
bolivia	guinea	montenegro	swaziland
bosnia and herzegovina	guinea-bissau	morocco	sweden
botswana	guyana	mozambique	switzerland
brazil	haiti	myanmar	syria
brunei	honduras	namibia	tajikistan
bulgaria	hong kong	nepal	tanzania
burkina faso	hungary	netherlands	thailand
burundi	iceland	new zealand	timor-leste
cambodia	india	nicaragua	togo
cameroon	indonesia	niger	trinidad and tobago
canada	iran	nigeria	tunisia
cape verde	iraq	north korea	turkey
central african republic	ireland	norway	turkmenistan
chad	islamic state	oman	uganda
chile	israel	pakistan	ukraine
china	italy	palestine, state of	united states
colombia	jamaica	panama	uruguay
congo	japan	papua new guinea	uzbekistan
costa rica	jordan	paraguay	venezuela
côte d’ivoire	kazakhstan	peru	vietnam
croatia	kenya	philippines	yemen
cuba	kuwait	poland	zambia
cyprus	kyrgyzstan	portugal	zimbabwe
czech republic	laos	puerto rico	

To analyze observations in $T_{country}$, we sequentially apply each country dictionary in *DICT*, to each observation (tweet) in $T_{country}$. By so doing we obtain a observation/feature matrix -of size $170806 * 163$-, called **F**, containing uniquely boolean values (zeros and ones). Each row of **F** corresponds to an observation, each column column of **F** corresponds to a feature (country dictionary in *DICT*). For any $i \in \{1, ..., 170806\} \subset \mathbb{Z}_{>0}$ and $j \in \{1, ...163\} \subset \mathbb{Z}_{>0}$, the $f_{i,j}$ element of **F** represents the output of the boolean function of dictionary *j*, applied to observation *i*. If the output of the boolean dictionary function j applied to observation i is TRUE $f_{i,j}$ will be equal to 1, if the output is FALSE $f_{i,j}$ will be equal to 0. For a given observation (row) the ones represent the countries that are mentioned in that observation, whereas the zeros represent those countries that are not mentioned in that particular observation.

Figure 3.1. Observation/Feature matrix F

$$\begin{array}{rc} & \overbrace{\begin{array}{ccc}\text{afghanistan} & \cdots & \text{zimbabwe}\end{array}}^{\textbf{DICT}\text{ (countries)}} \\ \textbf{Observations}\left\{\begin{array}{c}1\\ \vdots \\ 170\,806\end{array}\right. & \begin{pmatrix} f_{1,1} & \cdots & f_{1,163}\\ \vdots & \ddots & \vdots \\ f_{170806,1} & \cdots & f_{170806,163}\end{pmatrix}\end{array}$$

For example, if a tweet mentions a unique country (one or more times), like the following text:*"Great political uncertainty in the United Kingdom after the resignation of PM Cameron...Growth forecasts for the UK could be reviewed!"* it will be considered a country occurrence. Whereas, if a tweet mentions multiple countries (one or more times), like the following text *"Geo-political uncertainty grows worldwide as Russia, China and the United States do not find any agreed solution to the North-Korean crisis"* it will be considered a country co-occurrence.

Therefore, from a formal point of view, the **country co-occurrences** set $T_{country,CO} \subset T_{country}$ is composed by all those observations for which the corresponding row sum is strictly larger than one:

$$T_{country,CO} = \left\{ i \in T_{country} \middle| \sum_{j=1}^{163} f_{i,j} > 1 \right\}$$

Whereas, the **country occurrences** set $T_{country,OC} \subset T_{country}$ contains all those observations for which the corresponding row sum is equal to one:

$$T_{country,OC} = \left\{ i \in T_{country} \middle| \sum_{j=1}^{163} f_{i,j} = 1 \right\}$$

By construction we have that:

$$T_{country,OC} \cap T_{country,CO} = \emptyset$$

and

$$T_{country,OC} \cup T_{country,CO} = T_{country}$$

In the Table 3.2, that follows, we use the above defined $T_{country,CO}$ and $T_{country,OC}$ sets to count, for each country, the number of observations in which that country occurs alone or cooccurs with other countries. More specifically:

- The third column **(occurrences)** , is computed as the cardinality of the, by country, subsets of observations from $T_{country,OC}$. We call these country specific occurrence sets $T_{country,OC,k}$, where $k \in DICT$ identifies the country. We can compute the cardinality ($card$) of these sets as follows:

$$card\left(T_{country,OC,k}\right) = card\left(\left\{i \in T_{country,OC} \middle| f_{i,k} = 1\right\}\right)$$

- The fourth column **(co-occurrences)** , is computed as the cardinality of the, by country, subsets of observations from $T_{country,CO}$. We call these country specific cooccurrence sets $T_{country,CO,k}$, where $k \in DICT$ identifies the country. We can compute the cardinality ($card$) of these sets as follows:

$$card\left(T_{country,CO,k}\right) = card\left(\left\{i \in T_{country,CO} \middle| f_{i,k} > 1\right\}\right)$$

  In addition we further differentiate cooccurrences in two subcategories, for which we also compute the set cardinality by country:

  – The fifth column **(2 cliques)**, shows us, for each country $k \in DICT$, the cardinality of the subset of $T_{country,CO,k}$ that contains only those observations in which exactly two countries are mentioned together, we call this subset $T_{country,CO,k,2}$ and compute their cardinality as follows:

$$card\left(T_{country,CO,k,2}\right) = card\left(\left\{i \in T_{country,CO,k} \middle| \sum_{j=1}^{163} f_{i,j} = 2\right\}\right)$$

  – The sixth column **(3+ cliques)**, shows us, for each country $k \in DICT$, the cardinality of the subset of $T_{country,CO,k}$ that contains only those observations in which more than two countries are mentioned together, we call this subset $T_{country,CO,k,3+}$ and compute their cardinality as follows:

$$card\left(T_{country,CO,k,3+}\right) = card\left(\left\{i \in T_{country,CO,k} \middle| \sum_{j=1}^{163} f_{i,j} > 2\right\}\right)$$

- The second column **(N. Total)**, shows us the the cardinality of $T_{country,k}$, which is the subset of $T_{country}$ in which a given country $k \in DICT$ is mentioned. $T_{country,k}$ is the union of $T_{country,CO,k}$ and $T_{country,OC,k}$:

$$card\left(T_{country,k}\right) = card\left(T_{country,OC,k} \cup T_{country,CO,k}\right)$$

$$= card\left(T_{country,OC,k}\right) + card\left(T_{country,CO,k}\right) - card\left(T_{country,OC,k} \cap T_{country,CO,k}\right)$$

$$= card\left(\left\{ i \in T_{country,CO} \middle| f_{i,k} = 1 \right\}\right) + card\left(\left\{ i \in T_{country,CO} \middle| f_{i,k} > 1 \right\}\right) + card\left(\emptyset\right)$$

$$= card\left(\left\{ i \in T_{country,CO} \middle| f_{i,k} > 0 \right\}\right)$$

Table 3.2. Number of observations (tweets) in which country related tokens co/occur, by country and by typology of co/occurrence

Geo-Area Name	**N. total (number of observations by country)**	**N. (only) occurrences**	**N. (only) co-occurrences**	// *only 2 cliques*	// *only 3+ cliques*
GB/UK	69349	66253	3096	2774	322
united states	34015	28108	5907	5228	679
china	8750	5689	3061	2767	294
india	5848	4459	1389	1310	79
spain	4721	4385	336	294	42
italy	3981	3623	358	287	71
canada	3545	2870	675	637	38
france	3142	2485	657	494	163
turkey	3022	2713	309	257	52
colombia	2804	2779	25	21	4
australia	2705	2370	335	296	39
thailand	2695	2592	103	77	26
japan	2268	1408	860	601	259
greece	2064	1944	120	62	58
ireland	2029	1633	396	336	60
syria	1976	1693	283	188	95
germany	1634	793	841	421	420
brazil	1622	1499	123	68	55
singapore	1452	1191	261	249	12
russian federation	1304	802	502	373	129
philippines	1253	750	503	396	107
mexico	1236	1056	180	174	6
nigeria	1147	1012	135	118	17
south africa	1121	1020	101	66	35
pakistan	1076	943	133	125	8
iran	1051	954	97	66	31
ukraine	1003	709	294	60	234
cuba	991	534	457	383	74
south korea	862	787	75	64	11
islamic state	720	498	222	180	42
indonesia	703	237	466	455	11
hong kong	674	509	165	117	48
iraq	661	590	71	51	20

uganda	597	483	114	109	5
iceland	517	472	45	44	1
haiti	432	353	79	56	23
israel	423	263	160	154	6
zimbabwe	412	376	36	6	30
kenya	409	318	91	75	16
uzbekistan	389	343	46	40	6
nepal	344	280	64	55	9
north korea	340	236	104	92	12
saudi arabia	334	265	69	67	2
croatia	324	297	27	5	22
egypt	275	246	29	21	8
chad	267	247	20	20	0
new zealand	257	228	29	11	18
switzerland	245	217	28	10	18
jamaica	244	124	120	50	70
argentina	237	212	25	3	22
macedonia	230	227	3	1	2
georgia	228	196	32	6	26
bangladesh	226	93	133	133	0
jordan	222	203	19	18	1
south sudan	220	175	45	44	1
norway	214	108	106	94	12
libya	211	160	51	43	8
malaysia	191	142	49	26	23
venezuela	187	153	34	10	24
afghanistan	187	88	99	91	8
gambia	179	172	7	6	1
congo	179	169	10	2	8
ghana	177	154	23	22	1
vietnam	148	111	37	30	7
somalia	143	116	27	22	5
puerto rico	142	138	4	4	0
sweden	141	116	25	17	8
palestine, state of	133	40	93	93	0
qatar	129	120	9	7	2
cambodia	129	59	70	68	2
ethiopia	127	113	14	14	0
panama	120	68	52	52	0
poland	119	71	48	47	1
chile	117	100	17	16	1
austria	109	70	39	17	22
netherlands	108	71	37	16	21
zambia	105	101	4	1	3
bulgaria	105	93	12	10	2
morocco	101	16	85	66	19
belgium	95	50	45	24	21
cyprus	95	40	55	48	7
portugal	91	66	25	21	4
finland	91	57	34	11	23
algeria	90	84	6	5	1

yemen	88	79	9	6	3
myanmar	86	65	21	15	6
lebanon	85	49	36	32	4
gabon	81	72	9	9	0
serbia	76	26	50	50	0
rwanda	69	61	8	3	5
guinea	65	64	1	1	0
denmark	57	55	2	1	1
hungary	49	22	27	25	2
tanzania	48	37	11	6	5
mozambique	48	14	34	6	28
peru	46	42	4	3	1
romania	45	36	9	7	2
kazakhstan	44	42	2	2	0
bosnia herzegovina	42	4	38	38	0
burundi	41	35	6	1	5
sri lanka	40	36	4	2	2
niger	31	30	1	1	0
latvia	31	9	22	0	22
luxembourg	30	30	0	0	0
slovakia	29	22	7	6	1
tunisia	29	19	10	10	0
moldova	29	7	22	1	21
malawi	26	24	2	2	0
angola	26	14	12	6	6
malta	25	17	8	7	1
bhutan	25	2	23	23	0
liberia	24	23	1	0	1
mongolia	21	18	3	3	0
eritrea	21	15	6	6	0
kuwait	21	13	8	4	4
ecuador	19	19	0	0	0
guyana	19	3	16	16	0
lithuania	18	18	0	0	0
lesotho	17	17	0	0	0
bahrain	17	11	6	0	6
cameroon	17	7	10	2	8
czech republic	16	14	2	2	0
namibia	15	13	2	2	0
senegal	15	7	8	8	0
madagascar	14	14	0	0	0
mauritania	14	14	0	0	0
slovenia	14	13	1	0	1
armenia	14	12	2	2	0
laos	14	12	2	2	0
nicaragua	14	12	2	1	1
estonia	14	10	4	4	0
belarus	13	2	11	11	0
bolivia	10	9	1	0	1
guatemala	10	6	4	3	1
costa rica	8	7	1	0	1

honduras	8	7	1	1	0
arab emirates	7	7	0	0	0
burkina faso	7	5	2	0	2
mali	6	6	0	0	0
trinidad tobago	6	6	0	0	0
uruguay	6	3	3	3	0
albania	6	2	4	3	1
botswana	5	5	0	0	0
oman	5	2	3	2	1
azerbaijan	4	3	1	1	0
cape verde	3	3	0	0	0
montenegro	3	3	0	0	0
el salvador	3	2	1	1	0
benin	2	2	0	0	0
central african republic	2	2	0	0	0
kyrgyzstan	2	2	0	0	0
timor-leste	2	2	0	0	0
papua new guinea	2	1	1	1	0
swaziland	2	1	1	1	0
brunei	1	1	0	0	0
côte d'ivoire	1	1	0	0	0
guinea-bissau	1	1	0	0	0
suriname	1	1	0	0	0
tajikistan	1	1	0	0	0
togo	1	1	0	0	0
belize	1	0	1	1	0
paraguay	1	0	1	0	1
turkmenistan	1	0	1	1	0

*GAs have been ordered, in decreasing order, by **N.Total**, and, for equal **N.Total** by **N. occurrences***

As we can see from Table 3.2, the ten countries most mentioned -in Tweets about uncertainty- during the year 2016 are the United Kingdom, the United States, China, India, Spain, Italy, Canada, France, Turkey and Colombia. We remark that for each of these countries **N. occurrences** and **N. co-occurrences** represent rather different shares of **N.Total**. For example, both in absolute terms and relative to the value of **N.Total**, the United States have much more co-occurrences with other countries compared to GB/UK. This evidence supports the hypothesis that the the election of Trump as President of the United States, is considered -by English speaking Twitter users- a relatively more global/international uncertainty phenomenon with respect to the uncertainty implications of the EU-referendum results, whose expected effects appear to be concentrated within the European Union. This table also allows us to appraise that countries like Columbia *(FARC referendum held on October the 2nd)*, Italy *(constitutional referendum held on December the 4th)*, Spain *(general elections held on June the 26th)*, Turkey *(tentative of coup the 15th of July)*, Brazil *(impeachment of Dilma Rousseff)*, Australia *(general elections held on July the 2nd)*, Pakistan *(Panama papers leak*) and Syria *(civil war)*, for which **N. cooccurrences** is relatively low compared to **N. occurrences**, have been mainly subject to local (internal) phenomena/events causing uncertainty. Whereas countries like Germany, Russia, Cuba, North Korea and Israel, for which **N. cooccurrences** is relatively high compared to **N. occurrences**, appear to have been particularly subject to international uncertainty, or, uncertainty contagion from other countries during the period under investigation. Another important stylized fact that we can derive from this table, is that there are huge differences in terms of volume of Twitter activity in English concerning uncertainty among countries. At least some of these differences in Twitter activity by country could not be due to factual dissimilarities in the level of *civil society uncertainty* experienced in these countries, but to demographic factors *(size of countries)*, language factors *(share of the population able to write in English)*, as well as technological *(share of the population which has access to Internet)*, legal *(for example Twitter is censored in China)* and cultural factors *(like the habit of writing online about local and international facts)*, which may influence the level of Twitter activity about uncertainty for a given country.

The main reason for which, in Table 3.2, we distinguished co-occurrences in two subcategories of observations (**2 cliques** and **3+ cliques**), is that, we would like each observation in $T_{country}$ to have equal influence, in terms of overall contribution to the weights of **S-WUN**. Therefore, we cannot simply build our static network from the **F** matrix, using the counts of cooccurrences -by pairs of countries- to determine the edges' weights and counts of occurrences -by single country- to determine the nodes' weights, because by doing so, observations that mention a great number of countries (>2) would have a greater overall influence on the weights of our (**S-WUN**) network with respect to observations in which a single country or a pair of countries are mentioned.

Therefore, to obtain the desired final result: equal overall contribution of each observation; we must weight our observations' counts by edge/node. In such a way that each observation, whatever is the number of countries it refers to, may contribute to **S-WUN** for a mass (sum of effects on weights) equal to one (1). Our solution works as follows:

- **Occurrence observations**, i.e. those in which a single country is mentioned (one or several times), contribute to our network by adding one (1) to the mass of the country node to which the observation refers to.

For example, if the first observation in $T_{country}$ had the following textual content: *"Great political uncertainty in the United Kingdom after the resignation of PM Cameron...Growth forecasts for the UK could be reviewed!"* the first row in **F** would have a single non null element that corresponds to the to the UK/GB area, all other being zero. Since the dictionary of GB/US is the 53st element of the *DICT* list, the non null element would be $f_{1,53}$:

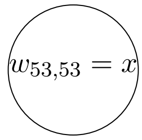
$$f_{1,j} = 0 \; for \; j \in \{1, ..., 163\} - \{53\}$$
$$and \; f_{1,j} = 1 for j \in \{53\}$$

Such an observation would affect the weight of the corresponding country node (GB/UK) by increasing its value by one (1). Call $w_{53,53}$ the weight of the GB/UK node in our static network, which is mathematically represented by the weighted matrix **W** of size $163 * 163$. The effect of the aforementioned observation, whose textual content is called 1^{st}_{Ob}, in terms of variation of **W**'s elements would be the following:

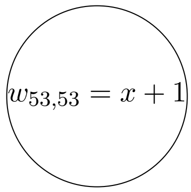
$$1_{Ob} \implies f_{1,53} = 1 \; and \; f_{1,j} = 0 for \; j \in \{1, ..., 163\} - \{53\} \implies \Delta w_{53,53} = (+)1$$

Which can be summarized in terms of network visualization of GB/UK node in **S-WUN** before and after discounting the effects of the observation 1^{st}_{Ob} as follows:

GB/UK node of **S-WUN** before considering 1^{st}_{Ob}:



GB/UK node of **S-WUN** after considering 1^{st}_{Ob}:



- **2 clique cooccurrence observations**, i.e. those in which two countries are mentioned (one or several times), contribute to our network by adding one (1) to the weight of the country edge that connects the two mentioned countries.

  For example, if the second observation in $T_{country}$ had the following textual content: *"Uncertainty reigns supreme in diplomatic relations between Italy and Egypt after the assassination of Giulio Regeni in Cairo"* the second row in **F** would have two non null elements, the ones that correspond to Egypt and Italy areas, all other being zero. Since the dictionary of Egypt and Italy are respectively the 44th and 74th elements of *DICT*, we would have:

$$f_{2,j} = 0\ for\ j \in \{1, ..., 163\} - \{44, 74\}$$

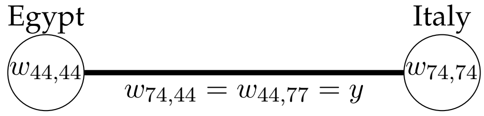
$$and\ f_{2,j} = 1 for j \in \{44, 74\}$$

  Such observation would affect the weight of the corresponding country edge (GB/UK) by increasing its value by one (1). Call $w_{44,74}$ the weight of the edge between Italy and Egypt in our static network. The effect of the aforementioned observation, whose textual content is called 2^{nd}_{Ob}, in terms of variation of **W**'s elements would be the following:

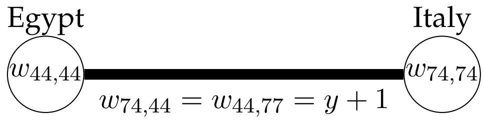
$$2_{Ob} \implies f_{2,44} = 1,\ f_{2,74} = 1\ and\ f_{2,j} = 0\ for\ j \in \{1, ..., 163\} - \{44, 74\}$$
$$\implies \Delta w_{44,77} = \Delta w_{77,44} = (+)1$$

  We have that both $w_{74,44}$ and $w_{44,77}$ change identically because **S-WUN** is an undirected network, therefore matrix **W** is symmetric. We can summarize the effect of 2^{nd}_{Ob} in terms of network visualization of the Egypt-Italy edge in **S-WUN** before and after discounting the effects of this observation:

Egypt-Italy edge of **S-WUN** before considering 2^{nd}_{Ob}:



Egypt-Italy edge of **S-WUN** after considering 2^{nd}_{Ob}:



- **3+ clique co-occurrence observations**, i.e. those in which more than two countries are mentioned (one or several times), contribute to the weight of each edge between pairs of countries mentioned in the observation,i.e. all possible 2-combinations of the observation's countries. It is like if each observation represented a n-clique, where n is the number of countries mentioned in that observation. Since the number of edges in a n-clique can be obtained by using the triangular number function (tri), we can obtain the number of edges in a n-clique observation as follows:

$$tri\,(n) = \frac{n * (n-1)}{2}$$

Hence the weight contribution of the observation to edges of the network, which must sum to one, will be split in equal parts among all the edges that constitute the n-clique, representing the n countries mentioned in the observation. Each edge weight contribution being equal to the inverse of the triangular number function previously defined:

$$tri^{-1}\,(n) = \frac{2}{n * (n-1)}$$

For example, if the third observation in $T_{country}$ had the following textual content: *"Geo-political uncertainty grows as Russia, Turkey, Syria and the United States do not find any agreed solution for the establishment of a Kurdish State"* the second row in **F** would have four non null elements, the ones that correspond to Russia, Turkey, Syria and the United States, all other being zero. Since the dictionary of these four areas are respectively the 126th,152th, 144th and 156th elements of *DICT*, we would have:

$$f_{3,j} = 0 \; for \; j \in \{1, ..., 163\} - \{126, 144, 152, 156\}$$

$$and \; f_{3,j} = 1 for j \in \{126, 144, 152, 156\}$$

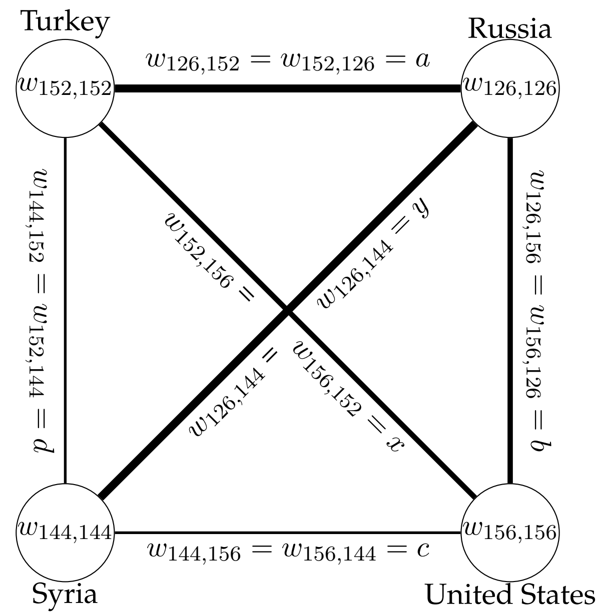
Such observation would affect the weight of the corresponding 4-clique's edges, by increasing each edges' weight by one sixth (1/6). Call $w_{126,144}$, $w_{126,152}$, $w_{126,156}$, $w_{144,152}$,$w_{144,156}$ and $w_{152,156}$ the weights of the edges in our static network belonging to this 4-clique. The effect of the aforementioned observation, whose textual content is called 3^{rd}_{Ob}, in terms of variation of $\mathbf{W}$'s elements would be the following:

$$3_{Ob} \implies f_{3,126} = 1, \; f_{3,144} = 1, \; f_{3,152} = 1, \; f_{3,156} = 1$$
$$and \; f_{3,j} = 0 \; for \; j \in \{1, ..., 163\} - \{126, 144, 152, 156\}$$
$$\implies \Delta w_{126,144} = \Delta w_{126,152} = \Delta w_{126,156} = \Delta w_{144,152} = \Delta w_{144,156} = \Delta w_{152,156}$$

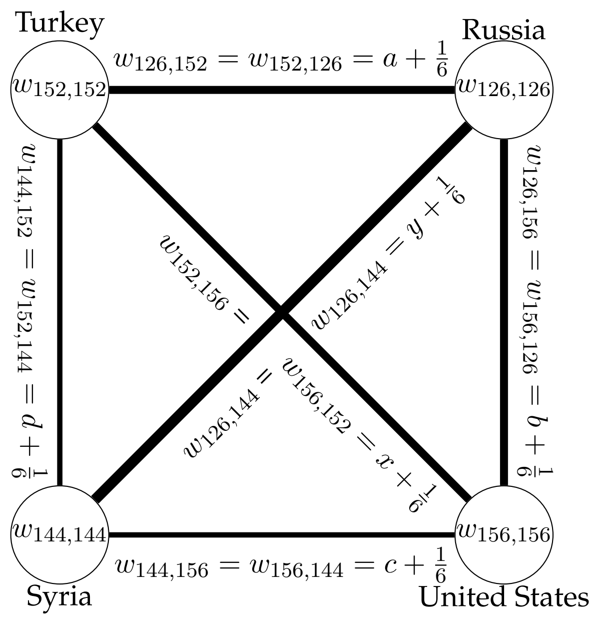
$$= \Delta w_{144,126} = \Delta w_{152,126} = \Delta w_{156,126} = \Delta w_{152,144} = \Delta w_{156,144} = \Delta w_{156,152} = (+)\frac{1}{6}$$

We can summarize the effect of 3^{rd}_{Ob}, in terms of network visualization of the corresponding 4-clique in **S-WUN** before and after discounting the effects of this observation, as follows:

corresponding 4-clique of **S-WUN** before considering 3^{rd}_{Ob}:



corresponding 4-clique of **S-WUN** after considering 3^{rd}_{Ob}:



We can compute the **W** matrix that represents **S-WUN**, including the weighing of cooccurrence observations as above specified, through the following procedure:

- We compute the row sums of each row of matrix **F**:

$$s_i = \sum_{j=1}^{163} f_{i,j} \; for \; i \in \{1, ..., 170806\}$$

and obtain the vector $\vec{S}$, which contains the 170806 s_i's. We transform the elements of vector $\vec{S}$ as follows:

$$for \; i \in \{1, ..., 170806\}$$

$$st_i = \begin{cases} 1 \; if \; s_i = 1 \\ \sqrt{tri^{-1}(s_i)} = \sqrt{\frac{2}{s_i*(s_i-1)}} \; if \; s_i > 1 \end{cases}$$

The vector which contains all the st_i's is called $\overrightarrow{ST}$.

- We then multiply each column of the **F** matrix (element-wise) by the vector $\overrightarrow{ST}$:

$$\mathbf{F} \odot \overrightarrow{ST} = (f_{i,j} \cdot st_i) = \begin{pmatrix} f_{1,1} \cdot st_1 & \cdots & f_{1,163} \cdot st_1 \\ \vdots & \ddots & \vdots \\ f_{170806,1} \cdot st_{170806} & \cdots & f_{170806,163} \cdot st_{170806} \end{pmatrix}$$

$$= \begin{pmatrix} f'_{1,1} & \cdots & f'_{1,163} \\ \vdots & \ddots & \vdots \\ f'_{170806,1} & \cdots & f'_{170806,163} \end{pmatrix} = \mathbf{F}'$$

- To obtain **W** we must multiply the transpose of the $\mathbf{F}'$ matrix by $\mathbf{F}'$ itself, and then, subtract from the elements in the diagonal of the resulting $(163 * 163)$ matrix the sum by row of out-of diagonal values of the same matrix ($\mathbf{F}'^T * \mathbf{F}'$):

$$\mathbf{W} = \begin{pmatrix} w_{1,1} & \cdots & w_{163,1} \\ \vdots & \ddots & \vdots \\ w_{1,163} & \cdots & w_{163,163} \end{pmatrix}$$

$$where \; w_{i,j} = \begin{cases} \sum_{k=1}^{170806} f'_{k,i} f'_{k,j} - \sum_{k=1|k \in T_{country,CO}}^{170806} f'_{k,i} f'_{k,j} \; if \; i = j \\ \sum_{k=1}^{170806} f'_{k,j} f'_{k,i} \; if \; i \neq j \end{cases}$$

By so doing we subtract from each element in the diagonal, which corresponds to a node of **S-WUN**, the weights of the edges that link to that node that are due to co-occurrence observations and therefore should not influence our node's weights.

$$and \; since \; for \; any \; i \in \{1, .., 163\} : \sum_{k=1}^{170806} f'_{k,i} f'_{k,i} - \sum_{k=1|k \in T_{country,CO}}^{170806} f'_{k,i} f'_{k,i}$$

$$= \sum_{k=1|k \in T_{country,OC}}^{170806} f'_{k,i} f'_{k,i}$$

$$we\ can\ rewrite\ \mathbf{W}\ as\ follows:$$

$$\mathbf{W} = \begin{pmatrix} \sum_{k=1|k \in T_{country,OC}}^{170806} f'_{k,1} f'_{k,1} & \cdots & \sum_{k=1}^{170806} f'_{k,163} f'_{k,1} \\ \vdots & \ddots & \vdots \\ \sum_{k=1}^{170806} f'_{k,163} f'_{k,1} & \cdots & \sum_{k=1|k \in T_{country,OC}}^{170806} f'_{k,163} f'_{k,163} \end{pmatrix}$$

The resulting **S-WUN** undirected network, represented by the symmetric matrix $\mathbf{W}$, is our first - and less sophisticated - tool to identify local, and transnational/international uncertainty events that occurred in the period under investigation: the 2nd, 3rd and 4th quarters of the year 2016. The main limit of this instrument are its static nature and the fact that weights of nodes and edges for countries which are more populated, or where a larger share of the population speaks in English and/or uses Twitter, will likely be overestimated compared to those areas in which the population size is smaller, or, where the share of English speaking Twitter users is smaller. Differently from standard co-occurrence matrices used in text mining applications, by being based country token dictionaries, the **S-WUN** network isn't a simple count of the number of occurrences and co-occurrences of a list of words in our set of observations (tweets) about uncertainty. But, it is a weighted matrix of the occurrences and co-occurrences of -more than one hundred and sixty- features, the countries contained in $DICT$, each of which is identified trough a different dictionary used to identify signals related to a country in Twitter observations. Another major difference compared to "classical" cooccurrence matrices is that the values in the diagonal, which represent the weights of the country nodes, have been pruned from the effects of those observations ($T_{country,CO}$) in which several geographic area cooccur. This point also distinguishes the methodology used build the TU indexes in the second article, from that presented in this article. As a result of the latter choice, through **S-WUN** nodes' weights we can appraise the relevance of internal/local uncertainty phenomena. Whereas, through edges' weights, we can appraise the relevance of international/transnational uncertainty phenomena, i.e. the percolation/contagion channels.

S-WUN has 163 nodes and 789 (non-null) undirected edges, i.e. the contagion channels. It is constituted by a big cluster (largest connected component) with 137 nodes, two (2) small clusters of size two (2) and 22 isolated nodes. The density of **S-WUN** is 0.05975915, while its transitivity is 0.3367068, this result shows clearly that **S-WUN** is very different from what would be a random network with the same density. As we can see at a first look from the representation of the whole static network (Fig. 3.2), the United States and the United Kingdom (GB/UK) were clearly the most central nodes of **S-WUN** during the year 2016. Both from the point of view of their connections and for the respective weights of their nodes, which represent local/internal uncertainty. Three distinct node centrality measures (betweenness, closeness and eigenvector) are available in section 3.0.4 of the Appendix. These centrality measures can be used to identify the nodes that played prominent roles in **S-WUN** during the year 2016. Here follow the list of the top10 countries (nodes) from the point of view of the (weighted) closeness centrality score:

(1) United States; (2) Cuba; (3) Morocco; (4) South Africa; (5) GB/UK; (6) Saudi Arabia; (7) Afghanistan; (8) North Korea; (9) Russian Federation (10) Turkey;

Many of the countries in this ranking, like Cuba, Saudi Arabia, Afghanistan and North Korea have strong uncertainty dependency relations with the United States. By being very close to the most central node of the network they rank high from the point of view of closeness centrality, which is a function of the shortest-path distances with respect to other non isolated nodes. These nodes represent potentially "hot" areas of the world, in terms of U.S. international affairs and conflicts, under the upcoming/new presidency. This finding supports our hypothesis that *civil society uncertainty* levels heavily depend on agents' expectations and their degree of divergence, and not simply on factual events.
The ranking of Morocco, in terms of closeness centrality, could at a first look seem surprising. However, in Morocco occured one of the most important global debates of the year 2016: the country hosted the *UN Climate Change Conference (COP22)* just after the the US presidential elections; as a result, the name of the country has been extensively and globally associated to uncertainty concerning climate change debates and expected worldwide policy changes under the Trump presidency.
From the point of view of the (weighted) betweenness centrality score, the top10 ranking countries are the following:

(1) United States; (2) GB/UK ; (3) India; (4) Russian Federation; (5) China; (6) Poland; (7) Turkey; (8) Germany; (9) Hungary; (10) Thailand;

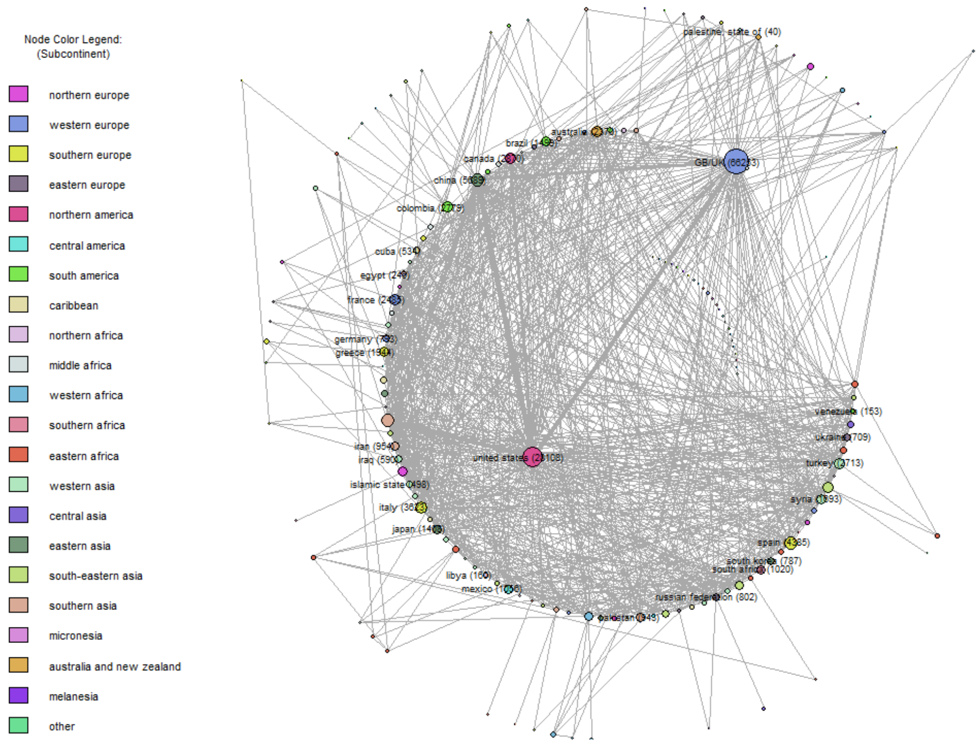
Most of the aforementioned top ranking countries -with the exception of Poland, Hungary and Thailand- represent worldwide *civil society uncertainty* hubs, they are seen as the key (political and economic) global/regional players in relation to those major events that raised globally *civil society uncertainty* in 2016. Hungary ranks high in relation to the Euoropean migration crisis and the Hungarian referendum on the European Union's migrant relocation plans, which was held on October the 2^{nd}. The rank of Poland is due to uncertainties in relation to the future geopolitical role and foreign policy strategy of Poland; being Poland a member of the NATO and given the rising military tensions with Russia around the Baltic Sea area in 2016. Whereas, the rank of Thailand is linked to rising geopolitical tensions in South-East Asia, in particular with China, after the proactive strengthening of diplomatic and commercial relations between Japan and Thailand.
Finally, the ten countries with the highest (weighted) eigenvector centrality score are the following:

(1) United States; (2) China; (3) GB/UK; (4) India; (5) Canada; (6) Japan; (7) Indonesia; (8) Cuba; (9) Philippines; (10) Ireland;

As we could expect, there is a clear overlapping between eigencentrality top ranking countries and countries in the aforementioned (top-ranking) lists. Canada appears in the aforesaid rank in relation to *civil society uncertainty* emerging in North America after the threats to the **North American Free Trade Agreement** (**NAFTA**) made by the new US presidency; but also, in relation to delays in the implementation of the **Comprehensive Economic and Trade Agreement** (**CETA**). Ireland ranks so high in relation to possible problems and difficulties in the Northern Ireland-Republic of Ireland commercial relations, after UK's EU-referendum vote in favour of Brexit. Whereas the presence of China, together with the Philippines, Indonesia and Japan is mostly due to territorial disputes in the South China Sea.

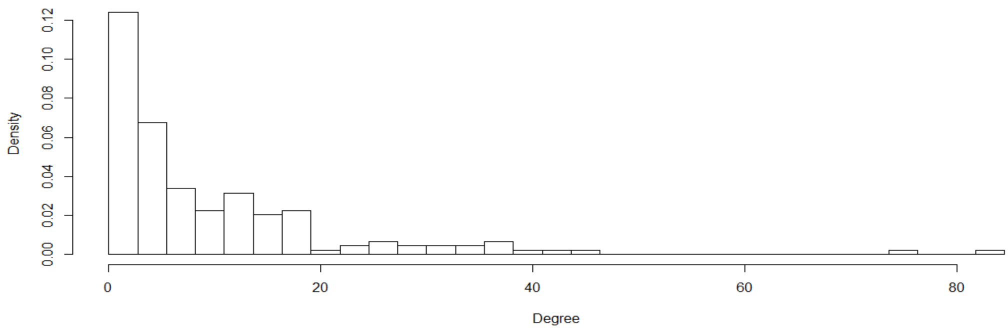
Given the observed characteristics of the **S-WUN** network, the EU-referendum and US-elections seem to have played a major worldwide disturbance effect on expectations of civil society members, all over the globe. We can safely claim that these two extreme uncertainty events propagated their *civil society uncertainty* effects at a worldwide scale: a very large group of countries, represented by the inner densely connected subnetwork in Fig.3.2, are attached to both the US and UK. This group of nodes constitutes the most highly connected component of **S-WUN**. Edge betweenness centrality measures, available in section 3.0.3 of the Appendix, show that among the ten most important edges of **S-WUN**, in terms of *civil society uncertainty* international percolation role, we find the following contagion channels: GB/UK - United States, Russian Federation - United States, India - United States, China - United States, Russian Federation -Turkey, and, India - Pakistan. The countries belonging to the inner circle of Fig.3.2, appear to be highly interdependent and therefore potentially more vulnerable, with respect to others, to the international/transnational propagation of uncertainty shocks. The edges among these countries are the most relevant channels of contagion, through which major uncertainty shocks were propagated globally. Their edges' weights and betweenness centrality scores, appear to be especially large when compared to those between countries at the periphery of **S-WUN**, which are very poorly and loosely connected to the rest of the world compared to this (inner circle) group. Finally, at the center of the network, as represented in Fig.3.2, there is a group of countries, arranged like an half-moon, which (with one exception) appear to be totally disconnected from the rest of the world from the point of view of uncertainty phenomena dependencies, these are the countries listed at the end of Table 3.2, like Tajikistan, Togo, Suriname, Brunei and many other "isolated" areas, with respect to international *civil society uncertainty* shocks identified through **S-WUN**.

Figure 3.2. Static Worldwide Uncertainty Network (S-WUN)



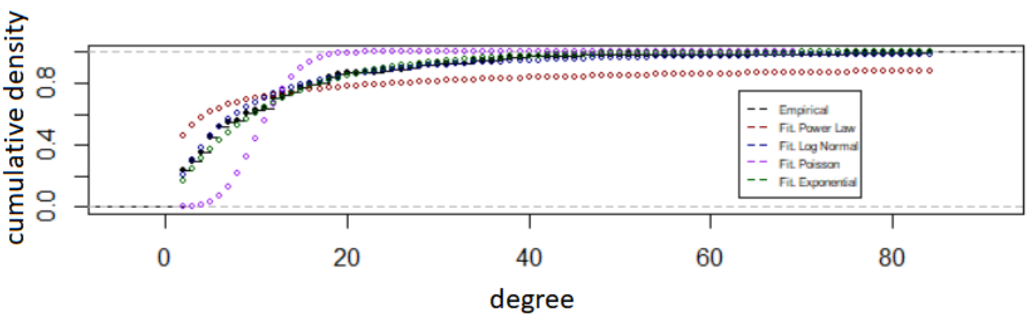
Here follows the empirical distribution of the degree of the nodes of our static network (Fig. 3.3).

Figure 3.3. Histogram of the empirical distribution of S-WUN nodes' degree



As we can see from Fig. 3.3, excluding isolated nodes (those with a degree=0), the empirical distribution of the degree of **S-WUN**'s nodes appears to follow either a power law or an exponential distribution: **S-WUN** has a few very large hub nodes and a great share of nodes with very few connections. We remind that, if the empirical distribution of the degree of nodes follows a power law, **S-WUN** is scale free. In Fig. 3.4 we fit, through MLE estimation, four distribution families: power-law, discretized log-normal, poisson and discretized exponential; to see which distribution among the four better approximates our empirical distribution, we exclude nodes with a degree equal to zero (0). The estimated parameter of the power-law approximation is $\alpha = 1.422871$, which means that the first and second moments of the fitted power law distribution are not finite. However, according to the Akaike information criterion, the fitted discretized exponential distribution (dots in green in Fig. 3.4) with parameter $\Lambda = 0.09360418$ approximates better than all other families our empirical distribution.

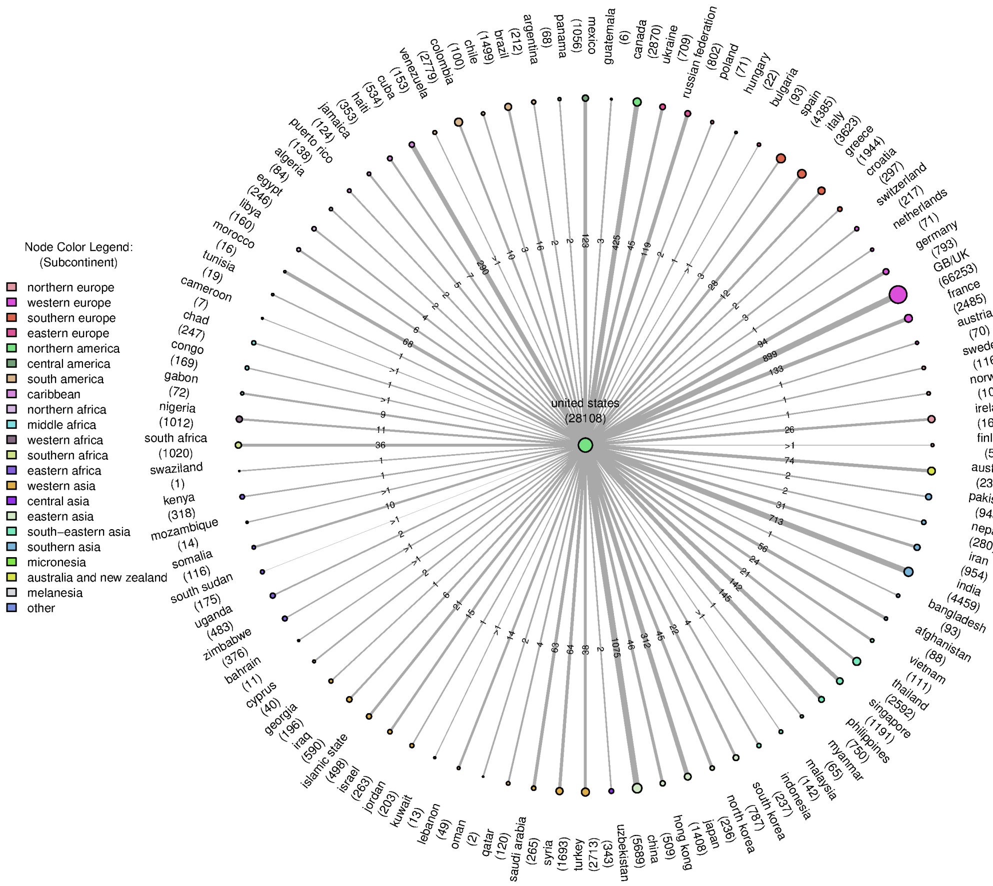
Figure 3.4. Empirical and fitted CDFs of S-WUN's nodes degree



Among nodes with the highest degree *(degree value in brackets)* in **S-WUN**, we find again the United States *(84)*, followed by GB/UK *(75)*, China *(46)*, Russia *(41)*, Turkey *(40)*, India *(38)*, Germany *(37)*, France *(37)*, Italy *(34)*, Canada *(34)*, Japan *(31)*, Brazil *(30)*, Syria *(28)* and Greece *(28)*. These countries can be considered the hubs of uncertainty contagion during the year 2016. They bind together almost the 90% of the remainder countries (with lower node degrees), constituting the skeleton of a worldwide integrated uncertainty system, through which international/transnational uncertainty events propagate.

Here follow two network visualizations, showing the neighbourhood of the United States (Fig. 3.5) and United Kingdom (Fig. 3.6) nodes in **S-WUN**.

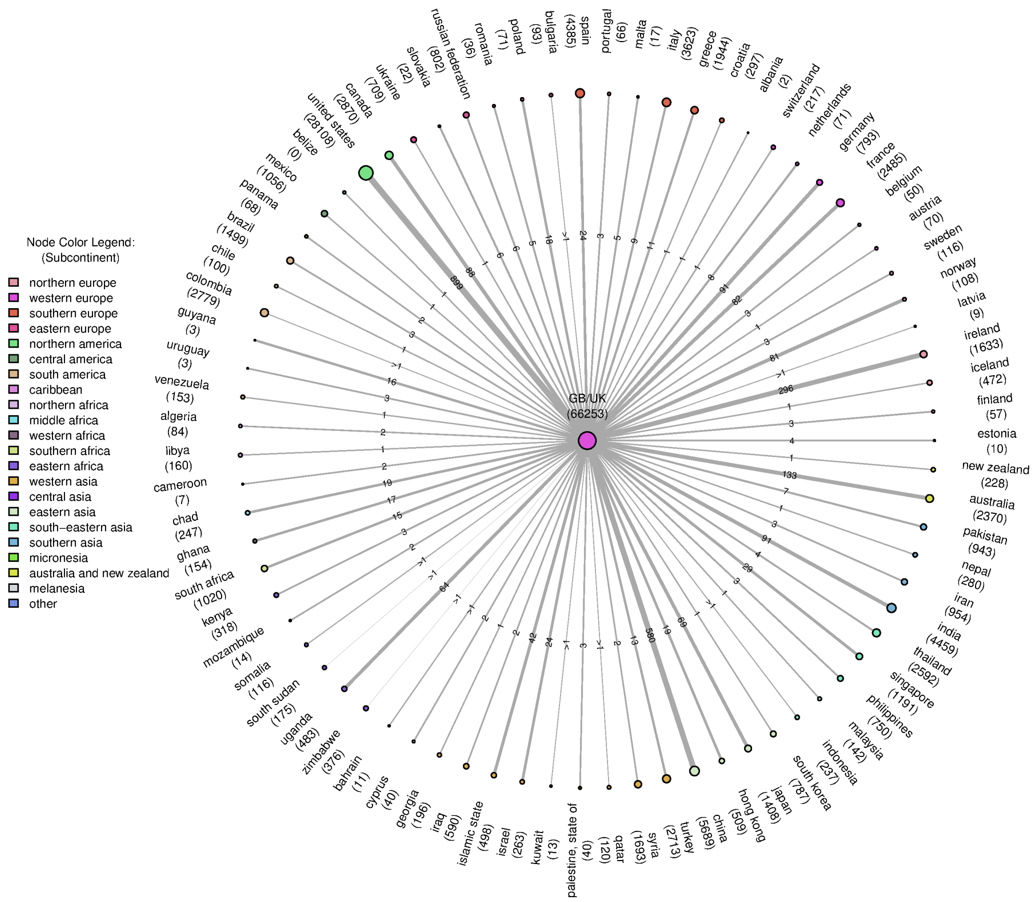
Figure 3.5. S-WUN neighborhood of the United States' node



Differently from the visualization of the whole **S-WUN** network, which was too large to allow us to plot the values of edges' weights, in the latter visualizations, by neighbourhood of a node, we can appraise the weights of edges and nodes in the neighbourhood of the United States and the United Kingdom.

As we can see from Fig. 3.5 the heaviest edges attached to the United States node are those that go towards the United Kingdom, China, India, Canada and Cuba. More reduced in terms of weight but still relevant are the edges that connect the United States to Russia, Japan, the Philippines and Singapore. Explanations of the possible causes of these heavy edge weights have been already stated. Many of the other relations are easily interpretable, for example the connection with Russia is certainly due to the conflict in Syria, in which both the US and Russia intervened, and, to the FBI inquiry on the intervention of Russia's intelligence and funding during the US elections in which Trump prevailed. Other uncertainty contagion channels, like that with Japan are linked to the expected/possible use of US bases in the Japanese archipelago under the Trump presidency, together with the (apparent) unpredictability of Trump's foreign policy and ensuing risks for the geopolitical stability of the North East Asia region.

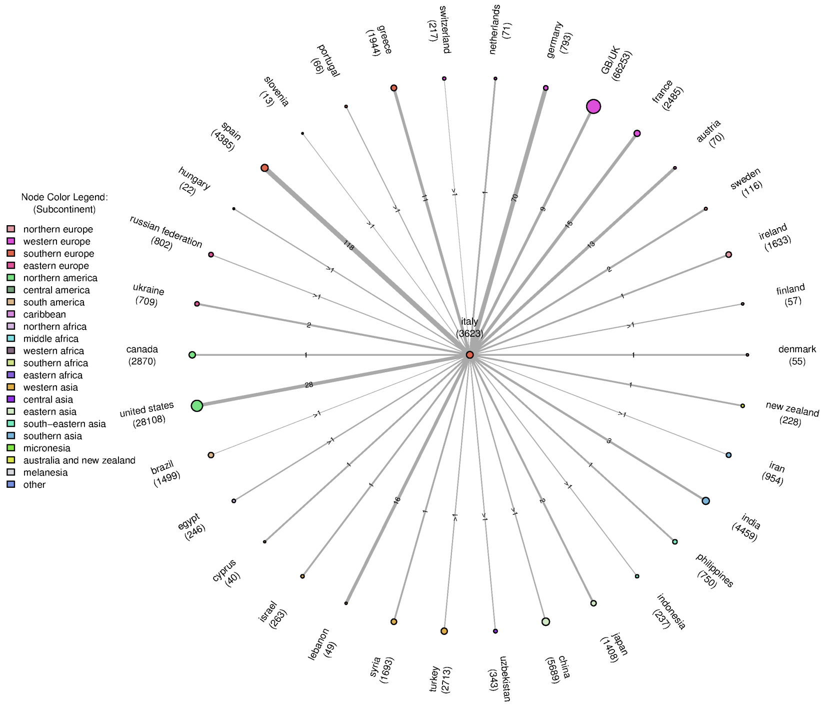
Figure 3.6. S-WUN neighborhood of GB/UK's node



Some of the most relevant edges for the United States are also among the most important ones for the United Kingdom. For example, as we can see from Fig 3.6 besides the US itself, China and -to a lesser extent also- India share an important contagion channel with the UK. Most of the uncertainty contagion channels of UK point to different countries with respect to US' edges. Among the most relevant edges for the UK we find those towards Ireland, Australia, France, Germany, Norway and Uganda. The UK shares also some uncertainty contagion channels with countries in southern Europe, like Italy, Spain and Greece, however the weights of these uncertainty contagion edges are smaller compared to those that are shared with northern European countries. Many countries in central Africa are also associated to UK, and to the effect of Brexit in states like Uganda, Chad, South Africa, Ghana and other countries in Africa. In these areas Brexit is seen both as a threat and an opportunity for the development of new trade agreements between the UK and African countries.

To give to the reader the possibility of exploring the various countries in **S-WUN** that couldn't be directly inserted in this document for reasons of space constraints, we have included here below a video stream from the Youtube channel of the author. The video 3.7 contains the visualizations of the neighbourhoods of all countries in **S-WUN** that have at lest one neighbour.

Figure 3.7. [Video] **S-WUN**, Neighborhood by Country (in alphabetic order)



CLICK ON THE IMAGE ABOVE TO START THE VIDEO
(You must be connected to the Internet)
If your PDF reader doesn't support flash video direct streaming, the video can be directly viewed at the following (Youtube) URL-link:
youtube.com/embed/AOLoPTpfEY0

By looking in detail to the neighbourhoods of countries we can appreciate the capacity of **S-WUN** of representing -rather correctly according to our prior knowledge- the occurrence and interdependencies of uncertainty phenomena in the real world.

3.3 Dynamic Worldwide Uncertainty Network (D-WUN)

The **Dynamic Worldwide Uncertainty Network** (**D-WUN**) is obtained by using the same methodology used to construct **S-WUN**, but instead of inferring a unique network for the entire period covered by the study, with a single matrix $\mathbf{W}$, as we did in the previous section, we will build a matrix and hence a network for each day of the time interval of this study, from April the 5th to December the 31st 2016. By doing so we obtain a tensor that contains (side by side) the various $\mathbf{W}_d$ where the d stands for the temporal dimension of the tensor, our first observed day ($d = 1$) being April the 5th 2016;

The analysis of this tensor will allow us to identify and appraise the dynamics of local and international events that cause uncertainty. As **S-WUN**, also this tool is potentially vulnerable to the overestimation bias for those countries whose population contains a larger share of English speakers, Twitter users, or for countries which are simply more populated compared to others. In addition, since twitter activity is on average less intense on weekends and holidays, the nodes and edges of this dynamic network will be oversized during working week days compared to weekend. The transformations that we will introduce later on, in the last three sections of this article will mitigate the effects of these potential sources of distortion.

To build **D-WUN** we start by subsetting the $TU_{country}$ dataset by time interval (day) in which observation have been published, fixing at midnight (00:00:00) UTC time the day time transition. The vector containing the meta-data the day of publication of our tweets is called $\overrightarrow{DAY}$, whose element day_i corresponds to the day in which observation i has been published online through Twitter. We use the vector $\overrightarrow{DAY}$ to subset $T_{country}$ for the different days observed in our dataset $T_{country}$. Daily subsets are called $T^d_{country}$ and defined as follows:

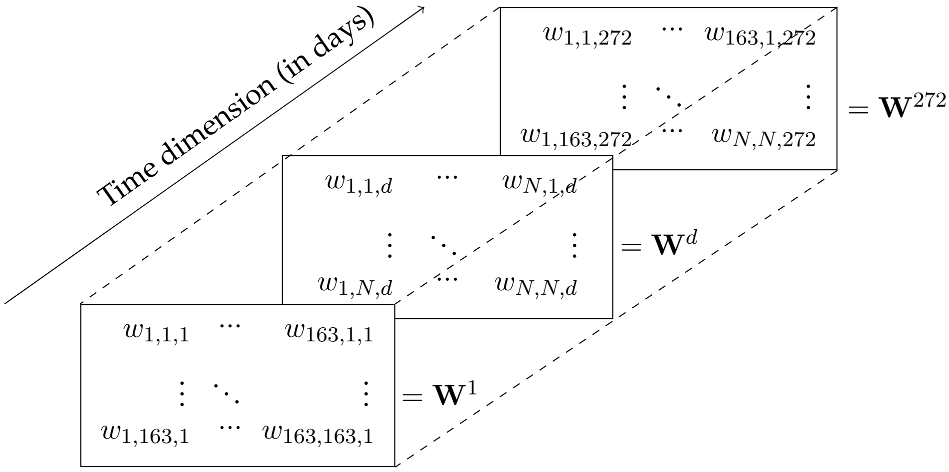
$$T^d_{country} = \{i \in T_{country} | day_i = d\}$$

The remainder of the process to build is identical to the one presented in the previous section for building **S-WUN** the only difference is that we are now using $T^d_{country}$ instead of $T_{country}$ and that we repeat the process for all $d \in \{1, ..., 272\}$. With each $T^d_{country}$ we obtain the corresponding matrix $\mathbf{W}^d$ of the dynamic network in day d. By repeating the process for all $d \in \{1, ..., 272\}$ we obtain a set of 272 $\mathbf{W}^d$ matrices that we join together in a three dimensional tensor $\overline{\mathbf{W}}$:

$$For\ all\ d \in \{1, ..., 272\} we\ have:$$

$$w_{i,j,d} = \begin{cases} \sum_{k=1|k \in T^d_{country,OC}}^{170806} f'_{k,i} * f'_{k,j}\ if\ i = j \\ \sum_{k=1|k \in T^d_{country}}^{170806} f'_{k,j} * f'_{k,i}\ if\ i \neq j \end{cases}$$

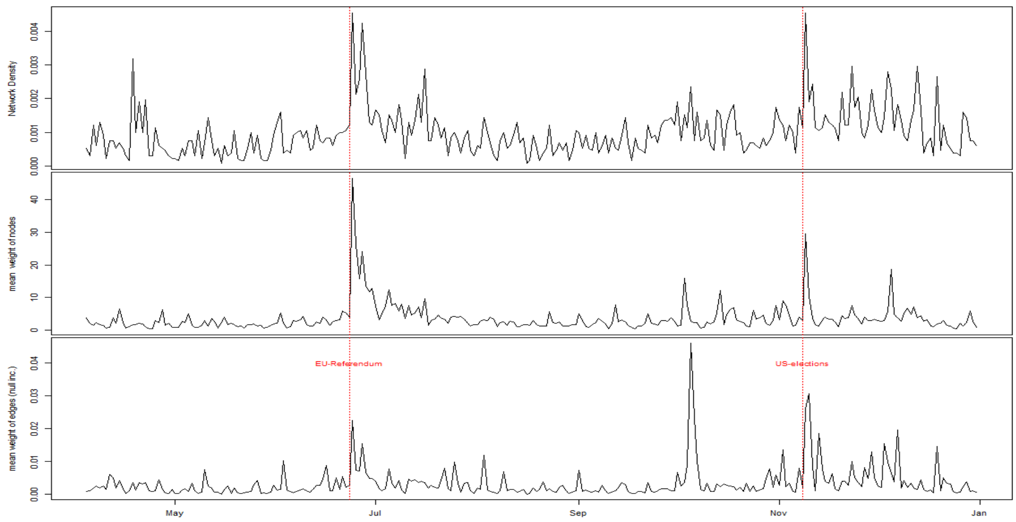
Figure 3.8. Dynamic Uncertainty Network Tensor $\overline{\mathbf{W}}$



Here follow three graphs describing some key aggregate features of the dynamic network given by tensor $\overline{\mathbf{W}}$:

1. **The network density by day -** the mean number of non-null edges per node of the network, it can be used as a proxy of the degree of connectivity/integration of the network, and hence of the likelihood of uncertainty contagions (all other parameters being equal);
2. **The mean value of nodes' weights by day-** can be used to evaluate the overall magnitude of local/internal uncertainty phenomena in the country network by day;
3. **The mean value of edges' weights by day-** including missing edges (those with zero/null weight), can be used to proxy the overall magnitude of international/-transnational uncertainty phenomena in **D-WUN** by day;

Figure 3.9. Dynamic Worldwide Uncertainty Network statistics across time



As we can see from Fig. 3.9, both the structure (density) and the weights of our dynamic network experienced major changes during and just after the EU-referendum and the US elections. The dates of these events are represented in the graphs by the two vertical red lines. Even though the two aforementioned events were accompanied by similar peaking patterns of the density of the network, their relative effects on the mean of the weights of edges and nodes were rather different. The EU referendum had a greater effect on the mean of nodes' weights that touched record high values the day after the vote, whereas the mean of edges' weights increased, but less compared to the nodes. Given that the density of the network also increased considerably after the EU-referendum it means that more contagion channels (edges) appeared in the network but their mean weight must have changed little. Whereas, after the US elections both the weights of nodes and those of the edges experienced similar aggregate changes (in terms of relative size of the variation). The dissipation of local uncertainty in the network (represented by the mean of the weights of nodes) appears to have been more rapid after the US elections than after the EU-referendum. However, for what concerns international/transnational uncertainty (represented by the mean of the weights of edges) the situation is the opposite. The international uncertainty effects of the election of Trump on **D-WUN** appear to last longer compared to those of the EU referendum. The period after the US elections also appears to be characterized by more variance of uncertainty in the network, from the point of view of both network density and the mean weight of the edges, with respect to the two months that followed the EU-referendum.

Therefore we can conclude that, even though the two major events of the year 2016 appear to be similar from the point of view of the maximum density reached by our uncertainty network during the two events, they are nevertheless extremely different for what concerns changes in the mean of the weights of nodes and edges, which represent respectively internal uncertainty and international uncertainty across the network. They are also different from the point of view of the uncertainty dissipation processes, through which, after the two events, the mean of the weights of edges and nodes, of the uncertainty network, decreased towards to their long run stationary levels. To tell it shortly, Brexit had a stronger and more persistent impact on the node's weights of our dynamic uncertainty network, whereas the US elections determined an almost specular effect on the edges' weights. In both cases the aforementioned variables continued oscillating, for more than one month, in a range of values far above their long term averages. The observed uncertainty phenomena around the US elections probably represents the impact, in terms of international/-transnational uncertainty, not only of the US elections' results, but also of US foreign policy making under the Trump presidency. Similarly that of the EU referendum also represents uncertainty linked to upcoming Brexit negotiations between the the UK and the EU.

3.4 Uncertainty Signals' decomposition: Redundancy and Intensity

In this final section, we wish to mitigate as much as possible those representativeness biases contained in $\overline{\mathbf{W}}$, which are due to the fact that we are working only with tweets in English. To do so we rescale the weights of all the nodes and edges of our dynamic network, using three distinct transformations:

1. The redundancy of uncertainty signals, which represents the variety of the sources that posted twitter uncertainty signals concerning a specific edge/node observed in a specific day;
2. The intensity of uncertainty in a node/edge in a given day with respect to uncertainty in that node/edge in past days;
3. The intensity uncertainty in a node/edge in a given day with respect to uncertainty in that same day in other nodes/edges;

Each transformation is applied separately to each element $w_{i,j,d}$ of $\overline{\mathbf{W}}$. For a given element $w_{i,j,d}$ of $\overline{\mathbf{W}}$, our transformations require us to use -only- the weights of other nodes/edges of $\overline{\mathbf{W}}$ of the same day d, and the weights of same node/edge (i, j) of $\overline{\mathbf{W}}$ in prior days $(t < d)$. We choose to use three transformations that share this property of using exclusively past and present values of the network and being applied to single $w_{i,j,d}$, to have the possibility of updating the contents of our tensor in real time, both by adding new daily matrices $\mathbf{W}^t$ and hence lengthening our time dimension, and also, to have the possibility of adding additional countries, lengthening our observed features dimension (adding columns to the $\mathbf{F}$ matrix), without having to rescale again all -past- values of our tensor $\overline{\mathbf{W}}$ at each update. In addition to facilitate the interpretation of network's weights and the merging of the the three uncertainty tensors that we obtain through the three transformations described in the next subsections, we want all our transformations to have as codomain subsets of $[0, 1]$. So that when we will merge them together, by element-wise multiplying the elements of these tensors, each Tensor will have equal influence on the final weight of the **Rescaled Dynamic Uncertainty Network** (**RD-WUN**). Final weights will be equivalent to the volumes in the three dimensional space represented by the three tensors. Given the properties of volumes, the edges/nodes that will have -for a given day- the highest rescaled uncertainty weights (closer to one), will be the ones that will exhibit the higher "cubic-volume" weight, represented by the element-wise multiplication of the three tensors. It is sufficient that one of these three weights is zero to obtain a "cubic-volume" weight equal to zero for the corresponding element in **RD-WUN**.

3.4.1 Redundancy Transformation and Tensor

The first transformation, called **Redundancy**, is motivated by the need of taking into account the variety of the sources that, in a given day, contribute to the values of the weights of a given node/edge by publishing one or more observations -Tweets- concerning that specific node/edge. Since on Twitter, as in other social networks, many signals of uncertainty are not based on the direct observation by the Twitter user of the "state of uncertainty" in a particular part of the system, i.e. local uncertainty (nodes) and transnational/international uncertainty (edges) in the network, but on simple imitation/repetition of the most common/widespread signals of uncertainty of the day. We want to find a way (a simple transformation) to represent in the interval [0,1[the redundancy of edge/node specific sets of daily signals in our dynamic uncertainty network. This in order to counteract/mitigate the impact of the imitative behaviour, i.e. its effects, in the daily weights of the different nodes/edges in our tensor $\overline{\mathbf{W}}$. Considering the fact that, the more there are observations regarding a given country (node) or relation among countries (edge) in a given day, the more it is likely that a great number (or share) of these observations will result from a process of imitation of other signals, rather than direct observation of uncertainty. We can assume that the likelihood of imitating others increases exponentially as a function of the number of signals pertaining to the same node/edge. In addition, we assume that when a single (only one) individual indicates/signals a state of uncertainty in a given node/edge in a given day, it is equally likely (probability of 0.5) that his signal corresponds to factual reality, i.e. in that country (or group of countries there has been an uncertainty event or a change in expectation that justifies/explains the signal, as it is likely (probability of 0.5) that his uncertainty signal is a "false flag" and doesn't correspond to reality, i.e. no event or information that may justify/explain a state of uncertainty in the mentioned country/countries actually occurred; In addition we assume that the marginal contribution of an observation (in terms of redundancy) must always be positive, but must decrease and tend to zero as the number of observations concerning that node/edge increases;

To summarize what we have previously said, to identify the redundancy of the uncertainty of the node (if $i = j$) or arc (if $i \neq j$) at time t, which we call $red_{i,j,d}$, we look for a sigmoid transformation (function) of the number of distinct sources x that in a given day mention a given node/edge and call this redundancy transformation $Re(x)$. Since $Re(x)$ must respect the conditions which have been previously justified, we are searching for a $Re(x)$ that:

1. Has as domain $[0, +\infty[= \mathbb{R}^+$ and as codomain ($cod(Re)$) a subset of $[0, 1[$:

$$[0, +\infty[\xrightarrow{Re(x)} cod(Re) \subseteq [0, 1[$$

2. Is increasing and concave where defined and continuous: $\frac{dRe(x)}{dx} > 0$ where the first order derivative is defined, and $\frac{d^2Re(x)}{dx^2} < 0$ where the second order derivative is defined.

3. Must tend to 1 as x tends to $+\infty$:

$$\lim_{x \to \infty} Re(x) = 1$$

4. It must respect the following constraints $Re(1) = 0.5$ and $Re(0) = 0$;

Up to our knowledge, the simplest function that respects all the above conditions is $Re(x) = x/(x + 1)$ Where for redundancy equal to zero (0) we mean a situation in which we are unable to evaluate whether an "event of uncertainty", is occurring or is expected to occur in the considered area/s, because of the absence of uncertainty signals for that area;

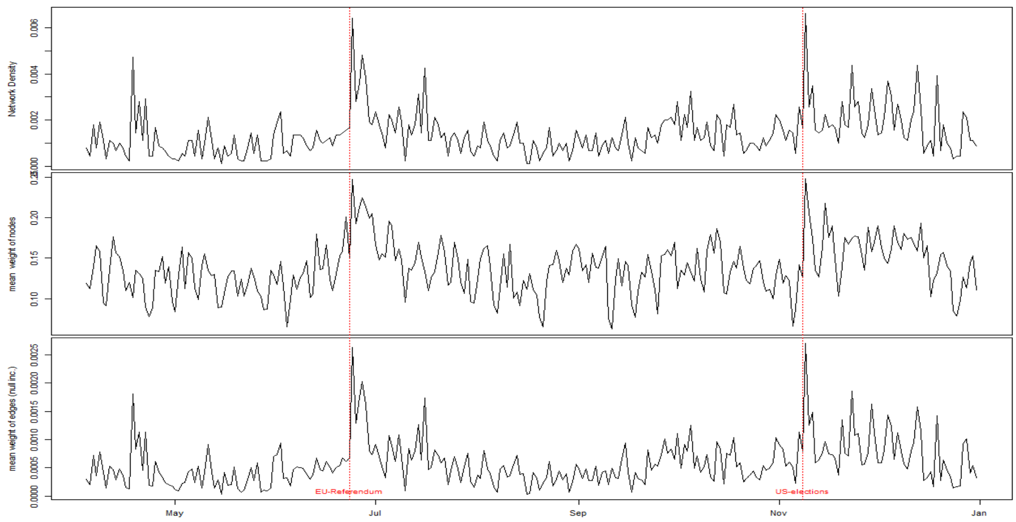
To count the distinct number of sources per day, per edge/node of the network we use three vectors containing the following metadata variables for each observation (Tweet):

- $\overrightarrow{UsrId}$: The unique Twitter identifier of the user that uploaded observation i is called $UsrId_i$;
- $\overrightarrow{DAY}$: The day in which the observation was uploaded/published on twitter, which we have previously defined and called day_i;
- $\mathbf{F}_{(i,)}$ (i^{th} *row of* $\mathbf{F}$): The occurrence or concurrence of countries to which the observation i refers to;

Call $T^{d,i,j}_{country}$ the set containing all observations of $T_{country}$ published in day d, concerning either a node i (if $i = j$) or an edge ij (if $i \neq j$). We have that the the number of distinct sources for that day and node/edge combination, is equal to the cardinality of the set containing unique combinations of $\overrightarrow{UsrId}$ and $\overrightarrow{DAY}$, among the subset of observations that are contained in that daily set of observations $T^{d,i,j}_{country}$ referring to that node/edge. by transforming the previously obtained value, by applying to it the Re transformation, we obtain the $red_{i,j,d}$ element of the redundancy tensor, called $\overline{\mathbf{RED}}$, that has the same size and structure (dimensions and features associated to the dimensions) of $\overline{\mathbf{W}}$.

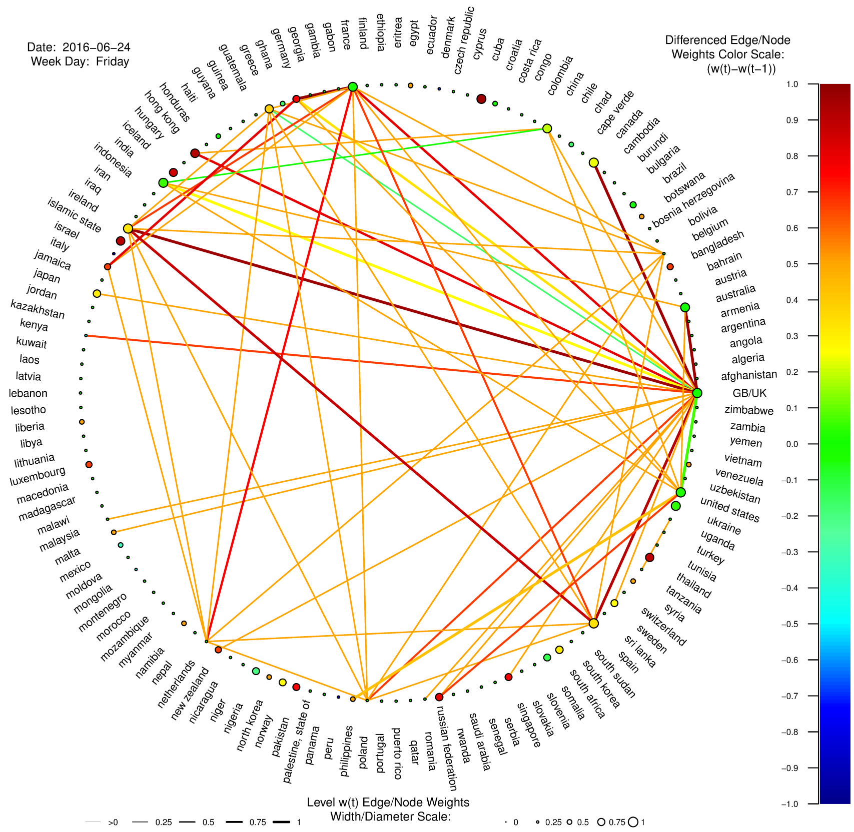
As we can see from Fig. 3.10 the structure (Density) isn't affected by the Redundancy transformation, as we expected. However the mean weights by day are rather different from before. By being dependent on the number of sources our weights now show us when, and for which nodes/edges our measurements are more robust. As we can see from the second and third graphs of Fig. 3.10, during major uncertainty events, not only the volumes of activity are larger, also the number of sources that produce this activity are relatively larger (with respect to the volumes) compared to normal times (when uncertainty is low). Therefore during major events our measurements of the degree of uncertainty in the node/edges of our network should be in average more robust and precise (less noisy) respect normal times, and hence less manipulable by single agents and BOTs. This quality is one of the strengths of our approach, that becomes more significant during crises, which exploits Twitter as a distributed uncertainty sensor and measurement system, whose aggregated sensitivity and precision grows during major events.

Figure 3.10. Redundancy Tensor - Statistics by day



By looking to the following video we can appraise the dynamics of redundancy of uncertainty signals by day and by edge/node. The higher is redundancy,i.e. the closer is a given $red_{i,j,d}$ to one, the more reliable are the uncertainty measurements concerning that specific node/edge (i, j) in that specific day (d).

Figure 3.11. [Video] Redundancy Tensor - Dynamic Network Visualization



CLICK ON THE IMAGE ABOVE TO START THE VIDEO
(You must be connected to the internet)
If your PDF reader doesn't support flash video direct streaming, the video can be directly viewed at the following (Youtube) URL-link:
youtube.com/embed/nLquhP30TfE

3.4.2 Impact Transformation and Tensor

The impact rescaling transformations applied to our our original tensor are two (distinct) functions of the elements of $\overline{\mathbf{W}}$:

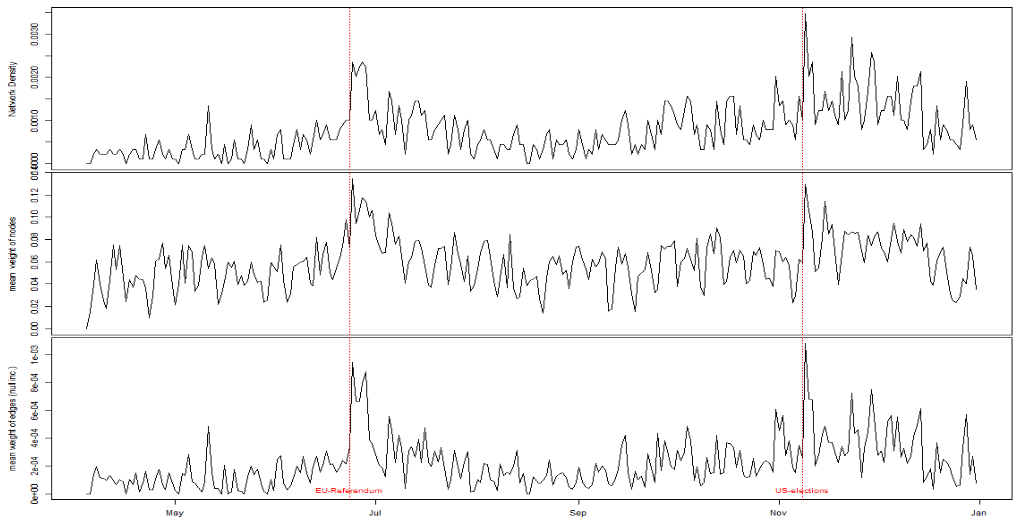
- One horizontal transformation, which exploits the country feature dimension, by rescaling the weight of each node/edge by comparing it (in terms of percentile rank) to the wights of other nodes/edges in the network at the same measurement time;
- One vertical transformation, which exploits the historical time-series dimension, by rescaling the weight of each node/edge by comparing it (in terms of percentile rank) to prior weights of the same node/edge;

Both transformations, which we call respectively $h\left(x\right)$ (horizontal impact rescaling) and $v\left(x\right)$ (vertical impact rescaling), are both applied to each element $w_{i,j,p}$ of tensor $\overline{\mathbf{W}}$.

$v\left(x\right)$ is used to appraise the relative relevance/magnitude of signals of uncertainty associated to a given node (geographic area) or edge (contagion channel among two areas) with respect to the past values of that same node/edge. For each combination of i, j and p, it is computed as the percentile rank (with strict sign of the inequality) of observation $w_{i,j,p}$ with respect to the set of observations, called $V_{i,j,p}$, with non null weights (for which $w_{i,j,t<p} > 0$) referring to the same node/edge identified through i and j, considering only past values $t < p$, i.e. the weights of the days before p.

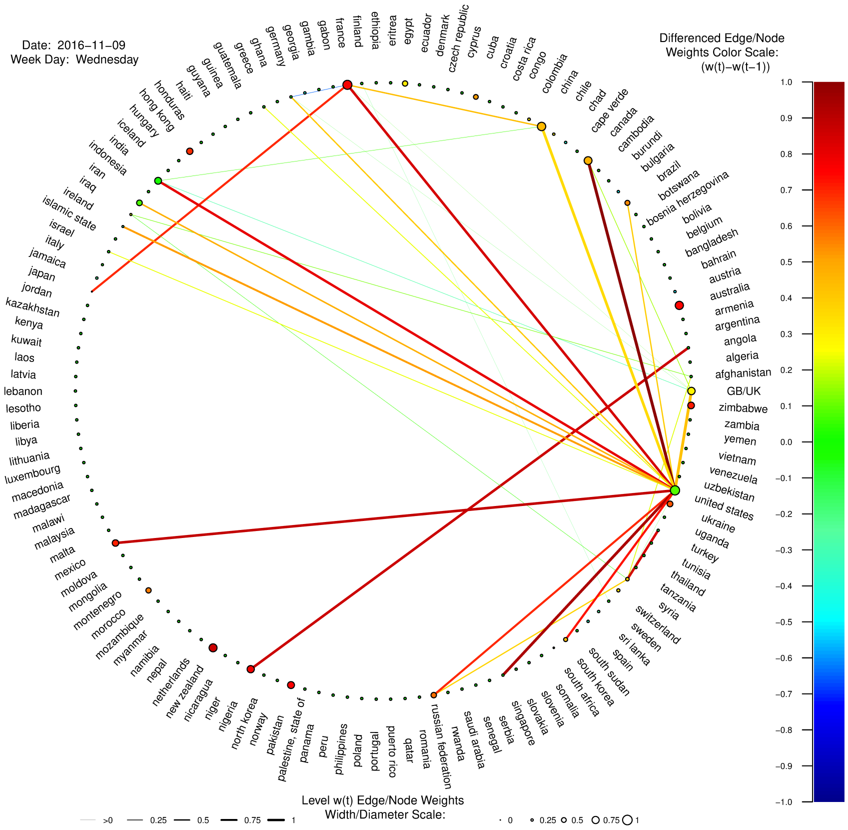
Whereas $h\left(x\right)$ is used to appraise the relative relevance/magnitude of signals of uncertainty associated to a given node (geographic area) or edge (contagion channel among two areas) in a given day, with respect to those of the other nodes/edges in that same day. It is computed as the percentile rank of $w_{i,j,p}$ with respect to the set of observations, called $H_{i,j,p}$, with non null weights, for which $w_{x\neq i,y\neq x,t=p} > 0$, referring to other nodes (if $i = j$) or edges (if $i \neq j$) for the same p. By applying the previously described transformations $v\left(x\right)$ and $h\left(x\right)$ to all $w_{i,j,p}$ we obtain the tensors called $\overline{\mathbf{V}}$ and $\overline{\mathbf{H}}$. By multiplying element-wise (Hadamard product) the elements of $\overline{\mathbf{V}}$ and $\overline{\mathbf{H}}$ we obtain the impact tensor $\overline{\mathbf{I}}$.

Figure 3.12. Impact Tensor - Statistics by day



In the following video 3.13 we can observe the dynamics of the network represented by $\bar{\mathbf{I}}$ across time.

Figure 3.13. [Video] Impact Tensor - Dynamic Network Visualization

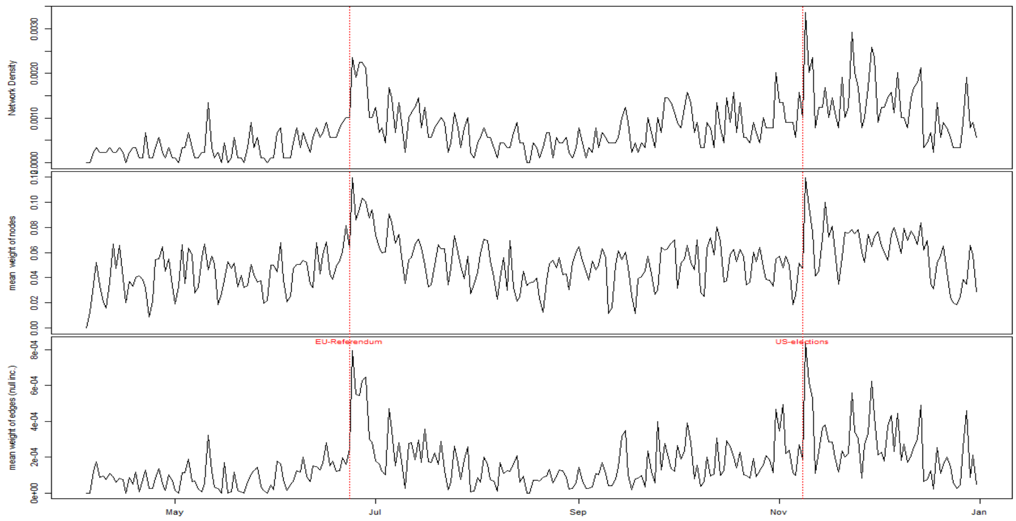


CLICK ON THE IMAGE ABOVE TO START THE VIDEO
(You must be connected to the internet)
If your PDF reader doesn't support flash video direct streaming, the video can be directly viewed at the following (Youtube) URL-link:
youtube.com/embed/htqIqX2ckPo

3.4.3 Rescaled Dynamic Worldwide Uncertainty Network (RD-WUN)

Finally, by element-wise multiplying (Hadamard product) tensor $\overline{\mathbf{I}}$ with tensor $\overline{\mathbf{RED}}$, we obtain our tensor $\overline{\mathbf{R}}$ representing our final **Rescaled Dynamic Worldwide Uncertainty Network** (**RD-WUN**) :

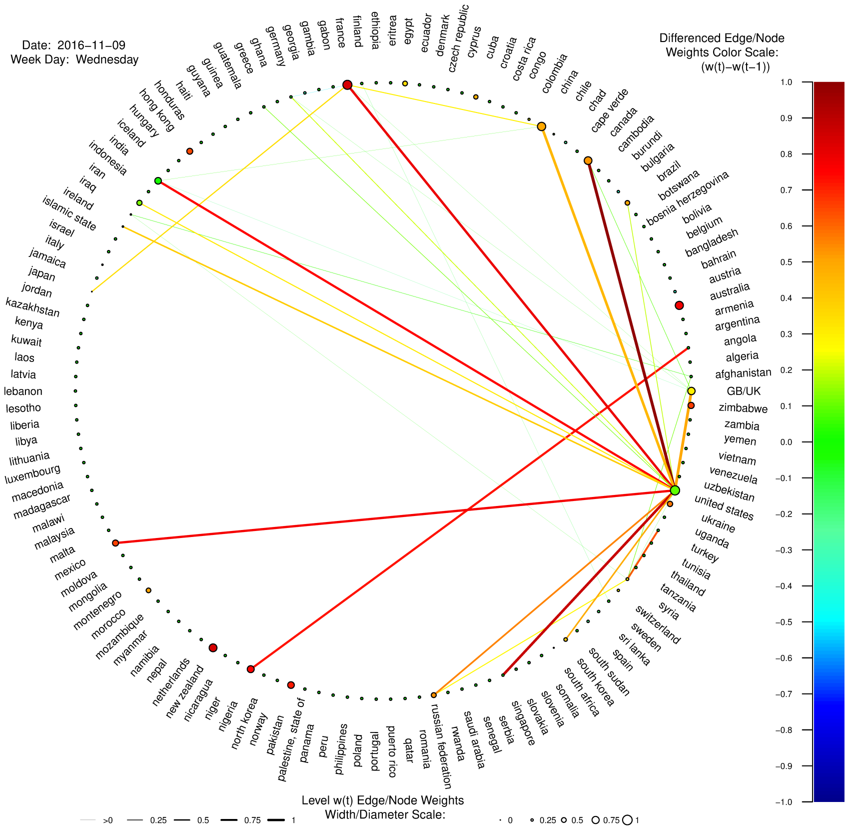
Figure 3.14. Rescaled Dynamic Worldwide Uncertainty Network: statistics by day



As we can see from Fig. 3.14 it looks like some of the aforementioned distortions and representativeness problems due to our sample of English tweets used to proxy uncertainty in the whole world, have been mitigated: mean values of nodes and edges appear to be less sensitive to events occurring in US and UK, which are the two English speaking countries from which a large share of our tweets come from. Therefore we fulfilled our objective of mitigating representativeness distortions, objective that pushed us to transform the weights of our original tensor $\overline{\mathbf{W}}$. In particular, the overestimation of uncertainty in/caused-by events occurring in countries where the English language, Internet and Twitter are relatively more widespread in the population, appear to be smaller after the transformations we have proposed in this article.

In the following video 3.15 we can observe the dynamics of the network represented by $\overline{\mathbf{R}}$ across time.

Figure 3.15. [Video] Rescaled Dynamic Network Visualization



CLICK ON THE IMAGE ABOVE TO START THE VIDEO
(You must be connected to the internet)
If your PDF reader doesn't support flash video direct streaming, the video can be directly viewed at the following (Youtube) URL-link:
youtube.com/embed/htqIqX2ckPo

Here follows a short list of interesting dates, with embedded links to their occurence in the aforementioned video, which illustrate the capacity of **RD-WUN** in capturing the dynamics of *civil society uncertainty* across the globe, potentially in real time:

- UKs EU-referendum (UK/GB, June 23rd 2016)
- Australian general elections (Australia, July the 2nd 2016)
- Failed Coup in Turkey (Turkey, July the 15th 2016)
- United States presidential elections (United States, November the 8th 2016)
- Italian Constitutional Referendum (Italy, December the 4th 2016)

3.5 Conclusion

The findings of this empirical work confirm that, thanks to our **Worldwide Uncertainty Network**, Twitter can be used, potentially in real time, as a worldwide distributed *civil society uncertainty* sensor system. Moreover, the observed patterns of *civil society uncertainty* in areas of the world relatively unreached by Twitter users, like Syria, North Korea and Afghanistan show us that, despite Twitter users may not physically and directly observe states of *civil society uncertainty* in specific areas in which civil or military conflicts occur, the volume and content of Twitter observations referring to uncertainty in these areas is far from being random. When considered collectively, Twitter users appear to be able to interpret and amplify those signals that are associated to factual states of *civil society uncertainty* in the different countries of the world and to recognize the dependencies among them.

3.6 Future Research

As we have seen in this final article, one of the most interesting qualities of Twitter data about uncertainty is its multidimensionality which renders it particularly suitable for those research applications that, by using a network approach exploit the high dimensionality of the data to enrich the interpretation of both content of tweets and volume of activity. Unfortunately, at the moment, the **Worldwide Uncertainty Network** is an undirected network, therefore even though we can say that an uncertainty contagion channel exists among two countries we cannot claim with certainty which is the direction of the contagion process.

One of possible solutions that could be used to overcome this problem/limitation, which is the subject of the next study by the author, is to assume that inter-area *civil society uncertainty* contagion is a process which is not instantaneous, but happens on a discrete daily scale following a particular process, here summarized:

1. **At time *t***, a local uncertainty event occurs in a country -called *i*- , i.e. a large number of agents from the civil society publicly signal the occurrence of an uncertainty event in that country, the weight of node *i* is close to 1. Meanwhile, uncertainty in another country -called *j*- is low, i.e. no or a very small number of agents signal the occurrence of an uncertainty event in *j*, the weight of node *j* is close to 0;
2. **Always at time *t***, agents evaluate the consequences of the *civil society uncertainty* peak (1) in *i* for the country *j*; A potential contagion channel among the two countries exists if a large number of agents publicly signal its existence, i.e. the higher is the weight of the edge between *i* and *j* - called *ij*- the larger is the contagion channel;
3. **At time *t+1***, which follows (1), contagion of *civil society uncertainty* from *i* to *j* may have occurred if its effects are visible in country *j*. Which, in terms of Twitter observations, is equivalent to a large number of agents that publicly signal the occurrence of an uncertainty event in the country *j*, while at the same time the contagion channel *ij* should still be different from zero;

We could exploit the above stated identification strategy, together with the concept of Granger Causation to identify and estimate the importance of these directed inter-area contagion mechanisms by using the data in our transformed tensors to distinguish among:

- **Uncertainty hysteresis of a node (intra-area AR dependency):** in a model where the dependent variable is uncertainty in nodes (observed nodes' weights), hysteresis can be identified through the autoregressive component on nodes' weights, by estimating the coefficients of an explanatory variable that contains lagged values of nodes' weights we can appraise the level of uncertainty hysteresis in a given node. We hypothesize that the degree of uncertainty hysteresis is node (country) specific;
- **Vulnerability of a country to international *civil society uncertainty* shocks (lagged dependencies on edges' weights):** in a model where the dependent variable is uncertainty in nodes (observed nodes' weights), vulnerability to international uncertainty shocks can be identified by estimating the coefficients of a set of proxy variables whose values are equal to the lagged weight of edges that lead to a given node;
- **Vulnerability of a country to transnational *civil society uncertainty* contagion (inter-area lagged dependencies):** in a model where the dependent variable is uncertainty in nodes (observed nodes' weights), inter-area directed contagion can be identified by estimating the coefficients of a set of proxy variables, whose values are equal to the lagged weights of edges -proxying contagion channels- that lead to a given node, multiplied by the lagged weights of the nodes at the other end of these edges -proxying the importance of uncertainty sources to which a given node has been connected to- ;
- **Country specific *civil society uncertainty* shocks (local uncertainty impulses):** identified through the residuals of a -estimated- model where the dependent variable is uncertainty in nodes (observed nodes' weights) and the explanatory variables are those presented here above;

  Since all weights in our rescaled tensors fall in the [0,1[interval, the tensors of our dynamic uncertainty networks appear to be particularly suitable statistical objects to fit VAR models of uncertainty, by country, with lagged contagion among nodes (countries) through edges (contagion channels), by using zero-inflated beta distributions to represent uncertainty expectations in the various nodes and edges of the network at a given moment in time.

Appendix

Country Dictionaries

In the following appendix are made available the country dictionaries' TOKENS (ordered alphabetically). For those countries who haven't been inserted directly in the Second Chapter of this work (UK/GB and US).

Table 3.3. Country Dictionaries' Tokens

Country Name	Tokens (CASE SENSITIVE)
Afghanistan	Afghanistan* afghanistan* AFGHANISTAN*
Albania	Albania* albania* ALBANIA*
Algeria	Algeria* algeria* ALGERIA*
Angola	Angola* angola* ANGOLA
Antarctica	Antarctica* antarctica* ANTARCTICA
Arab Emirates	Arab Emirates** arab emirates*** ARAB EMIRATES**
Argentina	Argentina* argentina* ARGENTINA*
Argentina	Armenia* armenia* ARMENIA*
Australia	Australia* australia* AUSTRALIA*
Austria	Austria* austria* AUSTRIA*
Azerbaijan	Azerbaijan* azerbaijan*

	AZERBAIJAN*
Bahrain	Bahrain* bahrain* BAHRAIN*
Bangladesh	Bangladesh* bangladesh* BANGLADESH*
Belarus	Belarus* belarus* BELARUS*
Belgium	Belgium* belgium* BELGIUM*
Belize	Belize* belize* BELIZE*
Belize	Benin* benin* BENIN*
Belize	Bhutan* bhutan* BHUTAN*
Bolivia	Boliva* bolivia* BOLIVIA*
Bosnia and Herzegovina	Bosnia* bosnia* BOSNIA* Herzegovina* herzegovina* HERZEGOVINA*
Botwana	Botswana* botswana* BOTSWANA*
Brazil	Brazil* brazil* BRAZIL*
Brunei	Brunei* brunei* BRUNEI*
Bulgaria	Bulgaria* bulgaria* BULGARIA*
Burkina Faso	Burkina Faso** burkina faso** BURKINA FASO**
Burundi	Burundi* burundi* BURUNDI*
	Cambodia*

Cambodia	cambodia* CAMBODIA*
Cameroon	Cameroon* cameroon CAMEROON*
Canada	Canada* canada* CANADA*
Cape Verde	Cape Verde** cape verde** CAPE VERDE**
Central African Republic	Central African Republic** central african republic** CENTRAL AFRICAN REPUBLIC**
Chad	Chad* chad* CHAD*
Chile	Chile* chile* CHILE*
China	China* china* CHINA*
Colombia	Colombia* colombia* COLOMBIA*
Congo	Congo* congo* CONGO*
Costa Rica	Costa Rica** costa rica** COSTA RICA**
Côte d'Ivoire	Côte d'Ivoire** côte d'ivoire** CÔTE D'IVOIRE**
Croatia	Croatia* croatia* CROATIA*
Cuba	Cuba* cuba* CUBA*
Cyprus	Cyprus* cyprus* CYPRUS*
Czech Republic	Czech Republic** czech republic** CZECH REPUBLIC**
Denmark	Denmark* denmark* DENMARK*

Ecuador	Ecuador* ecuador* ECUADOR*
Egypt	Egypt* egypt* EGYPT*
El Salvador	El Salvador** el salvador** EL SALVADOR**
England	England* england* ENGLAND*
Eritrea	Eritrea* eritrea* ERITREA*
Estonia	Estonia* estonia* ESTONIA*
Ethiopia	Ethiopia* ethiopia* ETHIOPIA*
European Union	EU* eu* European Union** european union** EUROPEAN UNION** E.U*** e.u*** Europe* europe* EUROPE*
Finland	Finland* finland* FINLAND*
France	France* france* FRANCE*
Gabon	Gabon* gabon* GABON*
Gambia	Gambia* gambia* GAMBIA*
Georgia	Georgia* georgia* GEORGIA*
Germany	Germany* germany* GERMANY*
	Gibraltar*

Gibraltar	gibraltar* GIBRALTAR*
Greece	Greece* greece* GREECE*
Greenland	Greenland* greenland* GREENLAND*
Guatemala	Guatemala* guatemala* GUATEMALA*
Guinea-bissau	Guinea-bissau** guinea-bissau** GUINEA-BISSAU**
Guinea	Guinea* guinea* GUINEA*
Guyana	Guyana* guyana* GUYANA*
Haiti	Haiti* haiti* HAITI*
Honduras	Honduras* honduras* HONDURAS*
Hong Kong	Hong Kong** hong kong** HONG KONG*
Hungary	Hungary* hungary* HUNGARY*
Haiti	Iceland* iceland* ICELAND*
India	India* india* INDIA*
Indonesia	Indonesia* indonesia* INDONESIA*
Iran	Iran* iran* IRAN*
Iraq	Iraq* iraq* IRAQ*
Ireland	Ireland* ireland* IRELAND*

Islamic State (of Iraq and the Levant)	ISLAMIC STATE** Islamic State** islamic state** ISIS* isis* Isis* isil* Isil* ISIL* daesh* Daesh* DAESH*
Israel	Israel* israel* ISRAEL*
Italy	Italy* italy* ITALY*
Jamaica	Jamaica* jamaica* JAMAICA*
Japan	Japan* japan* JAPAN*
Jordan	Jordan* jordan* JORDAN*
Kazakhstan	Kazakhstan* kazakhstan* KAZAKHSTAN*
Kenya	Kenya* kenya* KENYA*
Kuwait	Kuwait* kuwait* KUWAIT*
Kyrgyzstan	Kyrgyzstan* Kyrgyzstan* KYRGYZSTAN*
Laos	Laos* laos* LAOS*
Latvia	Latvia* latvia* LATVIA*
Lebanon	Lebanon* lebanon* LEBANON*
Lesotho	Lesotho* lesotho*

	LESOTHO*
Liberia	Liberia* liberia* LIBERIA*
Libya	Libya* libya* LIBYA*
Lithuania	Lithuania* lithuania* LITHUANIA*
Luxembourg	Luxembourg* luxembourg* LUXEMBOURG*
Macedonia	Macedonia* macedonia* MACEDONIA*
Madagascar	Madagascar* madagascar* MADAGASCAR*
Malawi	Malawi* malawi* MALAWI*
Malaysia	Malaysia* malaysia* MALAYSIA*
Mali	Mali* mali* MALI*
Malta	Malta* malta* MALTA*
Mauritania	Mauritania* mauritania* MAURITANIA*
Mexico	Mexico* mexico* MEXICO*
Moldova	Moldova* moldova* MOLDOVA*
Mongolia	Mongolia* mongolia* MONGOLIA*
Montenegro	Montenegro* montenegro* MONTENEGRO*
Morocco	Morocco* morocco* MOROCCO*
	Mozambique*

Mozambique	mozambique* MOZAMBIQUE*
Myanmar	Myanmar* myanmar* MYANMAR*
Namibia	Namibia* namibia* NAMIBIA*
Nepal	Nepal* nepal* NEPAL*
Netherlands	Netherlands* netherlands* NETHERLANDS*
New Zeland	New Zeland** new zeland** NEW ZELAND*
Nicaragua	Nicaragua* nicaragua* NICARAGUA*
Niger	Niger* niger* NIGER*
Nigeria	Nigeria* nigeria* NIGERIA*
North Korea	North Korea** north korea** NORTH KOREA** Pyongyang* pyongyang* PYONGYANG*
North Sudan	North Sudan* north sudan* NORTH SUDAN*
Norway	Norway* norway* NORWAY*
Oman	Oman* oman* OMAN*
Pakistan	Pakistan* pakistan* PAKISTAN*
Palestine	Palestine* palestine* PALESTINE*
Panama	Panama* panama* PANAMA*

Papua New Guinea	Papua New Guinea** papua new guinea** PAPUA NEW GUINEA**
Paraguay	Paraguay* paraguay* PARAGUAY*
Peru	Peru* peru* PERU*
Philippines	Philippines* philippines* PHILIPPINES*
Poland	Poland* poland* POLAND*
Portugal	Portugal* portugal* PORTUGAL*
Puerto Rico	Puerto Rico** puerto rico** PUERTO RICO*
Qatar	Qatar* qatar* QUATAR*
Romania	Romania* romania* ROMANIA*
Russian Federation	Russia* russia* RUSSIA*
Rwanda	Rwanda* rwanda* RWANDA*
Saudi Arabia	Saudi Arabia** saudi arabia** SAUDI ARABIA**
Scotland	Scotland* scotland* SCOTLAND*
Senegal	Senegal* senegal* SENEGAL*
Serbia	Serbia* serbia* SERBIA*
Singapore	Singapore* singapore* SINGAPORE*
Slovakia	Slovakia* slovakia*

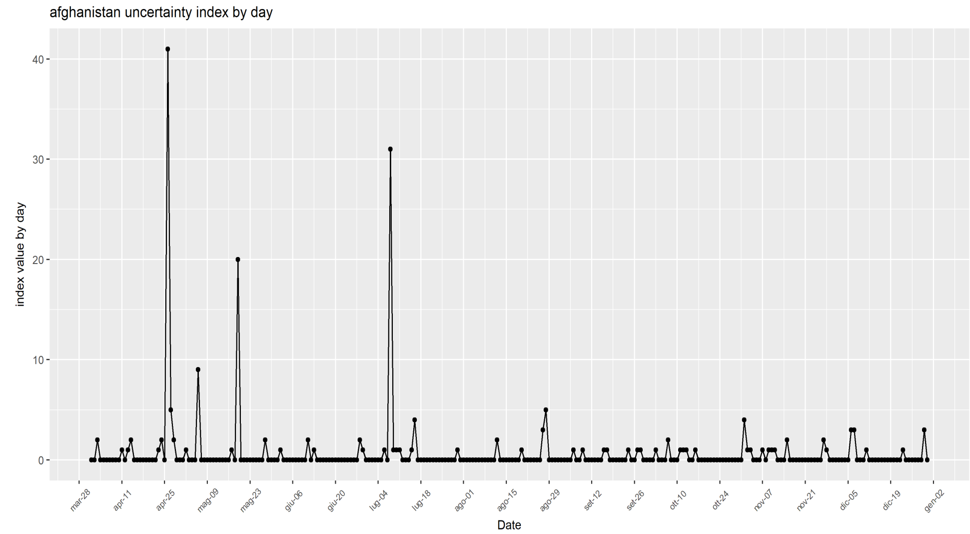
	SLOVAKIA*
Slovenia	Slovenia* slovenia* SLOVENIA*
Somalia	Somalia* somalia* SOMALIA*
South Africa	South Africa** south africa** SOUTH AFRICA**
South Korea	South Korea** south korea** SOUTH KOREA**
South Sudan	South Sudan** south sudan** SOUTH SUDAN**
Spain	Spain* spain* SPAIN*
Sri lanka	Sri Lanka** sri lanka** SRI LANKA*
Suriname	Suriname* suriname* SURINAME*
Swaziland	Swaziland* swaziland* SWAZILAND*
Sweden	Sweden* sweden* SWEDEN*
Switzerland	Switzerland* switzerland* SWITZERLAND*
Syria	Syria* syria* SYRIA*
Tajikistan	Tajikistan* tajikistan* TAJIKISTAN*
Tanzania	Tanzania* tanzania* TANZANIA*
Thailand	Thailand* thailand* THAILAND*
Thailand	Timor-Leste* timor-leste* TIMOR-LESTE*
	Togo*

Togo	togo* TOGO*
Trinidad and Tobago	Trinidad and Tobago** trinidad and tobago** TRINIDAN AND TOBAGO**
Tunisia	Tunisia* tunisia* TUNISIA*
Turkey	Turkey* turkey* TURKEY*
Turkmenistan	Turkmenistan* turkmenistan* TURKMENISTAN*
Uganda	Uganda* uganda* UGANDA*
Ukraine	Ukraine* ukraine* UKRAINE*
Uruguay	Uruguay* uruguay* URUGUAY*
Uzbekistan	Uzbekistan* uzbekistan* UZBEKISTAN*
Vatican	Vatican* vatican* VATICAN*
Venezuela	Venezuela* venezuela* VENEZUELA*
Vietnam	Vietnam* vietnam* VIETNAM*
Wales	Wales* wales* WALES*
Yemen	Yemen* yemen* YEMEN*
Zambia	Zambia* zambia* ZAMBIA*
Zimbabwe	Zimbabwe* zimbabwe* Zimbabwe*

Country Indexes' Time Series

In the following appendix are made avaliable the visualizatons of the time series of TU indexes by country, for all those countries of the world that weren't directly included in the second Chapter. Here follows an embedded video (streamed from the web) of these plots, in case of visualization problems related to the compatibility of your PDF reader, this video (uploaded on Youtube) is also available at the following URL-link: www.youtube.com/embed/JidFvtSJn2k. Plots of the TU indexes are presented in alphabetic order. We could not include them in the PDF document, because of document size constraints, which were imposed for the uploading this thesis online. Indexes' time series by country, as well as their visualizations, can also be requested directly to the author by writing at the following email address: carlo.santagiustina@unive.it

Figure 3.16. [Video] Twitter Uncertainty Indexes by Geographic Area



CLICK ON THE IMAGE ABOVE TO START THE VIDEO
(You must be connected to the internet)
If your PDF reader doesn't support flash video direct streaming, the video can be directly viewed at the following (Youtube) URL-link:
youtube.com/embed/JidFvtSJn2k

S-WUN Edge Betweenness Centrality Measures

The Betweenness Centrality of a edge is equal to the fraction of shortest paths between all pairs of (non isoloated) nodes in the S-WUN that pass through that edge of interest. In the weighted version the shortest path is computed using as distances the inverse of the edges' weights;

Table 3.4. S-WUN Edge Betweenness Centrality Measures

Contagion Channel Name	Unweighted Edge Betweenness Centrality	Weighted Edge Betweenness Centrality
GB/UK-albania	76.348	133.000
GB/UK-algeria	45.344	38.500
GB/UK-australia	24.525	269.000
GB/UK-austria	29.529	0.000
GB/UK-bahrain	58.977	38.500
GB/UK-belgium	86.925	0.000
GB/UK-belize	136.000	136.000
GB/UK-brazil	52.714	0.000
GB/UK-bulgaria	47.320	0.000
GB/UK-cameroon	47.212	136.000
GB/UK-canada	17.762	0.000
GB/UK-chad	61.582	136.000
GB/UK-chile	45.333	0.000
GB/UK-china	52.896	379.000
GB/UK-colombia	115.794	0.000
GB/UK-croatia	34.938	0.000
GB/UK-cyprus	27.326	0.000
GB/UK-estonia	136.000	136.000
GB/UK-finland	32.077	0.000
GB/UK-france	25.731	0.000
GB/UK-georgia	45.442	0.000
GB/UK-germany	37.937	210.000
GB/UK-ghana	128.673	394.000
GB/UK-greece	34.341	0.000
GB/UK-guyana	136.000	136.000
GB/UK-hong kong	35.913	0.000
GB/UK-iceland	78.112	0.000
GB/UK-india	28.524	0.000
GB/UK-indonesia	27.130	0.000
GB/UK-iran	28.703	0.000
GB/UK-iraq	53.040	0.000
GB/UK-ireland	42.524	270.000
GB/UK-islamic state	25.531	376.000
GB/UK-israel	29.816	397.000
GB/UK-italy	27.667	0.000
GB/UK-japan	33.048	0.000
GB/UK-kenya	101.478	0.000
GB/UK-kuwait	48.178	0.000
GB/UK-latvia	100.856	0.000
GB/UK-libya	34.076	0.000

GB/UK-malaysia	37.406	0.000
GB/UK-malta	85.078	136.000
GB/UK-mexico	28.222	0.000
GB/UK-mozambique	94.693	264.000
GB/UK-nepal	106.045	0.000
GB/UK-netherlands	23.203	136.000
GB/UK-new zealand	40.578	0.000
GB/UK-norway	35.960	267.000
GB/UK-pakistan	41.565	0.000
GB/UK-palestine, state of	81.216	0.000
GB/UK-panama	52.190	0.000
GB/UK-philippines	33.282	0.000
GB/UK-poland	41.672	883.000
GB/UK-portugal	66.033	0.000
GB/UK-qatar	43.973	0.000
GB/UK-romania	82.702	136.000
GB/UK-russian federation	47.839	0.000
GB/UK-singapore	28.528	0.000
GB/UK-slovakia	70.413	136.000
GB/UK-somalia	79.388	0.000
GB/UK-south africa	31.686	0.000
GB/UK-south korea	43.907	0.000
GB/UK-south sudan	31.997	0.000
GB/UK-spain	29.424	0.000
GB/UK-sweden	31.310	0.000
GB/UK-switzerland	27.979	0.000
GB/UK-syria	25.792	0.000
GB/UK-thailand	59.373	0.000
GB/UK-turkey	50.263	0.000
GB/UK-uganda	128.469	270.000
GB/UK-ukraine	31.223	0.000
GB/UK-united states	76.501	3030.000
GB/UK-uruguay	136.000	136.000
GB/UK-venezuela	38.809	0.000
GB/UK-zimbabwe	39.849	0.000
afghanistan-australia	16.863	0.000
afghanistan-china	36.672	0.000
afghanistan-iran	12.363	0.000
afghanistan-iraq	14.663	0.000
afghanistan-islamic state	10.323	0.000
afghanistan-pakistan	13.143	0.000
afghanistan-somalia	16.567	0.000
afghanistan-syria	16.933	0.000
afghanistan-turkey	29.897	0.000
afghanistan-turkmenistan	136.000	136.000
afghanistan-ukraine	13.231	0.000
afghanistan-united states	93.365	270.000
afghanistan-venezuela	11.134	0.000
albania-hong kong	10.175	1.000
albania-moldova	6.984	0.000
albania-norway	8.146	2.000

albania-turkey	34.744	0.000
albania-ukraine	11.538	0.000
algeria-canada	15.115	0.000
algeria-libya	6.836	0.000
algeria-nigeria	14.728	0.000
algeria-united states	55.579	97.500
angola-kenya	29.031	0.000
angola-nigeria	47.595	136.000
angola-portugal	25.946	0.000
angola-south africa	40.648	0.000
argentina-brazil	13.424	0.000
argentina-croatia	5.092	0.000
argentina-cuba	6.238	0.000
argentina-greece	13.588	0.000
argentina-japan	16.219	0.000
argentina-spain	13.091	0.000
argentina-united states	63.966	136.000
argentina-venezuela	8.589	0.000
armenia-china	77.823	127.000
armenia-germany	58.177	9.000
australia-austria	9.429	0.000
australia-canada	5.770	0.000
australia-china	10.603	0.000
australia-cyprus	5.928	0.000
australia-france	9.940	0.000
australia-hong kong	7.103	0.000
australia-india	5.699	0.000
australia-iran	5.099	0.000
australia-ireland	11.140	0.000
australia-japan	7.117	0.000
australia-malaysia	7.992	0.000
australia-new zealand	6.023	135.000
australia-norway	8.141	0.000
australia-pakistan	5.865	0.000
australia-russian federation	12.377	0.000
australia-singapore	6.248	0.000
australia-south africa	8.945	0.000
australia-south korea	8.460	0.000
australia-spain	9.447	0.000
australia-sri lanka	29.024	0.000
australia-switzerland	4.820	0.000
australia-syria	7.998	0.000
australia-turkey	13.324	0.000
australia-united states	29.903	0.000
australia-venezuela	6.950	0.000
austria-croatia	5.420	0.000
austria-finland	4.779	0.000
austria-france	10.042	0.000
austria-germany	9.788	0.000
austria-greece	9.459	0.000
austria-hungary	11.517	0.000

austria-iran	9.111	0.000
austria-italy	6.581	136.000
austria-latvia	17.379	0.000
austria-netherlands	5.037	0.000
austria-romania	12.982	0.000
austria-spain	7.015	0.000
austria-switzerland	4.455	0.000
austria-united states	43.352	0.000
austria-uzbekistan	10.078	0.000
azerbaijan-thailand	136.000	136.000
bahrain-egypt	7.165	0.000
bahrain-kuwait	4.221	0.000
bahrain-oman	3.282	0.000
bahrain-qatar	2.325	0.000
bahrain-united states	65.515	97.500
bangladesh-india	32.532	136.000
bangladesh-iraq	16.426	0.000
bangladesh-united states	87.321	0.000
belarus-poland	40.417	136.000
belarus-russian federation	96.063	0.000
belgium-bulgaria	7.759	0.000
belgium-canada	26.227	400.000
belgium-france	27.680	0.000
belgium-germany	26.212	0.000
belgium-greece	20.834	0.000
belgium-ireland	18.187	0.000
belgium-netherlands	10.913	0.000
belgium-poland	11.109	0.000
belgium-romania	5.986	0.000
belgium-senegal	113.418	268.000
belgium-spain	16.677	0.000
bhutan-nepal	136.000	136.000
bolivia-guatemala	78.649	135.000
bolivia-peru	57.351	1.000
bosnia and herzegovina-rwanda	42.441	268.000
bosnia and herzegovina-serbia	101.768	400.000
brazil-china	17.567	395.000
brazil-croatia	9.128	0.000
brazil-cuba	14.666	0.000
brazil-germany	20.481	0.000
brazil-greece	17.696	0.000
brazil-haiti	22.557	0.000
brazil-hungary	15.971	0.000
brazil-india	11.298	0.000
brazil-indonesia	8.524	0.000
brazil-iran	8.481	0.000
brazil-islamic state	9.309	0.000
brazil-italy	11.025	0.000
brazil-japan	9.165	0.000
brazil-malaysia	12.701	0.000
brazil-netherlands	10.382	0.000

brazil-nicaragua	66.934	0.000
brazil-nigeria	40.782	0.000
brazil-peru	72.772	135.000
brazil-poland	14.648	0.000
brazil-puerto rico	29.982	4.000
brazil-russian federation	17.789	0.000
brazil-slovenia	26.344	4.000
brazil-south africa	12.099	0.000
brazil-switzerland	6.632	0.000
brazil-syria	11.775	0.000
brazil-turkey	21.645	0.000
brazil-united states	36.122	0.000
brazil-venezuela	6.562	136.000
bulgaria-romania	5.960	0.000
bulgaria-russian federation	21.901	136.000
bulgaria-united states	66.170	0.000
burkina faso-colombia	34.579	8.000
burkina faso-turkey	101.421	128.000
burundi-kenya	78.210	0.000
burundi-rwanda	4.436	136.000
burundi-tanzania	2.517	0.000
burundi-uganda	50.836	0.000
cambodia-china	67.763	0.000
cambodia-japan	45.776	136.000
cambodia-philippines	22.461	0.000
cameroon-islamic state	7.727	0.000
cameroon-myanmar	6.566	0.000
cameroon-nigeria	15.963	0.000
cameroon-syria	12.503	0.000
cameroon-united states	53.193	0.000
canada-china	14.270	0.000
canada-cuba	19.794	0.000
canada-cyprus	7.043	0.000
canada-finland	9.118	0.000
canada-france	5.937	0.000
canada-georgia	15.099	0.000
canada-germany	10.174	0.000
canada-greece	11.157	0.000
canada-hong kong	10.423	0.000
canada-india	7.908	0.000
canada-iran	6.551	0.000
canada-ireland	9.208	0.000
canada-italy	6.230	0.000
canada-japan	7.055	0.000
canada-mexico	7.533	0.000
canada-new zealand	10.725	0.000
canada-nigeria	31.928	0.000
canada-norway	9.258	0.000
canada-pakistan	11.115	0.000
canada-philippines	7.349	0.000
canada-russian federation	9.153	0.000

canada-singapore	7.019	0.000
canada-slovakia	23.316	0.000
canada-sweden	7.836	0.000
canada-switzerland	6.289	0.000
canada-syria	8.803	0.000
canada-thailand	13.572	0.000
canada-turkey	14.954	0.000
canada-ukraine	8.442	0.000
canada-united states	25.816	530.000
chad-united states	74.418	0.000
chile-germany	18.512	136.000
chile-mexico	6.333	0.000
chile-pakistan	8.616	0.000
chile-united states	57.819	0.000
china-colombia	55.028	0.000
china-france	20.549	0.000
china-germany	21.704	0.000
china-hong kong	17.217	135.000
china-india	11.009	0.000
china-indonesia	10.555	136.000
china-iran	12.557	0.000
china-ireland	29.133	0.000
china-islamic state	12.120	0.000
china-israel	16.120	0.000
china-italy	14.486	0.000
china-japan	9.569	0.000
china-jordan	40.268	136.000
china-kenya	70.994	0.000
china-libya	17.054	0.000
china-malaysia	16.014	136.000
china-mexico	13.324	0.000
china-mongolia	99.713	0.000
china-myanmar	20.370	0.000
china-nepal	44.704	0.000
china-new zealand	16.508	0.000
china-north korea	24.442	0.000
china-pakistan	14.403	0.000
china-peru	114.751	0.000
china-philippines	11.710	10.000
china-russian federation	26.473	0.000
china-singapore	12.310	0.000
china-south africa	18.627	0.000
china-south korea	19.687	0.000
china-spain	21.668	0.000
china-switzerland	12.720	0.000
china-syria	15.901	0.000
china-thailand	25.677	0.000
china-turkey	28.919	0.000
china-ukraine	17.330	0.000
china-united states	43.290	864.000
china-uzbekistan	19.756	136.000

china-venezuela	15.261	0.000
china-vietnam	28.617	0.000
colombia-costa rica	135.000	135.000
colombia-haiti	16.741	0.000
colombia-hungary	18.502	20.000
colombia-india	35.460	0.000
colombia-mexico	14.516	0.000
colombia-paraguay	135.000	135.000
colombia-syria	24.291	0.000
colombia-turkey	40.494	0.000
colombia-united states	126.781	384.000
congo-germany	44.104	0.000
congo-ghana	15.315	134.000
congo-rwanda	40.329	2.000
congo-united states	112.748	0.000
costa rica-paraguay	1.000	1.000
croatia-france	11.034	0.000
croatia-haiti	9.672	0.000
croatia-russian federation	12.921	0.000
croatia-spain	8.051	0.000
croatia-switzerland	4.084	0.000
croatia-turkey	17.292	0.000
croatia-united states	39.147	136.000
cuba-france	23.546	0.000
cuba-haiti	9.362	135.000
cuba-jamaica	9.243	135.000
cuba-mexico	11.260	0.000
cuba-nicaragua	23.401	136.000
cuba-russian federation	21.687	0.000
cuba-united states	65.332	532.000
cuba-vietnam	11.656	0.000
cuba-yemen	16.295	0.000
cyprus-france	7.744	0.000
cyprus-germany	8.792	0.000
cyprus-greece	6.976	0.000
cyprus-india	10.927	136.000
cyprus-ireland	5.966	0.000
cyprus-italy	6.414	0.000
cyprus-malta	9.463	0.000
cyprus-spain	4.402	0.000
cyprus-turkey	15.710	0.000
cyprus-united states	38.749	0.000
czech republic-germany	78.503	129.000
czech republic-spain	57.497	7.000
denmark-france	47.745	0.000
denmark-germany	41.582	0.000
denmark-italy	37.818	136.000
denmark-portugal	8.855	0.000
egypt-france	15.731	0.000
egypt-greece	11.038	0.000
egypt-iraq	13.443	0.000

egypt-israel	8.863	0.000
egypt-italy	13.795	0.000
egypt-jordan	11.946	0.000
egypt-kuwait	6.084	0.000
egypt-lebanon	9.745	0.000
egypt-libya	10.672	2.000
egypt-oman	10.366	0.000
egypt-palestine, state of	14.064	134.000
egypt-panama	8.714	0.000
egypt-qatar	4.847	0.000
egypt-spain	13.141	0.000
egypt-turkey	24.632	0.000
egypt-united states	48.259	0.000
el salvador-honduras	1.000	1.000
eritrea-ethiopia	20.564	2.000
eritrea-somalia	117.089	134.000
ethiopia-israel	61.308	0.000
ethiopia-nepal	35.761	268.000
ethiopia-tanzania	18.168	136.000
ethiopia-uganda	27.885	0.000
ethiopia-uzbekistan	44.525	0.000
finland-germany	12.461	0.000
finland-iraq	13.612	0.000
finland-ireland	10.241	0.000
finland-italy	9.539	0.000
finland-latvia	11.892	0.000
finland-russian federation	12.206	136.000
finland-slovakia	6.997	0.000
finland-sweden	3.281	0.000
finland-united states	47.517	0.000
france-germany	8.845	10.000
france-greece	8.658	0.000
france-iceland	31.428	0.000
france-indonesia	8.527	0.000
france-iran	8.376	0.000
france-ireland	12.657	0.000
france-islamic state	8.832	0.000
france-italy	5.265	0.000
france-japan	10.120	0.000
france-mexico	10.662	0.000
france-morocco	35.730	0.000
france-netherlands	7.096	0.000
france-nigeria	39.301	0.000
france-poland	10.961	0.000
france-portugal	18.200	0.000
france-russian federation	13.957	0.000
france-south sudan	12.715	0.000
france-spain	6.905	261.000
france-sweden	9.289	0.000
france-switzerland	7.237	0.000
france-syria	10.135	0.000

france-thailand	20.538	0.000
france-turkey	16.968	0.000
france-united states	30.581	377.000
france-uzbekistan	15.772	0.000
france-zimbabwe	14.395	0.000
gabon-united states	136.000	136.000
gambia-ghana	22.888	0.000
gambia-liberia	5.733	2.000
gambia-nigeria	91.620	0.000
gambia-senegal	27.288	136.000
gambia-uganda	47.464	0.000
georgia-latvia	6.881	135.000
georgia-moldova	9.641	135.000
georgia-russian federation	16.807	402.000
georgia-ukraine	6.439	0.000
georgia-united states	61.797	0.000
germany-greece	11.972	0.000
germany-india	16.741	0.000
germany-indonesia	9.310	0.000
germany-ireland	14.674	0.000
germany-islamic state	12.805	18.000
germany-israel	14.937	0.000
germany-italy	5.540	380.000
germany-japan	13.426	0.000
germany-lebanon	24.916	0.000
germany-netherlands	6.863	0.000
germany-philippines	10.980	0.000
germany-poland	10.980	0.000
germany-portugal	16.634	0.000
germany-romania	30.634	0.000
germany-russian federation	18.367	0.000
germany-south africa	15.343	0.000
germany-spain	6.274	0.000
germany-sweden	9.991	0.000
germany-switzerland	8.567	0.000
germany-syria	15.170	0.000
germany-turkey	27.203	0.000
germany-ukraine	11.687	0.000
germany-united states	49.871	518.000
ghana-haiti	18.681	0.000
ghana-jordan	16.417	0.000
ghana-liberia	28.355	130.000
ghana-nigeria	25.928	0.000
greece-iceland	21.460	0.000
greece-ireland	13.813	0.000
greece-israel	12.301	0.000
greece-italy	7.813	6.000
greece-japan	14.945	0.000
greece-kuwait	14.486	4.000
greece-macedonia	93.250	136.000
greece-mozambique	28.223	6.000

greece-netherlands	6.717	0.000
greece-poland	8.641	0.000
greece-portugal	14.585	0.000
greece-qatar	11.206	0.000
greece-russian federation	18.902	0.000
greece-spain	6.793	0.000
greece-syria	12.417	0.000
greece-turkey	26.607	258.000
greece-united states	41.282	0.000
greece-venezuela	11.963	0.000
guatemala-peru	8.590	0.000
guatemala-united states	214.477	269.000
guinea-papua new guinea	1.000	1.000
haiti-jamaica	5.892	1.000
haiti-united states	85.762	0.000
hong kong-india	12.217	0.000
hong kong-japan	9.620	0.000
hong kong-united states	43.483	0.000
hong kong-vietnam	7.431	0.000
hungary-india	17.153	0.000
hungary-italy	13.171	0.000
hungary-japan	16.807	0.000
hungary-poland	13.058	633.000
hungary-portugal	12.447	0.000
hungary-serbia	39.515	530.000
hungary-slovenia	9.240	5.000
hungary-switzerland	6.731	0.000
hungary-united states	68.674	0.000
iceland-panama	6.351	136.000
india-indonesia	7.757	0.000
india-iran	6.954	0.000
india-islamic state	8.096	0.000
india-israel	12.159	0.000
india-italy	8.683	0.000
india-japan	5.710	0.000
india-libya	10.270	0.000
india-myanmar	15.619	0.000
india-nepal	32.256	526.000
india-netherlands	11.530	0.000
india-new zealand	9.443	0.000
india-nigeria	32.639	0.000
india-pakistan	11.041	532.000
india-russian federation	17.887	0.000
india-singapore	8.916	0.000
india-slovenia	32.665	117.000
india-south africa	10.586	0.000
india-south korea	15.197	0.000
india-south sudan	12.998	0.000
india-spain	13.554	0.000
india-sri lanka	41.292	135.000
india-switzerland	6.811	0.000

india-thailand	17.253	0.000
india-turkey	13.495	0.000
india-united states	25.721	1586.000
india-venezuela	10.121	0.000
india-zimbabwe	13.647	0.000
indonesia-italy	6.538	0.000
indonesia-malaysia	2.951	0.000
indonesia-pakistan	5.128	0.000
indonesia-philippines	3.424	0.000
indonesia-singapore	5.122	0.000
indonesia-south africa	5.987	0.000
indonesia-thailand	4.901	0.000
indonesia-turkey	12.695	0.000
indonesia-united states	30.113	0.000
iran-iraq	7.782	0.000
iran-islamic state	3.273	0.000
iran-italy	6.851	0.000
iran-lebanon	14.063	0.000
iran-libya	4.993	0.000
iran-nigeria	18.229	0.000
iran-pakistan	8.099	0.000
iran-russian federation	11.059	0.000
iran-saudi arabia	20.591	0.000
iran-singapore	6.857	0.000
iran-syria	5.864	0.000
iran-turkey	10.684	0.000
iran-united states	22.551	136.000
iran-uzbekistan	7.724	0.000
iran-venezuela	4.769	0.000
iraq-islamic state	6.309	0.000
iraq-kuwait	10.961	0.000
iraq-libya	8.197	0.000
iraq-nigeria	29.064	0.000
iraq-russian federation	21.662	0.000
iraq-serbia	61.983	0.000
iraq-somalia	15.259	0.000
iraq-syria	10.367	136.000
iraq-united states	46.251	0.000
iraq-venezuela	9.280	0.000
iraq-vietnam	16.896	0.000
iraq-yemen	22.840	0.000
ireland-israel	13.459	0.000
ireland-italy	12.476	0.000
ireland-malta	19.788	0.000
ireland-namibia	136.000	136.000
ireland-netherlands	7.435	0.000
ireland-new zealand	11.782	0.000
ireland-nigeria	36.973	0.000
ireland-norway	10.626	0.000
ireland-philippines	9.329	0.000
ireland-poland	11.237	0.000

ireland-singapore	9.844	0.000
ireland-spain	9.401	0.000
ireland-united states	61.836	0.000
ireland-vietnam	21.002	0.000
islamic state-libya	4.373	268.000
islamic state-nigeria	14.865	0.000
islamic state-philippines	7.121	0.000
islamic state-russian federation	11.267	0.000
islamic state-somalia	11.741	0.000
islamic state-syria	5.166	6.000
islamic state-turkey	10.785	0.000
islamic state-united states	24.383	0.000
islamic state-uzbekistan	7.419	0.000
israel-italy	8.593	0.000
israel-japan	10.400	0.000
israel-kuwait	10.362	3.000
israel-mexico	7.350	0.000
israel-pakistan	8.907	0.000
israel-palestine, state of	16.304	268.000
israel-thailand	11.611	0.000
israel-turkey	16.941	0.000
israel-ukraine	7.957	0.000
israel-united states	37.183	0.000
italy-japan	6.939	0.000
italy-lebanon	17.604	9.000
italy-netherlands	5.616	0.000
italy-new zealand	11.394	0.000
italy-philippines	8.216	0.000
italy-portugal	13.948	0.000
italy-russian federation	11.555	0.000
italy-slovenia	28.991	7.000
italy-spain	5.955	18.000
italy-sweden	8.431	0.000
italy-switzerland	4.423	0.000
italy-syria	9.409	0.000
italy-turkey	14.045	0.000
italy-ukraine	8.375	0.000
italy-united states	31.297	0.000
italy-uzbekistan	11.246	0.000
jamaica-united states	120.865	0.000
japan-myanmar	12.223	0.000
japan-nepal	25.320	0.000
japan-netherlands	8.327	0.000
japan-new zealand	9.703	0.000
japan-nigeria	37.195	0.000
japan-north korea	16.112	0.000
japan-philippines	7.010	0.000
japan-russian federation	13.998	0.000
japan-singapore	7.042	0.000
japan-slovenia	24.944	2.000
japan-spain	10.688	0.000

japan-switzerland	5.570	0.000
japan-thailand	15.737	0.000
japan-united states	30.440	268.000
japan-venezuela	8.588	0.000
japan-vietnam	15.513	0.000
jordan-lebanon	6.985	0.000
jordan-united states	77.060	0.000
kazakhstan-russian federation	136.000	136.000
kenya-netherlands	23.297	0.000
kenya-nigeria	32.848	140.000
kenya-rwanda	77.945	0.000
kenya-somalia	15.930	8.000
kenya-south africa	15.400	0.000
kenya-tanzania	72.926	0.000
kenya-uganda	7.026	0.000
kenya-united states	116.814	0.000
kuwait-oman	7.352	0.000
kuwait-qatar	4.020	0.000
kuwait-singapore	9.777	0.000
kuwait-united states	49.808	0.000
kuwait-vietnam	10.415	0.000
kuwait-yemen	16.356	129.000
laos-nepal	74.336	4.000
laos-vietnam	62.818	132.000
latvia-moldova	9.335	1.000
lebanon-macedonia	23.081	0.000
lebanon-saudi arabia	6.305	0.000
lebanon-ukraine	11.974	0.000
lebanon-united states	78.805	127.000
liberia-nigeria	101.912	4.000
libya-nepal	15.367	0.000
libya-nigeria	18.487	0.000
libya-russian federation	15.136	0.000
libya-singapore	6.108	0.000
libya-syria	8.290	0.000
libya-tunisia	18.768	136.000
libya-united states	28.892	0.000
libya-venezuela	6.551	0.000
macedonia-saudi arabia	21.673	0.000
malawi-mozambique	136.000	136.000
malaysia-philippines	4.026	0.000
malaysia-syria	9.455	0.000
malaysia-thailand	6.460	0.000
malaysia-united states	41.996	0.000
malta-spain	21.671	0.000
mexico-pakistan	6.395	0.000
mexico-philippines	4.625	0.000
mexico-russian federation	12.924	0.000
mexico-singapore	5.195	0.000
mexico-spain	9.185	0.000
mexico-thailand	7.381	0.000

mexico-united states	32.656	136.000
moldova-russian federation	47.292	0.000
moldova-turkey	55.052	0.000
moldova-ukraine	18.282	0.000
mongolia-pakistan	36.287	136.000
morocco-united states	100.270	136.000
mozambique-south africa	18.615	0.000
mozambique-united states	117.781	0.000
mozambique-zimbabwe	11.738	0.000
myanmar-singapore	6.457	0.000
myanmar-syria	13.398	0.000
myanmar-thailand	9.602	136.000
myanmar-united states	54.779	0.000
myanmar-vietnam	6.813	0.000
nepal-new zealand	13.454	0.000
nepal-somalia	14.908	0.000
nepal-united states	108.206	0.000
netherlands-poland	4.441	0.000
netherlands-russian federation	11.295	0.000
netherlands-spain	4.951	0.000
netherlands-switzerland	4.263	0.000
netherlands-ukraine	7.468	0.000
netherlands-united states	30.915	0.000
new zealand-pakistan	6.214	0.000
new zealand-sri lanka	14.497	1.000
new zealand-turkey	17.981	0.000
new zealand-uzbekistan	7.765	0.000
new zealand-zimbabwe	7.716	0.000
nicaragua-venezuela	46.066	0.000
niger-nigeria	136.000	136.000
nigeria-saudi arabia	28.596	0.000
nigeria-somalia	26.149	0.000
nigeria-south africa	16.078	0.000
nigeria-south sudan	12.307	0.000
nigeria-sri lanka	34.401	0.000
nigeria-turkey	56.456	0.000
nigeria-united states	109.678	520.000
north korea-russian federation	20.525	0.000
north korea-south korea	4.303	1.000
north korea-united states	60.645	135.000
north korea-uzbekistan	6.361	0.000
north korea-zimbabwe	8.433	0.000
norway-russian federation	14.781	0.000
norway-switzerland	6.524	135.000
norway-thailand	8.819	0.000
norway-united states	49.079	0.000
oman-qatar	5.668	0.000
oman-united states	109.331	136.000
pakistan-panama	11.685	270.000
pakistan-sri lanka	20.586	0.000
pakistan-thailand	9.489	0.000

pakistan-turkey	18.510	0.000
pakistan-united states	46.803	0.000
palestine, state of-syria	25.858	0.000
panama-qatar	4.011	0.000
panama-united states	65.203	0.000
philippines-russian federation	12.783	0.000
philippines-singapore	3.942	0.000
philippines-thailand	6.993	0.000
philippines-united states	39.160	126.000
poland-portugal	6.991	0.000
poland-spain	6.618	0.000
poland-sweden	5.224	0.000
poland-united states	54.438	0.000
portugal-spain	12.341	136.000
puerto rico-united states	106.018	132.000
qatar-spain	12.598	0.000
qatar-syria	13.278	0.000
qatar-united states	51.259	136.000
russian federation-singapore	12.150	0.000
russian federation-slovakia	28.494	0.000
russian federation-south africa	18.474	0.000
russian federation-spain	14.349	0.000
russian federation-sweden	11.715	0.000
russian federation-switzerland	9.012	0.000
russian federation-syria	14.278	0.000
russian federation-turkey	25.957	763.000
russian federation-ukraine	10.564	0.000
russian federation-united states	53.303	1589.000
russian federation-uzbekistan	17.759	0.000
russian federation-venezuela	11.346	0.000
russian federation-vietnam	25.482	0.000
rwanda-tanzania	6.811	0.000
rwanda-uganda	52.133	0.000
saudi arabia-united states	101.344	136.000
serbia-turkey	129.780	0.000
singapore-somalia	15.032	0.000
singapore-thailand	8.562	0.000
singapore-united states	28.442	136.000
singapore-vietnam	7.058	0.000
slovakia-sweden	6.781	0.000
slovenia-switzerland	13.815	1.000
somalia-united states	88.434	260.000
south africa-turkey	17.080	0.000
south africa-united states	35.203	402.000
south africa-zimbabwe	7.285	270.000
south korea-united states	48.216	135.000
south sudan-syria	9.753	0.000
south sudan-turkey	15.464	0.000
south sudan-uganda	13.230	136.000
south sudan-united states	39.033	0.000
south sudan-zimbabwe	5.058	0.000

spain-sweden	7.013	0.000
spain-turkey	22.061	0.000
spain-united states	41.400	0.000
swaziland-united states	136.000	136.000
sweden-thailand	9.621	136.000
sweden-united states	43.845	0.000
switzerland-turkey	12.346	0.000
switzerland-united states	30.498	0.000
syria-turkey	9.248	0.000
syria-ukraine	7.345	0.000
syria-united states	26.871	264.000
syria-venezuela	6.435	0.000
syria-yemen	31.818	0.000
syria-zimbabwe	10.327	0.000
tanzania-uganda	45.796	0.000
thailand-turkey	25.805	0.000
thailand-united states	70.372	532.000
thailand-zambia	35.410	0.000
tunisia-united states	117.232	0.000
turkey-ukraine	12.558	0.000
turkey-united states	56.128	0.000
turkey-uzbekistan	18.688	0.000
turkey-venezuela	13.837	0.000
turkey-yemen	53.535	263.000
turkey-zambia	76.812	0.000
turkey-zimbabwe	14.809	0.000
uganda-united states	149.787	0.000
ukraine-united states	38.824	136.000
united states-uzbekistan	47.327	0.000
united states-venezuela	34.525	0.000
united states-vietnam	66.776	266.000
united states-zimbabwe	47.075	0.000
uzbekistan-venezuela	8.787	0.000
uzbekistan-zimbabwe	7.537	0.000
zambia-zimbabwe	24.445	136.000

S-WUN Node Centrality Measures

Closeness and betweenness (node) centrality measures have been computed using as distances the inverse of the edges' weights. Whereas, the eigencentrality measure has been computed using the original weighted network matrix. Eigencentrality values have been then rescaled in the $[0, 1]$ *interval;*

Table 3.5. Weighted Node Centrality Measures

Country Name	Weighted Closeness Centrality	Weighted Betweenness Centrality	Rescaled Weighted Eigencentrality
GB/UK	0.00087063	4222	0.70987
afghanistan	0.00086982	135	0.02762
albania	0.00076506	0	0.00037
algeria	0.00082357	0	0.00162
angola	0.00083864	0	0.00002
arab emirates	NA	0	0.00033
argentina	0.00083124	0	0.00114
armenia	0.00076921	0	0.00042
australia	0.00086429	134	0.08842
austria	0.00081885	0	0.00113
azerbaijan	0.00077244	0	0.00001
bahrain	0.00071974	0	0.00044
bangladesh	0.00085550	0	0.02900
belarus	0.00084246	0	0.00010
belgium	0.00085647	266	0.00292
belize	0.00077906	0	0.00033
benin	NA	0	0.00033
bhutan	0.00082438	0	0.00007
bolivia	0.00041578	0	0
bosnia and herzegovina	0.00083211	266	0
botswana	NA	0	0.00033
brazil	0.00084928	269	0.01556
brunei	NA	0	0.00033
bulgaria	0.00085588	0	0.00037
burkina faso	0.00056774	0	0
burundi	0.00068348	0	0
cambodia	0.00086474	0	0.00625
cameroon	0.00081922	0	0.00070
canada	0.00086338	397	0.23512
cape verde	NA	0	0.00033
central african republic	NA	0	0.00033
chad	0.00085325	0	0.00679
chile	0.00082944	0	0.00221
china	0.00086552	1159	0.80218
colombia	0.00086046	273	0.00545
congo	0.00077177	0	0.00045
costa rica	0.00042227	0	0
croatia	0.00082295	0	0.00141
cuba	0.00087257	401	0.14309
cyprus	0.00084104	0	0.00752

czech republic	0.00076282	0	0.00005
côte d'ivoire	NA	0	0.00033
denmark	0.00076878	0	0.00001
ecuador	NA	0	0.00033
egypt	0.00084167	0	0.00193
el salvador	0.00005102	0	0
eritrea	0.00081518	0	0
estonia	0.00084577	0	0.00133
ethiopia	0.00078748	135	0.00003
finland	0.00085568	0	0.00150
france	0.00085387	256	0.10379
gabon	0.00086201	0	0.00422
gambia	0.00081153	1	0.00001
georgia	0.00081011	268	0.00102
germany	0.00085735	637	0.08661
ghana	0.00086501	261	0.00566
greece	0.00086167	137	0.00998
guatemala	0.00084075	134	0.00141
guinea	0.00005102	0	0
guinea-bissau	NA	0	0.00033
guyana	0.00086428	0	0.00532
haiti	0.00084884	0	0.00445
honduras	0.00005102	0	0
hong kong	0.00086184	0	0.04975
hungary	0.00084256	526	0.00051
iceland	0.00082137	0	0.00045
india	0.00086719	1516	0.46454
indonesia	0.00086495	0	0.15609
iran	0.00085654	0	0.01709
iraq	0.00084440	0	0.00411
ireland	0.00085959	135	0.11189
islamic state	0.00085802	266	0.02700
israel	0.00086059	266	0.01996
italy	0.00085781	278	0.02169
jamaica	0.00085085	0	0.00424
japan	0.00086622	135	0.19303
jordan	0.00085463	0	0.00611
kazakhstan	0.00082017	0	0.00007
kenya	0.00083662	6	0.00226
kuwait	0.00077420	0	0.00011
kyrgyzstan	NA	0	0.00033
laos	0.00077133	0	0.00001
latvia	0.00075320	0	0.00005
lebanon	0.00085652	0	0.00679
lesotho	NA	0	0.00033
liberia	0.00016601	0	0
libya	0.00084208	135	0.00389
lithuania	NA	0	0.00033
luxembourg	NA	0	0.00033
macedonia	0.00077188	0	0.00001
madagascar	NA	0	0.00033

malawi	0.00076763	0	0
malaysia	0.00085215	0	0.00826
mali	NA	0	0.00033
malta	0.00085044	0	0.00168
mauritania	NA	0	0.00033
mexico	0.00086141	0	0.06036
moldova	0.00076249	0	0.00004
mongolia	0.00080204	0	0.00039
montenegro	NA	0	0.00033
morocco	0.00087145	0	0.03203
mozambique	0.00081189	135	0.00105
myanmar	0.00081962	0	0.00253
namibia	0.00081245	0	0.00010
nepal	0.00082838	399	0.00694
netherlands	0.00084370	0	0.00358
new zealand	0.00083295	0	0.00146
nicaragua	0.00078062	0	0.00007
niger	0.00076747	0	0
nigeria	0.00085617	400	0.00637
north korea	0.00086844	0	0.02420
norway	0.00085570	134	0.02790
oman	0.00082469	0	0.00094
pakistan	0.00084887	401	0.01963
palestine, state of	0.00085841	133	0.00180
panama	0.00082602	135	0.00165
papua new guinea	0.00005102	0	0
paraguay	0.00042227	0	0
peru	0.00081126	0	0.00039
philippines	0.00085554	0	0.13420
poland	0.00085386	758	0.00702
portugal	0.00083551	0	0.00120
puerto rico	0.00082502	0	0.00095
qatar	0.00084663	0	0.00193
romania	0.00085065	0	0.00167
russian federation	0.00086823	1513	0.07443
rwanda	0.00074932	135	0
saudi arabia	0.00087043	0	0.02955
senegal	0.00083265	134	0.00001
serbia	0.00083457	397	0.00002
singapore	0.00086081	0	0.09583
slovakia	0.00085390	0	0.00200
slovenia	0.00001647	0	0
somalia	0.00085998	133	0.00499
south africa	0.00087126	268	0.02247
south korea	0.00086054	0	0.01216
south sudan	0.00085567	0	0.00064
spain	0.00084573	143	0.01644
sri lanka	0.00078510	0	0.00024
suriname	NA	0	0.00033
swaziland	0.00078120	0	0.00047
sweden	0.00083088	0	0.00185

switzerland	0.00083192	0	0.00204
syria	0.00085528	135	0.03307
tajikistan	NA	0	0.00033
tanzania	0.00077152	0	0.00001
thailand	0.00086237	402	0.01939
timor-leste	NA	0	0.00033
togo	NA	0	0.00033
trinidad and tobago	NA	0	0.00033
tunisia	0.00081269	0	0.00048
turkey	0.00086757	638	0.02858
turkmenistan	0.00077841	0	0.00001
uganda	0.00085922	135	0.02225
ukraine	0.00086094	0	0.02400
united states	0.00087330	7404	1
uruguay	0.00083780	0	0.00100
uzbekistan	0.00085806	0	0.01355
venezuela	0.00083195	0	0.00073
vietnam	0.00086686	131	0.01186
yemen	0.00082009	128	0.00010
zambia	0.00073857	0	0.00001
zimbabwe	0.00082041	135	0.00061

S-WUN Local Transitivity Measures

Table 3.6. S-WUN Local transitivity scores

Country Name	Local Transitivity
GB/UK	0.47705
afghanistan	0.76404
albania	0.40360
algeria	0.84574
angola	0.57400
arab emirates	NA
argentina	0.86429
armenia	1.00000
australia	0.77796
austria	0.69079
azerbaijan	NA
bahrain	0.80347
bangladesh	0.50376
belarus	0.00000
belgium	0.44471
belize	NA
benin	NA
bhutan	NA
bolivia	1.00000
bosnia and herzegovina	0.00000
botswana	NA
brazil	0.52161
brunei	NA
bulgaria	0.50532
burkina faso	1.00000
burundi	1.00000
cambodia	1.00000
cameroon	0.60635
canada	0.77477
cape verde	NA
central african republic	NA
chad	1.00000
chile	0.65753
china	0.65661
colombia	0.47900
congo	0.15686
costa rica	1.00000
croatia	0.77970
cuba	0.67771
cyprus	0.75546
czech republic	1.00000
côte d'ivoire	NA
denmark	1.00000
ecuador	NA
egypt	0.30559
el salvador	NA

eritrea	0.00000
estonia	NA
ethiopia	0.08571
finland	0.71745
france	0.69419
gabon	NA
gambia	0.08904
georgia	0.64780
germany	0.66079
ghana	0.01635
greece	0.62130
guatemala	0.03448
guinea	NA
guinea-bissau	NA
guyana	NA
haiti	0.42818
honduras	NA
hong kong	0.85921
hungary	0.15243
iceland	0.35846
india	0.73109
indonesia	0.95158
iran	0.76514
iraq	0.51575
ireland	0.83008
islamic state	0.70679
israel	0.41269
italy	0.60466
jamaica	1.00000
japan	0.67680
jordan	0.25305
kazakhstan	NA
kenya	0.36492
kuwait	0.28754
kyrgyzstan	NA
laos	0.00000
latvia	0.39035
lebanon	0.70918
lesotho	NA
liberia	1.00000
libya	0.47207
lithuania	NA
luxembourg	NA
macedonia	0.15385
madagascar	NA
malawi	NA
malaysia	0.87723
mali	NA
malta	1.00000
mauritania	NA
mexico	0.83560

moldova	0.43920
mongolia	1.00000
montenegro	NA
morocco	1.00000
mozambique	0.47467
myanmar	0.70978
namibia	NA
nepal	0.31053
netherlands	0.82614
new zealand	0.62311
nicaragua	0.54545
niger	NA
nigeria	0.24990
north korea	0.69542
norway	0.94233
oman	1.00000
pakistan	0.61799
palestine, state of	0.63082
panama	0.27308
papua new guinea	NA
paraguay	1.00000
peru	0.33333
philippines	0.85798
poland	0.51069
portugal	0.62599
puerto rico	1.00000
qatar	0.75646
romania	0.84328
russian federation	0.58952
rwanda	0.22500
saudi arabia	0.73723
senegal	0.00000
serbia	0.00000
singapore	0.90221
slovakia	1.00000
slovenia	1.00000
somalia	0.56250
south africa	0.78151
south korea	0.63159
south sudan	0.28141
spain	0.64061
sri lanka	0.84639
suriname	NA
swaziland	NA
sweden	0.64197
switzerland	0.61108
syria	0.62150
tajikistan	NA
tanzania	0.38911
thailand	0.62261
timor-leste	NA

togo	NA
trinidad and tobago	NA
tunisia	1.00000
turkey	0.52465
turkmenistan	NA
uganda	0.32481
ukraine	0.77121
united states	0.39076
uruguay	NA
uzbekistan	0.72431
venezuela	0.68949
vietnam	0.76889
yemen	0.27500
zambia	0.60000
zimbabwe	0.40729

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Estratto per riassunto della tesi di dottorato

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Dottorato: Economia

Ciclo: 30

Titolo della tesi : Talking About Uncertainty / Parlando dell'Incertezza

Estratto (Italiano):

Nel primo articolo di questo lavoro analizziamo la letteratura sull'incertezza. Dedichiamo particolare attenzione alla relazione tra metacognizione, incertezza ed aspettative. Analizziamo il ruolo del linguaggio e della comunicazione nell'emergere e nella risoluzione di stati di incertezza. Ipotizziamo che le persone si sentono incerte in relazione alla sorpresa attesa, che dipende dal grado di divergenza fra le aspettative probabilistiche di diversi agenti, elicitate pubblicamente durante processi di comunicazione e metacognizione sociale. Livelli di sorpresa attesa intollerabili o superiori alla media, resi espliciti attraverso il linguaggio, possono essere considerati segnali informativi utilizzabili per coordinare processi di revisione delle aspettative all'interno di un gruppo o della società civile. Il secondo articolo vuole dimostrare, in modo empirico, che possiamo estrarre ed aggregare i segnali di incertezza provenienti da fonti decentralizzate, come agenti di mercato e membri della società civile, utilizzando Internet e più specificatamente Twitter come archivio di informazioni che contiene la "saggezza delle folle" riguardo allo stato di incertezza in una specifica comunità o gruppo, in un dato momento. Estraiamo ed aggreghiamo questi segnali, costruendo un insieme di indici di incertezza della società civile per Paese. Modellizziamo la dipendenza tra i nostri indici di incertezza della società civile e alcuni proxy del livello di incertezze politica e di mercato preesistenti, evidenziando le differenze nella loro reattività ad eventi reali verificatesi nel 2016, come il Referendum sulla permanenza del Regno Unito nell'Unione Europea e le elezioni Presidenziali negli Stati Uniti. Infine, nel terzo articolo proponiamo un nuovo strumento, chiamato ***Worldwide Uncertainty Network****, per misurare ed analizzare le dinamiche ed interdipendenze dell'incertezza della società civile dei diversi Paesi del mondo. Questo strumento può essere utilizzato per identificare l'importanza sistemica dei diversi Paesi, in termini del loro ruolo nella percolazione sociale dell'incertezza a livello continentale e/o globale. I risultati di questo studio dimostrano che i segnali di incertezza provenienti da Twitter possono essere utilizzati per migliorare la nostra comprensione dei meccanismi di contagio e amplificazione dell'incertezza, sia fra Paesi che fra mercati, società civile e sistemi politici;*

Abstract (English):

In the first article we review existing theories of uncertainty. We devote particular attention to the relation between metacognition, uncertainty and expectations. We also analyse the role of natural language and communication for the emergence and resolution of states of uncertainty. We hypothesize that agents feel uncertainty in relation to their levels of expected surprise, which depends on probabilistic expectations-gaps elicited during communication processes. Under this framework above tolerance levels of expected surprise can be considered informative signals. These signals can be used to coordinate, at the group and social level, processes of revision of probabilistic expectations. When above tolerance levels of uncertainty are explicated by agents through natural language, in communication networks and public information arenas, uncertainty acquires a systemic role of coordinating

device for the revision of probabilistic expectations. The second article of this research seeks to empirically demonstrate that we can crowd source and aggregate decentralized signals of uncertainty, i.e. expected surprise, coming from market agents and civil society by using the web and more specifically Twitter as an information source that contains the "wisdom of the crowds" concerning the degree of uncertainty of targeted communities/groups of agents at a given moment in time. We extract and aggregate these signals to construct a set of civil society uncertainty proxies by country. We model the dependence among our civil society uncertainty indexes and existing policy and market uncertainty proxies, highlighting contagion channels and differences in their reactiveness to real-world events that occurred in the year 2016, like the EU-referendum vote and the US presidential elections. Finally, in the third article we propose a new instrument, called ***Worldwide Uncertainty Network****, to analyse the uncertainty contagion dynamics across time and areas of the world. Such an instrument can be used to identify the systemic importance of countries in terms of their civil society uncertainty social percolation role. Our results show that civil society uncertainty signals coming from the web may be fruitfully used to improve our understanding of uncertainty contagion and amplification mechanisms among countries and between markets, civil society and political systems;*

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