

# Handbook on Seasonal Adjustment

2018 edition





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## Foreword

Since the last decade of the 20th century, with the high priority attributed to the production of infra-annual statistics, Eurostat started to invest on the enhancement of the seasonal adjustment process at the European and national levels. In a first phase the focus was put on the identification of the method(s) to be recommended for the European Statistical System (ESS). After a long discussion and a detailed analysis only two methods, and the associated software, were retained: the ARIMA model-based approach implemented in TRAMO-SEATS and the moving average approach implemented in X-12-ARIMA with a slight preference for the former. Then Eurostat concentrated its activity on the development of a new and user friendly software for seasonal adjustment. The outcome of this period was the release in 1998 of Demetra, an interface to both TRAMO-SEATS and X-12-ARIMA. Finally Eurostat produced a set of recommendations to harmonize the seasonal adjustment process within the European Union. The first version of these recommendations was presented at the Seasonal Adjustment Methods Seminar (SAM-98) held in Bucharest in October 1998. A slightly revised version of the recommendations, amended to take into accounts the specificities of national accounts, was included as Chapter 9 of the Handbook on quarterly national accounts published in 1999 by Eurostat.

From the early 2000's Eurostat and the European Central Bank jointly worked to foster the harmonization of the seasonal adjustment process with particular attention to some specific series. During this period several task forces were organized to provide recommendations for the adjustment of key indicators like the Harmonized Index of Consumer Prices, the monetary aggregates, the quarterly national accounts, etc. A milestone in this harmonization process among countries and sectors was the constitution in 2005 of the Seasonal adjustment steering group (now Seasonal adjustment Expert Group) co-chaired by Eurostat and the ECB. The two main achievements of this group of high level European experts are: the European Statistical System (ESS) Guidelines on seasonal adjustment, first published in 2009 and revised in 2015 and the release of the innovative JDemetra+ software whose version 2.0 has been officially recommended, since 2 February 2015, to the members of the ESS and the European System of Central Banks as software for seasonal and calendar adjustment of official statistics.

This Handbook is in fact the third component of a triptych of products including also the guidelines and the JDemetra+ software. It aims to address all relevant aspects of the seasonal adjustment process also with a forward looking spirit so that aspects considered not particularly important for the regular production of seasonally adjusted data have been included. This is the case for example of the estimation of moving trading days, the improvements of asymmetric filters and the seasonal adjustment of daily and weekly data.

The Handbook is a collection of contributions written by high level experts in the field of seasonal adjustment from all around the world. The handbook has benefited of the comments and suggestions from ESS members in the context of a consultation organized by Eurostat and the Seasonal Adjustment Expert Group.

Eurostat would like to express its appreciation and gratitude to the Editor Gian Luigi Mazzi (formely Eurostat) and his co-editor Dominique Ladiray (French National Institute of Statistics and Economic Studies—INSEE) for the time and effort they dedicated to bring this Handbook to fruition; and to all authors having contributed to the various chapters. A special thanks goes to Dan A. Rieser, Eurostat, for his role as technical editor and the supervision of the entire publication process.



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# Table of contents

<b>1</b>	<b>Introduction</b>	<b>23</b>
	<i>by Dominique Ladiray and Gian Luigi Mazzi</i>	
1.1	Introduction	25
1.2	Seasonal adjustment in the production process of infra-annual statistics	26
1.3	Aim, scope and limitations of the handbook	27
1.4	Structure of the handbook	28
1.5	Conclusions	29
<b>I</b>	<b>General Aspects</b>	<b>32</b>
<b>2</b>	<b>Seasonal Adjustment: Objectives, Definitions, Costs and Benefits</b>	
	<i>by Estela Bee Dagum and Gian Luigi Mazzi</i>	<b>33</b>
2.1	Introduction	35
2.2	The Main Characteristics and Cost of Seasonality	36
2.3	Limitations of Same-Month Year-ago Comparisons	38
2.4	Seasonal Adjustment Methods	40
2.4.1	Moving Average Methods	40
2.4.2	Model-based Seasonal Adjustment Methods	41
2.4.2.1	Regression methods	41
2.4.2.2	Stochastic model based methods	42
2.5	The Calendar variations	42
2.5.1	The Moving-Holiday Component	42
2.5.2	The Trading-Day Component	44
2.5.2.1	Causes and Costs of Daily Patterns of Activity	44
2.5.2.2	A Classical Model for Trading-Day Variations	45
<b>3</b>	<b>Time Series Components</b>	<b>49</b>
	<i>by Estela Bee Dagum and Gian Luigi Mazzi</i>	
3.1	Introduction	51
3.2	Time Series Decomposition Models	51
3.3	The Secular or Long-Term Trend	53
3.3.1	Deterministic Trend Models	54
3.3.2	Stochastic Trends	54
3.4	The Business Cycle	55
3.4.1	Deterministic and Stochastic Models for the Business Cycle	56
3.5	The Seasonal Variations	57
3.5.1	Deterministic and Stochastic Models for Seasonality	58
3.6	Calendar Variations	59
3.6.1	The Moving-Holiday Component	59

3.6.2	The Trading-Day Component	60
3.6.3	A Classical Model for Trading-Day Variations	62
3.7	The Irregular Component	63
3.7.1	Redistribution Outliers and Strikes	64
3.7.2	Models for the Irregular Component and Outliers	64
<b>4</b>	<b>A Brief History of Seasonal Adjustment Methods and Software Tools</b>	
	<i>by Olivier Darné, Laurent Ferrara and Dominique Ladiray</i>	<b>67</b>
4.1	Introduction	69
4.2	Non-parametric methods (implicit models)	70
4.2.1	X-11 method	70
4.2.2	X-11 style methods	72
4.2.2.1	SEASABS	72
4.2.2.2	GLAS	72
4.2.2.3	STL	73
4.2.2.4	SABL	73
4.3	Semi-parametric methods (hybrid models)	73
4.3.1	X11-ARIMA	73
4.3.2	X12-ARIMA	74
4.4	Parametric methods (explicit models)	74
4.4.1	Determinist methods	74
4.4.1.1	DAINTES	75
4.4.1.2	BV4	75
4.4.2	Stochastic methods	75
4.4.2.1	ARIMA-model based approach	75
4.4.2.2	Structural time series approach	77
4.5	Conclusions	79
4.6	Annex: A short description of the JDemetra+ software	81
4.6.1	Statistical aspects	81
4.6.1.1	State space models	81
4.6.1.2	Seasonal adjustment	81
4.6.1.3	Other diagnostics	82
4.6.2	Technical aspects	82
<b>II</b>	<b>Pre-Treatment Methods</b>	<b>92</b>
<b>5</b>	<b>Calendar Effects</b>	
	<i>by Dominique Ladiray</i>	<b>93</b>
5.1	Introduction	95
5.2	The Various Calendar Effects	95
5.2.1	A few words on calendars	95
5.2.2	The Gregorian Calendar	96
5.2.3	The Trading-Day Effect	96
5.2.4	Moving Holidays	97
5.2.5	Impact models	97
5.2.6	Why should we take into account trading-day effects?	97
5.3	Detection and Estimation of Trading-Day Effects by Regression Analysis	98
5.3.1	The Basic Flow Day-of-Week Effect Model	98
5.3.2	The Basic Flow Week-Day Effect Model	100
5.4	Improving the Regression Model	101
5.4.1	Taking into Account National Holidays	101

5.4.2	Constructing a Relevant Set of Regressors	102
5.4.2.1	Example 1: The French Industrial Production Index	102
5.4.2.2	Example 2: The French Retail Trade Sector	102
5.4.2.3	Impact on the stability of the trading-day estimates	103
5.5	Impact Models	105
5.5.1	Models for Easter effect	105
5.5.1.1	The “SCeaster” model	106
5.5.1.2	The “Easter” model	107
5.5.1.3	Maillard’s Model	107
5.5.2	Models for the Ramadan effect	109
5.5.3	More general models	109
5.6	Spectral detection of trading-day effects	111
5.6.1	The Calendar Periodograms and the Trading-Day Frequencies	111
5.6.2	The USCB “Visual Test”	111
5.6.3	An example	112
5.7	Conclusions	114
<b>6</b>	<b>Moving Trading-Day Effects with X-13 ARIMA-SEATS and TRAMO-SEATS</b>	
	<i>by Ketty Attal-Toubert, Dominique Ladiray, Marco Marini and Jean Palate</i>	<b>117</b>
6.1	Introduction	119
6.2	The basic model with fixed coefficients	120
6.3	Stochastic models for time-varying trading-day coefficients	121
6.3.1	The basic structural model	121
6.3.2	Examples	124
6.3.3	RegArima Models	124
6.3.4	RegComponents Models	125
6.4	Improving the basic model for time-varying trading-day coefficients	126
6.4.1	The X-13ARIMA-SEATS “change-of-regime”, “history” and “slidingspans” specifications	127
6.4.2	Using rolling Windows to estimate moving trading-day effects	127
6.4.3	RegARIMA models with time-varying trading-day coefficients	129
6.5	Examples	130
6.5.1	Methodology	130
6.5.2	Results	130
6.6	Conclusions	133
<b>7</b>	<b>Outlier Detection and Correction</b>	
	<i>by Jens Mehrhoff</i>	<b>137</b>
7.1	The role of outlier modelling in seasonal adjustment	139
7.2	Aims of outlier modelling within the framework of seasonal adjustment	139
7.2.1	Elimination of factors that bias the estimation of the seasonal component	139
7.2.2	Adjustment for calendar effects with the aid of regARIMA modelling	140
7.2.3	Linearisation of the time series and Gaussian regARIMA residuals	140
7.2.4	Forecast of seasonal and calendar components	140
7.2.5	Estimation of the trend-cycle as a secondary aim	141
7.3	The different types of outliers	141
7.3.1	Overview of types of outliers	141
7.3.2	Outliers in the irregular component	142
7.3.3	Outliers in the trend-cycle	144
7.3.4	Outliers in the seasonal component	155
7.3.5	Two sides of the same coin: minimisation of false positives vs. false negatives	156
7.4	Estimation techniques for outlier modelling in the context of regARIMA models	158
7.5	Practical considerations as regards the detection and correction of outliers	161

7.5.1	Revisions of seasonally adjusted data	161
7.5.2	Treatment of (possible) outliers at the current end of the time series	162
7.6	Special issues in the treatment of outliers in seasonal adjustment	163
7.6.1	Extreme value detection in X-11	163
7.7	Conclusions	166
<b>8</b>	<b>Automatic Outlier Detection for the Basic Structural Time Series Model</b>	
	<i>by Stefano Grassi, Gian Luigi Mazzi and Tommaso Proietti</i>	<b>169</b>
8.1	Introduction	171
8.2	The Basic Structural Time Series Model	171
8.3	The detection of Outliers	173
8.3.1	Outlier types	173
8.4	Automatic outliers detection methods in seasonal adjustment routines	176
8.5	State space models and methods to detect outliers	178
8.5.1	The Kalman filter	179
8.5.2	State fixed interval smoother	179
8.5.3	Disturbance smoother	180
8.6	An automatic detection procedure	180
8.6.1	Critical values	181
8.6.2	Outline of the procedure	184
8.6.3	Comments	185
8.7	Illustrations	185
8.7.1	Italian industrial production: Sector CK	185
8.7.2	Italian industrial production: sector CH	186
8.7.3	Italian industrial production: sector CG	188
8.7.4	A tale of a seasonal outlier	188
8.8	Endogenous outliers and Non-Gaussian seasonal adjustment	192
8.9	Conclusions	192
<b>9</b>	<b>Transformations and Seasonal Adjustment of Economic Time Series</b>	
	<i>by Tommaso Proietti and Marco Riani</i>	<b>195</b>
9.1	Introduction	197
9.2	Box Cox family of transformations	198
9.3	Transformations and seasonal adjustment: state of the art	202
9.3.1	TRAMO/SEATS	202
9.3.2	X12 ARIMA	202
9.3.3	SABL	203
9.4	A unified approach to transformations and seasonal adjustment	203
9.4.1	Seasonal Adjustment and the Box-Cox Transformation	204
9.4.2	Analytical solutions	205
9.4.3	Approximate and computational solutions	206
9.5	Robustness issues in transformations	207
9.6	Conclusions	208
<b>III</b>	<b>Seasonal Adjustment Methods</b>	<b>214</b>
<b>10</b>	<b>Background and Perspectives for ARIMA Model-Based Seasonal Adjustment</b>	
	<i>by David F. Findley and Tucker McElroy</i>	<b>215</b>
10.1	Introduction	219
10.2	Conceptual Overview*	220
10.3	Fundamental Examples*	221



10.4	Linear Regression Applied for Signal Extraction*	222
10.4.1	Basic Examples of Covariance Properties Not Inherited by Estimates	223
10.5	Spectral Densities of Stationary Series*	223
10.5.1	Transfer Functions and the ARMA Spectral Density Formula*	224
10.5.1.1	ARMA Conventions	224
10.5.1.2	The Spectral Density Formula	225
10.5.2	Spectral Density Sums and Uncorrelated Decompositions*	225
10.5.2.1	The Canonical Sd Decomposition of an Invertible MA(1)	225
10.6	Canonical Decomposition of a First-Order Seasonal Autoregression*	226
10.6.1	Signal and Noise Filters of the Initial, Intermediate, and Final Years	228
10.6.2	The Error Variance Matrix of the Estimates	229
10.7	Pseudo-Spectral Densities of ARIMA Models*	230
10.8	Canonical Pseudo-Spectral Density Decompositions*	231
10.8.1	The Canonical Trend-Irregular Decomposition of the IMA(1,1)	231
10.8.2	A Sometimes Nonadmissible 3-Component Canonical Decomposition*	232
10.8.3	3-Component Pseudo-Spectral Density Decomposition of the Biannual Seasonal Random Walk	233
10.9	Matrix Formulation of Signal Extraction	234
10.9.1	Assumption A and Random Walk Forecasts and Backcasts*	234
10.9.2	Required Properties of the Stationarized Data, Signal, and Noise	234
10.9.3	The Two-Component Estimation Formulas	235
10.9.4	Filter and Error Variance Properties of The Canonical Decomposition*	236
10.10	Illustrative Seasonal Adjustment Filter and Standard Error Graphs*	236
10.11	Standard Errors of Change for Additive Estimates	237
10.11.1	Change from the Preceding Estimate	237
10.11.2	Revisions with Future Data*	239
10.12	Multiplicative Decomposition Estimates from Logs*	239
10.12.1	Standard Errors for Growth Rates*	240
10.13	ARMA and ARIMA Wiener-Kolmogorov Filters	241
10.14	W-K Filter Formula Examples	242
10.14.1	Rederiving the AR(1) <sub>r</sub> Symmetric Filters	242
10.14.2	Infinite W-K Filters	242
10.14.2.1	A Stationary Case: The Invertible Seasonal MA(1) <sub>r</sub>	242
10.14.2.2	Transfer Function Form and Coefficient Decay Rate of ARIMA W-K Filters	243
10.14.2.3	Nonstationary Case: IMA(1,1) Trend-Irregular Decomposition Filters	243
10.15	Biannual Seasonal Random Walk Filters*	244
10.15.1	The Symmetric Filters	245
10.15.2	The Asymmetric Filters	245
10.16	Differencing Operators of General ARIMA Filters	246
10.16.1	What Seasonal Decomposition Filters Annihilate or Preserve	246
10.17	Canonical Decomposition and Smoothing Trade-Offs*	246
10.18	Structural Models: A Trend Estimation Example	247
10.19	Spectral Factorization	247
10.19.1	The MA(1) Case	248
10.20	Regarding AMBSA Software Model Choices and Decomposition Components	248
10.21	Additional Components of Some AMBSA Software Decompositions	248
10.21.1	Stationary Components	249
10.21.2	A Trend-Cycle Decomposition Option for Long Series	249
10.22	Model-Based SA versus X-11 Filter SA	250
<b>11</b>	<b>Seasonality and Unobserved Components Models: An Overview</b>	
	<i>by Andrew Harvey</i>	<b>255</b>

## Table of contents

11.1	Introduction	257
11.2	Dummy variable and trigonometric seasonal models	257
11.2.1	Deterministic seasonality	257
11.2.2	Stochastic dummies	258
11.2.3	Trigonometric seasonality	259
11.2.4	Restricted seasonality and daily observations	259
11.3	Basic structural model and ARIMA models	260
11.3.1	Stationary form of the BSM	260
11.3.2	Seasonal ARIMA models	260
11.4	Trading day and calendar effects	261
11.4.1	Trading day variation	261
11.4.2	Moving festivals	262
11.5	Tests	263
11.5.1	Seasonal stationarity tests	263
11.5.2	Seasonality test	264
11.5.3	Seasonal unit root tests	265
11.5.4	Testing for trading day effects	266
11.6	Seasonal adjustment	266
11.7	Breaks in the seasonal pattern	267
11.8	Seasonal splines and weekly data	269
11.9	Seasonal specific models	271
11.9.1	Seasonal heteroscedasticity	271
11.9.2	Periodic models	272
11.9.3	Relationship between periodic models and trend plus seasonal models	272
11.9.4	Partly periodic models	274
11.10	Data irregularities and survey design	276
11.11	Robust seasonal adjustment	276
11.12	Conclusions	277
<b>12</b>	<b>Moving Average Based Seasonal Adjustment</b>	
	<i>by Dominique Ladiray</i>	<b>281</b>
12.1	Introduction	283
12.2	Definitions and useful tools	284
12.2.1	Definitions and example	285
12.2.2	Gain and phase shift functions	286
12.3	Desirable properties of a moving average	288
12.3.1	Trend preservation	288
12.3.2	Elimination of seasonality	289
12.3.3	Reduction of the irregular component	290
12.3.4	The Slutsky-Yule effect	290
12.3.5	An optimization problem	291
12.4	Usual moving averages	291
12.4.1	Simple moving averages	291
12.4.2	Composite simple moving averages	292
12.4.3	Spencer moving averages	292
12.4.4	Kendall and Stuart moving averages	293
12.4.5	Henderson moving averages	293
12.4.6	Musgrave asymmetric moving averages	295
12.4.7	Generalization of the Whittaker-Henderson approach	297
12.5	The X11 seasonal adjustment algorithm	297
12.5.1	Components and decomposition models	297
12.5.2	A simple seasonal adjustment algorithm	298

12.5.3	The basic algorithm of the X-11 method	299
12.5.4	Extreme observations and calendar effects	300
12.5.5	The iterative principle of X-11	302
12.5.6	From Census X-11 to X-11-ARIMA and X-12-ARIMA	303
12.6	The moving averages used in X-11	304
12.6.1	Composite simple moving averages	304
12.6.2	Estimation of trend: 2x4 and 2x12 averages	304
12.6.3	Estimation of Seasonality: $3 \times 3$ , $3 \times 5$ and $3 \times 9$ Averages	306
12.6.4	Henderson moving averages	307
12.6.5	Musgrave asymmetric moving averages	311
12.6.5.1	Musgrave asymmetric moving averages associated with Henderson symmetric moving averages	311
12.6.5.2	Comment about Musgrave moving averages	311
12.6.5.3	Asymmetric moving averages associated with composite moving averages	312
12.6.6	The X-11 moving average filter	315
12.7	Conclusions	318
<b>13</b>	<b>Quality of Seasonal Adjustment in the Model-Based Approach of TRAMO-SEATS</b>	
	<i>by Agustín Maravall</i>	<b>323</b>
13.1	Introduction	325
13.2	Program TRAMO	325
13.3	Decomposition of the Series	326
13.3.1	Derivation of the models for the components	326
13.3.2	Estimation of the components	328
13.3.2.1	Infinite realization $[x_{-\infty}, \dots, x_t, \dots, x_{+\infty}]$ and historical estimator	328
13.3.2.2	Finite realization $[x_1, x_2, \dots, x_T]$ and preliminary estimators	329
13.3.3	Example	330
13.3.3.1	Observed series $x_t$	330
13.3.3.2	Seasonally adjusted series $n_t$	330
13.3.3.3	Seasonal component $s_t$	331
13.4	Some Properties of the Estimators of the Components	331
13.4.1	Correlation between the estimators of the components	331
13.4.2	Non-linearity of the seasonally adjusted series	332
13.4.3	Non-invertibility	332
13.4.4	ARIMA modeling of SA series	332
13.5	Diagnostics	333
13.5.1	Quality Assessment and Diagnostics	333
13.5.2	Idempotency	339
13.6	Inference	340
13.6.1	Estimation error	340
13.6.2	Revision in SA series	340
13.7	Quality of Seasonal Adjustment	341
13.7.1	Good Model, Bad Seasonal	341
13.7.2	Bad Model, Good Seasonal	347
13.8	Conclusions	359
<b>IV</b>	<b>Improving End-Point Estimates for Seasonal Adjustment</b>	<b>364</b>
<b>14</b>	<b>Trend-Cycle Estimation</b>	
	<i>by Estela Bee Dagum</i>	<b>365</b>
14.1	Introduction	367

14.2	Deterministic and stochastic global trend models	368
14.3	Local trend-cycle models	370
14.4	Local trend cycle estimation	372
14.4.1	Locally Weighted Regression Smoother (LOESS)	372
14.4.2	Henderson Smoothing Filters	372
14.4.3	Dagum's modified 13-term Henderson Filter (DMH)	375
14.5	Gaussian Kernel Smoother	377
14.6	Cubic Smoothing Spline	378
14.7	Theoretical properties of the symmetric and asymmetric linear trend-cycle filters	381
14.8	Illustrative Results	382
<b>15</b>	<b>Asymmetric Moving Averages Minimizing Phase Shift</b>	
	<i>by Michel Grun-Rehomme, Fabien Guggemos and Dominique Ladiray</i>	<b>391</b>
15.1	Introduction	393
15.2	Moving averages: definitions and design	395
15.2.1	Definitions	395
15.2.2	Preserving trends and removing seasonalities	395
15.2.3	Gain and phase shift effects	396
15.3	Optimal moving averages based on revision criteria	397
15.3.1	Musgrave's approach	397
15.3.2	Towards the Generalized Direct Filter Approach	398
15.4	A general unifying framework to derive linear filters	399
15.4.1	From the revision criteria to a very general optimization problem	399
15.4.2	Classical particular cases	400
15.4.2.1	Fidelity	400
15.4.2.2	Smoothness	400
15.5	The Timeliness criterion	401
15.5.1	Minimizing the phase shift	401
15.5.2	The choice of a convenient penalty function	402
15.5.3	Variation on the Timeliness criterion	404
15.6	Operational procedure and applications	405
15.6.1	A mixed criterion	405
15.6.2	Application	406
15.6.2.1	Surrogate Henderson's filters	406
15.6.2.2	More on the asymmetric filters with 3 terms in the future.	407
15.7	Conclusions	409
15.8	Annex: Proofs	410
<b>16</b>	<b>Real Time Trend Extraction and Seasonal Adjustment</b>	
	<i>by Marc Wildi</i>	<b>415</b>
16.1	Introduction	417
16.2	Frequency Domain and Filter Effect	417
16.3	Data-Dependent Filters	419
16.3.1	Mean-Square Error Criterion	419
16.3.2	Controlling Speed and Reliability	420
16.3.3	New Generalized Criterion	421
16.4	Empirical Section	423
16.4.1	Introduction	423
16.4.2	Data and Modeling	424
16.4.3	T-DFA: Replication of Model-Based Mean-Square Performances (Airline Model)	425
16.4.3.1	Trend	426
16.4.3.2	Seasonal Adjustment	428

16.4.4	Customization: Trend Applications	429
16.4.4.1	introduction	429
16.4.4.2	Visualization of the Customization Effect	430
16.4.4.3	Model-Based Customization of Real-Time Model-Based Trend	431
16.4.4.4	Model-Based Customization of Real-Time Ideal Trend	433
16.4.4.5	T-DFA Customization of Seasonal Adjustment Filters	434
16.4.5	Customization beyond the Model Based Perspective	437
16.4.5.1	Seasonal Adjustment	437
16.4.5.2	Trend	440
16.5	Conclusions	448
<b>V</b>	<b>Seasonal Adjustment and Aggregation</b>	<b>452</b>
<b>17</b>	<b>Benchmarking and Temporal Consistency</b>	
	<i>by Benoit Quenneville and Susie Fortier</i>	<b>453</b>
17.1	Introduction	455
17.2	X-12-ARIMA FORCE spec	456
17.2.1	Benchmarking formulae	457
17.2.2	The type argument	457
17.3	Example	457
17.4	Alternative Benchmarking Software	460
17.5	Model-Based Methods	460
17.6	Conclusions	461
<b>18</b>	<b>Seasonal Adjustment of European Aggregates: Direct versus Indirect Approach</b>	
	<i>by Roberto Astolfi, Dominique Ladiray and Gian Luigi Mazzi</i>	<b>465</b>
18.1	Introduction	467
18.2	The direct versus indirect Problem	468
18.2.1	Direct, Indirect and Mixed Indirect Seasonal Adjustments	468
18.2.2	Could Direct and Indirect Approaches Coincide ?	469
18.2.3	A Priori Advantages and Drawbacks	470
18.3	Methodology	471
18.3.1	Various Causes of Revisions	471
18.3.2	Quality Measures	472
18.3.3	Software, Methods and Parameters	474
18.4	Gross Domestic Product, Geographical Aggregation	474
18.4.1	The Data	474
18.4.2	A first comparison between seasonally adjusted series	474
18.4.3	Concordance analysis of growth rates	479
18.4.4	Quality measures of seasonal adjustments	480
18.4.5	Roughness measures	480
18.4.6	Revision analysis	484
18.4.7	Analysis of the residuals	484
18.5	Gross Value Added, Aggregation by Sector	485
18.5.1	The Data	485
18.5.2	A first comparison between seasonally adjusted series	486
18.5.3	Concordance analysis of growth rates	489
18.5.4	Quality measures of seasonal adjustments	490
18.5.5	Roughness measures	490
18.5.6	Revision analysis	490
18.5.7	Analysis of the residuals	490

18.6	Conclusions	495
<b>19</b>	<b>Multivariate Benchmarking: The Direct Versus Indirect Problem</b> <i>by Benoit Quenneville and Susie Fortier</i>	<b>497</b>
19.1	Introduction	499
19.2	Seasonal adjustment of variables linked by aggregation constraints	499
19.2.1	Direct and indirect seasonal adjustment	500
19.2.2	Advantages and disadvantages	500
19.2.3	Guidelines	501
19.2.4	The two-way classification	501
19.2.5	SA of a two-way table	501
19.3	Techniques to force the respect of aggregation constraints of seasonally adjusted data when a direct adjustment is performed	502
19.3.1	Pro-rating	502
19.3.2	Generalization of pro-rating	503
19.3.3	Generalization of pro-rating to higher dimensional tables	503
19.3.4	Simple numerical examples	504
19.3.5	Specifying the alterability coefficients	504
19.4	Movement preservation principle based benchmarking techniques	505
19.4.1	The 2-step based methods	505
19.4.2	The simultaneous method of Di Fonzo and Marini (2010)	506
19.5	Example	506
19.5.1	Bottom-up	506
19.5.2	Top-bottom	506
19.5.3	Middle-up middle-down	513
19.6	Conclusions	513
<b>VI</b>	<b>Revision and Communication</b>	<b>516</b>
<b>20</b>	<b>Revisions</b> <i>by Duncan Elliott, Robert Kirchner and Craig H. McLaren</i>	<b>517</b>
20.1	Introduction	519
20.2	Defining revisions	519
20.3	Causes of revisions	520
20.3.1	Causes of revisions in observed data	521
20.3.1.1	Returns from survey respondents	521
20.3.1.2	Benchmarking data	521
20.3.1.3	Methodological and classification changes	522
20.3.2	Causes of revisions in seasonal adjustment outputs	522
20.3.2.1	Changes to the original estimates	522
20.3.2.2	Seasonal adjustment processes	522
20.3.2.3	Use of filters within seasonal adjustment	523
20.3.2.4	Use of models within seasonal adjustment	527
20.3.2.5	Use of concurrent or current seasonal adjustment	527
20.4	To revise or not to revise?	528
20.5	Revision policies	530
20.5.1	Revision policies for methodological changes	531
20.5.2	Revision policies for original estimates	531
20.5.3	Revision policies for seasonally adjusted estimates	532
20.5.4	Revision policies for specific outputs and methods	533
20.6	Presenting revisions	533

20.6.1	Revision triangles	533
20.6.2	Graphical approaches	534
20.6.2.1	Magnitude of revisions	535
20.6.2.2	Sources of revisions	537
20.6.2.3	Decomposing revisions	539
20.6.2.4	Representing uncertainty	539
20.7	Analysis of revisions	542
20.7.1	Revision diagnostics	542
20.7.1.1	Terminology and notation for revision measures	542
20.7.1.2	Different measures	542
20.7.2	Decomposing revisions	544
20.7.2.1	Decomposition methodology	544
20.7.2.2	Sources of Revisions	545
20.7.2.3	Mehrhoff's model	545
20.7.2.4	Case study: Decomposition of revisions	546
20.7.3	Using X-13-ARIMA-SEATS to assess revisions	548
20.7.3.1	Revision histories	548
20.7.3.2	Producing revision histories	548
20.7.3.3	Using revision histories	549
20.7.3.4	Comparing methods of estimation	553
20.7.3.5	Comparing direct and indirect estimation	553
20.8	Dealing with revisions in practice	554
20.8.1	Case study: Argentine currency in circulation	554
20.8.1.1	Seasonal adjustment during the crisis	556
20.8.1.2	Seasonal adjustment during the crisis revisited	557
20.8.1.3	Implications for seasonal adjustment	558
20.8.2	Case study: Real time databases	560
20.8.3	Case study: Revision analysis using History	560
20.9	Discussion	561
<b>21</b>	<b>Data Presentation Issues</b>	
	<i>by Mark Boxall, Craig H. McLaren and Mark Zhang</i>	<b>567</b>
21.1	Introduction	569
21.2	Data accessibility and availability	569
21.2.1	Data storage	570
21.2.2	Dissemination of metadata	571
21.3	Elements of a statistical release	572
21.3.1	Summary of the principal outputs	572
21.3.2	Tables and graphical presentation of headline information	572
21.3.3	Analysis and commentary on principal indicators	573
21.3.4	Explanatory notes	573
21.3.5	Supplementary information	573
21.3.6	Contact details	574
21.4	Choosing a headline indicator	574
21.4.1	Month on month indicators	574
21.4.2	Year on year indicators	575
21.4.3	Alternative indicators	576
21.4.4	A recommended approach	576
21.5	Graphically: Presenting time series estimates	576
21.5.1	Seasonally adjusted and/or trend-cycle estimates	577
21.5.2	Growth rates	580
21.5.3	Lollypop charts	581

21.5.4	Web based presentation	582
21.6	Statistical releases	582
21.6.1	Examples of different countries presentations	582
21.6.2	Presenting revisions to published estimates	583
21.6.2.1	Revision triangles	583
21.6.2.2	Trumpet graphs (what-if charts)	585
21.7	Press releases and the media	586
21.7.1	Case example: Media interpreting published estimates	586
21.8	Discussion	587
<b>VII</b>	<b>Seasonal Adjustment in Practice</b>	<b>590</b>
<b>22</b>	<b>Quality Measures and Reporting for Seasonal Adjustment</b>	
	<i>by Robert Kirchner, Dominique Ladiray and Gian Luigi Mazzi</i>	<b>591</b>
22.1	Introduction	593
22.2	Desirable properties for a seasonal adjustment method	593
22.2.1	Some properties of “optimal” seasonal adjustment	594
22.2.2	The ESS guidelines on seasonal adjustment	595
22.3	A list of quality measures for seasonal adjustment	596
22.3.1	Common quality measures	596
22.3.2	Specific quality measures	598
22.3.3	The JDEMETRA+ quality report	599
22.3.4	Limits of a pure numerical quality approach.	601
22.4	Examples of quality assessment and quality reporting	602
22.4.1	The quality report for seasonally adjusted data used at the Deutsche Bundesbank	602
22.4.1.1	Preliminary remarks	602
22.4.1.2	Quality control for headline indicators	602
22.4.1.3	Quality control in mass production	603
22.4.1.4	Basic plausibility checks	603
22.4.2	Quality assessment of seasonally adjusted series at the European Central Bank	606
22.4.3	Quality assessment of seasonally adjusted series at ISTAT	607
22.4.3.1	General considerations	607
22.4.3.2	Comments on the measures	611
22.4.4	A quality report used at Eurostat	611
22.4.5	Quality reporting at INSEE	612
22.4.6	Statistics Canada Dashboard	614
22.5	Conclusions	617
22.6	A Detailed List of Quality Measures for Seasonal Adjustment	618
<b>23</b>	<b>Short versus long time series: An empirical analysis</b>	
	<i>by Dario Buono, Enrico Infante and Gian Luigi Mazzi</i>	<b>629</b>
23.1	Introduction	631
23.2	Presentation of the study	633
23.2.1	Theoretical framework	633
23.2.2	Empirical design of the study	634
23.2.3	Tools and methods	634
23.3	Quality measures and assessment criteria	635
23.4	Analysis	637
23.4.1	Results for the EA19	637
23.4.2	Results for all the series	638
23.5	Conclusions	653



<b>24 Chain-linking of Quarterly National Accounts and Implications for Seasonal Adjustment</b>	<b>655</b>
<i>by Robert Kirchner and Marcus Scheiblecker</i>	
24.1 Conceptual considerations	657
24.1.1 Properties of unadjusted data	657
24.1.1.1 Introduction	657
24.1.1.2 Annual overlap method	658
24.1.1.3 Quarterly overlap method	660
24.1.1.4 Over-the-year technique	662
24.1.1.5 Summary	663
24.1.2 Seasonal Adjustment	663
24.1.2.1 Annual overlap method	664
24.1.2.2 Over-the-year-technique	664
24.1.2.3 Quarterly overlap method	664
24.2 Empirical evidence	665
<b>VIII Seasonal Adjustment and Business Cycles</b>	<b>668</b>
<b>25 The Effect of Alternative Seasonal Adjustment Methods on Business Cycle Analysis</b>	
<i>by Riccardo Gatto, Dominique Ladiray and Gian Luigi Mazzi</i>	<b>669</b>
25.1 Introduction	671
25.2 The data	671
25.3 The different seasonal adjustment methods	671
25.4 The graphical inspection and the cluster analysis of the different seasonal adjustment methods	672
25.5 Turning points dating analysis	676
25.6 Conclusions	679
<b>26 The Effect of Seasonal Adjustment on Turning-Point Detection</b>	
<i>by Monica Billio, Leonardo Carati, Dominique Ladiray and Gian Luigi Mazzi</i>	<b>681</b>
26.1 Introduction	683
26.2 Growth Cycle Coincident Indicator	683
26.2.1 Growth Cycle Coincident Indicator components	683
26.2.2 Revision policy	685
26.3 Coincident indicators from alternatively seasonal	685
26.3.1 Description of the time-series	685
26.4 Application	687
26.4.1 Research strategy	687
26.4.2 Evaluation criteria	687
26.4.3 Reference dating chronology of the growth cycle	688
26.4.4 GCCI: X-12-ARIMA, version 0.3, with the "automdl" specification	689
26.4.5 GCCI: X-12-ARIMA, version 0.3, with the "pickmdl" specification	692
26.4.6 GCCI: X-13-AS using X11 and the "pickmdl" specification	692
26.4.7 GCCI: Tramo-Seats	695
26.4.8 GCCI: Dainties	695
26.4.9 GCCI: Baysea	699
26.4.10 GCCI: Stamp	701
26.4.11 GCCI: Decomp	702
26.4.12 GCCI: Results discussion	704
26.5 Conclusions	706
<b>27 Seasonal Adjustment and Business Cycles</b>	
<i>by Gian Luigi Mazzi and Tommaso Proietti</i>	<b>709</b>

27.1	Introduction	711
27.2	Seasonality and Business Cycles Interactions	712
27.2.1	Nonparametric methods for trend-seasonal interaction	713
27.2.2	Parametric Models of Interactions	714
27.2.3	Formal decompositions	715
27.3	The effects of Seasonal Adjustment on Business Cycle Measurement	716
27.3.1	The effect on trend-cycle interactions	717
27.3.2	Seasonal Adjustment and the Reliability of the Cycle Estimates	718
27.3.3	Empirical Evaluation of the Variance Inflation Factor	722
27.3.4	Empirical illustration	722
27.3.5	Discussion	725
27.4	Conclusions	727
<b>IX</b>	<b>Seasonal Adjustment of High Frequency Data</b>	<b>734</b>
<b>28</b>	<b>Weekly Seasonal Adjustment: A Locally-weighted Regression Approach</b> <i>by William P. Cleveland, Thomas Evans and Stuart Scott</i>	<b>735</b>
28.1	Introduction	737
28.2	The Method	737
28.3	Holidays and Outliers	740
28.4	Diagnostics	741
28.5	Example: Initial Claims, U.S. Unemployment Insurance Program	743
28.6	Software	749
28.6.1	Description	749
28.6.2	Input and output files	751
28.6.3	Run instructions with the SAS interface program	753
<b>29</b>	<b>Seasonal Adjustment of Daily and Weekly Data</b> <i>by Dominique Ladiray, Gian Luigi Mazzi, Jean Palate and Tommaso Proietti</i>	<b>757</b>
29.1	Introduction	759
29.2	The Main Characteristics of High Frequency Data	760
29.2.1	Why to use High Frequency Data?	760
29.2.2	Characteristics of Daily and Weekly Data	760
29.2.2.1	Multiple and Non-integer Periodicities	760
29.2.2.2	Important Remarks	760
29.2.3	Checking for the Various Periodicities in the Data	762
29.2.3.1	Spectral Analysis	762
29.2.3.2	Statistical Tests	762
29.3	Pretreatment of Daily and Weekly Data	765
29.3.1	Decomposition Model	767
29.3.2	Calendar Effects	767
29.3.3	Dealing with outliers	769
29.4	Seasonal Adjustment Based on Non-parametric Methods	770
29.4.1	Adapting X11 to multiple and non integer periodicities	770
29.4.1.1	An X11-like algorithm for multiple periodicities	770
29.4.1.2	Dealing with non integer periodicities	772
29.4.2	Improving STL for multiple periodicities	773
29.4.3	Pros and cons of the X11 and STL approaches	774
29.5	Seasonal adjustment based on parametric models	775
29.5.1	Seasonal adjustment based on ARIMA Models	775
29.5.2	Canonical decomposition	775

29.5.3	Estimation of the components	775
29.5.4	Pros and Cons of the current implementation	776
29.5.5	Seasonal adjustment based on unobserved components models	776
29.5.5.1	Pros of the UC approach	776
29.5.5.2	Limitations of the UC approach	777
29.6	Application: Seasonal adjustment of the Electricity series	778
29.6.1	The seasonal patterns	778
29.6.2	Validation of the results	780
29.7	Conclusions	782

## **X ESS Guidelines on Seasonal Adjustment 786**

### **30 The ESS Guidelines on Seasonal Adjustment 787**

30.1	Introduction	791
30.1.1	Motivation for guidelines	791
30.1.2	Scope of guidelines	791
30.1.3	Costs and risks	791
30.1.4	Background to guidelines and basic definitions	792
30.2	Annex: Principles for seasonal adjustment	793
30.3	A policy for seasonal adjustment	794
30.3.1	A general seasonal adjustment policy	794
30.3.2	The need for domain specific seasonal adjustment policies	794
30.3.3	Consistency of general and domain specific policies	795
30.3.4	Stability of seasonal adjustment policies	796
30.3.5	Dissemination of general and domain specific seasonal adjustment policies	797
30.3.6	Quality framework for seasonal adjustment	797
30.4	Pre-treatment	799
30.4.1	General aspects for choosing between detailed and automatic pre-treatment	799
30.4.2	Graphical analysis of the series	800
30.4.3	Calendar adjustment	800
30.4.4	National and EU/euro area calendars	801
30.4.5	Choosing the frequency of time series for calendar adjustment	802
30.4.6	Other calendar related and weather effects	803
30.4.7	General principles of outlier detection and correction	805
30.4.8	Treatment of outliers at the end of the series and at the beginning of a major economic change	806
30.4.9	Model selection	807
30.4.10	Decomposition scheme	807
30.5	Seasonal adjustment	809
30.5.1	Choice of seasonal adjustment method	809
30.5.2	Choice of the software	809
30.5.3	Temporal consistency between unadjusted and seasonally adjusted data	810
30.5.4	Direct and indirect approaches	811
30.5.5	Direct versus indirect approach: dealing with data from different agencies	812
30.5.6	Different seasonal filters for different months/quarters	813
30.6	Revision policies	814
30.6.1	General revision policy and release calendar	814
30.6.2	Concurrent versus current adjustment	815
30.6.3	Length for routine revisions	816
30.6.4	Length for major revisions	816
30.7	Accuracy of seasonal adjustment	818

## Table of contents

30.7.1	Validation policy for seasonal adjustment	818
30.7.2	Measurement for individual series	818
30.7.3	Comparison of alternative approaches/strategies	819
30.8	Specific issues on seasonal adjustment	821
30.8.1	Seasonal adjustment of short and very short time series	821
30.8.2	Seasonal adjustment of long time series	821
30.8.3	Treatment of problematic series	822
30.8.4	Seasonal heteroskedasticity	823
30.8.5	Seasonal adjustment of annually chain-linked series (Laspeyres-type)	824
30.9	Data presentation issues	826
30.9.1	Data availability in databases	826
30.9.2	Press releases	826
30.9.3	Documenting metadata for seasonal adjustment	827

# 1

## Introduction



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## Table of Contents

1.1	Introduction	25
1.2	Seasonal adjustment in the production process of infra-annual statistics	26
1.3	Aim, scope and limitations of the handbook	27
1.4	Structure of the handbook	28
1.5	Conclusions	29



## 1.1 Introduction

The idea of decomposing a time series in unobserved components appeared already in the nineteenth century in the works of economists, some of whom did not hesitate to acknowledge that it came to them directly from astronomy or meteorology.<sup>1</sup> The aim of many studies was then to reveal “cycles”, the study and analysis of which might make it possible to explain and predict economic crises. In these conditions, short-term periodic components, in particular those related to the sequence of seasons, were of little interest and it was expedient to eliminate them:

*Every kind of periodic fluctuations, whether daily, weekly, monthly, quarterly, or yearly, must be detected and exhibited not only as a subject of study in itself, but because we must ascertain and eliminate such periodic variations before we can correctly exhibit those which are irregular or non-periodic and probably of more interest and importance.* Jevons (1862)

The lack of a precise definition of these “periodic fluctuations” and of efficient computational resources hampered for some times the development of relevant statistical methods for the estimation of the underlying trend-cycle component.

In the earlier twentieth century, a large part of time series studies were devoted to the estimation of the seasonal variations. Persons (1919) was certainly one of the first statistician proposing both a (qualitative) definition of the various components of a time series - the trend, the cycle, the seasonal and the irregular components - and a statistical algorithm to estimate them, the “link relatives method”. The process of decomposition was refined by Frederic R. Macaulay (1931), of the National Bureau of Economic Research, who in the 20’s introduced the “ratio-to-moving-average procedure”, variations of which are widely used today. But at the same period, King (1924) and other authors were still advocating the use of free-hand curves to eliminate unwanted components.

Massive or industrial seasonal adjustment started in the 50’s as a direct consequence of the computer revolution. In 1954, the manual “ratio-to-moving-average procedure” was replaced by a computer program (Census I), developed at the US Census Bureau which was modified and enlarged in 1955 and known as Census II method<sup>2</sup>. Since 1955, there have been several variants of Census II starting from X-1 to the famous X-11, and to the currently used X-13ARIMA-SEATS<sup>3</sup>. After the development of the Census II method, especially during the 60’s and 70’s, a number of alternative seasonal adjustment methods supported by computer’s programs was developed, see Chapter 4. Nevertheless they did not have a big impact to compete with Census II variants. In the 80’s, ARIMA-based methods<sup>4</sup> and methods based on structural models<sup>5</sup> were developed with a large impact both in the theoretical and applied worlds, and completed the statistician’s toolbox for seasonal adjustment.

Nowadays the situation has strongly evolved: the short-term evolution of the economy is monitored in quasi real time, and the emphasis is put on international and sectoral comparisons. In this context, infra-annual economic indicators represent nowadays a large part of the production of National statistical institutes and seasonally adjusted data, advocated by international organizations, have become a natural and essential input for policy making activities especially in the field of monitoring the cyclical movement, preventing or alleviate the effect of crises, supporting the monetary policy, ensuring a stable and balanced growth etc.

The structure of this chapter is as follow: section 1.2 briefly presents some aspects related to the seasonal

<sup>1</sup>One can hardly fail to cite the works of the meteorologist Buys-Ballot who, in 1847, studied periodic temperature variations by modelling the trend by a polynomial, seasonality by dummy variables and implicitly relying on linear regression techniques to estimate the parameters. See Buys-Ballot (1847)

<sup>2</sup>See Shiskin and Eisenpress (1957).

<sup>3</sup>See Findley and al (1998).

<sup>4</sup>See Burman (1980), Gómez and Maravall (1997).

<sup>5</sup>See Harvey (1989).

adjustment process; section 1.3 introduces the aim, scope and limitations of this handbook; section 1.4 provides a short overview of the structure of the handbook and the main content of each part while section 1.5 provides some general conclusions.

## 1.2 Seasonal adjustment in the production process of infra-annual statistics

Seasonal adjustment has become an essential part of the production process of most infra-annual statistics. Thanks to the implementation of seasonal adjustment methods into several IT tools and software, it has been integrated in the production process and can be considered nowadays as fully automated. Nevertheless, human intervention remains unavoidable both for the fine tuning of the seasonal adjustment process and the final quality assessment of seasonally adjusted figures. Such quality assessment is especially relevant for key economic indicators playing a crucial role in policy making activities.

In the European Union, both Eurostat and the European Central Bank (ECB) played a very important role in fostering the harmonization and industrialization of seasonal adjustment across European countries. Thanks to strong cooperation between Eurostat and the ECB two main achievements must be highlighted:

- The publication in 2009 of the first “European Statistical System (ESS) Guidelines on Seasonal Adjustment”. These guidelines, revised in 2015<sup>6</sup>, present theoretical aspects and practical implementation issues in a user friendly and easy to read framework, thereby addressing both experts and non-experts in seasonal adjustment. They describe the different steps in seasonal adjustment, highlight the main issues encountered during the adjustment, specify alternative solutions and identify the best practices.
- The release in February 2015 of JDemetra+ as software officially recommended by Eurostat and the ECB, for the seasonal and calendar adjustment of official statistics in the European Union. This software, which implements both the ARIMA-based method popularized by TRAMO-SEATS and the moving average based methods popularized by X-13ARIMA-SEATS, is nowadays used in all European countries and many other countries around the World.

Despite the fact seasonal adjustment has been the subject of hundred of papers during the last 150 years and despite the availability of many methods and tools, it still remains a difficult topic, not to say an open issue, at least for three reasons:

- The objectives of seasonal adjustment appear multiple and implicit. Is it to obtain the best estimate of the trend-cycle component? the best estimate of the seasonal component itself? or even a prediction of the next months or next year?
- The evaluation of the “seasonal component” provided by an adjustment method is hampered by the fact that the true seasonal component remains a theoretical and imprecise concept, never liable to direct observation.
- The information world evolves quickly and brings us new kinds of data. Numerous variables are nowadays collected weekly, daily or even hourly, that could bring valuable information to official statisticians in their evaluation of the state and short-term evolution of the Economy; and the techniques to seasonally adjust them have still to be designed and evaluated.

Problems the user faces are numerous, and here we just try to mention only few of them :

- The extraction of the seasonal component is usually performed using linear filters which are very sensitive to the presence of deterministic or non-linear elements like outliers, breaks and calendar effects. These effects are often detected and corrected before the decomposition of the series itself. Finding an

<sup>6</sup>See Eurostat (2015) and Chapter 30 of this handbook.



efficient balance between the need of cleaning the series as much as possible, and the need of avoiding smoothing the pattern too much, is still a challenge.

- Any seasonal adjustment method based on symmetric linear filters introduces a phase shift at the end of the series and delays the real time detection of a turning point. How can we minimize this delay? More generally, in the context of business cycle analysis, how can we minimize the bias introduced by the seasonal adjustment process?
- How could we integrate in usual univariate seasonal adjustment methods constraints within or between series?
- How should we adapt the methods or our practice to the various nature of raw “time series”? Some series are short, some others long. The first part of long time series has often be reconstructed, for example after a change of classification, using the dynamic of another but related series. Some indicators, like chain-linked indexes, are not strictly speaking time series, etc.
- As the components of a time series are never directly observed, how can the quality of an adjustment be assessed? Is it possible to define an optimal strategy, or set of parameters, to get the best seasonal adjustment?
- As already mentioned, high frequency data - i.e. data collected at hourly, daily or weekly intervals - bring new challenges to the statistician and cannot be easily seasonally adjusted with the currently available methods.

This non exhaustive list of still open problems and questions arising during the seasonal adjustment process show the usefulness both for data producers and users of a reference book helping them in better understanding what to do and how to read the data.

## 1.3 Aim, scope and limitations of the handbook

The present handbook, prepared by Eurostat, aims to conduct a wide review of seasonal adjustment methods and to provide users and producers with the necessary background information to produce high quality figures and to solve the variety of problems that could arise during the adjustment. the handbook is the last component of a triptych of products developed in the last years under the Eurostat responsibility in strong cooperation with the ECB:

- the ESS guidelines on seasonal adjustment: the reference guide to conduct a proper seasonal adjustment;
- The JDemetra+ software: the tool to perform the adjustment;
- The handbook on seasonal adjustment: the reference manual to understand the underlying methodologies.

The handbook is a collection of contributions from internationally recognized authors coming from various institutions and, **even if the ESS guidelines remains the main reference**, it is possible that some chapters do not completely follow the guidelines recommendations. This is for example the case for Chapter 21 that proposes a different publication strategy for the trend-cycle figures.

Various topics are presented in a didactic manner, making the reading easier for both non-expert and skilled users. Complex formalizations are also included in a way that they do not affect the understanding of the techniques by beginners and non-expert users. The handbook mainly focuses on well assessed and consolidated methodologies already implemented in current IT programs for the production of seasonally adjusted data. But several chapters go beyond and present for example new ideas and techniques to improve the real time detection of turning points or to seasonally adjust high frequency data.

The target readers of the handbook are mainly data producers from statistical offices and central banks, and also researchers, students and academics who can benefit from the content of this handbook.

## 1.4 Structure of the handbook

The handbook contains 30 chapters structured in 10 parts.

Part I “General aspects” discusses some general aspects of seasonal adjustment. In particular, Chapter 2 and 3 defines the various components of a time series (trend-cycle, seasonality, outliers, trading-day effects) and illustrate their contribution to the series and their impact on the short-term evaluation of the Economy. The various possible models for the long-term trend, the business cycle, the seasonal component and the irregular component are also discussed. Chapter 4 draws a brief history of seasonal adjustment.

Part II “Pre-Treatment Methods” deals with the necessary correction of the series, from deterministic and non-linear elements, before the decomposition per se and the estimation of the seasonal component. Chapters 5 and 6 are devoted to the trading day and moving holiday effects. Chapters 7 and 8 study the important problem of the automatic detection and correction of ruptures and outliers. The current seasonal adjustment methods allow for the adjustment of raw or log-transformed data only. Chapter 9 presents the impact of other Box-Cox transformations on seasonal adjustment.

Part III “Seasonal Adjustment Methods” presents the well assessed and consolidated methodologies already implemented in current IT programs. Chapters 10 and 13 are devoted to the ARIMA model-based approaches (in particular TRAMO-SEATS); Chapter 11 to the basic structural models and Chapter 12 to the moving average based methods popularized by X-13ARIMA-SEATS.

Part IV “Improving End-Point Estimates for Seasonal Adjustment” deals with the important problem of the phase shift introduced at the end of the series by the main seasonal adjustment methods, and the fact that turning points are consequently detected with a delay. Chapter 14 proposes methods to improve the estimation of the trend-cycle from a seasonally adjusted data. Chapters 15 and 16 develop filters that can minimize this phase shift and can therefore realize a compromise between accuracy, smoothness and timeliness.

Part V “Seasonal Adjustment and Aggregation” addresses the important point of consistency within and between the seasonally adjusted series. Chapter 17 shows how to assure temporal consistency with the annual figures, when such data officially exist as in the National Accounts, Balance of Payments, External Trade, where users’ needs for time consistency are stronger. Chapter 18 is devoted to a detailed presentation of the “direct versus indirect” problem, its implications and some non statistical guidelines for the choice of a strategy. Chapter 19 proposes simple techniques to force the respect of aggregation constraints based on multivariate benchmarking methods also known as raking, reconciliation or balancing.

Part VI “Revision and Communication” is devoted to the stability of the seasonal adjustment process and to the dissemination of seasonally adjusted data. Chapter 20 presents an in-depth analysis of the revisions inherent to the adjustment process and discusses the important question whether statistical results should be revised or not. In order to deal with the changes in the raw and adjusted data, different revision policies are also discussed. Chapter 21, building on existing practices and recommendations, provides a summary of relevant issues for the practitioner and publisher of official data. Topics covered include: elements of a statistical release, choosing an appropriate headline indicator, and graphically representing outputs.

Part VII “Seasonal Adjustment in Practice” deals with several practical problems faced by the producer of seasonally adjusted data. Chapter 22 deals with the quality assessment of the adjusted data. Quality measures are proposed and examples of existing quality reports are shown. Chapter 23 studies the impact of the length of the time series on the quality of the seasonally adjusted data and gives some guidelines to adjust short time series. Finally, Chapter 24 shows the implications for seasonal adjustment of the various available chain index concepts.

Part VIII “Seasonal Adjustment and Business Cycle” focuses on the consequences of the seasonal adjustment process on business cycle analysis. Chapter 25 addresses the hypothesis that an interaction between seasonal and business cycles can arise and presents an empirical study on the effects different seasonal adjustment algorithms might have on the business cycle analysis due to their different ability in separating the seasonal from the other frequencies. Chapter 26 aims at assessing whether and to what extent coincident indicators hinge upon the seasonal adjustment procedures applied to the time-series that are used to construct them. And Chapter 27 presents new statistical evidence on two related issues: the interactions between seasonal and business cycle fluctuations; the role of seasonal adjustment filters on the measurement and analysis of the characteristics of the business cycle (turning point estimation, assessment of cyclical stance, non-linearity of the business cycle, asymmetry, and so forth).

Part IX “Seasonal Adjustment of High Frequency Data” addresses the adjustment of daily and weekly data that cannot be currently adjusted by TRAMO-SEATS or X-13ARIMA-SEATS. Chapter 28 details a locally-weighted least squares procedure for weekly data having 52 or 53 observations in a year, a method which is currently being applied at the Bureau of Labor Statistics, the Federal Reserve, and the Bank of Canada. Chapter 29 shows how TRAMO-SEATS and X-13ARIMA-SEATS can be adapted to deal with high frequency data, and applies these new implementations on daily data.

Finally, Part X and its Chapter 30 is devoted to the “ESS Guidelines on Seasonal Adjustment”.

## 1.5 Conclusions

This short chapter presents the historical background which led to the decision of preparing this handbook and the approach followed in its preparation. There is no doubt that seasonally adjusted data constitute a very useful input for the short-term evaluation of the economic situation and contribute to relevant and effective responses to the current economic situation by policy makers and decision makers. In a globalized world, the availability of comparable seasonally adjusted indicators across countries and regions could be a further element in strengthening the implementation of quick and effective counter-cyclical measures, coordinated across countries.

The triptych “ESS guidelines on Seasonal Adjustment, JDemetra+ software, Handbook on Seasonal Adjustment” will certainly contribute to the dissemination of harmonized and high quality seasonally adjusted economic indicators. This triptych, mainly developed for the European Union countries, can constitute a world-wide recognized contribution to foster harmonisation and the use of best practices in seasonal adjustment with clear benefits at global level. The EDD guidelines on seasonal adjustment have already attracted the attention of several non-EU countries, non only European ones, but also emerging and developing countries. Same interested has attracted JDemetra+ software which is a really successful example of an open source statistical product as shown by the constantly growing number of downloads and requests about it. We hope that the handbook will also have a similar success confirming the validity of our idea to develop it by investing a lot of time and resources in this initiative.

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# General Aspects







# 2

## Seasonal Adjustment: Objectives, Definitions, Costs and Benefits



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## Table of Contents

2.1	Introduction	35
2.2	The Main Characteristics and Cost of Seasonality	36
2.3	Limitations of Same-Month Year-ago Comparisons	38
2.4	Seasonal Adjustment Methods	40
2.4.1	Moving Average Methods	40
2.4.2	Model-based Seasonal Adjustment Methods	41
2.5	The Calendar variations	42
2.5.1	The Moving-Holiday Component	42
2.5.2	The Trading-Day Component	44



## 2.1 Introduction

Time series of sub-yearly observations, e.g. monthly, quarterly, weekly, are often affected by seasonal variations. The presence of such variations in socio-economic activities has been recognized for a long time. Indeed seasonality usually accounts for most of the total variation within the year.

Seasonality is due to the fact that some months or quarters of the year are more important in terms of activity or level. For example, the level of unemployment is generally higher during the winter and spring months and lower in the other months. Yearly series cannot contain seasonality because the component is specified to cancel out over 12 consecutive months or 4 consecutive quarters. Flow series may also be affected by other kind of seasonal variations associated with the composition of the calendar. The most important calendar variations are the trading-day variations, which are due to the fact that some days of the week are more important than others. Trading-day variations imply the existence of a daily pattern analogous to the seasonal pattern but these daily factors are usually referred to as daily coefficients.

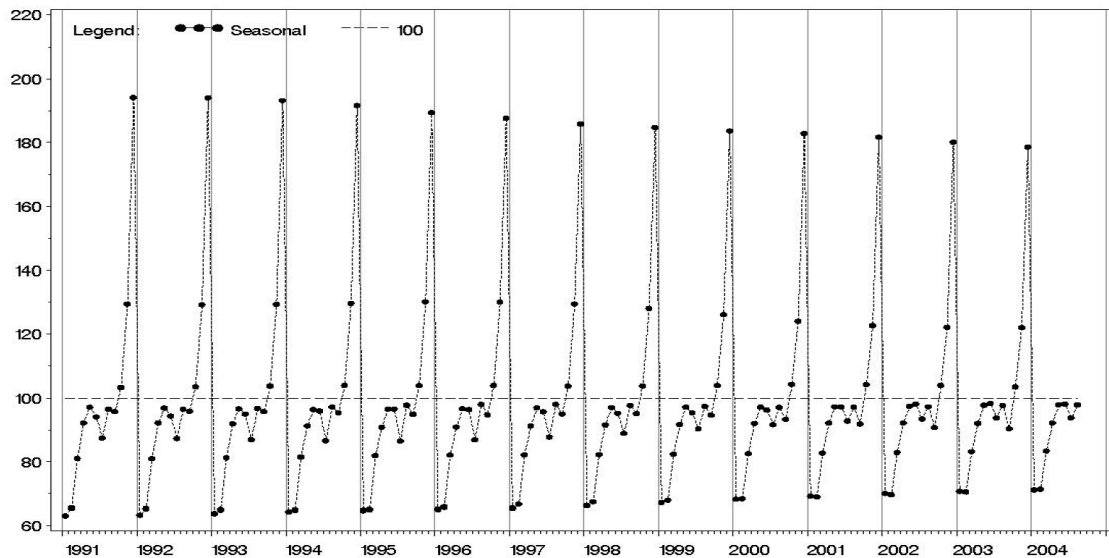
Similarly, the moving-holiday or moving-festival component is attributed to calendar variations, namely due to the fact that some holidays span consecutive months from year to year. For example, Easter can fall between March 23 to April 25. The Chinese New Year date depends on the lunar calendar. Ramadan falls eleven days earlier from year to year. In the Moslem world, Israel and in the Far East, there are many such festivals. For example, Malaysia contends with as many as eleven moving festivals, due to its religious and ethnic diversity. These festivals affect flow and stock variables and may cause a displacement of activity from one month to the previous or the following month. For example, an early date of Easter in March or early April can cause an important excess of activity in March and a corresponding short-fall in April, in variables associated to imports, exports, and tourism. When the Christian Easter falls late in April (e.g. beyond the 10-th), the effect is captured by the seasonal factor of April. Note that in the long run, Easter falls in April 11 times out of 14.

Some of these festivals have a positive impact on certain variables, for examples air traffic, sales of gasoline, hotel occupancy, restaurant activity, sales of flowers and chocolate in the case of Easter. The impact may be negative on other industries or sectors which close or reduce their activity during these festivals.

Trading day variations and moving calendar effects are seasonal, and are estimated by means of regression models for they are considered to be of a deterministic character. On the other hand, stochastic seasonal variations are often estimated either by means of moving averages or simple explicit Autoregressive Integrated Moving Average (ARIMA) models with a small number of parameters.

Seasonal adjustment means the removal of seasonal variations in the original series jointly with trading day variations and moving holiday effects. The main reason for seasonal adjustment is the need of standardizing socioeconomic series because seasonality affects them with different timing and intensity. Hence, the seasonally adjusted data reflect variations due only to the remaining components, namely trend-cycle and irregulars.

The information given by seasonally adjusted series plays a crucial role in the analysis of current economic conditions and provides the basis for decision making and forecasting, being of major importance around cyclical turning points.

**Figure 2.1: Seasonal pattern of Sales by Canadian Department Stores**

## 2.2 The Main Characteristics and Cost of Seasonality

The impact of seasonality in socioeconomic activities has long been recognized. Seasonal variations in agriculture, the low level of winter construction and high pre-Christmas retail sales are all well known.

Seasonality originates from climate and conventional seasons, like religious, social and civic events, which repeat from year to year.

The climatic seasons influence trade, agriculture, the consumption patterns of energy, fishing, mining and related activities. For example, the consumption of heating oil increases in winter, and the consumption of electricity increases in the summer months because of air conditioning.

Institutional seasons like Christmas, Easter, civic holidays, the beginning and ending of school and academic year, have a large impact on retail trade and on the consumption of certain goods and services, namely travel by plane, hotel occupancy, and consumption of gasoline.

In order to determine whether a series contains seasonality, it is sufficient to identify at least one month (or quarter) which tends to be systematically higher or lower than other months. Figure 2.1 exhibits the seasonal pattern of sales by Canadian Department Stores, where the values are much larger in December and much lower in January and February with respect to other months.

The seasonal pattern measures the relative importance of the months of the year. The constant 100% represents an average month or a nonseasonal month. The peak month is December, with sales almost 100% larger than on an average month; the trough months are January and February with sales almost 40% lower than on an average month. The seasonal amplitude, the difference between the peak and trough months of the seasonal pattern, reaches almost 140%.

The four main causes of seasonality are attributed to the weather, composition of the calendar, major institutional deadlines and expectations. Seasonality is largely exogenous to the economic system but can be partially offset by human intervention. For example, seasonality in money supply can be controlled by central bank decisions on interest rates. In other cases, the effects can be offset by international and inter-regional trade. For example Hydro Québec, a major Canadian electrical supplier, sells much of its excess power during

the summer seasonal trough months to the neighbouring Canadian provinces and U.S. states; and imports some of it during the winter seasonal peak months of electrical consumption in Québec. The scarcity of fresh fruits and vegetables in Germany and other Northern countries is handled in a similar manner. Some workers and businesses manage their seasonal pattern with complementary occupations: for example landscaping in the summer and snow removal in winter.

To some extent seasonality can evolve through technological and institutional changes. For example the developments of appropriate construction materials and techniques made it possible to continue building in winter. The developments of new crops, which better resist cold and dry weather, have influenced the seasonal pattern. The partial or total replacement of some crops by chemical substitutes, e.g. substitute of sugar, vanilla and other flavours, reduces seasonality in the economy.

As for institutional change, the extension of the academic year to the summer months in the 1970s affected the seasonal pattern of Canada unemployment for the population of 15 to 25 years of age. Similarly the practice of spreading holidays over the whole year impacted on seasonality. In the 1970s, December rapidly became a peak month for marriages in Canada. This was surprising because the month had always be a low month since the 1930s. Subject matter research determined that the Canadian law on income tax allowed the deduction of the spouse as a dependent for the whole year, even if the marriage had taken place in the last days of the year. In the 1980s, the Canadian government terminated that fiscal loophole, which resulted in a dramatic drop of marriages in December. The drop was so sudden that the series became difficult to seasonally adjust in the early 1980s. Indeed, seasonal adjustment methods assume gradual seasonal evolution. The same situation occurred in the United Kingdom. This kind of abrupt change in seasonal pattern is rather exceptional.

The changing industrial mix of an economy also transforms the seasonal pattern, because some industries are more seasonal than others. In particular, economies which diversify and depend less on "primary" industries (e.g. fishing, agriculture) typically become less seasonal.

In most situations, seasonality evolves slowly and gradually. Indeed the seasonal pattern basically repeats from year to year, as illustrated in Figure 2.1. Merely repeating the seasonal pattern of the last twelve months usually provides a reasonable forecast.

Another important characteristic of seasonality is the feasibility of its identification even if it is a latent (not directly observable) variable. The identification and estimation of seasonality, however, is not done independently of the other components affecting the time series under study. Traditionally, three other components have been distinguished, namely, the trend, the cycle and the irregulars. We shall here give only a brief description of each of these components, and refer the reader to Chapter 3 for further details. The trend reflects long term movements lasting many years. It is generally associated with structural causes, for example, institutional events, demographic and technological changes, new ways of organization. The cycle, usually referred to as the business cycle, it is a quasi-periodic oscillation characterized by periods of expansion and contraction, lasting from 3 to 5 years in average. In most analytical work the trend and the business cycle are combined because, for series covering medium or short periods of time, the long term trend cannot be estimated adequately. Finally, the irregulars represent unforeseeable movements related to events of all kinds. In general, the irregular component has a stable random appearance but, in some cases, it may include extreme variations or outliers with well identifiable causes, for example, flood, severe winters, and strikes. The seasonal variations can be distinguished from the trend by their oscillatory character, from the business cycle by having annual periodicity, and from the irregulars by being systematic. Seasonality entails large costs to society and businesses. One cost is the necessity to build warehouses to store inventories of goods to be sold as consumers require them, for example grain elevators. Another cost is the under-use and over-use of the factors of production: capital and labour.

Capital in the form of un-used equipment, buildings and land during part of the year has to be financed regardless. For example, this is the case in farming, food processing, tourism, electrical generation, and accounting. The cold climate increases the cost of buildings and infrastructure, e.g. roads, transportation systems, water and sewage systems, schools, hospitals; not to mention the damage to the same caused by the action of ice.

The labour force is over-used during the peak seasons of agriculture and construction for example; and, under-used in trough seasons sometimes leading to social problems.

A more subtle unwanted effect is that seasonality complicates business decisions by concealing the fundamental trend-cycle movement of the variables of interest.

## 2.3 Limitations of Same-Month Year-ago Comparisons

In the absence of seasonal adjustment, only the raw series, say  $x_t$  is available. In such cases, it is customary to use same-month comparisons from year to year,  $x_t - x_{t-12}$  to assess the stage of the business cycle. The rationale is that the seasonal effect in  $x_t$  is approximately the same as in  $x_{t-12}$  under the assumption of slowly evolving seasonality. Same-month year ago comparisons can be expressed as the sum of the changes in the raw series between  $x_t$  and  $x_{t-12}$ ,

$$x_t - x_{t-12} = (x_t - x_{t-1}) + (x_{t-1} - x_{t-2}) + (x_{t-2} - x_{t-3}) + \cdots + (x_{t+11} - x_{t-12}) \equiv \sum_{j=1}^{12} (x_{t-j+1} - x_{t-j}) \quad (2.1)$$

Equation 2.1 shows that same-month comparison display an increase, if the increases dominate the decreases over the 13 months involved, and conversely. The timing of  $x_t - x_{t-12}$  is  $t - 6$ , the average of  $t$  and  $t - 12$ . This points out a limitation of this practice: the diagnosis provided is not timely with respect to  $t$ . Furthermore,  $x_t$  and  $x_{t-12}$  may contain irregular variations affecting one observation positively and the other negatively, hence conveying instability to the comparison. Moreover, for flow data the comparison is systematically distorted by trading-day variations and moving holidays, if present.

As already mentioned, seasonal adjustment entails the removal of seasonality, trading-day variations and moving-holiday effects from the raw data, to produce a seasonally adjusted series, which consists of the trend-cycle and the irregular components. The irregular fluctuations in the seasonally adjusted series can be reduced by smoothing, to isolate the trend-cycle and to enable month-to-month comparisons.

Figure 2.2 displays the raw Canada Unemployment series along with the trend-cycle and seasonally adjusted estimates. In order to illustrate same month comparisons, note that the raw Canada Unemployment declined between Dec. 1978 and Dec. 1979 for instance. On the other hand, the trend-cycle,  $c_t$ , over the last months of 1979 was clearly rising. The difference in diagnosis results from the timing:  $t - 6$  (June 1979) for  $x_t - x_{t-12}$  and  $t - 0.5$  for  $c_t - c_{t-1}$ .

At the end of the series, the raw Unemployment series rose by about 50% between Dec. 1981 and Dec. 1982, which was a staggering and unprecedented growth in such a short time. On that basis, most observers anticipated even higher levels for 1983. By looking at the trend-cycle values however, there are signs that Unemployment was slowing down between Nov. and Dec. 1982. This suggests that unemployment could stabilize in early 1983. As shown by Figure 2.3, Unemployment did stabilize and started to decline in 1983. However, it is important to keep in mind that the most recent trend-cycle estimates are often subject to revisions. Hence any proper prognosis would require one or two more observations. Figure 2.3 also illustrates that business cycles vary in length and amplitude.

Figure 2.4 displays the trend-cycle of Sales by Canadian Department Stores, which are less sensitive to the current economic situation than Unemployment exhibited in Fig.2.3 Indeed, in the short run the unemployed people continue spending from their unemployment benefits and savings.

Figure 2.2: Raw unemployment, trend cycle and seasonally adjusted series 1978-92

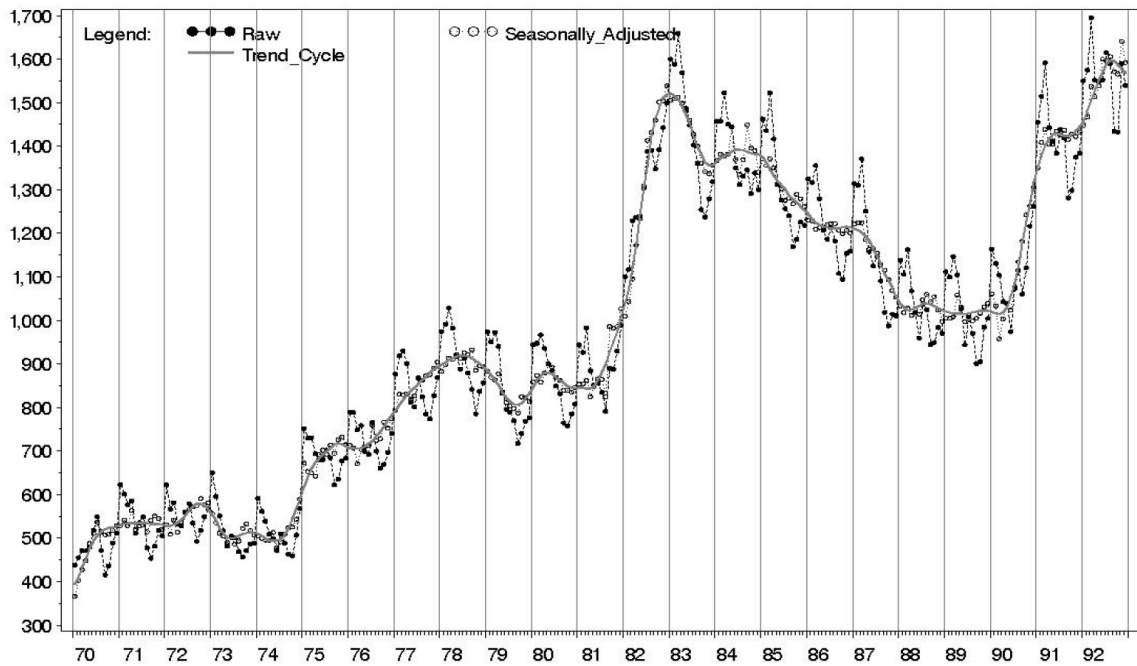
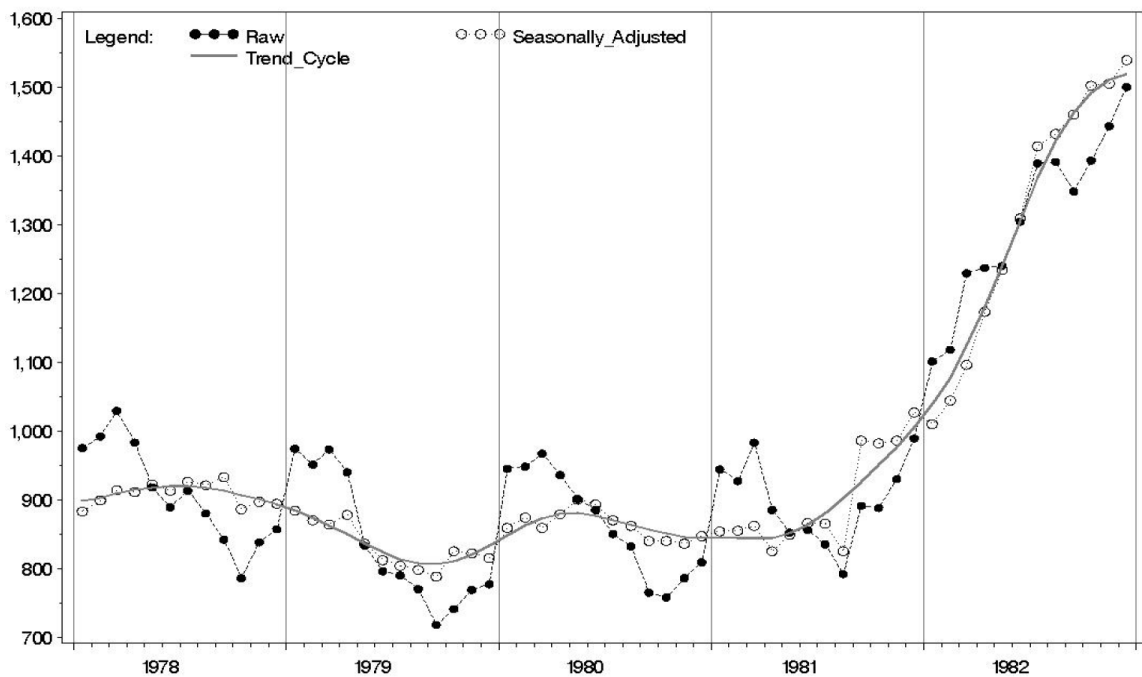
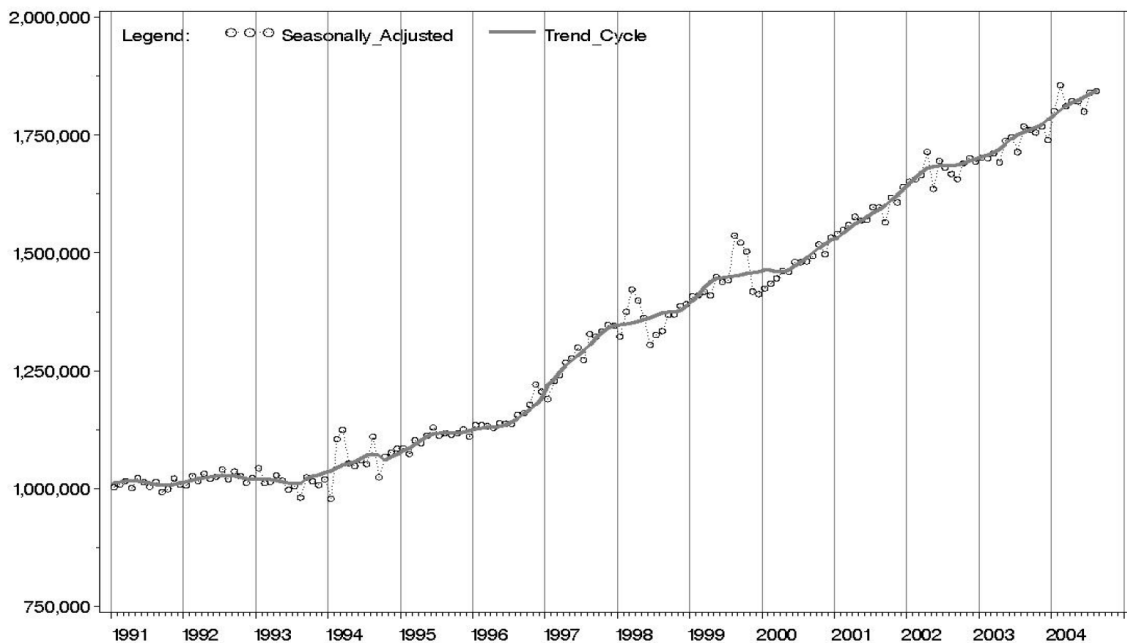


Figure 2.3: Raw Unemployment, trend-cycle and seasonally adjusted series 1978-82



**Figure 2.4:** Seasonally adjusted Sales by Canadian Department Stores and Trend-cycle Estimates, 1991-2004.



## 2.4 Seasonal Adjustment Methods

The seasonal adjustment methods developed so far are based on univariate time series decomposition models with no causal explanation for any of the components involved. It is difficult to classify existing methods into mutually exclusive categories. However, it is possible to group the majority of seasonal adjustment methods into two main classes, one based on moving averages or linear filters and the other on explicit models with a small number of parameters for each component.

### 2.4.1 Moving Average Methods

The best known and most often applied seasonal adjustment methods are based on moving averages or smoothing linear filters applied sequentially by adding ( and subtracting) one observation at a time. These methods assume that the time series components change through time in a stochastic manner. Given a time series,  $x_t, t = 1, \dots, T$ , for any  $t$  far removed from both ends, say  $m + 1 \leq t \leq T - m$ , the seasonally adjusted value  $x_t^a$ , is obtained by application of a symmetric moving average  $h_m(B)$

$$x_t^a = h_m(B)x_t = \sum_{j=-m}^m h_{m,j}x_{t-j} \quad (2.2)$$

where the weights  $h_{m,j}$  are symmetric, that is  $h_{m,j} = h_{m,-j}$ , and the length of the filter is  $2m + 1$ .

For current and recent data ( $T - m < t \leq T$ ) a symmetric filter cannot be applied, and therefore truncated asymmetric filters are used. For example, for the last available observation  $x_T$ , the seasonally adjusted value is given by

$$x_T^a = h_0(B)x_T = \sum_{j=0}^m h_{0,j}x_{T-j} \quad (2.3)$$



The asymmetric filters are time varying in the sense that different filters are applied for the  $m+1$  first and last observations. The end estimates are revised as new observations are added because of : (i) the new innovations , and (ii) the differences between the symmetric and asymmetric filters. The estimates obtained with symmetric filters are often called "final".

The development of electronic computers contributed to major improvements in seasonal adjustment based on moving averages and facilitated their massive application.

In 1954, Julius Shiskin of the US Bureau of Census developed software called Method I, based mainly on the works of Macaulay (1931) already being used by the US Federal Reserve Board. Census Method I was followed by Census Method II and eleven more experimental versions (X1 , X2, . . . , X11). The best known and widely applied was the Census X11 variant Shiskin et al. (1967) but produced poor seasonally adjusted data at the end of the series which is of crucial importance to assess the direction of the short-term trend and the identification of turning points in the economy.

Dagum (1980) developed the X11ARIMA to correct for this serious limitation. The X11ARIMA method consists of:

- (i) modelling the original series with an ARIMA model of the Box and Jenkins (1970) type,
- (ii) extending the original series 1 to 3 years with forecasts from the ARIMA model that fits and extrapolate well according to well defined acceptance criteria, and
- (iii) estimating each component with moving averages that are symmetric for middle observations and asymmetric for both end years. The latter are obtained via de convolution of Census X11 variant and the ARIMA model extrapolations.

The X11-ARIMA was extended by Dagum (1988) and, later by Findley et al. (1998) calling it X12ARIMA (see Chapter 7 for an extensive discussion). The X12-ARIMA offers a RegARIMA option that enables the estimation of deterministic components such as trading day variations and Moving Holiday effects simultaneously with the ARIMA model for extrapolation. It also includes new diagnostic tests and spectral techniques to assess the goodness of the results.

All the moving average methods here discussed are nonlinear; hence, the seasonally adjusted total of aggregated series is not equal to the algebraic sum of the seasonally adjusted series that enter into the aggregation. The main causes of nonlinearity are: (i) a multiplicative decomposition model for the unobserved components,(ii) the identification and replacement of extreme values, (iii) the ARIMA extrapolations, and (iv) automatic selection of moving average length for the estimation of the trend-cycle and seasonality. The properties of the combined linear filters applied to estimate the various components were originally calculated by Young (1968) for a standard option. Later, Dagum et al. (1996) calculated and analysed all possible filter combination of Census X11 and X11ARIMA. Cleveland and Tiao (1976) and Burrige and Wallis (1984) found ARIMA models that approximated well some of the linear filters used for the trend-cycle and seasonal component of Census X11.

## 2.4.2 Model-based Seasonal Adjustment Methods

The best known model-based methods are: (i) the regression methods with global or locally deterministic models for each component, and (ii) stochastic model based methods that use ARIMA models.

### 2.4.2.1 Regression methods

Seasonal adjustment by regression methods is based on the assumption that the systematic components of time series can be closely approximated by simple function of times over the entire span of the series. In general, two types of mathematical functions are considered. One is a polynomial of fairly low degree to represent

the trend component; the other, linear combinations of sine and cosine functions with different periodicity and fixed amplitude and phase, to represent business cycles and seasonality. To overcome the limitation of using global deterministic representations for the trend, cycle and seasonality, regression methods were extended to incorporate stochastic representations by means of local polynomials (spline functions) for successive short segments of the series and introducing changing seasonal amplitudes. A major breakthrough in this direction was made by Akaike (1980) who introduced prior constraints to the degree of smoothness of the various components and solved the problem with the introduction of a Bayesian model. Another important contribution is the regression method with locally deterministic models estimated by the LOESS (locally weighted regression) smoother developed by Cleveland et al. (1990).

#### 2.4.2.2 Stochastic model based methods

Stochastic model based methods were mainly developed during the 1980 following two different approaches. One originally known as seasonal adjustment by signal extraction (SIGEX) was developed by Burman (1980) or as ARIMA model based seasonal adjustment Hillmer and Tiao (1982) largely discussed in Bell and Hillmer (1984), the other is referred to as structural model decomposition method (see e.g. Harvey (1981) and Kittagawa and Gersch (1984)). The main difference between these two approaches is that in the latter simple ARIMA models are directly specified for each unobserved component whereas in the former an overall ARIMA model is obtained from observable data, and by imposing certain restrictions, models for each component are derived.

Since the components are unknown, to obtain a unique decomposition Hillmer and Tiao (1982) proposed a canonical decomposition which has the properties, among others, of maximizing the variance of the irregulars and minimizing the variance of the stochastic seasonal models. ARIMA models are sensitive to outliers or extreme values and cannot deal with deterministic components such as trading days and moving holidays. Therefore further developments were made by combining dummy variables regression models with ARIMA models to deal with these cases. In this regard, Gomez and Maravall (1996) developed TRAMO-SEATS.

On the other hand the structural model decomposition method starts directly with an observation equation (sometimes called measurement equation) consisting of the unobserved components. Simple ARIMA or stochastic trigonometric models are assumed for each component. Koopman et al. (1998) developed STAMP for structural model seasonal adjustment which includes several types of models for each component. Extensive discussions on TRAMO-SEATS and STAMP are given in Chapter 7.

## 2.5 The Calendar variations

We shall give here only a brief description of calendar variations, and refer the reader to Chapter 4 for further details.

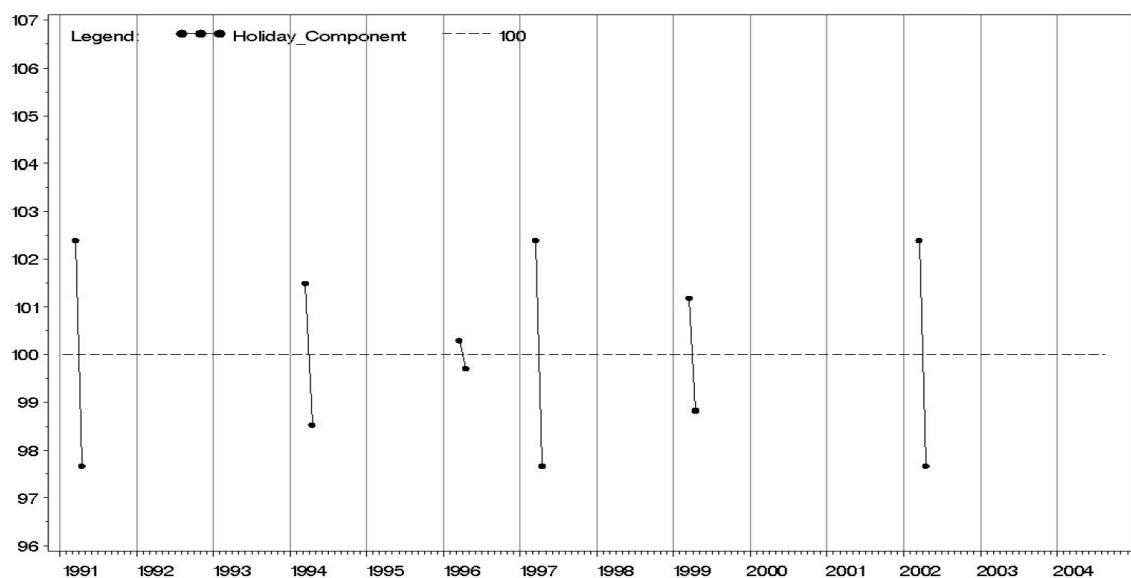
### 2.5.1 The Moving-Holiday Component

The moving-holiday or moving-festival component is attributed to calendar variations, namely the fact that some holidays spans between two consecutive months from year to year. For example, Easter can fall between March 23 to April 25, affecting March or April depending on the dates. An early Easter, falling in March or early April can cause an important excess of activity in March and a corresponding short-fall in April, in variables associated to imports, exports, tourism. When the Christian Easter falls late in April (e.g. beyond the 10-th), the effect is captured by the seasonal factor of April. In the long run, Easter falls in April 11 times out of 14.



**Table 2.1: Dates of Easter and presence of effect in March**

March 31 1991	April 12 1998, no effect
April 19 1992, no effect	April 4 1999
April 11 1993, no effect	April 23 2000, no effect
April 3 1994	April 15 2001, no effect
April 16 1995, no effect	March 31 2002
April 7 1996	April 20 2003, no effect
March 30 1997	April 11 in 2004, no effect.

**Figure 2.5: Moving Easter Holiday component**

Some of these festivals have a positive impact on certain variables, for examples air traffic, sales of gasoline, hotel occupancy, restaurant activity, sales of flowers and chocolate in the case of Easter. The impact may be negative on other industries or sectors which close or reduce their activity during these festivals.

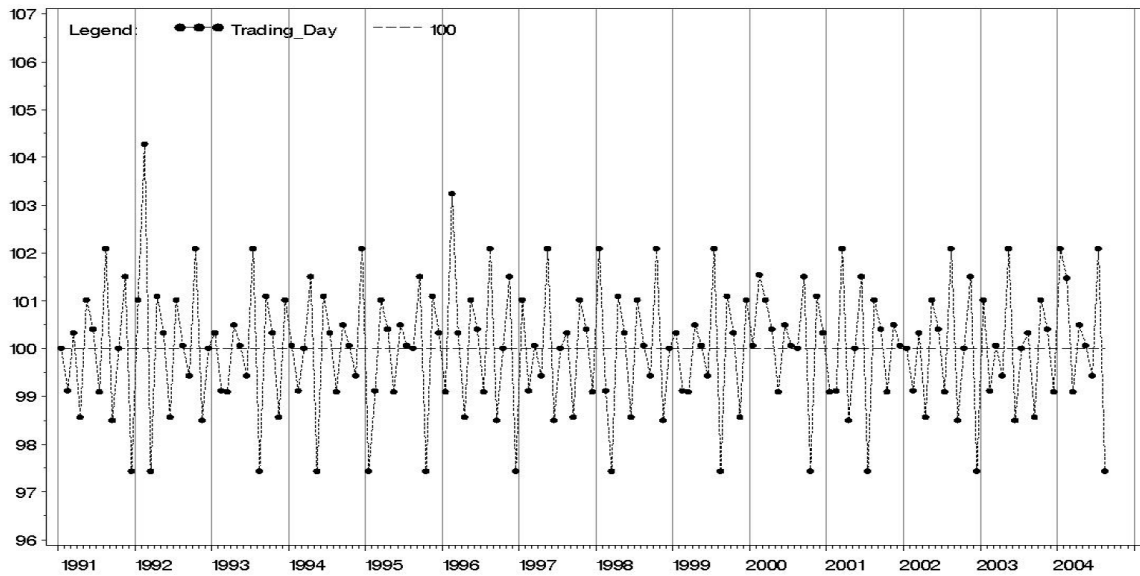
The festival effect may affect only the day of the festival itself, or a number of days preceding and/or following the festival. In the case of Easter, travellers tend to leave a few days before and return after Easter, which affects air traffic and hotel occupancy, etc., for a number of days. Purchases of flowers and other highly perishable goods, on the other hand, are tightly clustered immediately before the Easter date.

The effect of moving festivals can be seen as a seasonal effect dependent on the date(s) of the festival. Figure 2.5 displays the Easter effect on a given series where the Easter effect is rather mild. In some of the years, the effect is absent because Easter fell too late in April. The dates of Easter appear in Table 2.1.

In the case exemplified, the effect is felt seven days before Easter and on Easter Sunday but not after Easter. This is evidenced by years 1994, 1996 and 1999 where Easters falls early in April and impacts the month of March. Note that the later Easter falls in April, the smaller the displacement of activity to March; after a certain date the effect is entirely captured by the April seasonal factor. The effect here is rather moderate but may not be the case for other variables. For example, Imports and Exports are substantially affected by Easter, because Customs do not operate from Good Friday to Easter Monday. Easter can also significantly affect quarterly series, by displacing activity from the second to the first quarter.

There has been cases of complete reversal on the timing of the Easter effect. For example, Marriages in Canada were performed mainly by the Church during the 1940s up to the 1960s. The Church did not celebrate marriages during the Lent period, i.e. the 40 days before Easter. Some marriages therefore were celebrated

Figure 2.6: Trading-day estimates



before the Lent period, potentially affecting February and March. However, if Easter fell too early, many of these marriages were postponed after Easter. Generally, festival effects are difficult to estimate, because the nature and the shape of the effect are often not well known. Furthermore, there are few observations, i.e. one occurrence per year.

## 2.5.2 The Trading-Day Component

Flow series may be affected by other variations associated with the composition of the calendar. The most important calendar variations are the trading-day variations, which are due to the fact that some days of the week are more important than others. Trading-day variations imply the existence of a daily pattern analogous to the seasonal pattern. However, these daily factors are usually referred to as daily coefficients.

### 2.5.2.1 Causes and Costs of Daily Patterns of Activity

Depending on the socio-economic variable considered, some days may be 60% more important than an average day and other days, 80% less important.

If the more important days of the week appear five times in a month (instead of four), the month registers an excess of activity *ceteris paribus*. If the less important days appear five times, the month records a short-fall. As a result, the monthly trading-day component can cause increase of +8% or -8% (say) between neighbouring months and also between same-months of neighbouring years. The trading-day component is usually considered as negligible and very difficult to estimate in quarterly series.

Figure 2.6 displays the monthly trading-day component obtained from the following daily pattern: 90.2, 71.8, 117.1, 119.3, 97.6, 161.3 and 70.3 for Monday to Sunday (in percentage) respectively. The daily pattern indicates that Saturday is approximately 61% more important than an average day (100%); and that Tuesday and Sunday, 30% less important.

The monthly trading-day estimates shown in Figure 2.6 display a drop of 7% between Jan. and Feb. 1992 and a drop of 5% between Jan. 1992 and Jan. 1993. One can identify several instances where the change between same-months is significant. Indeed, same-month year-ago comparisons are never valid in the presence of trading-day variations, not even as a rule of thumb. Furthermore, it is apparent that the monthly

trading-day factors in the figure are identical for quite a few months. Indeed for a given set of daily coefficients, there are only 2 different monthly values for the trading-day component, for a given set of daily coefficients: seven values for 31-day months (depending on which day the month starts), seven for 30-day months, seven for 29-day months and one for 28-day months. In other words, there are at most 22 possible arrangements of days in monthly data.

Many goods and services are affected by daily patterns of activity, which entail higher costs for producers, namely through the need of higher inventories, equipment and staff on certain days of the week. For example, there is evidence that consumers buy more gasoline on certain days of the week, namely on Thursdays, Fridays, Saturdays and holidays, which results in line-ups and shortages at the pumps. In order to cope with the problem, gasoline retailers raise their price on those days to promote sales on other days. Furthermore, the elasticity of demand for gasoline is low. In other words, to reduce consumption by a small percentage, prices must be raised by a disproportionate percentage, which upsets some consumers. On the other hand, consumers can buy their gasoline on other days. The alternative for retailers is to acquire larger inventories, larger tanks, more pumps and larger fleets of tanker trucks, all of which imply higher costs and translate into much higher prices. In other words, there are savings associated with more uniform daily patterns; and costs, with scattered daily patterns.

A similar consumer behaviour prevails for the purchases of food, which probably results in more expensive prices, namely through higher inventories, larger refrigerators, more numerous cash registers and more staff, than otherwise necessary. Scattered daily patterns have been surprisingly observed for variables like births and deaths. Indeed, births are more frequent on certain days of the week, namely on working days to avoid overtime pay. This results from the practice of caesarean delivery and especially birth inducement now widely applied to encourage births on working days. In this particular case, an appropriately scattered daily pattern reduces costs. Deaths also occur more often on certain days of the week. Car accidents, drowning, skiing and other sporting accidents tend to occur on weekend days and on holidays. According to the Canadian Workmen Compensation Board, industrial accidents tend to occur more often on Friday afternoons when security is more lax. In principle, stock series pertaining to one day display a particular kind of trading-day variations. Among other things, inventories must anticipate the activity (flow) of the following day(s). For such stock series, the monthly trading-day factor coincides with the daily weight of the day.

### 2.5.2.2 A Classical Model for Trading-Day Variations

A frequently applied deterministic model for trading-day variations is that developed by Young (1965),

$$y_t = D_t + u_t, t = 1, 2, \dots, n, \quad (2.4)$$

$$D_t = \sum_{j=1}^7 \alpha_j N_{j,t} \quad (2.5)$$

where  $u_t \sim WN(0, \sigma_u^2)$ ,  $\sum_{j=1}^7 \alpha_j = 0$ ,  $\alpha_j = 1, \dots, 7$  denote the effects of the seven days of the week, Monday to Sunday, and  $N_{j,t}$  is the number of times day  $j$  is present in month  $t$ . Hence, the length of the month is  $N_t = \sum_{j=1}^7 N_{j,t}$ , and the cumulative monthly effect is given by Equation 2.5. Adding and subtracting

$$\bar{\alpha} = \left( \sum_{j=1}^7 \alpha_j \right) / 7 \text{ to 2.5 yields}$$

$$D_t = \bar{\alpha}N_t + \sum_{j=1}^7 (\alpha_j - \bar{\alpha})N_{j,t} \quad (2.6)$$

Hence, the cumulative effect is given by the length of the month plus the net effect due to the days of the week. Since  $\sum_{j=1}^7 (\alpha_j - \bar{\alpha}) = 0$ , model 2.6 takes into account the effect of the days present five times in the month. Model 2.6 can then be written as

$$D_t = \bar{\alpha}N_t + \sum_{j=1}^6 (\alpha_j - \bar{\alpha})(N_{j,t} - N_{7,t}) \quad (2.7)$$

Deterministic models for trading-day variations assume that the daily activity coefficients are constant over the whole range of the series. Stochastic model for trading-day variations have been rarely proposed. Dagum et al. (1992) developed a model where the daily coefficients change over time according to a stochastic difference equation.

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# 3

## Time Series Components



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## Table of Contents

3.1	Introduction	51
3.2	Time Series Decomposition Models	51
3.3	The Secular or Long-Term Trend	53
3.3.1	Deterministic Trend Models	54
3.3.2	Stochastic Trends	54
3.4	The Business Cycle	55
3.4.1	Deterministic and Stochastic Models for the Business Cycle	56
3.5	The Seasonal Variations	57
3.5.1	Deterministic and Stochastic Models for Seasonality	58
3.6	Calendar Variations	59
3.6.1	The Moving-Holiday Component	59
3.6.2	The Trading-Day Component	60
3.6.3	A Classical Model for Trading-Day Variations	62
3.7	The Irregular Component	63
3.7.1	Redistribution Outliers and Strikes	64
3.7.2	Models for the Irregular Component and Outliers	64



## 3.1 Introduction

This chapter is intended as a general introduction to time series components. A time series consists of a set of observations ordered in time, on a given phenomenon (target variable). Usually the measurements are equally spaced, e.g. by year, quarter, month, week, day. The most important property of a time series is that the ordered observations are dependent through time, and the nature of this dependence is of interest in itself.

Formally, a time series is defined as a set of random variables indexed in time,  $\{X_1, \dots, X_T\}$ . In this regard, an observed time series is denoted by  $\{x_1, \dots, x_T\}$ , where the sub-index indicates the time to which the observation  $x_t$  pertains. The first observed value  $x_1$  can be interpreted as the realization of the random variable  $X_1$ , which can also be written as  $X(t = 1, \omega)$  where  $\omega$  denotes the event belonging to the sample space. Similarly,  $x_2$  is the realization of  $X_2$  and so on. The  $T$ -dimensional vector of random variable can be characterized by different probability distribution.

For socio-economic time series the probability space is continuous, and the time measurements are discrete. The frequency of measurements is said to be high when it is daily, weekly or monthly and to be low when the observations are quarterly or yearly.

## 3.2 Time Series Decomposition Models

An important objective in time series analysis is the decomposition of a series into a set of non-observable (latent) components that can be associated to different types of temporal variations. The idea of time series decomposition is very old and was used for the calculation of planetary orbits by seventeenth century astronomers. Persons (1919) was the first to state explicitly the assumptions of unobserved components. As Persons saw it, time series were composed of four types of fluctuations:

1. A long-term tendency or secular trend.
2. Cyclical movements super-imposed upon the long-term trend. These cycles appear to reach their peaks during periods of industrial prosperity and their troughs during periods of depressions, their rise and fall constituting the business-cycle.
3. A seasonal movement within each year, the shape of which depends on the nature of the series.
4. Residual variations due to changes impacting individual variables or other major events such as wars and national catastrophes affecting a number of variables.

Traditionally, the four variations have been assumed to be mutually independent from one another and specified by means of an additive decomposition model:

$$X_t = T_t + C_t + S_t + I_t, \quad (3.1)$$

where  $X_t$  denotes the observed series,  $T_t$  the long-term trend,  $C_t$  the business-cycle,  $S_t$  seasonality and  $I_t$  the irregulars. If there is dependence among the latent components, this relationship is specified through a multiplicative model

$$X_t = T_t \times C_t \times S_t \times I_t, \quad (3.2)$$

where now  $S_t$  and  $I_t$  are expressed in proportion to the trend-cycle  $T_t C_t$ . In some cases, mixed additive-multiplicative models are used.

Whether a latent component is present or not in a given time series depends on the nature of the phenomenon and on the frequency of measurement. For example, seasonality is due to the fact that some months or quarters of a year are more important in terms of activity or level. Because this component is specified to

cancel out over 12 consecutive months or 4 consecutive quarters, or more generally over 365.25 consecutive days, yearly series cannot contain seasonality.

Flow series can be affected by other variations associated to the composition of the calendar. The most important are the trading-day variations, which are due to the fact that some days of the week are more important than others. Months with five of the more important days register an excess of activity (*ceteris paribus*) in comparison to months with four such days. Conversely, months with five of the less important days register a short-fall of activity. The length-of-month variation is usually assigned to the seasonal component. The trading-day component is usually considered as negligible in quarterly series and even more so in yearly data.

Another important calendar variation is the *moving-holiday* or *moving-festival* component. That component is associated to holidays which change date from year to year, e.g. Easter, causing a displacement of activity from one month to the previous or the following month. For example, an early date of Easter in March or early April can cause an important excess of activity in March and a corresponding short-fall in April, in variables associated to imports, exports, and tourism.

Under models (3.1) and (3.2), the trading-day and moving festival components (if present) are implicitly part of the irregular. In 1965, Young developed a procedure to estimate trading-day variations which was incorporated in the X-11 seasonal adjustment method (Shiskinat al. (1967)) and its subsequent versions, the X-11-ARIMA (Dagum (1980) and Dagum (1988)) and X-12-ARIMA (Findley et al. (1998)) methods. The later two versions also include models to estimate moving-holidays, e.g. Easter.

Considering these new components, the additive decomposition model becomes

$$X_t = T_t + C_t + S_t + D_t + H_t + I_t, \quad (3.3)$$

where  $D_t$  and  $H_t$  respectively denote the trading-day and moving-holiday components. Similarly, the multiplicative decomposition model becomes

$$X_t = T_t \times C_t \times S_t \times D_t \times H_t \times I_t, \quad (3.4)$$

where the components  $S_t$ ,  $D_t$ ,  $H_t$  and  $I_t$  are proportional to the trend-cycle  $T_t C_t$ .

Decomposition models (3.3) and (3.4) are traditionally used by seasonal adjustment methods. Other less used decomposition are the log additive model and the mixed models (additive and multiplicative) where, for example, the systematic relation among the components is multiplicative but the irregulars are additive.

Seasonal adjustment actually entails the estimation of all the time series components and the removal of seasonality, trading-day and holiday effects from the observed series. The rationale is that these components which are relatively predictable conceal the current stage of the business cycle which is critical for policy and decision making.

There is another time series decomposition often used for univariate ARIMA time series modelling and forecasting:

$$X_t = \eta_t + e_t, \quad (3.5)$$

where  $\eta_t$  and  $e_t$  are referred to as the signal and the noise, according to the electrical engineering terminology. The signal  $\eta_t$  comprises all the systematic components of models (3.1) to (3.4), i.e.  $T_t$ ,  $C_t$ ,  $S_t$ ,  $D_t$  and  $H_t$ .

Model (3.5) is classical in signal extraction where the problem is to find the best estimates of the signal  $\{\eta_t\}$  given the observations  $\{x_t\}$  corrupted by noise  $\{e_t\}$ . The best estimates are usually defined as minimizing the mean square error.

Finally, given its fundamental role in time series modelling, we summarize the famous decomposition theorem due to Wold (1938). Wold proved that any second-order stationary stochastic process  $\{X_t\}$  can be

decomposed in two mutually uncorrelated processes  $\{Z_t\}$  and  $\{V_t\}$ , such that

$$X_t = Z_t + V_t, \quad (3.6)$$

Where

$$Z_t = \sum_{j=0}^{\infty} \psi_j a_{t-j}, \quad \psi_0 \equiv 1, \quad \sum_{j=1}^{\infty} \psi_j^2 < \infty, \quad \text{with } \{a_t\} \sim WN(0, \sigma_a^2) \quad (3.7)$$

Component  $\{Z_t\}$  is a convergent infinite linear combination of the  $a_{t,s}$ , assumed to follow a white noise (WN) process of zero mean, constant variance  $\sigma_a^2$  and zero autocovariance. Model (3.7) is known as an infinite moving average  $MA(\infty)$  and the  $a_t$  are the innovations. The component  $\{Z_t\}$  is called the non-deterministic or purely linear component since only one realization of the process is not sufficient to determine future values  $Z_{t+l}$ ,  $l > 0$ , without error.

Component  $\{V_t\}$  can be represented by

$$V_t = \mu + \sum_{j=1}^{\infty} [\alpha_j \sin(\lambda_j t) + \beta_j \cos(\lambda_j t)], \quad -\pi < \lambda_j < \pi, \quad (3.8)$$

where  $\mu$  is the constant mean of process  $\{X_j\}$  and  $\{\alpha_j\}$ ,  $\{\beta_j\}$  are mutually uncorrelated white noise processes. The series  $\{V_t\}$  is called the deterministic part because it can be predicted in the future without error from a single realization of the process by means of an infinite linear combination of past values.

Wold theorem demonstrates that the property of stationarity is strongly related to that of linearity. It provides a justification for autoregressive moving average (ARMA) models (Box and Jenkins (1970)) and some extensions, such as the autoregressive integrated moving average (ARIMA) and regression-ARIMA models (RegARIMA).

A stochastic process  $\{X_t\}$  is second-order stationary or weakly stationary, if the first two moments are not time dependent, that is, the mean and the variance are constant, and the autocovariance function depends only on the time lag and not the time origin:

$$E(X_t) = \mu < \infty, \quad (3.9)$$

$$E(X_t - \mu)^2 = \sigma_X^2 < \infty, \quad E[(X_t - \mu)(X_{t-k} - \mu)] = \gamma(k) < \infty, \quad (3.10)$$

where  $k = 0, 1, 2, \dots$  denotes the time lag.

### 3.3 The Secular or Long-Term Trend

The concept of trend is used in economics and other sciences to represent long-term smooth variations. The causes of these variations are often associated with structural phenomena such as population growth, technological progress, capital accumulation, new practices of business and economic organization. For most economic time series, the trends evolve smoothly and gradually, whether in a deterministic or stochastic manner. When there is sudden change of level and/or slope this is referred to as a structural change. It should be noticed however that series at a higher levels of aggregation are less susceptible to structural changes. For example, a technological change is more likely to produce a structural change for some firms than for the whole industry.

The identification and estimation of the secular or long-term trend have posed serious challenges to statisticians. The problem is not of statistical or mathematical character but originates from the fact that the trend is a latent (non-observable) component and its definition as a long-term smooth movement is statistically vague. The concept of “long-period” is relative, since a trend estimated for a given series may turn out to be just a

long business cycle as more years of data become available. To avoid this problem statisticians have used two simple solutions. One is to estimate the trend and the business cycles, calling it the trend-cycle. The other solution is to estimate the trend over the whole series, and to refer to it as the longest non-periodic variation.

It should be kept in mind that many systems of time series are redefined every fifteen years or so in order to maintain relevance. Hence, the concept of long-term trend loses importance. For example the system of Retail and Wholesale Trade series was redefined in 1989 to adopt the 1980 Standard Industrial Classification (SIC), and in 2003 to conform to the North American Industrial Classification System (NAICS), following the North American Free Trade Agreement. The following examples illustrate the necessity of such reclassifications. The 1970 Standard Industrial Classification (SIC) considered computers as business machines, e.g. cash registers, desk calculators. The 1980 SIC rectified the situation by creating a class for computers and other goods and services. The last few decades witnessed the birth of new industries involved in photonics (lasers), bioengineering, nano-technology, electronic commerce. In the process, new professions emerged, and Classification systems had to keep up with these new realities.

There is a large number of deterministic and stochastic models which have been proposed for trend estimation (see Dagum and Dagum (1988)). Deterministic models are based on the assumption that the trend can be well approximated by mathematical functions of time such as polynomials of low degree, cubic splines, logistic functions, Gompertz curves, modified exponentials. Stochastic trends models assume that the trend can be better modelled by differences of low order together with autoregressive and moving average errors.

### 3.3.1 Deterministic Trend Models

The most common representation of a deterministic trend is by means of polynomial functions. The observed time series is assumed to have a deterministic non-stationary mean, i.e. a mean dependent on time. A classical model is the regression error model where the observed data is treated as the sum of the trend and a random component such that,

$$Y_t = T_t + u_t, \quad (3.11)$$

where  $T_t$  denotes the trend and  $u_t$  is assumed to follow a stationary process, often white noise. The polynomial trend can be written as

$$T_t = \alpha_0 + \alpha_1 t + \dots + \alpha_n t^n, \quad (3.12)$$

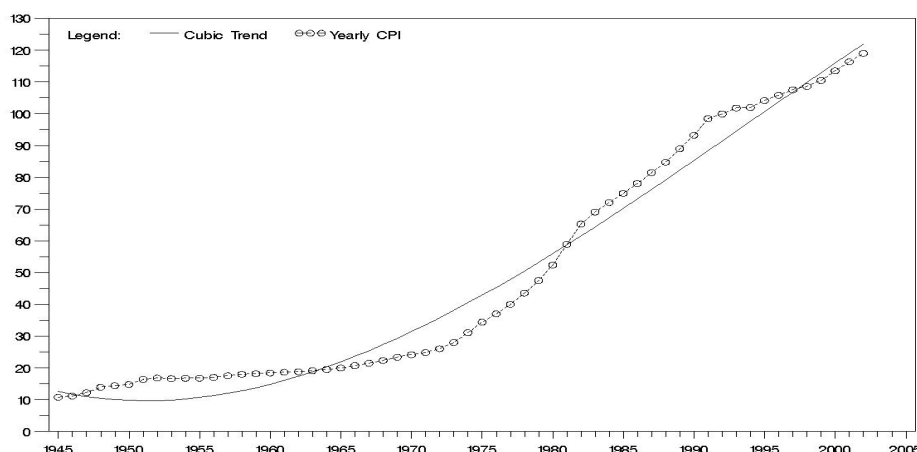
where generally  $n \leq 3$ . The trend is said to be of a deterministic character because the observed series is affected by random shocks which are assumed to be uncorrelated with the systematic part. Besides polynomial of time, three very well known growth functions have been widely applied in population and economic studies, namely the modified exponential, the Gompertz and the logistic. Historically, the first growth model for time series was proposed by Malthus (1798) in the context of population growth. He stated two time path processes, one for the supply of food and the other for population. According to Malthus, the supply of food followed an arithmetic progression and population a geometric one.

Figure 3.1 shows a deterministic cubic trend fitted to the yearly averages of the Canadian Consumer Price Index.

### 3.3.2 Stochastic Trends

Stochastic models are appropriate when the trend is assumed to follow a non-stationary stochastic process. The non-stationarity is modelled with finite differences of low order (cf. Harvey (1985), Maravall (1993)).

A typical stochastic trend model often used in structural time series modelling, is the so-called random walk with constant drift. In the classical notation the model is

**Figure 3.1:** Cubic trend fitted to the yearly averages of the Canadian Consumer Price Index

$$\mu_t = \mu_{t-1} + \beta + \xi_t, \quad t = 1, 2, \dots, n; \quad \xi_t \sim N(0, \sigma_\xi^2), \quad \text{or} \quad \Delta\mu_t = \beta + \xi_t \quad (3.13)$$

where  $\mu_t$  denotes the trend,  $\beta$  a constant drift and  $\{\xi_t\}$  is a normal white noise process. Solving the difference equation 3.13 and assuming  $\xi_0 = 0$ , we obtain

$$\mu_t = \beta t + \Delta^{-1}\xi_t = \beta t + \sum_{j=0}^{\infty} \xi_{t-j}, \quad t = 1, \dots, n, \quad (3.14)$$

which shows that a random walk with constant drift consists of a linear deterministic trend plus a non-stationary infinite moving average.

Another type of stochastic trend belongs to the  $\text{ARIMA}(p, d, q)$  class, where  $p$  is the order of the autoregressive polynomial,  $q$  is the order of the moving average polynomial and  $d$  the order of the finite difference operator  $\Delta = (1 - B)$ . The backshift operator  $B$  is such that  $B^n z_t = z_{t-n}$ . The  $\text{ARIMA}(p, d, q)$  model is written as

$$\phi_p(B)(1 - B)^d z_t = \theta_q(B)a_t, \quad a_t \sim N(0, \sigma_a^2), \quad (3.15)$$

where  $z_t$  now denotes the trend,  $\phi_p(B)$  the autoregressive polynomial in  $B$  of order  $p$ ,  $\theta_q(B)$  stands for the moving average polynomial in  $B$  of order  $q$ , and  $\{a_t\}$  denotes the innovations assumed to follow a normal white noise process. For example, with  $p = 1$ ,  $d = 2$ ,  $q = 0$ , model (3.15) becomes

$$(1 - \phi_1 B)(1 - B)^2 z_t = a_t, \quad (3.16)$$

which means that after applying first order differences twice, the transformed series can be modelled by an autoregressive process of order one.

## 3.4 The Business Cycle

The business cycle is a quasi-periodic oscillation characterized by periods of expansion and contraction of the economy, lasting on average from three to five years. Because most time series are too short for the identification of a trend, the cycle and the trend are estimated jointly and referred to as the trend-cycle. As a result the concept of trend loses importance. The trend-cycle is considered a fundamental component, reflecting the underlying socio-economic conditions, as opposed to seasonal, trading-day and transient irregular

fluctuations.

The proper identification of cycles in the economy requires a definition of contraction and expansion. The definition used in capitalistic countries to produce the chronology of cycles is based on fluctuations found in the aggregate economic activity. A cycle consists of an expansion phase simultaneously present in many economic activities, followed by a recession phase and by a recovery which develops into the next expansion phase. This sequence is recurrent but not strictly periodic. Business cycles vary in intensity and duration. In Canada for example, the 1981 recession was very acute but of short duration, whereas the 1991 recession was mild and of long duration. Business cycles can be as short as 13 months and as long as 10 years.

A turning point is called a peak or downturn when the next estimate of the trend-cycle indicates a decline in the level of activity; and a trough in the opposite situation. There are many ways to determine when a downturn occurs, but in general, (see e.g. Dagum and Luati (2000), Chhab et al. (1999), Zellner et al. (1991)) a downturn is deemed to occur at time  $t$  in the trend-cycle of monthly series, if

$$c_{t-3} \leq c_{t-2} \leq c_{t-1} > c_t \geq c_{t+1}; \quad (3.17)$$

and an upturn, if

$$c_{t-3} \geq c_{t-2} \geq c_{t-1} < c_t \leq c_{t+1}. \quad (3.18)$$

Thus a single change to a lower level  $c_t$ , between  $t + 1$  and  $t$ , qualifies as a downturn, if  $c_{t+1} \leq c_t$  and  $c_{t-3} \leq c_{t-2} \leq c_{t-1}$ ; and conversely for an upturn.

The dating of downturns and upturns is based on a set of economic variables related to production, employment, income, trade and so on.

### 3.4.1 Deterministic and Stochastic Models for the Business Cycle

Similarly to the trend, the models for cyclical variations can be deterministic or stochastic. Deterministic models often consist of sine and cosine functions of different amplitude and periodicities. For example, denoting the cycle by  $c_t$ , a deterministic model is

$$c_t = \sum_{j=1}^{j=2} [\alpha_j \cos(\lambda_j t) + \beta_j \sin(\lambda_j t)], \quad (3.19)$$

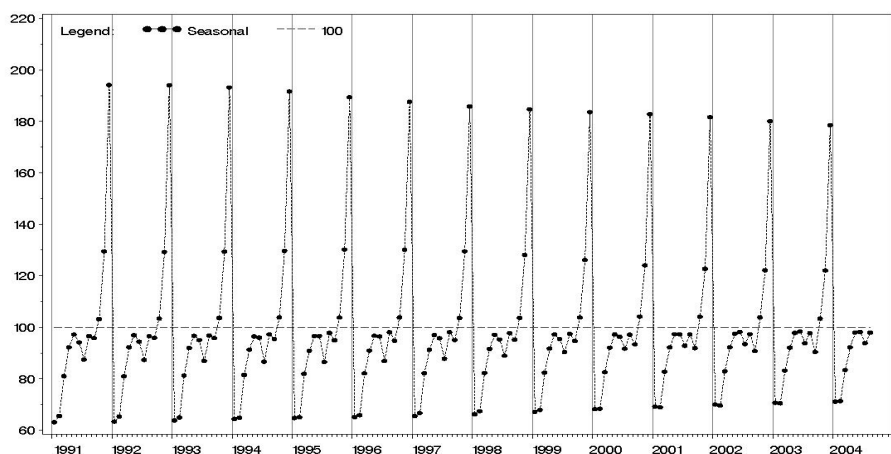
where  $\lambda_1 = 2\pi/60$  and  $\lambda_2 = 2\pi/40$ . Model (3.27) takes into consideration two dominant cycles found in the European and American economies, those of 60 and 40 months respectively.

Stochastic models of the ARIMA type, involving autoregressive models of order 2 with complex roots, have also been used to model the trend-cycle.

For example,

$$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + a_t, \quad a_t \sim N(0, \sigma^2) \quad (3.20)$$

where  $c_t$  denotes the cycle,  $a_t$  is assumed Normal white noise, and the following conditions apply to the parameters:  $\phi_1 + \phi_2 < 1$ ,  $\phi_2 - \phi_1 < 1$  and  $-1 < \phi_2 < 0$  (see Box and Jenkins (1970)).

**Figure 3.2: Seasonal pattern of Sales by Canadian Department Stores**

## 3.5 The Seasonal Variations

Seasonality originates from climate and conventional seasons, like religious, social and civic events, which repeat from year to year.

The climatic seasons influence trade, agriculture, the consumption patterns of energy, fishing, mining and related activities. For example, the consumption of heating oil increases in winter, and the consumption of electricity increases in the summer months because of air conditioning. Institutional seasons like Christmas, Easter, civic holidays, the school and academic year have a large impact on retail trade and on the consumption of certain goods and services, namely travel by plane, hotel occupancy, consumption of gasoline.

The four main causes of seasonality are attributed to the weather, composition of the calendar, major institutional deadlines and expectations. Seasonality is largely exogenous to the economic system but can be partially offset by human intervention. For example, seasonality in money supply can be controlled by central bank decisions on interest rates. In other cases, the seasonal effects can be offset by international and inter-regional trade. To some extent seasonality can evolve through technological and institutional changes. For example the developments of appropriate construction materials and techniques made it possible to continue building in winter. The development of new crops, which better resist cold and dry weather, have influenced the seasonal pattern. The partial or total replacement of some crops by chemical substitutes, e.g. substitute of sugar, vanilla and other flavours, reduces seasonality in the economy.

As for institutional change, the extension of the academic year to the summer months affected the seasonal pattern of unemployment for the population of 15 to 25 years of age. Similarly the practice of spreading holidays over the whole year impacted on seasonality.

The changing industrial mix of an economy also transforms the seasonal pattern, because some industries are more seasonal than others. In particular, economies which diversify and depend less on "primary" industries (e.g. fishing, agriculture) typically become less seasonal.

In most situations, seasonality evolves slowly and gradually as illustrated in Figure 3.2 for the sales by Canadian Department Stores.

In order to determine whether a series contains seasonality, it is sufficient to identify at least one month (or quarter) which tends to be systematically higher or lower than other months. In Figure 3.2 the sales values are much larger in December and much lower in January and February with respect to other months. The seasonal pattern measures the relative importance of the months of the year. The constant 100% represents



an average month or a non-seasonal month. The *peak* month is December, with sales almost 100% larger than on an average month; the *trough* months are January and February with sales almost 40% lower than on an average month. The *seasonal amplitude*, the difference between the peak and trough months of the seasonal pattern, reaches almost 140%.

### 3.5.1 Deterministic and Stochastic Models for Seasonality

The simplest deterministic seasonal model for monthly series can be written as

$$S_t = \sum_{j=1}^{j=12} \alpha_j d_{jt} + u_t, \quad d_{jt} = \begin{cases} 1 & \text{if } j = t \pm 12k, \quad k = 0, 1, 2, \dots, 11 \\ 0 & \text{otherwise,} \end{cases} \quad (3.21)$$

subject to  $\sum_{j=1}^{j=12} \alpha_j = 0$ , and where  $\{u_t\}$  is assumed to be a white noise. The  $\alpha_j$  are the seasonal effects and the  $d_{jt} = s$  are dummy variables.

Model (3.21) can be equivalently written by means of sines and cosines

$$S_t = \sum_{j=1}^{j=6} [\alpha_j \cos(\lambda_j t) + \beta_j \sin(\lambda_j t)], \quad (3.22)$$

where  $\lambda_j = 2\pi j/12$ , for  $j = 1, 2, \dots, 6$  and  $\beta_6 = 0$ . The  $\lambda_j$  are known as the seasonal frequencies, with  $j$  corresponding to cycles lasting 12, 6, 4, 3, 2.4 and 2 months respectively.

In order to represent stochastic seasonality, the  $\alpha_j$  of equation 3.21 are specified as random variables instead of constant coefficients (see Dagum (2001)). Such a model is

$$S_t = S_{t-12} + \omega_t, \quad (3.23)$$

or

$$(1 - B^{12})S_t = \omega_t, \quad (3.24)$$

subject to constraints  $\sum_{j=0}^{j=11} S_{t-j} = \omega_t$  where  $\omega_t$  is assumed white noise.

Model (3.22) specifies seasonality as a non-stationary random walk process. Since  $(1 - B^s \equiv (1 - B)(1 + B + \dots + B^{s-1}))$ , model-based seasonal adjustment method assigns  $(1 - B)$  to the trend and  $S(B) = \sum_{j=s-1}^j 0B^j$  to the seasonal component. Hence, the corresponding seasonal model is

$$\sum_{j=0}^{j=s-1} S_{t-j} = \omega_t, \quad (3.25)$$

which entails a volatile seasonal behavior, because the sum is not constrained to 0 but to the value of  $\omega_t$ . Indeed, the spectrum of  $\sum_{j=0}^{j=s-1} B^j$  (not shown here) displays broad bands at the high seasonal frequencies, i.e. corresponding to cycles of 4, 3, and 2.4 months.

Model (3.25) has been used in many structural time series models (see e.g. Harvey (1981), Kitagawa and Gersch (1984)). A very important variant to model (3.25) was introduced by Hillmer and Tiao (1982) and largely discussed in Bell and Hillmer (1984), that is

$$\sum_{j=0}^{j=s-1} S_{t-j} = \eta_s(B)b_t, \quad (3.26)$$

where  $\eta_s(B)$  is a moving average of  $s - 1$  minimum order and  $b_t \sim WN(0, \sigma_b^2)$ . The moving average com-



ponent enables seasonality to evolve gradually. Indeed, the moving average eliminates the aforementioned bands at the high seasonal frequencies.

Another stochastic seasonality model is based on trigonometric functions (see Harvey (1989)) defined as

$$S_t = \sum_{j=1}^{j=[s/2]} \gamma_{j,t}, \quad (3.27)$$

where  $\gamma_{j,t}$  denotes the seasonal effects generated by

$$\begin{bmatrix} \gamma_{j,t} \\ \gamma_{j,t}^* \end{bmatrix} = \begin{bmatrix} \cos \lambda_j & \sin \lambda_j \\ -\sin \lambda_j & \cos \lambda_j \end{bmatrix} \begin{bmatrix} \gamma_{j,t-1} \\ \gamma_{j,t-1}^* \end{bmatrix} + \begin{bmatrix} \omega_{j,t} \\ \omega_{j,t}^* \end{bmatrix} \quad (3.28)$$

and  $\lambda_j = 2\pi j/s$ ,  $j = 1, \dots, [s/2]$  and  $t = 1, \dots, T$ . The seasonal innovation  $\omega_{j,t}$  and  $\omega_{j,t}^*$  are mutually uncorrelated with zero means and common variance  $\sigma_\omega^2$ .

## 3.6 Calendar Variations

We shall give here only a brief description of calendar variations, and refer the reader to Chapter 5 for further details.

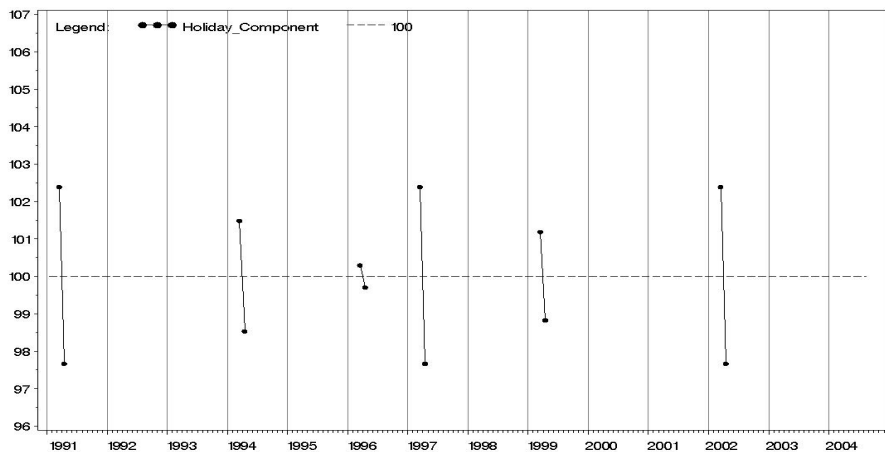
### 3.6.1 The Moving-Holiday Component

The moving-holiday or moving-festival component is attributed to calendar variations, namely the fact that some holidays change date from year to year. For example, Easter can fall between March 23 to April 25. The Chinese New Year date depends on the lunar calendar. Ramadan falls eleven days earlier from year to year. In the Moslem world, Israel and in the Far East, there are many such festivals. For example, Malaysia contends with as many as eleven moving festivals, due to its religious and ethnic diversity. These festivals affect flow and stock variables and may cause a displacement of activity from one month to the previous or the following month. For example, an early date of Easter in March or early April can cause an important excess of activity in March and a corresponding shortfall in April, in variables associated to imports, exports, and tourism. When Easter falls late in April (e.g. beyond the 10-th), the effect is captured by the seasonal factor of April. In the long run, Easter falls in April 11 times out of 14. Some of these festivals have a positive impact on certain variables, for examples air traffic, sales of gasoline, hotel occupancy, restaurant activity, sales of flowers and chocolate (in the case of Easter). The impact may be negative on other industries or sectors which close or reduce their activity during these festivals.

**Table 3.1:** Dates of Easter and presence of effect in March.

March 31 1991	April 12 1998, no effect
April 19 1992, no effect	April 4 1999
April 11 1993, no effect	April 23 2000, no effect
April 3 1994	April 15 2001, no effect
April 16 1995, no effect	March 31 2002
April 7 1996	April 20 2003, no effect
March 30 1997	April 11 in 2004, no effect

The festival effect may affect only the day of the festival itself, or a number of days preceding and/or following the festival. In the case of Easter, travelers tend to leave a few days before and return after Easter, which

**Figure 3.3: Moving Holiday component of the Sales by Canadian Department Stores**

affects air traffic and hotel occupancy, etc., for a number of days. Purchases of flowers and other highly perishable goods, on the other hand, are tightly clustered immediately before the Easter date.

The effect of moving festivals can be seen as a seasonal effect dependent on the date(s) of the festival. Figure 3.3 displays the Easter effect on the sales by Canadian Department Stores. In this particular case, the Easter effect is rather mild. In some of the years, the effect is absent because Easter fell too late in April. The dates of Easter appear in Table 3.1.

In the case exemplified, the effect is felt seven days before Easter and on Easter Sunday but not after Easter. This is evidenced by years 1994, 1996 and 1999 where Easter falls early in April and impacts the month of March. Note that the later Easter falls in April, the smaller the displacement of activity to March; after a certain date the effect is entirely captured by the April seasonal factor. The effect is rather moderate for Department Stores. This may not be the case for other variables. For example, Imports and Exports are substantially affected by Easter, because Customs do not operate from Good Friday to Easter Monday. Easter can also significantly affect quarterly series, by displacing activity from the second to the first quarter.

There has been cases of complete reversal on the timing of the Easter effect. For example, Marriages in Canada were performed mainly by the Church during the 1940s up to the 1960s. The Church did not celebrate marriages during the Lent period, i.e. the 40 days before Easter. Some marriages therefore were celebrated before the Lent period, potentially affecting February and March. However, if Easter fell too early, many of these marriages were postponed *after* Easter.

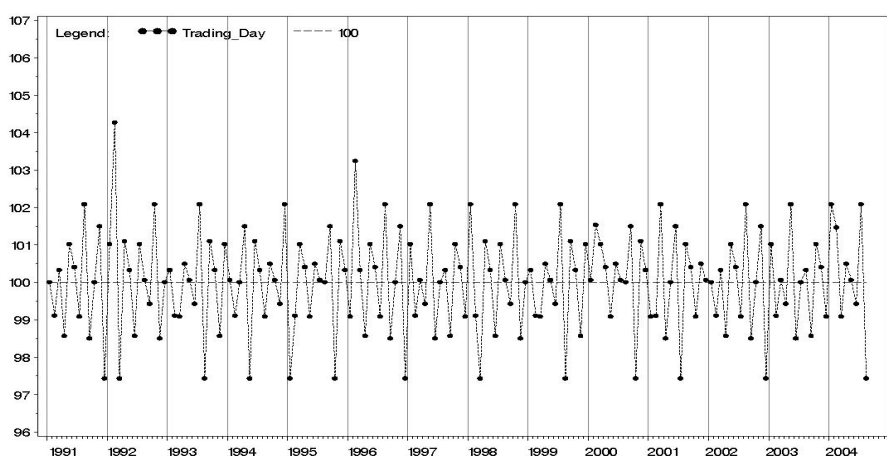
Generally, festival effects are difficult to estimate, because the nature and the shape of the effect are often not well known. Furthermore, there are few observations, i.e. one occurrence per year.

### 3.6.2 The Trading-Day Component

Flow series may be affected by other variations associated with the composition of the calendar. The most important calendar variations are the trading-day variations, which are due to the fact that some days of the week are more important than others. Trading-day variations imply the existence of a daily pattern analogous to the seasonal pattern. However, these daily factors are usually referred to as *daily coefficients*.

Depending on the socio-economic variable considered, some days may be 60% more important than an

Figure 3.4: Trading-day estimates of the Sales by Canadian Department Stores



average day and other days, 80% less important.

If the more important days of the week appear five times in a month (instead of four), the month registers an excess of activity *ceteris paribus*. If the less important days appear five times, the month records a short-fall. As a result, the monthly *trading-day component* can cause increase of +8% or -8% (say) between neighbouring months and also between same-months of neighbouring years. The trading-day component is usually considered as negligible and very difficult to estimate in quarterly series.

Figure 3.4 displays the monthly trading-day component obtained from the following daily pattern: 90.2, 71.8, 117.1, 119.3, 97.6, 161.3 and 70.3 for Monday to Sunday (in percentage) respectively. The daily pattern indicates that Saturday is approximately 61% more important than an average day (100%); and that Tuesday and Sunday, 30% less important. For the multiplicative, the log-additive and the additive time series decomposition models, the monthly trading-day component is respectively obtained in the following manner

$$D_t = \sum_{\tau \in t} d_{\tau}/n_t \equiv (2800 + \sum_{\tau \in t, 5 \text{ times}} d_{\tau})/n_t, \quad (3.29)$$

$$D_t = \exp\left(\sum_{\tau \in t} d_{\tau}/n_t\right) \equiv \exp\left(\sum_{\tau \in t, 5 \text{ times}} d_{\tau}/n_t\right), \quad (3.30)$$

$$D_t = \sum_{\tau \in t} d_{\tau} \equiv \left(\sum_{\tau \in t, 5 \text{ times}} d_{\tau}\right), \quad (3.31)$$

where  $d_{\tau}$  are the daily coefficients in the month. The preferred option regarding  $n_t$  is to set it equal to the number of days in month  $t$ , so that the length-of-month effect is captured by the multiplicative seasonal factors, except for Februaries<sup>1</sup> The other option is to set  $n_t$  equal to 30.4375, so that the multiplicative trading-day component also accounts for the length-of-month effect. The number 2800 in equation 3.29 is the sum of the first 28 days of the months expressed in percentage.

The monthly trading-day estimates of the Sales by Canadian Department Stores shown in Figure 3.4 were obtained with the log-additive model (3.30). They display a drop of 7% between Jan. and Feb. 1992 and a drop of 5% between Jan. 1992 and Jan. 1993. One can identify several instances where the change between

<sup>1</sup>To adjust Februaries for the length-of-month, the seasonal factors of that month are multiplied by 29/28.25 and 28/28.25 for the leap and non-leap years respectively.

same-months is significant. Indeed, same-month year-ago comparisons are never valid in the presence of trading-day variations, not even as a rule of thumb. Furthermore, it is apparent that the monthly trading-day factors in the figure are identical for quite a few months. Indeed for a given set of daily coefficients, there are only 22 different monthly values for the trading-day component, for a given set of daily coefficients: seven values for 31-day months (depending on which day the month starts), seven for 30-day months, seven for 29-day months and one for 28-day months. In other words, there are at most 22 possible arrangements of days in monthly data.

Many goods and services are affected by daily patterns of activity, which entail higher costs for producers, namely through the need of higher inventories, equipment and staff on certain days of the week.

For example, there is evidence that consumers buy more gasoline on certain days of the week, namely on Thursdays, Fridays, Saturdays and holidays, which results in line-ups and shortages at the pumps. In order to cope with the problem, gasoline retailers raise their price on those days to promote sales on other days. Furthermore, the elasticity of demand for gasoline is low. In other words, to reduce consumption by a small percentage, prices must be raised by a disproportionate percentage, which upsets some consumers. On the other hand, consumers can buy their gasoline on other days. The alternative for retailers is to acquire larger inventories, larger tanks, more pumps and larger fleets of tanker trucks, all of which imply higher costs and translate into much higher prices. In other words, there are savings associated with more uniform daily patterns; and costs, with scattered daily patterns.

A similar consumer behaviour prevails for the purchases of food, which probably results in more expensive prices, namely through higher inventories, larger refrigerators, more numerous cash registers and more staff, than otherwise necessary.

Scattered daily patterns have been surprisingly observed for variables like births and deaths. Indeed, births are more frequent on certain days of the week, namely on working days to avoid overtime pay. This results from the practice of caesarean delivery and especially birth inducement now widely applied to encourage births on working days. A time series decomposition of monthly data for Québec in the 1990s revealed that 35% more births took place on Thursdays. A similarly analysis of the Ontario data revealed the same phenomenon. In this particular case, an appropriately scattered daily pattern reduces costs.

Deaths also occur more often on certain days of the week. Car accidents, drowning, skiing and other sporting accidents tend to occur on weekend days and on holidays. According to the Canadian Workmen Compensation Board, industrial accidents tend to occur more often on Friday afternoons when security is more lax.

In principle, stock series pertaining to one day display a particular kind of trading-day variations. Among other things, inventories must anticipate the activity (flow) of the following day(s). For such stock series, the monthly trading-day factor coincides with the daily weight of the day.

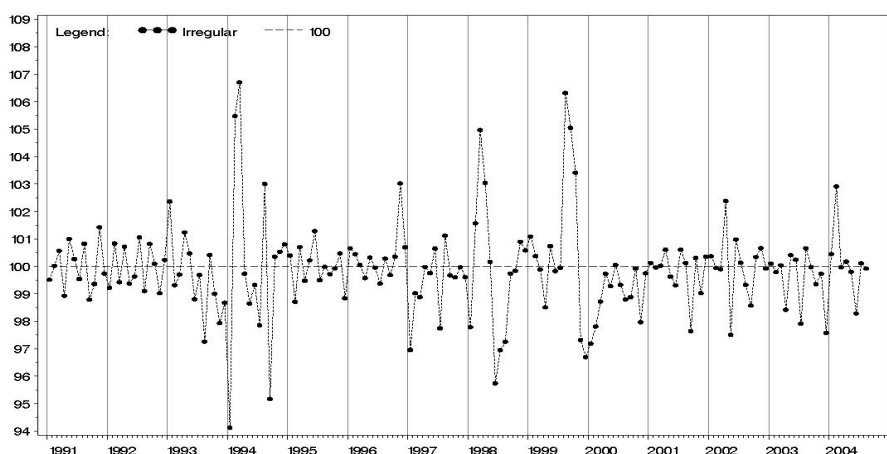
### 3.6.3 A Classical Model for Trading-Day Variations

A classical deterministic model for trading-day variations was developed by Young (1965),

$$y_t = D_t + u_t, \quad t = 1, \dots, n \quad (3.32)$$

$$D_t = \sum_{j=1}^{j=7} \alpha_j N_{jt}, \quad (3.33)$$

where  $u_t \sim WN(0, \sigma_u^2)$ ,  $\sum_{j=1}^{j=7} \alpha_j = 0$ , and  $\alpha_j$ ,  $j = 1, \dots, 7$ , denote the effects of the seven days of the week, Monday to Sunday, and  $N_{jt}$  is the number of times day  $j$  is present in month  $t$ . Hence, the length of the month is  $N_t = \sum_{j=1}^{j=7} N_{jt}$ , and the cumulative monthly effect is given by Equation 3.33.

**Figure 3.5:** Irregular component of Sales by Canadian Department Stores

Adding and subtracting  $\bar{\alpha} = \sum_{j=1}^{j=7} \alpha_j / 7$  to Equation 3.33 yields

$$D_t = \bar{\alpha}N_t + \sum_{j=1}^{j=7} (\alpha_j - \bar{\alpha})N_{jt} \quad (3.34)$$

Hence, the cumulative effect is given by the length of the month plus the net effect due to the days of the week. Since  $\sum_{j=1}^{j=7} (\alpha_j - \bar{\alpha}) = 0$ , model 3.33 takes into account the effect of the days present five times in the month. Model (3.34) can then be written as

$$D_t = \bar{\alpha}N_t + \sum_{j=1}^{j=6} (\alpha_j - \bar{\alpha})(N_{jt} - N_{7t}), \quad (3.35)$$

with the effect of Sunday being  $\alpha_7 = -\sum_{j=1}^{j=6} \alpha_j$ .

Deterministic models for trading-day variations assume that the daily activity coefficients are constant over the whole range of the series.

Stochastic model for trading-day variations have been rarely proposed. Dagum et al. (1992) developed a model where the daily coefficients change over time according to a stochastic difference equation.

## 3.7 The Irregular Component

The irregular component in any decomposition model represents variations related to unpredictable events of all kinds. Most irregular values have a stable pattern, but some extreme values or outliers may be present. Outliers can often be traced to identifiable causes, for example strikes, droughts, floods, data processing errors. Some outliers are the result of displacement of activity from one month to the other.

Figure 3.5 displays the irregular component of Sales by Canadian Department Stores, which comprises extreme values, namely in 1994, 1998, 1999 and Jan 2000. Most of these outliers have to do with the closure of some department stores and the entry of a large department store in the Canadian market.

As illustrated by Figure 3.5, the values of the irregular component may be very informative, as they quantify the effect of events known to have happened.

Note that it is much easier to locate outliers in the irregular component than in the raw series because the presence of seasonality hides the irregular fluctuations.

### 3.7.1 Redistribution Outliers and Strikes

Some events can cause displacements of activity from one month to the next months, or vice versa. This phenomenon is referred to as redistribution outliers. We also deal with the strike effect under this headline. The outliers must be modelled and temporally removed from the series in order to reliably estimate the systematic components, namely the seasonal and trading-day components.

In January 1998, there was a severe Ice Storm which brought down major electrical transmission lines, causing black outs lasting from a few days to several weeks in populated parts of eastern Ontario and western Quebec. The irregular component of New Motor Vehicles provided a measure of the effect, in the amount of 15% drop in sales. This drop was compensated in February.

The termination of a government grant program, e.g. for the purchase of a first house, can displace activity from a month to previous months. For example, in the 1990s the Province of Ontario had a grant program to encourage households to purchase a first house. At one point, the termination of the program was announced. Some buyers who considered purchasing a house decided to take advantage of the program. This caused a surge of activity in the months preceding the termination date and a shortfall in months following. Conversely, activity can be delayed, when a government program is expected to take effect at a given date.

Events like major snow storms and power black outs usually postpone activity to the next month, without much longer term effect.

### 3.7.2 Models for the Irregular Component and Outliers

The irregulars are most commonly assumed to follow a white noise process defined by

$$E(u_t) = 0, \quad E(u_t)^2 = \sigma_u^2 < \infty, \quad E(u_t u_{t-k}) = 0 \text{ if } k \neq 0 \quad (3.36)$$

If  $\sigma_u^2$  is assumed constant (homoscedastic condition),  $u_t$  is referred to as white noise in the strict sense. If  $\sigma_u^2$  is finite but not constant (heteroscedastic condition),  $u_t$  is called white noise in the weak sense. For inferential purposes, the irregular component is often assumed to be normally distributed and not correlated, which implies independence. Hence,  $u \sim NID(0, \sigma_u^2)$ .

There are different models proposed for the presence of outliers depending on how they impact the series under question. If the effect is transitory, the outlier is said to be additive; and if permanent, to be multiplicative. Box and Tiao (1975) introduced the following intervention model to deal with different types of outliers,

$$y_t = \sum_{j=0}^{\infty} h_{t-j} x_{t-j} + \eta_t = \sum_{j=0}^{\infty} h_j B^j x_j + \eta_t = h(B)x_t + \eta_t \quad (3.37)$$

where the observed series  $\{y_t\}$  consists of an input series  $\{x_t\}$  considered a deterministic function of time and a stationary process  $\{\eta_t\}$  of zero mean and non-correlated with  $\{x_t\}$ . In such a case the mean of  $\{y_t\}$  is given by the deterministic function  $\sum_{j=0}^{\infty} h_{t-j} x_{t-j}$ . The type of function assumed for  $\{x_t\}$  and weights  $\{h_t\}$  depend on the characteristic of the outlier or unusual event and its impact on the series.

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# 4

## A Brief History of Seasonal Adjustment Methods and Software Tools



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## Table of Contents

4.1	Introduction	69
4.2	Non-parametric methods (implicit models)	70
4.2.1	X-11 method	70
4.2.2	X-11 style methods	72
4.3	Semi-parametric methods (hybrid models)	73
4.3.1	X11-ARIMA	73
4.3.2	X12-ARIMA	74
4.4	Parametric methods (explicit models)	74
4.4.1	Determinist methods	74
4.4.2	Stochastic methods	75
4.5	Conclusions	79
4.6	Annex: A short description of the JDemetra+ software	81
4.6.1	Statistical aspects	81
4.6.2	Technical aspects	82

## 4.1 Introduction

The main objective of seasonal adjustment methods is the elimination of periodic fluctuations that do not contribute to the understanding of the basic trend and short-term movements of any time series. In macroeconomics, major non-seasonal movements, especially trend and cycles, which are of great importance for decision-makers and policy-makers, may be hindered by the presence of a seasonal behavior. The idea to identify and delete seasonality in time series is not recent and can be dated back to the 19<sup>th</sup> century. For example, Jevons (1862) writes:

*“Every kind of periodic fluctuations, whether daily, weekly, monthly, quarterly, or yearly, must be detected and exhibited not only as a subject of study in itself, but because we must ascertain and eliminate such periodic variations before we can correctly exhibit those which are irregular or nonperiodic and probably of more interest and importance.”*

Regarding macroeconomics, as noted by Hylleberg (1992), the seasonal adjustment issue occurs in at least two major economic situations, namely (i) historical business cycle analyses (Burns et al. (1946), Kaiser and Maravall (2000)) and (ii) the assessment of current economic conditions (Moore (1961), Dagum (2001)). It is also pointed out by Hylleberg (1992) that the use of seasonally adjusted data for econometric modelling may imply a misspecification risk for the models, leading in turn to spurious relationship estimation and poor forecasting performances. Having said that, it turns out that the main macroeconomic models used by practitioners, for example in international institutions, governments and central banks, are estimated using seasonally adjusted data provided by official statistical offices.

The underlying idea of seasonal adjustment procedures is that any raw time series can be divided into components, individually non-observed but distinct, as proposed by Persons (1919). For example the Harvard Barometer in the early 1920s is a nice application of this idea.<sup>1</sup> The unobserved components into which it seems natural to divide an economic time series may represent heterogeneous and complex groups of causal factors. Now seasonal adjustment has to be considered as a signal extraction procedure, or estimation of one or more unobserved components, and also as a form of prediction. Following this approach, four typical unobserved components have been identified in the literature: (i) a long term or secular trend ( $T_t$ ), (ii) a cyclical movement superposed to the trend ( $C_t$ ), (iii) a seasonal movement ( $S_t$ ) defined as a regular movement with a fixed infra-annual period, and (iv) a residual variation ( $I_t$ ) representing the unexplained part. Those four unobserved components can be added or multiplied according to the preferred decomposition scheme.

Actually, this decomposition idea is an old one and finds its origin in meteorology and astronomy. For example, the seminal works of Buys Ballot (1847) are often quoted as the first references on seasonal adjustment procedures. For discussions on historical aspects on this topic, we refer for example to Nerlove et al. (1979), Dagum (1979, 1986, 2001), Bell and Hillmer (1984), Armatte (1992), Hylleberg (1992), Ladiray and Quenneville (2001), Darné (2003) or Dagum and Bianconcini (2016). Nowadays the decomposition schemes can take deterministic components into account such as the number of working days within the considered period of time or moving holidays effects (Eastern effects, Thanksgiving effects, . . .) that can affect various time series. For example, using the additive scheme, such effects are integrated in the following way:

$$X_t = T_t + C_t + S_t + WD_t + HE_t + I_t, \quad (4.1)$$

where ( $WD_t$ ) is the working days component and ( $HE_t$ ) is the holidays effect component.

As noted before, seasonal adjustment can be seen as a signal extraction exercise. Thus the main issue when estimating a given component is to not affect the other ones. Under this constraint, seasonal adjustment methods were initially developed in the 1920s and 1930s as a tool for seasonal economic variable analysis in the absence of suitable statistical models (see Mendershausen (1937)). The methods were developed empirically, using non-parametric tools such as moving averages. Adequate parametric models for seasonal

<sup>1</sup> See Armatte (2003) for a description of the Harvard Barometer for business cycle analysis.

models were not used until the 1950s.<sup>2</sup> Then, advances in that domain has grown enormously, using the subsequent development of computer software for time series modeling.

Seasonal adjustment procedures are generally classified into two categories: *non-parametric* methods based on linear smoothing filters and *parametric* methods in which unobserved components are explicitly specified and estimated.<sup>3</sup> In turn the specification of parametric models can be done either by assuming a deterministic behavior for each component in equation (4.1) or by considering that each component of this equation follows a stochastic process, as for example an ARIMA-type process. However, the recent development of procedures mixing approaches belonging to both categories questions this classification. Consequently *semi-parametric* methods combining explicit and implicit modelling of each components are now considered by researchers.

In this chapter, starting from this classification, we propose to return on the various seasonal adjustment methods proposed in the literature from an historical perspective.

## 4.2 Non-parametric methods (implicit models)

As regards macroeconomic time series, smoothing techniques based on linear filtering, especially moving averages, were already known in early 1920s but were rarely used in practice (Falkner (1924), King (1924)). The main tool to eliminate seasonal fluctuations, namely the centered moving average of order 12 for monthly series (or of order 4 for quarterly ones), turned out to be a poor estimate of the trend cycle component ( $T_t + C_t$  in equation (4.1)). Indeed, it has been shown that this specific moving average was not able to track accurately turning points of cycles with a period longer than 5 years. In addition moving averages are very sensitive to the presence of outliers and therefore need a pre-treatment of extreme values. This limit has led some researchers as King (1924) to introduce more robust estimates of seasonal factors like the moving median. At the end of the 1920s, the development of new smoothing filters allowed a broader diffusion of non-parametric or "empirical" methods (see for example Macaulay (1931)). At the same time, Joy A. and Thomas (1928) put forward an approach based on moving averages enabling to take into account long term variations of seasonal fluctuations present in many series followed by the Federal Reserve Board. Progressively, governments and statistical offices started to apply such approaches to get seasonally adjusted time series. But this was costly, long and subjective, as noted by Dagum (1979), because adjustments were carried out manually.

In the years following the Second World War, computer science developments largely contributed to the diffusion and improvement of non-parametric methods. In this respect, the most famous approach is certainly X-11 that has generated a great variety of approaches for which the basic principles are identical to X-11 and are referred to as X-11 *style*, in the terminology of Sutcliffe (1999), including GLAS, STL, SABL and SEASABS.

### 4.2.1 X-11 method

In 1954, Julius Shiskin developed at the US Bureau of the Census a seasonal adjustment method called *Method I* (see Shiskin (1957), Shiskin (1978)). This technique was then followed by a sequence of eleven experimental versions of *Method II* (X-0, X-1, ...), still developed by the same team, to finally led to the X-11 version of the program in 1965 (Shiskin et al. (1967)). Directly inspired from moving average smoothing techniques and the works of Macaulay (1931), these versions were the first automatic approaches for seasonal adjustment and X-11 became rapidly a worldwide reference.

The family of X-11 programs is a seasonal adjustment method based on different kinds of weighted (symmetric and asymmetric) moving averages, without an underlying explicit model and it was developed mostly on

<sup>2</sup>For detailed description of seasonal adjustment methods we refer to Fase et al. (1973), Baron (1973), Burman (1979), Pierce (1980) and Den Butter et al. (1991). More recent studies have been carried out by Fischer (1995), Bianchi (1996), National Statistics (1996), Eurostat (1998), European Central Bank (2000), Darné (2003) or Dagum and Bianconcini (2016).

<sup>3</sup>This classification was introduced by Dagum (1986), see also Sutcliffe (1999) on this point.

an empirical basis. In addition, the X-11 program enables to identify and to delete working days effects. However, the integration of holidays effects is not authorized. Each estimated component is obtained through various iterations detailed by Dagum (1988) and Ladiray and Quenneville (2001) (see also Laroque (1977), for quarterly data, and Hylleberg (1992)). Statistical properties of filters can be found in Dagum (1978) while the study of filter convolutions is detailed in Dagum et al. (1996).

The estimates of trend and seasonal components are obtained from cascade filtering that results from the convolution of various individual linear filters: (i) 12-term centered seasonal moving average; (ii) two  $3 \times (2n + 1)$  seasonal moving averages; and (iii) the Henderson moving average. The 12-term centered seasonal moving average is defined as

$$D(B) = (1/24)B^{-6}(1 + B)(1 + B + B^2 + \dots + B^{11})$$

where  $B$  is the backshift operator defined as  $B^m y_t = y_{t-m}$  and  $B^0 = 1$ , and the seasonal moving averages as follows

$$S_t^{3 \times (2n+1)} = \frac{1}{3}(S_{t-12}^{(2n+1)} + S_t^{(2n+1)} + S_{t+12}^{(2n+1)})$$

$$\text{with } S_t^{(2n+1)} = \frac{1}{2n+1} \sum_{j=-n}^n SI_{t+12j}$$

where  $S_t^{3 \times (2n+1)}$  denotes the  $(3 \times (2n + 1))$  seasonal moving averages, and  $SI$  is the seasonal-irregular component (*de-trended value of the series*). The order of the moving average is selected from the irregular-seasonal ratio (I/S) among the following moving averages:  $3 \times 3$ ,  $3 \times 5$  and  $3 \times 9$ .

The estimate of the trend-cycle is made by the application of one of three different Henderson linear filters available in the computer package, namely, the 9-, 13-, and 23-term. These filters developed by Henderson (1916) are based on summation formula which makes the sum of squares of the third differences of the smoothed series a minimum for any numbers of terms. In other words, the  $\sum (\Delta^3 y_t)^2$  is minimized, where  $\Delta$  is the difference operator and  $y_t$  is the output or smoothed series, if and only if  $\sum (\Delta^2 h_k)^2$  is minimized, where  $h_k$ ' are the weights, subject to the constraints that  $\sum h_k = 1$ ,  $\sum kh_k = 0$  and  $\sum k^2 h_k = 0$  (Dagum (1978)).

The Henderson symmetric weight system of length  $2n + 1$ , where  $m = n + 2$ , is given by

$$h_k = \frac{315[(m-1)^2 - k^2][m^2 - k^2][(m+1)^2 - k^2][3m^2 - 16 - 11k^2]}{8m(m^2 - 1)(4m^2 - 1)(4m^2 - 9)(4m^2 - 25)} \quad (4.2)$$

To derive a set of 13 weights from (4.2), 8 is substituted for  $m$  and the values are obtained for each  $n$  from -6 to 6. The Henderson 13-term trend-cycle filter is thus given by<sup>4</sup>,

$$H_{13}(B) = -0.019B^{-6} - 0.028B^{-5} + 0.00B^{-4} + 0.065B^{-3} + 0.147B^{-2} \\ + 0.214B^{-1} + 0.24B^{-0} + 0.214B^1 + 0.147B^2 + 0.065B^3 \\ + 0.00B^4 - 0.028B^5 - 0.019B^6 \quad (4.3)$$

For the observations at the beginning and the end of the series the so called asymmetric Henderson filters were really derived by Musgrave (1964). The calculation of the asymmetric weights is based on the minimization of the mean squared revision between the final estimate (obtained by the application of the symmetric filter) and the preliminary estimate (obtained by the application of an asymmetric filter) subject to the constraint

<sup>4</sup>The Henderson 9-term and 23-term trend-cycle filters are obtained in substituting  $m$  by 6 and 13, respectively. These filters proposed by Henderson (1916) are solutions of a recurrence linear equation (Gouriéroux and Monfort (1997)). See Bell and Monsell (1992) for graphs of Henderson filter weights and transfer function squared gains employed in X-11.



that the sum of the weights is equal to one (Laniel (1985), Dagum (2001)).<sup>5</sup> The assumption made is that at the end of the series, the seasonally adjusted values are equal to a linear trend-cycle plus a purely random irregular, i.e.  $Y_t = c_0 + c_1 t + \varepsilon_t$  with  $\varepsilon_t \sim IID(0, \sigma^2)$ . The equation used is:

$$E[r_t^{i,m}]^2 = c_1^2 \left( t - \sum_{j=-i}^m h_{ij}(t-j) \right)^2 + \sigma^2 \sum_{j=-m}^m (h_{mj} - h_{ij})^2 \quad (4.4)$$

where  $h_{mj}$  and  $h_{ij}$  are the weights of the symmetric (central) filter and the asymmetric filters, respectively;  $h_{ij} = 0$  for  $j = -m, \dots, -i - 1$ ,  $c_1$  is the slope of the line and  $\sigma^2$  denotes the noise variance. There is a relation between  $c_1$  and  $\sigma^2$  such that the noise to signal ratio ( $I/C$ ) is given by

$$(I/C) = (4\sigma^2/\pi)^{\frac{1}{2}}/|c_1| \text{ where } D = \frac{c_1^2}{\sigma^2} = \frac{4}{\pi(I/C)^2} \quad (4.5)$$

The  $I/C$  ratio is given by the absolute mean of the first difference of the irregulars over the absolute mean of the first differences of the trend-cycle estimates. The  $I/C$  ratio determines the length of the Henderson trend-cycle filter appropriate for the series under question.<sup>6</sup> The weights are calculated such that  $I/C$  equal to 3.50, or 0.99 or 4.50 for the 13-term, 9-term and 23-term Henderson filters, respectively. Thus, setting  $t = 0$  and  $m = 6$  for the end weights of the 13-term Henderson, we have,

$$\frac{E[r_0^{i,6}]^2}{\sigma^2} = \frac{4}{\pi(I/C)^2} \left( \sum_{j=-i}^6 h_{ij}(t-j) \right)^2 + \sum_{j=-6}^6 (h_{6j} - h_{ij})^2 \quad (4.6)$$

The standard Henderson trend estimation consists of applying the automatically selected Henderson filter to a robust seasonally adjusted series. The robustification is done by the default replacement of extreme values where irregulars falling between  $\pm 1.5\sigma$  and  $\pm 2.5\sigma$  are scaled down linearly and beyond  $\pm 1.5\sigma$  are replaced by their mean.

## 4.2.2 X-11 style methods

### 4.2.2.1 SEASABS

SEASABS (*SEASonal Analysis at Australian Bureau of Statistics*) has been proposed by the Australian Bureau of Statistics (1987) (see also Australian Bureau of Statistics (1999a,b)) which is a knowledge-based seasonal analysis and adjustment tool. The seasonal adjustment algorithm is based on the X-11 package. SEASABS keeps records of the previous analysis of a series so it can compare X-11 diagnostics over time and "knows" what parameters led to the acceptable adjustment at the last analysis.

### 4.2.2.2 GLAS

GLAS (*General Linear Abstraction of Seasonality*) was developed by Young (1992) at the Bank of England to get seasonally adjusted monetary variables. The trend and seasonal components are estimated and smoothed using a moving average of data with a triangular shaped weighting pattern. GLAS is designed to provide only additive linear adjustments. GLAS implements a *minimum revision* algorithm put forward by

<sup>5</sup>Kenny and Durbin (1982), Laniel (1985), Dagum (2001), Ladiray and Quenneville (2001), Gray and Thompson (2002) and Quenneville et al. (2003) studied and compared different methods to generate the weights of asymmetric moving averages.

<sup>6</sup>Gray and Thompson (2002) proposed different methods to estimate the parameter  $D$  in order to determinate the asymmetric linear filters.



Lane (1972) to evaluate end-of-sample weights. This program also proposes to take working days into account, while the management of holidays effects is not possible.

#### 4.2.2.3 STL

STL (*Seasonal-Trend decomposition based on Loess*)<sup>7</sup> has been proposed by Cleveland et al. (1990) at Bell Laboratories. STL shares the same basic principle as GLAS, namely the idea of non-parametric regression based on locally weighted averaging of the data. Estimate and smoothing of trend and seasonality are obtained by fitting a polynomial by weighted least squares using a tri-cube weight function, more robust to outliers. Note that a linear or quadratic adjustment is used for the trend, while it is a constant or linear one for the seasonal component. As for GLAS, this program also proposes to take working days into account, while the management of holidays effects is not possible.

#### 4.2.2.4 SABL

SABL (*Seasonal Adjustment at Bell Laboratories*) developed by Cleveland et al. (1978) at Bell Laboratories is in principal of similar construction to X-11, and therefore has no underlying explicit model (Cleveland et al. (1981); Cleveland et al. (1982)). SABL was especially created to handle anomalous data (outliers) by using a non-linear filter based on  $M$ -estimation, before applying the linear filter, to reach more robustness. The principles of  $M$ -estimation were put forward by Huber (1964), such minimax estimates are proved to be robust to outliers. The smoothing of trend and seasonal components is based on a rather complex system of moving averages and regressions (on weighted moving medians). In addition, the SABL program integrates working days effects as in X-11.

## 4.3 Semi-parametric methods (hybrid models)

Following the work of Box and Jenkins in the 1970s (see Box and Jenkins (1970)) on the ARIMA (AutoRegressive Integrated Moving Average) models a new variant of X-11 is developed, called X11-ARIMA. It used an ARIMA model to forecast beyond the current series and backcast before the beginning of the series in order to replace the missing data which results at the beginning and end of the series to allow the use of less asymmetric filters. The original X-11 only arbitrarily extrapolated the missing values. The result was that revisions were significantly reduced when finally the missing data becomes available. These semi-parametric models integrated in X11-ARIMA and X12-ARIMA use linear filters of the X-11 method combined with ARIMA-model filters which are adjusted globally to the data. Nevertheless, the ARIMA models can not be applied directly on the economic time series without a previous analysis of them, especially due to the problems of stationarity and determinist effects.

### 4.3.1 X11-ARIMA

X11-ARIMA is an improvement of X-11 program, proposed by Dagum (1975, 1978, 1979), which has been automated in X11-ARIMA/80 by Dagum (1980) at Statistics Canada. The major improvement is the possibility to adjust an ARIMA model to the times series, which allows to forecast until three years additional (one year in the default option), leading to a best seasonal adjustment of the data at the end of the series. This software has been improved by proposing (i) estimation and correction of calendar effects, (ii) statistical tests on the presence of moving seasonality, (iii) indicators to evaluate the quality of the seasonal adjustment, and (iv) a

<sup>7</sup>Loess or Lowess stands for "LOcally WEighted Scatterplot Smoother".

selection of seasonal filters defined in function of the characteristics of the data. These new developments have been implemented in X11-ARIMA/88 and X11-ARIMA/2000 at Statistics Canada (Dagum (1988)).

Another extension of the X-11 method is the UK-version of X-11 by introducing forecasts for twelve months as X-11-ARIMA/88. The forecasts are made by using the Kenny-Durbin autoregressive procedure (Kenny and Durbin (1982)). However, National Statistics (1996) and Fischer (1995) showed that the forecasting performance is better from the X-11-ARIMA than the X-11-UK. The Dutch Central Bureau of Statistics also proposed an extension by developing the CPBX11 software (Central Bureau of Statistics (1981); Van Der Hoeven and Hundepool (1986)), which combines the characteristics of the X-11 method and the CPB-1 method developed by the Central Planning Bureau (Central Bureau of Statistics (1976)).

### 4.3.2 X12-ARIMA

X12-ARIMA is basically based on X11-ARIMA (1980 and 1988 versions) and has been developed at the US Bureau of the Census, see Findley et al. (1988), Findley et al. (1998), US Bureau of the Census (2000). It uses the same method than X-11 and integrates most of the improvements introduced in X11-ARIMA and some news developments. The main change is an additional pre treatment for the data called RegARIMA (*Regression ARIMA*).<sup>8</sup> This pre-program allows to simultaneously estimate outliers, trading-day variations and calendar effects, with a seasonal ARIMA model. This latter is selected from an automatic procedure based on five models, similar to X11-ARIMA, and used to extrapolate the original series. Others seasonal and Henderson filters are added in X12-ARIMA. It also provides the same diagnostics available in X11-ARIMA to test the quality of the seasonal adjustment and some news, such as spectral tests on the seasonality and the residual trading-day variations or diagnostics on the stability of the seasonal adjustment.

## 4.4 Parametric methods (explicit models)

Some authors criticized the non-parametric seasonal adjustment approaches based on linear filters or moving averages. For example, Slutsky (1927) and Yule (1927) showed that using moving averages can introduce artificial cycles in the data, and Fisher (1937) deplored the application of *ad hoc* "empirical" procedures even though appropriate mathematic tools are available. Therefore, the disaffection in these methods conducted to the use of explicit models for the seasonal adjustment. Two types of methods have been developed: (i) methods based on determinist models, and (ii) methods based on stochastic models.

### 4.4.1 Determinist methods

The regression models provide the first seasonal adjustment model-based approaches (MBA).<sup>9</sup> At the end of the 1930s, Fisher (1937) and Mendershausen (1939) proposed to adjust polynomial by the least squares method to remove the seasonal component. In the 1960s, the use of multiple regression techniques to seasonal adjust the economic time series widely diffuses for two reasons: First, the construction of econometric models for monthly and quarterly time series, and, second, the development of the computers. These methods are based on modelling the original series and each component by simple parametric functions, and on the estimations of parameters by the ordinary least squares method. For majors contributions on these methods see Hannan (1960), Lovell (1963), Rosenblatt (1963), Ladd (1964), Jorgenson (1964), Henshaw (1966), Stephenson and Farr (1972), and Wallis (1978).

At the present time this approach has not a large interest for the seasonal adjustment because it does not allow to take into account the stochastic properties of the economic time series. Nevertheless, some extensions

<sup>8</sup>This is similar to the TRAMO part included in SEATS.

<sup>9</sup>Buys Ballot (1847) has been the first to propose a determinist seasonal adjustment method based on global regressions.

of the determinist methods have been developed by using local regressions, such as DAINITIES and BV4 softwares.

#### 4.4.1.1 DAINITIES

DAINTIES was the official seasonal adjustment method of the European Commission and was developed in 1979 as a successor of the SEABIRD method.<sup>10</sup> It is based on moving regression methods and assumes an additive or a multiplicative decomposition based on a log-additive decomposition on base 10. DAINITIES uses only asymmetric filter, so that no revisions are necessary. This does however lead to drawbacks. Asymmetric filters lead to phase shifts in the estimator and to distortions of the estimator (Fischer (1995)).

#### 4.4.1.2 BV4

BV4 (*Berliner Verfahren 4*, Procedure of Berlin version No 4) is a seasonal adjustment method based on moving filter applications derived from approximating functions by a regression approach, proposed by Nourney (1983, 1984). It was developed originally by the Technical University Berlin (*Technische Universität Berlin*) and the German Institute for Economic Research (*Deutsches Institut für Wirtschaftsforschung*, DIW) (Nullau et al. (1969)). It is the official seasonal adjustment method of the Central Statistical Office of Germany, see Statistisches Bundesamt (2006). BV4 is theoretically better founded than ad hoc methods due to the use of local regression methods. Nevertheless this method lead to identification problems and can only sound theoretically better after some specifications are done, such as using polynomial of order three and trigonometric functions for estimating the trend and seasonal components, respectively.

### 4.4.2 Stochastic methods

The development of the theory on the stochastic processes as well as the development of the computers after the World War II strongly contributed to the improvement of the parametric methods, and thus to the seasonal adjustment tools. These methods are based on the specification of unobserved component ARIMA (UCARIMA) models and on the signal extraction techniques.<sup>11</sup> We distinguish two types of approaches. One type is called the ARIMA model-based (AMB) approach. The other type is called the structural time series (STS) approach.

#### 4.4.2.1 ARIMA-model based approach

The aim of the AMB approach is to model the observed time series from a seasonal ARIMA (SARIMA)<sup>12</sup> model in order to deduce the components from the structure of the model by using the spectral estimations.<sup>13</sup> The major contributions on this approach are Box et al. (1978), Burman (1980), Hillmer and Tiao (1982), Bell (1984), and Maravall and Pierce (1987).

<sup>10</sup>Seasonal adjustment in Eurostat was done for a number of years in the database CRONOS using a method called DAINITIES. As Eurostat had to abandon CRONOS to migrate to modern databases, DAINITIES became unavailable (Fischer (1995)).

<sup>11</sup>The problem of the signal extraction is to estimate the signal  $S_t$  in the observations  $Z_t = S_t + N_t$  where  $N_t$  is the "noise". Kolmogorov (1939) and Wiener (1949) resolved this problem for stationary time series, by obtaining a linear function  $\hat{S}_t$  to minimize  $E[(S_t - \hat{S}_t)^2]$ . Hannan (1967) and Bell (1984), among others, extended this results to non-stationary time series. The signal extraction est employed in seasonal adjustment by identifying  $S_t$  and  $N_t$  as the seasonal and non-seasonal components, respectively, and by modelling suitably  $Z_t$ ,  $S_t$  and  $N_t$  (Bell (1984)).

<sup>12</sup>Hannan (1964) is the first to propose a seasonal adjustment method based on stationary stochastic models. This approach is extended to nonstationary models by Hannan (1967) and Hannan et al. (1970).

<sup>13</sup>The seasonal adjustment methods based on the spectral estimations have been suggested by Melnick and Moussourakis (1974) and Geweke (1978).

Since the components are unobservable and in order to obtain an unique decomposition from the general ARIMA fitted to the original time series, Hillmer and Tiao (1982) proposed the so-called *canonical decomposition*. It has the properties, among others, to maximize the variance of the irregular component and to minimize the variance of the seasonal component. The ARIMA models are very sensitive to the outliers and can not estimate correctly the determinist components. Therefore, others developments have been proposed by combining the regression models with dummy variables and ARIMA errors. TRAMO-SEATS has been developed in this way.

### TRAMO-SEATS software

TRAMO (*Time Series Regression with ARIMA Noise, Missing Observations, and Outliers*) and SEATS (*Signal Extraction in Arima Time Series*) have been developed by Maravall and Gómez (1992) and Gómez and Maravall (1992, 1997) at the Bank of Spain (Maravall and Caporello (2004)).

The SEAT part of the software makes the assumption that the original series is stationary or can be made stationary applying differences of a finite order. The requisite of stationarity is basic to find an ARIMA model for the original series. Since most socioeconomic data are often affected by the presence of deterministic effects, such as outliers, structural breaks, trading day variations, moving holidays, which do not accept ARIMA modeling, the TRAMO part of the software is employed to remove those deterministic effects and make the series stationary.<sup>14</sup> TRAMO performs estimation, forecasting, and interpolation using a mix model consisting of regression variables with ARIMA errors.

SEATS starts by fitting an ARIMA model to the stationary linear series produced by TRAMO. This model is determined by an automatic procedure of model identification based on constraints concerning the seasonal and nonseasonal polynomial orders as well as on SIC criterion. Then, SEATS uses the AMB approach to decompose the time series into trend-cycle, seasonal and irregular components. This method is based on the MSX (*Minimum Seasonal eXtraction*) method originally developed by Burman (1980) at the Bank of England.<sup>15</sup> Let  $x_t$  denote the original series, and  $z_t = \delta(B)x_t$  represent the differenced data, where  $\delta(B)$  stands for the difference operators applied on  $x_t$  to make it stationary, such that

$$\delta(B) = \Delta^d \Delta_S^D \quad (4.7)$$

where  $\delta = 1 - B$  and  $\Delta_S^D = (1 - B^S)^D$  represent the regular and seasonal difference operators. The model for the differenced series  $z_t$  can be expressed as

$$\phi(B)(z_t - \hat{z}) = \theta(B)a_t \quad (4.8)$$

where  $\hat{z}$  is the mean of  $z_t$ , and  $a_t$  is assumed to be Gaussian white noise with zero mean and variance  $\sigma_a^2$ .  $\phi(B)$  and  $\theta(B)$  are autoregressive and moving average polynomials, respectively, satisfying stationarity and invertibility conditions. It should be noted that although explicitly written as a function of  $B$ , the autoregressive and moving average polynomials are also in  $B^s$ . The generalized model can be written as

$$\Phi(B)x_t = \theta(B)a_t + c \quad (4.9)$$

where  $\Phi(B) = \phi(B)\delta(B)$  represents the generalized autoregressive polynomial. The factorization of  $\Phi(B)$  can be written as:  $\Phi(B) = \phi_T(B)\phi_S(B)\phi_C(B)$  where  $\phi_T(B)\phi_S(B)$  and  $\phi_C(B)$  are the AR polynomials with the trend, seasonal, and cyclical roots, respectively.

<sup>14</sup>This is similar to the RegARIMA part included in X12-ARIMA.

<sup>15</sup>The MSX method, also called SIGEX (*SIGnal EXtraction*), which is based on the signal extraction, uses the spectral analysis to decompose the time series. Burman (1995) developed the PROPHET software based on this procedure.

We can then decomposed  $x_t$  as follows:

$$\frac{\theta(B)}{\Phi(B)}a_t = \frac{\theta_T(B)}{\Phi_T(B)}a_{Tt} + \frac{\theta_S(B)}{\Phi_S(B)}a_{St} + \frac{\theta_C(B)}{\Phi_C(B)}a_{Ct} + u_t \quad (4.10)$$

where  $u_t$  is a Gaussian white noise. Then, the models of the unobserved components are

$$\begin{aligned} \phi_T(B)T_t &= \Phi_T(B)a_{Tt} \\ \phi_S(B)S_t &= \Phi_S(B)a_{St} \\ \phi_C(B)C_t &= \Phi_C(B)a_{Ct} \end{aligned} \quad (4.11)$$

There is no unique decomposition from an ARIMA model of the original series unless we use the canonical decomposition proposed by Hillmer and Tiao (1982). This decomposition maximizes the variance of the irregular component and minimizes the variances of the others components, and thus, produces highly stable seasonal and trend components. Maravall and Planas (1996) showed that, with the set of admissible decompositions, the mean squared error (MSE) of the component estimators are always minimized for the canonical decomposition. SEATS estimates the parameters of trend-cycle and seasonal components using the Wiener-Kolmogorov filter.

#### 4.4.2.2 Structural time series approach

The STS approach consists to specify directly ARIMA models for each unobserved component. It follows the principle of the regression methods but instead of using global determinist models or local stochastic regression models to estimate each component the STS approach employs simple stochastic models being in the ARIMA models with an IMA (Integrated Moving Average) prevalence. The majors references on this approach are Engle (1978), Abrahams and Dempster (1979), Harvey and Todd (1983) and Kitagawa and Gersch (1984).

The decomposition method of the structural approach begins by a measurement equation including each unobservable component, namely the trend-cycle, seasonal and irregular components. One assumes a very simple ARIMA for each unobservable variable which are defined by state equations. The structural model is put into state space form and is often estimated from Kalman filter. STAMP is the more used software integrating the STS approach.

#### STAMP software

Koopman et al. (1995, 2010) developed the STAMP (*Structural Time series Analyser, Modeller and Predictor*) software at the London School of Economics and Political Science.<sup>16</sup>

STAMP, in its default option, suggests to use the so-called basic structural model (BSM) developed by Harvey (1981). This model consists of a trend assumed to follow a random walk with a drift, stochastic trigonometric seasonal components and a Gaussian white noise,  $N(0, \sigma_\epsilon^2)$ , irregular component. That is, the trend component is generally given by,

$$T_t = T_{t-1} + \beta_{t-1} + \eta_t \quad \beta_t = \beta_{t-1} + \zeta_t \quad (4.12)$$

where  $\eta_t \sim N(0, \sigma_\eta^2)$  and  $\zeta_t \sim N(0, \sigma_\zeta^2)$  are two uncorrelated white noise processes.

The seasonal component is a stochastic trigonometric model proposed by Harvey (1989)<sup>17</sup>, defined as

<sup>16</sup>The current version 8.30 of STAMP is fully integrated with OxMetrics software.

<sup>17</sup>A seasonal model based on a dummy variable form is also proposed in STAMP (Harvey and Todd (1983)).

$$S_t = \sum_{j=1}^{[s/2]} \gamma_{j,t} \quad (4.13)$$

where each  $\gamma_{j,t}$  is generated as

$$\begin{bmatrix} \gamma_{j,t} \\ \gamma_{j,t}^* \end{bmatrix} = \begin{bmatrix} \cos \lambda_j & \sin \lambda_j \\ -\sin \lambda_j & \cos \lambda_j \end{bmatrix} \begin{bmatrix} \gamma_{j,t-1} \\ \gamma_{j,t-1}^* \end{bmatrix} + \begin{bmatrix} \omega_{j,t} \\ \omega_{j,t}^* \end{bmatrix}$$

with  $\lambda_j = 2\pi j/s$ ,  $j = 1, \dots, [s/2]$ ,  $s$  denotes the number of observation per year, and  $t = 1, \dots, T$ . The seasonal innovations  $\omega_{j,t}$  et  $\omega_{j,t}^*$  are mutually uncorrelated with mean zero and common variance  $\sigma_\omega^2$ . The variances of innovations,  $\sigma_\eta^2$ ,  $\sigma_\zeta^2$ ,  $\sigma_\omega^2$  and  $\sigma_\varepsilon^2$ , are called the *hyperparameters* of the model. These hyperparameters govern the amount of smooth for the construction of the estimations of trend and seasonal components from their  $q$ -ratios:  $q_T = \sigma_\eta^2/\sigma_\varepsilon^2$  and  $q_S = \sigma_\omega^2/\sigma_\varepsilon^2$ , respectively. These  $q$ -ratios are estimated from likelihood maximum method. Then, the estimation of the components is based on the use of Kalman filter.

The model must be beforehand transformed into a state space from consisting of two equations: the first, called *measurement equation*, gives the relationship between the observed and unobserved variable, describing the dynamic of the system

$$y_t = (1 \ 0 \ 1 \ 0)\alpha_t + (\sigma_\varepsilon \ 0 \ 0 \ 0)u_t$$

where  $\alpha_t$  follows the *transition equation*, the second equation, which analytically describes the dynamic pattern governing the evolution of the unobserved variable:

$$\alpha_t = \begin{pmatrix} T_t \\ \beta_t \\ \gamma_{j,t} \\ \gamma_{j,t}^* \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos \lambda_j & \sin \lambda_j \\ 0 & 0 & -\sin \lambda_j & \cos \lambda_j \end{pmatrix} \alpha_{t-1} + \begin{pmatrix} 0 & \sigma_\eta & 0 & 0 \\ 0 & 0 & \sigma_\zeta & 0 \\ 0 & 0 & 0 & \sigma_\omega \\ 0 & 0 & 0 & 0 \end{pmatrix} u_{t-1}$$

with  $u_t = (\varepsilon_t \ \eta_t \ \zeta_t \ \omega_t)'$ . The estimators of components are optimal in the sense of minimum mean squared error (MMSE).

Others parametric seasonal adjustment methods based on stochastic models have been proposed.<sup>18</sup> The BAYSEA and DECOMP softwares have been developed at the Japan Institute of Mathematical Statistics: BAYSEA (*BAYesian SEasonal Adjustment*) proposed by Akaike and Ishiguro (1980) is based on a optimization function with some constraints on the trend and the seasonality, and use the Bayesian modeling suggested by Akaike (1980); DECOMP proposed by Kitagawa and Gersch (1984) adapts the Bayesian model of Akaike by developing a state space method for the seasonal adjustment. The MING (*Mixture based Non-Gaussian method*) software developed by Bruce and Jurke (1993, 1996) use the STS approach as well as the methodology proposed by Kitagawa (1994), by assuming that the innovations of the models of each component are non Gaussian. The MicroCAPTAIN (*Micro Computer Aided Program for Time-series Analysis and Identification of Noisy Systems*) software proposed par Young and Benner (1991) also uses the STS approach, but with time-varying coefficient models that are estimated using the frequency domain method of Dynamic Harmonic Regression (Young et al. (1999)), as opposed to the Maximum Likelihood estimation method applied in the other software. We also mention the regCMPNT software developed by Bell (2004) at the US Census Bureau, based on the RegComponent models which uses the STS approach with ARIMA models for which the error term follow a standard ARIMA model.

<sup>18</sup>Breitung (1994, 1999) proposed a seasonal adjustment method based on the Beveridge and Nelson (1981) decomposition. This method decomposes an ARIMA model into the sum of a permanent component (stochastic trend as a random walk) and a transitory component (stationary ARMA process) which are perfectly correlated.

## 4.5 Conclusions

In this chapter, we presented various seasonal adjustment methods according to the classification we proposed (non-parametric, parametric and semi-parametric) from an historical perspective, as well as the main statistical software in which such methods are implemented (see Figure 4.1 for a synoptic summary).

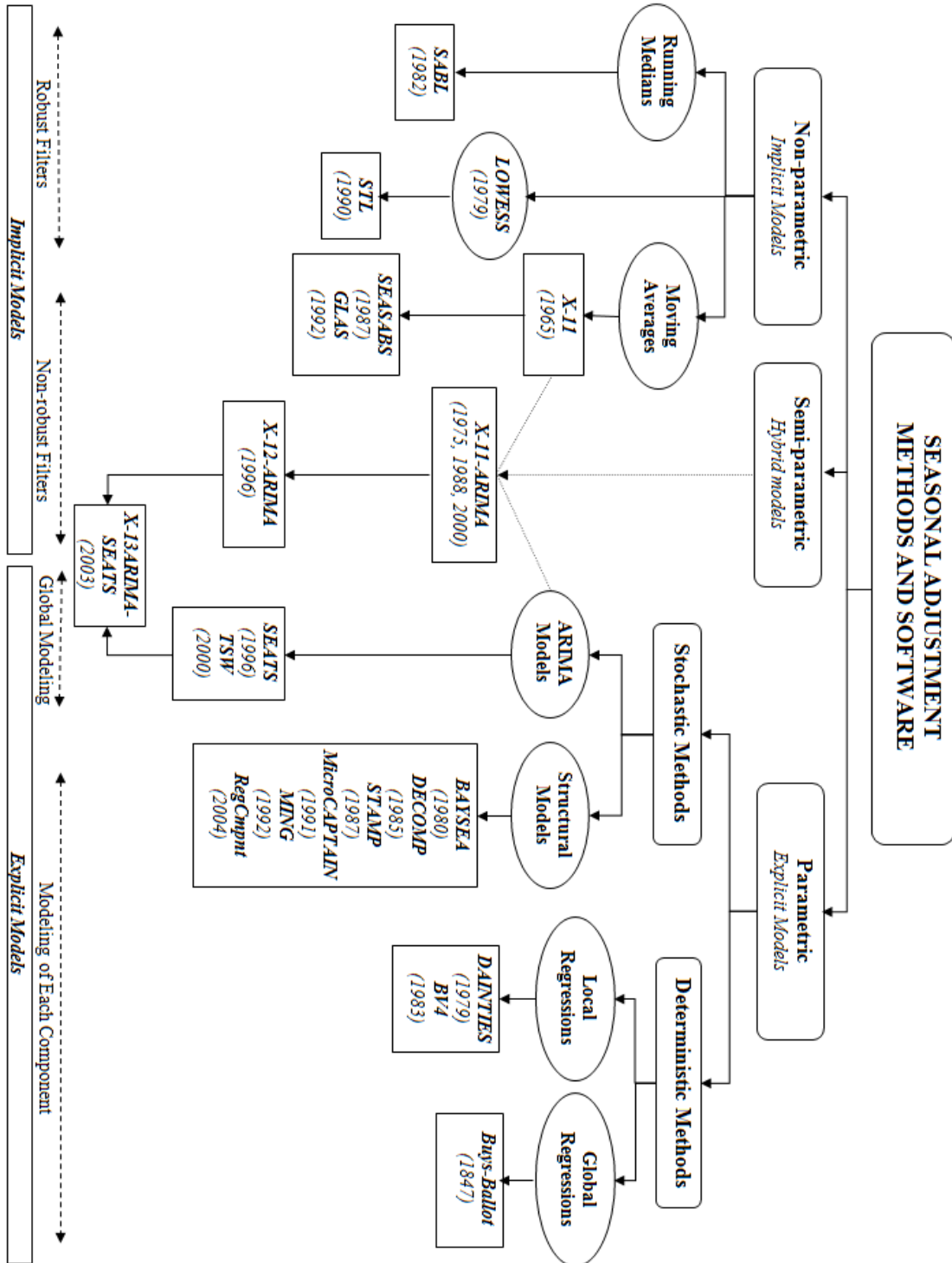
Obviously each method possesses its own advantages and drawbacks. For example, it is often advocated that non-parametric approaches do not rely on statistical models hindering thus confidence intervals estimation for each unobserved component. In opposition, it is well known that the model specification step involved in parametric approaches is quite touchy and may sometimes lead to misspecification issues. For example, the Gaussian assumption may be rejected in the presence of extreme values in the data, especially for highly disaggregated variables.

Recent developments on seasonal adjustment methods build on those drawbacks. The current lines of research on this topic focus mainly on edge effects, that is estimation issues at the end of the sample, the most interesting values for economic assessment, and on optimal elimination of specific effects that could disturb the standard methods such as structural breaks, extreme values, non-linearities, calendar effects, ... In addition, the development of empirical criteria to assess the quality of seasonal adjustment procedures are of great interest for practitioners.

To our knowledge, there are no clear conclusions on the fact that one method outperforms systematically the others. The simultaneous use of several types of methods may be viewed as a robustness check. For example, the JDemetra+ software developed by the National Bank of Belgium (NBB) in cooperation with the Deutsche Bundesbank under the auspices of Eurostat and the European Central Bank (Grudkowska (2017)) as well as the X-13ARIMA-SEATS program developed by the US Census of Bureau with the collaboration of the Bank of Spain (Monsell (2007), Monsell (2009), US Bureau of the Census (2017)) go in this direction in the sense that they allow an easy implementation of X12-ARIMA and TRAMO-SEATS.



Figure 4.1: A classification of seasonal adjustment methods and software



## 4.6 Annex: A short description of the JDemetra+ software

JDemetra+ is the new European tool for seasonal adjustment (SA) developed by the National Bank of Belgium (NBB) in cooperation with the Deutsche Bundesbank and Eurostat in accordance with the Guidelines of the European Statistical System (ESS).

JDemetra+ has been officially recommended by Eurostat and the European Central Bank, since 2 February 2015, to the members of the ESS and the European System of Central Banks as software for seasonal and calendar adjustment of official statistics.

Besides seasonal adjustment, JDemetra+ bundles other time series methods that are useful in the production or analysis of official economic statistics, including for instance outlier detection, nowcasting, temporal disaggregation or benchmarking. The object-oriented design of the software allows alternative uses of the routines and the development of extensions. So, more than a close collection of algorithms, it should also be considered as a toolbox that can help solving time series problems encountered in large-scale production as well as in research.

In a first paragraph, we shortly review the main statistical methods implemented in JDemetra+. We introduce in a second paragraph some technical aspects of the tool.

### 4.6.1 Statistical aspects

Before considering seasonal adjustment and the other time series methods, we briefly present the state space framework of JDemetra+, which plays a central role in the whole application.

#### 4.6.1.1 State space models

Many statistical problems, ranging from seasonal adjustment to temporal disaggregation can be solved by means of state space models and by their related filtering and smoothing algorithms. JDemetra+ relies on a flexible and extensible state space framework that can be used in an efficient way on a large panel of univariate or multivariate models. The models considered in JDemetra+ are very similar to those described in Durbin and Koopman (2012). Besides the usual filtering and smoothing algorithms, JDemetra+ provides some advanced solutions, like for instance fast Chandrasekhar recursions, diffuse square root initialization, fixed point smoothers or array algorithms. It also provides optimized implementations of several specific models, including AR(I)MA models, structural models, time-varying regressions and tools to combine them efficiently.

#### 4.6.1.2 Seasonal adjustment

The core of the tool is devoted to seasonal adjustment. JDemetra+ provides new implementations of the two leading SA methods: TRAMO-SEATS and X-13ARIMA-SEATS. Those methods have been re-engineered using an object-oriented approach that enables easier handling, extensions and modifications.

The pre-processing parts of the algorithms are implemented using a common framework. Roughly speaking, that framework can be viewed as a tool for identifying and for estimating a common regression model, which generalizes the models considered in the original programs. For design, performance and/or coherence issues, JDemetra+ uses occasionally specific algorithmic solutions, which can explain some (usually small) discrepancies in comparison with the original software.

The X11 decomposition of JDemetra+ mimics the original FORTRAN implementation and the canonical decomposition popularized by SEATS has been redesigned to deal with a larger family of UCARIMA models. In addition to the Burman's algorithm of the original program, JDemetra+ also provides an estimation of the latter models by means of the Kalman smoother.

Besides those methods recommended in the ESS guidelines for Seasonal adjustment edited by Eurostat, JDemetra+ contains implementations of basic structural models, using different seasonal components, and of more advanced solutions, like for instance seasonal specific structural time series or generalized airline models.

Finally, JDemetra+ offers a large set of SA diagnostics that can be used independently on most decompositions.

All these tools are available through a very user-friendly graphic interface, a screenshot of which is shown in Figure 4.2.

#### 4.6.1.3 Other diagnostics

JDemetra+ provides various algorithms for benchmarking and temporal disaggregation. The Denton and the Cholette's benchmarking routines are available in their univariate and multi-variate forms. The most popular temporal disaggregation methods, like Chow-Lin, Fernandez or Litterman are also included in the tool. All those routines are implemented by means of state space forms.

In the domain of nowcasting, JDemetra+ contains routines to specify and estimate mixed frequency dynamic factor models and to measure how the real-time dataflow updates expectations, as for instance in Banbura and Modugno (2010). The software can also be used to perform pseudo out-of-sample forecasting evaluation.

### 4.6.2 Technical aspects

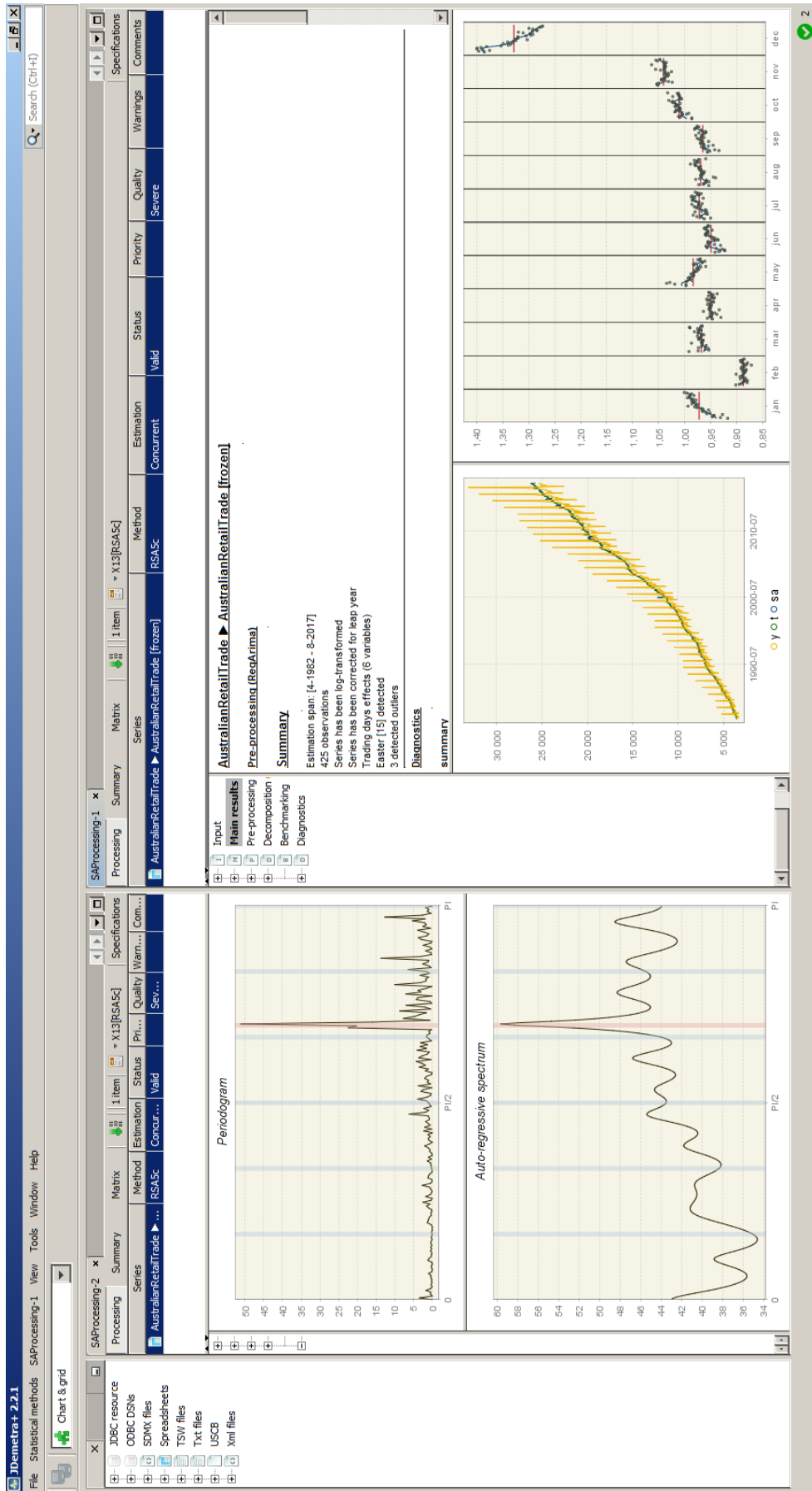
Though the acronym is usually used to refer to the graphical interface, JDemetra+ consists in fact of several software components, designed for different uses.

The core of the tool is a collection of Java packages, which contain mainly the statistical algorithms and the I/O routines. Those packages can be used independently in home-made applications.

On top of the core libraries we will find the main graphical interface, a command line tool for batch SA processing (called the "Cruncher"), a WEB service around the main statistical methods and extensions for the "R" language. The graphical interface, designed for end-users, is built on the NetBeans platform. The application is extensible in a decentralised manner through plugins. The `SRJ` packages provide, through the `rJava` package, a transparent access to numerous routines of JDemetra+.

All the tools are free and open-source software (FOSS), developed under the EUPL licence. They are available on Github at the following URL: <https://github.com/jdemetra>.

Figure 4.2: A screenshot of the Jdemetra+ graphical interface.



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# Pre-Treatment Methods







# 5

## Calendar Effects





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## Table of Contents

5.1	Introduction	95
5.2	The Various Calendar Effects	95
5.2.1	A few words on calendars	95
5.2.2	The Gregorian Calendar	96
5.2.3	The Trading-Day Effect	96
5.2.4	Moving Holidays	97
5.2.5	Impact models	97
5.2.6	Why should we take into account trading-day effects?	97
5.3	Detection and Estimation of Trading-Day Effects by Regression Analysis	98
5.3.1	The Basic Flow Day-of-Week Effect Model	98
5.3.2	The Basic Flow Week-Day Effect Model	100
5.4	Improving the Regression Model	101
5.4.1	Taking into Account National Holidays	101
5.4.2	Constructing a Relevant Set of Regressors	102
5.5	Impact Models	105
5.5.1	Models for Easter effect	105
5.5.2	Models for the Ramadan effect	109
5.5.3	More general models	109
5.6	Spectral detection of trading-day effects	111
5.6.1	The Calendar Periodograms and the Trading-Day Frequencies	111
5.6.2	The USCB “Visual Test”	111
5.6.3	An example	112
5.7	Conclusions	114

## 5.1 Introduction

Almost if not all economic time series are nowadays computed and published according to the Gregorian calendar, a calendar based on the motion of the Earth around the Sun. This solar calendar schedules our lives and usually reflects a deep impact on the economy. The most well known and important calendar effect is seasonality often defined as fluctuations observed during the year (each month, each quarter) and that appear to repeat themselves with a more or less regular magnitude from one year to the other.

But, most of economic indicators are also linked, directly or indirectly, to a daily activity which is usually summed up and recorded each month or each quarter. In this case, the number of working days, which varies from a month to another in a quasi predetermined way, can explain some short-term movements in the time series. One more Saturday in a month for example significantly impacts the retail trade turnover in European countries. Apart the day composition of the month, other calendars effects such as public holidays or religious events may also affect the series. Religious events are often closely linked to other calendars and their dates, expressed in the Gregorian calendar, move through our solar year. This is the case for Easter whose date, linked to the full moon, is usually expressed in the Gregorian calendar for Catholic countries and in the Julian calendar for Orthodox countries.

These periodic fluctuations, as well as the seasonality, are usually detected and eliminated in order to exhibit the irregular or non-periodic movements which are probably of most interest and importance. The ESS Guidelines for Seasonal Adjustment, see Eurostat (2015), give some precise recommendations concerning calendar effects:

- *To avoid misleading results, seasonal adjustment should be applied only when seasonal and/or calendar effects can be properly explained, identified and estimated. Where none of these effects can be identified and estimated, unadjusted and calendar/seasonally adjusted series are identical.*
- *It is recommended to use regression model with ARIMA errors to calculate calendar adjustment factors. These calendar adjustment factors should take into account the different characteristics of national calendars.*

The objective of this chapter is to present the main notions and techniques to apply these principles efficiently. The chapter is therefore dedicated to the detection, the modeling and the estimation of these calendar effects.

Section 5.2 presents the various calendar effects and the Gregorian calendar whose specificities and relationships with other calendars, generate these effects. Section 5.3 is devoted to the detection and estimation of trading-day effects by means of a regression model with ARIMA errors, for flow series. Section 5.4 shows how to improve the regression model taking into account National calendars and selecting the relevant set of regressors. Section 5.5 focuses on modeling the impact of specific holidays. Finally, section 5.6 presents some results on the spectral detection of trading-day effects.

## 5.2 The Various Calendar Effects

Most economic series are recorded and published according to the Gregorian calendar and calendar effects are directly linked to the characteristics of this calendar.

### 5.2.1 A few words on calendars

Most calendars currently in use are based on the motion, or apparent motion, of the earth, moon and sun. These calendars can be divided in three main categories:

- Solar calendars, like the Gregorian and the Julian calendars, are based on the earth's motion around the sun. In these calendars the year approximates the “tropical year”, the time the earth takes to go from one fixed point, such as a solstice or equinox, to the next.
- Lunar calendars, like the Islamic calendar, are on the opposite based on the moon's motion around the earth. In these calendars, the month approximates the “synodic month”, the time from one new moon to the next.
- Lunisolar calendars, like the Chinese and Hebrew calendars, are mainly based on the moon and months reflect the lunar cycle, but then new months (e.g. “second Adar” in the Hebrew calendar) are inserted to bring the calendar year into synchronization with the solar year.

Even though the Gregorian calendar is in common and legal use, lunar and lunisolar calendars as well as other solar calendars serve to determine traditional or religious holidays in many parts of the world. Such holidays include Easter, Ramadan, Diwali, Chinese New Year etc.

### 5.2.2 The Gregorian Calendar

In the Gregorian calendar a usual year is made of 12 months - January, February, March, April, May, June, July, August, September, October, November and December - with respective number of days 31, 28, 31, 30, 31, 30, 31, 31, 30, 31, 30 and 31. A usual year therefore contains 365 days, which is unfortunately a bit too short as the earth takes roughly 365 days and six hours to complete one revolution around the sun. The approximation of the tropical year is achieved by having in the Gregorian calendar 97 special years every 400 years (the leap years) in which February has 29 days.

A leap year is a year divisible by 4 but not by 100, unless the year is also divisible by 400. So 1900 was not a leap year, 2000 was a leap year, and 2100 will not be a leap year. There remains, nevertheless, a slight error considered to be one day every 4000 years but our Gregorian calendar does not correct for it.

Each period of 400 years thus contains  $400 \times 365 + 97 = 146,097$  days, which is exactly 20,871 weeks. Therefore the Gregorian calendar is periodic with period 400 years. The average length of a year over this cycle is  $146,097 / 400 = 365.2425$  days. The average length of a month is  $365.2425 / 12 = 30.436875$  days.

### 5.2.3 The Trading-Day Effect

Based on the motion of the earth around the sun, our calendar is directly linked to the seasons and **seasonality** is by nature the most important and well-known calendar effect.

Furthermore, months (or quarters) are not directly comparable. They do not have the same number of days (mainly a seasonal effect) and the day composition of months varies from one month to another and from one year to another. For example, May 2015 had 5 Saturdays, one more than May 2014, April 2015 and June 2015. In the retail trade sector, this extra Saturday can make more difficult the year-to-year and month-to-month turnover comparisons.

This effect directly linked to the day composition of the month is called the **trading-day effect**.

National holidays are often linked to a date, not to a specific day. For example, in catholic countries, Christmas is always the 25<sup>th</sup> of December, but not always a Sunday. As these National days are usually off, they might impact the activity of some sectors of the economy and they are usually taken into account in the trading-day effect.

## 5.2.4 Moving Holidays

Some National holidays celebrate religious events. These events are often closely linked to other calendars and their dates, expressed in the Gregorian calendar, move through our solar year. This is the case for Easter whose date, linked to the full moon, is usually expressed in the Gregorian calendar for Catholic countries and in the Julian calendar for Orthodox countries. This is also the case for Ramadan whose date is also linked to the moon but expressed in the Hegire calendar.

These religious events are known to have an impact on some sectors of the economy. For example, Bessa and al. (2008) evaluated the effect of Ramadan on the Tunisian economy and also demonstrated that the catholic Easter has a significant impact on the tourism sector.

Dating these events in the Gregorian calendar could be quite complex and algorithms have been proposed<sup>1</sup> that permit to convert a date from a specific calendar to another.

## 5.2.5 Impact models

Moving holidays might have two different impacts:

1. Days off are usually observed to celebrate the event and these days are usually taken into account in the trading-day effect.
2. We might observe a change in the activity the days before or after the event. For example, the days before Easter and the Chinese New Year or the last days of Ramadan usually have an impact on the retail trade sales.

Specific “impact models” have been developed to estimate this “before or after” effect on the activity. Of course, this effect on the economy can also be observed for fixed National holidays like Labor day, Christmas day etc ... But in the case of moving holidays, the effect might have a significant non-seasonal impact and the series presenting this kind of effect are usually corrected for.

## 5.2.6 Why should we take into account trading-day effects?

**Table 5.1: Number of working days (Monday to Friday) in France by quarter.**

Year	Gregorien calendar					National calendar				
	Q1	Q2	Q3	Q4	Total	Q1	Q2	Q3	Q4	Total
2012	65	65	65	66	261	65	60	64	64	253
2013	64	65	66	66	261	63	60	65	63	251
2014	64	65	66	66	261	63	60	64	64	251
2015	64	65	66	66	261	63	60	65	64	252
2016	65	65	66	65	261	63	63	64	63	253
2017	65	65	65	65	260	65	60	63	63	251
2018	65	65	65	66	261	64	60	64	64	252
2019	64	65	66	66	261	63	60	65	63	251
2020	65	65	66	66	262	64	60	65	64	253

Table 5.1 shows the number of working days (Monday to Friday) by quarter in France, taking into account or not the National calendar. This number, quite regular in the generic Gregorian calendar, can be very different from a quarter to another and from one year to another if we take into account the National specificities:

<sup>1</sup> See for example Dershowitz and Reingold (2008)

- You usually have 3, 4 or even 5 more working-days in Q1 than in Q2. 2016 was a very special year with the same number of working-days in Q1 and Q2;
- In 2016, Q2 had 3 more working-days than the other second quarters between 2012 and 2020.

The impact on the adjusted series can be important as shown by Figure 5.1 were the French GDP (chain-linked values) is represented both seasonally and working-day adjusted (SWDA) and seasonally adjusted (SA) only. The quarter to quarter growth rate can be very different when the trading-day effect is accounted for, even if the National specificities were not included in the regressors.

## 5.3 Detection and Estimation of Trading-Day Effects by Regression Analysis

In this section, we focus on trading-day regressors for flow series. Regression models for the detection and estimation of these trading-day effects have been proposed by Young (1965) in the context of the X-11 software for seasonal adjustment and in Bell and Hillmer (1983). Trading day regressors for end-of-month stock series were proposed in Cleveland and Grupe (1983), Bell (1984), Bell (1995) and in Findley and Monsell (2009).

### 5.3.1 The Basic Flow Day-of-Week Effect Model

Following the notation of Soukup and Findley (1999), we assume that the  $j^{th}$  day of the week has an effect  $\alpha_j$  where, for example,  $j = 1$  refers to Monday,  $j = 2$  refers to Tuesday etc., and  $j = 7$  refers to Sunday. Each  $\alpha_j$  represents for example the average sales for one day  $j$ .

If  $N_{jt}$  represents the number of days  $j$  in the month  $t$ , the length of the month will be  $N_t = \sum_{j=1}^{j=7} N_{jt}$  and the cumulative effect for that month, the total sales of the month, will be:  $TD_t = \sum_{j=1}^{j=7} \alpha_j N_{jt}$ .

A first idea to detect and evaluate the trading-day effects in a series is to explain the series by the seven regressors  $N_{jt}$ . Unfortunately the corresponding model presents some defaults:

- Problem 1: It appears that the  $N_{jt}$  regressors are highly correlated and therefore the  $\alpha_j$  estimates (the  $\hat{\alpha}_j$ ) are unstable;
- Problem 2: The regressors are seasonal - on average you will always have more Monday in February than in March - and you include in the trading-day effect a part of the seasonal effect;
- Problem 3: As there is no reason why  $\sum_{j=1}^{j=7} \hat{\alpha}_j$  should be equal to 0, the trading-day effect does not vanish on a week.

We can use a different but equivalent formulation of the trading-day effect to solve these problems

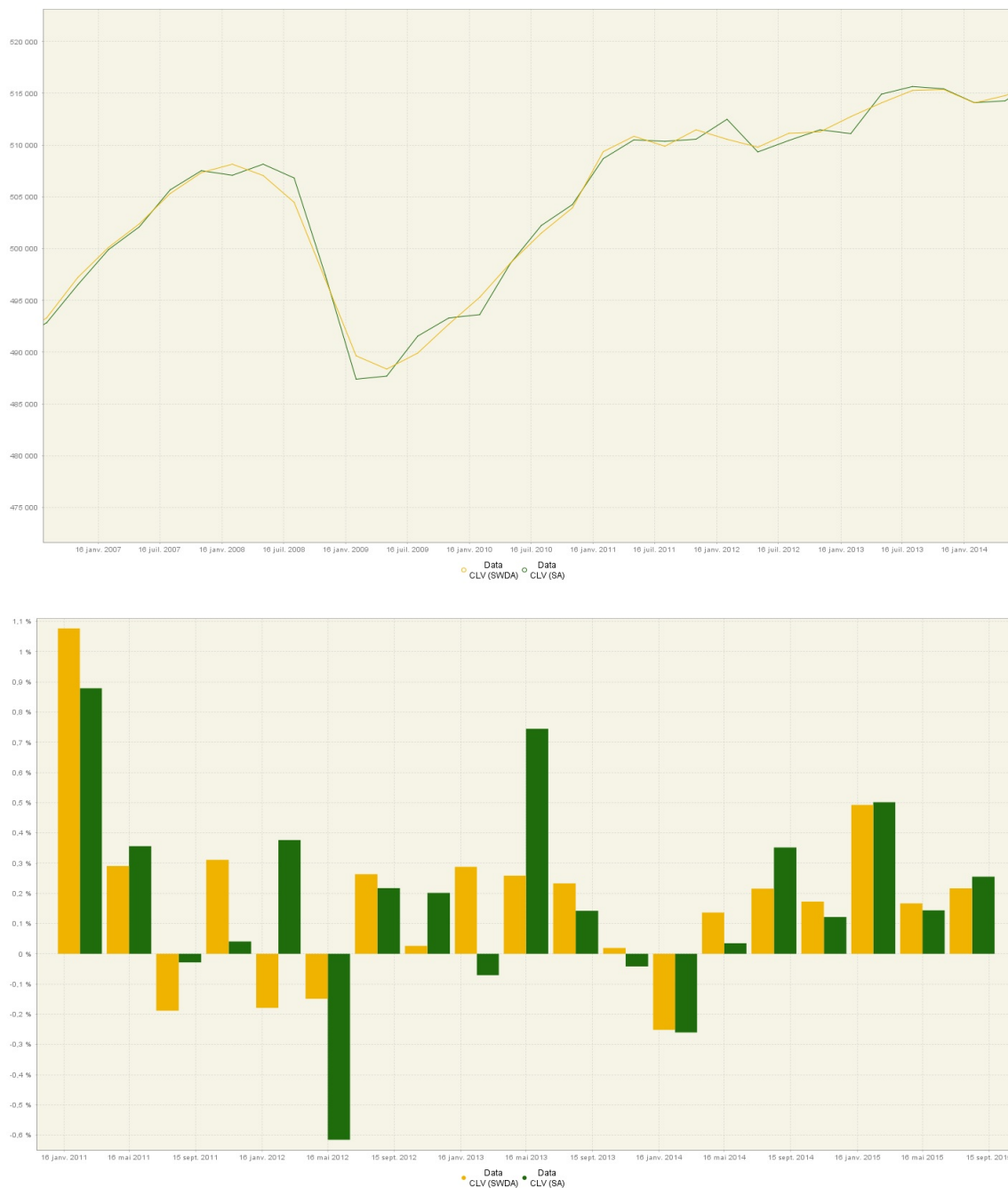
The mean daily effect, the average sales for one day, is  $\bar{\alpha} = \frac{1}{7} \sum_{j=1}^{j=7} \alpha_j$ . As, by design,  $\sum_{j=1}^{j=7} (\alpha_j - \bar{\alpha}) = 0$ , we may write:

$$TD_t = \sum_{j=1}^{j=7} \alpha_j N_{jt} = \bar{\alpha} N_t + \sum_{j=1}^{j=7} (\alpha_j - \bar{\alpha}) N_{jt} = \bar{\alpha} N_t + \sum_{j=1}^{j=6} (\alpha_j - \bar{\alpha}) (N_{jt} - N_{7t}) \quad (5.1)$$

And the cumulative monthly trading-day effect is decomposed into an effect directly linked to the length of the month and a net effect for each day of the week.

Note that the sum  $\sum_{j=1}^{j=7} (\alpha_j - \bar{\alpha}) N_{jt}$  involves only the days of the week occurring five times in a month; every month contains four complete weeks, for which by definition the effect linked to the days is cancelled out, plus

**Figure 5.1:** French GDP Seasonally adjusted (SA) and Seasonally and working-day adjusted (SWDA); in levels (upper panel) and quarterly growth rates (lower panel).



Notes: The computations were done using Tramo-Seats (JDemetra+ version 2.1<sup>a</sup>), with an automatic model identification and the default trading-day model with 6 regressors that do not take into account the National calendar. The SA series is in green, the SWDA series in orange.

<sup>a</sup> JDemetra+ is the European software for seasonal adjustment that implements the 2 most popular seasonal adjustment methods: TRAMO-SEATS and X13-ARIMA-SEATS.

0, 1, 2 or 3 days which contribute to the trading-day effect for the month.

Since the seasonal part is already captured by the seasonal adjustment filters, it should not be removed during calendar adjustment. Trading-day effects, in the narrow sense, should therefore be associated with the non-seasonal part of the effect and equation (5.1) must be adjusted to remove possible seasonality and trend.

- Potentially, part  $\bar{\alpha}N_t$  of the equation contains such components because the months vary in length and because, as we have seen, variable  $N_t$  is periodic (period of 400 years). These effects can be summarized by the quantity  $\bar{\alpha}N_t^*$  where  $N_t^*$  represents the average, over 400 years, of the length of the month  $t$ . In other words,  $N_t^*$  is equal to 30 or 31 if the month in question is not the month of February, and is equal to 28.2425 otherwise. Thus, we have:  $\bar{\alpha}N_t = \bar{\alpha}N_t^* + \bar{\alpha}(N_t - N_t^*)$  an equation whose second part is zero except for the month of February.
- The second part of the equation includes  $N_{jt}$ , the number of times that day  $j$  is present in month  $t$ . These variables are periodic (period of 4800 months or 400 years) with equal means for a given month. In the second part of the equation, the difference  $N_{jt} - N_{7t}$  is used, and since these variables show the same behaviour, the difference presents no seasonality and no trend.
- The new formulation solves Problem 2 as there is no more seasonality and trend in the regressors.

The current versions of both X-13ARIMA-SEATS (U.S. Census Bureau (2015)) and TRAMOSEATS (Gómez and Maravall (1996)) use the following model to estimate the tradingday effect:

$$X_t = \beta_0 LY_t + \sum_{j=1}^{j=6} \beta_j (N_{jt} - N_{7t}) + Z_t \quad (5.2)$$

where:

- $X_t$  is the raw series (monthly retail trade sales);
- $Z_t$  follows an ARIMA model  $(p, d, q)(P, D, Q)_s$ ;
- $\beta_0 = \bar{\alpha}$  and  $LY_t = (N_t - N_t^*)$  is the leap year regressor;
- $\beta_j = \alpha_j - \bar{\alpha}$  for  $1 \leq j \leq 6$
- and  $\hat{\beta}_7 = -\sum_{j=1}^{j=6} \hat{\beta}_j$ , which assures that the trading-day effect vanishes on a complete week and solves Problem 3<sup>2</sup>

Other specifications of the model, week-day regressor and no leap year regressor, are available in both software<sup>3</sup>.

### 5.3.2 The Basic Flow Week-Day Effect Model

Model (5.2) is not very parsimonious as it implies the estimation of 7 regressors plus the parameters of the ARIMA model.

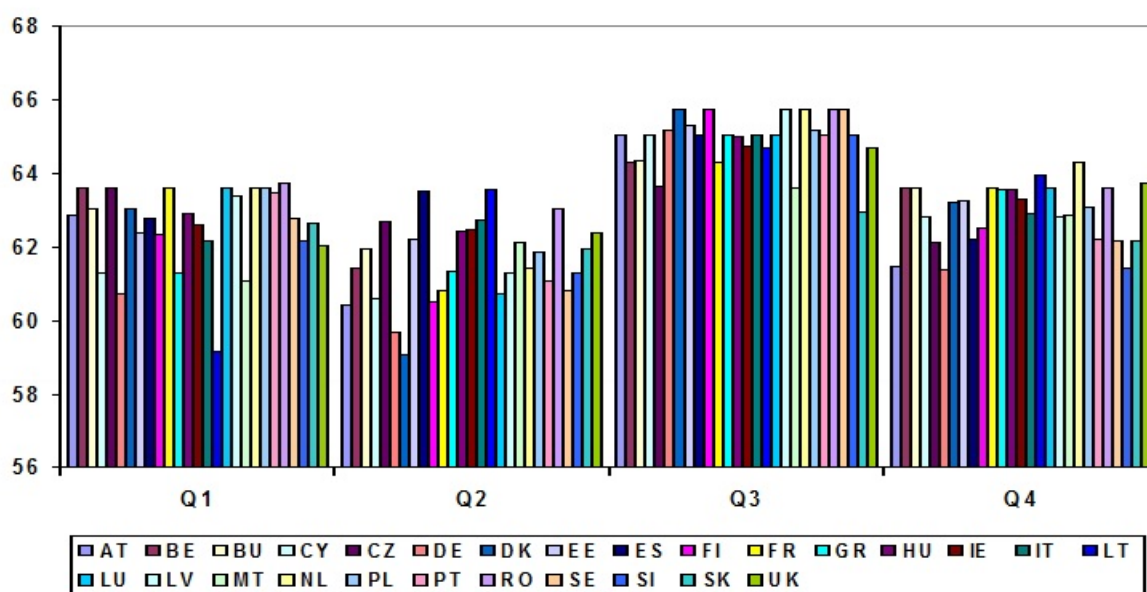
The number of trading-day regressors can be reduced making some hypothesis on the various days. The Week-Day model supposes that the week days (Monday, Tuesday, Wednesday, Thursday and Friday) have the same behavior as well as Saturday and Sunday. This implies some constraints on the parameters:

- $\beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5$
- $\beta_6 = \beta_7$

<sup>2</sup> Using contrasts also improves the stability of the estimates (Problem 1). This point will be detailed in Section 5.4.

<sup>3</sup>X13-ARIMA-SEATS proposes also to estimate the trading-day effect from an estimation of the irregular effect. In this case, the residual  $Z_t$  is supposed to be a white noise



**Figure 5.2:** Average number of working days by quarter for European Countries

As  $\beta_7 = -\sum_{j=1}^{j=6} \beta_j$  we obtain, using basic algebra, the following “Week-Day” model with two regressors only:

$$X_t = \beta_0 LY_t + \beta_1 \left[ \sum_{j=1}^{j=6} N_{jt} - \frac{5}{2}(N_{6t} + N_{7t}) \right] + Z_t \quad (5.3)$$

## 5.4 Improving the Regression Model

### 5.4.1 Taking into Account National Holidays

A default of models (5.2) and (5.3) is that the regressors are computed from the generic Gregorian calendar and therefore do not take into account National specificities.

Figure 5.2 shows the number of working days for European countries when National specificities are taken into account. The situation appears very different according to the country and the National specificities can reduce the number of working days significantly for a given quarter. For example, May in France is a very special month that can have up to 4 days off (Labor Day, Victory day, Ascension Thursday and Pentecost Monday).

It seems therefore important to take into account the National specificities when estimating the trading-day effect.

The basic idea to build a generic trading-day model is to consider that the week is composed by 14 different days: the usual seven days “in” and seven days “off”. Then  $N_{1t}$  and  $N_{8t}$  are respectively the number of Mondays “in” and “off” in month  $t$ ,  $N_{2t}$  and  $N_{9t}$  are the number of Tuesdays “in” and “off” in month  $t$  etc. To define the contrasts you need a “reference day”, for example Sunday, and you have the following general model:

$$X_t = \beta_0 LY_t + \sum_{j=1}^{j=6} \beta_j (N_{jt} - N_{7t}) + \sum_{j=8}^{j=14} \beta_j (N_{jt} - N_{7t}) + Z_t \quad (5.4)$$

Then you have to make some hypothesis to get a more parsimonious model. For example, if you assume that all “days off” behave like a Sunday, you have:

$$\beta_7 = \beta_8 = \beta_9 = \beta_{10} = \beta_{11} = \beta_{12} = \beta_{13} = \beta_{14},$$

and with some basic algebra, you obtain the following 7-regressor model:

$$X_t = \beta_0 LY_t + \sum_{j=1}^{j=6} \beta_j \left[ N_{jt} - \frac{1}{8} \sum_{k=7}^{k=14} N_{kt} \right] + Z_t \quad (5.5)$$

### Remark 1

Unfortunately, the contrasts do not remove the seasonality anymore and you have to adjust them, removing from  $N_{jt}$  the average number of days  $j$  in the month corresponding to date  $t$ . There is no difficulty to compute these long term averages on the 400-year cycle of the Gregorian calendar if you suppose that the holidays remain the same on the complete period<sup>4</sup>.

### Remark 2

The coefficient 1/8 which appears in the contrast is useless from the mathematical point of view: if you remove it, you will get the same estimation of the trading-day effect as this new set of regressors generates the same vector space. But the  $\beta_j$  coefficients will not have the same interpretation.

## 5.4.2 Constructing a Relevant Set of Regressors

Available programs give you the choice between 2 different models only, the Trading-Day model with 6 regressors and the Week-Day model with 1 regressor, to which you can add the LeapYear regressor.

But the 6-regressor model often appears unstable and can be not parsimonious enough, in particular for short series. On the other hand, the 1-regressor model is for some sectors of the economy irrelevant: in the retail trade sector, it is unlikely that Saturday behaves like Sunday.

A more optimal set of regressors for a specified sector of the economy can be found from the various shapes of observed Trading-Day effects using for example a hierarchical cluster analysis (HCA)<sup>5</sup>.

### 5.4.2.1 Example 1: The French Industrial Production Index

For this example, we consider the 91 Industrial Production indexes at the NACE2 group level. The trading-day effect for each series has been estimated with a 6-regressor model. 45 series present a significant trading-day effect and for these series, the 7 coefficients, one for each day of the week, have been classified using a HCA. The dendrogram of this clustering is presented in Figure 5.3.

Two groups clearly appear: (Saturday, Sunday) and (Monday, Tuesday, Wednesday, Thursday, Friday). In this case, the Week-Day model with 1-regressor is appropriate.

### 5.4.2.2 Example 2: The French Retail Trade Sector

For this second example, we analyze 105 turnover indexes from the French retail trade sector. 73 series present a significant trading-day effect. The dendrogram of the HCA done on the 7 coefficients is presented in

<sup>4</sup>In fact, this 400-year cycle does not take into account the moving holidays. As an example, the dates of Easter and related events are periodical of period 5 700 000 years in the Gregorian calendar!

<sup>5</sup>see for example Everitt et al. (2011)

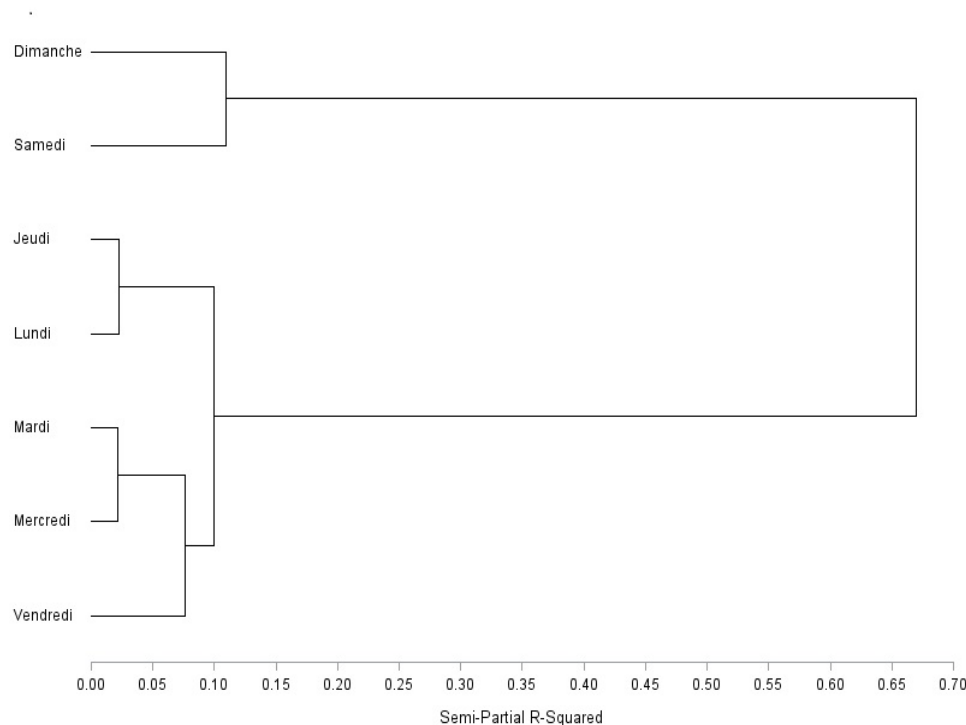
**Figure 5.3:** Dendrogram from a HCA on the 91 IPI trading-day effects (NACE2 Group level)

Figure 5.4.

It is clear in this case that Sunday and Saturday have a very different behavior. The Week-day model, with 1-regressor is inappropriate and 2 other sets of regressors can be considered:

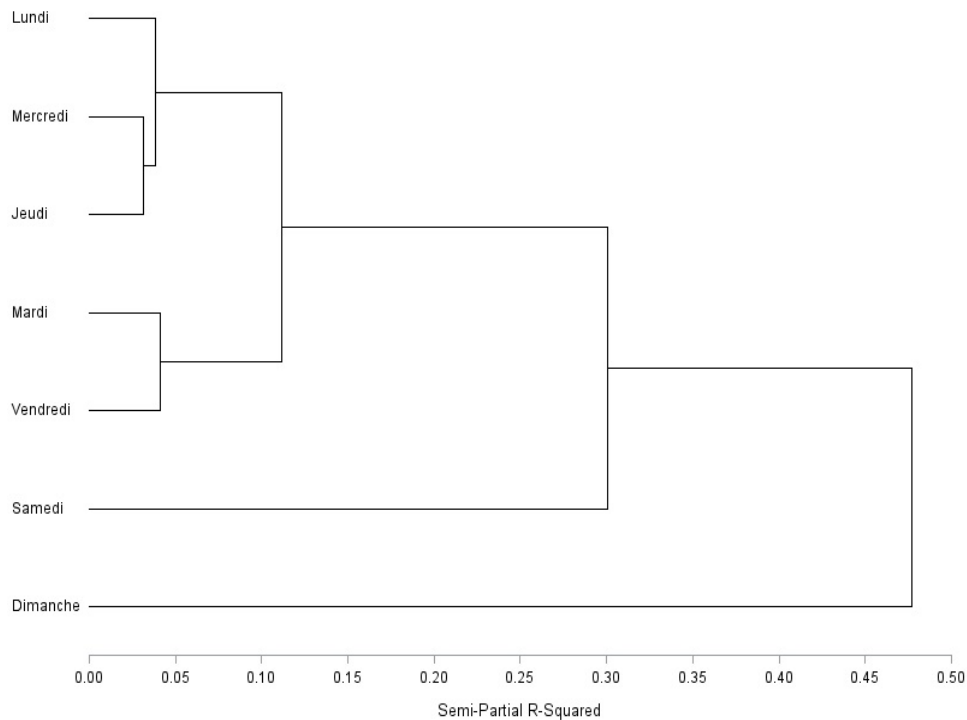
- A 2-regressor model with (Saturday) and (Monday, Tuesday, Wednesday, Thursday, Friday) in contrast with Sunday;
- A 3-regressor model with (Saturday), (Tuesday, Friday) and (Monday, Wednesday, Thursday) in contrast with Sunday.

#### 5.4.2.3 Impact on the stability of the trading-day estimates

As shown by Salinas and Hillmer (1987), the stability of the trading-day estimates is linked to the high correlation that usually exists between the regressors. The correlation matrix is then ill-conditioned and one eigenvalue is close to 0.

This instability can be summarized by the ratio between the smallest and the largest eigenvalues:  $R = \text{Min}(\lambda)/\text{Max}(\lambda)$ ; the smaller the ratio, the higher the instability problems. A simulation study was done to evaluate the distribution of the ratio using the following methodology for monthly data:

- A complete 400-year cycle of the generic Gregorian calendar is generated;
- The 7 trading-day regressors, the numbers of each day for each month, are computed and seasonally adjusted;
- The corresponding 6 contrasts to Sunday are derived as well as the 2 regressors (Saturday) and (Monday, Tuesday, Wednesday, Thursday, Friday) in contrast with Sunday;
- These contrasts are also computed taking into account the specificities of the French calendar;

**Figure 5.4:** Dendrogram from a HCA on the 105 Retail trade trading-day effects**Table 5.2:** Average “Min-Max” Ratio according to the calendar, the set of regressors and the length of the series.

Length (in years)	Gregorian Calendar			French National Calendar	
	7 regressors	6 contrasts	2 contrasts	6 contrasts	2 contrasts
5	0.0005	0.0321	0.0243	0.1773	0.1444
10	0.0005	0.0329	0.0243	0.1935	0.1442
15	0.0005	0.0331	0.0243	0.1975	0.1440
20	0.0005	0.0331	0.0243	0.1997	0.1437
All	0.0005	0.0328	0.0243	0.1919	0.1441

- For each possible subset of length 5, 10, 15 and 20 years, the correlation matrix is derived and the R ratio computed.

From the results of the study presented in Table 5.2, we can draw some conclusions:

- Using contrasts improves the ratio and therefore the stability of the estimates (the ratio is multiplied by 60);
- Using the National specificities improves drastically the ratio, against multiplied by 6;
- The length of the series does not really impacts the ratio.

## 5.5 Impact Models

Some holidays might cause a change in the level of activity the days before and/or after the event. For example, Easter has an impact on the sales of flowers, chocolate, lamb etc. Specific “impact models” have been developed to estimate this “before and/or after” effect on the economy.

We review some of the basic models included in the main seasonal adjustment programs and propose a generic form of impact models. Even if these models are presented for specific cases (Easter and Ramadan<sup>6</sup>), they can be used for any fixed or moving special day<sup>7</sup>.

### 5.5.1 Models for Easter effect

Easter is a Christian holiday commemorating the resurrection of Christ. This event, linked to the full moon<sup>8</sup>, is celebrated by all Christian countries but its date, is usually expressed in the Gregorian calendar for Catholic countries and in the Julian calendar for Orthodox countries. Determining in advance the dates of Easter has been the subject of works by famous mathematicians, and Gauss himself is the author of scholarly, but unfortunately complex algorithms<sup>9</sup>.

For example, the following algorithm gives you the date of Easter in both Gregorian and Julian calendars for a given year<sup>10</sup>:

$$G = \text{year} \bmod 19$$

For the Julian calendar:

$$I = (19G + 15) \bmod 30$$

$$J = \left(\text{year} + \frac{\text{year}}{4} + I\right) \bmod 7$$

For the Gregorian calendar:

$$C = \frac{\text{year}}{100}$$

$$H = \left(C - \frac{C}{4} - \frac{8C+13}{25} + 19G + 15\right) \bmod 30$$

$$I = H - \frac{H}{28} \left(1 - \frac{29}{H+1} \frac{21-G}{11}\right)$$

$$J = \left(\text{year} + \frac{\text{year}}{4} + I + 2 - C + \frac{C}{4}\right) \bmod 7$$

Thereafter, for both calendars:

$$L = I - J$$

$$\text{EasterMonth} = 3 + \frac{L+40}{44}$$

$$\text{EasterDay} = L + 28 - 31 \frac{\text{EasterMonth}}{4}$$

The Sunday Catholic Easter can fall between March 22<sup>nd</sup> and April 25<sup>th</sup>, i.e. in different months and different quarters, when the Sunday Orthodox Easter can fall between April 4<sup>th</sup> and May 9<sup>th</sup>.

Several models have been proposed to model the Catholic Easter effect and details on the models used in the X-11-family software can be found in Ladiray and Quenneville (2001). We focus here on models available in the current versions of X-13ARIMA-SEATS and TRAMO-SEATS.

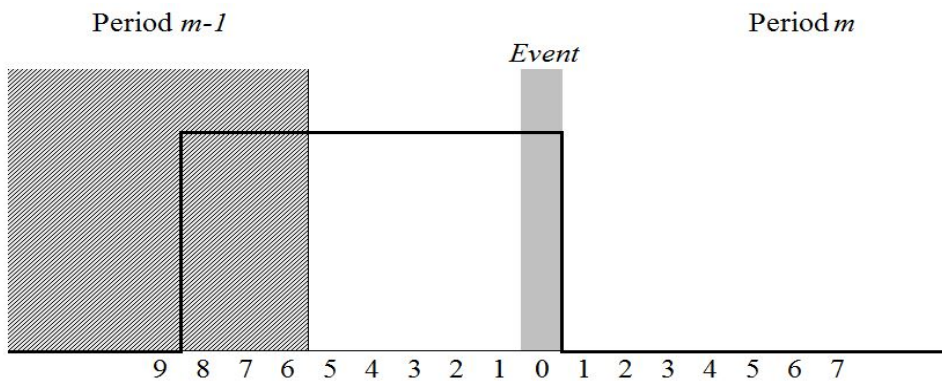
<sup>6</sup> Models for the Chinese new year have been proposed by Lin and Liu (2002).

<sup>7</sup> In particular, X-13ARIMA-SEATS proposes models for Labor day and Thanksgiving.

<sup>8</sup> A short, but approximate, definition of the date of Easter is “the first Sunday after the first full moon on or after the vernal equinox”.

<sup>9</sup> For more details, see <https://en.wikipedia.org/wiki/Computus>.

<sup>10</sup> In this algorithm, only integer divisions are allowed. For example:  $11 = 2 * 4 + 3$ . So,  $11/4 = 2$  and  $3 = (11) \bmod 4$ .

Figure 5.5: The S Ceaster[w] model (X-13ARIMA-SEATS),  $w = 9$ 

### 5.5.1.1 The “S Ceaster” model

This model is used in the more general context of the estimation of calendar effects proposed by the X-13ARIMA-SEATS method.

The Easter holiday, in this gradual effect model, is assumed to have an impact on the  $w$  days ( $1 \leq w \leq 24$ ) leading up to Easter Sunday, and the model  $Y_{ij} = a + bX_{ij}(w) + \varepsilon_{ij}$  is posited where:

- $Y_{ij}$  is the value of the irregular or of the raw series corresponding to year  $i$  and period (month or quarter)  $j$ .
- For a given year  $i$ , let  $n_i$  be the number of days, of the  $w$  days before Easter (including Easter), that fall in March (or in the first quarter). Therefore:

$$X_{ij}(w) = \begin{cases} n_i/w & \text{for a month of March or a first quarter (j=3 or j=1)} \\ -n_i/w & \text{for a month of April or a second quarter (j=4 or j=2)} \\ 0 & \text{otherwise} \end{cases}$$

- The residuals  $\varepsilon_{ij}$  are supposed to be a white noise when the effect is estimated from the irregular component or an ARIMA model  $(p, d, q)(P, D, Q)$  when the effect is estimated in the raw data, before the decomposition part.

The constraints imposed on  $w$ ,  $1 \leq w \leq 24$ , mean that only the values of the regressor for March and April (or of the first and second quarter) are not nil<sup>11</sup>

Several comments may be made.

- First of all, if Easter falls in March, the associated value  $X_{i3}(w)$  is equal to 1. Similarly, if Easter falls after April  $w$ , this value  $X_{i3}(w)$  is equal to 0.
- Given the simple form of the explanatory variable, the estimator  $\hat{b}$  can therefore be given a more explicit form. Assuming that the series is made up of  $T$  observations, we have:

$$\hat{b} = \frac{Cov(X, Y)}{Var(X)} \text{ with } Cov(X, Y) = \frac{1}{T} \sum_i \sum_j X_{ij} Y_{ij} - \overline{XY} \text{ and } Var(X) = \frac{1}{T} \sum_i \sum_j X_{ij}^2 - \overline{X}^2$$

The only non-nil values of variable  $X$  are those for the months of March and April that are, moreover, opposites. If it is assumed that the series does not begin a month of April and does not end in March,

<sup>11</sup>In fact, this is not exactly true since, for example, in the year 2008 Easter fall on March 23, and if  $w = 24$ , there will then be one day in February.

for every month of March there will be a corresponding month of April and the sum of the two values of variable  $X$  of these months will be nil. The average  $\bar{X}$  is therefore also nil in this case, and in the general case, nearly 0.

We will therefore have:

$$Var(X) = \frac{1}{T} \sum_i \sum_j X_{ij}^2 - \bar{X}^2 = \frac{1}{T} \sum_i \sum_j X_{ij}^2 = \frac{2}{T} \sum_i \left(\frac{n_i}{w}\right)^2$$

$$Cov(X, Y) = \frac{1}{T} \sum_i \sum_j X_{ij} Y_{ij} - \bar{X}\bar{Y} = \frac{1}{T} \sum_i \sum_j X_{ij} Y_{ij} = \frac{1}{T} \sum_i \frac{n_i}{w} [Y_{i3} - Y_{i4}]$$

And we therefore see the values of the differences appear, for each year, of the irregulars of March and April.

- Finally, as  $\hat{a} = \bar{Y} - \hat{b}\bar{X}$  and the average  $\bar{X}$  is close to 0,  $\hat{a}$  is very close to  $\bar{Y}$  the average of the irregular component (or close to the theoretical average a equal to 0 for an additive model and to 1 for a multiplicative model).

The Easter effect is derived by  $\hat{E}_{ij} = \hat{Y}_{ij} = \hat{a} + \hat{b}X_{ij}(w)$  or, more precisely, by  $\hat{E}_{ij} = a + \hat{b}X_{ij}(w)$ , making it possible to ensure that Easter has no effect beyond the months of March and April.

The model proposed by X-13ARIMA-SEATS therefore closely resembles the gradual effect model of X-11-ARIMA/88<sup>12</sup>, but here the estimation of the regression model is performed using all available years, not just those years in which Easter falls in March or after April  $w$ .

### 5.5.1.2 The “Easter” model

This model is used in the more general context of the estimation of calendar effects proposed by the X-13ARIMA-SEATS and TRAMO-SEATS methods.

Here again, the Easter holiday is assumed to have an impact on the  $w$  days ( $1 \leq w \leq 25$ ) leading up to Easter Sunday, and the model  $Y_{ij} = a + bX_{ij}(w) + \varepsilon_{ij}$  is posited where:

- $Y_{ij}$  is the value of the irregular or of the raw series corresponding to year  $i$  and period (month or quarter)  $j$ .
- For a given year  $i$ , let  $n_{ij}$  be the number of days, of the  $w$  days before Easter (excluding Easter), that fall in month (or quarter)  $j$ . So, we first define the variable  $Z_{ij}(w) = \frac{n_{ij}}{w}$ .

Given the constraints on  $w$ , this variable is nil except for the months of February, March and April. However, it has a certain seasonality: the values pertaining to the month of February will, for example, be structurally weaker. The variable  $X_{ij}(w)$  is obtained by subtracting from  $Z_{ij}(w)$  the average  $\bar{Z}_j(w)$  corresponding to month  $j$  and calculated for the available years. We therefore have:  $X_{ij} = \frac{n_{ij}}{w} - \bar{Z}_j(w)$ . Thus, it becomes possible to retain the series level by cancelling the Easter effect for the set of months concerned. The explanatory variable therefore has a nil average and no seasonality.

The constraints on the value of  $w$  mean that corrections can be made only for the months of February, March and April. As previously, the values of  $a$  and  $b$  are estimated by ordinary least squares, and the Easter effect is derived by  $\hat{E}_{ij} = a + \hat{b}X_{ij}(w)$

### 5.5.1.3 Maillard's Model

Maillard (1994) used a different model, considering a linear “increase-decrease” model illustrated in Figure 5.7. The effect starts  $a$  days before the event (date  $t_0$ ) and stops  $b$  days after.

<sup>12</sup> Moreover, this is why this modelling is referred to in X-12-ARIMA by the keyword SCEaster, the SC standing for Statistics Canada



Figure 5.6: The Easter[w] model (X-13ARIMA-SEATS and TRAMO-SEATS),  $w = 9$

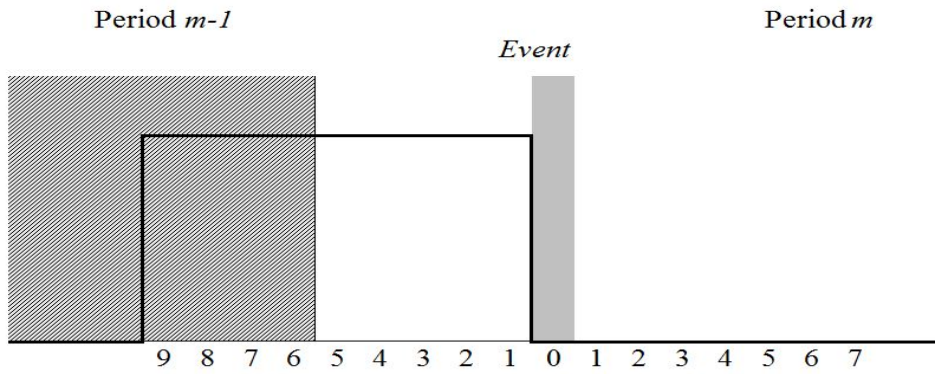
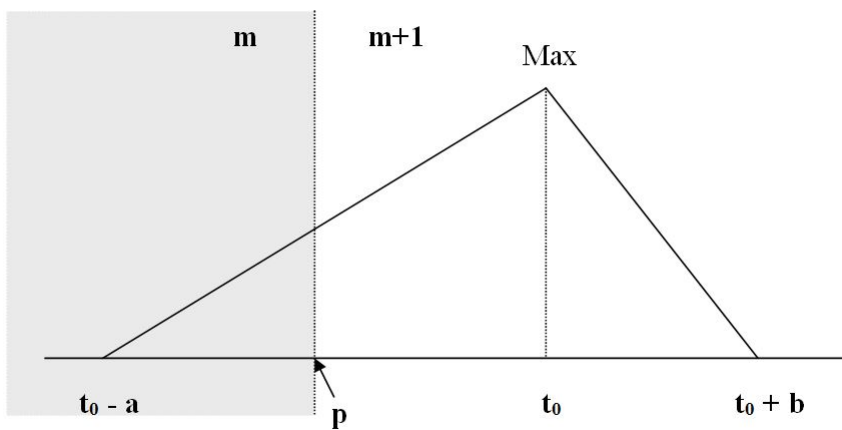


Figure 5.7: A linear "increase-decrease" model for the impact of Easter.



The value of the Easter effect  $X_t$  regressor for months  $t$  and  $t + 1$  is proportional to the part of the triangle area in the months and determined by:

$$X_t = \begin{cases} 0 & \text{if } p \leq (t_0 - a) \\ \frac{1}{a(a+b)}(p + a - t_0)^2 & \text{if } (t_0 - a) < p < t_0 \\ \frac{a}{(a+b)} & \text{if } p = t_0 \\ 1 - \frac{1}{b(a+b)}(t_0 + b - p)^2 & \text{if } t_0 < p < (t_0 + b) \\ 1 & \text{if } (t_0 + b) \leq p \end{cases}$$

And  $X_{t+1} = 1 - X_t$ .

### 5.5.2 Models for the Ramadan effect

The effect of Muslim festivals have been studied for example in Bessa and al. (2008), Sarhani and El Afia (2014) etc.

Muslim societies follow the Hijri (Islamic) calendar, which is strictly lunar based and composed of twelve lunar months, of length 29 or 30 days in a year of 354 (or 355 days in leap year). The Islamic calendar is used to date events in many Muslim countries (concurrently with the Gregorian calendar), and used by Muslims everywhere in the world to determine Islamic holidays and festivities.

A characteristic of this calendar is that in most countries, the beginning of a month is stated from a human observation. For our purposes, the modeling of the effect for the seasonal adjustment of the series, this makes essential the precise record of the festival dates which could change from a country to another. When forecasts are needed, algorithms are used to provide approximate dates for the future.

In the Islamic calendar, years follow a 30-year cycle and on this cycle, the year average length is very close to the length of the lunar year. Several rules (algorithms) exist to determine the leap years in which the last month (Jumada al Oula) counts 30 days:

- 2, 5, 7, 10, 13, 15, 18, 21, 24, 26, et 29 ; (Kuwait algorithm)
- 2, 5, 7, 10, 13, 16, 18, 21, 24, 26, et 29 ; (common version)
- 2, 5, 8, 10, 13, 16, 19, 21, 24, 27, et 29 ; (Indian tables)

Bessa and al. (2008) use several regressors to model the effect of the Islamic festivals. All these regressors are based on the same principle: the value of the regressor for a month  $t$  is proportional to the number of days of month  $t$  linked to the considered holiday. More precisely:

- Immediate impact models, only counting the number of days off, were considered for Aid el-Fitr, Aid el-Adha, Ras el-Am el-Hijiri and Mawlid.
- For the effect of Ramadan, the Islamic holy month of fasting, they considered gradual impact models and also used different regressors for the first and second parts of Ramadan. This model is illustrated in Figure 5.8.

It has to be noted that these regressors do not present any trend and seasonality in the Gregorian calendar, mostly because the Islamic year goes through the complete solar year and because the Islamic events are therefore not linked to one or several Gregorian months<sup>13</sup>.

### 5.5.3 More general models

It is quite easy to derive more general impact models. For a given event, the parameters of a model are:

<sup>13</sup>This would be different for the Chinese New Year that falls in January or February in the Gregorian calendar.

Figure 5.8: A general constant impact model

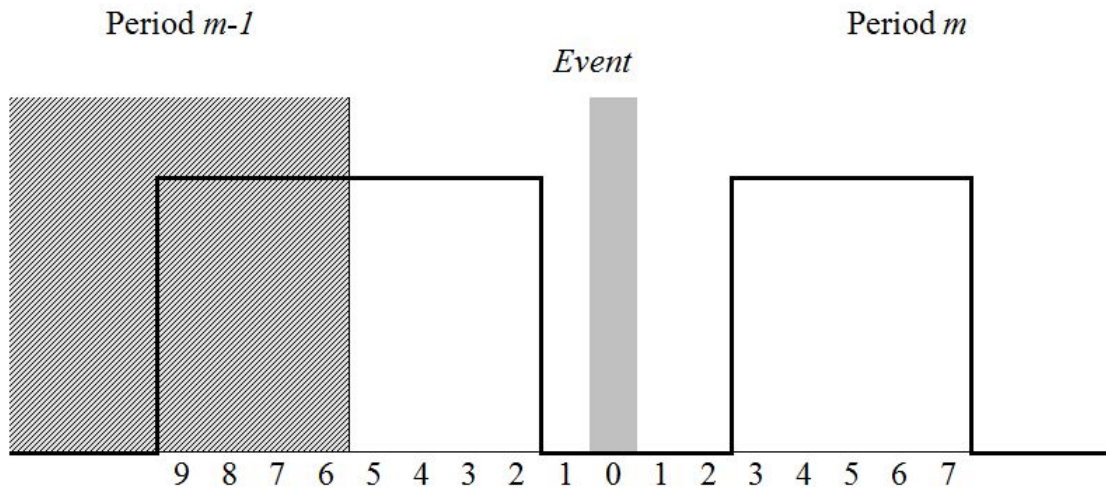
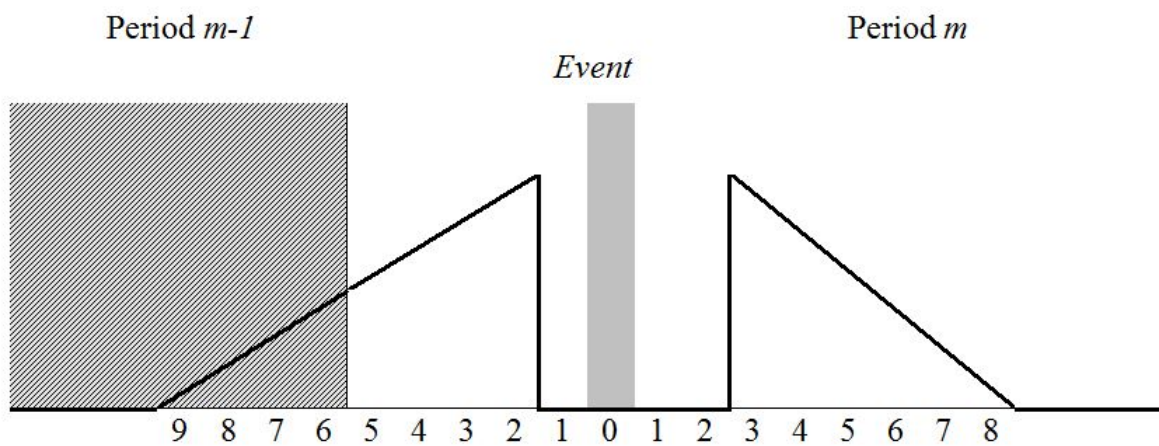


Figure 5.9: a general "increase-decrease" linear impact model



- The date of the event in the Gregorian calendar;
- The period on which the event is supposed to have an impact that can be determined by 4 parameters: the starting date of the impact before the event, the ending date of the impact before the event, the starting date of the impact after the event and the ending date of the impact after the event;
- The nature of the impact: constant, linear etc.

Figure 5.8 and Figure 5.9 illustrate a general constant impact model and a general “increase-decrease” linear model.

## 5.6 Spectral detection of trading-day effects

In seasonal adjustment software like X-13ARIMA-SEATS, TRAMO-SEATS and JDemetra+, periodograms and spectra are used to check for the presence of trading-day effects. Their performances in the detection of these effects have been studied and evaluated by Soukup and Findley (1999) Soukup and Findley (2000) and Ladiray (2012).

### 5.6.1 The Calendar Periodograms and the Trading-Day Frequencies

Using the complete 400-year cycle of the Gregorian calendar permits to get very good estimates and it appears that the 7 days have very similar periodograms<sup>14</sup>. Figure 5.10 (left panels) shows the periodograms of the number of Sundays per month and quarter. These periodograms computed on the raw data present some seasonality mainly due to a length-of-month, or length-of-quarter, effect. This seasonality is even more evident on the week-day periodogram (see Figure 5.6). The raw data can be easily seasonally adjusted by removing for each day its long-term average observed for each month (or quarter). The periodograms computed on these centered distributions show the main trading-day frequencies (see right panels). The real trading-day effect is in fact a linear combination of the basic day effects. As the day-of-the-week distributions are not independent, the frequencies attached to a real trading-day effect may be different from the theoretical ones. Ladiray (2012) made a large scale study on real time series and it appears from the experiment that:

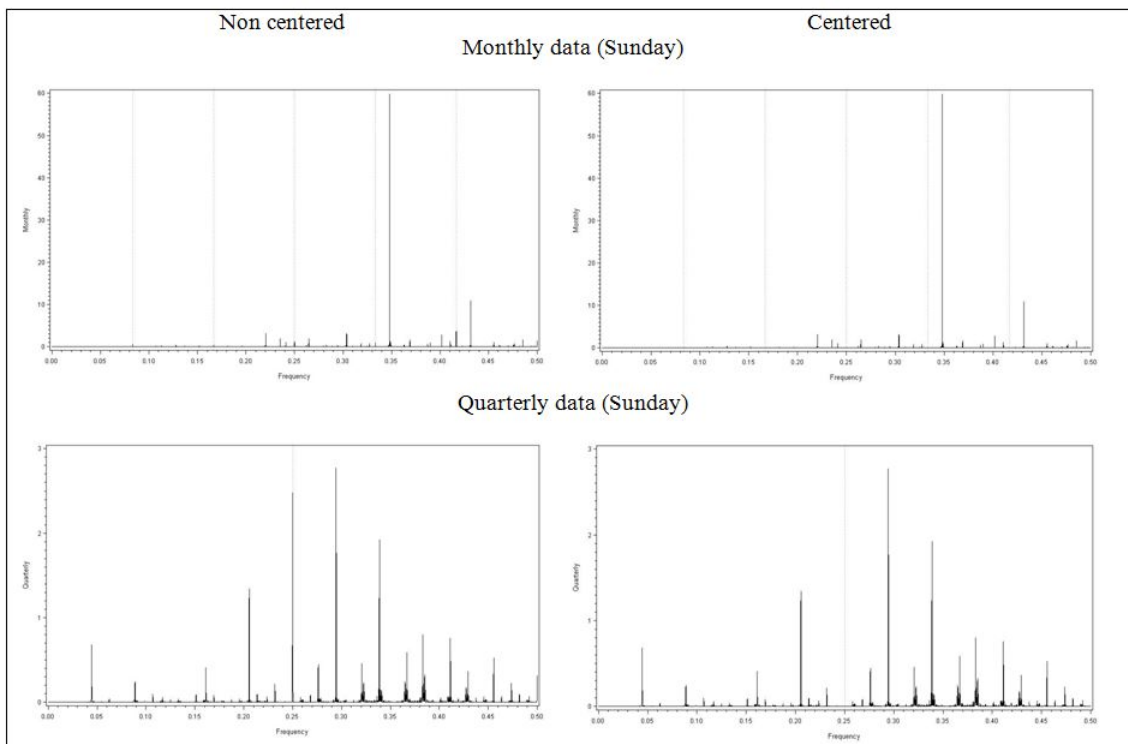
- For monthly series, trading-day effects are characterized by mainly 2 frequencies, 0.348125 and 0.431458, but it is difficult to distinguish between the other main frequencies as their importance changes according to the number of regressors used in the regression. Note that these 2 frequencies are those exhibited and used for monthly series by X-13ARIMA-SEATS in its “visual spectral test” and JDemetra+ in its diagnostics.
- The situation is less clear with the quarterly series and it is quite difficult to distinguish between the 4 following frequencies: 0.294375, 0.338750, 0.383125 and 0.429375.

### 5.6.2 The USCB “Visual Test”

X-13ARIMA-SEATS (U.S. Census Bureau (2015), section 6.1) provides spectral plots and associated interpretative messages to alert the user to the presence of seasonal and trading day effects in the original series and to the presence of residual effects in the residuals of the Reg-Arima modelling, the seasonally adjusted series and the irregular component. Because of difficulties associated with statistical significance tests for periodic components in autocorrelated data, such tests are not used in the program and the warning messages issued by the software are based on an empirically obtained criterion of “visual significance”. The program

<sup>14</sup> McNulty and Huffman (1989) reached a quite different conclusion but they only used a 28-year cycle which is a very crude estimate of the real periodicity of the calendar.

**Figure 5.10:** Periodogram of a day-of-the-week series (here # of Sundays)



calculates values of the spectrum (or periodogram) at 61 frequencies, including the seasonal and trading day frequencies, and graphs the results in the output file using line printer plots.

Both seasonal and trading-day frequencies are marked on the graph. When larger than the median of the spectral estimates, a “6 -star” peak at one of the seasonal or trading-day frequencies is considered “visually significant” with one star corresponding to 1/52nd of the range between the maximum and minimum spectral values Soukup and Findley (1999), Soukup and Findley (2000). By default, the program uses the last 96 observations (8 years for a monthly series) in its calculations of the spectrum. In the case of trading day, a peak in the spectrum at one of the trading day frequencies shows the need for trading day estimation. Several spectrum estimators are used to detect trading day effects: an autoregressive spectral estimator (the default for X-13ARIMA-SEATS) and a nonparametric Tukey estimator.

### 5.6.3 An example

Figure 5.12 presents an example of the spectral diagnostics output by JDemetra+. It concerns the French retail trade turnover seasonally adjusted series. Both periodogram and autoregressive spectrum show a peak at the main trading-day frequency (pink line) when no correction for trading-day was done. On the contrary, no peak is visible when a trading-day correction using the National calendar was performed.

Figure 5.11: Periodogram of the week-day series (# Monday + # Tuesday + ... + # Friday)

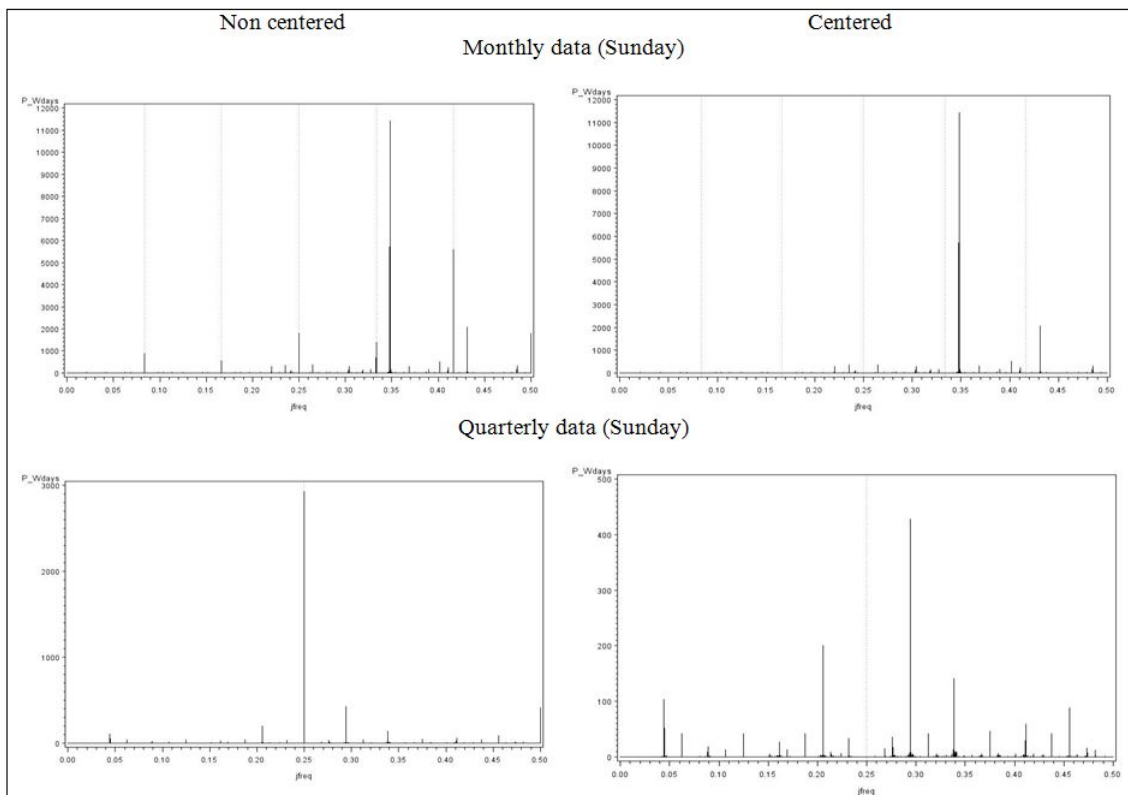
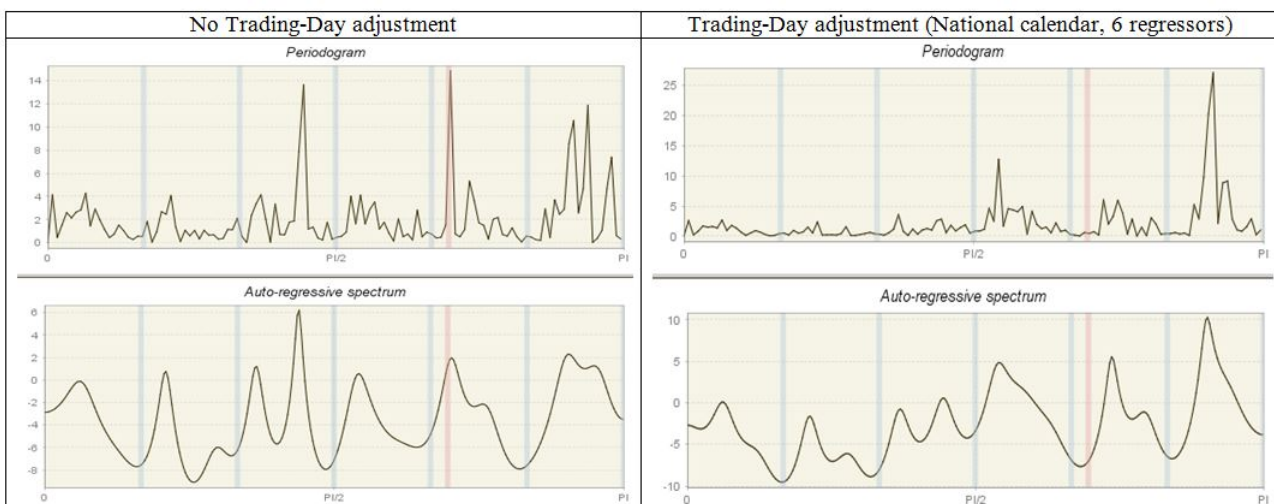


Figure 5.12: Checking for residual trading-day effects in the French retail trade turnover SA series



## 5.7 Conclusions

This chapter presents the definitions and the main methodologies useful to perform a correct detection and estimation of calendar effects in flow series.

However, two important points are missing.

1. The regressors to use to correct stock series, such as inventories at month's end, from Trading-day effects. Findley and Monsell (2009), by deriving an invertible linear relation between stock and flow trading day regression coefficients, show how flow day-of-week effect constraints can be imposed upon the day-of-week effect component of the stock trading day model of Bell used in X-12-ARIMA Bell (1984), Bell (1995). Findley et al. (2012) use a similar idea to derive impact model regressors ("holiday regressors") for stock series from cumulative sums of flow-series regressors.
2. The models presented in the chapter assume that the regressor coefficients are constant over time. As long as the relative weight of daily activities is fixed on the span of the series, this deterministic model gives reasonable estimates. However, this is not always a realistic assumption. In the European Union, Member states legislations used to prohibit the opening of retail trade stores on Sunday. Stochastic models for time-varying trading-day coefficients have been proposed in the literature. Monsell (1983) used random walk models for the coefficients. Daqu and al. (1992) and Daqu and Quenneville (1993) considered a more general formulation, including seasonal, trend and irregular components in the model along with time-varying trading-day effects. Bell (2004) introduced the RegComponent model, a regression model whose errors follow an ARIMA component time series model. This class of models is quite general and can be used to allow for stochastic time-varying regression coefficients.

To conclude, it has to be stressed that calendar effects are usually a small component of a time series and it is sometimes difficult to get a good and stable estimate of these effects. It is therefore good to keep in mind the following principle highlighted in the ESS guidelines for Seasonal Adjustment: "*To avoid misleading results, seasonal adjustment should be applied only when seasonal and/or calendar effects can be properly explained, identified and estimated.*"



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# 6

## Moving Trading-Day Effects with X-13 Arima-Seats and Tramo-Seats



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## Table of Contents

6.1	Introduction	119
6.2	The basic model with fixed coefficients	120
6.3	Stochastic models for time-varying trading-day coefficients	121
6.3.1	The basic structural model	121
6.3.2	Examples	124
6.3.3	RegArima Models	124
6.3.4	RegComponents Models	125
6.4	Improving the basic model for time-varying trading-day coefficients	126
6.4.1	The X-13ARIMA-SEATS “change-of-regime”, “history” and “slidingspans” specifications	127
6.4.2	Using rolling Windows to estimate moving trading-day effects	127
6.4.3	RegARIMA models with time-varying trading-day coefficients	129
6.5	Examples	130
6.5.1	Methodology	130
6.5.2	Results	130
6.6	Conclusions	133

## 6.1 Introduction

A large part of economic indicators related to production, imports-exports, inventories and sales are affected by trading-day or calendar variations. Trading-day effects reflect variations in monthly and quarterly time series due to the changing composition of months or quarters with respect to the numbers of times each day of the week occurs in the month or quarter. These variations are systematic and can strongly influence the short-term variations of the series and the period-to-period comparisons.

A trading-day regression model with ARIMA errors (regARIMA), derived from the simple model proposed by Young (1965), is currently used by X-13ARIMA-SEATS and TRAMO-SEATS. This model assumes that the trading-day coefficients are constant over time. As long as the relative weight of daily activities is quite stable over the span of the series, this deterministic model gives reasonable estimates. However, this is not always a realistic assumption. In the European Union, Member states regulations used to prohibit the opening of retail trade stores on Sunday. This situation, as well as consumers' shopping patterns, has changed substantially in the last decade. Seasonal adjustment practitioners sometimes deal with this issue by restricting the span of the series to which the trading-day model is fitted. However, this strategy might only provide a crude approximation to trading-day effects that vary through time and a higher instability of the estimates.

Akaike and Ishiguro (1980) proposed the use of a Bayesian model for seasonal adjustment and implemented the method in a computer program called BAYSEA in which Ishiguro and Akaike (1981) introduced a time-varying trading-day adjustment procedure. Since, proposing a way to integrate in the main seasonal adjustment methods an efficient strategy to detect and estimate time-varying trading-days has been a challenge. Research has been following different paths:

- Monsell (1983), Dagum and Quenneville (1988), Dagum et al. (1992) propose models to estimate these stochastic effects from a series with no trend and no seasonal component, typically the irregular component issued from a seasonal adjustment process like X11. Dagum and Quenneville (1993) explicitly introduce a stochastic model for trading-day variations jointly estimated with the remaining components, namely the trend-cycle and the seasonal component. In these papers, the authors take advantage of the Basic Structural Model popularized by Harvey (1989).
- Quenneville et al. (1999), Zhang and Poskitt (2006) and Maravall and Pérez (2011) propose to incorporate stochastic trading-day effects in regARIMA models, a class of models already used in X-13ARIMA-SEATS and TRAMO-SEATS.
- Bell (2004) introduces the RegComponent models, an approach that encompasses both structural and regARIMA approaches, and Bell and Martin (2004) show how they can be used to estimate time-varying trading-day effects.
- Sutcliffe (1996), Sutcliffe (2003) and Campbell and Chen (2015) propose a semi-parametric method based on "rolling windows" that estimates moving effects on subspans of the irregular component. This strategy is already used by the Australian Bureau of Statistics in SEASABS, a variant of X-13ARIMA-SEATS.

All these approaches use models, state space representations of which allow a fast and efficient estimation of the parameters and an easier integration to recommended seasonal adjustment programs<sup>1</sup>

The chapter is devoted to the estimation of these time-varying trading-day effects. Section 6.2 presents the regression models currently implemented in the "X11 family" (X-12-ARIMA and X-13ARIMA-SEATS) and TRAMO-SEATS for the detection and estimation of fixed trading-day effects. Section 6.3 presents stochastic

<sup>1</sup>Eurostat, the Statistical Office of the European Union, and the European Central Bank suggest using ARIMA-based methods, like TRAMO-SEATS, or moving-average based methods, like X-13ARIMA-SEATS. JDemetra+, the European software for seasonal adjustment implements the two methods and will be used for the applications of Section 6.5. The program and documentation are available at [https://ec.europa.eu/eurostat/cros/content/seasonal-adjustment\\_en](https://ec.europa.eu/eurostat/cros/content/seasonal-adjustment_en)



models for time-varying trading-day coefficients that have been proposed in the literature: the basic structural model, regARIMA models and regComponent models. Section 6.4 presents two different strategies to implement time-varying coefficients models in X-13ARIMA-SEATS and TRAMO-SEATS programs: the “Rolling window” approach and a regARIMA approach that includes stochastic trading-day effects. Various strategies are applied to real time series and compared in Section 6.5; Section 6.6 concludes.

## 6.2 The basic model with fixed coefficients

It will be assumed below, following the notation of Findley et al. (1998), that the  $j^{th}$  day of the week has an effect  $\alpha_j$  where, for example,  $j = 1$  refers to Monday,  $j = 2$  refers to Tuesday, etc., and  $j = 7$  refers to Sunday. Each  $\alpha_j$  represents for example the average sales for one day  $j$ . If  $D_{jt}$  represents the number of days  $j$  in the month  $t$ , the length of the month will be

$$N_t = \sum_{j=1}^7 D_{jt}$$

and the cumulative effect for that month, the total sales of the month, will be:

$$\sum_{j=1}^7 \alpha_j D_{jt}.$$

We also have

$$\bar{\alpha} = \frac{1}{7} \sum_{j=1}^7 \alpha_j$$

the mean daily effect, the average sales for one day. Since by design we have  $\sum_{j=1}^7 (\alpha_j - \bar{\alpha}) = 0$ , we may write:

$$\sum_{j=1}^7 \alpha_j D_{jt} = \bar{\alpha} N_t + \sum_{j=1}^7 (\alpha_j - \bar{\alpha}) D_{jt} = \bar{\alpha} N_t + \sum_{j=1}^6 (\alpha_j - \bar{\alpha}) (D_{jt} - D_{7t}) \quad (6.1)$$

Thus, the cumulative monthly effect is decomposed into an effect directly linked to the length of the month and a net effect for each day of the week.

Note that the sum

$$\sum_{j=1}^7 (\alpha_j - \bar{\alpha}) D_{jt}$$

involves only the days of the week occurring five times in a month; every month contains four complete weeks, for which by definition the effect linked to the days is canceled out, plus 0, 1, 2 or 3 days which contribute to the trading-day effect for the month. Equation 6.1 must be adjusted to remove possible seasonality and trend.

- Potentially, part  $\bar{\alpha} N_t$  of the equation contains such components because the months vary in length and because variable  $N_t$  is periodic (period of 400 years). These effects can be summarized by the quantity  $\bar{\alpha} N_t^*$  where  $N_t^*$  represents the average, over 400 years, of the length of the month  $t$ . In other words,  $N_t^*$  is equal to 30 or 31 if the month in question is not the month of February, and is equal to 28.25 otherwise. Thus, we have:  $\bar{\alpha} N_t = \bar{\alpha} N_t^* + \bar{\alpha} (N_t - N_t^*)$ , an equation whose second part is zero except for the month of February.
- The second part of the equation includes  $D_{jt}$ , the number of times that day  $j$  is present in month  $t$ . These variables are periodic (period of 4800 months, 1600 quarters or 400 years) with equal means for

a given month. In the second part of the equation, the difference  $D_{jt} - D_{7t}$  is used, and since these variables show the same behaviour, the difference involves no seasonality and no trend.

The procedure used to adjust a series  $y_t$  for these effects depends on the decomposition model used and on the series used to estimate the effects.

- The effects can be estimated from an estimation of the irregular component, after the decomposition of the series. For an additive decomposition, the model is the following:

$$\hat{I} = \beta_0(N_t - N_t^*) + \sum_{j=1}^{j=6} \beta_j T_{jt} + e_t,$$

where  $\bar{\alpha}N_t^*$  has been subtracted logically from equation 6.1 to remove the seasonal length-of-month effect,  $T_{jt} = D_{jt} - D_{7t}$ ,  $\beta_0 = \bar{\alpha}$  and  $\beta_j = \alpha_j - \bar{\alpha}$ , for  $1 \leq j \leq 6$ . This model is estimated by Ordinary Least Squares and implemented in X-12-ARIMA or X-13ARIMA-SEATS using the “X11regression” specification.

- The trading-day effects can also be estimated before the decomposition of the series, and this is what is recommended, using a RegARIMA model:

$$(1 - B)^d(1 - B^{12})^D y_t = \beta_0(N_t - N_t^*) + \sum_{j=1}^{j=6} \beta_j T_{jt} + z_t,$$

where  $z_t$  follows an ARMA model. This second possibility is implemented in X-12-ARIMA or X-13ARIMA-SEATS using the “Regression” specification and in TRAMO-SEATS using the “ $TD = 7$ ” parameter.

Other specifications of the model, namely the week-day and no leap year regressors, are available in the programs; see for example USCB (2017) and Gómez and Maravall (1997).

## 6.3 Stochastic models for time-varying trading-day coefficients

### 6.3.1 The basic structural model

A structural time series model is based on the principle that a time series consists of interpretable unobserved components such as trend, seasonal, cycle and irregular (Harvey (1989), Harvey (2018)). One particular useful model for seasonal adjustment is the Basic Structural Model (BSM). Let  $y_t$  be a (monthly) time series. The BSM is given by:

$$y_t = \mu_t + \gamma_t + \epsilon_t, \quad t = 1, \dots, n, \quad (6.2)$$

where  $\mu_t$  is the trend,  $\gamma_t$  the seasonality and  $\epsilon_t$  the irregular component. Such components are unobserved and modelled by stochastic processes.

The trend component  $\mu_t$  is usually specified as:

$$\begin{aligned} \mu_{t+1} &= \mu_t + v_t + \eta_t, \quad \eta_t \approx NID(0, \sigma_\eta^2) \\ v_{t+1} &= v_t + \zeta_t, \quad \zeta_t \approx NID(0, \sigma_\zeta^2), \end{aligned} \quad (6.3)$$

with  $\mu_1 \approx (0, \kappa)$  and  $v_1 \approx (0, \kappa)$  where  $\kappa$  is large (Koopman et al. (1998)). The initial conditions for  $\mu_1$  and  $\eta_1$  indicate that no information is available. Model 6.3 is called a local linear trend. The term  $v_t$  is the slope of the trend: when  $\sigma_\zeta^2 = 0$ ,  $v_{t+1} = v_t = v$  and 6.3 becomes a local trend model. When also  $\sigma_\eta^2 = 0$ , then the



trend is linear deterministic and 6.3 reduces to a deterministic linear trend model.<sup>2</sup>

The seasonal component  $\gamma_t$  can be specified in various ways. The trigonometric seasonal model, used for example in Koopman et al. (1998), and Koopman and Franses (2001), is given by:

$$\gamma_t = \sum_{j=1}^6 \gamma_{j,t}, \quad (6.4)$$

where

$$\begin{pmatrix} \gamma_{j,t+1} \\ \gamma_{j,t+1}^* \end{pmatrix} = \begin{bmatrix} \cos \lambda_j & \sin \lambda_j \\ -\sin \lambda_j & \cos \lambda_j \end{bmatrix} \begin{pmatrix} \gamma_{j,t} \\ \gamma_{j,t}^* \end{pmatrix} + \begin{pmatrix} \omega_{j,t} \\ \omega_{j,t}^* \end{pmatrix}, \quad (6.5)$$

with frequencies  $\lambda_j = \pi j/6$ , for  $j = 1, \dots, 6$ . The disturbances are mutually uncorrelated and normally distributed with mean zero and variance matrix:

$$\text{Var} \begin{pmatrix} \omega_{j,t} \\ \omega_{j,t}^* \end{pmatrix} = \begin{bmatrix} \sigma_{\omega_j}^2 & 0 \\ 0 & \sigma_{\omega_j}^2 \end{bmatrix}$$

The terms associated with different frequencies have different variances. Each initial seasonal value  $\gamma_{j,1}$  and  $\gamma_{j,1}^*$ , for  $j = 1, \dots, 6$  is initialized with a diffuse prior, that is  $\gamma_{j,1} \approx N(0, \kappa)$  and  $\gamma_{j,1}^* \approx N(0, \kappa)$ . The trigonometric seasonal model 6.4 has the property to evolve very smoothly over time. Finally, the irregular term  $\epsilon_t$  follows a normal random variable with mean zero and variance  $\sigma_\epsilon^2$ .

The BSM can be written in state space form, which is particularly useful for estimating time-varying models. The following state space representation is chosen (adopted by the `SsfPack` package for example, Koopman et al. (1998)):

$$\begin{aligned} \begin{pmatrix} \alpha_{t+1} \\ y_t \end{pmatrix} &= \delta_t + \phi_t \alpha_t + u_t, \quad u_t \approx NID(0, \Omega_t) \\ \delta_t &= \begin{pmatrix} d_t \\ c_t \end{pmatrix}, \quad \phi_t = \begin{pmatrix} T_t \\ Z_t \end{pmatrix}, \quad u_t = \begin{pmatrix} H_t \\ G_t \end{pmatrix} \epsilon_t, \\ \Omega_t &= \begin{pmatrix} H_t H_t' & H_t G_t' \\ G_t H_t' & G_t G_t' \end{pmatrix} \\ \alpha_1 &\approx N(a, P), \quad t = 1, \dots, n \end{aligned} \quad (6.6)$$

The  $(m \times 1)$  vector  $\alpha_t$  is the state of the system, containing unobserved stochastic processes and fixed effects. The  $(N \times 1)$  vector  $y_t$  contains the observations at time  $t$  of the observed variables. The matrix  $\phi_t$ , of dimension  $[(m + N), m]$  changed to, defines the state and measurement equations. The deterministic matrices  $T_t$ ,  $Z_t$ ,  $H_t$  and  $G_t$  are referred to as system matrices. In our case the state vector  $\alpha_t$  is defined as:

$$\alpha_t = (\mu_t \quad v_t \quad \gamma_{1,t} \quad \gamma_{1,t}^* \quad \dots \quad \gamma_{5,t}^* \quad \gamma_{6,t})'$$

and has dimension  $(13 \times 1)^3$ , while the observational vector  $y_t$  is one-dimensional. The vector  $\delta_t$  is null, while

<sup>2</sup>Another popular parametrization is the smooth trend, obtained with  $\sigma_\zeta^2 > 0$  and  $\sigma_\eta^2 = 0$ .

<sup>3</sup>The seasonal coefficient  $\gamma_{6,t}^*$  is excluded from the state because  $\gamma_6 = \pi$  and  $\sin \gamma_6 = 0$ .

$\phi$  is defined as:

$$\phi = \begin{pmatrix} T \\ Z \end{pmatrix}$$

$$T = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos \lambda_1 & \sin \lambda_1 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & -\sin \lambda_1 & \cos \lambda_1 & 0 & \dots & 0 & 0 & 0 & 0 \\ \dots & & & & 0 & & & & & \dots \\ 0 & 0 & 0 & 0 & 0 & \dots & \cos \lambda_5 & \sin \lambda_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \dots & -\sin \lambda_5 & \cos \lambda_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$Z = [1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1]$$

The matrix  $\Omega$  is diagonal with elements

$$(\sigma_\eta^2 \ \sigma_\zeta^2 \ \sigma_{\omega_1}^2 \ \sigma_{\omega_1}^2 \ \dots \ \sigma_{\omega_5}^2 \ \sigma_{\omega_5}^2 \ \sigma_{\omega_6}^2) \quad (6.7)$$

Note that the time index has been dropped by the notation of  $\phi$  and  $\Omega$ . The initial state vector is assumed to follow a diffuse distribution, that is  $\alpha_1 \sim N(0, \kappa I_{13})$  with  $\kappa$  arbitrarily large.

The classical BSM can be extended to include time-varying calendar effects. Model (6.2) is modified as follows:

$$y_t = x_t' \beta_t + \mu_t + \gamma_t + \epsilon_t, \quad t = 1, \dots, n, \quad (6.8)$$

where  $x_t$  is the  $(k \times 1)$  vector of regressors with calendar effects at time  $t$  and  $\beta_t$  is the  $(k \times 1)$  vector containing the corresponding time-varying coefficients. We assume that these follow independent random walk models:

$$\beta_{i,t+1} = \beta_{i,t} + \xi_{i,t}, \quad i = 1, \dots, k \quad t = 1, \dots, n. \quad (6.9)$$

The  $\xi_{i,t}$ 's are mutually independent normally distributed processes with variance  $\sigma_{\xi,i}^2$ . When  $\sigma_{\xi,i}^2 = 0$ ,  $\beta_{i,t+1} = \beta_{i,t} = \beta_i$ : a coefficient is thus fixed when the corresponding innovation variance is zero. The hypothesis of a random walk is particularly appealing for capturing possible time variation in calendar effects: in fact, it avoids too much erratic variation around the average level, instead allowing the coefficients to change more smoothly over long periods of time without being tied to fixed means (Bell and Martin (2004)).

The state space representation of the BSM needs to be changed to introduce the regression effects  $x_t$ . The state vector is augmented at the top with the calendar effects:

$$\alpha_t = (x_t \ \mu_t \ v_t \ \gamma_{1,t} \ \gamma_{1,t}^* \ \dots \ \gamma_{6,t})'$$

With  $T$  and  $Z$  defined as above, the new matrix  $\phi_t$  becomes:

$$\phi_t = \begin{bmatrix} I_8 & 0 \\ 0 & T \\ x_t' & Z \end{bmatrix}$$

which is a time-varying matrix, for the presence of  $x_t$  in the measurement equation. Time-varying regression coefficients are introduced in the state space model by defining the diagonal matrix  $\Omega$  as

$$(\sigma_{x_1}^2 \ \sigma_{x_2}^2 \ \sigma_{x_3}^2 \ \dots \ \sigma_{x_k}^2 \ \sigma_\eta^2 \ \sigma_\epsilon^2 \ \sigma_{\omega_1}^2 \ \sigma_{\omega_1}^2 \ \dots \ \sigma_{\omega_5}^2 \ \sigma_{\omega_5}^2 \ \sigma_{\omega_6}^2), \quad (6.10)$$

i.e. by augmenting the matrix  $\phi_t$  with the variances of each calendar effect.

The BSM augmented with time-varying calendar effects  $x_t$  can be estimated by maximum likelihood using the software STAMP or the OX package *SsfPack*, which is a collection of routines for implementing, fitting and analysing models in state space forms (Koopman et al. (1998)).

### 6.3.2 Examples

One can find in the literature several proposals of models with time-varying coefficients for trading-day effects that can be considered as examples of the previous BSM model.

- Monsell (1983) works with simulated data with no trend or seasonal components, just a trading day component and an error term. The model can be written, using our previous notations:  $y_t = TD_t + \epsilon_t$  where the error term is normally distributed with mean zero and nonzero variance. The author simulates various moving trading-day patterns and error variances, and uses the Kalman filter and smoother to get estimates for the trading day coefficients.
- Dagum and Quenneville (1988), still using series with no trend or seasonal components, typically the irregular component issued from an adjustment with X-11-ARIMA, use a stochastic model where the daily coefficients follow either a random walk or a random walk with a random drift. This last model gives a local approximation to a linear trend in the daily coefficients. The estimation of the stochastic models is also made using the Kalman filter and the fixed interval smoother.
- Dagum et al. (1992) extend previous works by introducing a general stochastic model for the trading-day coefficients, the parameters of which are estimated by maximum likelihood, and a test to assess whether the trading-day variations are deterministic or stochastic. They work also on data with no trend-cycle and seasonal components, i.e. data seasonally adjusted using the X-11-ARIMA, and suppose the trading-day coefficients  $\beta_{it}$  follow the model:  $(1 - B)^k \underline{\beta}_t = \underline{\chi}_t$  where  $\underline{\beta}_t = [\beta_{1t}, \beta_{2t}, \dots, \beta_{6t}]'$  is the vector of trading-day coefficients in month  $t$  and  $\underline{\chi}_t$  is the vector of errors which follows a  $NID(\underline{0}_6, \sigma_\chi^2 I_6)$ . The rationale behind the model is to adjust locally a polynomial of degree  $k$  to the trading-day coefficients. The degree  $k$  adequate for each series is selected using Akaike's criterion (AIC).
- Dagum and Quenneville (1993) explicitly introduce a stochastic model for trading-day variations jointly estimated with the remaining components, namely the trend-cycle and the seasonal component. In this basic structural model, the stochastic trading-day component follows the same model than in Dagum et al. (1992).

### 6.3.3 RegARIMA Models

RegARIMA models are used in both X-13ARIMA-SEATS (see USCB (2017)) and TRAMO-SEATS (see Gómez and Maravall (1997)) to perform an automatic detection and correction of additive, transitory, level shift or seasonal outliers and of calendar effects such as the trading-day and Easter ones, before doing the decomposition of the series.

If  $y_t$  denotes the observed series (perhaps log-transformed), the programs fit a model of the type:

$$y_t = \mu + x_t' \beta + z_t \quad (6.11)$$

where  $\mu$  is a constant term,  $x_t' \beta$  is the regression term, and  $z_t$  follows an ARIMA model which is simultaneously identified by the programs. As far as trading-day effects are concerned, the regressors used in the model are those described in Section 6.2 and the effects are supposed constant on the complete span of the series.

Some studies have been done to incorporate stochastic trading-days in the regARIMA context:

- Quenneville et al. (1999), in order to measure the effect of the de-regulation on the daily pattern, introduce two sets of trading-day variables. The first set of regressors is defined over the whole series when the second is only defined over the de-regulated months. A deterministic Easter effect is also introduced in the model. At the end, the estimated model is a regARIMA model with both stochastic and deterministic effects and where the error term is supposed to follow a ARIMA model  $(0, 1, 1)(0, 1, 1)_{12}$ . One weakness of the model is that the trading-day component of the de-regulated regime requires the

extra six coefficients to be estimated from a quite small number of observations (23 months in the study) which leaves few degrees of freedom. In order to overcome this problem, the authors use an alternative parametrization of the trading-day coefficients, based on trigonometric functions of time at the daily frequencies  $2\pi j/7, j = 1, 2, 3$ . This specification is an adaptation of the well-known trigonometric specification of monthly seasonal patterns presented in Section 6.3.1.

- Zhang and Poskitt (2006) explore a way to utilize the regARIMA approach to capture an evolving trading-day effect rather than estimating evolving trading-day effect from the irregular component of the X-11 decomposition or an unobserved component in a structural state space model. They consider splitting the time-varying trading-day effect into constant and stochastic parts:  $\tau_t = \eta_t + \mu_t$  where  $\eta_t = \sum_{j=1}^{j=6} \beta_j T_{jt} = T_t' \beta$  is the constant component and  $\mu_t = T_t' \gamma_t$ ;  $\gamma_t = \gamma_{t-1} + \kappa_t$  is the stochastic component.

They propose to estimate the constant and stochastic components of the trading-day effect sequentially in two steps.

- Step 1: Estimate the constant trading-day effect using the regARIMA framework. The model residuals ( $e_t$ ) are supposed to still contain the evolving trading-day effect.
- Step 2: Estimate the random coefficient regression model for the stochastic trading-day component from the model residuals ( $e_t$ ) using a Kalman filter method to estimate the random coefficient with the following state-space representation:  $e_t = T_t' \gamma_t + \epsilon_t$  and  $\gamma_t = \gamma_{t-1} + \kappa_t$ , where  $\epsilon_t$  follows a NID( $0, \sigma_\epsilon^2$ ) and  $\kappa_t$  follows a NID( $0, \sigma_\kappa^2 W$ ). There are many different specifications for the covariance structure for state variable vector. Bell and Martin (2004) provide a comprehensive review of the different structures. Due to the constraints between the 7 day type effects, the authors use the covariance structure proposed by Harvey (1989) and specified as:

$$W = \begin{pmatrix} 6/7 & -1/7 & \cdots & -1/7 \\ -1/7 & 6/7 & \cdots & -1/7 \\ \vdots & \vdots & \ddots & \vdots \\ -1/7 & -1/7 & \cdots & 6/7 \end{pmatrix}_{6 \times 6}.$$

- In recent versions of TRAMO-SEATS, Maravall and Pérez (2011) add a stochastic trading-day component. The trading-day adjustment is done in two steps:
  - In a first step a deterministic trading-day effect is removed through regression;
  - In a second step, when the resulting series still exhibits a spectral peak at the trading-day frequency, the peak is removed through the estimation of an ARMA(2,2) component. In this case, the AR part contains the roots close to the trading-day frequency, and the MA part is obtained from the model decomposition. In this approach, the stochastic trading-day effect is captured with a single random variable with different deterministic monthly means.

### 6.3.4 RegComponents Models

The RegComponent approach proposed by Bell (2004) bases seasonal adjustment on a dynamic regression model with errors that follow an unobserved components (UC-)ARIMA model. It is implemented in a computer program called REGCOMPNT developed by the US Census Bureau, see Bell (2011). The approach encompasses both the structural approach presented in Section 6.3.1 and the regARIMA approach used in X-13ARIMA-SEATS and TRAMO-SEATS in particular to estimate the trading-day effect.

The general specification of RegComponent models is

$$y_t = x_t' \delta + \sum_{i=1}^m h_{it} z_{it},$$

with  $y_t$  denoting an univariate time series observed at times  $t = 1, \dots, n$  (or a transformation thereof);  $x_t$  is an  $r \times 1$  vector of deterministic regression variables,  $\delta$  is the corresponding vector of coefficients;  $h_{it}$ ,  $i = 1, \dots, m$  are scale factors and  $z_{it}$ ,  $i = 1, \dots, m$ , are independent unobserved components with ARIMA representation.

The components  $\mu_{it}$  are formulated as  $\phi_i(L)\Delta_i(L)\mu_{it} = \theta_i(L)\xi_t$ , where  $\phi_i(L)$  is a stationary AR polynomial,  $\Delta_i(L)$  is a lag polynomial featuring all roots on the unit circle,  $\theta_i(L)$  is an invertible moving average polynomial, and finally,  $\xi_t$  is white noise.

Among the deterministic regressors the REGCOMPNT program makes available:

- Trading-day Variables for modeling trading-day effects in flow or stock series, as well as for modeling length-of-month (or quarter) effects or leap-year effects.
- Holiday Variables for modeling Easter, Labor Day, or Thanksgiving effects.
- Outliers and interventions. Variables for modeling additive outliers, level shifts, and ramp effects.

Bell (2004), Bell and Martin (2004) and Bell (2011) let in their examples the trading-day coefficients follow random walk models.

Bell (2004) and Bell (2011) discusses the statistical treatment in detail. This rests on the state space representation of the models which opens the way to their statistical treatment via the Kalman filter and related smoothing algorithms. Estimation is carried out by maximum likelihood via the prediction error decomposition and smoothing will enable diagnosing outliers and structural change.

Although the REGCOMPNT program is a very flexible tool for the analysis of seasonal time series, automatic handling of outliers is not yet available.

## 6.4 Improving the basic model for time-varying trading-day coefficients

Previous examples demonstrated that models already exist to take into account time-varying trading-day effects. Unfortunately, these models often require a precise and manual tuning, and efficient algorithms to be estimated. This is therefore still a challenge to integrate them in seasonal adjustment packages used for mass production which performs an automatic detection and correction of outliers, trading-day and moving holiday effects as well as the decomposition in trend-cycle, seasonal component and irregular for hundreds of series in the same run.

In the following, after looking at what already exists in recommended software to check for the presence of moving trading-day effects, we focus on two ideas:

- to incorporate stochastic trading-day effects in regARIMA models, a class of models already used in X-13ARIMA-SEATS and TRAMO-SEATS;
- and to implement a semi-parametric method based on “rolling windows” that estimates moving effects using usual regARIMA models on subspans of the series.

Both strategies have been implemented in JDemetra+ whose regARIMA module is natively expressed in State Space form.

### 6.4.1 The X-13ARIMA-SEATS “change-of-regime”, “history” and “slidingspans” specifications

X-13ARIMA-SEATS already provides the user with several possibilities to check for moving trading-day effects: the “change-of-regime” specification, the “history” specification and the “slidingspans” specification.

“Change-of-regime” regression variables allow checking if the seasonal or trading-day behavior of the series being modeled change at a specific date. The date specified for the change of regime divides the series into two spans, an early span containing the data for times prior to the change date and a late span containing the data from on and after this date. Fixed trading-day effects are then estimated on the two spans. A usual output of this change of regime is shown in Figure 6.1. In this example, the series shows no significant trading-day effect before 1990. From January 1990 onwards, Sunday for example has a clear negative effect when Friday and Saturday have a positive effect.

**Figure 6.1:** An example of a “Change-of-regime” regression.

Regression Model			
Variable	Parameter Estimate	Standard Error	t-value
<b>Trading Day (after 1990.Jan)</b>			
Mon	-0.3896	0.17535	-2.22
Tue	0.4149	0.17616	2.36
Wed	0.0911	0.17713	0.51
Thu	0.0025	0.17535	0.01
Fri	0.6747	0.17529	3.85
Sat	0.9055	0.17303	5.23
*Sun (derived)	-1.6991	0.17576	-9.67
<b>&amp;&amp;Trading Day (change for before 1990.Jan)</b>			
Mon I	0.0134	0.27093	0.05
Tue I	-0.3483	0.27153	-1.28
Wed I	0.0852	0.27175	0.31
Thu I	0.3141	0.26742	1.17
Fri I	-0.0353	0.27087	-0.13
Sat I	-0.1543	0.27180	-0.57
*Sun I (derived)	0.1252	0.27137	0.46
Leap Year	2.2634	0.40489	5.59
Easter[8]	0.9349	0.27089	3.45

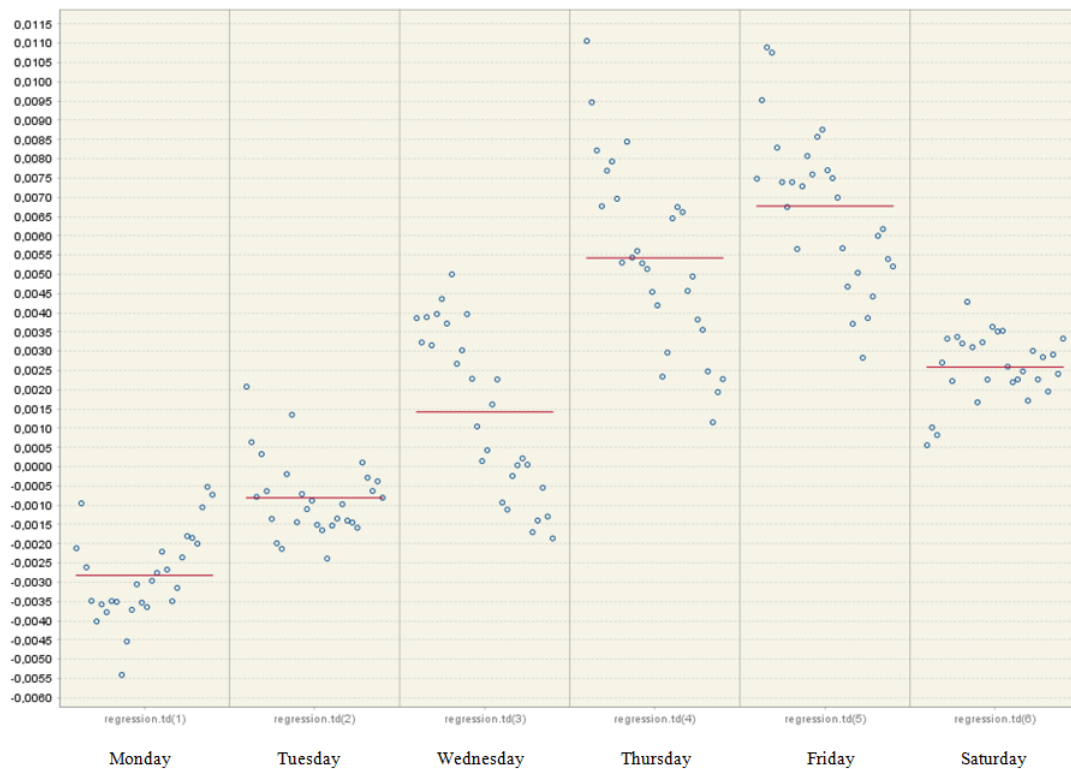
The “history” and “slidingspans” specifications allow testing the stability of the regARIMA model and of other features of the seasonal adjustment by performing the adjustment on a sequence of truncated versions of the time series, adding each time a new point for example (“history”) or using overlapping spans (“slidingspans”). Figure 6.2 shows that the trading-day pattern of the analyzed series is strongly moving across time: the weights of Wednesday, Thursday and Friday are decreasing and, by difference, the weight of Sunday strongly increases.<sup>4</sup>

### 6.4.2 Using rolling Windows to estimate moving trading-day effects

The basic idea, which is very simple, is a direct extension of the X-13ARIMA-SEATS “slidingspans” specification. It can therefore easily be integrated in both TRAMO-SEATS or X-13ARIMA-SEATS. In fact, this strategy

<sup>4</sup>This stability analysis is an output from JDemetra+.

Figure 6.2: Evolution of the trading-day coefficients.



has already been implemented in JDemetra+ in its diagnostics to check the stability of the seasonal adjustment. This is also a similar idea which is used by the Australian Bureau of Statistics in its SEASABS adaptation of X-13ARIMA-SEATS.

Let us suppose for example a monthly time series with  $N$  observations and a span of length  $n$ .

- The estimation is first done on the complete time series to obtain the ARIMA model followed by the series, the decomposition model, the outliers and the estimation of the fixed trading day effect and other calendar effects like the Easter effect;
- The estimation is then done on the first  $n$  observations,  $t = 1, \dots, n$ , using or not the same ARIMA model and the previously detected outliers;
- You add the next observation to your span (observation  $n + 1$ ), remove the first one and estimate the trading-day effect on this new series of  $n$  observations, using or not the same ARIMA model and the previously detected outliers;
- You do it iteratively and get at the end  $N - n + 1$  estimations of the trading day coefficients.

Of course, it is possible to use any trading-day regressors, for example to take into account the specificities of the national calendar.

If the principle of the “rolling window” estimation is simple, some parameters have still to be specified. For examples:

- The length  $n$  of the window which should allow for a good estimation of the trading-day effect. For example, if an estimation of the leap year effect is required,  $n$  should be large enough to observe 3 leap years in each span;
- The date at which the trading-day estimate will be affected. It can be the last date or the middle of the period. In any case, a strategy to estimate the missing observations has to be precised.



- The status of the ARIMA model. Moving trading-days are usually observed on long time series for which it could be difficult to find a stable model.

### 6.4.3 RegARIMA models with time-varying trading-day coefficients

As seen in Section 6.2, the trading-day effect can be expressed as:

$$TD_t = \sum_{i=1}^6 (\alpha_i - \bar{\alpha})(D_{it} - D_{7t}) = \sum_{i=1}^6 \beta_i T_{it}$$

Stochastic time varying trading-day effects can be modeled assuming for example that the coefficients  $\alpha_i$  or  $\beta_i$  follow a multivariate random walk, which gives 2 models:

- Model I (Bell)  $\beta_{t+1} = \beta_t + \varepsilon_t$  where the covariance of the innovations is diagonal. Usually, we will use a single parameter (same variance for all the coefficients).

In this model, the contrast day plays a special role. More specifically, the innovation of its coefficient is negatively correlated with all the other innovations and it has a larger variance (the sum of all the other variances).

- Model II (Harvey)  $\alpha_{t+1} = \alpha_t + \varepsilon_t$  where the covariance of the innovations is diagonal.

Using  $\beta_{it} = \alpha_{it} - \bar{\alpha}_t$  we can easily derive the covariance of  $\Delta\beta$ . In the case of a single parameter (same variance for all coefficients, that we set to 1 to simplify), we have:

$$\text{Cov}(\Delta\beta) = \begin{pmatrix} 6/7 & -1/7 & \dots & -1/7 \\ -1/7 & 6/7 & \dots & -1/7 \\ \vdots & \vdots & \ddots & \vdots \\ -1/7 & -1/7 & \dots & 6/7 \end{pmatrix}_{6 \times 6}.$$

The generalization to different variances for each day is immediate.

In this model, all the days play the same role, which seems more attractive.

- Other generalizations

This approach can easily be extended to other definitions of the trading days effects, obtained by putting the days in different groups and by defining the contrasts variables accordingly (see for instance the usual working days).

- State space form

A linear regression component with time varying coefficients can be added in a straightforward way to any state space model. Using the usual notations (see 6.3.1), we have:

$$\text{State: } \tilde{\alpha}_t = \begin{pmatrix} \alpha_t \\ \beta_t \end{pmatrix}$$

$$\text{Measurement: } \tilde{Z}_t = (Z_t \quad C_t), \tilde{H}_t = H_t$$

$$\text{Transition: } \tilde{T}_t = \begin{pmatrix} T_t & 0 \\ 0 & I \end{pmatrix}, \tilde{V}_t = \begin{pmatrix} V_t & 0 \\ 0 & V_\beta \end{pmatrix}$$

$$\text{Initialization: } \tilde{\alpha}_0 = \begin{pmatrix} \alpha_0 \\ 0 \end{pmatrix}, \tilde{P}_* = \begin{pmatrix} P_* & 0 \\ 0 & 0 \end{pmatrix}, \tilde{P}_\infty = \begin{pmatrix} P_\infty & 0 \\ 0 & I \end{pmatrix}$$

The model can be estimated using the usual algorithms (prediction error decomposition by means of the diffuse Kalman filter and smoothing of the results by means of the Kalman smoother).

## 6.5 Examples

### 6.5.1 Methodology

In order to have enough observations to perform a relevant analysis, we will use a long time series: the Australian monthly retail trade from April 1982 to August 2017, 425 observations.

In the estimations, we use:

- The 6 trading-day contrast regressors calculated on the default Gregorian calendar, i.e without taking into account the specificities of the national calendar;
- The X-13ARIMA-SEATS implementation of the JDemetra+ program;
- A 10-year running window to compute the semi-parametric estimate. The “rolling window” effect is also smoothed using the loess smoother;
- The Harvey’s covariance structure and an airline model for the regARIMA estimation.

We also estimate a time-varying TD effect from a basic structural model, using PROC UCM in the SAS system.

The results are presented in graphs showing the evolution of the various estimates for the various day effects.

- The horizontal black line is the fixed effect; the dotted black lines are the confidence limits of this fixed effect;
- The green line shows the moving effect estimated with the basic structural model described in section 6.3.1;
- The orange line is the “rolling window” effect described in section 6.4.2. The red line is the smoothed “rolling window” effect;
- The blue line is the stochastic regARIMA estimate described in section 6.4.3.

### 6.5.2 Results

Fixed daily effects in the Australian retail trade series are presented in Figure 6.3, and the various moving daily effects in Figure 6.4.

From the statistical point of view, it can be noted that:

- The 3 different approaches (BSM, regARIMA and rolling windows) tell roughly the same story;
- The direct estimates of the “rolling window” approach are very unstable and require some smoothing. This can be done as here using a specific smoother or by increasing the number of points to take in the window;
- It seems that for several days (Monday, Tuesday, Friday and Saturday), the hypothesis of a constant effect in time is correct. Statistical tests to choose between the 2 hypothesis (fixed and moving) have been proposed, for example by Zhang and Poskitt (2006) but still have to be implemented and evaluated.

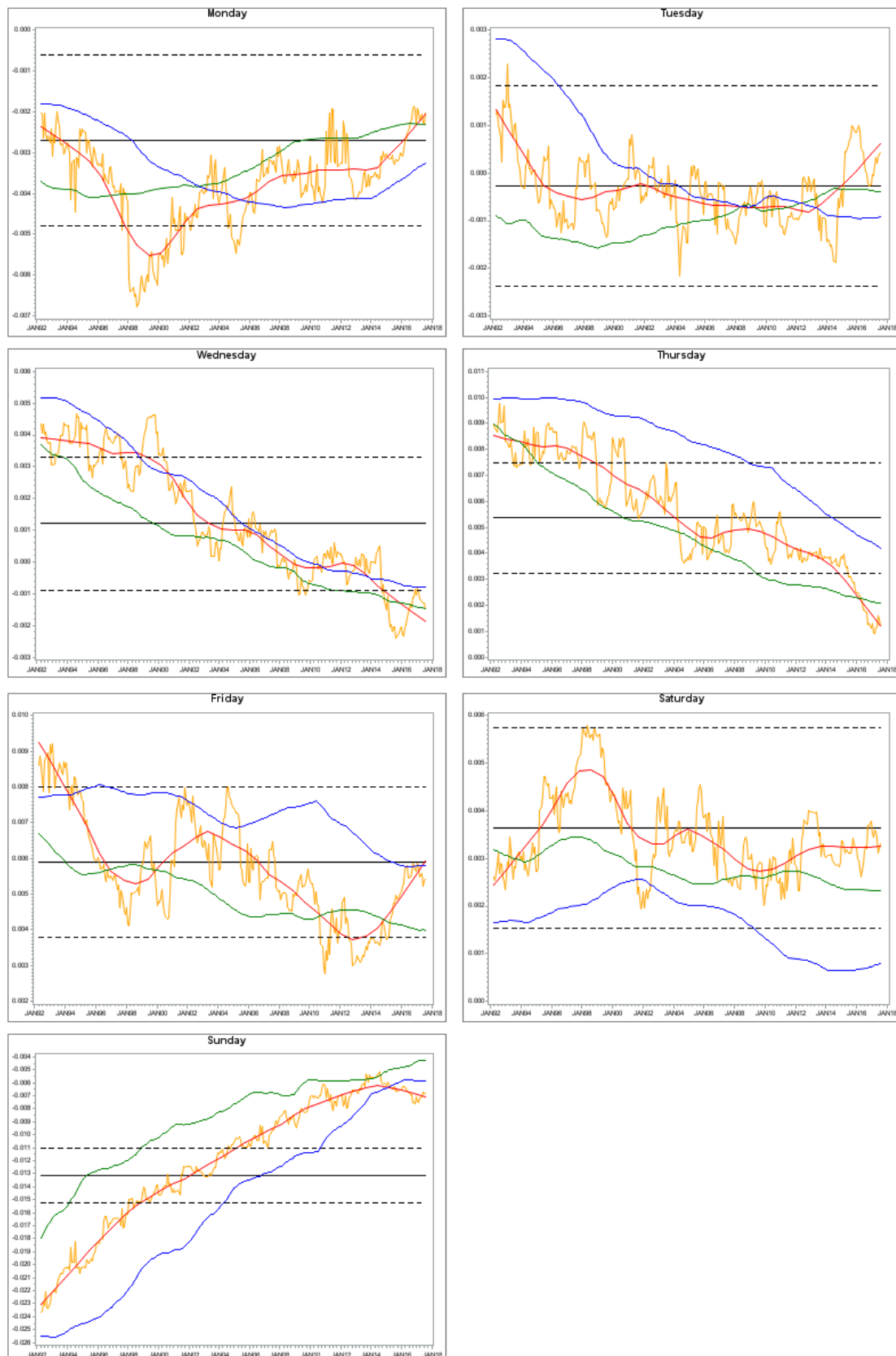
According to the fixed daily effects, Thursday, Friday and Saturday have a clear positive effect on the sales. On the opposite, Sunday shows a strong negative effect. But the results presented in Figure 6.4 tell a more informative story:

- As more shops are opened on Sunday, the negative impact of this day on the sales was much more important at the beginning of the estimated period (-0.025 for the regARIMA estimation) than at the end (-0.006 for the regARIMA estimation);
- At the same time, it seems that less people do their shopping on Wednesday and Thursday.
- Simultaneously, Wednesday which had a positive impact at the beginning of the period might soon present a negative impact on the sales.

**Figure 6.3:** Fixed daily effects in the Australian monthly retail trade series.

Regression Model			
Variable	Parameter Estimate	Standard Error	t-value
Trading Day			
Mon	-0.0031	0.00109	-2.81
Tue	0.0003	0.00109	0.24
Wed	0.0010	0.00109	0.92
Thu	0.0054	0.00111	4.91
Fri	0.0060	0.00109	5.51
Sat	0.0035	0.00110	3.15
*Sun (derived)	-0.0131	0.00110	-11.90
Easter[8]	0.0114	0.00232	4.93
Automatically Identified outliers			
A01986.Mar	-0.0466	0.01001	-4.65
Ls2000.Jun	0.0688	0.01129	6.09
Ls2000.Jul	-0.0840	0.01127	-7.45

**Figure 6.4:** Moving daily effects in the Australian monthly retail trade series. The horizontal black line is the fixed estimate; The green line shows the BSM estimate; The orange line is the raw “rolling window” estimate, the red line is the smoothed “rolling window” estimate; The blue line is the stochastic regARIMA estimate.



## 6.6 Conclusions

This chapter describes several models to estimate time-varying trading-day effects: the basic structural model, the regARIMA approach and a semi-parametric approach based on rolling windows. All these approaches, including the semi-parametric one that also uses regARIMA models, can be put in a state space form; they could therefore be easily implemented in recommended seasonal adjustment software like X-13ARIMA-SEATS and TRAMO-SEATS.

In some occasions, like for the adjustment of the Australian retail trade series, they appear very useful and informative. But there is still a lot of work to use them on a regular basis and in production.

- These techniques should be compared and evaluated as regards their impact on the seasonally and calendar adjusted series to be finally published;
- Statistical tests have still to be proposed and evaluated to choose between fixed and moving effects;
- As we saw in the proposed example, the days of the week might require different modeling which will make the adjustment also more complex.

To conclude, it has to be stressed that calendar effects are usually a small component of a time series and it is sometimes difficult to get a good and stable estimate of these effects. It is therefore good to keep in mind the following principle highlighted in the ESS guidelines for Seasonal Adjustment: *“To avoid misleading results, seasonal adjustment should be applied only when seasonal and/or calendar effects can be properly explained, identified and estimated.”*

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# 7

## Outlier Detection and Correction



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## Table of Contents

7.1	The role of outlier modelling in seasonal adjustment	139
7.2	Aims of outlier modelling within the framework of seasonal adjustment	139
7.2.1	Elimination of factors that bias the estimation of the seasonal component	139
7.2.2	Adjustment for calendar effects with the aid of regARIMA modelling	140
7.2.3	Linearisation of the time series and Gaussian regARIMA residuals	140
7.2.4	Forecast of seasonal and calendar components	140
7.2.5	Estimation of the trend-cycle as a secondary aim	141
7.3	The different types of outliers	141
7.3.1	Overview of types of outliers	141
7.3.2	Outliers in the irregular component	142
7.3.3	Outliers in the trend-cycle	144
7.3.4	Outliers in the seasonal component	155
7.3.5	Two sides of the same coin: minimisation of false positives vs. false negatives	156
7.4	Estimation techniques for outlier modelling in the context of regARIMA models	158
7.5	Practical considerations as regards the detection and correction of outliers	161
7.5.1	Revisions of seasonally adjusted data	161
7.5.2	Treatment of (possible) outliers at the current end of the time series	162
7.6	Special issues in the treatment of outliers in seasonal adjustment	163
7.6.1	Extreme value detection in X-11	163
7.7	Conclusions	166

## 7.1 The role of outlier modelling in seasonal adjustment

In general, there exists broad consensus that abrupt, strong and atypical changes of a time series have to be handled separately. But, how can such observations be treated adequately in the context of seasonal adjustment? The answer to this question is the content of this chapter.

First, it has to be understood that outlier detection and correction pursue multiple aims. They are of particular necessity for preventing the estimation of seasonal and calendar effects from being biased. Moreover, linearisation of the time series is important for identifying the correct model. Last but not least, good forecasts rely on consistent parameter estimates. The outline of the chapter is as follows. Section 7.2 gives an introduction to the aims of outlier modelling. The standard types of outliers are presented in great detail in Section 7.3. Section 7.4 covers the technical aspects of outlier identification. Practical considerations are to be found in Section 7.5. Special issues are dealt with in Section 7.6. The final section concludes.

## 7.2 Aims of outlier modelling within the framework of seasonal adjustment

### 7.2.1 Elimination of factors that bias the estimation of the seasonal component

What is the ultimate goal of seasonal adjustment? According to item 0 of the ESS Guidelines on Seasonal Adjustment, the purpose of seasonal adjustment is to filter out the usual seasonal fluctuations, i.e. those movements that recur with similar intensity in the same season each year, from the unadjusted time series.<sup>1</sup>

To achieve this goal, it is important that atypical observations (outliers) do not distort the estimate of the seasonal component. Both X-12-ARIMA and TRAMO/SEATS take this into consideration and use a two-step approach for the treatment of outliers and the estimation of the seasonal component. In the first step, outliers are identified and their estimated impact is removed from the unadjusted time series (in X-12-ARIMA this happens in the regression part while in TRAMO/SEATS this task is performed by the TRAMO program). Based on outlier-adjusted data from the first step, the seasonal component is estimated in the second step (in the X-11 part and the SEATS program, respectively). Therefore, unique atypical effects do not become part of the seasonal component. After removing the usual seasonal fluctuations from unadjusted data, unusual movements continue to be visible in seasonally adjusted data. This is in line with the definition of what the latter data should represent: the “news” in the time series, i.e. changes of the trend-cycle and irregular component. From this perspective, it is hence necessary that these unusual movements are treated as outliers, and thus are not attributed to the seasonal component.

Most of the time series relevant for short-term business cycle analysis contain atypical observations which do not follow the usual pattern of the time series. These outliers can best be dealt with if their sources can be identified. Examples are the consequences of economic policy, large-scale orders, strikes, exceptional seasonal influences such as extreme weather conditions or atypical holiday constellations.

There are a couple of explanations for why the results without outlier modelling are regularly well apart from those with outlier modelling. Firstly, regARIMA estimates and forecasts are biased as regards the seasonality of the series if outliers are not modelled. This has a greater impact on TRAMO/SEATS than on X-12-ARIMA because the decomposition of SEATS is based on regARIMA modelling alone. In the X-12-ARIMA program, the seasonal filters are selected in its X-11 subroutine. Secondly, without outlier modelling, the effects of

<sup>1</sup> See Eurostat (2009), ESS Guidelines on Seasonal Adjustment, Luxembourg: Office for Official Publications of the European Communities.

an abrupt change of the time series are in part wrongly allocated to the seasonal component rather than completely to the trend-cycle or irregular component. That means they do not remain fully visible in seasonally adjusted data — especially at the end of the time series. This can put major difficulties on the analysis of the current economic development. Furthermore, even a year after that, problems may arise at the current end of the time series. This is because the estimate of the seasonal component has partially absorbed the past special one-off effect. Hence, influences are adjusted for which actually no longer exist. This results in a loss of the informative value of seasonally adjusted data.

### 7.2.2 Adjustment for calendar effects with the aid of regARIMA modelling

RegARIMA modelling contains a regression part of the (transformed) unadjusted time series and an ARIMA part for the error term. In addition to outlier treatment, the regression part allows for the estimation of calendar effects.<sup>2</sup> The calendar component is the result of the linear combination of the calendar coefficients and the respective regressor values.<sup>3</sup>

As this component contains no news on the underlying economic development either, it can be interpreted as seasonality in the broader sense. Eventually, outlier detection and correction prevents calendar component estimates from being distorted.

### 7.2.3 Linearisation of the time series and Gaussian regARIMA residuals

Non-linearities of the stochastic process used for approximation of the time series and non-Gaussian residuals from the regARIMA model often arise from the presence of outliers. In order to linearise the time series and arrive at Gaussian residuals it is of great importance to use exogenous regressors to model outliers. If outliers remain unidentified — and thus are not treated properly — it will not be possible to identify the correct regARIMA model. It is a necessary (though not sufficient) condition for identification of the model to have a linearised time series which can be described completely by its own past and not by other (exogenous) variables.<sup>4</sup>

Without outlier modelling the model which is finally chosen may be wrongly specified. This is due to a spurious autocorrelation structure introduced by outliers which in turn is then attributed to autoregressive or moving average parameters.<sup>5</sup>

### 7.2.4 Forecast of seasonal and calendar components

Ill-specified regARIMA models and inadequate parameter estimates result in biased forecasts of the time series and negatively influence the estimation of seasonal and calendar components. Therefore, at the current end of the time series, the quality of seasonal adjustment may be severely deteriorated. But since the most recent observations are in the focus of users of seasonally adjusted data, outlier modelling is a crucial issue.

<sup>2</sup>See Findley, D., Monsell, B., Bell, W., Otto, M., and Chen, B.-C. (1998), “New Capabilities and Methods of the X-12-ARIMA Adjustment Program”, *Journal of Business and Economic Statistics*, 16, 127-152. Even if the correct model can be identified, it is likely that parameter estimates are distorted in the presence of outliers.

<sup>3</sup>See Chapter 5 of this handbook for an in-detail exposition on the treatment of calendar effects.

<sup>4</sup>According to the Wold decomposition theorem, the non-deterministic part of every stationary time series can be expressed as an  $MA(\infty)$  process of which a finite-order  $ARMA(p,q)$  process is an approximation. See Wold, H. (1938), *A Study in the Analysis of Stationary Time Series*, Stockholm, Sweden: Almqvist & Wiksell.

<sup>5</sup>The effect can also be the other way round so that ARMA parameters cannot be identified any longer as these are masked by outliers. A more parsimonious parametrisation mainly with respect to the seasonal part, for instance, might be due to the inability of the misspecified model to take account of the seasonality correctly.

**Table 7.1:** Characteristics of different types of outliers

Type of outlier	Component	Durability of impact	Visible in SA data?	Frequency of use
Additive outlier	Irregular	Temporary	Yes	Common
Temporary change	Irregular	Temporary	Yes	Rare
Level shift	Trend-cycle	Permanent	Yes	Common
Ramp	Trend-cycle	Permanent	Yes	Rare
Temporary level shift	Trend-cycle	Temporary	Yes	Rare
Seasonal outlier	Seasonal	Permanent	No	Sometimes

## 7.2.5 Estimation of the trend-cycle as a secondary aim

For the seasonally adjusted time series it does not matter whether the effect of an outlier is assigned to the trend-cycle or irregular component as it consists of these two. Even less important is whether sudden changes in the level of the time series are modelled as temporary or permanent outliers, as a level shift or multiple additive outliers and so on. However, it plays a role that these effects are modelled in order to prevent distortion of the estimate of the seasonally adjusted time series. Apart from that, it makes a big difference which type of outlier is modelled for the attribution of the effect to the trend-cycle or irregular component. Level shifts (permanent and temporary) and ramps belong to the former component while additive outliers and temporary changes belong to the latter one.<sup>6</sup>

## 7.3 The different types of outliers

### 7.3.1 Overview of types of outliers

In the current versions of X-12-ARIMA and TRAMO/SEATS, the regARIMA modelling facility enables consideration of six types of outliers in the context of seasonal adjustment to deal with abrupt, strong and atypical temporary or permanent changes of the time series. On the one hand, additive outliers and — less commonly — temporary changes are frequently related to the irregular component of the time series. On the other hand, level shifts and — less commonly — ramps and temporary level shifts are generally associated with the trend-cycle. Seasonal outliers or “breaks” are neither attributed to the trend-cycle nor irregular component but to the seasonal component. Thus, while the first five types of outliers remain visible in the seasonally adjusted time series — which comprises the trend-cycle and irregular component — the last type is adjusted for. In the following table all six types of outliers are assigned to the unobserved component they relate to, it is stated whether the effect of the outlier is temporary or permanent, whether or not the effect remains visible in seasonally adjusted (SA) data and how likely it is to observe the outlier in practice.

In the following three parts of this section outliers in the irregular component, the trend-cycle and the seasonal component are discussed. For each type the formula of the regressor for an outlier occurring at date  $t_0$  is given.<sup>7</sup> For all outliers a plot of the associated regressor is provided along with a visualisation of the impact of the outlier on a stylised time series (see Table 7.1). Moreover, it is compared what the consequences on the unobserved components are of no outlier treatment as opposed to the correct treatment of outliers. In addition, real world examples are presented for the more common types of outliers: additive outliers, level shifts and seasonal outliers/breaks.<sup>8</sup>

A stylised time series is generated in order to display the full effects of the different outliers most clearly. The trend-cycle of this time series is equal to  $100 \exp(0.01t)$ ,  $t = 1, 2, \dots$ , while the seasonal component is

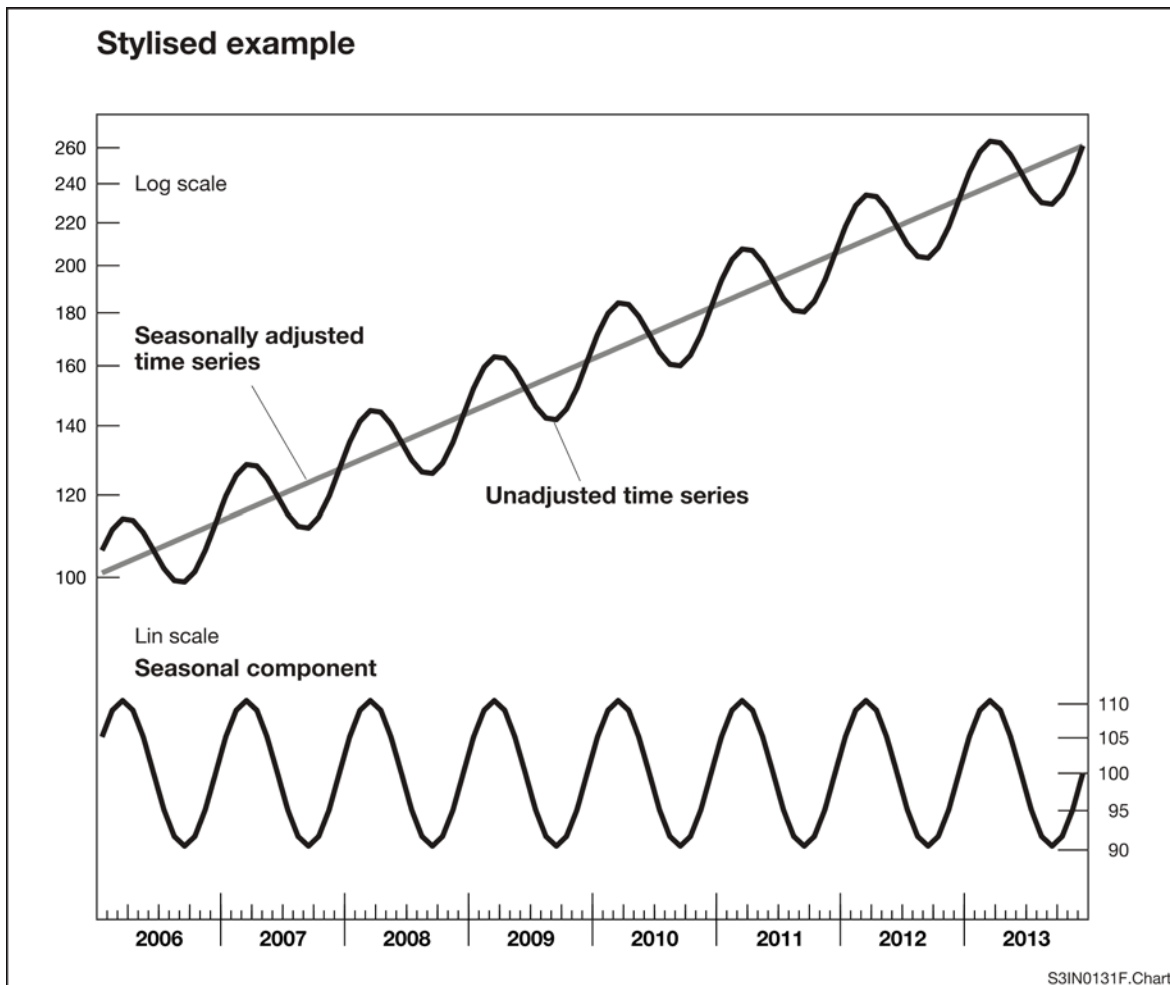
<sup>6</sup>These types of outliers are discussed in depth in the following section.

<sup>7</sup>See US Census Bureau (2009), X-13ARIMA-SEATS Reference Manual, Version 0.1 (Beta).

<sup>8</sup>A set of selected German time series is employed here. Therefore, the results are not generally valid for any other time series.



Figure 7.1: Stylised example



generated via  $\exp(0.1 \sin(\pi t/6))$ . For the sake of simplicity of the exposition, the time series does not consist of an irregular or a calendar component. The next figure shows the unadjusted and seasonally adjusted time series as well as the seasonal component.

In what follows the time series ( $y_t$ ) containing the outlier ( $x_t$ ) is constructed as  $y_t \exp(x_t)$ . For computational simplicity, seasonal adjustment is performed using the ratio to moving average method. Initially, the trend-cycle is removed by applying a centred  $2 \times 12$  moving average to the unadjusted time series. Thereafter, the seasonal component of each month is estimated as the normalised average<sup>9</sup> of the trend-cycle adjusted time series for that specific month.

### 7.3.2 Outliers in the irregular component

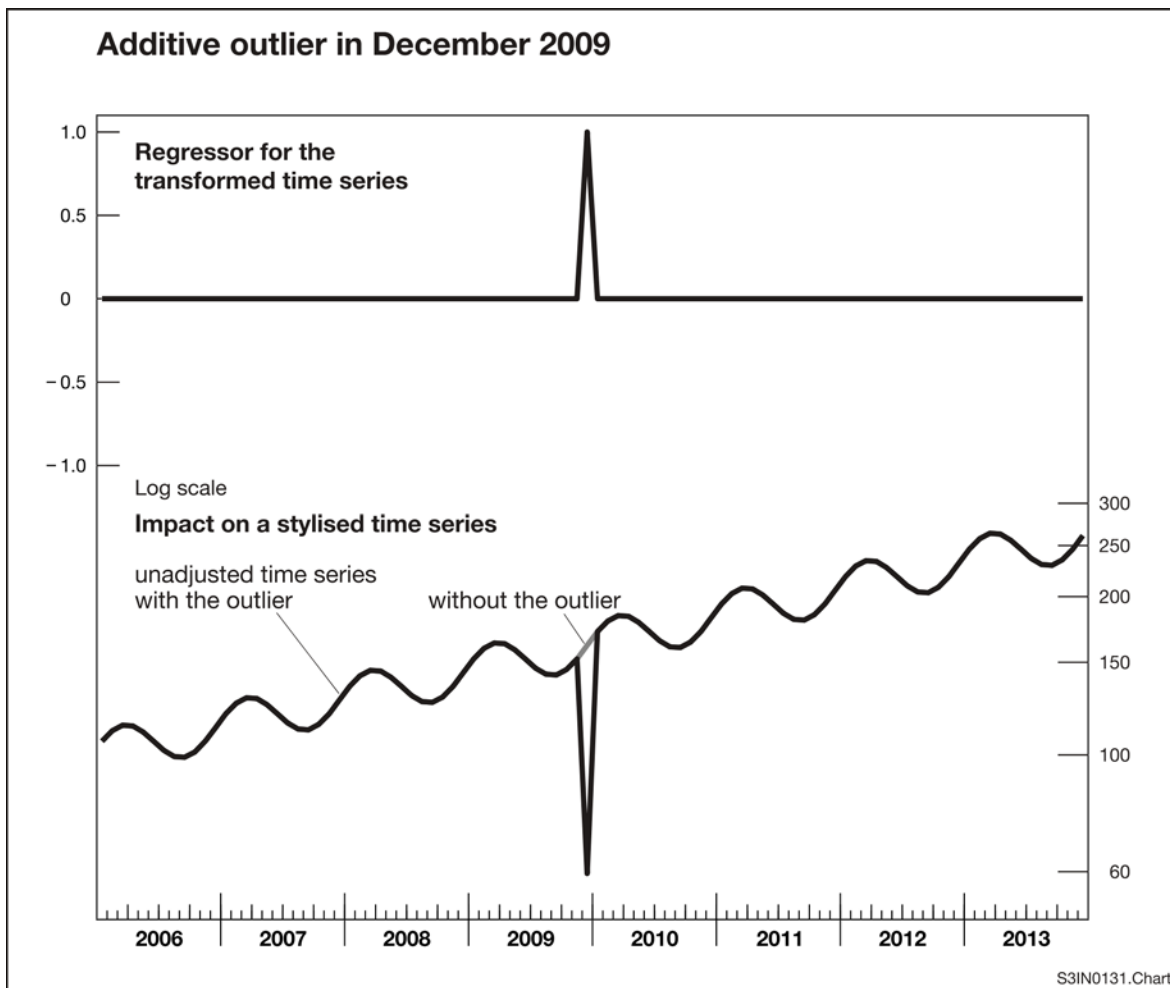
First, additive outliers ( $AO$ ) which represent a one-off peak or trough in the time series at a single observation are defined as:

$$AO_t = \begin{cases} 1 & \text{if } t = t_0 \\ 0 & \text{if } t \neq t_0 \end{cases} \quad (7.1)$$

<sup>9</sup>On the issue of normalisation see the explanation on p. 16 in Ladiray, D., and Quenneville, B. (2001), Seasonal Adjustment with the X-11 Method, Lecture Notes in Statistics, 158, New York, NY: Springer.



Figure 7.2: Additive outlier in December 2009



The occurrence of large orders is one of many possible examples for a time series which can contain additive outliers. The example chosen here is that of German orders received for capital goods from abroad. In June 2007, Airbus received 425 firm orders from 19 customers at the Paris Air Show, worth several billion euro at its German subsidiaries.<sup>10</sup>

Thus, the component manufacture of aircraft led to the unusual peak in the time series.

That outlier adjustment makes an important difference in the decomposition of the unadjusted time series into its unobserved components can be seen from the figure below. If the outlier is not taken into account, it is partially attributed to the seasonal component for December. This results in hiding the full effect of the outlier in the seasonally adjusted time series in December 2009 and spurious seasonal adjustment of December values in all other years.

Second, temporary changes ( $TC$ ) where the change in the level of the time series is not of permanent but temporary nature, i.e. the effect decreases exponentially ( $0 < \alpha < 1$ ) with the course of time, have the following representation:

$$TC_t = \begin{cases} 0 & \text{if } t < t_0 \\ \alpha^{t-t_0} & \text{if } t \geq t_0 \end{cases} \quad (7.2)$$

<sup>10</sup>See Airbus press release, "Renewed momentum for Airbus' leading products, ends Paris Air Show with 425 firm orders", 22 June 2007.

**Figure 7.3:** Orders received in Germany from abroad, capital goods

It is somewhat difficult to find an appropriate real world example for a temporary change. The number of new vehicle registrations after a VAT increase is a candidate. But the rather strong assumption of an exponential decay associated with the temporary change regressor makes it hard to empirically observe such an effect.

The effect of erroneous seasonal adjustment of a time series with a temporary change is similar to that of a time series with an additive outlier. However, the estimate of the seasonal component is affected not only just in December. Yet, the inaccuracy is smaller in December compared to the additive outlier effect. The error in the trend-cycle estimate is larger in size and prolonged in duration to that of an additive outlier.

### 7.3.3 Outliers in the trend-cycle

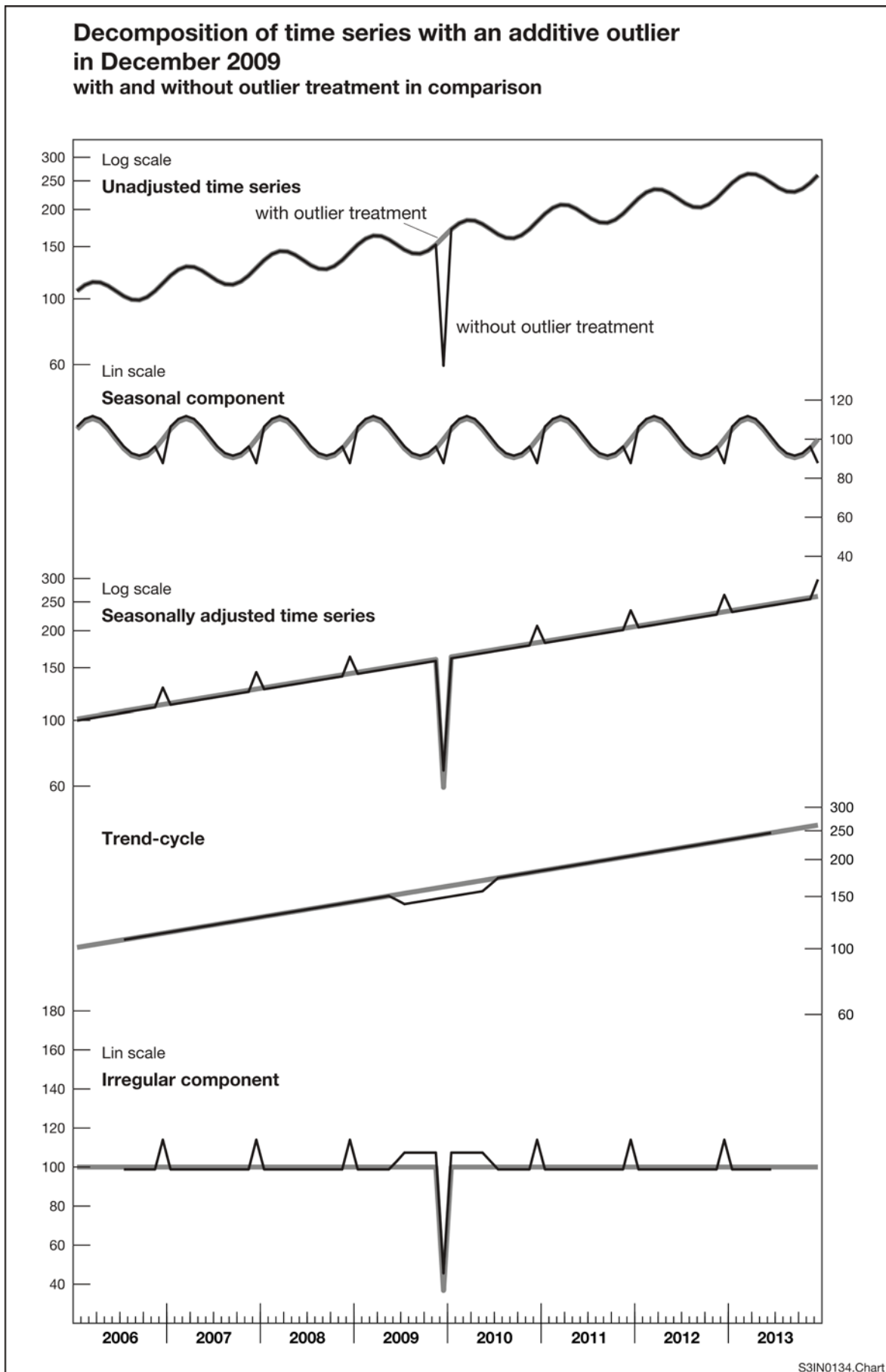
Third, level shifts ( $LS$ ) that permanently increase or decrease the (transformed) data by some constant factor prior to a certain observation can be written as:

$$LS_t = \begin{cases} -1 & \text{if } t < t_0 \\ 0 & \text{if } t \geq t_0 \end{cases} \quad (7.3)$$

The introduction of new NACE Rev. 1 led to a permanent shift in the level of the time series of output in Germany in the manufacture of consumer electronics after its introduction in 1995.

An unidentified level shift in seasonal adjustment leads to a misinterpretation of the true seasonal pattern of the underlying time series. If a downward level shift occurs in December, the seasonal component for this month and the first half of the year is underestimated, while being overestimated for the rest of the year. At first, seasonal adjustment assigns too much of the outlier effect to the trend-cycle estimate and then too little, i.e. the change in the trend-cycle is too smooth, and therefore not as abrupt as it should be.

Figure 7.4: Decomposition of time series with an additive outlier in December 2009



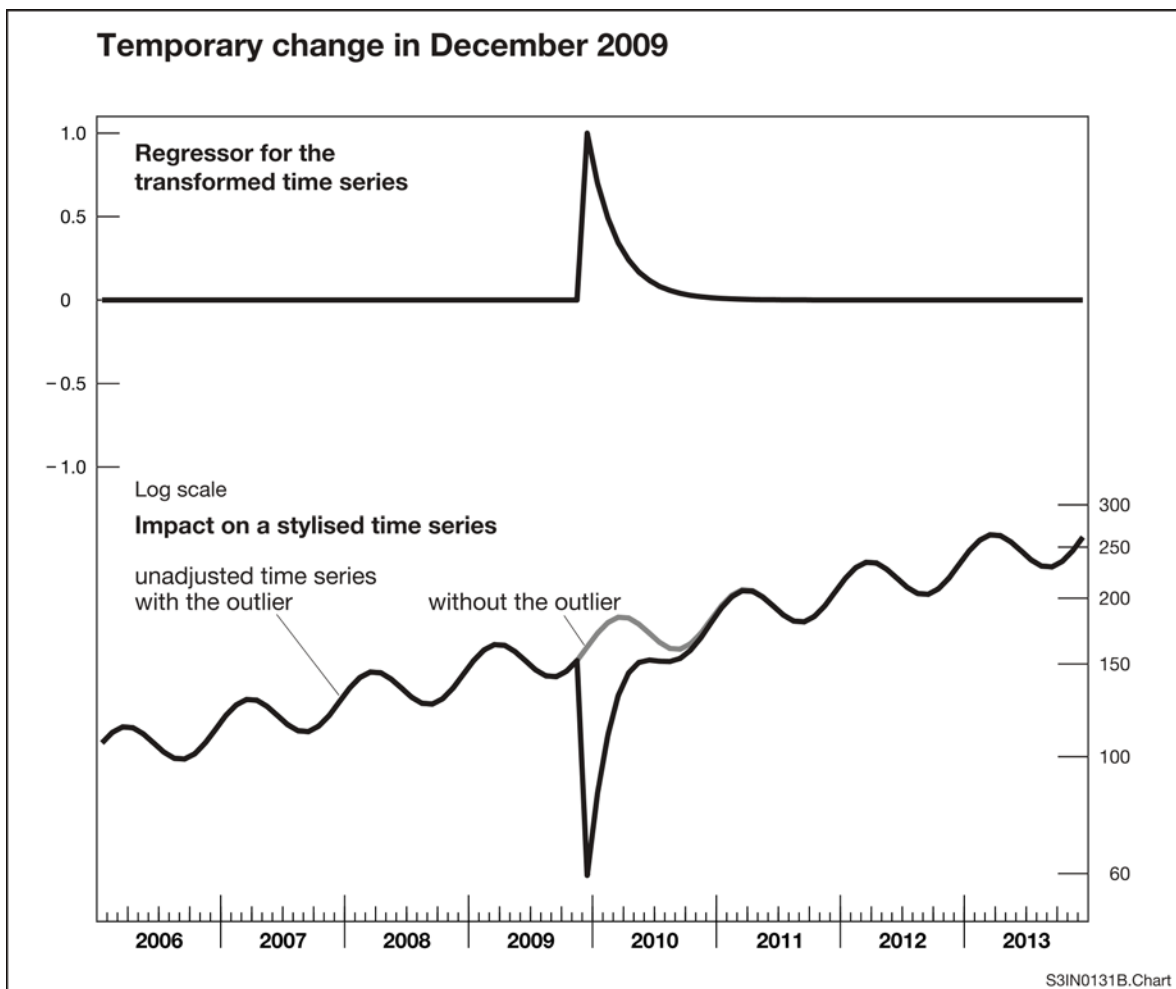
**Figure 7.5:** Temporary change in December 2009

Figure 7.6: Decomposition of time series with a temporary change in December 2009 with and without outlier treatment in comparison

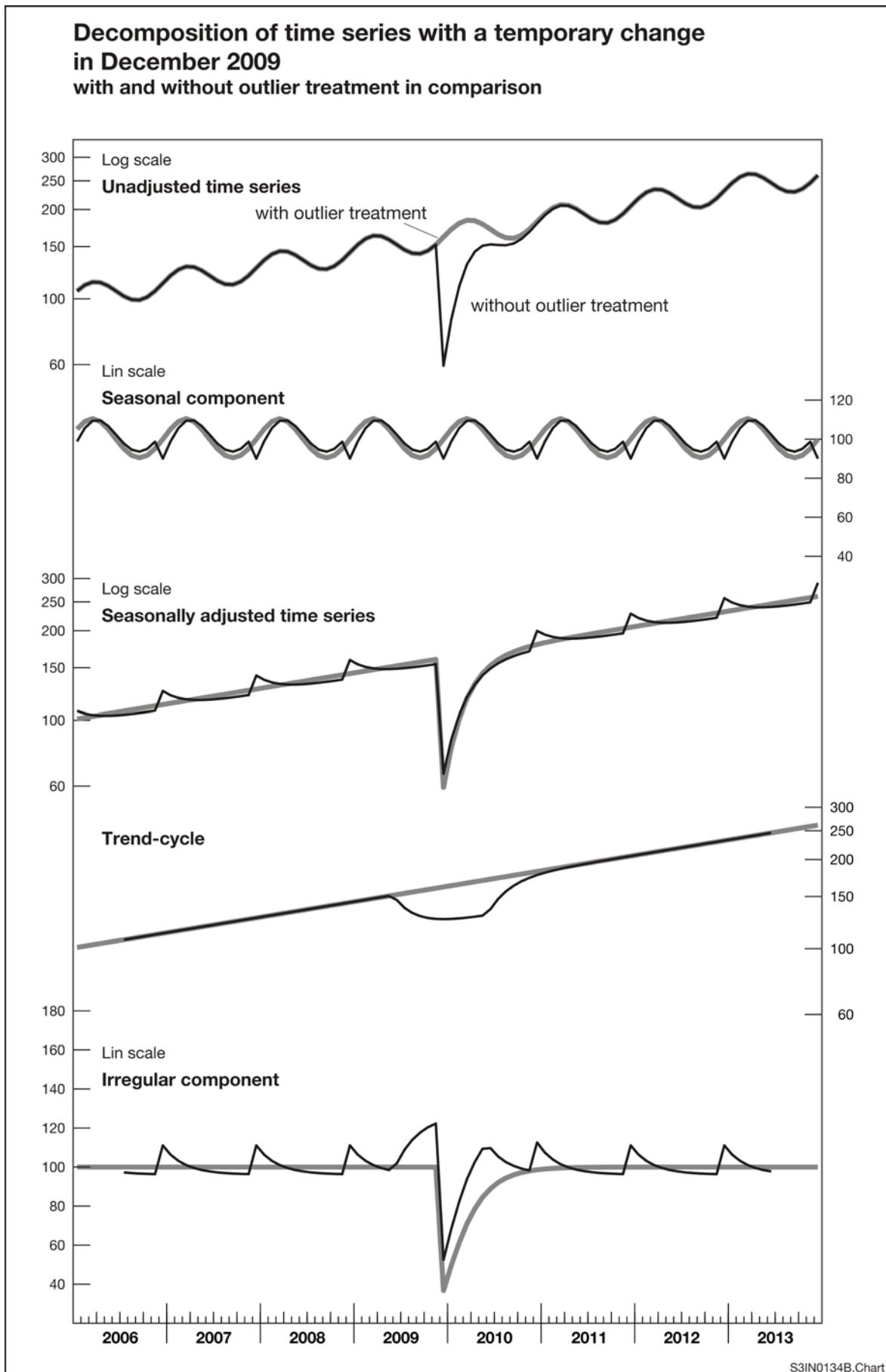


Figure 7.7: Level shift in December 2009

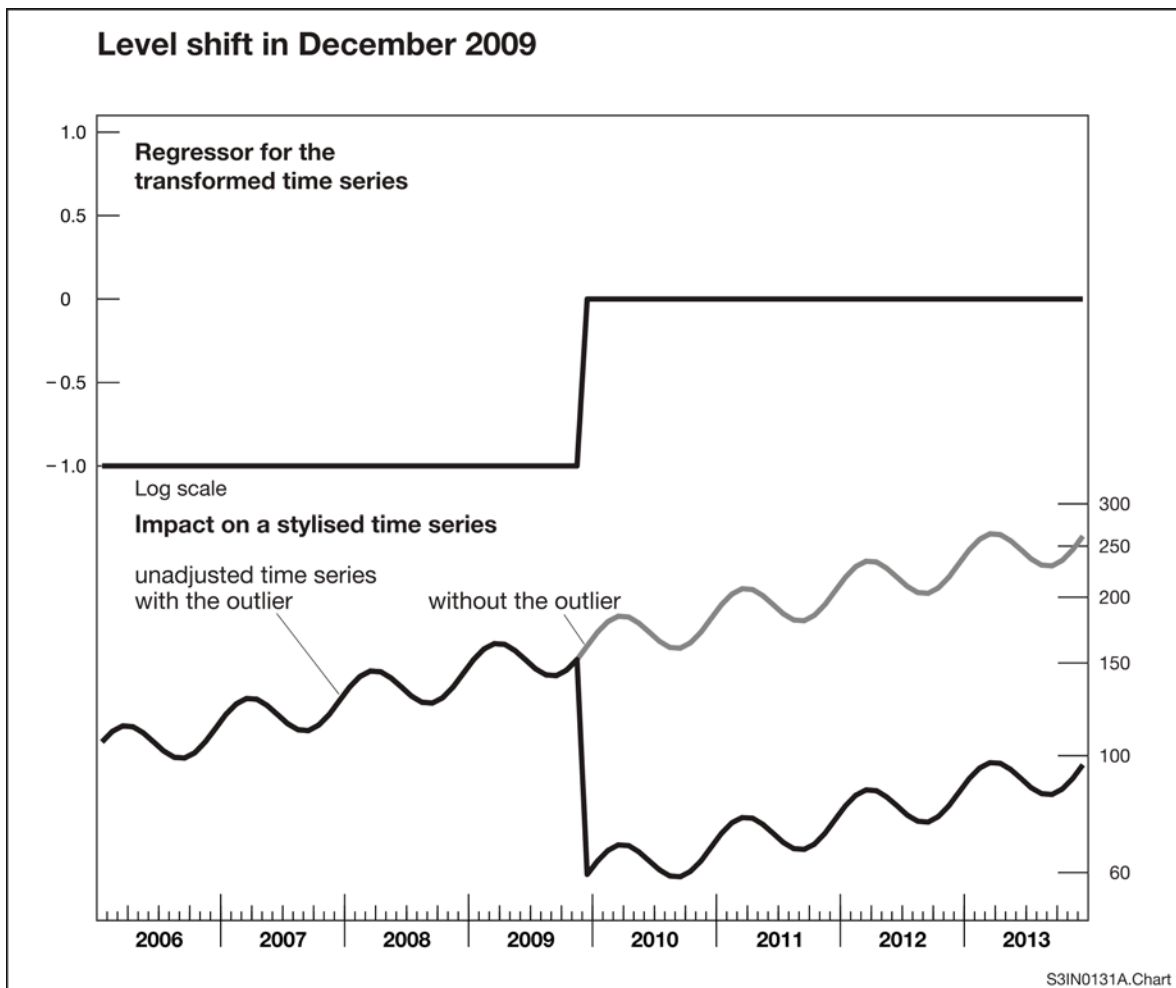
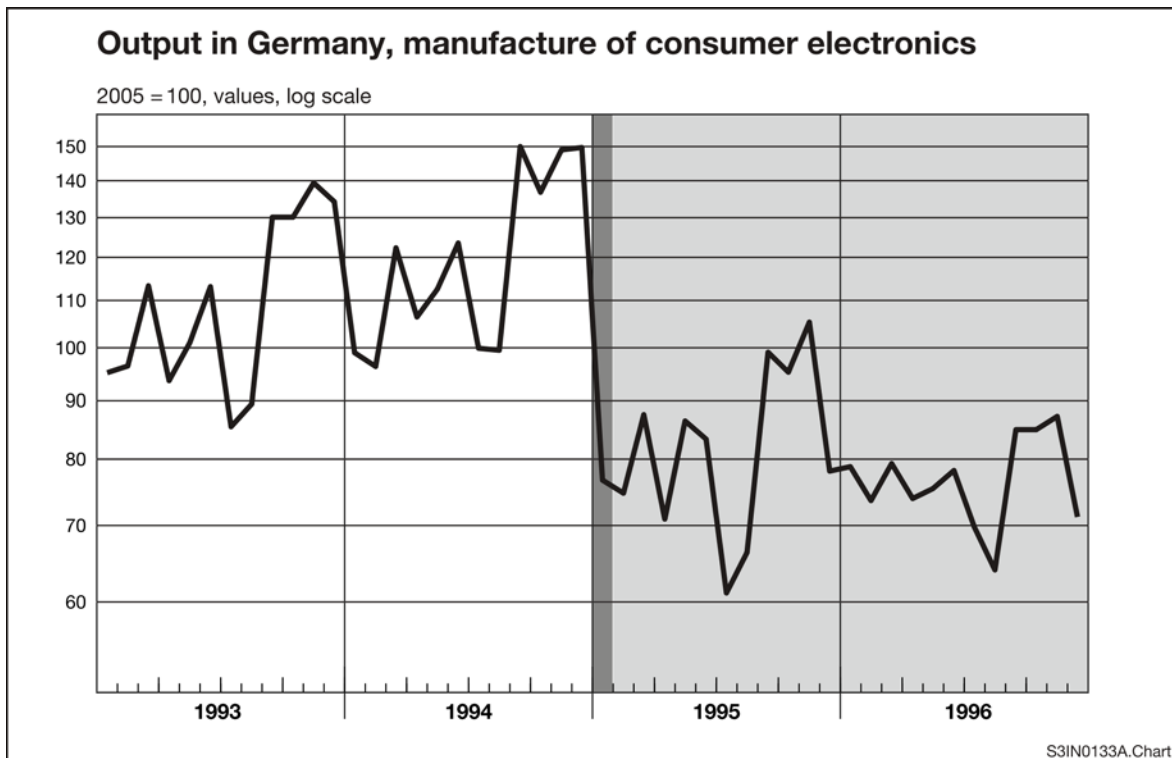


Figure 7.8: Output Germany , manufacture of consumer electronics



Fourth, ramps ( $RP$ ) that allow for a smooth, linear transition between two time points ( $t_0$  is the start date and  $t_1$  the end date) unlike the abrupt change associated with level shifts<sup>11</sup> are given by:

$$RP_t = \begin{cases} -1 & \text{if } t \leq t_0 \\ (t - t_0)/(t_1 - t_0) - 1 & \text{if } t_0 < t < t_1 \\ 0 & \text{if } t \geq t_1 \end{cases} \quad (7.4)$$

The ramp is nothing else than a level shift with the notable difference that the effect does not emerge immediately from one period to the other but evolves linearly over time. There is no shock on the time series at  $t_0$  and the influence increases constantly until the effect reaches its final size at  $t_1$ . Very much like the effect of a temporary change fades away only gradually compared to the one of an additive outlier, the full effect of a ramp becomes visible only after a few periods compared to the one of a level shift.

That the outcomes of seasonal adjustment of level shifts and ramps without outlier treatment are very similar comes at no surprise because of the definition of a ramp (see Figure 7.11). Still, the seasonal component distortion is quite different. While the erroneous level shift adjustment saw a break in December in the estimate of the seasonal component, for the ramp the seasonal component displays a phase shift. This means that December values, for example, are adjusted with the January seasonal component. The trend-cycle estimates based on untreated level shifts and ramps are broadly comparable.

Fifth, for temporary level shifts ( $TL$ ) where the level shift is of temporary rather than permanent nature it follows that:

<sup>11</sup>Usage of the terms “ramp” and “temporary level shift” is not standardised in the literature. Depending on the authors, it is common to use “ramps” and “temporary level shifts” interchangeably. Here, the definition follows the programs X-12-ARIMA and TRAMO/SEATS.



Figure 7.9: Decomposition of a time series with a level shift

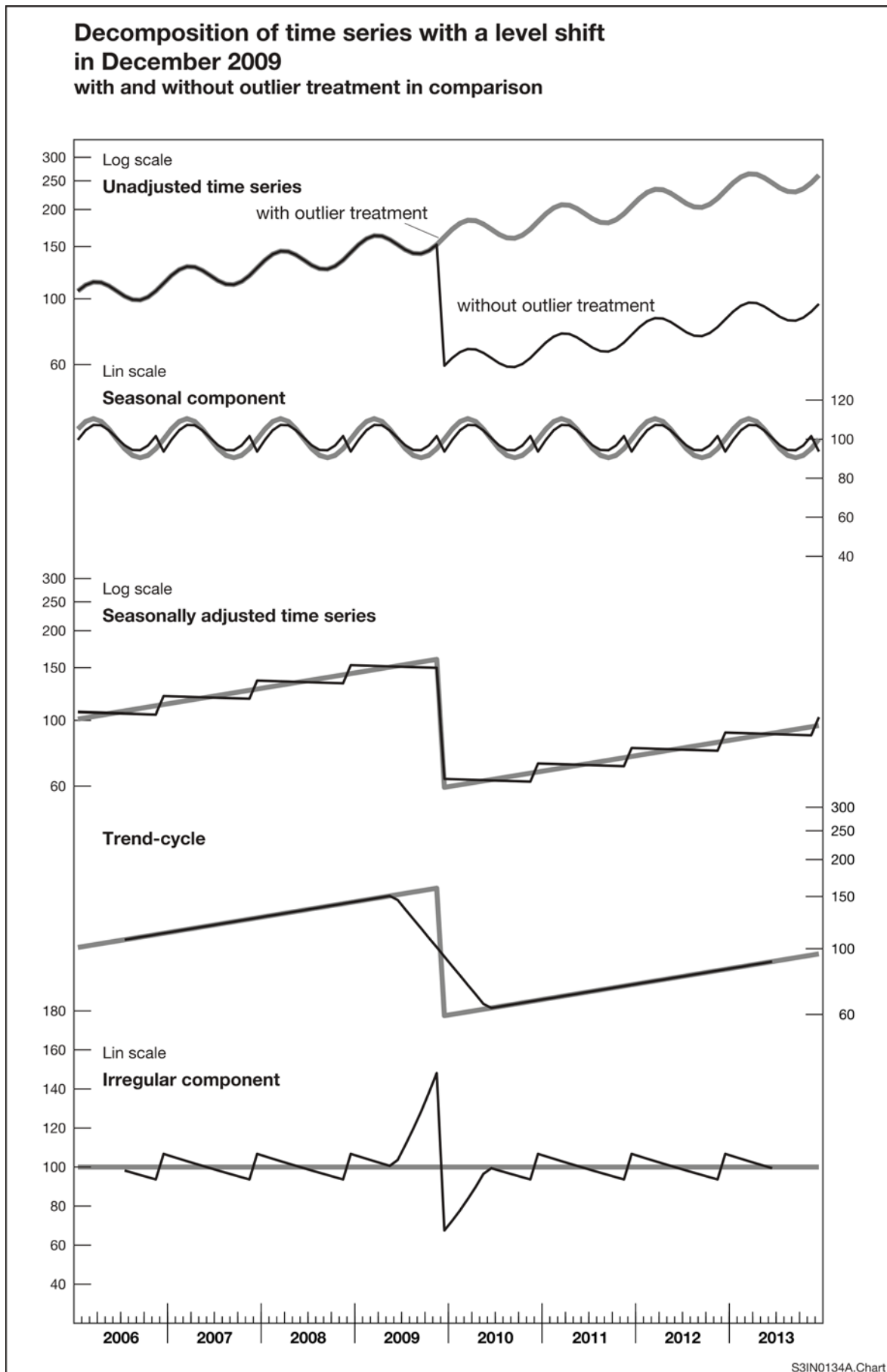


Figure 7.10: Ramp from December 2009 to May 2010

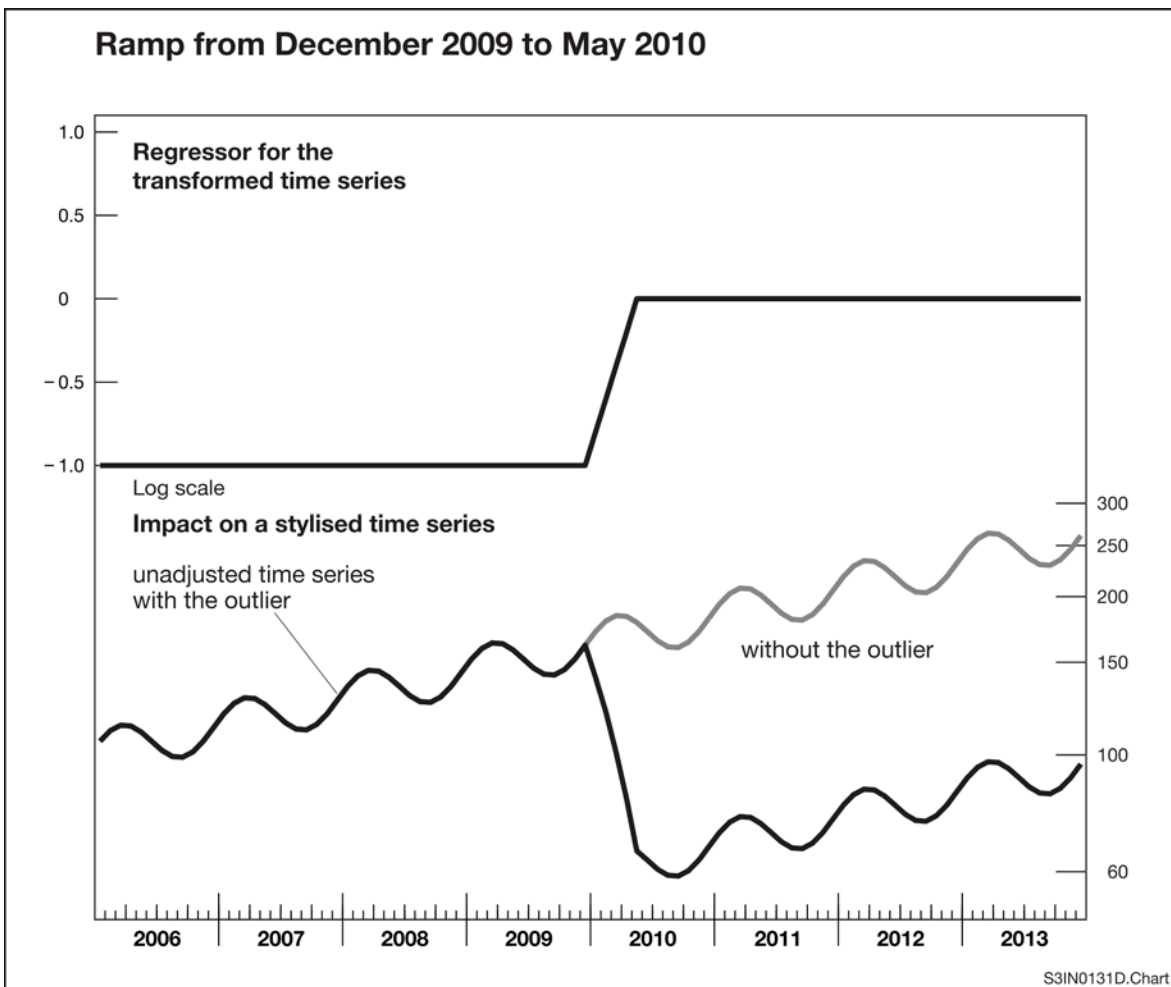


Figure 7.11: Decomposition of time series with a ramp from December 2009 to May 2010

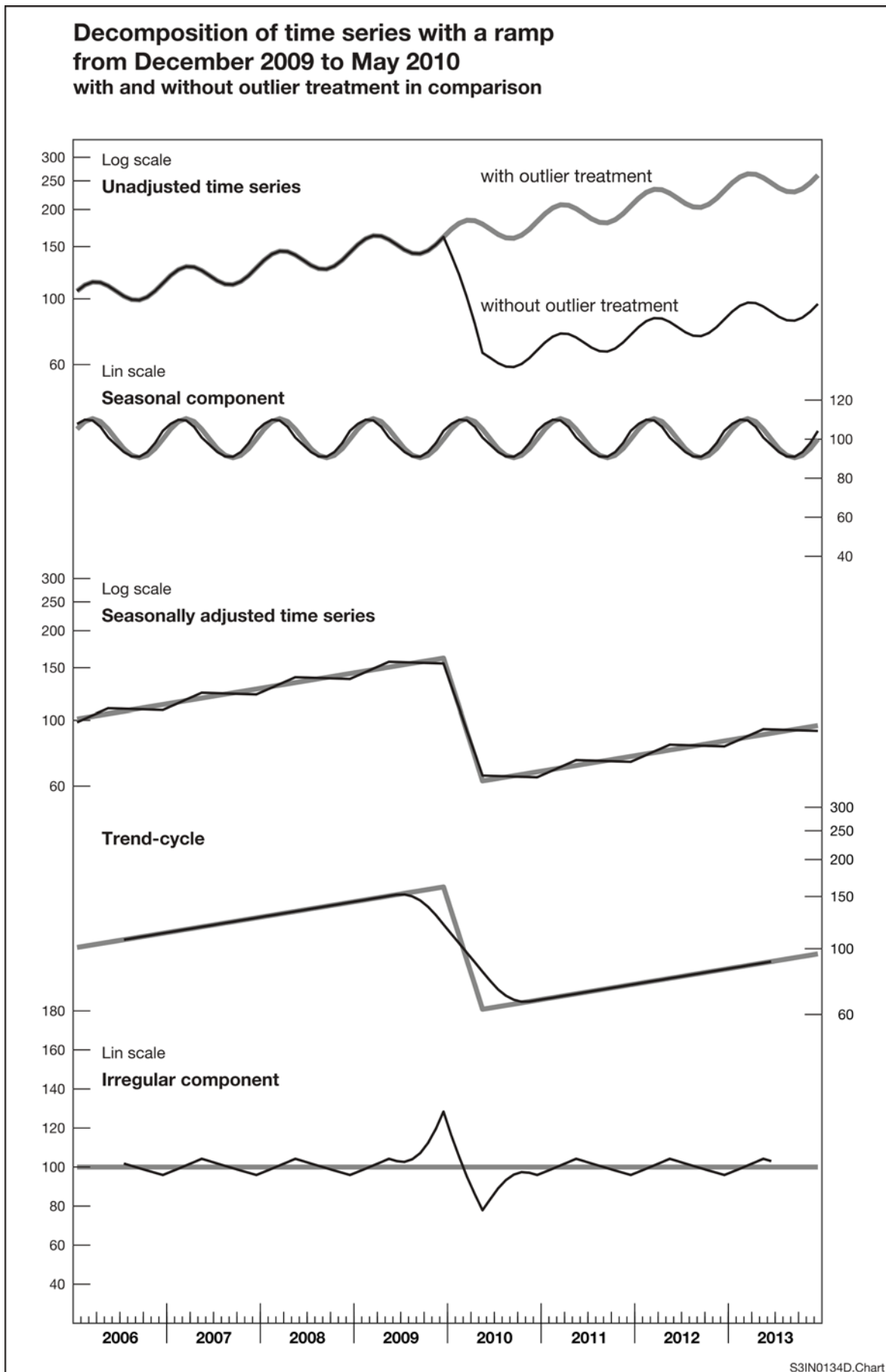
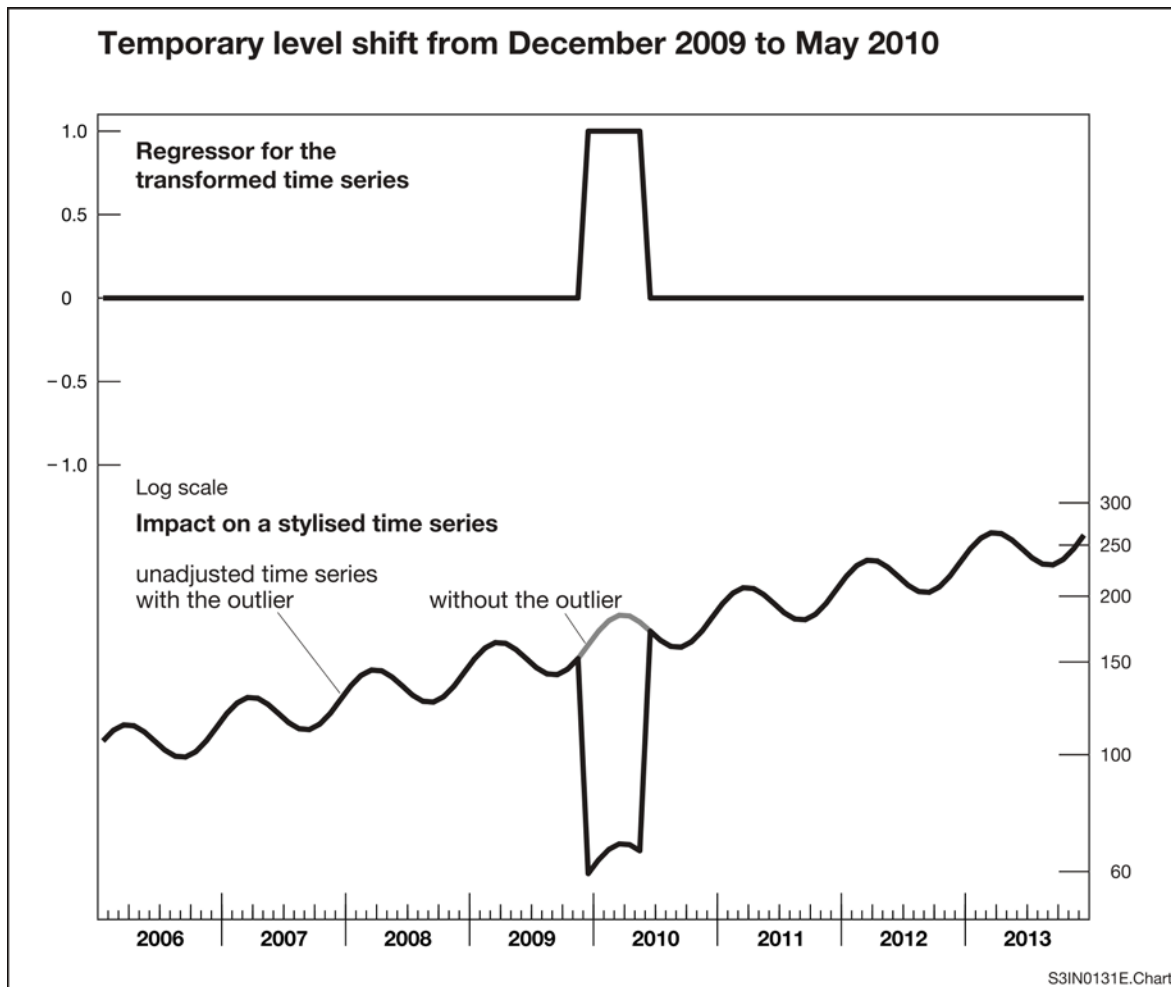


Figure 7.12: Temporary level shift from December 2009 to May 2010



$$TL_t = \begin{cases} 0 & \text{if } t \leq t_0 \\ 1 & \text{if } t_0 < t \leq t_1 \\ 0 & \text{if } t > t_1 \end{cases} \quad (7.5)$$

If a level shift is of temporary rather than permanent nature, the temporary level shift comes into play. It is produced by two level shifts in  $t_0$  and  $t_1$  that cancel each other. This is one of the, in a statistical sense exact, possible interpretations of a temporary level shift. The other one is that of multiple additive outliers from  $t_0$  to  $t_1$ . Specifically, a temporary “level shift” in a single period is the same as an additive outlier. What makes all the difference between the two effects is their attribution to the trend-cycle and irregular component, respectively. Any temporary measure, such as an exemption, might generate a temporary level shift if the effects are equal in  $t_0$  and  $t_1$ .

The effect of inappropriate seasonal adjustment of a temporary level shift is best understood when one re-considers that a temporary level shift is actually two level shifts of the same size with opposite signs. Hence, the break described for the level shift occurs twice, at the beginning and at the end of the temporary level shift. In addition to the misalignment of the seasonal component’s estimate, the trend-cycle effect is heavily underestimated. The picture that emerges is much worse than that for the misspecified level shift.

**Figure 7.13:** Decomposition of time series with a temporary level shift from December 2009 to May 2010

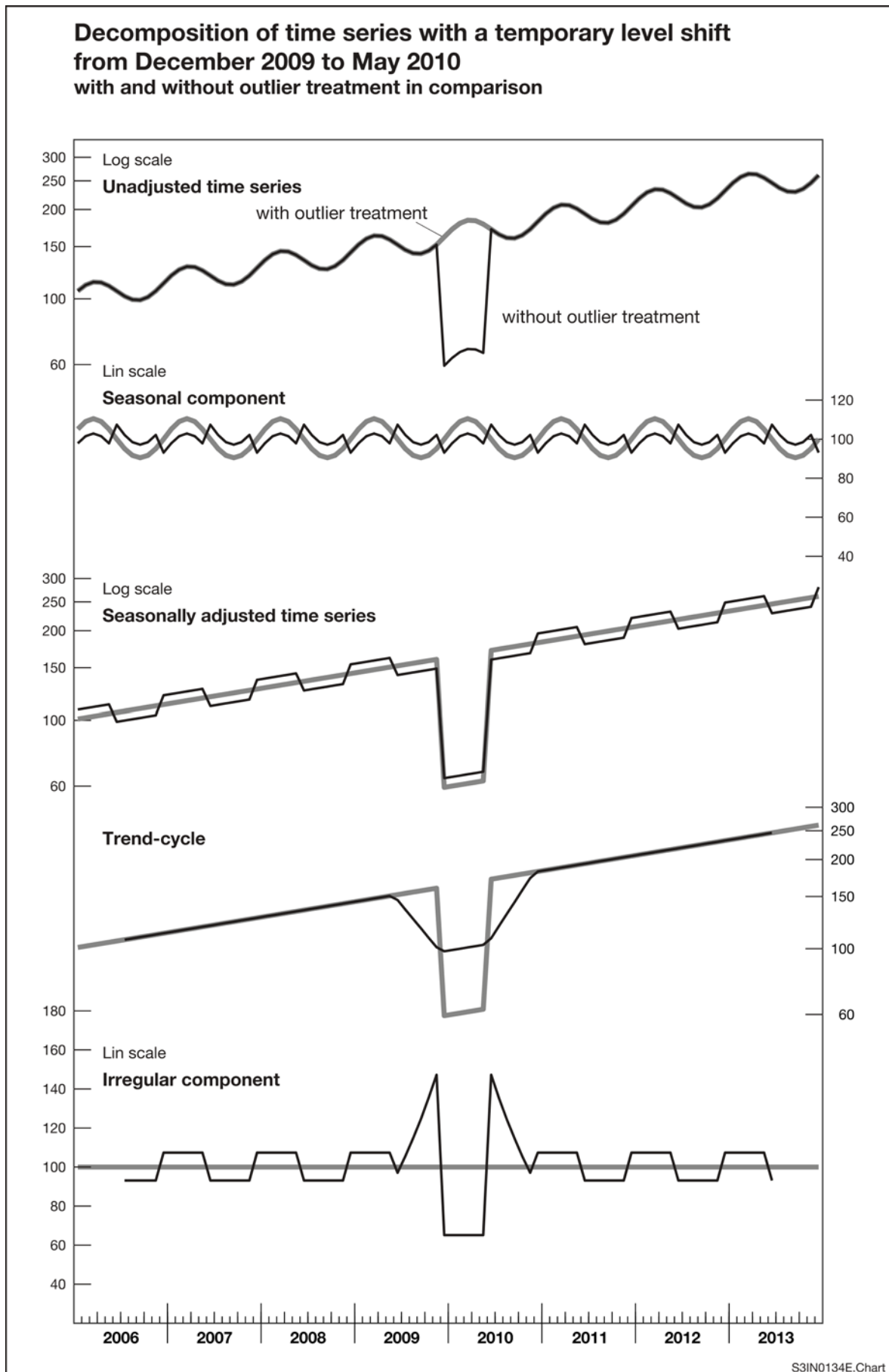
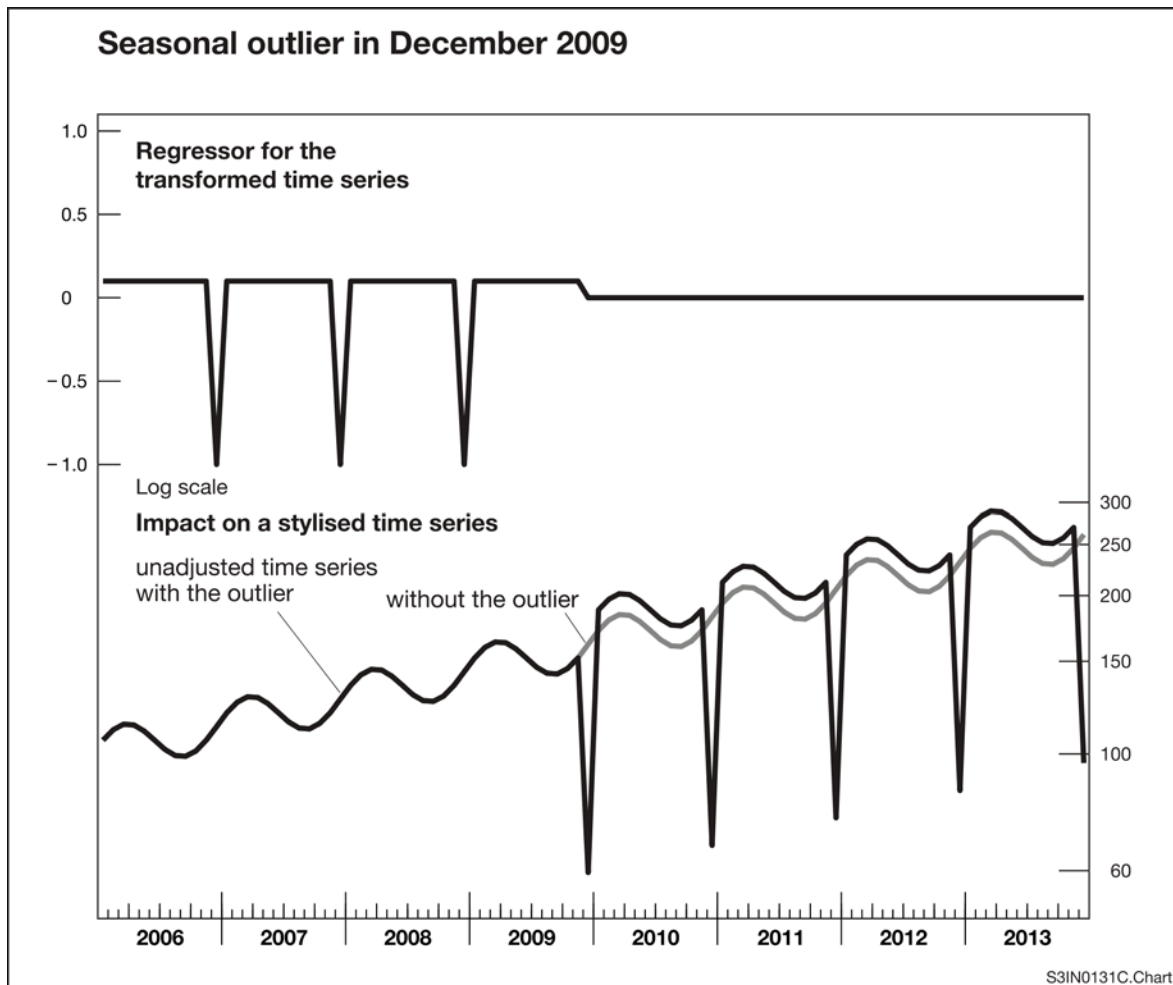


Figure 7.14: Seasonal outlier in December 2009



### 7.3.4 Outliers in the seasonal component

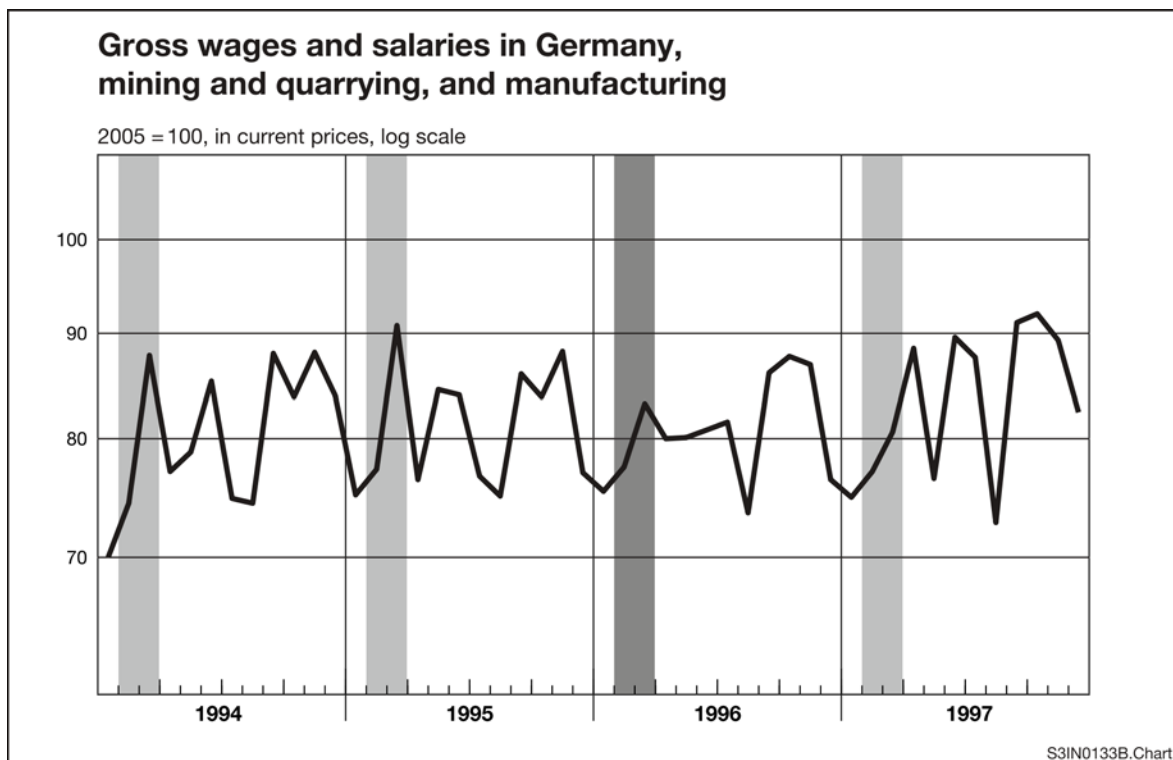
Sixth, seasonal outliers ( $SO$ ) which do not affect the level of the time series but the seasonal pattern by allowing for an abrupt increase or decrease of the seasonal component for a specific month ( $s = 12$ ) or quarter ( $s = 4$ ), hence a permanent seasonal break<sup>12</sup>, of which the formula is:

$$SO_t = \begin{cases} 0 & \text{if } t \geq t_0 \\ -1 & \text{if } t < t_0 \text{ and } t \text{ same month/quarter as } t_0 \\ 1/(s-1) & \text{otherwise} \end{cases} \quad (7.6)$$

When in 1996 a large German industrial company permanently shifted its bonus payments from March to February this led to a seasonal break.<sup>13</sup> Hence, from 1996 onwards the seasonal component for February became higher and the one for March lower. In the above terminology, this is a dual seasonal outlier in February and March 1996.

<sup>12</sup>“Seasonal break” would be the term that describes this type of outlier more appropriately. In the literature, the effect of a “seasonal outlier” in one month/quarter is fully absorbed in a single other month/quarter unlike a “seasonal level shift” where it is spread over the rest of the calendar year, which is the case with the above regressor. See Kaiser, R., and Maravall, A. (2001), “Seasonal Outliers in Time Series”, *Journal of the Inter-American Statistical Institute (Revista Estadística)*, 53, 101-142.

<sup>13</sup>See Kirchner, R. (1999), “Auswirkungen des neuen Saisonbereinigungsverfahrens X-12-ARIMA auf die aktuelle Wirtschaftsanalyse in Deutschland”, Research Centre of the Deutsche Bundesbank Discussion Paper, 7.

**Figure 7.15:** Gross wages and salaries in Germany, mining and quarrying, and manufacturing

If a seasonal outlier is not modelled, this will have a big effect on the decomposition of the time series. The estimate of the seasonal component cannot account for the break in seasonality. The effect is spread over the entire time span, so that the month of the drop of the seasonal component (December) is over-adjusted before the break and under-adjusted thereafter, while the converse holds true for all other months. However, the trend-cycle estimates are similar between correct and no treatment of the outlier.

### 7.3.5 Two sides of the same coin: minimisation of false positives vs. false negatives

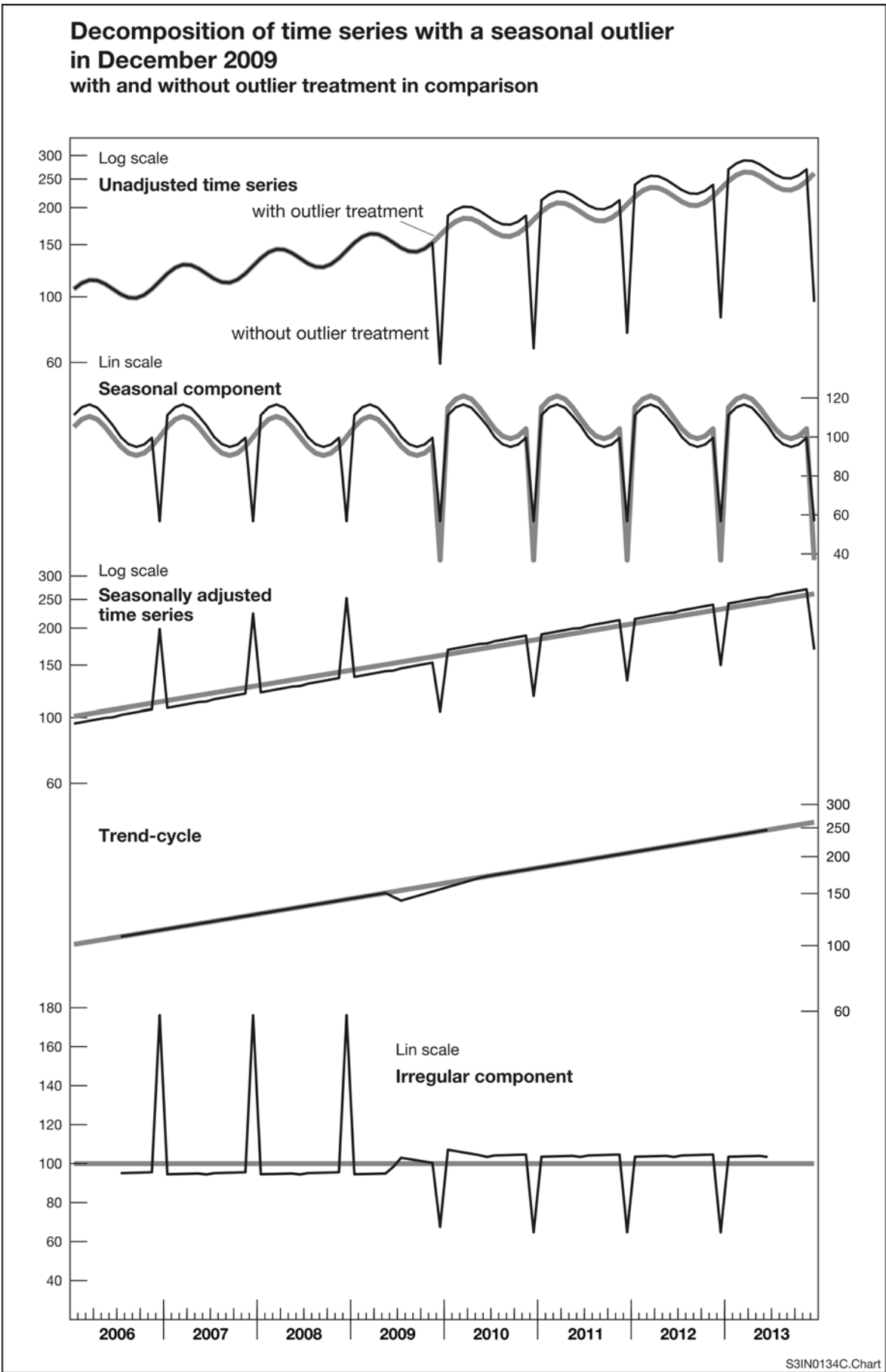
The problem of modelling or not modelling a possible outlier can be perceived as that with the errors associated with a statistical hypothesis test. The rate of false positives of this test is what is known as the probability of a type I error. Here, it means that an outlier is modelled although the observation in question is actually not an outlier. For the rate of false negatives the argument is inverted, it is the probability of a type II error. Translated to outlier modelling, it stands for the case that no outlier is modelled when the observation in question is in fact an outlier.<sup>14</sup> Thus, the question of “is this particular observation truly an outlier or not?” is posed to producers of seasonally adjusted data. The table 7.2 contrasts these two sides of the same coin.

In general, the two aforementioned aims are mutually exclusive. If one is afraid of too many false negatives, the solution would be to model more possible outliers. At best, this will not change the number of false positives but in the worst case, this approach will increase it by the same amount as additional outliers are modelled. The same rationale works in the other direction. Thus, consideration is given to which type of error is more severe in Section 7.5 of this chapter.

<sup>14</sup>One can also think of a type III error, that is the observation is an outlier which is modelled — hence, a true positive — but the wrong type of outlier is used or the effect is falsely estimated.



Figure 7.16: Decomposition of time series with a seasonal outlier in December 2009



**Table 7.2:** Characteristics of different types of outliers

Type I and type II errors*		
		Observation is . . .
An outlier is . . .	. . . an outlier	. . . not an outlier
. . . modelled	True positive	False positive (type I error)
. . . not modelled	False negative (type II error)	True negative

\* The (ambiguous) “null hypothesis” reads “observation is not an outlier”. Note that probabilities in columns but not in rows add up to unity.

## 7.4 Estimation techniques for outlier modelling in the context of regARIMA models

As stated previously, the treatment of outliers and calendar effects is preliminary to seasonal adjustment. Their effects on the level of the series are estimated using a time series regression model with ARIMA disturbances (a regARIMA model), which is specified as follows:

$$y_t = \sum_j \beta_j x_{jt} + u_t, \quad \phi(B)\Phi(B)\delta^d\delta_s^D u_t = \theta(B)\Theta(B)\epsilon_t, \quad (7.7)$$

where the  $x_{jt}$  are predetermined variables selected by the user (intervention effects, calendar regressors, etc.). The disturbance term  $u_t$  is a seasonal ARIMA  $(p, d, q)(P, D, Q)$  process, where

$$\phi(B) = 1 - \phi_1 B - \dots - \phi_p B^p, \quad \Phi(B) = 1 - \Phi_1 B^s - \dots - \Phi_P B^{Ps} \quad (7.8)$$

are the non-seasonal and seasonal autoregressive (AR) polynomials, respectively, in the backshift operator  $B$ , such that  $B^r y_t = y_{t-r}$ ,

$$\delta^d = (1 - B)^d, \quad \delta_s^D = (1 - B^s)^D \quad (7.9)$$

are the non-seasonal and seasonal differencing operators, and

$$\theta(B) = 1 - \theta_1 B - \dots - \theta_q B^q, \quad \Theta(B) = 1 - \Theta_1 B^s - \dots - \Theta_Q B^{Qs} \quad (7.10)$$

are the non-seasonal and seasonal moving average (MA) polynomials, respectively.

Conditional on the orders of the ARIMA model and the predetermined variables, estimation is carried out by maximum likelihood (ML). The regression coefficients can be concentrated out of the likelihood function, which amounts to estimating them by generalised least squares. Hence, for given values of the ARIMA parameters, the ML estimates of the  $\beta_j$  are available in closed form and are delivered by fast and reliable methods. Selection of the orders can be carried out by the inspection of the global and partial autocorrelation functions of the differenced series or by information methods such as the Akaike information criterion (AIC), the corrected AIC, or the Bayesian information criterion (BIC). Typical values for the differencing orders are  $d = 0, 1, 2$  and  $D = 1$ , respectively.

It can immediately be seen that the user faces a circularity problem here. On the one hand, efficient estimation of the effect of the outliers requires the correct specification of the model. Whereas on the other hand, outliers and structural breaks may potentially hinder both the identification and the estimation of the model (e.g. a level shift may indicate the need to difference the series; see Fox (1972), Tsay (1986) and Balke (1993), among others).

The two main seasonal adjustment programs offer the user a practical way out of this dilemma that relies on two fundamental options:

- Automatic model identification
- Automatic outlier detection and estimation

TRAMO has a built-in automatic model selection routine that is similar to the one available in X-12-ARIMA in the “automdl” spec, which both go essentially through three stages. In the first stage, the series is corrected for potential candidate calendar and outlier effects by fitting a default model, represented by the airline model, i.e. the seasonal ARIMA process with orders  $(0, 1, 1) \times (0, 1, 1)$ . The second stage proceeds to the identification of the differencing orders  $d$  and  $D$ , using threshold rules applied to the autoregressive roots estimated from the time series. Finally, the AR orders  $p$  and  $P$  as well as the MA orders  $q$  and  $Q$  are selected according to the BIC using a general-to-specific approach. More details can be found in Gómez and Maravall (2001) and Maravall (2009). For the selection of the Box-Cox transformation parameter see Box and Cox (1964) and Chapter 9 of this handbook. X-12-ARIMA also features a model selection procedure patterned after the X-11-ARIMA/88 procedure. For a comparison of these automatic selection procedures see Farooque et al. (2001), and McDonald-Johnson et al. (2007).

Outlying observations that have been identified by the users using their previous knowledge of the phenomena are handled by including the appropriate intervention variables as further regression effects in the regARIMA framework. Anyway, both X-12-ARIMA and TRAMO/SEATS have a built-in automatic outlier detection method that hinges on the following assumptions:

- There exists a linear and Gaussian model for the data, possibly after a transformation of the type proposed by Box and Cox (1964), which provides a coherent and plausible representation for the great majority of the available data, except for a few outlying observations.
- The outliers are due to exogenous and possibly unidentified causes. Signal extraction and forecasting rule out the occurrence of extreme events in the future.
- The outliers affect only the conditional mean of the series and not the conditional variance.
- The outlier types belong to a predefined set of categories: additive outliers (*AO*), level shifts (*LS*), temporary changes (*TC*) and innovative outliers (*IO*).
- The location and type of the outliers (among the above-specified categories) are unknown.

Automatic outlier detection is based on iterative procedures that were originally proposed and implemented by Chang et al. (1988) and Tsay (1988), and further refined by Chen and Liu (1993). Hillmer et al. (1983) applied these methods in the context of seasonal adjustment. The overall aim is to locate the timing of the outlier, select the most likely type and adjust for its effects, so as to formulate the final model for seasonal adjustment or a preliminary adjustment for subsequent filtering.

The current implementations in TRAMO/SEATS and X-12-ARIMA take the following steps:

1. Initialisation step: a seasonal regARIMA model is estimated by maximum likelihood, possibly featuring calendar effects and user intervention variables.
2. Forward addition step: this step proceeds to the addition of outliers one at a time.
  - a) Calculate the robust standard error  $\sigma = 1.4826 \times \text{Median}(|v_t|)$ , where  $v_t$  is the innovations sequence (computed by the Kalman filter adapted to the state space representation of the regARIMA model), using the current parameter estimates. This is known as the median absolute deviation (MAD) estimate of the standard deviation based on the normality assumption (for a normal random variable, the MAD is proportional to the standard deviation where the factor of proportionality is the 3rd quartile of the standard normal distribution, which is 0.6745). It provides a measure which is more robust to outlying observations.

- b) Using the robust standard error, calculate the sequence of regression t-statistics for the presence of *AO*, *LS*, *TC* and *IO* outliers which are currently not in the model for all time periods  $\tau$ , denoted  $t_{j\tau}$ ,  $\tau = 1, 2, \dots, T$ , and  $j = AO, LS, TC, IO$ .
  - c) As proposed by Chang, Tiao and Chen (1988), locate the outlier regressor with the maximum t-value by the statistic  $\bar{t}_{j\tau} = \max_{j,\tau} t_{j\tau}$ , with  $t_{j\tau}$ ,  $\tau = 1, 2, \dots, T$ , and  $j = AO, LS, TC, IO$ .
  - d) If  $|\bar{t}_{j\tau}| > C$ , add the relevant regressor  $x_{jt}$  to the model to correct the series for the effects of the outlier (one can optionally re-estimate the parameters of the regARIMA model). Otherwise, stop. The critical value  $C$  has to take into account the simultaneous nature of the testing problem. With the support of Monte Carlo simulations, Chang et al. (1988) recommended three values:  $C = 3.0$  for high sensitivity to outliers,  $C = 3.5$  for medium sensitivity, and  $C = 4.0$  for low sensitivity.
  - e) Repeat steps a.-d. until no additional outliers are found.
3. Backward deletion step: this step aims at removing potential outliers identified in the previous stage whose t-statistics do not exceed the threshold any more.
- a) Start with the model including all outlier regressors identified in the forward addition stage. As shown in Chen and Liu (1993), the estimates of the outlier effects obtained simultaneously from a multivariate regression may differ substantially from the ones obtained by the forward addition step.
  - b) Calculate the maximum likelihood estimates of the parameters, including the prediction error variance  $\sigma^2$ .
  - c) Using the estimated parameters, calculate the t-statistics for all of the outliers identified in the forward addition step that remain in the model. Determine which of these regressors has the minimum value of  $|t_{j\tau}|$ .
  - d) If  $\min_{j,\tau} |t_{j\tau}| < C$ , delete this regressor from the model and go to a. Otherwise, stop.

The procedure is open to a number of criticisms. First and foremost, the procedure is tailored to detect single outliers. However, outliers may occur in patches of consecutive observations and masking is likely to take place. Secondly, the distribution of the test statistics arising from sequential detection is unknown and the critical values currently in use are a coarse simulation-based approximation. A third limitation lies with the assumption that outliers are exogenous and due to unidentified causes. An alternative modelling strategy is to view structural change as endemic, or endogenous, and to estimate a seasonal model with disturbances characterised by heavy-tailed distributions, such as Student's t-distribution or scale mixtures of Gaussians. This approach has not yet found its way into the literature. It requires a heavier computational burden, due to the fact that estimation and signal extraction require Monte Carlo simulation methods. An early move towards this research area is represented by Bruce and Jurke (1996). A maximum likelihood approach based on importance sampling has been proposed by Aston and Koopman (2006). Yet another limitation is that certain crises that hit the world economy, such as the most recent recession, cannot be captured fully by a combination of the intervention variables currently available in the programs.

Despite all criticism, the automatic outlier detection procedure outlined in this section seems to work well in most practical situations.

## 7.5 Practical considerations as regards the detection and correction of outliers

### 7.5.1 Revisions of seasonally adjusted data

Revisions of seasonally adjusted data stemming from different treatment of outliers are analysed in what follows. If identification of outliers is altered from one period to the next, revisions will likely increase. This is due to the fact that the impact of the outlier is partially allocated to the seasonal component in one period (if not adjusted) and to the trend-cycle or the irregular component in another period (if adjusted), i.e. it remains fully visible in seasonally adjusted data only in the latter but not in the former case.

Sections 7.2 and 7.5 of this chapter give a more detailed discussion on the necessity of outlier modelling, in particular on the interplay of identification of possible sources of outliers and their modelling with appropriate variables. However, it is not always possible to get hold of sufficient information. This may be because of mass production of seasonally adjusted data within a limited time frame or just because nothing (or not enough) can be known about the movements of the particular time series. In both cases an automatic procedure for outlier detection and correction may be applicable.<sup>15</sup>

Both leading seasonal adjustment programs, X-12-ARIMA and TRAMO/SEATS, have an automatic outlier identification tool. It is an iterative procedure which tests stepwise for the presence of outliers and adds significant variables to the regARIMA model in the forward pass. In the backward pass, variables that are no longer significant are deleted. Hence, identification of outliers depends on the critical value to be set in the specification. When the t-statistic of a variable exceeds this value, the observation in question is considered as being an outlier. Consequently, the probability of false positives and false negatives varies with the choice of the critical value.

Technically, an outlier is an observation which is further away from the expectation than the critical value times the standard deviation of the residuals. But since both expectation and standard deviation depend on the model, the identified outliers vary with the model. So, if the model or its parameters change with the course of time, the detection of outliers different from the earlier ones may be the result, which, in turn, will lead to revisions of seasonally adjusted data.

Unfortunately, outliers can be automatically identified because of an inadequate ARIMA model or missing calendar adjustment. In these circumstances, the ARIMA model should be changed accordingly and appropriate calendar regressors be included, respectively.

If outlier detection is automated, one has to live — to a certain extent — with revisions. Moreover, extra care is necessary for observations whose outlier t-statistics are just below the critical value as it can be exceeded with the revision of old or addition of new data.

This does not mean, however, that one cannot learn something from automation of outlier detection. On the contrary, automatically identified outliers should be checked for an economic reasoning. If good reasons for outliers are found, their dates should be fixed in the regression part. Fixation of outlier modelling has the big advantage that identification of these outliers does not change, and thus seasonally adjusted data are revised less over time. In addition to the fixation, metadata — i.e. why this observation is modelled as an outlier — should accompany the specification.

<sup>15</sup>See Section 7.4 of this chapter for a detailed discussion of estimation techniques for outlier modelling in the context of regARIMA models.

### 7.5.2 Treatment of (possible) outliers at the current end of the time series

Outlier modelling at the current end of the time series is a vital question. In a certain way, one can think of not modelling outliers as modelling them and fix the parameter associated to this variable to zero. To carry the idea further, eventually, even if outliers are not explicitly modelled, they are implicitly. Thus, begging the new question, “is it appropriate to fix parameters of outliers to zero?” After having made clear that the question of outlier modelling is not “if” but “how”, the role of additional information becomes apparent.

In general, a change in the level of the time series does not come out of the blue. In many cases, additional information is available to producers of seasonally adjusted data, giving details about what has happened. This can help to explain which type of outlier is probably appropriate as well as of what size the effect can be expected to be approximately. In this sense, outlier modelling relies on the coincidence of statistical significance of an identified outlier and additional information that supports this view.

However, outliers may be identified spuriously (false positives) or are not identified due to masking effects (false negatives).<sup>16</sup> Which type of error is more severe? This question is not easy to answer. Purely statistical considerations point to false negatives being more severe because for a false positive the parameter estimate should be statistically insignificant. On the other hand, degrees of freedom are lost by modelling too many outliers which are in truth valid observations. Still, false negatives are subsumed in the error term which, in turn, may no longer fulfill the conditions for unbiased parameter estimation — rendering forecasts biased and calendar adjustment invalid.<sup>17</sup>

To get back to the role of additional information, next, several cases are revisited where additional information sheds light on outlier modelling.

- Changes in legislation: e.g. a VAT increase will generally cause a level shift of consumer price indices
- Statistical changes: the introduction of a new classification system may result in a structural break, usually modelled with a level shift
- Leading indicators: during the economic and financial crisis, for instance, industrial production indices were available before the publication of GDP figures
- Outside information: the results from business surveys are capable of providing information for seasonal adjustment of related business statistics
- Requests at companies: the occurrence of large orders or the shift of bonus payments for example can be identified by directly contacting the companies in question and applying appropriate outliers such as additive outliers and the like
- Other sources: extraordinary circumstances such as major strikes or extreme weather conditions can be inferred from the daily press or even special recordings

Mehrhoff<sup>18</sup> deals with the question of outlier modelling in times of crisis and how observations should be treated at the beginning of such a one. Using data on Argentine currency in circulation, which was affected by the 2001/2002 Argentine crisis, he is able to evaluate the final effects of the crisis on seasonality and seasonal adjustment — unlike with time series which cover the 2008/2009 financial and economic crisis. The results point to largely different estimates in the short run, i.e. during and shortly after the crisis, between seasonal adjustment with and without outliers. But, as time passes, these differences fade away and the results become more similar in the longer run, i.e. long after the crisis. In the end, the preliminary results obtained with outlier modelling during the crisis are close to the final results, no matter whether outliers are identified or not in the long run. Then again, seasonal adjustment in the short run without outlier treatment is remarkably distinct

<sup>16</sup>See Section 7.3 of this chapter for a discussion of type I and type II errors, respectively.

<sup>17</sup>See Section 7.2 of this chapter for a review of the requirements of regARIMA estimation.

<sup>18</sup>See Mehrhoff, J. (2010), “Seasonal Adjustment in Times of Strong Economic Changes”, 2nd Eurostat/ECB Workshop on the Implementation of the ESS Guidelines on Seasonal Adjustment, Frankfurt am Main, 6-7 July 2010, and “Ajuste Estacional en Tiempos de Cambios Económicos Fuertes”, Ensayos Económicos (Economics Essays, in Spanish), 59, 57-73.

from the final results. This fact makes a good case for outlier modelling right from the beginning of a crisis. A condition is that the reasons for seasonality continue to exist — even in times of strong economic changes. Then seasonal adjustment is justified in order to facilitate the uncovering of “news” in economic developments during this period.

Lastly, some considerations are given as to the choice of the type of outlier independently of additional information. First, the case of a single observation which is suspected of being an outlier. All outlier variables will inevitably result in complete ignorance of this observation as nothing can be learnt from the most recent development of the time series — that particular observation is effectively deleted and replaced with a forecast. Second, the question of whether the effect of an outlier should be temporary or permanent. Recall that one cannot tell the difference between an additive outlier, a level shift, a temporary change and a seasonal outlier at the last observation. In the same manner, one also cannot distinguish between a (permanent) level shift and a temporary level shift if the current economic development is still in the level shift phase, i.e.  $t < t_1$ . Third, on the issue of observational equivalence of different combinations of outliers for multiple observations. Just one example: two level shifts in consecutive periods cannot be distinguished from a) an additive outlier in the first period plus a level shift in the second period and b) an additive outlier along with a level shift in the first period. The t-values of the outlier variables from the three different specifications — although observationally equivalent — can differ remarkably. Depending on the threshold critical value, it can even be the case that in one specification outliers are not identified, in the next specification only one of them is identified and in the third specification both outliers are identified.

The biggest risk in modelling of seasonal data seems to be the misspecification of a seasonal outlier at the end of time series as an additive outlier.<sup>19</sup> To reiterate, while the effect of a seasonal outlier is adjusted for, that of an additive outlier is not. The thing is that it is not possible to discriminate statistically between the two. Hence, additional information is necessary for a well-founded decision. However, experiences over decades at the Deutsche Bundesbank with German time series point to the fact that seasonal outliers are extremely rare in practice. Most of the reasons for seasonality, such as the length of months, public holidays, school holidays, climatic conditions and so on, change if at all only very gradually. Hence, the occurrence of an additive outlier or a level shift is much more likely than that of a seasonal outlier. Notwithstanding, seasonal outliers do appear.<sup>20</sup> In order to prevent outright biases in seasonal adjustment, it would be desirable that the producers of seasonally adjusted data check whether or not the conditions for seasonality remained valid and use this external information in the estimation of the seasonal component.

## 7.6 Special issues in the treatment of outliers in seasonal adjustment

### 7.6.1 Extreme value detection in X-11

In X-12-ARIMA, outliers are identified and treated both during cleaning of the data (the A-cycle) and during seasonal adjustment itself (B-cycle onwards). The justification of also identifying and treating outliers during the X-11 algorithm is that the underlying homoskedasticity assumption may be restrictive for some time series.

- During the A-cycle, the “outlier” spec is used to detect regARIMA outliers based on the variance of the whole span of the time series of interest. Once these outliers have been assessed, and the decision has been made whether to implement them or not, they are fixed in the “regression” spec. The regARIMA adjustments define the prior-adjusted series B1, which the “x11” spec seasonally adjusts via the X-11 algorithm.

<sup>19</sup>See also the discussion of type I and type II — and type III — errors in Section 7.3 of this chapter.

<sup>20</sup>See the description of seasonal outliers in Section 7.3 of this chapter and the example herein.



- From the B-cycle onwards, prior to estimation of the seasonal component, the “x11” spec identifies outliers in the irregular by comparing individual values with a moving five year standard deviation based on the irregular series available at the time. For the first two years, the standard deviation centred on the third year is used, and for the last two years, that centred on the third latest year. Each irregular is then assigned a weight, and the seasonal-irregular series is modified by the respective weight prior to estimation of the seasonal component. This process is involved in a number of intermediate tables, such as B4 and B9, but the final outcome is figure 7.18 “Final weights for irregular component” (originally labelled table C.17).

Going one step further, in the case of existence of seasonal heteroskedasticity like in the weather and snow-dependent output of the construction sector in Germany during the winter time, it may even be sensible to take into account the possibility of period-specific (e.g. month-specific) standard deviations using the “calendarsigma=all” option in order to detect extreme values.

The “sigmalim=(lowerlim upperlim)” option in the “x11” spec defines how extreme values are identified and treated based on the rolling five year and period-specific standard deviation, respectively. If not specified, the default thresholds are 1.5 and 2.5, and the treatment is as follows: (It is demeaned in the case of multiplicative seasonal adjustment)

- If  $|I_t| < 1.5\sigma$ , then  $I_t$  has weight 100 and  $(SI)_t$  is unmodified.
- If  $1.5\sigma \leq |I_t| \leq 2.5\sigma$ , then  $I_t$  has weight  $100 * (2.5\sigma - |I_t|) / (2.5\sigma - 1.5\sigma)$  and  $(SI)_t$  is partially adjusted.
- If  $|I_t| > 2.5\sigma$ , then  $I_t$  has weight 0 and  $(SI)_t$  is fully adjusted.

The weighting scheme is also depicted in the following figure 7.17. For  $I_t$  with weights less than 100,  $(SI)_t$  becomes the weighted average of itself and its four nearest seasonal neighbours with weight 100. The weight assigned to each neighbour is one quarter of the difference between the weight of  $I_t$  and 100. In the example below, the respective seasonal-irregular series would be fully adjusted at 1990 Q4, 1991 Q1 and 1991 Q4, and partially adjusted at 1992 Q1 and 1996 Q3.

Outlier identification and treatment in regARIMA and the X-11 algorithm differ in a conceptual sense. The underlying rationale for X-11 is insufficient knowledge about sources, whereas for regARIMA it is assumed that sufficient knowledge or statistical criteria can determine exact critical values for outlier detection. Accordingly, the two approaches lead to different decisions — a continuous one for X-11, and a binary one for regARIMA — and different adjustments — downweighting of the outlier in X-11, and deletion in regARIMA.

As a consequence of the different rationales, the regARIMA outlier identification procedure is prone to produce relatively high revisions, whereas the X-11 algorithm has a much weaker influence. This is because whilst new observations can lead to a binary change in the treatment of a value as an outlier in regARIMA, they will only marginally change the treatment of an observation as an outlier in the X-11 algorithm.

Figure 7.17: Weighting of the irregular component in the X-11 replacement procedure

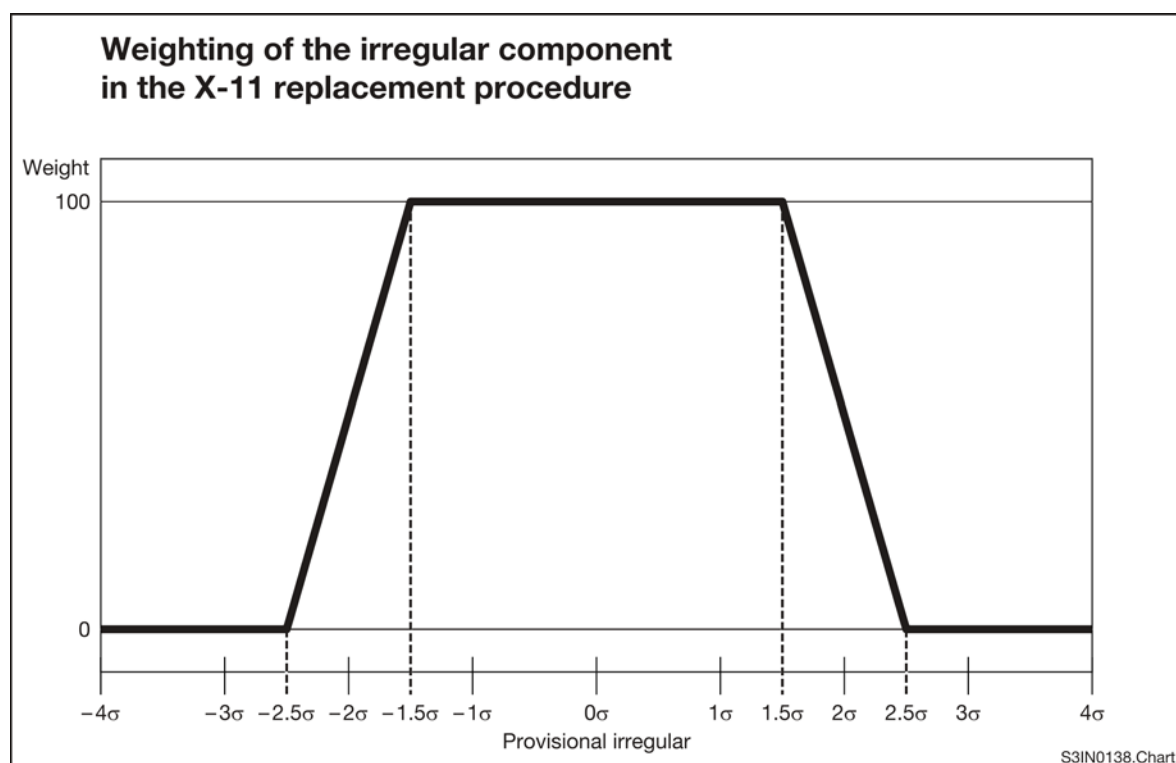


Figure 7.18: Table C17: Final weights for irregular component

**Table C17: Final weights for irregular component**

From 1990.1 to 1997.4

Observations 32

Lower sigma limit 1.50

Upper sigma limit 2.50

	1st	2nd	3rd	4th	S.D.
1990	100.00	100.00	100.00	0.00	1.55
1991	0.00	100.00	100.00	0.00	1.55
1992	41.89	100.00	100.00	100.00	1.55
1993	100.00	100.00	100.00	100.00	1.46
1994	100.00	100.00	100.00	100.00	1.00
1995	100.00	100.00	100.00	100.00	0.72
1996	100.00	100.00	3.77	100.00	0.70
1997	100.00	100.00	100.00	100.00	0.70

## 7.7 Conclusions

This chapter has clearly highlighted “the importance of being earnest” with outlier modelling. While the treatment of outliers itself is of utmost significance, for seasonal adjustment purposes it does not matter that much whether the effect of an outlier is assigned to the trend-cycle or irregular component. The decisive thing is that unusual movements that are readily understandable in economic terms are not attributed to the seasonal component.

However, the question of relying on automatism or on user intervention is not that clear cut. The role of additional information is crucial here. With no information at hand, one has to rely on automatic outlier detection and correction. Then again, the use of information can significantly decrease revisions.

The ESS Guidelines suggest in item 1.4 as best alternative that outliers for whom a clear interpretation exists, regressors are included in the model. An acceptable approach is performing outlier detection and correction in a completely automatic procedure. No pre-treatment of outliers is to be avoided.

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# 8

## Automatic Outlier Detection for the Basic Structural Time Series Model



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## Table of Contents

8.1	Introduction	171
8.2	The Basic Structural Time Series Model	171
8.3	The detection of Outliers	173
8.3.1	Outlier types	173
8.4	Automatic outliers detection methods in seasonal adjustment routines	176
8.5	State space models and methods to detect outliers	178
8.5.1	The Kalman filter	179
8.5.2	State fixed interval smoother	179
8.5.3	Disturbance smoother	180
8.6	An automatic detection procedure	180
8.6.1	Critical values	181
8.6.2	Outline of the procedure	184
8.6.3	Comments	185
8.7	Illustrations	185
8.7.1	Italian industrial production: Sector CK	185
8.7.2	Italian industrial production: sector CH	186
8.7.3	Italian industrial production: sector CG	188
8.7.4	A tale of a seasonal outlier	188
8.8	Endogenous outliers and Non-Gaussian seasonal adjustment	192
8.9	Conclusions	192



## 8.1 Introduction

Outlier adjustment forms integral part of seasonal adjustment, which is relevant in an economic environment subject to large shocks and structural change. The approach to seasonal adjustment and outlier detection considered in this paper is the structural time series approach proposed by Harvey (1989) and West and Harrison (1997), according to which a parametric model for the series is formulated directly in terms of unobserved components. This appears to us as the most natural framework for the two operations. As a matter of fact, the model provides a specific representation for the trend component, the seasonal component, the calendar component and the irregular component. Our reference model for the adjustment purpose will be the *Basic Structural Model* (BSM, Harvey (1989)).

The latter admits a state space representation, which opens the way to its statistical treatment via the Kalman filter and related smoothing algorithms. Estimation is carried out by maximum likelihood via the prediction error decomposition and smoothing will enable diagnosing outliers and structural change.

Although structural time series modelling and seasonal adjustment using the BSM is well established and can in fact be performed using specialised software, namely STAMP 8 (Koopman et al. (2009)) and Ox (Doornik (2009)), the routine treatment of the above specification issues and the automatic handling of outliers is yet not available.

On the contrary, official seasonal adjustment procedures like Tramo-Seats and X-12-ARIMA have in-built facilities for automatic outlier detection. While these procedures are applied successfully to a large number of cases, there are some important drawbacks associated with them. The first is that the automatic procedures are conditional on the maximum likelihood estimates of the parameters; with the exception of the scale parameter, for which a robust estimate is obtained from the MAD, the initial parameter estimates carried out under the null model can be badly biased, affecting the results of the procedure. The second is the confusion between outliers types. In particular, the distribution of the test statistic for an outlier depends on the outlier type and the signal to noise ratios.

The original contributions of this paper to a well established literature can be sketched as follows. First, we use a single statistic to detect the timing of an outlier. We simulate the critical values of the statistic, which depend solely on the sample size (thus they are independent of the signal to noise ratios). Secondly, we can handle seasonal outliers, i.e. breaks in the seasonal pattern.

The plan of the paper is the following. Section 8.2 reviews the reference model the BSM. Most of our treatment is dedicated to monthly time series, although the methods are readily extended to other regular seasonal periods. In section 8.3 we discuss the main type of outliers: additive, level shifts, slope changes, innovative outliers and discuss the nature of a seasonal outlier. Section 8.4 review the automatic detection procedures implemented in standard seasonal adjustment routines. Section 8.5 deals with the state space methodology and the algorithm (disturbance smoother) that are used to detect outliers. In section 8.6 we outline our automatic detection procedure and in section 8.7 we provide some illustrations.

## 8.2 The Basic Structural Time Series Model

The basic structural model (BSM henceforth), proposed by Harvey and Todd (1983) for univariate time series, and extended by Harvey (1989) to the multivariate case, postulates an additive decomposition of the series into a trend, a seasonal and an irregular component. Its name stems from the fact that it provides a satisfactory fit to a wide range of seasonal time series, thereby playing a role analogous to the Airline model in an unobserved components framework.

The BSM postulates an additive and orthogonal decomposition of a time series into unobserved components representing the trend, seasonality and the irregular component.

Let  $y_t$  denote a time series observed at  $t = 1, 2, \dots, n$ ; the BSM is formulated as follows:

$$y_t = \mu_t + \gamma_t + \sum_{k=1}^K \delta_k x_{kt} + \epsilon_t, \quad t = 1, \dots, n, \quad (8.1)$$

where  $\mu_t$  is the trend component,  $\gamma_t$  is the seasonal component, the  $x_{kt}$ 's are appropriate regressors that account for any known intervention as well as calendar effects, namely trading days, moving festivals (Easter) and the length of the month, and  $\epsilon_t \sim \text{NID}(0, \sigma_\epsilon^2)$  is the irregular component.

The trend component has a local linear representation:

$$\begin{aligned} \mu_{t+1} &= \mu_t + \beta_t + \eta_t, & \eta_t &\sim \text{NID}(0, \sigma_\eta^2) \\ \beta_{t+1} &= \beta_t + \zeta_t, & \zeta_t &\sim \text{NID}(0, \sigma_\zeta^2) \end{aligned} \quad (8.2)$$

where  $\eta_t$  and  $\zeta_t$  are mutually uncorrelated.

The seasonal component can be modelled using a trigonometric representation, such that the seasonal effect at time  $t$  arises from the combination of a set of stochastic cycles whose common variance is  $\sigma_\omega^2$ . Alternatively, it is possible to use the so called Harrison and Stevens (HS) specification which is formulated directly in terms of the effect of a particular season, thereby enhancing flexibility needed to model seasonal heteroscedasticity. For a comparison of the various representations of a seasonal component and a discussion of the implications for forecasting, see Proietti (2000).

The trigonometric representation is such that  $\gamma_t$  arises from the combination of six stochastic cycles defined at the seasonal frequencies  $\lambda_j = 2\pi j/12$ ,  $j = 1, \dots, 6$ ,  $\lambda_1$  representing the fundamental frequency (corresponding to a period of 12 monthly observations) and the remaining being the five harmonics (corresponding to periods of 6 months, i.e. two cycles in a year, 4 months, i.e. three cycles in a year, 3 months, i.e. four cycles in a year, 2.4, i.e. five cycles in a year, and 2 months):

$$\gamma_t = \sum_{j=1}^6 \gamma_{jt}, \quad \begin{bmatrix} \gamma_{j,t+1} \\ \gamma_{j,t+1}^* \end{bmatrix} = \begin{bmatrix} \cos \lambda_j & \sin \lambda_j \\ -\sin \lambda_j & \cos \lambda_j \end{bmatrix} \begin{bmatrix} \gamma_{j,t} \\ \gamma_{j,t}^* \end{bmatrix} + \begin{bmatrix} \omega_{j,t} \\ \omega_{j,t}^* \end{bmatrix}, \quad j = 1, \dots, 5, \quad (8.3)$$

and  $\gamma_{6,t} = -\gamma_{6t} + \omega_{6t}$ . The disturbances  $\omega_{jt}$  and  $\omega_{jt}^*$  are normally and independently distributed with common variance  $\sigma_\omega^2$  for  $j = 1, \dots, 5$ , whereas  $\text{Var}(\omega_{6t}) = 0.5\sigma_\omega^2$ ; see Proietti (2000), for further details.

Calendar effects are handled by adding regression effects in the model equation for  $y_t$ . Three sets of regressors are defined to account for each of the three sources of variation mentioned in the introduction.

Trading day (working day) effects occur when the level of activity varies with the day of the week, e.g. it is lower on Saturdays and Sundays. Letting  $D_{jt}$  denote the number of days of type  $j$ ,  $j = 1, \dots, 7$ , occurring in month  $t$  and assuming that the effect of a particular day is constant, the differential trading day effect for series  $i$  is given by:

$$TD_{it} = \sum_{j=1}^6 \delta_{ij} (D_{jt} - D_{7t})$$

The regressors are the differential number of days of type  $j$ ,  $j = 1 \dots, 6$ , compared to the number of Sundays, to which type 7 is conventionally assigned. The Sunday effect on the  $i$ -th series is then obtained as  $(-\sum_{j=1}^6 \delta_{ij})$ . This expedient ensures that the TD effect is zero over a period corresponding to multiples of the weekly cycle.

As far as moving festivals are concerned, the Easter effect is modelled as  $E_t = \delta h_t$  where  $h_t$  is the proportion of 7 days before Easter that fall in month  $t$ . Subtracting the long run average, computed over the first 400 years of the Gregorian calendar (1583-1982), from  $h_t$  yields the regressor  $h_t^* = h_t - \bar{h}_t$ , where  $\bar{h}_t$  takes the values 0.354 and 0.646 respectively in March and April, and zero otherwise. Finally, the length of month (LOM) regressor results from subtracting from the number of days in each month,  $\sum_j D_{jt}$ , its long run average, which

is 365.25/12.

## 8.3 The detection of Outliers

Automatic outlier detection is based on the following fundamental assumptions:

- There exists a linear and Gaussian model for the data that provides a plausible representation for the great majority of the available data, except for a few exceptional observations.
- The outliers are due to exogenous and possibly unidentified causes. Signal extraction and forecasting rules out the occurrence of extreme events in the future.
- The location and type outlier is unknown.

An alternative modelling strategy is to view structural change as endemic, or endogenous and to estimate a seasonal model with disturbances characterised by heavy tailed distributions, such as Student's  $t$ , or scale mixtures of Gaussians.

Iterative procedures for the detection and the correction of various types of outliers were implemented by Chang et al. (1988) and Tsay (1988), and further refined by Chen and Liu (1993a). Hillmer et al. (1983) applied these methods in the context of seasonal adjustment.

The next section reviews the main outlier types considered in the literature; seasonal outliers will also be discussed, while they are often neglected by both the literature and seasonal adjustment. Estimation of the outliers effects is carried out by generalised least squares.

Finally we outline a detection procedure that is based on the auxiliary residuals and the innovations that result from fitting a Basic Structural model to the series and validate its performance on a set of Italian industrial production time series.

### 8.3.1 Outlier types

The effect of a single outlier on the time series  $y_t$  can be represented as follows:

$$y_t = \delta x_t + w_t. \quad (8.4)$$

Here,  $x_t$  denotes the outlier signature,  $\delta$  is the effect of the outlier, and  $w_t$  is a BSM, not contaminated by the outlier effect.

Let  $I_t(\tau)$  denote a pulse indicator, defined as

$$I_t(\tau) = \begin{cases} 1, & t = \tau \\ 0, & t \neq \tau \end{cases}$$

where  $1 \leq \tau \leq n$ . Most outlier signature types considered in the literature can be defined by applying a rational polynomial to  $I_t(\tau)$ . In general, we can write

$$x_t = \frac{1}{\varpi(L)} I_t(\tau), \quad \varpi(L) = 1 - \varpi_1 L - \dots - \varpi_g L^g. \quad (8.5)$$

In particular we can distinguish several types of outlier according to their effects on  $y_t$ .

### Additive outliers

An additive outlier (AO) affects a single observation. Its intervention signature is obtained by setting  $\varpi(L) = 1$ , so that  $x_t = I_t(\tau)$ . Hence, an AO is associated with non persistent events, like strikes or natural events, e.g. the eruption of the Eyjafjallajökull volcano, or an electricity blackout, that affect the level of the series *una tantum*, without further effects on the future values of the series.

The effect on an additive outlier on the autocorrelation function (ACF) and the spectral density level of the series has been discussed in the literature; see e.g. Peña (2001), and the references therein. As it can be seen from (8.4), the effects are comparable to those obtained by contaminating the series by an additive white noise component,  $\epsilon_t$ , with variance approximately equal to  $\delta^2/n$ .

### Level shifts

The intervention signature of a level shift is obtained setting  $\varpi(L) = 1 - L$

$$x_t = \frac{1}{1-L} I_t(\tau) = I(t \geq \tau)$$

where  $I(\cdot)$  is the indicator function. The intervention  $I(t \geq \tau)$  is also known as a step dummy.

The effects of a level shift on the pseudo ACF and spectral density of  $y_t$  are comparable to contaminating the series by a random walk component  $\mu_t = \mu_{t-1} + \eta_t$  with  $\text{Var}(\eta_t) = \delta^2/n$ .

### Slope changes

The slope change (SC) intervention (also referred to as a ramp shift in Chen and Tiao (1990) arises from integrating a step dummy intervention:

$$x_t = \frac{1}{(1-L)^2} = \frac{1}{1-L} I(t \geq \tau).$$

In the spline literature, this intervention is denoted  $x_t = (t-\tau)_+$  where  $(t-\tau)^p$  is a truncated power polynomial of degree  $p$ .

### Transitory changes

The effect of a transitory change (TC) lies somewhat between that of AO and LS. The intervention signature is in fact

$$x_t = \frac{1}{1-\phi L} I_t(\tau), \quad 0 < \phi < 1.$$

Obviously, if  $\phi = 0$ , we face an AO; at the other extreme,  $\phi = 1$ , we face a LS. The coefficient  $\phi$  poses a nonlinear estimation problem and thus it is usually fixed, rather than estimated.

### Mixed types

Combining a transitory change and with level shift we get a temporary ramp (TR), whose intervention signature is

$$x_t = \frac{1}{(1-L)(1-\phi L)} I_t(\tau), \quad 0 < \phi < 1.$$

**Table 8.1:** Intervention signatures according to outlier type.

Type	Time						
	$\tau$	$\tau + 1$	$\tau + 2$	$\tau + 3$	$\dots$	$\tau + s$	$\tau + s + 1$
AO	1	0	0	0	$\dots$	0	0
LS	0	1	1	1	$\dots$	1	1
SC	0	1	2	3	$\dots$	$s$	$s + 1$
TC	0	1	$\phi$	$\phi^2$	$\dots$	$\phi^s$	$\phi^{s+1}$
SO $c_{jt}$	0	$\cos(\lambda_j)$	$\cos(2\lambda_j)$	$\cos(3\lambda_j)$	$\dots$	$\cos(s\lambda_j)$	$\cos(2s\lambda_j)$
SO $s_{jt}$	0	$\sin(\lambda_j)$	$\sin(2\lambda_j)$	$\sin(3\lambda_j)$	$\dots$	$\sin(s\lambda_j)$	$\sin(2s\lambda_j)$
IO	1	$\psi_1$	$\psi_2$	$\psi_3$	$\dots$	$\psi_s$	$\psi_{s+1}$

### Seasonal outliers

In principle, nothing prevents the definition of a seasonal outlier using the intervention signature

$$\frac{1}{S(L)}I_t(\tau).$$

We however face the following problem: for a monthly time series there are 11 potential interventions which satisfy the defining relationship  $S(L)x_t = I_t(\tau)$ . Their intervention signature is represented in the table below for 12 consecutive time points. This pattern is repeated modulo 12

Dummy	Time											
	$\tau$	$\tau + 1$	$\tau + 2$	$\tau + 3$	$\tau + 4$	$\tau + 5$	$\tau + 6$	$\tau + 7$	$\tau + 8$	$\tau + 9$	$\tau + 10$	$\tau + 11$
SO $d_1$	1	-1	0	0	0	0	0	0	0	0	0	0
SO $d_2$	1	0	-1	0	0	0	0	0	0	0	0	0
SO $d_3$	1	0	0	-1	0	0	0	0	0	0	0	0
SO $d_4$	1	0	0	0	-1	0	0	0	0	0	0	0
SO $d_5$	1	0	0	0	0	-1	0	0	0	0	0	0
SO $d_6$	1	0	0	0	0	0	-1	0	0	0	0	0
SO $d_7$	1	0	0	0	0	0	0	-1	0	0	0	0
SO $d_8$	1	0	0	0	0	0	0	0	-1	0	0	0
SO $d_9$	1	0	0	0	0	0	0	0	0	-1	0	0
SO $d_{10}$	1	0	0	0	0	0	0	0	0	0	-1	0
SO $d_{11}$	1	0	0	0	0	0	0	0	0	0	0	-1
SO sum	11	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

One possibility is to include them all in the linear model with the same coefficient, which amounts to include a unique  $x_t$  given by the last row of the table. Notice, however, that  $S(L)x_{\tau+j} = 11 - j \neq 0$  for  $j = 1, \dots, 10$ . The most complete option is to include the 11 regressors with different coefficients.

If seasonality is modelled using trigonometric cycles a more direct set of options is available. A monthly deterministic seasonal component can be written as the sum of six trigonometric cycles,

$$\gamma_t = \sum_{j=1}^5 [\alpha_j \cos(\lambda_j t) + \alpha_j^* \sin(\lambda_j t)] + \alpha_6 \cos(\pi t), \quad \lambda_j = \frac{2\pi}{12}j.$$

The  $j$ -th seasonal cycle can be also defined by the deterministic difference equation  $S_j(L)\gamma_{jt} = 0$ , where  $S_j(L) = 1 - 2\cos\lambda_j L + L^2$ . Hence, a seasonal outlier affecting the  $j$ -th trigonometric cycle, is defined by the intervention signature  $S_j(L)x_t = I_t(\tau)$ . There are two such  $x_t$ 's satisfying the difference equation, listed in table 8.1. Hence, the statistical treatment of a SO at frequency  $\lambda_j$  requires the introduction of two intervention variables, so that it is handled by generalised regression of  $y_t$  on  $\delta_j c_{jt} + \delta_j^* s_{jt}$ .

If  $\delta_j^* = 0$ , then the outlier affects only the amplitude of the seasonal pattern. If  $\delta_j = 0$  only the phase effect

will be present.

### Innovational outliers

An innovational, or innovative, outlier (IO) is an outlier whose effects propagate according to the impulse response function of the model.

Let  $\pi(L)w_t = \xi_t$  be the autoregressive representation of the linear process  $w_t$ , where  $\pi(L) = 1 - \pi_1 L - \dots$ ;  $\xi_t$  is the innovation at time  $t$ ,  $\xi_t = w_t - E(w_t | w_{t-j}, j \geq 1)$ . The impulse response function of  $y_t$  describes the dynamic response of  $y_{t+j}$  to a unit innovation taking place at time  $t$  and it is provided by the coefficients of  $[\pi(L)]^{-1} = \psi(L)$ .

An IO is defined by the intervention signature

$$x_t = \psi(L)I_t(\tau) = \frac{1}{\pi(L)}I_t(\tau).$$

Replacing in (8.4) we obtain the dynamic regression model  $\pi(L)y_t = \delta I_t(\tau) + \xi_t$ .

It is difficult to associate this type of outliers with a particular real life event. The structural approach to time series analysis makes it clear that such a shock affects all the components in a given proportion. For nonstationary models, such that for instance  $\Delta \Delta_{12}y_t$  has a stationary and invertible representation, IO will produce a change in the seasonal pattern as well as level shifts and slope changes in the usual proportion represented by the Kalman gain.

The effects on the ACF and the spectral density are less dramatic, with respect to other outliers type and also the effects on parameter estimates are less severe. This is so since  $x_t$  has the same ACF and a spectrum proportional to that of  $w_t$ .

The problem of distinguishing between innovation outliers and level shifts has been studied by Balke (1993). As a matter of fact, if, for instance, the model is a non seasonal ARIMA with order of integration equal 1, the intervention signature of an innovation outlier (provided by the impulse response function) is very similar to that of a level shift. The same applies to a pure AR(1) when the AR parameter is positive.

## 8.4 Automatic outliers detection methods in seasonal adjustment routines

In this section we discuss the automatic outlier detection methods adopted by the two most popular seasonal adjustment methods, Tramo-Seats and X-12-ARIMA. The overall aim is locating the timing of the outlier, select the most likely type and adjust for its effects, so as to formulate the final model for seasonal adjustment or a preliminary adjustment.

The automatic detection of outliers has a long history. The earliest attempt was Chang et al. (1988), who proposed likelihood ratio (LR) tests for detecting and discriminating AO and IO. Let  $t_{j,\tau} = \hat{\delta}\tau / s.e.(\hat{\delta}\tau)$  denote the  $t$ -statistic for an outlier of type  $j$  at time  $\tau$ ,  $\tau = 1, \dots, n$ . The LR approach for detecting an outlier of type  $j$  at unknown locations leads to the maximum statistics  $\max_{\tau} |t_{j,\tau}|$ ,  $\tau = 1, \dots, n$ , which is the maximum of the absolute  $t$ -statistics for the different outlier types.

Automatic detection of outliers faces three main challenges. The first deals with the effect of outliers on parameter estimates, as it is unfeasible to estimate the models parameters  $nJ$  times, where  $J$  denotes the number of outlier types. As a result, the  $t$ -ratios will be conditional on the maximum likelihood estimates of the parameters assuming no outlier contamination  $y_t = w_t$  (or including only the interventions known a

priori). Both Tramo-Seats and X-12-ARIMA postulate a seasonal ARIMA (SARIMA) model for  $w_t$ , and thus in the above expressions,  $\pi(L)$  and  $\Sigma$  depend on these parameter estimates and on the chosen orders of the difference polynomials. The second is that outliers may occur in patches of consecutive observations, so that masking is likely to take place, i.e. some outliers are undetected because of the presence of others. The third is that the automatic methods are based on sequential detection procedures (the potential outliers are detected one after the other) so that distribution of the test statistics are unknown and has to be simulated for different sample sizes.

The outlier detection procedures implemented in Tramo-Seats and X-12-ARIMA consider the following outlier types: innovation outliers additive outliers (AOs), temporary change outliers (TC) (with  $\phi$  usually fixed, rather arbitrarily, at 0.7) and level shifts (LS). The procedures are iterative and take the following steps:

1. Initialisation: the parameters of the SARIMA model are estimated by maximum likelihood.
2. Forward addition step.
  - a) Calculate the robust standard error<sup>1</sup>,  $\hat{\sigma} = 1.4826 \times \text{median}(|\nu_t|)$ , where  $\nu_t$  are the innovations computed by the Kalman filter adapted to the state space representation of the SARIMA model, using the current parameter estimates.
  - b) Calculate the  $t$ -statistics for the presence of AO and LS outliers not currently in the model.
  - c) As proposed by Chang et al. (1988), locate the outlier regressor with maximum  $t$ -value by the statistic  $\bar{t}_{j,\tau} = \max_{j,\tau} \{|t_{j,\tau}|, j = \text{AO, LS, TC, IO}\}$
  - d) If  $\bar{t}_{j,\tau} > C$ , add the relevant regressor  $x_t$  to the model to correct the series from the effects of the outlier (one could optionally reestimate the parameters of the SARIMA model). Otherwise, stop. The critical value  $C$  has to take into account the simultaneous nature of the testing problem. With the support of Monte Carlo experimentation, Chang et al. (1988) recommended three values:  $C = 3.0$  for high sensitivity to outliers,  $C = 3.5$  for medium sensitivity, and  $C = 4.0$  for high sensitivity<sup>2</sup>.

Repeat steps (a)-(d) until there are no additional outliers are found.

3. Backward Deletion step. Start with the model including all outlier regressors added in the forward addition stage. As shown in Chen and Liu (1993b), the estimates of the outlier effects obtained simultaneously from a multivariate regression may differ substantially from the ones obtained by the forward addition.
  - a) Calculate maximum likelihood estimates of the parameters, including the p.e.v..
  - b) Using the estimated parameters, calculate the  $t$ -value for all the outliers identified in forward addition that remain in the model. Determine which of these regressors has  $\min |t_{j,\tau}|$ .
  - c) If  $\min |t_{j,\tau}| < C$ , delete this regressor from the model and go to (a). Otherwise, stop.

<sup>1</sup>The mean absolute deviation (MAD) estimate of the standard deviation  $\sigma$  is based on the fact that for a normal random variable,  $y \sim N(\mu, \sigma^2)$ ,  $\text{median}(|y - \mu|) \approx 0.6745\sigma$ , and thus a robust estimate of  $\sigma$  is provided by the MAD of the innovations multiplied by the reciprocal of 0.6745 (1.4826).

<sup>2</sup>The critical value depends on the sample size; these suggestions are valid for a sample size of 200 observations.



## 8.5 State space models and methods to detect outliers

The intervention signature of a given outlier type can be obtained from the state space representation of the BSM model. Let  $\beta$  denote a vector of shocks, we can represent the model as

$$y_t = Z\alpha_t + X_t\beta + G\epsilon_t, \quad t = 1, 2, \dots, n, \quad \epsilon_t \sim \text{NID}(0, \sigma^2 I). \quad (8.6)$$

$$\alpha_{t+1} = T\alpha_t + W_t\beta + H\epsilon_t, \quad (8.7)$$

where  $T_t$  is  $m \times m$  and  $H_t$  is  $m \times g$ ,  $g$  being the dimension of the vector  $\epsilon_t$ , and  $W_t$  is a fixed and known matrices of dimension  $m \times k$ .

The specification of the state space model is completed by the *initial conditions* concerning the distribution of  $\alpha_1$ : when nonstationary and regression effects are present, we express the initial state vector in terms of the vector  $\beta$  as follows:

$$\alpha_1 = \tilde{\alpha}_{1|0}^* + W_0\beta + H_0\epsilon_0, \quad \epsilon_0 \sim \text{N}(0, \sigma^2 I), \quad (8.8)$$

where  $\tilde{\alpha}_{1|0}^*$ ,  $W_0$  and  $H_0$  are known quantities.

According to the terminology of de Jong and Penzer (1998),  $X_t$  and  $W_t$  are referred to as shock design matrices. An AO is such that  $X_t$  is a pulse dummy taking value 1 at time  $\tau$  and zero elsewhere, and  $W_t$  is zero; for a level shift,  $X_t$  is always zero and  $W_t$  is zero except at time  $\tau$  where it has a unit element in the first row and column.

The intervention signature corresponding to a shock at time  $\tau$  is

$$D_t(\tau) = \begin{cases} 0, & t = 1, \dots, \tau \\ X_\tau & t = \tau \\ ZT^{t-\tau-1}W_\tau & t = \tau + 1, \dots, n. \end{cases}$$

For a slope change intervention,  $X_t = 0, \forall t$ , and  $W_\tau = e_2$ , where  $e_k$  is the  $k$ -th column of the identity matrix (i.e. the shock design matrix selects the second column of the transition matrix).

The intervention signature of an IO can be obtained from the innovation form of the state space model:

$$\begin{aligned} y_t &= Z\tilde{\alpha}_{t|t-1} + X_t\beta + \nu_t, \quad \nu_t \sim \text{NID}(0, \sigma^2 F_t) \\ \tilde{\alpha}_{t+1|t} &= T\tilde{\alpha}_{t|t-1} + W_t\beta + K_t\nu_t. \end{aligned} \quad (8.9)$$

Setting  $X_t = I_t(\tau)$  and  $W_t = K_t \cdot I_t(\tau)$ :

$$D_t(\tau) = \begin{cases} 0, & t = 1, \dots, \tau \\ 1 & t = \tau \\ ZT^{t-\tau-1}K_\tau & t = \tau + 1, \dots, n. \end{cases}$$

The state space representation of the model for seasonal adjustment opens the way to a variety of algorithms that simplify the evaluation of the test statistics that are needed for outlier detection. Tramo-Seats and X-12-ARIMA use the state space representation of the SARIMA model for computing the innovations,  $\pi(L)y_t$ , and thus the GLS estimate of the intervention effect.

Maravall (1987) and Kohn and Ansley (1989) established that the test for an AO at time  $\tau$  is obtained from the so-called irregular auxiliary residuals, which is an estimate of the measurement error in  $y_t$ , conditional on the full sample. Harvey and Koopman (1992) proposed diagnostics obtained from the estimate of the disturbances, conditional on the full sample, for the class of so-called structural time series models, which are

formulated directly in terms of unobserved components such as level, slope seasonal. In this framework, a particular outlier type originates from adding pulse intervention on the transition equation of the corresponding component.

Atkinson and al (1997) investigate the effect on the hyperparameters estimates of an intervention modelled by including an additional regressor. The literature has focuses instead on the effect on the conditional expectation of the dependent variable. These authors use the score statistic to approximate the changes in hyperparameters.

Any type of intervention statistics can be computed from the output of the KFS, in particular, from the smoothing errors, as shown in de Jong and Penzer (1998). The test statistics for any number of interventions can be generated by a single run of the KF and smoother under the null model, i.e. without interventions.

The following subsections provide a review of the relevant algorithms associated to the state space model.

### 8.5.1 The Kalman filter

The Kalman filter (KF) is a recursive algorithm which processes the observations one by one and returns the innovation,  $\nu_t = y_t - E(y_t|Y_{t-1})$ ,  $Y_t = \{y_1, \dots, y_t\}$  and their variance, the one-step-ahead prediction of the state vector and its prediction error variance.

The KF also enables the evaluation of the likelihood function by performing a prediction error decomposition.

The Kalman filter (see Anderson and Moore (1979), Harvey (1989) and Durbin and Koopman (2001)) is a fundamental algorithm for the statistical treatment of a state space model. Under the Gaussian assumption it produces the minimum mean square estimator of the state vector along with its mean square error matrix, conditional on past information; this is used to build the one-step-ahead predictor of  $y_t$  and its mean square error matrix. Due to the independence of the one-step-ahead prediction errors, the likelihood can be evaluated via the prediction error decomposition.

Let us start from the simple case when the vector  $\beta$  is fixed and known and  $\alpha_1 \sim N(\alpha_{1|0}, P_{1|0})$ , where  $\alpha_{1|0}, P_{1|0}$  are known. Then, defining  $Y_t = \{y_1, y_2, \dots, y_t\}$ , the information set up to and including time  $t$ , and the conditional mean and variance of the state vector,  $\tilde{\alpha}_{t|t-1} = E(\alpha_t|Y_{t-1})$ ,  $\text{Var}(\alpha_t|Y_{t-1}) = \sigma^2 P_{t|t-1}$ , the KF is given by the following recursive formulae and definitions for  $t = 1, \dots, n$ :

$$\begin{aligned} \nu_t &= y_t - Z_t \tilde{\alpha}_{t|t-1} - X_t \beta, & F_t &= Z_t P_{t|t-1} Z_t' + G_t G_t', \\ & & K_t &= (T_t P_{t|t-1} Z_t' + H_t G_t') F_t^{-1}, \\ \tilde{\alpha}_{t+1|t} &= T_t \tilde{\alpha}_{t|t-1} + W_t \beta + K_t \nu_t, & P_{t+1|t} &= T_t P_{t|t-1} T_t' + H_t H_t' - K_t F_t K_t'. \end{aligned} \quad (8.10)$$

### 8.5.2 State fixed interval smoother

Smoothing deals with the estimation of the components and the disturbances based on the full sample of observations. A comprehensive account of the classic algorithms is given in Anderson and Moore (1979), ch. 7. In the Gaussian case the fixed interval smoother provides the minimum mean square estimator of  $\alpha_t$  using  $Y_n$ ,  $\tilde{\alpha}_{t|n} = E(\alpha_t|Y_n)$ , along with its MSE matrix  $P_{t|n} = E[(\alpha_t - \tilde{\alpha}_{t|n})(\alpha_t - \tilde{\alpha}_{t|n})'|Y_n]$ . The computations can be carried out efficiently using the following backwards recursive formulae, given by Bryson and Ho (1969) and de Jong (1989), starting at  $t = n$ , with initial values  $r_n = 0$  and  $N_n = 0$ :

$$\begin{aligned} r_{t-1} &= L_t' r_t + Z_t' F_t^{-1} \nu_t, & N_{t-1} &= Z_t' F_t^{-1} Z_t + L_t' N_t L_t, \\ \tilde{\alpha}_{t|n} &= \tilde{\alpha}_{t|t-1} + P_{t|t-1} r_{t-1}, & P_{t|n} &= P_{t|t-1} - P_{t|t-1} N_{t-1} P_{t|t-1}, \end{aligned} \quad (8.11)$$

where  $L_t = T_t - K_t Z_t'$ . A preliminary forward KF pass is required to store the quantities  $\tilde{\alpha}_{t|t-1}$ ,  $P_{t|t-1}$ ,  $\nu_t$ ,  $F_t$  and  $K_t$ . The proof of (8.11) is found in de Jong (1989).

In the state space framework two other sets of residuals can be defined, complementing the KF innovations in providing further diagnostic quantities. The *smoothing errors* (de Jong (1988a), Kohn and Ansley (1989), sometimes referred to as *smoothations*,

$$u_t = F_t^{-1}v_t - K_t' r_t,$$

with variance

$$M_t = F_t^{-1} + K_t' N_t K_t,$$

play an essential role in interpolation and cross-validation, since it can be shown that the residual arising from deletion of the observation at time  $t$  (cross-validatory, or jackknifed, or deletion residual) is:

$$y_t - E(y_t | y_1, \dots, y_{t-1}, y_{t+1}, \dots, y_n) = M_t^{-1} u_t.$$

Algorithms for multiple deletion and cross-validation are considered by Proietti (2003).

### 8.5.3 Disturbance smoother

The disturbance smoother provides estimates (via the conditional mean) of the disturbances that drive the evolution of the states and the measurement error, conditional on the full sample. These estimates are also known as auxiliary residuals and, suitably standardized, provide test statistics for the presence of outliers.

The estimation of the disturbances  $\epsilon_t$  and  $\eta_t$  associated with the various components, referred to as *disturbance smoothing* (Koopman (1993)), is built upon the quantities  $u_t$ ; de Jong and Penzer (1998) have shown that the output of the smoothing algorithm is crucial for diagnostic checking. The irregular auxiliary residual,

$$E(\epsilon_t | Y_n) = y_t - Z_t \tilde{\alpha}_{t|n} = G_t G_t' u_t,$$

standardised by the square root of the diagonal elements in  $H_t M_t H_t$ , corresponds to what is known in the regression literature as an internally studentised residuals. The standardised smoothed estimates of the disturbances are referred to as *auxiliary residuals*.

Also, the auxiliary residuals associated with the unobserved components in  $\alpha_t$  are the elements of  $H_{t-1} H_{t-1}' r_t$ , scaled by the square root of the diagonal elements of  $H_{t-1} H_{t-1}' N_t H_{t-1} H_{t-1}'$ . They are employed to provide test statistics for outliers and structural change in the various components of interest; see de Jong and Penzer (1998) for further details.

## 8.6 An automatic detection procedure

The automatic outlier detection procedure proposed in this section draws substantially from those currently implemented in Tramo-Seats and X-12-ARIMA. In particular, it will be structured so as to feature a forward addition step followed by a backward deletion step. However, it introduces two new features:

- The null model is the BSM, rather than a seasonal ARIMA.
- Rather than monitoring the maximum of 4 outlier statistics according to type, a single statistic will highlight the presence of an outlying observation at time  $\tau$ .
- The type of outliers considered are: AO, LS, IO and seasonal shifts.
- The critical values of the outlier tests are obtained by simulation and are compared to those obtained via a Bonferroni correction.

As far as the second point is concerned, the statistic of the presence of an outlier of any type is:

$$\varrho_{\tau}^2 = \frac{\nu_{\tau}}{\sigma^2 F_{\tau}} + \sigma^{-2} r'_{\tau} N_{\tau}^{-1} r_{\tau}. \quad (8.12)$$

The rationale for  $\varrho_{\tau}^2$  has been provided by de Jong and Penzer (1998), who show that the (8.12) is the maximal statistic for any type of intervention occurring at time  $\tau$ . In fact, the statistic combines the test for an IO at time  $\tau$  (the first summand) with a test for a state intervention, which includes, for the BSM, a test for a level shift, a slope change, seasonal shifts. Under the normality assumption  $\varrho_{\tau}^2 \sim \chi(m+1)$ . When the hyperparameters and  $\sigma^2$  are estimated, the results holds asymptotically. If the hyperparameters are known, and  $\sigma^2$  is estimated without considering the  $\tau$ -th observation, the distribution is Fisher's  $F(m+1, n-k-1)$ . This provides a reference distribution if the location  $\tau$  of the outlier is known.

Monitors the statistic  $\varrho_{\tau}^2$  leads to the detection of a strange observation. The reason why  $y_{\tau}$  is outlying will be later investigated by envisaging likely causes: AO, IO, LS, SC and seasonal changes are obvious candidates in the BSM framework.

### 8.6.1 Critical values

Outlier detection provides a case of multiple testing. The number of tests is larger than the sample size  $n$  (it is typically a multiple of it by a factor representing the number of outlier types), and the test statistics are correlated. The problem of controlling the type I error rate arises. One possibility is to estimate the marginal distribution of the test statistic by simulation; the latter, however, depends at least on the sample size. It is yet to be explored the role of the correlation (induced by the parameters of the generating model).

The Bonferroni correction provides a way of casting a lower bound on the size of a sequential test of hypothesis. Assume for simplicity that there exists a single outlier type with signature  $x_t$ . The null hypothesis of no outlier at a specific time  $\tau$ ,  $H_0 : \delta = 0$ , can be tested by the test statistic  $t_{\tau}$ , whose null distribution is Students'  $t$  with  $n^*$  degrees of freedom,  $t \sim T$ . The null hypothesis that there is no outlier in the sample can be tested by the statistic  $\max\{|t_{\tau}|, \tau = 1, \dots, n\}$ . The distribution of the maximum statistic is not easy to derive, due to the lack of independence of the individual sample statistics  $t_{\tau}$ . However, a conservative critical value can be obtained by the following argument.

Let  $\alpha$  be the size of the test. Then, we aim at determining the critical value  $c_{\alpha}$  such that

$$P\left(\bigcap_{\tau=1}^n \{|t_{\tau}| \leq c_{\alpha}\}\right) > 1 - \alpha$$

Equivalently, by de Morgan's law,

$$P\left(\bigcup_{\tau=1}^n \{|t_{\tau}| > c_{\alpha}\}\right) \leq \alpha.$$

By Bonferroni inequality,

$$nP(|t_{\tau}| > c_{\alpha}) \geq P\left(\bigcup_{\tau=1}^n \{|t_{\tau}| > c_{\alpha}\}\right),$$

If  $c_{\alpha}$  is chosen as the  $1-\alpha/(2n)$  percentile of the  $T$  distribution, i.e.  $P(|t_{\tau}| > c_{\alpha}) \leq \alpha/n$ , then  $P(|\max\{|t_{\tau}|, \tau = 1, \dots, n\}| > c_{\alpha}) = P(\bigcup_{\tau=1}^n \{|t_{\tau}| > c_{\alpha}\}) \leq \alpha$ .

The problem with the Bonferroni approach is that the actual size can be much smaller than the nominal. To obtain further insight on the issue, we conducted a simulation experiment which aims at estimating the sampling distribution of the maximal statistic and its quantiles by Monte Carlo simulation for different sample sizes. In particular, we simulated  $M$  time series of length  $n = 12 \times m$ , where  $m = 6, 8, 10, 12, 15, 18, 22, 25, 30, 35$ , is the number of years in the sample, and computed  $\max \varrho_{\tau}^2$ , that is the maximum of the maximal test statistic.

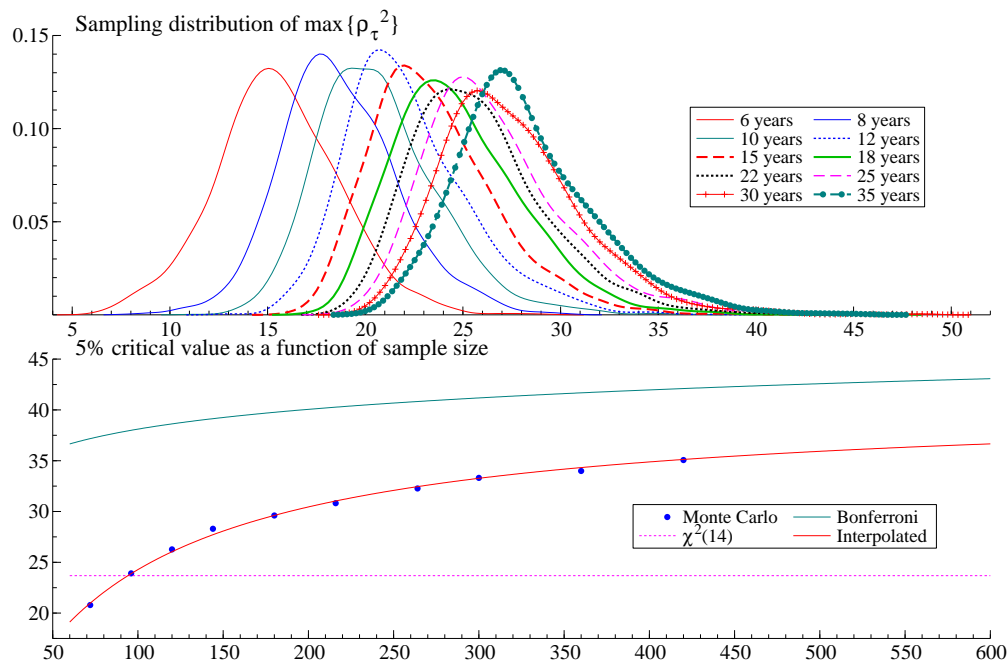
Figure 8.1: Monte Carlo Distribution of  $\max\{\rho\tau^2\}$ 

Table 8.2: Estimated 5% critical value for the maximal test statistic

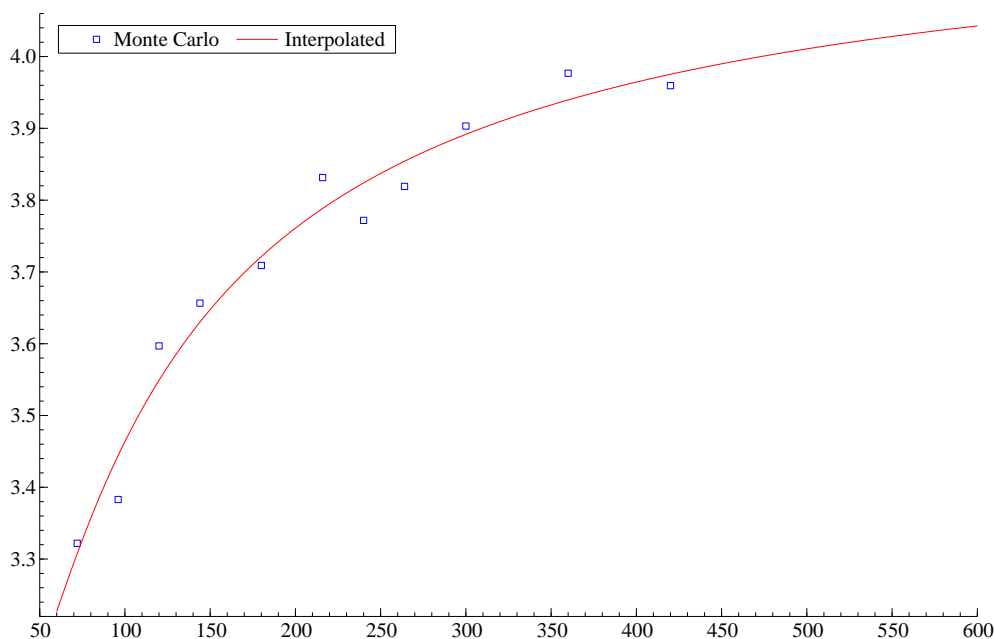
$n$	$q_{95}(n)$
60	19.118
120	26.040
180	29.614
240	31.796
300	33.266
360	34.325
420	35.123
480	35.746
540	36.247
600	36.657

Figure 8.1 presents the estimated distribution of the maximal statistic in the first panel. As expected, the mean and the quantiles of the distribution increase with the sample size. The empirical 95th percentiles are plotted in the bottom panel against the sample size  $n$  and are compared with the 5% Bonferroni critical value and that of the  $\chi^2$  distribution with 14 degrees of freedom. The solid red line is the 95th quintile interpolated by a hyperbolic regression model, giving the fitted values

$$q_{95}(n) = \left[ 0.024499 + 1.6685 \frac{1}{n} \right]^{-1}.$$

Table 8.2 displays the values of  $q_{95}(n)$  for typical values of  $n$ .

The same experiment Monte Carlo experiment delivered the sampling distribution of the  $\max\{t\tau\}$  statistic. The latter is less sensitive to the sample size, as it is evident from the plot of the estimated critical value at the 5% level versus  $n$  (see figure 8.2). Again the Bonferroni critical values are larger than the estimated ones.

**Figure 8.2:** 5% critical value of the  $\max\{|t_\tau|\}$  statistic**Table 8.3:** Estimated 5% critical value for the  $\max |t_{j\tau}|$  test statistic

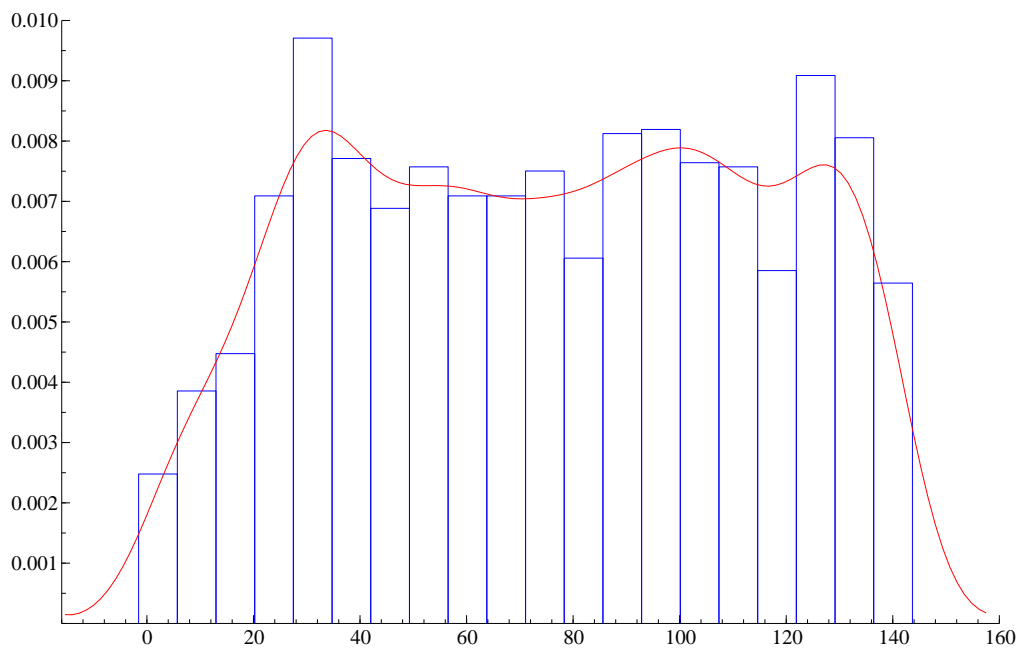
$n$	$q_{t\tau,95}(n)$
60	3.23
120	3.55
180	3.72
240	3.82
300	3.89
360	3.94
420	3.98
480	4.00
540	4.02
600	4.04

The interpolation for any value of  $t$  is done by hyperbolic regression

$$q_{t\tau,95}(n) = \left[ 0.23710 + 6.3538 \frac{1}{n} - 119.26 \frac{1}{n^2} \right]^{-1}.$$

The location of the maximum is not uniform across the time range. In fact, the probability of (mistakenly) detecting an outlier at the beginning and the end of the series is lower at the extremes of the series. See figure 8.3, which presents the distribution of  $\tau$  for  $\tau \in 1, \dots, 144$ .

It should be noticed that  $\tau$  which maximizes  $\max\{|t_\tau|\}$  need not coincide with that maximizing  $|t_{j\tau}|$ ; this is so since the latter looks at specific interventions (e.g. pulse interventions, level shifts, etc.), whereas the former looks at a combination of interventions and thus considers the interactions between single interventions.

**Figure 8.3:** Monte Carlo distribution of  $\tau$  (outlier location) for which  $\{|t_{\tau}|\}$  is maximum.

### 8.6.2 Outline of the procedure

The proposed procedure takes the following steps:

1. Initialisation: estimate the parameters of the BSM with calendar effects and any known intervention by maximum likelihood. The scale parameter  $\sigma^2$  is concentrated out of the likelihood function.
2. Forward addition step. Outliers are added one by one.
  - a) Calculate a robust estimate of the prediction error variance,  $\hat{\sigma} = 1.4826 \times \text{median}(|\tilde{\nu}_t|)$ , where  $\tilde{\nu}_t = \nu_t / \sqrt{F_t}$  are the scaled innovations computed by the Kalman filter, using the current parameter estimates.
  - b) Calculate the  $\varrho\tau^2$  statistics for the presence of outliers not currently in the model, and compute its maximum:  $\bar{\varrho}\tau^2 = \max_{\tau} \{\varrho\tau^2\}$ .
  - c) An outlier is identified and located if  $\bar{\varrho}\tau^2 > c\alpha$ , where  $c\alpha$  is the critical value of the test statistic at the  $\alpha$  level. Otherwise, the procedure stops.
  - d) If  $\bar{t}_{j,\tau} > c\alpha$ , determine the outlier type at time  $\tau$  by selecting the type which is associated to the largest  $t$  statistic at time  $\tau$ :  $\max_j \{|t_{j\tau}|, j = AO, LS, SC, SO_1, \dots, SO_6\}$ .
  - e) Add the relevant regressor  $x_t$  to the model to correct the series from the effects of the outlier (one could optionally re-estimate the parameters of the BSM).

Repeat steps (a)-(e) until there are no additional outliers are found.

3. Backward Deletion step. Start with the model including all outlier regressors added in the forward addition stage.
  - a) Calculate maximum likelihood estimates of the parameters, including the p.e.v..
  - b) Using the estimated parameters, calculate the  $t$ -value for all the outliers identified in forward addition that remain in the model. Determine which of these regressors has  $\min |t_{j,\tau}|$ .



c) If  $\min |t_{j,\tau}| < C$ , delete this regressor from the model and go to (a). Otherwise, stop.

### 8.6.3 Comments

- The critical values for the  $\max\{\varrho_\tau^2\}$  statistic obtained by Bonferroni method result very conservative. This is due to the high serial autocorrelation characterising the statistic.
- One drawback of the  $\{\varrho_\tau^2\}$  statistic is that it is not available for the initial stretch of observations. One possibility is to use the GLS residuals rather than the innovations (generalised recursive residuals)  $y_t - E(y_t|Y_{t-1})$ , as they are defined also for  $t = 1, \dots, K$ , where  $K$  is the number of elements of the vector  $\beta$ . Essentially, the GLS residual is conditional on the GLS estimates of the initial and regression effects. Alternatively, we can continue to use the recursive residuals, but for the initial stretch of observations we use  $\{\varrho_\tau^2\} = \sigma^{-2}r_\tau' N_\tau^{-1}r_\tau$ . In this case an IO cannot be identified at the beginning of the series.
- The MAD estimator assumes normality. When normality is violated, it may actually result larger than the ML estimator. We always consider the smallest between the two in our procedure.
- In the forward addition step the effects of the outliers on the hyperparameters estimates is neglected. One possibility to overcome this situation is to adopt the estimation strategy proposed in Atkinson and al (1997), which consists of performing a single iteration of the scoring algorithm.
- For the backward deletion step the critical value for the minimum  $t$ -statistic is the same used for the forward addition step. This may be inappropriate and the issue arises whether standard critical values ought to be used, which implies that we are now assuming that the location of the outliers is known.

## 8.7 Illustrations

The recent economic crises provides an interesting testbed for outlier detection procedures. As a particularly dramatic form of structural break we are interested in dating its inception and what type of change is detected. The data set used in the illustration consists of the industrial production index for Italy. The series are available for the period starting in January 1990 and ending in December 2009.

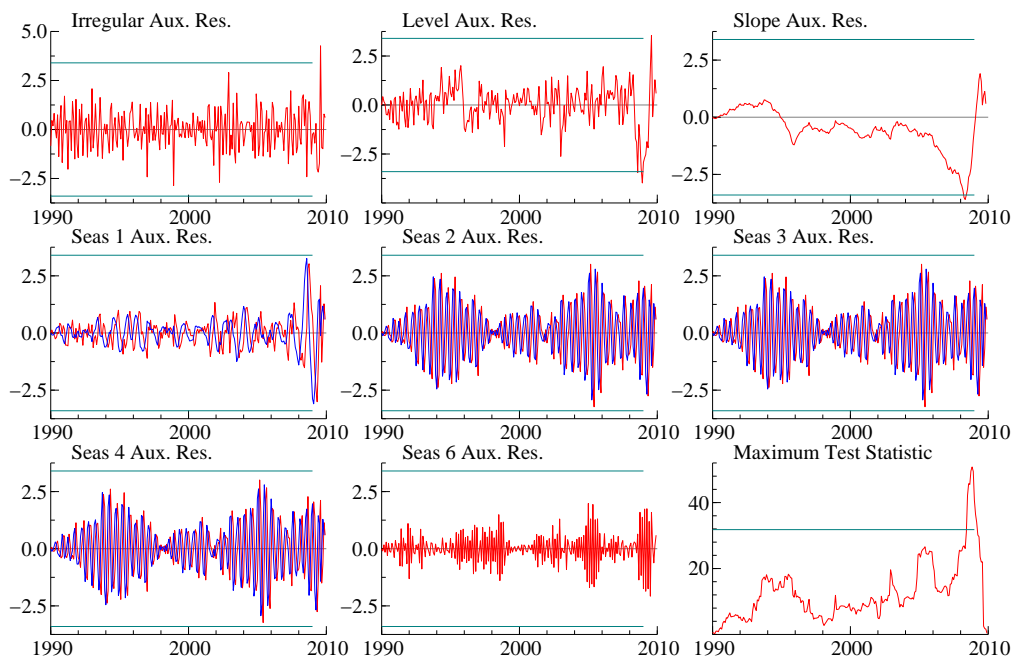
### 8.7.1 Italian industrial production: Sector CK

Our first illustrations deals with the Italian industrial production index for the sector CK of the NACE Rev. 1.1 classification, available for the sample period 1990.1-2009.12.

The automatic outlier detection routine of X-12-ARIMA identifies two outliers (the estimated model is the Airline model). Six trading days and Easter regressors were included in the model.

Variable	Parameter Estimate	Standard Error	t-value
-----			
Automatically Identified Outliers			
LS2008.Dec	-20.8641	3.67616	-5.68
A02009.Aug	19.8054	4.38898	4.51

When the BSM with calendar effects is fitted to the series assuming no outlier, the hyperparameter estimates are:  $\hat{\sigma}_\epsilon^2 = 5.1509$ ,  $\hat{\sigma}_\eta^2 = 5.6353$ ,  $\hat{\sigma}^2_\eta = 0.0021$ ,  $\hat{\sigma}_\omega^2 = 0.0704$ ; the prediction error variance is estimated  $\hat{\sigma}^2 = 29.309$ .

**Figure 8.4:** Italy, Index of Industrial Production (sector CK). Standardized auxiliary residuals.

The largest value of the statistic  $\max\{\varrho_{\tau}^2\}$  corresponds to December 2008; the selected type is LS. Figure 8.4 presents the standardised auxiliary residuals and the maximal test statistic. Notice that the highest  $t$  statistic would point out to an AO in August 2009. However, the  $\max\{\varrho_{\tau}^2\}$  statistic for that period is not significant. The forward addition step, thus detects a LS and then a seasonal change in August 2008 affecting the first trigonometric cycle. We include two interventions corresponding to a truncated sine and cosine function at frequency  $\pi/6$ . The subsequent backward deletion step deletes the last two interventions. Hence, the only outlier identified is a LS. The final components are presented in figure 8.5.

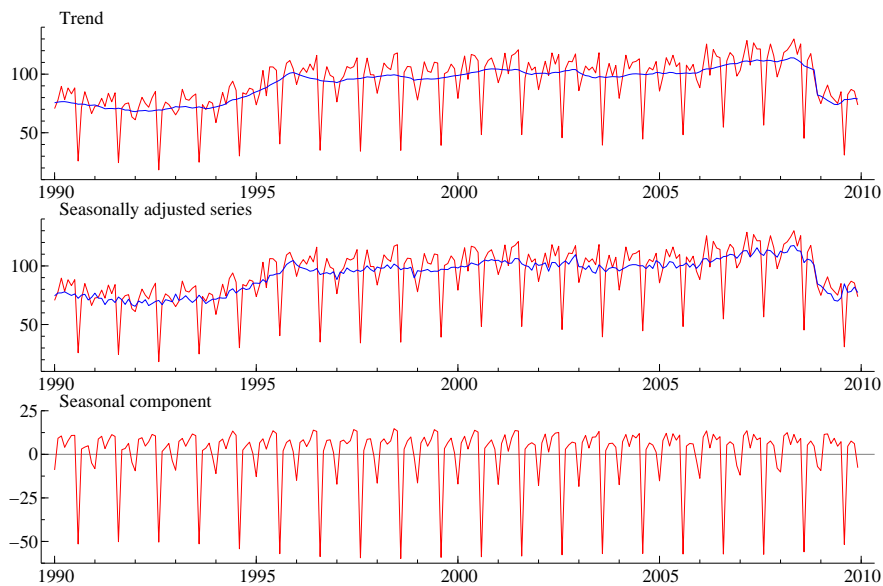
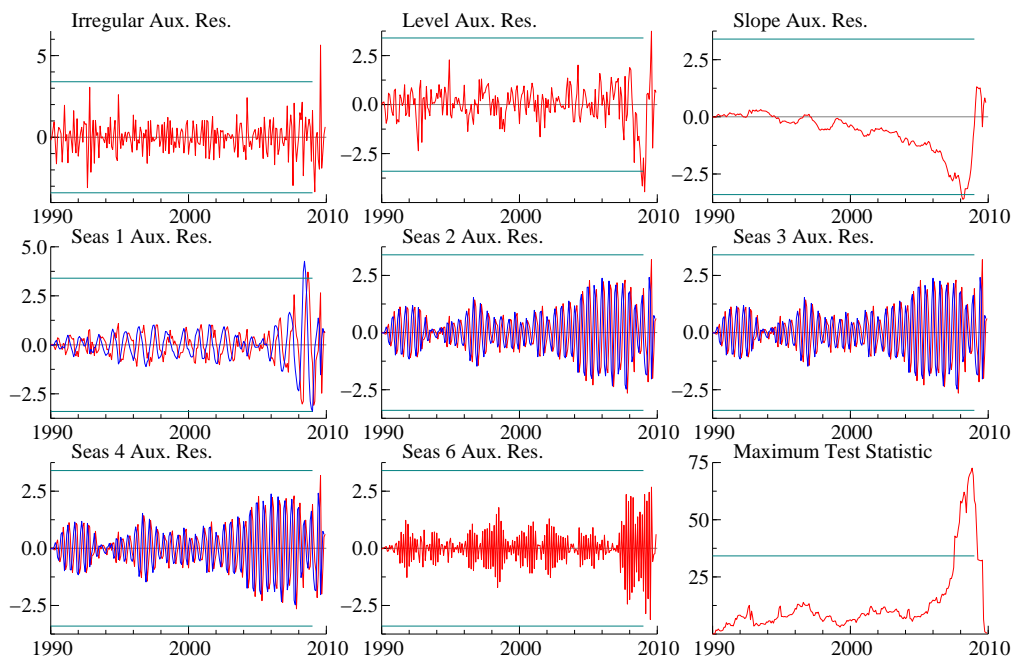
### 8.7.2 Italian industrial production: sector CH

Our second illustration deals with the industrial production series for the sector CH. X-12-ARIMA identifies several level shifts and an AO in August 2009.

Variable	Parameter Estimate	Standard Error	t-value
Automatically Identified Outliers			
LS2008.Sep	-11.1705	2.54327	-4.39
LS2008.Dec	-13.8589	2.66424	-5.20
LS2009.Feb	-14.5382	2.80202	-5.19
A02009.Aug	21.6003	2.96474	7.29

When the BSM is fitted, our procedure identifies two level shifts occurring in September and December 2008, which coincide with those identified by X-12-ARIMA. Furthermore, two seasonal outliers, corresponding to the interventions  $SO_{s_{3\tau}}$  (see table 8.1) for May 2008 and  $SO_{c_{1\tau}}$  for October 2008.

It should be noticed from figure 8.6 that the maximum of the statistic  $\max\{\varrho_{\tau}^2\}$  (occurring in December 2008) does not correspond to the maximum of the  $|t_{j\tau}|$  statistic, which points out to an AO occurring in August 2009. Hence, our procedure takes a different route, identifies a LS in December 2008 and, once its effects are accounted for the evidence for an AO in August 2009 becomes less strong and seasonal change is captured.

**Figure 8.5:** Italy, Index of Industrial Production, sector CK. Final estimated components.**Figure 8.6:** Italy, Index of Industrial Production (sector CH). Standardized auxiliary residuals.

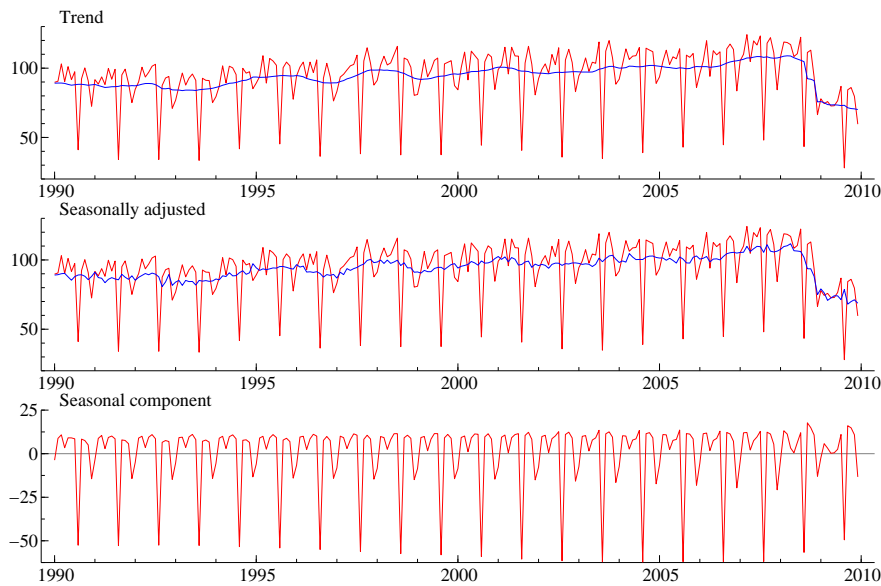
**Figure 8.7:** Italy, Index of Industrial Production, sector CH. Final estimated components.

Figure 8.7 displays the final estimates of the components.

### 8.7.3 Italian industrial production: sector CG

When we automatic detection procedure is based on  $\max\{t_{j,\tau}, j = IO, A0, LS, SC\}$ , the results are the same obtained by Tramo-Seats and X-12-ARIMA. This can be seen for the industrial production series for the sector CH. Both X-12-ARIMA and our procedure based on fitting the BSM identify two level shifts occurring in September and December 2008, and an AO in December 2006. which coincide with those identified by X-12-ARIMA.

Figure 8.8 displays the maximal statistic  $\max\{\rho_{\tau}^2\}$  and the auxiliary residuals for the irregular and the level components, whereas figure 8.9 displays the final estimates of the components.

### 8.7.4 A tale of a seasonal outlier

This illustration deals with a genuine seasonal outlier.

The European Union (EU) member countries compute the Harmonized Index of Consumer Prices (HICP) according to a harmonized methodology which enhances the international comparisons of inflation.

The HICP is a chain index of the Laspeyres-type, tracking the changing price of a fixed basket of goods and services over time. In compliance with the methodology of chain indices, the basket and weights are updated on an annual basis. Although the basket is fixed the weights are country specific.

The Italian HICP suffered a major structural break in January 2001, since when, the HICP has taken into account temporary price reductions (i.e. sales), which were previously excluded.

The series is available for the period Jan. 1996- June 2010. The X-12-ARIMA automatic outlier detection procedure does not identify any outlying observations.

On the contrary, our automatic procedure identifies a seasonal outlier referring to the first harmonic cycle (2 cycles per year, period equal to six months). This is plausible, since there are two price reduction periods

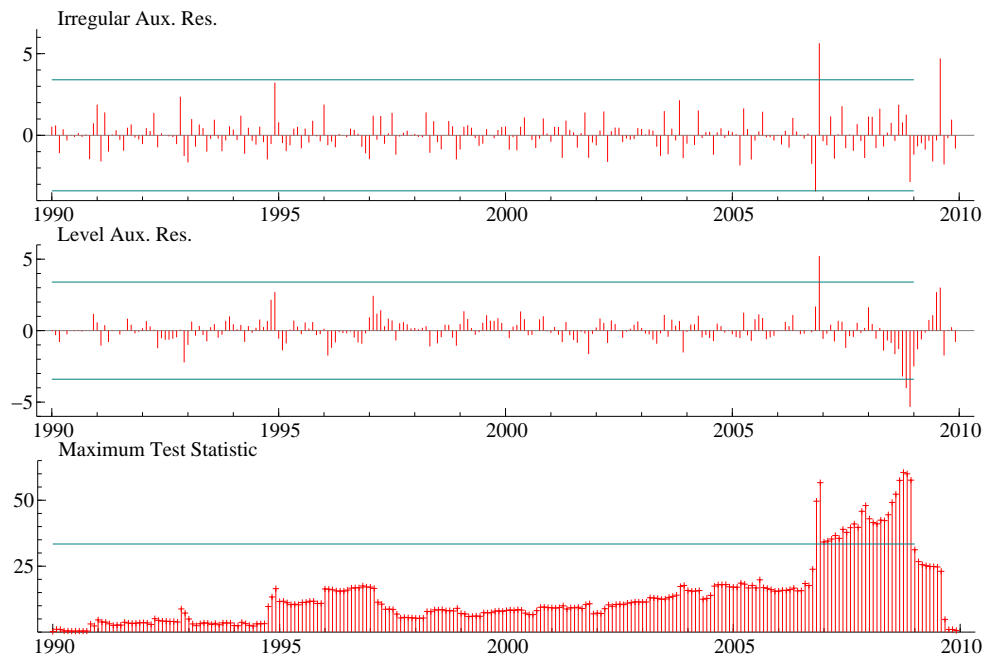
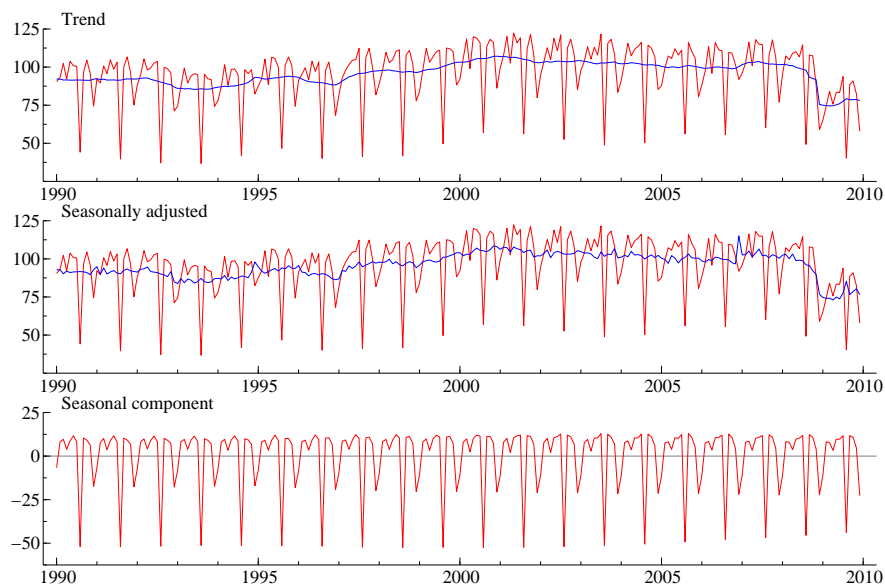
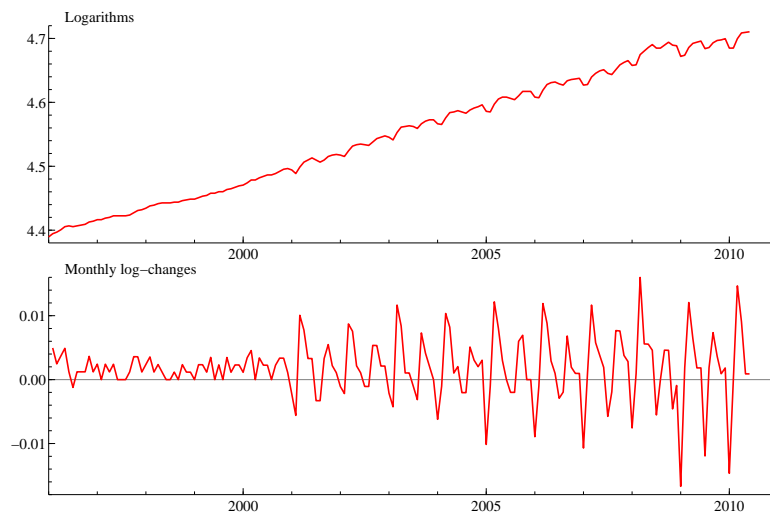
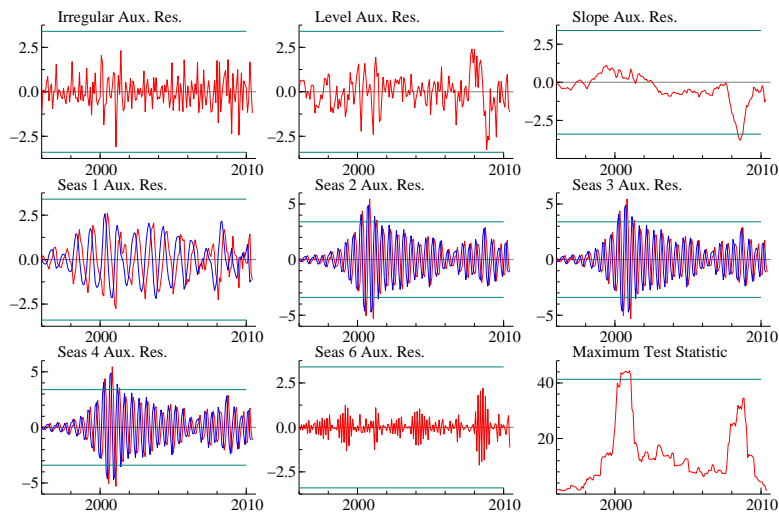
**Figure 8.8:** Italy, Index of Industrial Production (sector CG). Standardized auxiliary residuals.**Figure 8.9:** Italy, Index of Industrial Production, sector CG. Final estimated components.

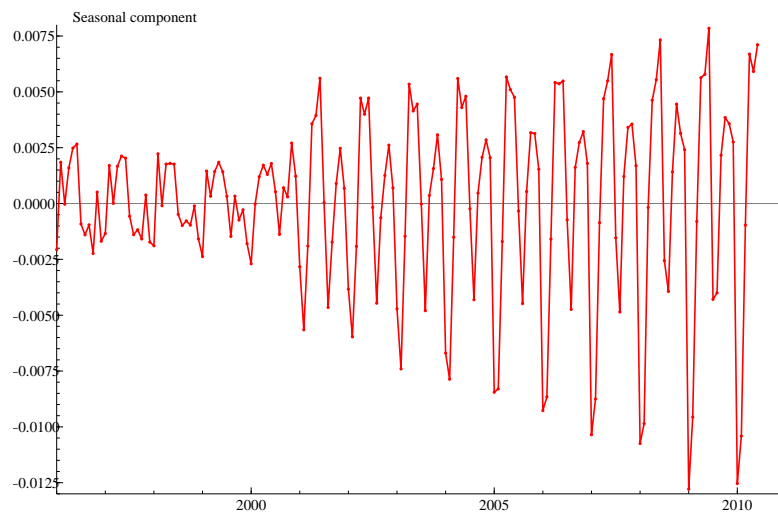
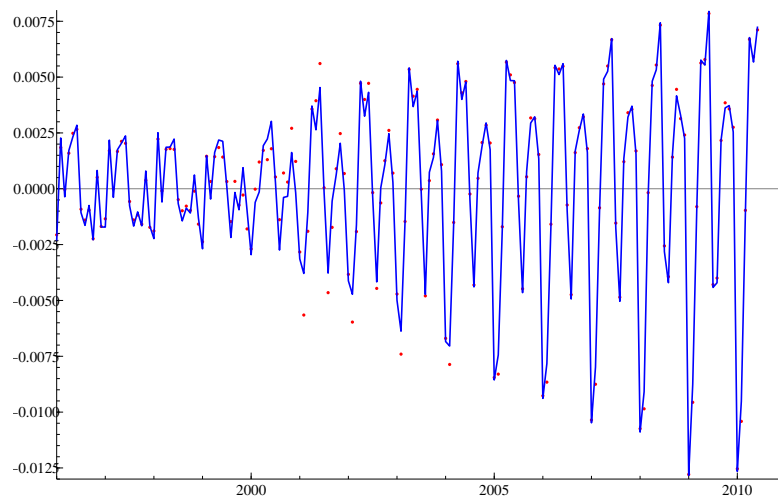
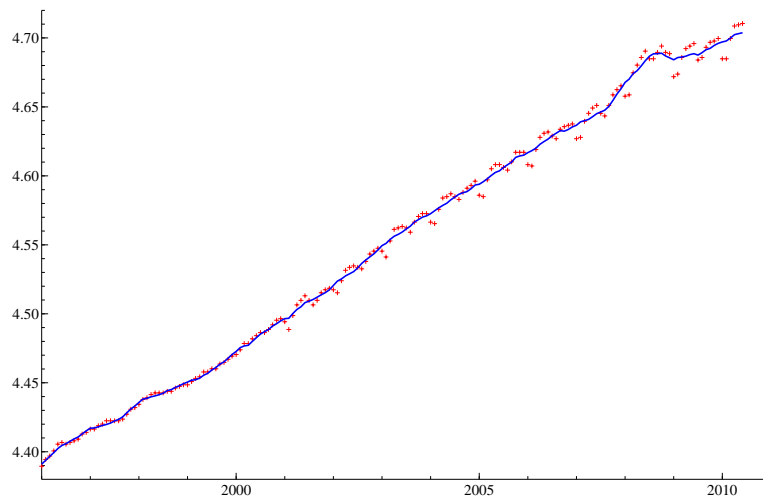
Figure 8.10: Italy, Harmonized Index of Consumer Prices.

Figure 8.11: Italy, Harmonized Index of Consumer Prices. Standardized auxiliary residuals and  $\rho_T$  statistic.

corresponding to January and July.

Figure 8.12 displays the seasonal component resulting from the stochastic seasonal component and the effect of the outlier, whereas figure 8.12 compares it to the seasonal component extracted when no allowance is made for a structural change. It is evident from the plot that the effect of the change is smeared across subsequent observations.

Figure 8.14 overlays the seasonal adjusted series to the original one.

**Figure 8.12:** Final seasonal component.**Figure 8.13:** Seasonal component. No outlier fitted.**Figure 8.14:** Final Seasonally adjusted series



## 8.8 Endogenous outliers and Non-Gaussian seasonal adjustment

A different route to handle outliers and breaks within seasonal adjustment is to allow the disturbances of the BSM to have heavy-tailed distributions. In a sense the outliers are made endogenous and their estimation does not require a separate procedure.

The earliest papers proposing handling structural breaks by non-Gaussian models are due to Kitagawa (1987), Kitagawa (1989). Two main approaches have been explored by the literature:

- Robust seasonal adjustment with Student's  $t$  disturbances.
- Normal scale mixture models for the disturbances

The first approach has been investigated by Durbin and Koopman (1999). Aston and Koopman (2006) consider it with respect to the canonical decomposition of the Airline mode. Estimation is carried out by maximum likelihood, using Monte Carlo importance sampling to estimate the likelihood and the components. The second by Bruce and Jurke (1996), among others. That reference adopts an approximate likelihood estimation method based on the Gaussian sum smoother of Kitagawa, and provided an extensive examination of the feasibility of robust seasonal adjustment based on mixtures. The mixture approach is also considered by Giordani et al. (2005).

While the  $t$  distribution is chosen because it depends on a single parameter, the merits of the mixture approach is that it enables to locate the outliers by the posterior probability of the mixture indicator.

A more thorough analysis of the non-Gaussian approach is beyond the scope of this report, but we plan to investigate its merits in comparison with the classical procedures.

## 8.9 Conclusions

The structural approach provides a natural framework for detecting structural change and outlying observations in economic time series. The necessary statistics are available immediately from the classical Kalman filter and smoothing equations. Handling special features, such as seasonal changes and outliers is immediate. This paper has evaluated a sequential procedure for the detection of outliers which has several new features, being based on a maximal statistic and contemplating the possibility of detecting seasonal breaks.

There are two main aspects that we think should be investigated further: sequential detection procedures are prone to masking, which is produced by patches of unusual observations. Approaches to the detection of multiple outliers in an unobserved components framework are Proietti (2000), who deals with leave- $k$ -out diagnostics, and Penzer (2007a), who proposes put- $k$ -shocks-in statistics. Secondly, the maximum likelihood estimators of the parameters are not robust to the outliers. Their influence may cause a swamping effect, such that a good observation appears to be outlying. A very promising solution to both problems has been recently proposed by Riani et al. (2009).

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# 9

## Transformations and Seasonal Adjustment of Economic Time Series



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## Table of Contents

9.1	Introduction	197
9.2	Box Cox family of transformations	198
9.3	Transformations and seasonal adjustment: state of the art	202
9.3.1	TRAMO/SEATS	202
9.3.2	X12 ARIMA	202
9.3.3	SABL	203
9.4	A unified approach to transformations and seasonal adjustment	203
9.4.1	Seasonal Adjustment and the Box-Cox Transformation	204
9.4.2	Analytical solutions	205
9.4.3	Approximate and computational solutions	206
9.5	Robustness issues in transformations	207
9.6	Conclusions	208



## 9.1 Introduction

Transformations aim at establishing a scale, different from the original measurements, for which the linear Gaussian model holds. The analysis of time series data is often improved by using a transformation of the response rather than the original response itself. There are physical reasons why a transformation might be expected to be helpful in some examples. If the data arise from a counting process, they often have a Poisson distribution and the square root transformation will provide observations with an approximately constant variance, independent of the mean. Most time series are non-negative, show variations which increase as the level of the series increases and therefore cannot be subject to additive errors of constant variance. Such effects are most noticeable if there are observations both close to, and far from, zero as it frequently happens in real time series with strong seasonal movements.

In this chapter we address the issue of data transformations in the context of seasonal adjustment. In adjusting the data for seasonal variations, a fundamental question is whether to decompose the series with additive decomposition or multiplicative decomposition, i.e., whether to use no transformation or a log transformation or an intermediate transformation like the square root.

Seasonal adjustment rests upon two basic pillars: additivity and orthogonality of the seasonal and non seasonal components. This point is made strongly by Bell and Hillmer (1984, sec. 4.2), who state that “someone who does not want to make these assumptions is working on a different problem”. In order to achieve additivity of the components most institutions (like for example the U.S. Census Bureau) adjust time series data for seasonal variations using logarithmic transformations for forecast extension and a multiplicative model for the forecast-extended series. The use of the log transformation is motivated by the fact that the seasonal variations in most seasonal economic time series increase and decrease proportionally with increases and decreases in the level of the series.

Clearly, the decision of transforming or not transforming the data substantially affects the forecast error, especially if the variation in the series occurs at the end of the series, because most forecast methods place more weight on the most recent data. In addition, taking logs when the seasonal pattern does not change with the level, has the effect of producing a series whose seasonal variability does change with the level.

The above considerations show it is very important to choose the most appropriate transformation parameter before modeling time series and carrying out seasonal adjustment. It must also be remarked that applying different transformations to components can raise questions about the implied seasonal component of the aggregate. Finally, the choice of the best transformation is often complicated by the presence of outliers. It is well known in the statistical literature that the presence of atypical observations may lead to wrong conclusions about the transformation parameter. In general, prior outlier detection is performed on the original scale before estimating the transformation parameter. It is clear, however, that observations which seem atypical on the original scale may fit completely inside the bulk of the data once the data have been transformed.

This chapter tackles all the above issues, has the purpose both to review the state of the art of transformations and seasonal adjustment and to show how it is possible at the same time: 1) to robustly estimate the transformation parameter; 2) to evaluate the effect that the different seasons exert on this estimate; 3) to obtain, in a closed form solution, the posterior mean and variance of the nonseasonal component in the original scale, once the data have been transformed.

The structure of the chapter is as follows. In section 9.2 we review the normalized Box and Cox family of transformations and recall the score test for transformation. In section 9.3 we review the procedures which are currently used by the most popular softwares to estimate the transformation parameter and to carry out seasonal adjustment. More in detail, we recall the approaches to data transformation used by X12 ARIMA, TRAMO SEATS, STAMP and SABL. In section 9.4 we discuss why in our opinion, the transformation problem has not received sufficient recognition in the current seasonal adjustment practice and discuss the seasonal balance constraint. We also show how it is possible from a theoretical and computation point of view, to obtain the posterior mean and variance of the nonseasonal component in the original scale, once the data have been

transformed. In Section 9.5 we deal with the problem of robust estimation of the transformation parameter. Section 9.6 concludes and provides food for thought for additional research. Mathematical details are left to the two appendices at the end of the chapter.

## 9.2 Box Cox family of transformations

We initially recall the Box Cox family for the linear regression model  $y = X\beta + \epsilon$  with uncorrelated errors ( $\epsilon \sim N(0, \sigma^2 I_n)$ ) and then extend it to the time series (ARIMA or structural) framework.

For transformation of just the response  $y$  in the linear regression model, Box and Cox (1969) analyze the normalized power transformation

$$z(\lambda) = \begin{cases} \frac{y^\lambda - 1}{\lambda \dot{y}^{\lambda-1}} & \lambda \neq 0 \\ \dot{y} \log y & \lambda = 0, \end{cases} \quad (9.1)$$

where the geometric mean of the observations is written as  $\dot{y} = \exp(\Sigma \log y_i / n)$ . The model fitted is multiple regression with response  $z(\lambda)$ ; that is,

$$z(\lambda) = X\beta + \epsilon. \quad (9.2)$$

When  $\lambda = 1$ , there is no transformation:  $\lambda = 1/2$  is the square root transformation,  $\lambda = 0$  gives the log transformation and  $\lambda = -1$  the reciprocal. These are the most widely used transformations, frequently supported by some empirical reasoning. For example, time series often have a standard deviation proportional to the mean, so that the variance of the logged response is approximately constant (See Annex A). For this form of transformation to be applicable, all observations need to be positive. For it to be possible to detect the need for a transformation the ratio of largest to smallest observation should not be too close to one.

The purpose of the analysis is to find an estimate of  $\lambda$  for which the errors in the  $z(\lambda)$  (9.2) are, at least approximately, normally distributed with constant variance and for which a simple linear model adequately describes the data. This is achieved by finding the maximum likelihood estimate of  $\lambda$ , assuming a normal theory linear regression model.

Once a value of  $\lambda$  has been decided upon, the analysis is the same as that using the simple power transformation

$$y(\lambda) = \begin{cases} (y^\lambda - 1)/\lambda & \lambda \neq 0 \\ \log y & \lambda = 0 \end{cases}. \quad (9.3)$$

However the difference between the two transformations is vital when a value of  $\lambda$  is being found to maximize the likelihood, since allowance has to be made for the effect of transformation on the magnitude of the observations<sup>1</sup>

The likelihood of the transformed observations relative to the original observations  $y$  is

$$(2\pi\sigma^2)^{-n/2} \exp\{-(y(\lambda) - X\beta)^T (y(\lambda) - X\beta) / 2\sigma^2\} J,$$

where the Jacobian

$$J = \prod_{i=1}^n \left| \frac{\partial y_i(\lambda)}{\partial y_i} \right| \quad (9.4)$$

allows for the change of scale of the response due to transformation (Annex B). A simpler, but identical, form

<sup>1</sup> The geometric mean has the same dimension as the arithmetic mean and as  $y$ , so the dimension of  $z(\lambda)$  is that of  $y$ . The same is true for  $z(0)$  since changing the scale of measurement of the  $y$  merely adds a constant to this  $z$ . Therefore the response  $z(\lambda)$  in the regression model has the dimension of  $y$  whatever the value of  $\lambda$ . Sums of squares can therefore be directly compared (see also Bickel and Doksum (1981) and Box and Cox (1982)).



for the likelihood is found by working with the normalized transformation, defined in general as

$$z(\lambda) = y(\lambda)/J^{1/n},$$

for which the Jacobian is one. The likelihood is therefore now

$$(2\pi\sigma^2)^{-n/2} \exp\{-(z(\lambda) - X\beta)^T(z(\lambda) - X\beta)/2\sigma^2\}, \quad (9.5)$$

a standard normal theory likelihood for the response  $z(\lambda)$ . For the power transformation (9.3),

$$\frac{\partial y_i(\lambda)}{\partial y_i} = y_i^{\lambda-1}$$

so that

$$\log J = (\lambda - 1) \sum \log y_i = n(\lambda - 1) \log \dot{y}.$$

The maximum likelihood estimates of the parameters are found in two stages. For fixed  $\lambda$  the likelihood (9.5) is maximized by the least squares estimates

$$\hat{\beta}(\lambda) = (X^T X)^{-1} X^T z(\lambda),$$

with the residual sum of squares of the  $z(\lambda)$ ,

$$R(\lambda) = z(\lambda)^T (I - H) z(\lambda) = z(\lambda)^T A z(\lambda). \quad (9.6)$$

Division of (9.6) by  $n$  yields the maximum likelihood estimator of  $\sigma^2$  as

$$\hat{\sigma}^2(\lambda) = R(\lambda)/n.$$

Replacement of this estimate by the mean square estimate  $s^2(\lambda)$  in which  $n$  is replaced by  $(n - p)$  does not affect the development that follows.

For fixed  $\lambda$  we find the loglikelihood maximized over both  $\beta$  and  $\lambda$  by substitution of  $\hat{\beta}(\lambda)$  and  $s^2(\lambda)$  into (9.5). If an additive constant is ignored this partially maximized, or profile, loglikelihood of the observations is

$$L_{\max}(\lambda) = -(n/2) \log\{R(\lambda)/(n - p)\} \quad (9.7)$$

so that  $\hat{\lambda}$  minimizes  $R(\lambda)$ . To repeat what has already been stressed, it is important that  $R(\lambda)$  in (9.7) is the residual sum of squares of the  $z(\lambda)$ , a normalized transformation with the physical dimension of  $y$  for any  $\lambda$ . Comparisons of residual sums of squares of the simple power transformation  $y(\lambda)$  are misleading. Let

$$S(\lambda) = y(\lambda)^T (I - H) y(\lambda) \quad (9.8)$$

be the residual sum of squares of the unnormalized  $y(\lambda)$ . Suppose, for example, that the observations are of order  $10^3$ , the residual sum of squares  $S(1)$  will be of order  $10^6$ , whereas, when  $\lambda = -1$ , the reciprocal transformation, the observations and  $S(-1)$  will be of order  $10^{-6}$ . However relatively well the models for  $\lambda = 1$  and  $\lambda = -1$  explain the data,  $S(-1)$  will be very much smaller than  $S(1)$ . Comparison of these two residual sums of squares will therefore indicate that the reciprocal transformation is to be preferred. This bias is avoided by the use of  $R(\lambda)$  in (9.7), since the magnitude of  $z(\lambda)$  does not depend on  $\lambda$ .

For inference about the transformation parameter  $\lambda$ , Box and Cox suggest likelihood ratio tests using (9.7), that is, the statistic

$$T_{LR} = 2\{L_{\max}(\hat{\lambda}) - L_{\max}(\lambda_o)\} = n \log\{R(\lambda_o)/R(\hat{\lambda})\}. \quad (9.9)$$

A disadvantage of this likelihood ratio test is that a numerical maximization is required to find the value of  $\hat{\lambda}$ .

For regression models a computationally simpler alternative test is the approximate score statistic derived by Taylor series expansion of (9.1) as

$$\begin{aligned} z(\lambda) &\doteq z(\lambda_0) + (\lambda - \lambda_0) \left. \frac{\partial z(\lambda)}{\partial \lambda} \right|_{\lambda=\lambda_0} \\ &= z(\lambda_0) + (\lambda - \lambda_0)w(\lambda_0), \end{aligned} \quad (9.10)$$

which only requires calculations at the hypothesized value  $\lambda_0$ . In (9.10)  $w(\lambda_0)$  is the constructed variable for the transformation. Differentiation of  $z(\lambda)$  for the normalized power transformation yields

$$\begin{aligned} w(\lambda) &= \frac{\partial z(\lambda)}{\partial \lambda} \\ &= \frac{y^\lambda \log y}{\lambda y^{\lambda-1}} - \frac{y^\lambda - 1}{\lambda y^{\lambda-1}} (1/\lambda + \log y). \end{aligned} \quad (9.11)$$

The combination of (9.10) and the regression model  $y = X\beta + \epsilon$  leads to the model

$$\begin{aligned} z(\lambda_0) &= X\beta - (\lambda - \lambda_0)w(\lambda_0) + \epsilon \\ &= X\beta + \gamma w(\lambda_0) + \epsilon, \end{aligned} \quad (9.12)$$

where  $\gamma = -(\lambda - \lambda_0)$ . The approximate score statistic for testing the transformation  $T_p(\lambda_0)$  is the  $t$  statistic for regression on  $w(\lambda_0)$  in (9.12). This can either be calculated directly from the regression in (9.12), or from the formulae for added variables (see for example Atkinson and Riani (2000)).

The  $t$  test on the additional constructed variable  $w(\lambda_0)$ , which we denote with symbol  $T_p(\lambda)$  is known in the statistical literature as “score test statistic for transformation”.

*Remark:* because  $T_p(\lambda)$  is the  $t$  test for regression on  $-w(\lambda)$ , large positive values of the statistic mean that  $\lambda_0$  is too low and that a higher value should be considered.

Up to now, we have assumed uncorrelated observations. In presence of a time series  $y_t$ ,  $t = 1, \dots, T$ , the above theory can be easily extended both if we consider the ARIMA (Box, Jenkins and Reinsel, 1994) or the structural framework (Harvey (1989)). In the case of a seasonal ARIMA specification  $ARIMA(p, d, q)(P, D, Q)_s$  for the error term  $\epsilon_t$  we can write the model as:

$$z_t(\lambda) = \sum_{k=1}^K \delta_k x_{kt} + \epsilon_t(\lambda) \quad (9.13)$$

where  $x_{kt}$ 's are appropriate regressors that account for calendar effects, namely trading days, moving festivals (Easter) and the length of the month and  $\epsilon_t(\lambda)$  is defined as

$$\phi_p(L)\Phi_P(L^s)\Delta^d\Delta_s^D\epsilon_t(\lambda) = \theta_q(L)\Theta_Q(L^s)\epsilon_t^*$$

where  $\phi_p(L)$ ,  $\theta_q(L)$ ,  $\Phi_P(L^s)$ ,  $\Theta_Q(L^s)$  are polynomials respectively of order  $p$ ,  $q$ ,  $P$  and  $Q$  in the lag operator  $L$  satisfying the usual stationarity and invertibility conditions,  $\Delta = 1 - L$ ,  $\Delta_s = 1 - L^s$ , and  $\epsilon_t^* \sim N(0, \sigma_*^2)$ . If, on the contrary, the model is formulated directly in terms of the unobserved components (as it happens in the structural approach) we obtain<sup>2</sup>

$$z_t(\lambda) = \sum_{k=1}^K \delta_k x_{kt} + \mu_t + \gamma_t + \epsilon_t, \quad t = 1, \dots, T, \quad (9.14)$$

<sup>2</sup>Sometimes to the formulation given in equation (9.14), which is known in the literature as basic structural model, an additional component called  $\psi_t$ , which has the purpose of capturing the cyclical movements of the series, is added.

where  $\mu_t$  is the trend component,  $\gamma_t$  is the seasonal component and  $\epsilon_t = N(0, \sigma_\epsilon^2)$  is the irregular component.

The trend component has a local linear representation:

$$\begin{aligned}\mu_{t+1} &= \mu_t + \beta_t + \eta_t, & \eta_t &\sim \text{NID}(0, \sigma_\eta^2), \\ \beta_{t+1} &= \beta_t + \zeta_t, & \zeta_t &\sim \text{NID}(0, \sigma_\zeta^2),\end{aligned}$$

where  $\beta_t$  is the stochastic slope, that in turn evolves as a random walk; the disturbances  $\eta_t, \zeta_t$ , are independent of each other and of any remaining disturbance in the model.

The seasonal component has a trigonometric representation, such that the seasonal effect at time  $t$  arises from the combination of  $[s/2]$  stochastic cycles where  $s$  is the seasonal period. For example, in presence of monthly data ( $s = 12$ ) we obtain:  $\gamma_t = \sum_{j=1}^6 \gamma_{jt}$ , where, for  $j = 1, \dots, 5$ ,

$$\begin{aligned}\gamma_{j,t+1} &= \cos \lambda_j \gamma_{j,t} + \sin \lambda_j \gamma_{j,t}^* + \omega_{j,t} & \omega_{j,t} &\sim \text{NID}(0, \sigma_\omega^2) \\ \gamma_{j,t+1}^* &= -\sin \lambda_j \gamma_{j,t} + \cos \lambda_j \gamma_{j,t}^* + \omega_{j,t}^* & \omega_{j,t}^* &\sim \text{NID}(0, \sigma_\omega^2)\end{aligned}$$

and  $\gamma_{6,t+1} = -\gamma_{6t} + \omega_{6t}$ ,  $\omega_{6t}^* \sim \text{NID}(0, \sigma_\omega^2/2)$ ;  $\lambda_j = \frac{2\pi}{12}j$  is the seasonal frequency. The disturbances  $\omega_{jt}$  and  $\omega_{jt}^*$  are assumed to be normally and independently distributed with common variance  $\sigma_\omega^2$ . All the disturbances are assumed to be mutually uncorrelated.

An alternative approach to model stochastic seasonality is derived by writing

$$\gamma_t = x_t' \xi_t \quad \xi_t = \xi_{t-1} + \omega_t$$

where  $x_t' = [D_{1t}, \dots, D_{st}]$ , with  $D_{jt} = 1$  in season  $j$  and 0 otherwise. The vector  $\xi_t$  contains the effects associated to each season and changes over time according to a multivariate random walk;  $\omega_t$  is a zero-mean multivariate white noise with covariance matrix

$$\text{Var}(\omega_t) = \sigma^2 \omega [I_s - \frac{1}{s} i_s i_s'],$$

where  $i_s$  is a vector of 1s of order  $s$ , which enforces the constraint  $i_s' \text{Var}(\omega_t) = 0$ . This formulation is known in the literature as the Harrison and Stevens (HS) specification. The distinguishing feature of this approach is that it is formulated directly in terms of the effect of a particular season, thereby enhancing flexibility needed to model seasonal heteroscedasticity.

Both in the structural or in the ARIMA approach, if we want to apply the score test for transformation, the augmented model is estimated adding to the previous regressors  $\sum_{k=1}^K \delta_k x_{kt}$  the additional variable  $\delta^* w_t(\lambda_0) = (\lambda_0 - \lambda) w_t(\lambda_0)$ . For example in the structural framework the augmented model becomes:

$$z_t(\lambda_0) = \mu_t + \gamma_t + \sum_k \delta_k x_{kt} + \delta^* w_t(\lambda_0) + \epsilon_t,$$

Significant regression on  $w_t(\lambda_0)$  denotes the need for a transformation different from  $\lambda_0$  and provides a preliminary estimate of the correct  $\lambda$  as  $\hat{\lambda} = \lambda_0 - \hat{\delta}$ .

## 9.3 Transformations and seasonal adjustment: state of the art

The purpose of this section is to analyze how the issue of preliminary transforming the series is tackled by the most popular existing softwares which enable us to perform seasonal adjustment.

### 9.3.1 TRAMO/SEATS

TRAMO/SEATS (Gómez V. and Maravall (1997)) is a fully automatic procedure that extracts the trend-cycle, seasonal, irregular and certain transitory components of high frequency time series via the so-called ARIMA-model-based (AMB) method (see Burman (1980) and Hillmer and Tiao (1982), among others). The system is composed of two programs, the TRAMO (Time series Regression with ARIMA noise, Missing observations and Outliers) and the SEATS (Signal Extraction in ARIMA Time Series). TRAMO is a program for estimation and forecasting of regression models with possibly nonstationary ARIMA noise of the kind given in equation (9.13) and any sequence of missing values. SEATS is the program designed to handle the estimation of the aforementioned unobserved components of the data following the AMB methodology. The programs are structured to be used together for in-depth analysis of one or more series. TRAMO pre-adjusts the series for signal extraction by SEATS.

As concerns the possibility of transforming the data, TRAMO has an option called LAM which is the manual is explained as follows:

LAM = 0 Takes logs of data.

LAM = 1 (Default) No transformation of data.

LAM = -1 The program tests for the log-level specification

The help guide of the program explains that “the test is based, first, on the slope ( $b$ ) of a range-mean regression, trimmed to avoid outlier distortion. This slope  $b$  is compared to a constant ( $\beta$ ), close to zero, that depends on the number of observations. When the results of the regression are unclear, the value of LAM is chosen according to the BIC of the default model, using both specifications<sup>3</sup>.

### 9.3.2 X12 ARIMA

X-12-ARIMA is the Census Bureau's latest program in the X-11 line of seasonal adjustment programs. X-12-ARIMA uses signal-to-noise ratios to choose between a fixed set of moving-average filters, often called X-11-type filters (U.S. Census Bureau (2009)).

In the X-12- Arima programme the user can supply a prespecified value of transformation parameter  $\lambda^4$  or the logistic transformation. On the other hand, if the user decides to use the automatic option, the program chooses between no transformation and a log transformation by fitting a regARIMA model of the kind given

<sup>3</sup>The manual of the program also contains the two following remarks:

Note 1: The value LAM = -1 is recommended for automatic modelling of many series.

Note 2: The value  $\beta$  increases, so as to favor the choice of the log transformation, when a large number of series are routinely adjusted.

<sup>4</sup>From section 7.16 of the X-12 ARIMA documentation, (version 0.3 December 2009, <http://www.census.gov/srd/www/x12a/>) there is an option which is called *power* ( $\lambda$  in our notation) which transforms the input series  $y_t$  using the following modified Box-Cox version:

$$\begin{cases} \log(y_t) & \lambda = 0 \\ \lambda^2 + \frac{y_t^{\lambda-1}}{\lambda} & \lambda \neq 0 \end{cases}$$

The reason for using this modification in the manual is explained as follows: this formula for the Box-Cox power transformation is constructed so that its values will be close to  $y_t$  when  $\lambda$  is near 1 and close to  $\log y_t$  when  $\lambda$  is near zero. It also has the property that the transformed value is positive when  $y_t$  is greater than 1.

in equation (9.13) to the untransformed and transformed series, using an Akaike's Information Criterion Corrected for sample size (AICC). More in detail, The AICC of the log transformation needs to be bigger by 2 than the AICC of no transformation for X-12-ARIMA to choose no transformation over the log transformation (U.S. Census Bureau (2009)). In the X12 context, the ARIMA model which is chosen has simply the specific purpose of obtaining forecast and backcast extensions by simple inversion of the extrapolations made on the transformed scale in order to use symmetric seasonal and trend filters.

### 9.3.3 SABL

SABL, a nonparametric seasonal adjustment procedure developed at Bell Laboratories and documented in Cleveland et al. (1978), which performs the selection of a preliminary power transformation parameter that minimises the covariance between the level and the seasonal components. The issue of transforming the seasonally adjusted estimates on the original scale is not addressed explicitly. Shulman and McKenzie (1984) illustrate that the SABL estimates of the transformation parameter may not be optimal and may differ significantly from those obtained using maximum likelihood.

## 9.4 A unified approach to transformations and seasonal adjustment

In the previous section we have clearly seen that current seasonal adjustment practice does not fully take into account the problem of seasonal adjustment under transformation.

The question has to be raised as to why the transformation problem has not received sufficient recognition in the current seasonal adjustment practice. We can envisage three arguments: the first deals with the seasonal balance constraint, by which the expectation of the sum of the seasonal component over a calendar year is zero. According to a well established view the constraint should be enforced on the original measurement scale; to put it differently, the seasonally adjusted series should have the same expectation (average) as the original series over twelve consecutive monthly observations. This view is at the root of the treatment of the problem of seasonal adjustment under transformations by Thomson and Ozaki (2002), who propose ad hoc solutions with the specific intent of enforcing the seasonal balance constraint on the original scale.

A second argument deals with contemporaneous aggregation: the seasonally adjusted aggregate should be equal to the aggregated sum of the seasonally adjusted sub-series. The consistency in aggregation requires that the series are not transformed as a necessary (though not sufficient) condition, and thus would not hold for the Box-Cox transformation. A third argument concerns the difficulties and the computational burden linked with the detection of influential observations and or of the outliers on the transformed scales.

None of these arguments is compelling. Multiplicative adjustment, which is used frequently for economic time series already incorporates a different seasonal balance constraint, which refers to the geometric average, rather than the arithmetic. The view taken in this chapter is that the possibly stochastic seasonal balance constraint needs to hold only on the transformed scale. The transformation parameter uniquely defines what type of seasonal balance constraint is enforced on the original scale; roughly speaking, if the power transformation parameter is 1, then the balance constraint is additive, if it is equal to 0 it is multiplicative, if it is  $-1$  is harmonic, i.e. it is defined on the reciprocal of the series. Moreover, the conditions for consistency in cross-sectional aggregation are so stringent that the indirect seasonal adjustment of an aggregate is almost never used in practice. As concerns the third argument, in Section 9.5 we show how it is possible to robustly estimate the transformation parameter and at the same way to evaluate the effect that the different seasons exert on this estimate.

### 9.4.1 Seasonal Adjustment and the Box-Cox Transformation

Given a particular value of the transformation parameter both equations (9.13) and (9.14) can be cast in state space form. The Kalman filter enables the evaluation of the likelihood via the prediction error decomposition. See Durbin and Koopman (2001) and Harvey and Proietti (2005) for a review. The maximum likelihood estimates can be obtained by a quasi-Newton algorithm, such as the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm (see Press et al. (1992), sec. 10.7). Once the estimates of the underlying components are obtained (directly using the structural approach or passing through the canonical decomposition of the ARIMA model in the TRAMO-SEATS framework), we end up with an estimate of the seasonal and non seasonal part of the model. Let us write the series in the transformed scale as

$$y_t(\lambda) = u_t^* + S_t,$$

where  $S_t$  is the estimate of the seasonal component obtained with the different methods and  $u_t^*$  is the seasonally adjusted series on the transformed scale. Now, let us denote by  $\tilde{u}_t^* = E(u_t^*|\mathcal{F}_T)$  and  $V_t = \text{Var}(u_t^*|\mathcal{F}_T)$ , respectively the posterior mean and variance of  $u_t^*$ ,  $\mathcal{F}_t$  being the information set at time  $t$ . These quantities are delivered by the Kalman filter and smoother (KFS) applied to the relevant linear state space model (see e.g. Durbin and Koopman (2001) for details). In the structural approach the KFS directly uses the non seasonal part of the model ( $\mu_t + \epsilon_t$ ), in the AMB method the KFS is applied to non seasonal part of the decomposed ARIMA model.

We define the seasonally adjusted series on the original scale as the inverse transformation of the nonseasonal component  $u_t^*$ ,  $y_t^* = u^{-1}(u_t^*)$ , where  $u^{-1}(\cdot)$  is the inverse transformation. For the Box-Cox transformation (eq. 9.3):

$$y_t^* = \begin{cases} (1 + \lambda u_t^*)^{1/\lambda}, & \lambda \neq 0, \\ \exp(u_t^*), & \lambda = 0. \end{cases}$$

The estimator of the seasonally adjusted series is thus

$$\tilde{y}_t^* = E(y_t^*|\mathcal{F}_T) = \int u^{-1}(u_t^*) f(u_t^*|\mathcal{F}_T) du_t^*. \quad (9.15)$$

whereas the conditional variance of the estimation error for the seasonally adjusted series is defined as:

$$\text{Var}(y_t^*|\mathcal{F}_T) = \int [u^{-1}(u_t^*) - \tilde{y}_t^*]^2 f(u_t^*|\mathcal{F}_T) du_t^* = E(y_t^{*2}|\mathcal{F}_T) - \tilde{y}_t^{*2}. \quad (9.16)$$

As is well-known, the conditional expectation is the optimal estimator under quadratic loss. The above integrals do have a closed form solution only in particular cases, namely  $\lambda = 0$ , and  $\lambda = 1/p$ ,  $p = 1, 2, 3, \dots$ , as it will be seen shortly.

Notice that the naïve estimator of the SA series,

$$\hat{y}_t^* = \begin{cases} (1 + \lambda \tilde{u}_t^*)^{1/\lambda}, & \lambda \neq 0, \\ \exp(\tilde{u}_t^*), & \lambda = 0, \end{cases} \quad (9.17)$$

provides the *median* of the conditional distribution of  $y_t^*$ , given the observations.

For  $\lambda = 0$  using the properties of the lognormal distribution we have that:

$$E(y_t^*|\mathcal{F}_T) = \exp\left(\tilde{u}_t^* + \frac{V_t}{2}\right) = \hat{y}_t^* \exp\left(\frac{V_t}{2}\right). \quad (9.18)$$

$$\text{Var}(y_t^*|\mathcal{F}_T) = \exp(2\tilde{u}_t^* + V_t) \cdot (\exp(V_t) - 1). \quad (9.19)$$

### 9.4.2 Analytical solutions

The purpose of this section is to show how to obtain the exact values of the posterior mean and variance of the seasonally adjusted series in the original scale for a general value of  $\lambda \neq 0$ . Proietti and Riani (2009) prove the following theorem.

*Theorem 1: the mean and the variance of the seasonally adjusted series in the original scale are given by the two following expressions:*

$$E(y_t^* | \mathcal{F}_T) = \tilde{y}_t^* = \hat{y}_t^* \left[ 1 + \sum_{j=1}^{\infty} k_{2j}(t) a_j(t) \right] \quad (9.20)$$

$$\text{Var}(y_t^* | \mathcal{F}_T) = \hat{y}_t^{*2} \left[ \sum_{j=1}^{\infty} k_j^2(t) a_j(t) + 2 \sum_{j=1}^{\infty} \sum_{r=1}^{\infty} k_j(t) k_{j+2r}(t) a_{j+r}(t) - \left( \sum_{j=1}^{\infty} k_{2j}(t) a_j(t) \right)^2 \right] \quad (9.21)$$

where<sup>5</sup>

$$a_j(t) = E[(u_t^* - \tilde{u}_t^*)^{2j} | \mathcal{F}_T] = \frac{(2j)!}{j! 2^j} V_t^j \quad \text{and} \quad k_j(t) = \frac{1}{j!} \left( \prod_{k=1}^{j-1} (1 - \lambda k) \right) \hat{y}_t^{*-\lambda j}$$

The results follow from the Taylor series expansion of the reverse transformation. Notice that for  $\lambda = 0$   $k_j(t) = (j!)^{-1}$  and the term of (9.20) is simply the expansion of  $\exp(V_t/2)$ .

The expressions in square brackets in equations (9.20) and (9.21) are the multiplicative correction terms that have to be applied to the naïve estimator of the SA series or to its square in order to produce the conditional mean and the conditional variance in the original scale.

An alternative expression for the variance is derived as follows. Defining  $\hat{V}_t^*$  the naïve estimate of the variance resulting from the application of the Delta method,

$$\hat{V}_t^* = V_t \left[ \frac{du^{-1}(u_t^*)}{du_t^*} \Big|_{u_t^* = \tilde{u}_t^*} \right]^2 = V_t \hat{y}_t^{*2(1-\lambda)},$$

then we can rewrite (9.21) as:

$$\text{Var}(y_t^* | \mathcal{F}_T) = \hat{V}_t^* \left[ 1 + \sum_{j=2}^{\infty} \bar{k}_j^2(t) \bar{a}_j(t) + 2 \sum_{j=1}^{\infty} \sum_{r=1}^{\infty} \bar{k}_j(t) \bar{k}_{j+2r}(t) \bar{a}_{j+r}(t) - V_t \left( \sum_{j=1}^{\infty} \bar{k}_{2j}(t) \bar{a}_j(t) \right)^2 \right], \quad (9.22)$$

where  $\bar{k}_j(t) = k_j(t) \hat{y}_t^{*\lambda}$  and  $\bar{a}_j(t) = a_j(t)/V_t$ . According to expression (9.22), the exact variance can be seen as the product of the naïve variance resulting from the Delta method and a correction factor.

For  $\lambda = 1/p, p = 1, 2, \dots$ , it is immediate to see that the series  $k_1(t), k_2(t), \dots$  contains only  $p$  terms different from zero. For example, for  $\lambda = 1$ ,  $k_1(t) = 1/\hat{y}_t^*$  and  $k_2(t) = k_3(t) = \dots = 0$  so that  $\tilde{y}_t^* = \hat{y}_t^*$  and  $\text{var}(y_t^* | \mathcal{F}_T) = V_t$  as obvious. In the case of the square root transformation ( $\lambda = 0.5$ )  $k_1(t) = 1/\sqrt{\hat{y}_t^*}$ ,  $k_2(t) = 1/(4\hat{y}_t^*)$ ,  $a_1(t) = V_t$ ,  $a_2(t) = 3V_t^2$  and  $k_3(t) = k_4(t) = \dots = 0$  so that

$$\tilde{y}_t^* = \hat{y}_t^* \left[ 1 + \frac{1}{4} \frac{V_t}{\hat{y}_t^*} \right] \quad (9.23)$$

<sup>5</sup>We adopt the convention that when  $j = 1$  the product in brackets in  $k_j$  equals 1,  $\prod_{i=1}^0 x_i = 1$ .



**Table 9.1:** Exact correction factors which have to be applied to the naïve estimator of the seasonally adjusted series and of the variance, in order to obtain the conditional mean and the conditional variance in the original scale for the most important fractional values of  $\lambda$ .

	Mean	Variance
$\lambda$	Correction factor for $\hat{y}_t^*$	Correction factor for $\hat{V}_t^*$
1/2	$1 + \frac{1}{4} \frac{V_t}{\hat{y}_t^*}$	$1 + \frac{1}{8} V_t \hat{y}_t^{*-1}$
1/3	$1 + \frac{1}{3} \frac{V_t}{\hat{y}_t^{*2/3}}$	$1 + \frac{4}{9} V_t \hat{y}_t^{*-2/3} + \frac{5}{243} V_t^2 \hat{y}_t^{*-4/3}$
1/4	$1 + \frac{3}{8} \frac{V_t}{\hat{y}_t^{*1/2}} + \frac{3}{256} \frac{V_t^2}{\hat{y}_t^{*}}$	$1 + \frac{21}{32} V_t \hat{y}_t^{*-1/2} + \frac{3}{32} V_t^2 \hat{y}_t^{*-1} + \frac{3}{32} V_t^3 \hat{y}_t^{*-3/2}$

and

$$\text{Var}(y_t^* | \mathcal{F}_T) = \hat{y}_t^* V_t + \frac{1}{8} V_t^2.$$

Using similar arguments we give in Table 9.1, for the most common values of  $\lambda$ , the exact correction factors for the mean and the variance which must be applied to the naïve estimator of the seasonally adjusted series  $\hat{y}_t^*$  in order to find the true conditional mean and variance in the original scale.

This table clearly shows that the correction term depends on the ratio between the variance (raised to some power) of the SA series on the transformed scale and the value of the naïve estimator (raised to some power of  $\lambda$ ). If this is small, the correction is negligible.

### 9.4.3 Approximate and computational solutions

In the previous section we have dealt with the analytical solution of the posterior mean and variance of the seasonally adjusted series in the original scale. In this section we discuss how to obtain it in practice. For general  $\lambda$  there are three possible ways of evaluating  $E(y_t^* | \mathcal{F}_T)$  and  $\text{Var}(y_t^* | \mathcal{F}_T)$ :

- Monte Carlo evaluation using the simulation smoother: the latter is used to draw repeated samples from the conditional distribution of the seasonally adjusted series in the transformed scale  $u^* = \{u_1^*, \dots, u_T^*\}$ , given the available observations.
- Numerical integration with respect to the normal density,  $f(u_t^* | \mathcal{F}_T)$ , whose moments  $\tilde{u}_t^*$  and  $V_t^*$  are provided by the Kalman filter and smoother.
- Direct application of equations (9.20) and (9.21) truncating the summations to a particular order.

As concerns the first method (Monte Carlo evaluation), sampling from the posterior distribution of the latent components or disturbances has been considered in detail: Carlin et al. (1992) proposed a single move state sampler, which however usually is very inefficient due to the high correlation between the unobserved components, especially when they are weakly evolutive. Gamerman (1998) proposed a single move disturbance sampler, which is more efficient since the disturbances driving the components are much less persistent and autocorrelated over time. Along with reparameterization, an effective strategy is blocking, through the adoption of a multimove sampler as in Carter, C.K. and Kohn (1994) and Frühwirth-Schnatter (1994), who focus on sampling the unobserved components. Again, a more efficient multimove sampler can be constructed by focusing on the disturbances, rather than the states. This is the idea underlying the simulation smoother proposed by de Jong and Shephard (1995). More precisely, letting  $\varsigma_t$  denote the vector of disturbances that drive the nonseasonal component of the series,  $u_t^*$ . In the structural approach  $\varsigma_t = [\eta_t, \zeta_t, \epsilon_t]'$  while in the canonical decomposition framework  $\varsigma_t$  is a function of ARIMA model for the canonical trend and the canonical

noise. The simulation smoother hinges on the following factorisation of the joint posterior density:

$$f(\varsigma_0, \dots, \varsigma_T | \mathcal{F}_T) = f(\varsigma_T | \mathcal{F}_T) \prod_{t=0}^{T-1} f(\varsigma_t | \varsigma_{t+1}, \dots, \varsigma_T; \mathcal{F}_T).$$

Conditional random vectors are generated recursively. In the forward step we run the KF and the innovations, their covariance matrix and the Kalman gain are stored. In the backwards sampling step conditional random vectors are generated recursively from  $\varsigma_t | \varsigma_{t+1}, \dots, \varsigma_T; \mathcal{F}_T$ ; the algorithm keeps track of all the changes in the mean and the covariance matrix of these conditional densities. The simulated disturbances are then integrated into the trend using the transition equation and a draw from  $u_1^*, \dots, u_T^* | \mathcal{F}_T$  is obtained. A computationally faster simulation smoother has been recently developed by Durbin and Koopman (2002).

As concerns numerical integration, a routine like QuadPack function QAGS, available in most softwares, can be used<sup>6</sup>. According to our experience the finite integration interval can be defined as  $[\tilde{u}_t^* - 8\sqrt{V_t}, \tilde{u}_t^* + 8\sqrt{V_t}]$ , where  $\tilde{u}_t^*$  and  $V_t$  are evaluated by the Kalman filter and smoother applied to the transformed observations

Proietti and Riani (2009) compare for some time series the estimates of the SA series arising for the estimated transformation parameter with those arising in the case of untransformed observations or the logarithms. Their graphs clearly highlight that the differences can be relevant and the Box-Cox transformation is indeed an issue in seasonal adjustment. As concerns the performance of the different methods to obtain the seasonally adjusted series on the original scale, Proietti and Riani (2009) find that numerical integration is the most accurate and has an excellent performance also for the estimation of the conditional variance; the performance of Monte Carlo integration depends on the number of replications that are used. The convergence to the true conditional mean is not very fast. This is due to the correlation between the random draws that results from the persistence of the nonseasonal component of the series. The use of an antithetic variable greatly improves the performance. In any case, both numerical and Monte Carlo integration outperform the naïve estimate given in equation (9.17).

## 9.5 Robustness issues in transformations

Robust transformations of seasonal time series is both feasible, relevant and necessary, as argued in Proietti and Riani (2007) and Proietti and Riani (2009). It is feasible, since there are computationally efficient and accurate methods of estimating the conditional mean and variance of the seasonally adjusted series that are applicable in the absence of a closed form solution. It is relevant, since the estimates may differ relevantly from those obtained using either the untransformed observations or the logarithms. Finally, it is necessary to estimate the transformation parameter in a robust way because the presence of outliers can have strong effects on its estimate. Also this aspect is feasible, because nowadays using the forward search it is possible to know how many observations are in agreement with a particular value of the transformation parameter so it is easy to choose the best  $\lambda$ . The approach of the monitoring the values of the score test (fan plot) in order to find in a robust automatic way the best value of the transformation parameter has already been applied successfully in the context of the Italian Agricultural Census realized by the Italian National Institute of Statistics.

<sup>6</sup>QuadPack is a Fortran library for univariate numerical integration (quadrature) using adaptive rules (see Piessens et al. (1983) for more details).

## 9.6 Conclusions

The rationale behind the transformation is to enhance several desirable features of the maintained measurement model: linearity, additivity and orthogonality of components, normality of the disturbances driving the components.

This chapter has investigated the issue of seasonal adjustment under the Box-Cox power transformation of time series which are bounded from below by 0. For lack of space we could only discuss the univariate time series. The reader interested in the extension of the current approach to multivariate time series and or to other classes of transformation such as the Aranda-Ordaz, can refer to the research report by the same authors (Proietti and Riani (2007)).

Our suggested strategy is to impose the seasonal constraint on the transformed scale, perform seasonal adjustment using structural or canonical approach and then transform back into the original series. Although in this chapter we have concentrated on the Box Cox transformation, the extension to other parametric classes that are continuous in the transformation parameters and invertible is straightforward. Continuity is required for likelihood based inferences on the transformation parameter; invertibility is necessary to re-express the nonseasonal component on the original scale.

Finally, it is worthwhile to remark that even if the focus of this paper was seasonal adjustment, our method can be easily extended to find the estimate of all the other components on the original scale (e.g. the detrended series). In other words, once the two conditional moments of the detrended series in the transformed scaled are found using the KFS, the detrended series on the original scale can be computed using numerical or Monte Carlo integration or the exact analytic solution described in the chapter.

A relevant topic that we did not address in this chapter concerns the assessment of the reliability of the seasonally adjusted series, taking into account the additional source of uncertainty determined by the selection of the transformation parameter from the data. In fact, all the proposed inferences were conditional on the scale selected. In the context of regression analysis Bickel and Doksum (1981) showed that the spread of the marginal distribution of the estimators of the regression parameters is much larger than that of the conditional distribution, given the estimated transformation parameter. In the wake of this results one might want to investigate and quantify the increase in the variance due to the transformation parameter uncertainty. Actually, this point is highly controversial. However Carroll and Ruppert (1981) give a general result which indicates that the cost of estimating extra nuisance parameters such as  $\lambda$  for prediction is not large. Furthermore, Box and Cox (1982) and Hinkley and Runger (1984) argue that the variance inflation is irrelevant and illusory, as the linear model parameters have meaning only with reference to a particular scale and thus all relevant inferences can only be conditional on the selected transformation parameter.

## Annex A: variance stabilizing transformation

In this annex we show, given a sample of observations  $y = (y_1, \dots, y_n)'$  for which

$$\text{var}(y_i) \propto \{E(y_i)\}^{2\alpha} = \mu^{2\alpha},$$

how to find a variance stabilizing transformation  $g(y)$  such that the variance of the transformed variable  $\text{var}\{g(y_i)\}$  is approximately constant.

Using a first-order Taylor expansion about  $\mu$

$$g(y_i) \approx g(\mu) + g'(\mu)(y_i - \mu).$$

Consequently

$$\begin{aligned} \text{var}[g(y_i)] &\approx \{g'(\mu)\}^2 \text{var}(y_i) \\ &\approx \{g'(\mu)\}^2 \mu^{2\alpha}. \end{aligned}$$

Now for  $\text{var}\{g(y_i)\}$  to be approximately constant,  $g(y_i)$  must be chosen so that

$$\{g'(\mu)\}^2 \mu^{2\alpha} = \text{const}_1$$

$$g'(\mu) = \text{const}_2 \times \mu^{-\alpha}.$$

So that, on integration,

$$g(\mu) = \begin{cases} \mu^{-\alpha+1} & \text{if } \alpha \neq 1 \\ \log \mu & \text{if } \alpha = 1, \end{cases}$$

since the constant does not matter. For example, if the standard deviation of a variable is proportional to the mean ( $\alpha = 1$ ) a logarithmic transformation (the base is irrelevant) will give a constant variance. If the variance is proportional to the mean ( $\alpha = 1/2$ ), the square root transformation will give a constant variance and so on. Table 9.2 reports the transformation required to stabilize the variance for different values of  $\alpha$ . More generally,

**Table 9.2:** Transformations to constant variance when the variance depends on the mean

$\alpha$	$\text{var}[y] = k\mu^{2\alpha}$	Transformation
0	$k$	$y$
1/2	$k\mu$	$\sqrt{y}$
1	$k\mu^2$	$\log y$
3/2	$k\mu^3$	$1/\sqrt{y}$
2	$k\mu^4$	$1/y$

to stabilize the variance we can use  $g(y) = y(\lambda)$  defined in equation (9.3).

## Annex B: the Jacobian

The Jacobian of the transformation from  $y(\lambda)$ , defined in equation (9.3), to  $y$ , is the determinant of the matrix

$$J = \begin{vmatrix} \frac{\partial y_1(\lambda)}{\partial y_1} & \frac{\partial y_1(\lambda)}{\partial y_2} & \dots & \frac{\partial y_1(\lambda)}{\partial y_n} \\ \frac{\partial y_2(\lambda)}{\partial y_1} & \frac{\partial y_2(\lambda)}{\partial y_2} & \dots & \frac{\partial y_2(\lambda)}{\partial y_n} \\ \dots & \dots & \ddots & \vdots \\ \frac{\partial y_n(\lambda)}{\partial y_1} & \frac{\partial y_n(\lambda)}{\partial y_2} & \dots & \frac{\partial y_n(\lambda)}{\partial y_n} \end{vmatrix}$$

$$= \begin{vmatrix} y_1^{\lambda-1} & 0 & \dots & 0 \\ 0 & y_2^{\lambda-1} & \dots & 0 \\ \dots & \dots & \ddots & \vdots \\ 0 & 0 & \dots & y_n^{\lambda-1} \end{vmatrix}.$$

So  $J = \prod_{i=1}^n y_i^{\lambda-1} = y^{\lambda-1}$ .

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# Seasonal Adjustment Methods





# 10

## Background and perspectives for ARIMA Model-Based Seasonal Adjustment



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## Table of Contents

10.1	Introduction	219
10.2	Conceptual Overview*	220
10.3	Fundamental Examples*	221
10.4	Linear Regression Applied for Signal Extraction*	222
10.4.1	Basic Examples of Covariance Properties Not Inherited by Estimates	223
10.5	Spectral Densities of Stationary Series*	223
10.5.1	Transfer Functions and the ARMA Spectral Density Formula*	224
10.5.2	Spectral Density Sums and Uncorrelated Decompositions*	225
10.6	Canonical Decomposition of a First-Order Seasonal Autoregression*	226
10.6.1	Signal and Noise Filters of the Initial, Intermediate, and Final Years	228
10.6.2	The Error Variance Matrix of the Estimates	229
10.7	Pseudo-Spectral Densities of ARIMA Models*	230
10.8	Canonical Pseudo-Spectral Density Decompositions*	231
10.8.1	The Canonical Trend-Irregular Decomposition of the IMA(1,1)	231
10.8.2	A Sometimes Nonadmissible 3-Component Canonical Decomposition*	232
10.8.3	3-Component Pseudo-Spectral Density Decomposition of the Biannual Seasonal Random Walk	233
10.9	Matrix Formulation of Signal Extraction	234
10.9.1	Assumption A and Random Walk Forecasts and Backcasts*	234
10.9.2	Required Properties of the Stationarized Data, Signal, and Noise	234
10.9.3	The Two-Component Estimation Formulas	235
10.9.4	Filter and Error Variance Properties of The Canonical Decomposition*	236
10.10	Illustrative Seasonal Adjustment Filter and Standard Error Graphs*	236
10.11	Standard Errors of Change for Additive Estimates	237
10.11.1	Change from the Preceding Estimate	237
10.11.2	Revisions with Future Data*	239
10.12	Multiplicative Decomposition Estimates from Logs*	239
10.12.1	Standard Errors for Growth Rates*	240
10.13	ARMA and ARIMA Wiener-Kolmogorov Filters	241
10.14	W-K Filter Formula Examples	242
10.14.1	Rederiving the AR(1) <sub>r</sub> Symmetric Filters	242
10.14.2	Infinite W-K Filters	242

10.15	Biannual Seasonal Random Walk Filters*	244
10.15.1	The Symmetric Filters	245
10.15.2	The Asymmetric Filters	245
10.16	Differencing Operators of General ARIMA Filters	246
10.16.1	What Seasonal Decomposition Filters Annihilate or Preserve	246
10.17	Canonical Decomposition and Smoothing Trade-Offs*	246
10.18	Structural Models: A Trend Estimation Example	247
10.19	Spectral Factorization	247
10.19.1	The MA(1) Case	248
10.20	Regarding AMBSA Software Model Choices and Decomposition Components	248
10.21	Additional Components of Some AMBSA Software Decompositions	248
10.21.1	Stationary Components	249
10.21.2	A Trend-Cycle Decomposition Option for Long Series	249
10.22	Model-Based SA versus X-11 Filter SA	250





## 10.1 Introduction

We present methodology and implementation details of ARIMA model-based seasonal adjustment as developed by Tiao and Hillmer (1978) and Hillmer and Tiao (1982), with important implementation contributions by Burman (1980), Gómez and Maravall (1996) and others. We use the abbreviation SA for seasonal adjustment and AMBSA for this ARIMA-Model-Based SA method. Typically this refers to the decomposition of a span of Seasonal-ARIMA-modeled time series data into component time series for the same time span, usually seasonal, trend and irregular component series, with the irregular obtained in such a way that the decomposition is “canonical”. This term means that it conforms to Tiao and Hillmer’s attractive way of specifying a unique decomposition by requiring the white noise irregular component to have maximal variance (possibly zero). Most commonly, the ARIMA model’s differencing operator has a seasonal sum factor and a trend differencing factor. When only the latter is present, the decomposition has trend and irregular components but no seasonal component.

AMBSA is applied to time series which, in the course of obtaining the ARIMA (sometimes ARMA) model, have been preadjusted for identified effects of outliers, holidays, and other calendar effects. *The model obtained is treated as correct in all calculations.* In particular, autocovariances (and spectral densities) calculated for components from the model, after any model-specified differencing, ignore uncertainties in the ARIMA model’s selection and estimation procedures. Similarly, coefficients of the component estimates and the mean square errors calculated for component estimates are treated as non-stochastic.

After preadjustment, if only the seasonal adjustment is wanted, the task and methods can be viewed as those of basic *statistical signal processing* applied to the preadjusted series in order to statistically suppress the *seasonal* “noise” to better reveal the *nonseasonal* “signal” (or the non-trend to better reveal the trend if it is the signal, etc.).

For AMBSA in such two-component decomposition situations, under weak assumptions, mean square optimal, i.e. minimum mean square error linear unobserved component estimation can be formulated as a linear regression problem with refinements to accommodate ARIMA differencing operations, as displayed in the easily programmed matrix formulas of McElroy (2008) shown in Subsection 10.9.3.

Calculation of these estimates and their mean square error variance matrix via the matrix formulas is an option in some AMBSA software. However, the matrix calculations are not as numerically efficient and stable as those of the two traditional, less elementary calculation methods, the “Wiener-Kolmogorov” method presented in Section 10.13, and the state space method presented in Durbin and Koopman (2012).

*AMBSA software* refers to the software currently in wide use by National Statistical Institutes and Central Banks. This includes JDemetra+, see Eurostat (2015), TRAMO-SEATS, see Gómez and Maravall (1996) and X-13ARIMA-SEATS, see U.S. Census Bureau (2017).

AMBSA should not be a black box procedure to its users because default software procedures are sometimes seriously inadequate. Also user decisions regarding software options and model choice can strongly impact the results obtained, for better or worse. A seasonal adjuster who understands the basic facets of the method and some of its diagnostics, as outlined and then detailed in this document, will have a greater capacity to obtain successful adjustments. Maravall (2016) has alternative treatments of some of the topics we consider and treatments of further important topics relevant to the AMBSA methods described in Gómez and Maravall (1996) and adopted in the software of the U.S. Bureau of the Census and Eurostat.

The reader is assumed to be familiar ARIMA time series models. For an understanding of the nature of the component estimates, only a basic background in linear regression sufficient for the review of regression in Section 10.4 is needed. The regression formulas are first illustratively applied in Section 10.6 to a stationary case, to obtain the canonical decomposition of a span of data from an  $AR(1)_r$ , a first-order seasonal autoregressive model with seasonal period  $r$  ( $r$  observations per year). In this case, they yield simple revealing time-varying filter formulas for the signal and noise component estimates and for their error variance matrix.



Thereafter technical background for nonstationary ARIMA data with multiple unobserved components is developed, moving step by step through the simplest formulas that provide concrete representations of key features of AMBSA.

The reader might start by perusing the Sections whose titles have a \*, and later reading for details as their content becomes more directly relevant.

## 10.2 Conceptual Overview\*<sup>1</sup>

### Estimating Two-Component Decompositions.

Two-component decompositions can illustrate the main concepts of AMBSA. We begin with a span  $Z_1, \dots, Z_n$  of  $n$  stationary zero-mean data, in vector form  $Z = (Z_1, \dots, Z_n)'$ , that has an ARMA model and consequently an autocovariance matrix  $\Sigma_{ZZ} = EZZ'$  that is positive definite<sup>2</sup>.

Suppose  $Z_t$  is considered to be the sum of two unobserved, mutually uncorrelated, stationary component series.

$$Z_t = S_t + N_t. \quad (10.1)$$

Thus there is an autocovariance decomposition,

$$\Sigma_{ZZ} = \Sigma_{SS} + \Sigma_{NN}. \quad (10.2)$$

In the cases we consider, the unobserved components can be usually be estimated with the aid of appropriate properties specified or inferred for  $\Sigma_{SS}$  and  $\Sigma_{NN}$ , as we first illustrated simply with (10.4).

Henceforth,  $I$  denotes the identity matrix of order  $n$ . From (10.2), the linear regression formulas of Section 10.4 provide the  $n \times n$  coefficient matrices  $\beta_S = [\beta_S(j, k)]$ ,  $1 \leq j, k \leq n$  and  $\beta_N = I - \beta_S$  of the *minimum mean square error* (MMSE) linear estimates,  $\beta_S Z = \hat{S} = (\hat{S}_1, \dots, \hat{S}_n)'$  of  $S = (S_1, \dots, S_n)'$ , and  $\beta_N Z = \hat{N} = I - \hat{S} = (\hat{N}_1, \dots, \hat{N}_n)'$  of  $N = (N_1, \dots, N_n)'$  in (10.3). The  $j$ -th row of  $\beta_S$  shows the data coefficients of the linear estimate  $\hat{S}_j = \sum_{k=1}^n \beta_S(j, k) Z_k$  of the decomposition of  $Z_j$  and correspondingly for  $\hat{N}_j$  in

$$Z = \hat{S} + \hat{N}. \quad (10.3)$$

With a two-component decomposition, the component of greater interest can be labeled *signal*  $S_t$  and the other labeled *noise*  $N_t$ . The simplest case is that of *white noise*, uncorrelated and constant variance  $N_t$ , resulting in  $\Sigma_{NN} = \sigma^2 I$ . A specification of  $\sigma^2 > 0$  small enough that  $\Sigma_{SS} = \Sigma_{ZZ} - \sigma^2 I$  is a covariance matrix, i.e. positive semi-definite (all eigenvalues nonnegative, one or more positive) will provide a decomposition (10.2) that yields estimates (10.3).

$$\Sigma_{ZZ} = \Sigma_{SS} + \sigma^2 I. \quad (10.4)$$

### A Limited Elementary Stationary-Case Specification of $\sigma^2$ .

Here is a possible specification of  $\sigma^2$  in (10.4) that only uses standard matrix concepts: Specify  $\sigma^2$  as the maximal white noise variance compatible with (10.4). From  $\Sigma_{SS} = \Sigma_{ZZ} - \sigma^2 I$  one sees that this  $\sigma^2$  is the smallest eigenvalue  $\alpha_{\min}$  of  $\Sigma_{ZZ}$ . It is positive because every ARMA covariance matrix  $\Sigma_{ZZ}$  is positive definite<sup>3</sup>. The resulting  $\hat{N}$  and  $\hat{S} = Z - \hat{N}$  from the regression formulas (10.6) of Section 10.4 are the MMSE linear estimates of the white noise  $\hat{N}$  and the signal  $\hat{S}$  for the specified decomposition (10.4). (The estimated  $\hat{N}_t$  are not white noise, see (10.9)).

<sup>1</sup>\* indicates especially fundamental material.

<sup>2</sup>See Wikipedia Contributors (2017b).

<sup>3</sup>Every positive definite matrix has such a  $\sigma^2 = \alpha_{\min}$  decomposition. No model connection is required.

A choice of  $\sigma^2$  different from (smaller than)  $\alpha_{\min}$  produces a different decomposition (10.4) with different estimated components. So the choice  $\alpha_{\min}$  is just one of many possibilities for stationary data.

For ARMA  $Z_t$ , AMBSA software uses the less elementary  $\sigma^2$  specification of Hillmer and Tiao (1982) described in the next paragraph. Differently from  $\sigma^2 = \alpha_{\min}$ , its definition of  $\sigma^2$  admits a generalization for ARIMA  $Z_t$ , the usual case for AMBSA. So it provides some useful conceptual consistency between stationary and nonstationary cases. See Section 10.7.

### The Canonical Two-Component Stationary-Case Decomposition.

The generalizable definition of  $\sigma^2$  requires the ARMA model for  $Z_t$  to be invertible, which is the typical case in practice. This means that the *spectral density*  $g_Z(\lambda)$ , defined by (10.22) and abbreviated sd (or Sd, plural sds or Sds) has a positive minimum value,  $\min_{\lambda} g_Z(\lambda) > 0$ . For ARMA  $Z_t$ , the *canonical decomposition*, see Tiao and Hillmer (1978), specifies  $\sigma^2 = \min_{\lambda} g_Z(\lambda)$  in (10.4).

Sd functions are autocovariance generating functions, see (10.14). The matrix decomposition (10.4) is generated for all series lengths  $n$  by the sd decomposition  $g_Z(\lambda) = g_S(\lambda) + \sigma^2 = g_S(\lambda) + g_N(\lambda)$  with  $g_S(\lambda) = g_Z(\lambda) - \sigma^2$ . Invertibility of the ARMA model guarantees that  $g_S(\lambda) = g_Z(\lambda) - \sigma^2 \geq 0$ . It follows that  $g_S(\lambda)$  specifies a stationary component  $S_t$  having a non-invertible ARMA model, such that the variance  $\sigma^2$  of the white noise component  $N_t$  is maximal. Such maximality is the defining property of the *canonical decomposition*, also when there are several non-white-noise components.

### Nonstationary Case.

For invertible ARIMA  $Z_t$ ,  $\sigma^2$  is specified as the minimum of the *pseudo-spectral density* (p-sd) of the model, defined by (10.43). The trend plus irregular decomposition (10.55) of Subsection 10.8.1, with trend component denoted  $p_t$ , illustrates this.

The sd and p-sd formulas (10.22) and (10.43) are sources of the great versatility of AMBSA. They reflect the model structure in ways that facilitate the specification of decompositions with appropriate components, see Section 10.8 and Subsection 10.21.

If decomposition calculations provide an sd or a p-sd for each component, and a nonnegative value for the constant calculated to be the maximal variance, then the decomposition is *admissible* (or *acceptable*). If this constant is negative, the decomposition is *nonadmissible*. Section 10.8 provides two examples of fundamental ARIMA models with admissible decompositions for all model parameter values and one ARIMA model whose decomposition is admissible only for a subinterval of model coefficient values.

## 10.3 Fundamental Examples\*

Section 10.10 offers graphs of central and concurrent (latest observation time) MMSE filters from a standard seasonal model, and also graphs of the associated time-varying error variances of their AMBSA seasonal adjustments. Subsections 10.11.2 and 10.12.1 illustrate how error variances and covariances can be used to obtain probability intervals for an estimated growth rate and also for its revised value that will be obtained by reestimating with additional later data.

Sections 10.10–10.12 illustrate, in various ways, how greater AMBSA smoothing is associated with greater instability of the AMBSA estimates, a theme relevant for Subsection 10.17.

The seasonal random walk is the nonstationary analogue of the seasonal  $AR(1)_r$ . Its  $r = 2$  biannual model is the only nonstationary model for which we show exact formulas for seasonal, trend and irregular filters, symmetric and asymmetric. The symmetric filters of this model are derived in Section 10.15. The asymmetric filters are obtained using MMSE forecasts and backcasts of data required by the symmetric filters but not available.

Reflecting the complexity of most economic indicators that are seasonally adjusted, the trend component has often been called the *trend-cycle component* and regarded as consisting of a long-term trend component plus one or more higher frequency cyclical components. An approach to providing such a decomposition with one cycle component, available in some AMBSA software, is described in Subsection 10.21.2.

## 10.4 Linear Regression Applied for Signal Extraction\*

As in Section 10.2, we start from zero mean data  $Z_1, \dots, Z_n$  whose  $n \times n$  autocovariance matrix  $\Sigma_{ZZ}$ , is positive definite and focus on two-unobserved-component decompositions  $Z_t = S_t + N_t$  with uncorrelated components,  $ES_j N_k = 0$ ,  $j, k = 1, \dots, n$ . With  $Z = (Z_1, \dots, Z_n)'$ , the task is to use a specified autocovariance decomposition (10.2) to obtain  $n \times n$  coefficient matrices  $\beta_S, \beta_N$  of linear estimates  $\hat{S} = \beta_S Z$ ,  $\hat{N} = \beta_N Z$  of a decomposition (10.3) that have minimum mean square error. To accomplish this, setting  $Z = (Z_1, \dots, Z_n)'$  and  $S = (S_1, \dots, S_n)'$ , we seek the  $n \times n$  coefficient matrix  $\beta_S$  such that the error  $e = \hat{S} - S$  of the linear estimate  $\hat{S} = \beta_S Z$  is uncorrelated with  $Z$ ,  $Ee'Z = 0$ , which then also holds for the error  $\hat{N} - N = -e$ . This property characterizes linear estimates  $\beta_S Z$  whose mean square error  $Ee'e = \sum_{j=1}^n e_j^2$  is minimal, as Section 4.1 of Whittle (1963) shows<sup>4</sup>. Other MMSE characterizations derived include: For every positive definite  $n \times n$  matrix  $Q$ , the MMSE error  $e_t$  minimizes  $\sum_{j,k=1,\dots,n} E\{e_j Q_{j,k} e_k\}$ . Also, with Gaussian  $Z$ , the MMSE  $\hat{S}$  and  $\hat{N}$  are the conditional expectations of  $S$  and  $N$  given  $Z$ .

Let  $0_n$  denote the zero matrix of order  $n$ . Because  $\Sigma_{SN} = ES N' = 0_n$  yields  $\Sigma_{SZ} = \Sigma_{SS}$ , we have, with  $\iff$  denoting equivalence,

$$E(S - \beta_S Z) Z' = 0_n \iff \Sigma_{SZ} - \beta_S \Sigma_{ZZ} = 0_n \iff \Sigma_{SS} = \beta_S \Sigma_{ZZ}, \quad (10.5)$$

resulting in  $\hat{S} = \beta_S Z$  with  $\beta_S = \Sigma_{SS} \Sigma_{ZZ}^{-1}$ . Analogously,  $\hat{N} = \beta_N Z$  with  $\beta_N = \Sigma_{NN} \Sigma_{ZZ}^{-1}$ , whence  $\beta_S + \beta_N = I$ .

In summary,

$$\hat{S} = \beta_S Z, \quad \beta_S = \Sigma_{SS} \Sigma_{ZZ}^{-1}, \quad \hat{N} = \beta_N Z, \quad \beta_N = \Sigma_{NN} \Sigma_{ZZ}^{-1}, \quad \beta_S + \beta_N = I. \quad (10.6)$$

It follows that covariance matrices of the estimates have the formulas

$$\Sigma_{\hat{S}\hat{S}} = \Sigma_{SS} \Sigma_{ZZ}^{-1} \Sigma_{SS}, \quad \Sigma_{\hat{N}\hat{N}} = \Sigma_{NN} \Sigma_{ZZ}^{-1} \Sigma_{NN}. \quad (10.7)$$

The last formula in (10.6) shows that the estimates provide a decomposition (10.3). For a specified decomposition (10.2), the estimate  $\hat{S} = Z - \hat{N}$  can be regarded as an optimally “de-noised” version of the data for revealing the signal  $S$ .

For  $1 \leq t \leq n$ , the  $t$ -th row of  $\beta_S$  provides the *coefficients* of the MMSE linear estimate  $\hat{S}_t = \sum_{j=1}^n \beta_{S,t,j} Z_j$  and correspondingly for  $\beta_N$  and  $\hat{N}_t$ , see examples in Subsection 10.6.1.

From (10.1) and (10.3), the estimation error  $e = S - \hat{S}$  is equal to  $\hat{N} - N$ , so both estimates have the same *error variance matrix*,

$$\begin{aligned} \Sigma_{ee} &= E(S - \hat{S})(S - \hat{S})' = E(N - \hat{N})(N - \hat{N})' \\ &= \Sigma_{NN} \Sigma_{ZZ}^{-1} \Sigma_{SS} = \Sigma_{SS} \Sigma_{ZZ}^{-1} \Sigma_{NN} = \hat{\Sigma}_{\hat{N}\hat{S}} = \Sigma_{\hat{S}\hat{N}}. \end{aligned} \quad (10.8)$$

<sup>4</sup>Wikipedia Contributors (2013) derives and applies the analogous MMSE estimate identifying property for simpler non-time series contexts.

### Change of Scale Results.

It follows from the preceding formulas that if  $Z$  and its components are multiplied by scalar  $\alpha \neq 0$ , then  $\Sigma_{ee}$  and other covariance matrices become multiplied by  $\alpha^2$  but the filter coefficient vectors  $\beta_S$  and  $\beta_N$  are unchanged, they are *scale invariant*.

#### 10.4.1 Basic Examples of Covariance Properties Not Inherited by Estimates

The final formulas for  $\Sigma_{ee}$  show that, whereas  $\Sigma_{SN} = \Sigma_{NS} = 0_n$ , the estimates are cross-correlated,  $\Sigma_{\hat{S}\hat{N}} = \Sigma_{\hat{N}\hat{S}} = \Sigma_{ee}$ , a positive definite matrix if both  $\Sigma_{SS}$  and  $\Sigma_{NN}$  have this property, otherwise positive semidefinite. This generalizes to AMBSA. Estimates of uncorrelated components are cross-correlated because all are linear functions of the data  $Z$  (after differencing to stationarity in nonstationary cases). Also, a component estimate has covariance properties different from the component. Most basically, for a white noise component  $N$  of non-white-noise  $Z$ , the estimate  $\hat{N}$ , is not white noise. From (10.7),

$$\Sigma_{\hat{N}\hat{N}} = \sigma^4 \Sigma_{ZZ}^{-1} = \sigma^4 (\Sigma_{SS} + \sigma^2 I)^{-1}. \quad (10.9)$$

## 10.5 Spectral Densities of Stationary Series\*

*Optional Review of Complex Numbers.*

Spectral densities can be defined without using complex numbers as we show, but then formulas and important seasonal decomposition calculations lose simplicity. We use standard notation,  $z = a + ib$  with  $a$  and  $b$  real and  $i^2 = -1$ . The number  $a$  is the real part of  $z$ ,  $a = \text{Re}(z)$ , and  $b$  is the imaginary part,  $b = \text{Im}(z)$ .  $\bar{z} = a - ib$  is the complex conjugate of  $z$ . Its properties are  $z + \bar{z} = 2 \text{Re}(z)$ ,  $z - \bar{z} = 2i \text{Im}(z)$  and  $\sqrt{z\bar{z}} = \sqrt{a^2 + b^2}$ , which is the magnitude of  $z$ , denoted  $|z|$  (the distance from  $(a, b)$  to  $(0, 0)$  in the coordinate plane). Euler's formula  $e^{i\theta} = \cos \theta + i \sin \theta$  for real  $\theta$  shows that  $e^{-i\theta}$  is the complex conjugate of  $e^{i\theta}$  and that  $|e^{i\theta}|^2 = \cos^2 \theta + \sin^2 \theta = 1$ . The *polar* representation of  $z$  is  $|z| e^{i\theta}$ , with phase  $\theta$ . Especially relevant are calculations like

$$\left| 1 \pm \theta e^{i\lambda} \right|^2 = \left( 1 \pm \theta e^{i\lambda} \right) \left( 1 \pm \theta e^{-i\lambda} \right) = 1 + \theta^2 \pm \theta \left( e^{i2\pi\lambda} + e^{-i2\pi\lambda} \right) = 1 + \theta^2 \pm 2\theta \cos \lambda. \quad (10.10)$$

For more information, see Wikipedia Contributors (2012).

### Notational Convention:

Hereafter,  $w_t$  denotes a covariance stationary series, sometimes ARMA, possibly the stationary transform  $w_t = \delta(B) Z_t$  of an ARIMA  $Z_t$  with differencing operator  $\delta(B)$ . When  $Z_t$  is stationary, then  $w_t = Z_t$ .

With  $\gamma_j = E w_t w_{t-j}$ ,  $j = 0, \pm 1, \dots$ , the *spectral density* of  $w_t$  is the function defined for  $-1/2 \leq \lambda \leq 1/2$  by

$$g_w(\lambda) = \sum_{j=-\infty}^{\infty} \gamma_j e^{i2\pi j\lambda}. \quad (10.11)$$

Because  $\gamma_{-j} = \gamma_j$ ,

$$g_w(\lambda) = \gamma_0 + \sum_{j=1}^{\infty} \gamma_j \left( e^{i2\pi j\lambda} + e^{-i2\pi j\lambda} \right), \quad (10.12)$$

$$= \gamma_0 + 2 \sum_{j=1}^{\infty} \gamma_j \cos 2\pi j\lambda. \quad (10.13)$$

$g_w(\lambda)$  is also called the *autocovariance generating function* of  $w_t$  due to

$$\gamma_j = \int_{-1/2}^{1/2} e^{-i2\pi j\lambda} g_w(\lambda) d\lambda = 2 \int_0^{1/2} \cos 2\pi j\lambda g_w(\lambda) d\lambda, \quad j = 0, \pm 1, \dots \quad (10.14)$$

Thus  $g_w(\lambda)$  is a frequency domain re-expression of the autocovariance properties of  $w_t$ . It is an even function,  $g_w(-\lambda) = g_w(\lambda)$ ,  $-1/2 \leq \lambda \leq 1/2$ , which is nonnegative, a property expressed in the ARMA spectral density formula (10.22). Any integrable function with these properties is the spectral density of a stationary time series, see Brockwell and Davis (1991).

White noise,  $w_t = a_t$ , with  $\sigma_a^2 = E a_t^2$  has the simplest sd. From (10.12), its spectral density is a constant,

$$g_a(\lambda) = \sigma_a^2, \quad -1/2 \leq \lambda \leq 1/2. \quad (10.15)$$

Conversely, if (10.15) holds, it follows from (10.14) that the autocovariances of  $a_t$  are zero at nonzero lags, i.e.  $a_t$  is white noise. To illustrate (10.11), an MA(1),  $w_t = (1 - \theta B)a_t$ , has autocovariances  $\gamma_0 = \sigma_a^2(1 + \theta^2)$ ,  $\gamma_{\pm 1} = -\sigma_a^2\theta$  and  $\gamma_j = 0$  for  $|j| \geq 2$ , so from (10.12) and (10.10),

$$g_w(\lambda) = \sigma_a^2(1 + \theta^2) - \sigma_a^2\theta(e^{i2\pi j\lambda} + e^{-i2\pi j\lambda}) = \sigma_a^2 |1 - \theta e^{i2\pi\lambda}|^2. \quad (10.16)$$

The final formula in (10.16) is an instance of the general ARMA sd formula (10.22).

## 10.5.1 Transfer Functions and the ARMA Spectral Density Formula\*

### 10.5.1.1 ARMA Conventions

For a stationary ARMA(p,q) series  $w_t$ , by definition

$$\phi(B) w_t = \theta(B) a_t, \quad (10.17)$$

with white noise  $a_t$ . Unlike some AMBSA software, we use the sign convention of Box and Jenkins (1976), with the basic ARMA(1,1) expressed as  $(1 - \phi_1 B) w_t = (1 - \theta_1 B) a_t$ . Stationarity requires  $\phi(z) = 1 - \phi_1 z + \dots - \phi_p z^p$  to satisfy

$$\phi(z) \neq 0, \quad |z| \leq 1, \quad (10.18)$$

( $|\phi_1| < 1$  if  $p = 1$ ). With no loss of generality under Gaussian assumptions,  $\theta(z)$  is assumed to satisfy

$$\theta(z) \neq 0, \quad |z| < 1, \quad (10.19)$$

a condition that is always imposed by the ARIMA coefficient estimation routines of AMBSA software. If also  $\theta(z) \neq 0$  whenever  $|z| = 1$ , then  $w_t$  and its model are said to be *invertible*. Both are *noninvertible* if  $\theta(z) = 0$  for a  $z$  with  $|z| = 1$ .

When  $w_t$  is a *seasonal* ARMA process,  $\phi(B) \Phi(B^r) w_t = \theta(B) \Theta(B^r) a_t$  with seasonal period  $r \geq 2$ , then (10.18) and (10.19) apply to the *total* AR and MA polynomials<sup>5</sup>  $\varphi(z) = \phi(z) \Phi(z^r)$  and  $\vartheta(z) = \theta(z) \Theta(z^r)$ .

For simplicity, we usually use this total notation for all ARMA and ARIMA models,

$$\varphi(B) Z_t = \vartheta(B) a_t, \quad (10.20)$$

seasonal or nonseasonal, referring to  $\vartheta(z)$  as the MA polynomial and  $\varphi(z)$  as the AR polynomial, which can include the differencing polynomial in the nonstationary case.

<sup>5</sup> $\vartheta$  is script  $\theta$  "theta" and  $\varphi$  is script  $\phi$  "phi".

The formula (10.22) of the spectral density of such a  $w_t$  follows from (10.17) via a fundamental fact: When a stationary series  $y_t$  is the output of a *linear filter*  $\beta(B) = \sum_j \beta_j B^j$ , i.e.  $y_t = \sum_j \beta_j x_{t-j}$  for some stationary  $x_t$ , then the spectral densities of the input series  $x_t$  and the output series  $y_t$  are related by

$$g_y(\lambda) = \left| \beta(e^{i2\pi\lambda}) \right|^2 g_x(\lambda), \quad (10.21)$$

see Theorems 4.4.1 and 4.10.1 of Brockwell and Davis (1991). The function  $\beta(e^{-i2\pi\lambda}) = \sum_j \beta_j e^{-i2\pi j\lambda}$  is the *transfer function* of the filter and  $|\beta(e^{i2\pi\lambda})|^2$  is the *squared gain* of the filter. The filter is *symmetric*,  $\beta_j = \beta_{-j}$  for  $j \neq 0$ , when  $\beta(e^{-i2\pi\lambda}) = \beta(e^{i2\pi\lambda})$  for all  $\lambda$ .

### 10.5.1.2 The Spectral Density Formula

Using (10.21), it follows from (10.15) and (10.17) that  $|\varphi(e^{i2\pi\lambda})|^2 g_w(\lambda) = |\vartheta(e^{i2\pi\lambda})|^2 \sigma_a^2$  and therefore that an ARMA sd has the form

$$g_w(\lambda) = \sigma_a^2 \frac{|\vartheta(e^{i2\pi\lambda})|^2}{|\varphi(e^{i2\pi\lambda})|^2}, \quad -1/2 \leq \lambda \leq 1/2. \quad (10.22)$$

Thus an ARMA(1,1)  $w_t$  has sd

$$w(\lambda) = \sigma_a^2 \frac{|1 - \theta e^{i\lambda}|^2}{|1 - \phi e^{i\lambda}|^2}.$$

Invertibility is equivalent to the sd having a positive minimum,  $\sigma^2 = \min_{\lambda} g_w(\lambda) > 0$ .

## 10.5.2 Spectral Density Sums and Uncorrelated Decompositions\*

If stationary times series  $x_t$  and  $\tilde{x}_t$  are uncorrelated, then

$$E(x_t + \tilde{x}_t)(x_{t-j} + \tilde{x}_{t-j}) = Ex_t x_{t-j} + E\tilde{x}_t \tilde{x}_{t-j} \quad j = 0, \pm 1, \dots$$

It follows that the sd of the sum series  $w_t = x_t + \tilde{x}_t$  is the sum of the component sds,

$$g_w(\lambda) = g_x(\lambda) + g_{\tilde{x}}(\lambda). \quad (10.23)$$

Conversely, if the spectral density  $g_w(\lambda)$  of a stationary series is found to have a decomposition (10.23), then as regards its autocovariance properties, one can treat  $w_t$  as admitting a decomposition  $w_t = x_t + \tilde{x}_t$  with uncorrelated components having spectral densities  $g_x(\lambda)$  and  $g_{\tilde{x}}(\lambda)$ . (The possible correlated decompositions yielding (10.23) lack practical value, see Findley (2012).)

As a fundamental example of (10.23), in the invertible case,  $\sigma^2 = \min_{\lambda} g_w(\lambda) > 0$ , then with  $g_S(\lambda) = g_w(\lambda) - \sigma^2$ , the decomposition

$$g_w(\lambda) = \{g_w(\lambda) - \sigma^2\} + \sigma^2 \equiv g_S(\lambda) + g_N(\lambda) \quad (10.24)$$

specifies a canonical two-component decomposition,  $w_t = S_t + N_t$ , with white noise  $N_t$ .

### 10.5.2.1 The Canonical Sd Decomposition of an Invertible MA(1)

For an MA(1)  $w_t$  with  $|\theta| < 1$ , from (10.10),  $\sigma^2 = \min_{\lambda} \sigma_a^2 (1 + \theta^2 - 2\theta \cos \lambda) = \sigma_a^2 (1 + \theta^2 - 2|\theta|)$ . Therefore

$$g_S(\lambda) = g_w(\lambda) - \sigma^2 = \sigma_a^2 (2|\theta| - 2\theta \cos \lambda).$$

## 10.6 Canonical Decomposition of a First-Order Seasonal Autoregression\*

The time-varying filters and MMSE errors for (10.24) can given in detail for data from the first-order seasonal autoregressive model  $AR(1)_r$ , with seasonal period  $r \geq 2$ ,

$$w_t = \Phi w_{t-r} + a_t, \quad -1 < \Phi < 1. \quad (10.25)$$

From Box and Jenkins (1976) p. 329,

$$\gamma_j = Ew_{t+j}w_t = \sigma_a^2 \begin{cases} (1 - \Phi^2)^{-1} \Phi^k, & |j| = kr, \quad k = 0, 1, \dots \\ 0, & \text{otherwise.} \end{cases} \quad (10.26)$$

From (10.22) and (10.24), we obtain  $g_w(\lambda)$  and its canonical decomposition components:

$$g_w(\lambda) = \sigma_a^2 \left| 1 - \Phi e^{i2\pi r \lambda} \right|^{-2}, \quad (10.27)$$

$$g_N(\lambda) = \sigma^2 = \min_{\lambda} g_w(\lambda) = g_w(0) = \sigma_a^2 (1 + |\Phi|)^{-2}. \quad (10.28)$$

Formula (19) of Findley et al. (2015) shows that for  $\Phi > 0$ ,

$g_S(\lambda) = g_w(\lambda) - g_N(\lambda)$  can be expressed as

$$g_S(\lambda) = \sigma_a^2 \Phi (1 + \Phi)^{-2} \frac{|1 + e^{i2\pi \lambda}|^2}{|1 - \Phi e^{i2\pi \lambda}|^2}.$$

For simplicity, we only consider  $\Phi > 0$ . Then the minimum in (10.28) occurs at the frequencies in  $-1/2 \leq \lambda \leq 1/2$  where  $\cos 2\pi r \lambda = -1$ , such as  $\lambda = \pm (2r)^{-1}$ .

The peaks in Figure 10.1 are at  $\lambda = 0$  and at each seasonal frequency,  $k/12$  cycles per year,  $1 \leq k \leq 6$ , always with amplitude  $\sigma_a^2 (1 - \Phi)^{-2} = (1 + \Phi)(1 - \Phi)^{-1}$ . The peaks for  $\Phi = 0.70$  are broader and much lower than those for  $\Phi = 0.95$ . The minimum value  $(1 - \Phi)(1 + \Phi)^{-1}$  occurs midway between each pair of peaks.

The canonical sd decomposition  $g_w(\lambda) = (g_w(\lambda) - \sigma_a^2 (1 + \Phi)^{-2}) + \sigma_a^2 (1 + \Phi)^{-2}$  identifies the matrix decomposition

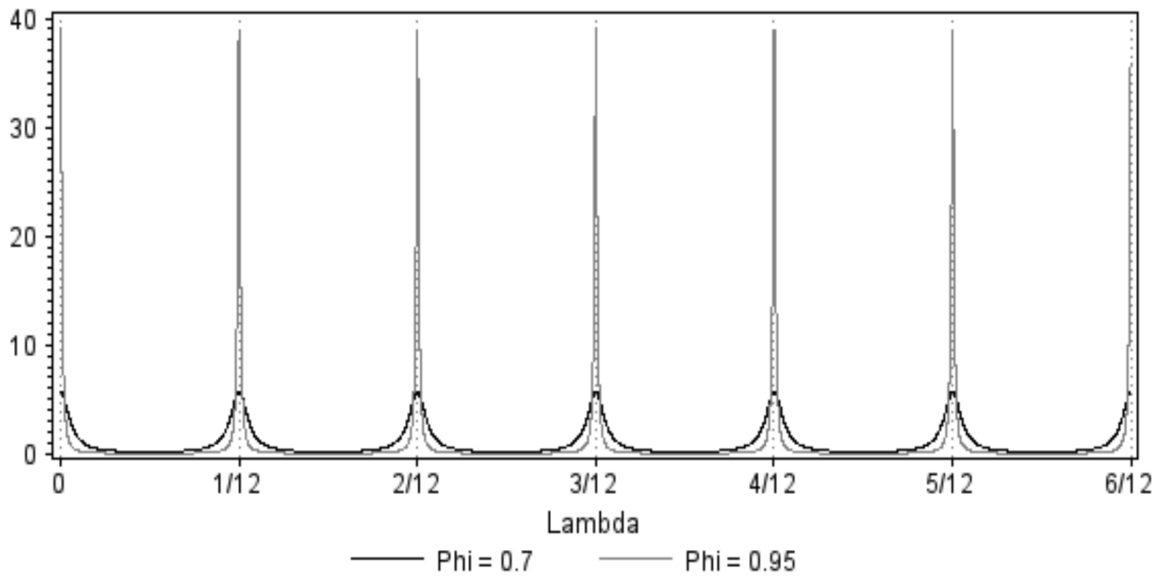
$$\Sigma_{ww} = (\Sigma_{ww} - \sigma_a^2 (1 + \Phi)^{-2} I) + \sigma_a^2 (1 + \Phi)^{-2} I \equiv \Sigma_{SS} + \Sigma_{NN}. \quad (10.29)$$

Substitution from (10.26) into the regression formulas (10.6) yields the estimates  $\hat{S}_t$  and  $\hat{N}_t$  of the canonical decomposition, as we illustrate.

For the seasonal  $AR(1)_r$  model, the entries of the inverse matrix  $\Sigma_{ww}^{-1}$  have known, relatively simple formulas,



Figure 10.1: Spectra of SAR(1) with Phi=0.70 and 0.95



Note: Two  $r = 12$  SAR(1) sds for  $0 \leq \lambda \leq 1/2$ , for  $\Phi = 0.70$  (darker line) and  $\Phi = 0.95$ , with  $\sigma_a^2 = 1 - \Phi^2$  to have  $\gamma_0 = 1$ , hence area 1/2 below each graph.

see Wise (1955) and Zinde-Walsh (1988). For example, when  $r = 2, n = 7$ ,

$$\Sigma_{ww}^{-1} = \sigma_a^{-2} \begin{bmatrix} 1 & 0 & -\Phi & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\Phi & 0 & 0 & 0 \\ -\Phi & 0 & 1 + \Phi^2 & 0 & -\Phi & 0 & 0 \\ 0 & -\Phi & 0 & 1 + \Phi^2 & 0 & -\Phi & 0 \\ 0 & 0 & -\Phi & 0 & 1 + \Phi^2 & 0 & -\Phi \\ 0 & 0 & 0 & -\Phi & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -\Phi & 0 & 1 \end{bmatrix}. \quad (10.30)$$

For  $r \geq 2$  and all  $n \geq 2r + 1$ , as (10.30) indicates,  $\Sigma_{ww}^{-1}$  has a tridiagonal symmetric form, with nonzero values only on the main diagonal and the  $r$ -th diagonals above and below. The sub- and superdiagonals have the entries  $-\Phi\sigma_a^{-2}$ . The first and last  $r$  entries of the main diagonal are  $\sigma_a^{-2}$  and the rest are  $\sigma_a^{-2}(1 + \Phi^2)$ .

For  $\beta_N = \Sigma_{NN}\Sigma_{ww}^{-1} = \sigma_N^2\Sigma_{ww}^{-1} = (1 + \Phi)^{-2}\sigma_a^2\Sigma_{ww}^{-1}$ , one has, when  $r = 2, n = 7$ ,

$$\beta_N = (1 + \Phi)^{-2} \begin{bmatrix} 1 & 0 & -\Phi & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\Phi & 0 & 0 & 0 \\ -\Phi & 0 & 1 + \Phi^2 & 0 & -\Phi & 0 & 0 \\ 0 & -\Phi & 0 & 1 + \Phi^2 & 0 & -\Phi & 0 \\ 0 & 0 & -\Phi & 0 & 1 + \Phi^2 & 0 & -\Phi \\ 0 & 0 & 0 & -\Phi & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -\Phi & 0 & 1 \end{bmatrix}, \quad (10.31)$$

Further, from  $\beta_S = I - \beta_N$ ,

$$\beta_S = \Phi(1 + \Phi)^{-2} \begin{bmatrix} (2 + \Phi) & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & (2 + \Phi) & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & (2 + \Phi) & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & (2 + \Phi) \end{bmatrix}. \quad (10.32)$$

### 10.6.1 Signal and Noise Filters of the Initial, Intermediate, and Final Years

For general  $r \geq 2$  and  $n \geq 2r + 1$ , the  $\Sigma_{ww}^{-1}$  formula of Wise (1955) yields the filter formulas for  $\hat{N}_t$  and  $\hat{S}_t = w - \hat{N}_t$  shown in (10.33)–(10.37) and (10.38)–(10.40). For the *intermediate times*  $r + 1 \leq t \leq n - r$ , the noise component estimate  $\hat{N}_t$  is given by a symmetric filter (10.33) with equal negative initial and final coefficients smaller in magnitude than the positive central coefficient.

$$\hat{N}_t = \frac{1}{(1 + \Phi)^2} (-\Phi w_{t-r} + (1 + \Phi^2) w_t - \Phi w_{t+r}). \quad (10.33)$$

The filters for the initial and final years are asymmetric. For the *initial year*  $1 \leq t \leq r$ ,

$$\hat{N}_t = \frac{1}{(1 + \Phi)^2} (w_t - \Phi w_{t+r}) \quad (10.34)$$

$$= \frac{1}{(1 + \Phi)^2} (-\Phi \{\Phi w_t\} + (1 + \Phi^2) w_t - \Phi w_{t+r}). \quad (10.35)$$

The filter for the *final year*  $n - r + 1 \leq t \leq n$  is the time-reverse of the initial year filter,

$$\hat{N}_t = (1 + \Phi)^{-2} (-\Phi w_{t-r} + w_t) \quad (10.36)$$

$$= \frac{1}{(1 + \Phi)^2} (-\Phi w_{t-r} + (1 + \Phi^2) w_t - \Phi \{\Phi w_t\}). \quad (10.37)$$

In comparison with (10.33), the value  $\{\Phi w_t\}$  in the re-expression (10.35) appears as the MMSE  $\text{AR}(1)_r$  *backcast* of the missing  $w_{t-r}$  and in (10.37) as the MMSE  $\text{AR}(1)_r$  *forecast* of the missing  $w_{t+r}$ .

Also for  $\hat{S}_t$  at intermediate times  $r + 1 \leq t \leq n - r$  the filter formula is symmetric,

$$\hat{S}_t = \frac{\Phi}{(1 + \Phi)^2} (w_{t-r} + 2w_t + w_{t+r}) = \frac{4\Phi}{(1 + \Phi)^2} \left( \frac{1}{4} w_{t-r} + \frac{1}{2} w_t + \frac{1}{4} w_{t+r} \right). \quad (10.38)$$

As with  $\hat{N}_t$ , for the initial and final years, the  $\hat{S}_t$  filters are asymmetric. For  $1 \leq t \leq r$ ,

$$\hat{S}_t = \frac{\Phi}{(1 + \Phi)^2} ((\Phi + 2) w_t + w_{t+r}) = \frac{4\Phi}{(1 + \Phi)^2} \left( \frac{1}{4} \{\Phi w_t\} + \frac{1}{2} w_t + \frac{1}{4} w_{t+r} \right), \quad (10.39)$$

and for  $n - r + 1 \leq t \leq n$ , the filter is the time reverse of the initial year filter,

$$\hat{S}_t = \frac{\Phi}{(1 + \Phi)^2} (w_{t-r} + (\Phi + 2) w_t) = \frac{4\Phi}{(1 + \Phi)^2} \left( \frac{1}{4} w_{t-r} + \frac{1}{2} w_t + \frac{1}{4} \{\Phi w_t\} \right). \quad (10.40)$$

The role of  $\{\Phi w_t\}$  in (10.39) and (10.40) is as in (10.35) and (10.37).

The classical nonregression approach, illustrated in Subsection 10.14.1, quickly yields the symmetric filters

but explicitly needs forecasts and backcasts to obtain the asymmetric filters.

Figure 10.2 shows the extracted signal  $\hat{S}_t$  from an  $n = 144$  simulated  $r = 12$  (monthly)  $Z_t$  with  $\Phi = 0.95$ , after MMSE suppression of  $Z_t$ 's white noise component. The  $\hat{S}_t$  track all but the most rapid movements of the  $Z_t$  series, but with fewer changes of direction over the 12 years. Findley et al. (2016) gives a formal sense in which the calendar month series of the  $\hat{S}_t$  are smoother than those of the  $Z_t$ .

**Figure 10.2:** The 12 calendar-month subseries, their averages (horizontal lines), and canonical  $\hat{S}_t$  (darker line) of a length 144 simulated  $\Phi = 0.95$  SAR(1)  $Z_t$ .



## 10.6.2 The Error Variance Matrix of the Estimates

From (10.8), for  $q = 2$  and  $n = 7$ , the error variance matrix has the formula

$$\Sigma_{ee} = \sigma_a^2 \frac{\Phi}{(1 + \Phi)^4} \begin{bmatrix} 2 + \Phi & 0 & \Phi & 0 & 0 & 0 & 0 \\ 0 & 2 + \Phi & 0 & \Phi & 0 & 0 & 0 \\ \Phi & 0 & 2 & 0 & \Phi & 0 & 0 \\ 0 & \Phi & 0 & 2 & 0 & \Phi & 0 \\ 0 & 0 & \Phi & 0 & 2 & 0 & \Phi \\ 0 & 0 & 0 & \Phi & 0 & 2 + \Phi & 0 \\ 0 & 0 & 0 & 0 & \Phi & 0 & 2 + \Phi \end{bmatrix}. \quad (10.41)$$

The error variances of the initial and final years are larger than the error variance  $2\sigma_a^2(1 + \Phi)^{-4}\Phi$  at intermediate times by the amount  $\sigma_a^2\Phi^2(1 + \Phi)^{-4}$ , which is the mean square error<sup>6</sup> of using  $\Phi(1 + \Phi)^{-2}\{\Phi w_t\}$  to forecast/backcast  $\Phi(1 + \Phi)^{-2}w_{t\pm q}$  in (10.34) and (10.37), since from (10.26) we have

$$E(w_{t\pm q} - \Phi w_t)^2 = (1 + \Phi^2)\gamma_0 - 2\Phi\gamma_q = (1 - \Phi^2)\gamma_0 = \sigma_a^2. \quad (10.42)$$

The fact that the intermediate-time mean square error has the same positive value for all  $n \geq 5$  shows that the mean square error does not become negligible with large  $n$ . *Unobserved components can be estimated*

<sup>6</sup>With model-based estimates from more general models for  $Z_t$ , more forecasts and backcasts are needed and their error cross-covariances occur in the mean square error formulas, which are less simple.

only to limited precision.

## 10.7 Pseudo-Spectral Densities of ARIMA Models\*

With nonstationary ARIMA  $Z_t$ , the *pseudo-spectral density* (p-sd) takes over the role of the sd in decomposition calculations. Its *partial fraction decomposition* (e.g., Wikipedia Contributors (2011)) is often the starting place for deriving the canonical decomposition as is shown with examples below.

Let  $Z_t$  denote a nonstationary ARIMA time series with differencing operator  $\delta(B) = 1 + \sum_{j=1}^d \delta_j B^j$  for  $d \geq 1$  such that  $w_t = \delta(B) Z_t$  has the model (10.17). We always assume that  $\delta(B)$  and  $\vartheta(B)$  have no common factors: there is no overdifferencing.

With  $g_w(\lambda)$  as in (10.22), the *pseudo-spectral density* (p-sd, plural p-sds) of  $Z_t$  is defined by

$$g_Z(\lambda) = \frac{g_w(\lambda)}{|\delta(e^{i2\pi\lambda})|^2} = \sigma_a^2 \frac{|\vartheta(e^{i2\pi\lambda})|^2}{|\delta(e^{i2\pi\lambda})|^2 |\varphi(e^{i2\pi\lambda})|^2}, \quad -1/2 \leq \lambda \leq 1/2. \quad (10.43)$$

The p-sd is a non-integrable function,  $\int_{-1/2}^{1/2} g_Z(\lambda) d\lambda = \infty$ , as each zero of  $\delta(e^{i2\pi\lambda})$  occurs where  $g_w(\lambda) > 0$ .

The most basic p-sd is that of the (0,1,0) or *random walk*,

$$(1 - B)Z_t = a_t \iff Z_t = Z_{t-1} + a_t, \quad (10.44)$$

where  $a_t$  is white noise with variance  $\sigma_a^2$ . From (10.43),

$$g_Z(\lambda) = \frac{\sigma_a^2}{|1 - e^{i2\pi\lambda}|^2}. \quad (10.45)$$

The model (10.44) is the special case  $\theta = 0$  of the *invertible IMA(1,1) trend model*,

$$(1 - B)Z_t = (1 - \theta B)a_t, \quad -1 < \theta < 1, \quad (10.46)$$

which, by (10.16) and (10.43), has the p-sd

$$g_Z(\lambda) = \sigma_a^2 \left| \frac{1 - \theta e^{i2\pi\lambda}}{1 - e^{i2\pi\lambda}} \right|^2, \quad -1 < \theta < 1. \quad (10.47)$$

Especially informative will be the  $r \geq 2$  *seasonal (1,1)<sub>r</sub>* generalization of (10.46),

$$(1 - B^r)Z_t = (1 - \Theta B^r)a_t, \quad -1 < \Theta < 1, \quad (10.48)$$

considered in Subsection 10.8.2. Its p-sd is

$$g_Z(\lambda) = \sigma_a^2 \left| \frac{1 - \Theta e^{i2\pi r\lambda}}{1 - e^{i2\pi r\lambda}} \right|^2, \quad -1 < \Theta < 1. \quad (10.49)$$

## 10.8 Canonical Pseudo-Spectral Density Decompositions\*

### 10.8.1 The Canonical Trend-Irregular Decomposition of the IMA(1,1)

The differencing operator  $1 - B$  of (10.46), the IMA(1,1), is a trend differencing, so the "signal" of its p-sd decomposition is a trend. We use  $p_t$  for trend (later trend-cycle) and  $N_t$  for the white noise irregular of the canonical two-component decomposition, as in Section 10.6,

$$Z_t = p_t + N_t. \quad (10.50)$$

The minimum value of the p-sd  $g_Z(\lambda)$  in (10.47) is not obvious, so we calculate the partial fraction decomposition,

$$\sigma_a^{-2} g_Z(\lambda) = \frac{|1 - \theta e^{i2\pi\lambda}|^2}{|1 - e^{i2\pi\lambda}|^2} = \frac{a}{|1 - e^{i2\pi\lambda}|^2} + b. \quad (10.51)$$

The constants  $a, b$  are obtained by multiplying (10.51) by  $|1 - e^{i2\pi\lambda}|^2$  and expanding the resulting expressions,

$$\begin{aligned} |1 - \theta e^{i2\pi\lambda}|^2 &= a + b |1 - e^{i2\pi\lambda}|^2 \\ (1 + \theta^2) - \theta (e^{i2\pi\lambda} + e^{-i2\pi\lambda}) &= a + b (2 - (e^{i2\pi\lambda} + e^{-i2\pi\lambda})) \\ &= (a + 2b) - b (e^{i2\pi\lambda} + e^{-i2\pi\lambda}). \end{aligned}$$

Equating constants and coefficients of  $(e^{i2\pi\lambda} + e^{-i2\pi\lambda})$  on both sides yields  $a + 2b = 1 + \theta^2$ ,  $b = \theta$ . Thus  $a = (1 - \theta)^2$  and we have

$$\sigma_a^{-2} g_Z(\lambda) = \frac{(1 - \theta)^2}{|1 - e^{i2\pi\lambda}|^2} + \theta, \quad (10.52)$$

revealing that the variance of the canonical white noise irregular component is

$$\sigma_a^{-2} \sigma_N^2 = \min_{\lambda} \sigma_a^{-2} g_Z(\lambda) = \frac{(1 - \theta)^2}{4} + \theta = \frac{(1 + \theta)^2}{4} \quad (10.53)$$

in units of  $\sigma_a^2$ . Therefore, for the p-sd of the trend, from (10.47),

$$\begin{aligned} \sigma_a^{-2} g_p(\lambda) &= \sigma_a^{-2} (g_Z(\lambda) - \sigma_N^2) = \frac{(1 - \theta)^2}{|1 - e^{i2\pi\lambda}|^2} - \frac{(1 - \theta)^2}{4} \\ &= \frac{1}{4} (1 - \theta)^2 \frac{4 - |1 - e^{i2\pi\lambda}|^2}{|1 - e^{i2\pi\lambda}|^2} = \frac{1}{4} (1 - \theta)^2 \frac{|1 + e^{i2\pi\lambda}|^2}{|1 - e^{i2\pi\lambda}|^2}. \end{aligned} \quad (10.54)$$

In summary, from (10.53) and (10.54), the canonical IMA(1,1) p-sd decomposition

$$g_Z(\lambda) = g_p(\lambda) + g_N(\lambda)$$

has

$$g_p(\lambda) = \frac{1}{4} (1 - \theta)^2 \frac{|1 + e^{i2\pi\lambda}|^2}{|1 - e^{i2\pi\lambda}|^2} \sigma_a^2, \quad g_N(\lambda) = \frac{1}{4} (1 + \theta)^2 \sigma_a^2, \quad (10.55)$$

and is admissible for all  $-1 \leq \theta < 1$ .

The canonical trend's model is the noninvertible IMA(1,1),

$$(1 - B)p_t = (1 + B)b_t, \quad (10.56)$$

with white noise  $b_t$  having variance  $\sigma_b^2 = \sigma_a^2(1 - \theta)^2/4$ .

For  $0 < \theta < 1$ , the *noncanonical*, admissible, IMA(1,1) trend-irregular *Structural Model* decomposition is derived in Section 10.18.

### 10.8.2 A Sometimes Nonadmissible 3-Component Canonical Decomposition\*

For three-component *seasonal-trend-irregular* decompositions, we follow the notation of Maravall (2016) in using  $s_t$ ,  $p_t$  and  $u_t$  for the respective components,

$$Z_t = s_t + p_t + u_t. \quad (10.57)$$

We start with our most revealing example, the canonical (10.57) decomposition of (10.48) for  $r = 2$  (biannual data). Partial fraction calculations for the p-sd (10.49) like those used for (10.52) yield

$$\sigma_a^{-2}g_Z(\lambda) = \frac{(1 - \Theta)^2}{4|1 + e^{i2\pi\lambda}|^2} + \frac{(1 - \Theta)^2}{4|1 - e^{i2\pi\lambda}|^2} + \Theta. \quad (10.58)$$

If  $\Theta$  is nonnegative, this is an admissible p-sd decomposition, but not the canonical decomposition: the condition  $\Theta \geq 0$  is too restrictive.

To obtain the candidate canonical p-sd decomposition, the positive (equal) minimum values of the nonconstant p-sds in (10.58),

$$\min_{\lambda} \frac{(1/4)(1 - \Theta)^2}{|1 + e^{i2\pi\lambda}|^2} = \min_{\lambda} \frac{(1/4)(1 - \Theta)^2}{|1 - e^{i2\pi\lambda}|^2} = (1 - \Theta)^2/16, \quad (10.59)$$

are shifted to the constant term  $\Theta$  in (10.58) to yield the larger constant,

$$\Theta + 2(1 - \Theta)^2/16 = \frac{(1 - \Theta)^2 + 8\Theta}{8} = \frac{1 + 6\Theta + \Theta^2}{8},$$

and the noninvertible seasonal and trend p-sds

$$\begin{aligned} \frac{(1/4)(1 - \Theta)^2}{|1 + e^{i2\pi\lambda}|^2} - (1 - \Theta)^2/16 &= \frac{(1 - \Theta)^2}{16} \frac{4 - |1 + e^{i2\pi\lambda}|^2}{|1 + e^{i2\pi\lambda}|^2} = \frac{(1 - \Theta)^2}{16} \frac{|1 - e^{i2\pi\lambda}|^2}{|1 + e^{i2\pi\lambda}|^2}, \\ \frac{(1/4)(1 - \Theta)^2}{|1 - e^{i2\pi\lambda}|^2} - (1 - \Theta)^2/16 &= \frac{(1 - \Theta)^2}{16} \frac{4 - |1 - e^{i2\pi\lambda}|^2}{|1 - e^{i2\pi\lambda}|^2} = \frac{(1 - \Theta)^2}{16} \frac{|1 + e^{i2\pi\lambda}|^2}{|1 - e^{i2\pi\lambda}|^2}. \end{aligned}$$

These result in the new decomposition

$$\sigma_a^{-2}g_Z(\lambda) = \frac{(1 - \Theta)^2}{16} \frac{|1 - e^{i2\pi\lambda}|^2}{|1 + e^{i2\pi\lambda}|^2} + \frac{(1 - \Theta)^2}{16} \frac{|1 + e^{i2\pi\lambda}|^2}{|1 - e^{i2\pi\lambda}|^2} + \frac{1 + 6\Theta + \Theta^2}{8}. \quad (10.60)$$

This is the canonical p-sd decomposition when the admissibility condition  $1 + 6\Theta + \Theta^2 \geq 0$  holds, which is equivalent to  $\Theta \geq -3 + 2\sqrt{2} \doteq -0.1716$ . For such  $\Theta$ , the canonical p-sd decomposition for (10.57) has

$$g_s(\lambda) = \sigma_a^2 \frac{(1 - \Theta)^2}{16} \frac{|1 - e^{i2\pi\lambda}|^2}{|1 + e^{i2\pi\lambda}|^2}, \quad g_p(\lambda) = \sigma_a^2 \frac{(1 - \Theta)^2}{16} \frac{|1 + e^{i2\pi\lambda}|^2}{|1 - e^{i2\pi\lambda}|^2}, \quad g_u(\lambda) = \sigma_u^2, \quad (10.61)$$

with the maximal irregular component variance

$$\sigma_u^2 = \frac{(\Theta^2 + 6\Theta + 1)}{8} \sigma_a^2. \quad (10.62)$$

Thus, for these  $\Theta$ , we have a canonical decomposition (10.57) with white noise  $u_t$  having variance (10.62) and with ARIMA  $s_t$  and  $p_t$  having the noninvertible biannual seasonal and trend models,

$$\begin{aligned} (1 + B) s_t &= (1 - B) c_t, \\ (1 - B) p_t &= (1 + B) b_t, \end{aligned}$$

respectively, with

$$\sigma_b^2 = \sigma_c^2 = (1 - \Theta)^2 \sigma_a^2 / 16. \quad (10.63)$$

For output tables of AMBSA software, the white noise variance  $\sigma_a^2$  of  $Z_t$ 's ARIMA model is often set equal to 1.0 in the calculation of seasonal decomposition components' white noise variances like  $\sigma_b^2$  and  $\sigma_c^2$ . These are then identified as being in "units of  $\text{var}(a)$ ".

For the subinterval  $-1 < \Theta < -3 + 2\sqrt{2}$ , (10.62) yields a negative  $\sigma_u^2$ . For these  $\Theta$ , the p-sd decomposition is *nonadmissible* and there is no AMBSA decomposition from the estimated model of  $Z_t$ . The automatic nonadmissible decomposition model replacement option<sup>7</sup> of some AMBSA software replaces  $\Theta$  in this one-parameter case with the closest  $\Theta$  from an admissible decomposition,  $\Theta \doteq -.1716$  for the model (10.48) with  $r = 2$ . Then  $\sigma_u^2 = 0$  so there is no irregular component.

Hillmer and Tiao (1982), pp. 66-67, also provide results for  $r > 2$  and for more general seasonal models, but only for  $r = 2$  (biannual data) do they obtain simple formulas. In practice, estimated  $\Theta$  are usually positive for an Airline i.e.,  $(0,1,1)(0,1,1)_r$  model, and  $\Theta \geq 0$  results in admissibility for this and some similar seasonal models, as Hillmer and Tiao show. Our final canonical p-sd decomposition example will provide a revealing collection of filter formulas for component estimation.

### 10.8.3 3-Component Pseudo-Spectral Density Decomposition of the Biannual Seasonal Random Walk

Setting  $\Theta = 0$  and  $r = 2$  in (10.48) yields the biannual Seasonal Random Walk, the  $(0,1,0)_2$  or  $\text{SRW}_2$ ,

$$(1 - B^2)Z_t = a_t, \quad (10.64)$$

whose p-sd is given by

$$\sigma_a^{-2} g_Z(\lambda) = \frac{1}{|1 - e^{i2\pi 2\lambda}|^2} = \frac{1}{|1 + e^{i2\pi\lambda}|^2} \frac{1}{|1 - e^{i2\pi\lambda}|^2}. \quad (10.65)$$

Setting  $\Theta = 0$  in (10.61) and (10.62) yields the p-sd formulas for its 3-component decomposition (10.57),

$$g_s(\lambda) = \frac{\sigma_a^2 |1 - e^{i2\pi\lambda}|^2}{16 |1 + e^{i2\pi\lambda}|^2}, \quad g_p(\lambda) = \frac{\sigma_a^2 |1 + e^{i2\pi\lambda}|^2}{16 |1 - e^{i2\pi\lambda}|^2}, \quad g_u(\lambda) = \frac{\sigma_a^2}{8}. \quad (10.66)$$

Thus  $s_t$  and  $p_t$  are nonstationary, with respective differencing polynomials  $\delta_s(B) = 1 + B$  and  $\delta_p(B) = 1 - B$ , with  $\delta_p(B)$  also the differencing polynomial of the seasonal adjustment  $sa_t = p_t + u_t$ . Section 10.15 provides the symmetric filter formulas, also obtained by Maravall and Pierce (1987), whose tutorial study of (10.64)

<sup>7</sup>One should check that the software's replacement model has acceptable goodness-of-fit diagnostics. If it does not, the user could explore other models and/or shorter data spans to try to obtain an estimated model with an admissible decomposition having acceptable diagnostics. Nonadmissibility is generally associated with data whose graph shows quite erratic movements or strong trend movements.



displays the infinitely many decompositions different from the canonical that can result from apportioning p-sd minima like (10.59) differently between two or more component p-sds (or sds).

We now consider methods for estimating components of nonstationary decompositions and their filters' properties.

## 10.9 Matrix Formulation of Signal Extraction: Difference-Stationary Case

For data with a known ARIMA model, three approaches for calculating MMSE estimates (providing identical estimates) are available among the main AMBSA programs. The only elementary approach, also the easiest to program for two-component decompositions, is presented in Subsection 10.9.3. To best reveal to the reader how the nonstationary case differs from the stationary case, we start with the need for an assumption that provides MMSE optimality of forecasts, backcasts and component estimates.

### 10.9.1 Assumption A and Random Walk Forecasts and Backcasts\*

Purely autocovariance-based formulas like those above, e.g. in (10.6), are not applicable to the inherently more complex case of nonstationary ARIMA  $Z_t$  because variances and autocovariances cannot be estimated for nonstationary variates. We illustrate why this is so with the simplest ARIMA process, the random walk (10.44). Its  $Z_t, t \geq 2$ , are generated recursively from  $Z_1$  and future white noise  $a_t, t \geq 2$ ,

$$Z_t = Z_{t-1} + a_t = Z_{t-2} + a_{t-1} + a_t = \cdots = Z_1 + \sum_{j=2}^t a_j, t \geq 2. \quad (10.67)$$

Assumption A of Bell (1984) for (10.44) is that the initial value  $Z_1$  of the data generating formula (10.67) is uncorrelated with all  $a_t$ , including the  $a_t, t \leq 1$  which generate earlier  $Z_t$  via  $Z_{t-1} - Z_t = -a_t, t \leq 1$ .

$EZ_1^2$  cannot be estimated consistently from one datum  $Z_1$ . Nor can the covariance  $EZ_1a_t$ . So Assumption A can neither be verified nor contradicted. It can be replaced by other assumptions but doing so leads to more complex formulas with no advantages, see Bell (1984), where it is also indicated why Assumption A justifies the standard ARIMA forecast formulas by guaranteeing that they provide MMSE forecasts. We establish this for the random walk.

For (10.44), the "well known" (if rarely fully derived) result is that *for all  $h \geq 1$ , the MMSE  $h$ -step forecast of  $Z_{t+h}$  from data  $Z_1, \dots, Z_t$  is the latest datum,  $\hat{Z}_{t+h} = Z_t$* . To establish this result, note from (10.67) that the forecast error  $Z_{t+h} - Z_t = a_{t+1} + a_{t+2} + \cdots + a_{t+h}$  is uncorrelated with  $Z_1$  by Assumption A and with  $a_2, \dots, a_t$  by the white noise property. Consequently, the error of forecast  $\hat{Z}_{t+h} = Z_t$  is uncorrelated with the data, which is the MMSE characterizing property, see Section 10.4. Analogous calculations show that  $Z_1$  is the MMSE backcast of  $Z_1, \dots, Z_t$  for all  $h \geq 1$ .

### 10.9.2 Required Properties of the Stationarized Data, Signal, and Noise

When signal and noise components are ARIMA, there are three differencing operators  $\delta(B) = 1 + \delta_1 B + \cdots + \delta_d B^d$  for the series  $Z_t$ ,  $\delta_S(B) = 1 + \delta_1^S B + \cdots + \delta_{d_S}^S B^{d_S}$  for  $S_t$ , and  $\delta_N(B) = 1 + \delta_1^N B + \cdots + \delta_{d_N}^N B^{d_N}$  for  $N_t$ . If either  $S_t$  or  $N_t$  is a stationary component, such as an irregular, transitory, or cycle component (see

Subsection 10.21.2) one sets  $\delta_S(B) = 1$  or  $\delta_N(B) = 1$ . For MMSE component estimates from series of length  $n > d$ , the formulas of McElroy (2008) below and most of those of Section 10.13 require:

1.  $\delta(B) = \delta_S(B) \delta_N(B)$
2.  $\delta_S(B)$  and  $\delta_N(B)$  have no common zeroes
3. The stationary processes

$$U_t = \delta_S(B) S_t, \quad V_t = \delta_N(B) N_t, \quad (10.68)$$

are uncorrelated<sup>8</sup>:  $EU_s V_t = 0$ , for  $s = d_s + 1, \dots, n, t = d_N + 1, \dots, n$ .

4. The  $d = \deg \delta(B)$  initial values  $Z_1, \dots, Z_d$  are uncorrelated with the series  $U_t$  and  $V_t$ . (Assumption A of Bell (1984)).

We refer to these as Requirements 1–3 and Assumption A. Regarding 1 and 2, when  $\delta(B) = (1 - B)^d (1 - B^{12})$ , as is common with monthly data  $Z_t$ , with  $S_t$  the seasonal and  $N_t = Z_t - S_t$  the nonseasonal component, these Requirements are met by  $\delta_S(B) = 1 + B + \dots + B^{11}$  and  $\delta_N(B) = (1 - B)^{d+1}$ .

### 10.9.3 The Two-Component Estimation Formulas

For  $U_t = \delta_S(B) S_t$ , define  $U = (U_{d_S+1}, \dots, U_n)'$  and let  $\Delta_S$  be the  $(n - d_S) \times n$  matrix implementing  $\delta_S(B)$ , i.e. such that

$$U = \Delta_S S.$$

Thus, when  $\delta_S(B) = 1 + \sum_{j=1}^{d_S} \delta_j^S B^j$ , the matrix  $\Delta_S$  has the form

$$\Delta_S = \begin{bmatrix} \delta_{d_S}^S & \delta_{d_S-1}^S & \dots & 1 & 0 & \dots & 0 \\ 0 & \delta_{d_S}^S & \delta_{d_S-1}^S & \dots & 1 & \dots & 0 \\ \vdots & & & & & & \vdots \\ 0 & & & & & 1 & 0 \\ 0 & 0 & \dots & \dots & \delta_1^S & 1 \end{bmatrix}.$$

Set  $\Sigma_U = EUU'$  and let  $\Sigma$ ,  $\Sigma_V$  and  $\Delta_N$  be the analogous matrices for  $\delta(B) Z_t$ ,  $V_t = \delta_N(B) N_t$ , and  $\delta_N(B)$ . With the autocovariance matrices  $\Sigma_U$  and  $\Sigma_V$  and differencing matrices  $\Delta_S$  and  $\Delta_N$ , McElroy (2008) shows, under Requirements 1–4, that the MMSE linear estimate

$$\hat{S} = \beta_S Z \quad (10.69)$$

is obtained with

$$\beta_S = (\Delta_S' \Sigma_U^{-1} \Delta_S + \Delta_N' \Sigma_V^{-1} \Delta_N)^{-1} \Delta_N' \Sigma_V^{-1} \Delta_N, \quad (10.70)$$

and the variance matrix  $\Sigma_{ee}$  of the signal extraction error  $e = S - \hat{S}$  has the formula

$$\Sigma_{ee} = (\Delta_S' \Sigma_U^{-1} \Delta_S + \Delta_N' \Sigma_V^{-1} \Delta_N)^{-1}. \quad (10.71)$$

For each  $1 \leq t \leq n$ , the  $t$ -th row of  $\beta_S$  consists of the filter coefficients  $\beta_{S,t,j}$  used to obtain  $\hat{S}_t = \sum_{j=1}^n \beta_{S,t,j} Z_j$ . The *change of scale results* below formulas (10.8) also apply in the ARIMA case, e.g., the filters do not depend on  $\sigma_a^2$ . McElroy (2008) establishes various properties of the filters, including their *reverse symmetry*: the coefficients for  $t = n$  are those of  $t = 1$  in reverse order, and similarly for  $t = n - 1$  and  $t = 2$ ,

<sup>8</sup>Findley (2012) shows how this requirement can be weakened.

etc. If  $n$  is odd,  $n = 2m + 1$ , then the filter for the midpoint  $t = m + 1$  is *symmetric*,  $\beta_{S,m+1,j} = \beta_{S,m+1,n-j}$  for  $1 \leq j \leq m$ . Otherwise, the AMBSA seasonal adjustment filters are asymmetric. As in the stationary case,  $\hat{N} = \beta_N Z$  with  $\beta_N = I - \beta_S$ .

A conspicuous feature of the formula for  $\beta_S$  is the noise differencing operator  $\Delta_N$  on the right: When  $N_t$  is nonstationary, signal extraction starts by stationarizing the noise component.

Extensions of the matrix formulas are developed in McElroy (2006) and McElroy and Holan (2012) for the long-term trend and cycle estimates of Subsection 10.21.2.

### 10.9.4 Filter and Error Variance Properties of The Canonical Decomposition\*

Remark 1 of McElroy (2008) describes how the formulas (10.70) and (10.71) simplify when one component is stationary. For the canonical decomposition case, with  $\Sigma_{NN} = \sigma^2 I$ ,  $\sigma^2 > 0$ , and  $W = \Delta Z$ , the result is

$$\begin{aligned}\beta_S &= I - \sigma^2 \Delta' \Sigma_W^{-1} \Delta, \\ \Sigma_{ee} &= \sigma^2 \beta_S.\end{aligned}$$

This  $\Sigma_{ee}$  formula is a generalization of (10.8) to nonstationary  $Z_t$  and  $S_t$ . When, for example,  $S_t$  is the trend component of the IMA(1,1) model considered in Subsection 10.8.1, then  $\sigma^2 = (1 + \theta)^2 / 4$  in units of  $\sigma_a^2$ , see (10.53).

The formulas reveal that, with white noise  $N$ ,  $\beta_S = \sigma^{-2} \Sigma_{ee}$  is a positive definite matrix, since  $\Sigma_{ee}$  is. It follows that the filter coefficient  $\beta_{S,t,t}$  of  $Z_t$  in  $\hat{S}_t = \sum_{j=1}^n \beta_{S,t,j} Z_j$  is positive for each  $1 \leq t \leq n$ . Further, the largest magnitude coefficient in  $\beta_S$  is on the main diagonal, i.e., is the coefficient  $\beta_{S,t,t}$  of  $Z_t$  in  $\hat{S}_t$  for some  $t$ , and thus positive, see Theorem 12.4 of Noble (1969). Also,  $\beta_N = 1 - \beta_S = \sigma^2 \Delta' \Sigma_W^{-1} \Delta$  has these properties.

The largest magnitude coefficient property does not generalize to 3-component decompositions: In Subsection 10.15.2, the seasonal component filter (10.91) of the canonical three-component decomposition of a seasonal random walk is displayed. Its largest magnitude coefficient is negative and not on the main diagonal.

Section 5.2 of McElroy (2008) shows a way, which is implemented in some AMBSA software to, use matrix formulas to obtain AMBSA estimates for decompositions with more components. But usually the state space method or the W-K filter-based calculation method described in Burman (1980) is used. Each is available in widely used AMBSA software and can handle any number of components. Both have important computational efficiency advantages over the matrix formulas. The different methods produce the same finite-sample estimates and filter coefficients (up to rounding error). Only the matrix and state space calculations produce finite-sample variances and covariances rather than infinite-sample-based approximations.

## 10.10 Illustrative Seasonal Adjustment Filter and Standard Error Graphs\*

For a monthly time series of length  $n = 131$  from the  $\theta = 0$  Airline model,

$$(1 - B)(1 - B^{12}) \log Z_t = (1 - \Theta B^{12}) a_t, \quad (10.72)$$

with  $\Theta = 0.3$ , Figure 10.3 shows the filter coefficients for the symmetric midpoint SA filter ( $t = 66$ , solid lines) and for the one-sided concurrent SA filter ( $t = 131$ , dashed lines). Focusing on the larger coefficients, both filters have effective lengths of about two years, with large coefficients of different signs between time  $t$  and the adjacent same-calendar-month times, also for other  $t$  not shown. As a consequence, their SA estimates can be adaptive to short-term changes in the features of the series, potentially providing considerable smoothing.

But with future data, the SA, especially its concurrent value, can have large *revisions*, i.e., large changes in value from its initial estimate, in comparison to revisions from Figure 10.4's  $\Theta = 0.9$  filters, as the standard errors in Figure 10.5 indicate.

**Figure 10.3:** Symmetric central ( $t = 66$ ) and one-sided concurrent ( $t = 131$ ) seasonal adjustment filter coefficients for  $n = 131$  from the canonical decomposition of the  $(0,1,0)(0,1,1)_{12}$  model with  $\Theta = 0.3$ .

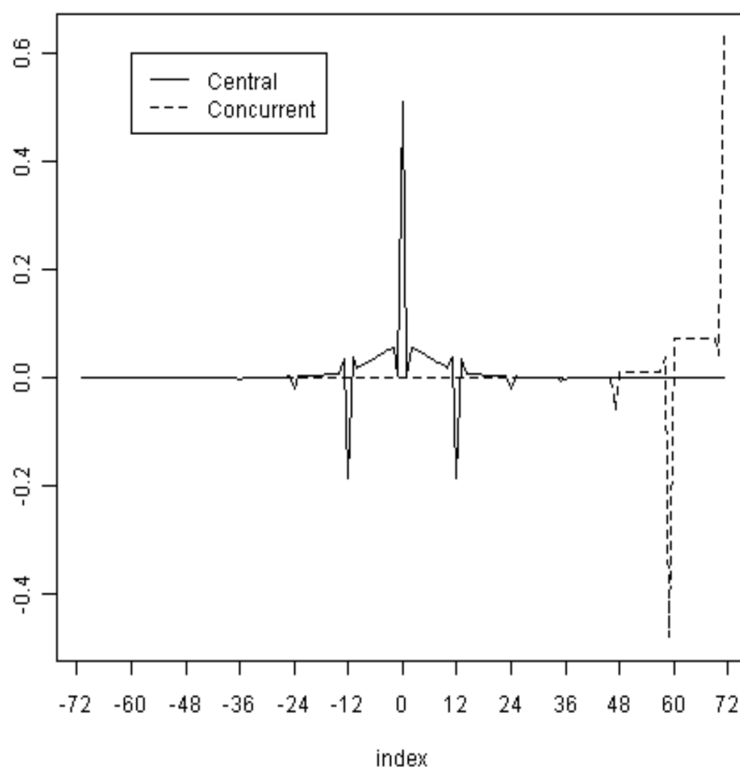


Figure 10.5 shows that additional smoothing from the  $\Theta = 0.3$  model results in more than twice the standard error of the less smooth seasonal adjustment from the  $\Theta = 0.9$  model. Note too how the standard errors increase with the distance from the center of the series. The same  $\sigma_a^2$  is used in (10.72) for both models.

## 10.11 Standard Errors of Change for Additive Estimates

### 10.11.1 Change from the Preceding Estimate

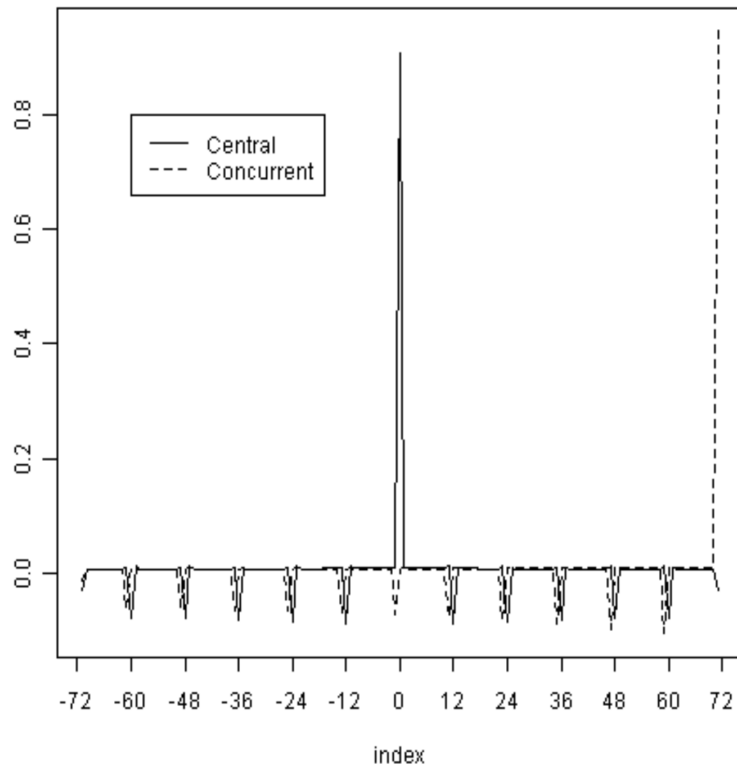
Error variances and covariances from  $\Sigma_{ee}$  can be used to describe the uncertainty in measures of change. Let  $S_t$  denote the seasonal adjustment component of interest (e.g., the seasonal adjustment or the trend). With an additive decomposition, the error of the one-month change estimate  $\hat{S}_t - \hat{S}_{t-1}$  is the difference of the errors of the two estimates,

$$(S_t - S_{t-1}) - (\hat{S}_t - \hat{S}_{t-1}) = (S_t - \hat{S}_t) - (S_{t-1} - \hat{S}_{t-1}) = e_t - e_{t-1},$$

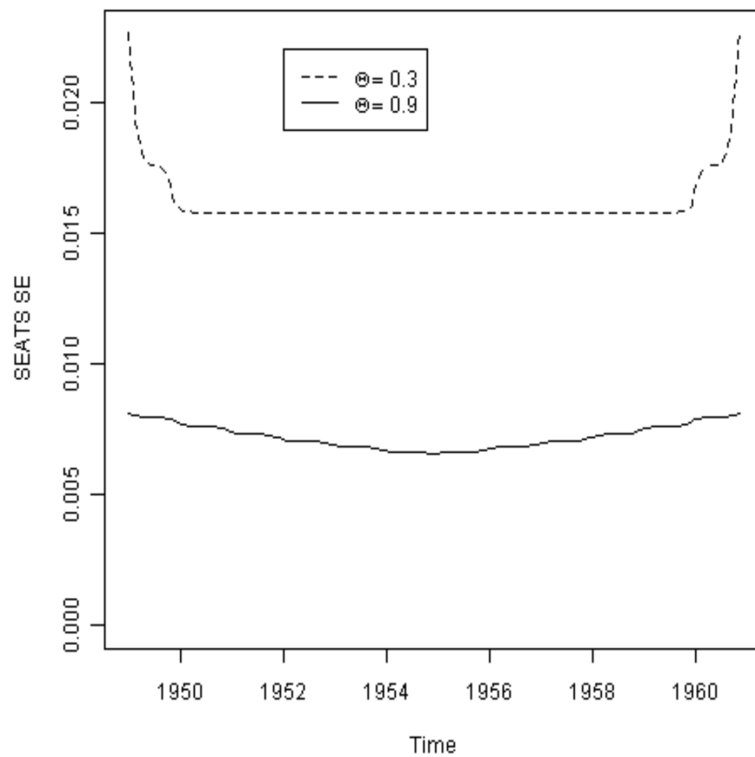
so the error variance is

$$E(e_t - e_{t-1})^2 = (\Sigma_{ee})_{t,t} + (\Sigma_{ee})_{t-1,t-1} - 2(\Sigma_{ee})_{t,t-1}. \quad (10.73)$$

**Figure 10.4:** The  $\Theta = 0.9$  analogue of the preceding Figure. The coefficients decrease slowly, so the filters are not very responsive to short-term data fluctuations.



**Figure 10.5:** The finite-sample standard errors of seasonal adjustments from the models of Figures 10.3 and 10.4, increase with the asymmetry of the filters as more forecasts and backcasts are used.



Hence the standard error of  $\hat{S}_t - \hat{S}_{t-1}$  is given by

$$\sqrt{(\Sigma_{ee})_{t,t} + (\Sigma_{ee})_{t-1,t-1} - 2(\Sigma_{ee})_{t,t-1}}, \tag{10.74}$$

from which probability intervals for  $S_t - S_{t-1}$  can be calculated under standard assumptions.

We illustrate such an interval for the more complex case of future revisions of an estimate.

### 10.11.2 Revisions with Future Data\*

The  $n \times n$  matrix  $\Sigma_{ee}$  depends only on the models for  $S_t$  and  $N_t$ , not on  $Z_t$  data, see (10.71). So it can be calculated for future series lengths  $n+h$  for any  $h = 1, 2, \dots$  when only  $n$  observations are available. Let  $\hat{S}_{t|1}^n$  and  $\hat{S}_{t|1}^{n+h}$  denote the estimates of  $S_t$  from these two series lengths, resulting in the revision  $\hat{S}_{t|1}^{n+h} - \hat{S}_{t|1}^n$ , and let  $\Sigma_{ee}^{(n)}$  and  $\Sigma_{ee}^{(n+h)}$  denote the corresponding error variance matrices. The main result is that  $R_t(h) = E\left(\hat{S}_{t|1}^{n+h} - \hat{S}_{t|1}^n\right)^2$ , called the *revision variance* or the *mean square revision*, is given by the nonnegative quantity

$$R_t(h) = \left(\Sigma_{ee}^{(n)}\right)_{t,t} - \left(\Sigma_{ee}^{(n+h)}\right)_{t,t}. \quad (10.75)$$

Hence, assuming normality, a 95 percent probability interval for the revised estimate  $\hat{S}_{t|1}^{n+h}$  with additional data  $Z_{n+1}, \dots, Z_{n+h}$  is given by  $\hat{S}_{t|1}^n - 1.96\sqrt{R_t(h)} \leq \hat{S}_{t|1}^{n+h} \leq \hat{S}_{t|1}^n + 1.96\sqrt{R_t(h)}$ .

With  $R_t(\infty) = \lim_{h \rightarrow \infty} R_t(h)$ , McElroy and Gagnon (2008) show how to calculate the analogue  $1 - \sqrt{1 - R_t(h)/R_t(\infty)}$  of a revision measure produced by AMBSA software for revisions of estimates from the infinite past  $\hat{S}_{t|-\infty}^n - \hat{S}_{t|-\infty}^{n+h}$ ,  $1 \leq h < \infty$ .

McElroy and Gagnon further show how this is a measure of the proportion of the total root mean square revision  $\sqrt{R_t(\infty)}$  of  $\hat{S}_{t|1}^n$  that is obtained after  $h$  months. Their numerical comparisons indicate that the software-default infinite-data-based measures, which are computationally less expensive (especially for large  $n$ ), differ little from  $1 - \sqrt{1 - R_t(h)/R_t(\infty)}$  except when  $n$  is small.

## 10.12 Multiplicative Decomposition Estimates from Logs\*

Most often, it is the logs of positive economic data  $z_t$ ,  $Z_t = \log z_t$ , that can be modeled with an ARIMA model. In this case, the conceptual two-component signal and noise decomposition model is a multiplicative decomposition,  $z_t = s_t n_t$ , estimated from the additive decomposition  $Z_t = S_t + N_t$ , with  $S_t = \log s_t$  and  $N_t = \log n_t$ .

If lognormality (e.g., Wikipedia Contributors (2017a)) is assumed, the MMSE estimate of  $s_t = \exp S_t$  is

$$\exp\left(\hat{S}_t + \frac{1}{2}(\Sigma_{ee})_{t,t}\right) = \left(\sqrt{\exp(\Sigma_{ee})_{t,t}}\right) \exp \hat{S}_t,$$

whose mean square error is  $\exp 2\hat{S}_t \left(\exp\left(2(\Sigma_{ee})_{t,t}\right) - \exp\left((\Sigma_{ee})_{t,t}\right)\right)$ , and analogously for  $n_t = \exp N_t$ .

The product of the optimal estimates is thus not  $z_t$  but  $z_t (\Sigma_{ee})_{t,t}$ . In AMBSA practice however,  $\hat{s}_t = \exp \hat{S}_t$  and  $\hat{n}_t = \exp \hat{N}_t$  are taken as the estimates, giving up idealized mean square optimality for the practical goal of having a multiplicative decomposition  $z_t = \hat{s}_t \hat{n}_t$ .

Another effect of estimating components by exponentiating estimates made from log transformed data is that the resulting *level estimates are downwardly biased*, as a consequence of the geometric-arithmetic mean inequality:  $\exp\left(n^{-1}\sum_{t=1}^n \log x_t\right) < n^{-1}\sum_{t=1}^n x_j$  for  $n \geq 2$  unless all  $x_j$  have the same value. Thus trend estimates obtained by exponentiating the trend estimates of the log transformed data are downwardly biased.

A simple often effective procedure of Maravall to reduce this bias is implemented in all AMBSA software. See Proietti and Riani (2009) for a more encompassing analysis and discussion of transformations.

### 10.12.1 Standard Errors for Growth Rates\*

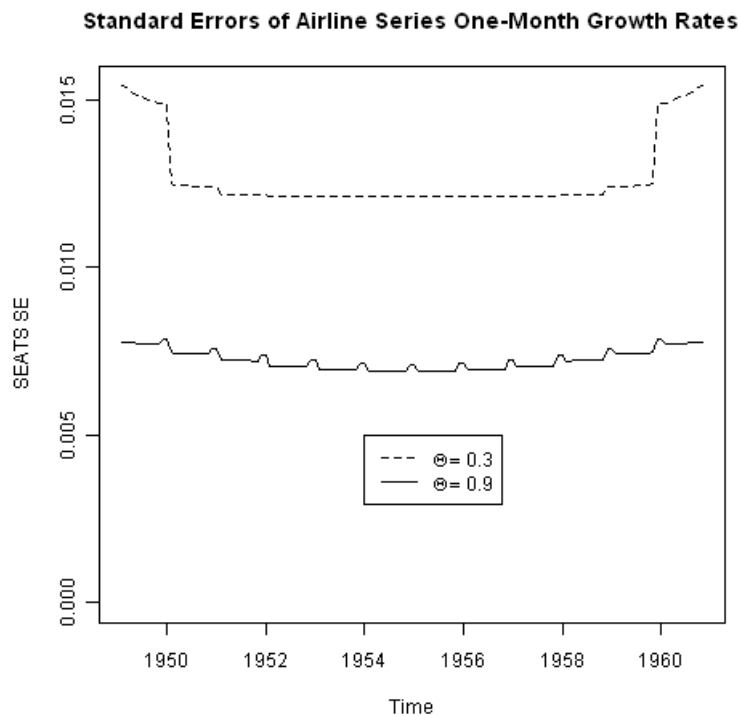
Growth rates from multiplicative decompositions are calculated as

$(s_t - s_{t-1}) / s_{t-1}$  in the one-period case. When they are reasonably small, e.g.  $< 0.10$ , then

$$\hat{S}_t - \hat{S}_{t-1} = \log\left(\frac{\hat{s}_t}{\hat{s}_{t-1}}\right) = \log\left(1 + \frac{\hat{s}_t - \hat{s}_{t-1}}{\hat{s}_{t-1}}\right) \doteq \frac{\hat{s}_t - \hat{s}_{t-1}}{\hat{s}_{t-1}}, \quad (10.76)$$

and the error variance of  $(\hat{s}_t - \hat{s}_{t-1}) / \hat{s}_{t-1}$  can be estimated by the error variance of  $\hat{S}_t - \hat{S}_{t-1}$ , i.e. by (10.73), and its standard error by (10.74). The standard errors of  $\hat{S}_t - \hat{S}_{t-1}$  shown in Figure 10.6 for the models of Figures 3 and 4 reveal that those for  $\Theta = 0.3$  are substantially larger and increase much more sharply near the ends of the series than those for  $\theta = 0.9$ .

**Figure 10.6:** Finite-sample ( $n = 131$ ) standard errors of the approximate one-month growth rates (10.76) from (10.74) for the canonical decompositions of the (0,1,0)(0,1,1) models with  $\Theta = 0.3$  and  $\Theta = 0.9$ . The value used for the white noise variance  $\sigma_a^2$  is the same as for Figure 10.3 and the interpretation of the graphs is analogous.





## 10.13 ARMA and ARIMA Wiener-Kolmogorov Filters

MMSE component estimation for the case of bi-infinite data  $Z_m, -\infty < m < \infty$  is the conceptual starting point for the approach of Hillmer and Tiao (1982) approach to MMSE finite-sample component estimation. For signal plus noise decompositions of stationary series, the symmetric MMSE filter formulas (10.77) and associated transfer function formulas below were independently published by Kolmogorow (1939) and Wiener (1949). Bell (1984) provided conceptual foundations for the ARIMA generalization. As Subsection 10.6.1 illustrated with finite symmetric filters for stationary data and Subsection 10.15 illustrates for nonstationary data, forecasts and backcasts replace the unavailable data required by the symmetric filter, thereby defining a time-varying asymmetric filter. The resulting estimates are referred to as Wiener-Kolmogorov or W-K estimates.

Starting from the always symmetric, usually bi-infinite filters introduced here, AMBSA software that does not apply state space methods or matrix formulas will apply W-K filters and the algorithm of Tunnicliffe-Wilson published in the Appendix of Burman (1980). The algorithm exploits the fact that, because recursion relations from the model can be used, the desired finite-sample MMSE estimates can be obtained from rapidly from a relatively small number of forecasts and backcasts from the finite available ARMA or ARIMA data.

We first consider estimation of a two-component decomposition,  $Z_t = S_t + N_t$  for  $Z_t$  with an invertible ARIMA model.

To obtain the MMSE estimates  $\hat{S}_t$  and  $\hat{N}_t, -\infty < t < \infty$ , from bi-infinite data<sup>9</sup>  $Z_m, -\infty < m < \infty$ , the Requirements 1 and 2 and Assumption A of Subsection 10.9.2 are retained. Requirement 3 is reformulated for  $U_t = \delta_S(B) S_t$  and  $V_t = \delta_N(B) N_t$  as  $EU_t V_{t-j} = 0$  for all  $j = 0, \pm 1, \pm 2, \dots$

Under these conditions, the symmetric filters

$$\beta_S(B) = \beta_{S,0} + \sum_{j=1}^{\infty} \beta_{S,j} (B^{-j} + B^j), \quad \beta_N(B) = \beta_{N,0} + \sum_{j=1}^{\infty} \beta_{N,j} (B^{-j} + B^j)$$

of the bi-infinite data estimates,

$$\hat{S}_t = \beta_S(B) Z_t = \beta_{S,0} Z_t + \sum_{j=1}^{\infty} \beta_{S,j} (Z_{t+j} + Z_{t-j})$$

and  $\hat{N}_t = \beta_N(B) Z_t$ , with  $\beta_N(B) = 1 - \beta_S(B)$ , have W-K transfer functions defined by ratios (10.77) of p-sds of  $S_t, N_t$  and  $Z_t$ . It is now helpful to use the fact that the defining formulas (10.22) of sds and (10.43) of p-sds are each functions of  $e^{i2\pi\lambda}$ . We will write  $g_Z(e^{i2\pi\lambda})$  instead of  $g_Z(\lambda)$  and similarly for all other p-sds. The W-K formulas are

$$\beta_S(e^{i2\pi\lambda}) = \frac{g_S(e^{i2\pi\lambda})}{g_Z(e^{i2\pi\lambda})}, \quad \beta_N(e^{i2\pi\lambda}) = \frac{g_N(e^{i2\pi\lambda})}{g_Z(e^{i2\pi\lambda})}. \quad (10.77)$$

With stationary  $Z_t$ , sds replace p-sds in these formulas and the  $S_t$  series is assumed to be uncorrelated with the  $N_t$  series. Appendix A of Findley et al. (2015) provides an elaboration of the derivation of Whittle (1963) for the stationary case formula  $\beta_S(e^{i2\pi\lambda}) = g_S(e^{i2\pi\lambda}) / g_Z(e^{i2\pi\lambda})$  and of the estimation error spectral density formula: in the original notation  $g_e(\lambda) = g_S(\lambda) g_N(\lambda) / g_Z(\lambda)$ , an analogue of the formula  $\Sigma_{ee} = \Sigma_{SS} \Sigma_{ZZ}^{-1} \Sigma_{NN}$  of (10.8).

The ps-d generalization of (10.21) shows that multiplying the squares of the transfer functions (10.77) by

<sup>9</sup>Bell (1984) shows how the infinite past  $Z_m, m \leq 0$  can be generated recursively from the degree  $d$  differencing polynomial  $\delta(B)$  of  $Z_t$ , the starting values  $Z_1, \dots, Z_d$ , and the stationary process  $w_t = \delta(B) Z_t, -\infty < t < \infty$ . This was illustrated with the random walk, which has  $d = 1$  and  $\delta(B) = (1 - B)$ , in Subsection 10.9.1.

the p-sd or sd of  $Z_t$  yields the p-sds or sds of the component estimates. From these the estimates' ARIMA or ARMA models are revealed, see Findley et al. (2016) for examples. These models provide seasonal adjustment quality diagnostics, see Maravall (1987), Findley et al. (2005) and their references.

Analogous W-K formulas apply for decompositions with more components, e.g., with seasonal  $s_t$ , trend  $p_t$ , and irregular  $u_t$ ,

$$\beta_s(e^{i2\pi\lambda}) = \frac{g_s(e^{i2\pi\lambda})}{g_Z(e^{i2\pi\lambda})}, \quad \beta_p(e^{i2\pi\lambda}) = \frac{g_p(e^{i2\pi\lambda})}{g_Z(e^{i2\pi\lambda})}, \quad \beta_u(e^{i2\pi\lambda}) = \frac{g_u(e^{i2\pi\lambda})}{g_Z(e^{i2\pi\lambda})}. \quad (10.78)$$

## 10.14 W-K Filter Formula Examples

### 10.14.1 Rederiving the $AR(1)_r$ Symmetric Filters

From (10.77), we can quickly re-obtain (10.33): First, from (10.28) and (10.27),

$$\beta_N(e^{i2\pi\lambda}) = \frac{g_N(e^{i2\pi\lambda})}{g_w(e^{i2\pi\lambda})} = \frac{(1 + \Phi)^{-2}}{|1 - \Phi e^{i2\pi r\lambda}|^{-2}} = (1 + \Phi)^{-2} |1 - \Phi e^{i2\pi r\lambda}|^2. \quad (10.79)$$

Next, for translations from transfer functions of the form  $|\sum_j \alpha_j e^{i2\pi r\lambda}|^2$  to filters, we can adopt a device of Maravall and Pierce (1987), replacing  $e^{\pm i2\pi j\lambda}$  by  $B^{\pm j}$  to obtain symmetric filter formulas,

$$|\sum_j \alpha_j B^j|^2 = (\sum_j \alpha_j B^j) (\sum_j \alpha_j B^{-j}). \quad (10.80)$$

From (10.79) and (10.80),

$$\beta_N(B) = (1 + \Phi)^{-2} (1 - \Phi B^r) (1 - \Phi B^{-r}) = (1 + \Phi)^{-2} (-\Phi B^r + (1 + \Phi^2) - \Phi B^{-r}).$$

This is the filter that produces (10.33).

### 10.14.2 Infinite W-K Filters

A W-K filter is infinite if and only if the model for  $Z_t$  has a moving average component.

#### 10.14.2.1 A Stationary Case: The Invertible Seasonal $MA(1)_r$

Suppose  $Z_t = (1 - \theta B^r) a_t$ , with  $r \geq 2$ ,  $\sigma_a^2 = 1$ ,  $0 < |\theta| < 1$ . Then  $g_Z(\lambda) = |1 - \theta e^{i2\pi r\lambda}|^2$  and the white noise sd  $g_N(\lambda) = \sigma^2$  has

$$\sigma^2 = \min_{\lambda} |1 - \theta e^{i2\pi r\lambda}|^2 = (1 - |\theta|)^2.$$

Thus its W-K transfer function is

$$\beta_N(e^{i2\pi\lambda}) = \frac{\sigma^2}{g_Z(e^{i2\pi\lambda})} = (1 - |\theta|)^2 |1 - \theta e^{i2\pi r\lambda}|^{-2}.$$

From (10.22), this has the form of the sd of a seasonal  $AR(1)_r$  with AR coefficient  $\theta$ , white noise variance  $(1 - |\theta|)^2$ , and thus variance  $(1 - |\theta|)^2 (1 - \theta^2)^{-1} = (1 - |\theta|) (1 + |\theta|)^{-1}$ . Hence, from (10.26) and (10.12),

the filters are symmetric and infinite:

$$\beta_N(B) = \frac{1 - |\theta|}{1 + |\theta|} \left( 1 + \sum_{j=1}^{\infty} \theta^j (B^{jr} + B^{-jr}) \right),$$

$$\beta_S(B) = 1 - \beta_N(B) = \frac{2|\theta|}{1 + |\theta|} + \sum_{j=1}^{\infty} \left\{ -\frac{1 - |\theta|}{1 + |\theta|} \theta^j \right\} (B^{jr} + B^{-jr}).$$

Therefore

$$\hat{N}_t = \beta_N(B) Z_t = \frac{1 - |\theta|}{1 + |\theta|} \left( Z_t + \sum_{j=1}^{\infty} \theta^j (Z_{t-jr} + Z_{t+jr}) \right), \quad (10.81)$$

$$\hat{S}_t = \beta_S(B) Z_t = \frac{2|\theta|}{1 + |\theta|} Z_t + \sum_{j=1}^{\infty} \left\{ -\frac{1 - |\theta|}{1 + |\theta|} \theta^j \right\} (Z_{t-jr} + Z_{t+jr}). \quad (10.82)$$

The filter coefficients are nonzero only at lag zero and seasonal lags. Lag zero has the largest magnitude coefficient. The magnitudes at seasonal lags decrease exponentially with increasing seasonal lag. For  $\theta > 0$ , the seasonal lag  $jr \geq 1$  coefficients are positive for the white noise estimate  $\hat{N}_t$  and negative for the signal estimate  $\hat{S}_t$ . The coefficients decay exponentially at the rate  $|\theta|^j$  starting at  $j = 1$ .

Next we consider the nonstationary case, starting with a general result.

#### 10.14.2.2 Transfer Function Form and Coefficient Decay Rate of ARIMA W-K Filters

All W-K transfer functions like those in (10.78) have the form of the sd of an ARMA model because the differencing operator factor of  $g_Z(e^{i2\pi\lambda})$ , e.g.,  $|\delta_S(e^{i2\pi\lambda})|^{-2} |g_N(e^{i2\pi\lambda})|^{-2}$  in the general two-component case, becomes the numerator factor  $|\delta_S(e^{i2\pi\lambda})|^2 |g_N(e^{i2\pi\lambda})|^2$  in  $g_Z(e^{i2\pi\lambda})^{-1}$ , cancelling any differencing factors of  $g_S(e^{i2\pi\lambda})$  and  $g_N(e^{i2\pi\lambda})$ . It follows that the coefficients of the W-K filter  $\beta(B)$  coincide with the autocovariances  $\gamma_j$  of this ARMA model,

$$\beta(B) = \gamma_0 + \sum_{j=1}^{\infty} \gamma_j (B^{-j} + B^j). \quad (10.83)$$

This formula reveals the important property that, as in Figures 3 and 4, the weight  $\gamma_0$  given by W-K filters to the contemporaneous datum  $Z_t$  in the estimate  $\hat{\beta}_t = \beta(B) Z_t$  is positive and greater in magnitude than all other coefficients,  $\gamma_0 > |\gamma_j|$  for all  $j > 1$ , because  $|\gamma_j/\gamma_0| < 1$  holds for ARMA autocorrelations. These properties were established for the filters of every canonical two-component finite-sample decomposition in Subsection 10.9.4.

For specific examples of an ARIMA model's component filters, we return to the fundamental model of Subsection 10.8.1.

#### 10.14.2.3 Nonstationary Case: IMA(1,1) Trend-Irregular Decomposition Filters

For the trend-irregular decomposition (10.50) of the model  $(1 - B) Z_t = (1 - \theta B) a_t$ ,  $-1 < \theta < 1$ , the sd and p-sd functions of the irregular and trend estimates,  $g_u(\lambda)$  and  $g_p(\lambda) = 1 - \beta_u(\lambda)$ , were obtained in (10.55). Here we derive the formulas of the components' estimation filters and note some features that generalize. From the formulas of (10.55) for  $g_u(\lambda)$  and of (10.47) for  $g_Z(\lambda)$ , the transfer function (10.78) for the bi-infinite

data estimate  $\hat{u}_t = \beta_u(B) Z_t$  of the white noise irregular component is

$$\beta_u(\lambda) = \sigma_u^2 g_Z(\lambda)^{-1} = \frac{1}{4} (1 + \theta)^2 \frac{|1 - e^{i2\pi\lambda}|^2}{|1 - \theta e^{i2\pi\lambda}|^2}. \quad (10.84)$$

From (10.53), this is the sd of an ARMA(1,1) with AR coefficient  $\theta$ , MA coefficient 1, and white noise variance  $(1 + \theta)^2 / 4$ . The transfer function of the trend component,

$$\beta_p(\lambda) = g_p(\lambda) g_Z(\lambda)^{-1} = \frac{1}{4} (1 - \theta)^2 \frac{|1 + e^{i2\pi\lambda}|^2}{|1 - \theta e^{i2\pi\lambda}|^2}, \quad (10.85)$$

is the sd of an ARMA(1,1) with AR coefficient  $\theta$ , MA coefficient  $-1$  and innovation variance  $(1 - \theta)^2 / 4$ . By (10.83), each has as filter coefficients the autocovariances of its sd's model. We calculate the formula for  $\beta_p(B)$  in this way and then obtain  $\beta_u(B)$  as  $1 - \beta_p(B)$ . Applying the recursions (3.4.7) of Box and Jenkins (1976) for ARMA(1,1) autocovariances to (10.85), we obtain

$$\begin{aligned} \beta_p(B) &= \frac{2}{1 - \theta} \left\{ 1 + \frac{1}{2} (B + B^{-1}) + \frac{1}{2} \sum_{j=2}^{\infty} \theta^{j-1} (B^j + B^{-j}) \right\}. \\ \beta_u(B) &= 1 - \beta_p(\theta) = \frac{2}{1 - \theta} \left\{ \frac{1 + \theta}{2} - \frac{1}{2} (B + B^{-1}) - \frac{1}{2} \sum_{j=2}^{\infty} \theta^{j-1} (B^j + B^{-j}) \right\}. \end{aligned}$$

The coefficients decay exponentially at the rate  $|\theta|^j$  starting<sup>10</sup> at  $j = 2$ . For  $\theta > 0$ , the midpoint ( $j = 0$ ) coefficients,  $2(1 - \theta)^{-1}$  for  $\beta_p(B)$  and  $(1 + \theta)(1 - \theta)^{-1}$  for  $\beta_u(B)$ , are the largest in magnitude. Also, for the trend estimate, all coefficients are positive. For the irregular estimate, except at the midpoint, all are negative.

## 10.15 Biannual Seasonal Random Walk Filters\*

It is useful to consider detailed results for the 3-component decomposition of (10.64) with  $r = 2$ . The results illustrate AMBSA for the rather rare case of a model with no MA component. Here, unless the data interval is short, symmetric filters can be applied in an interior interval and asymmetric filters are required only near the two ends of the series. So one wants to be aware of differences in the properties of the two kinds of filters.

From (10.65),

$$g_Z(\lambda)^{-1} = \sigma_a^{-2} |1 - e^{i2\pi\lambda}|^2 = \sigma_a^{-2} |1 + e^{i2\pi\lambda}|^2 |1 - e^{i2\pi\lambda}|^2. \quad (10.86)$$

Multiplication into (10.66) provides the filter transfer functions of the canonical decomposition,

$$\beta_s(e^{i2\pi\lambda}) = \frac{1}{16} |1 - e^{i2\pi\lambda}|^4, \quad \beta_p(e^{i2\pi\lambda}) = \frac{1}{16} |1 + e^{i2\pi\lambda}|^4, \quad \beta_u(e^{i2\pi\lambda}) = \frac{1}{8} |1 - e^{i2\pi\lambda}|^2, \quad (10.87)$$

with  $\beta_{sa}(e^{i2\pi\lambda}) = 1 - \beta_s(e^{i2\pi\lambda})$  for the seasonal adjustment filter.

<sup>10</sup>In general, when the model of  $Z_t$  has a total AR polynomial  $\varphi(B)$  (including any differencing operator) and an MA polynomial with no zeroes of order greater than one, it can be shown that decay rate  $\tau^j$  begins at  $j = \deg \varphi(B) + 1$  with  $\tau$  equal to the maximum magnitude of the reciprocals of the zeroes of the MA polynomial  $\Theta(z)$ . For example, in the MA(1) case,  $\Theta(z) = 1 - \theta z$  is zero for  $z = \theta^{-1}$ , whose reciprocal is  $\theta$ .

### 10.15.1 The Symmetric Filters

For the symmetric filters, (10.87) yields the length 5 formulas

$$\beta_s(B) = \frac{1}{16} |1 - B|^4 = \frac{1}{16} (B^2 - 4B + 6 - 4B^{-1} + B^{-2}). \quad (10.88)$$

$$\beta_p(B) = \frac{1}{16} |1 + B|^4 = \frac{1}{16} (B^2 + 4B + 6 + 4B^{-1} + B^{-2}). \quad (10.89)$$

$$\beta_{sa}(B) = 1 - \beta_s(B) = \frac{1}{16} (-B^2 + 4B + 10 + 4B^{-1} - B^{-2}).$$

$$\beta_u(B) = \frac{1}{8} (1 - B^2) (1 - B^{-2}) = \frac{1}{8} (-B^2 + 2 - B^{-2}). \quad (10.90)$$

For  $n \geq 6$ , these filters produce estimates for times  $3 \leq t \leq n - 2$ .

### 10.15.2 The Asymmetric Filters

Forecasts of the series values at times  $n + 1$  and  $n + 2$  are needed for component estimates at  $t = n - 1, n$ . So are backcasts for  $t = 1, 2$ . We illustrate with odd  $n$ ,  $n = 2m + 1$ , for which  $\hat{Z}_{2m+2} = Z_{2m}$ ,  $\hat{Z}_{2m+3} = Z_{2m+1}$  and  $\hat{Z}_{-1} = Z_1$ ,  $\hat{Z}_0 = Z_2$  are needed for component estimates. The resulting filters are asymmetric. Subsection 8.2 of Findley et al. (2016) derives these results and the asymmetric filter formulas for the initial year and final year. Only final year filters are displayed below. We identify forecasts  $\hat{Z}$  only for  $\hat{u}_{2m}$ ,  $\hat{u}_{2m+1}$  and  $\hat{s}_{2m+1}$ .

$$\begin{aligned} \hat{u}_{2m} &= \frac{1}{8} \{-Z_{2m-2} + 2Z_{2m} - \hat{Z}_{2m+2}\} = \frac{1}{8} \{-Z_{2m-2} + Z_{2m}\} = \frac{1}{8} (1 - B^2) Z_{2m}. \\ \hat{u}_{2m+1} &= \frac{1}{8} \{-Z_{2m-1} + 2Z_{2m+1} - \hat{Z}_{2m+3}\} = \frac{1}{8} \{-Z_{2m-1} + Z_{2m+1}\} = \frac{1}{8} (1 - B^2) Z_{2m+1}. \\ \hat{s}_{2m+1} &= \frac{1}{16} \{Z_{2m-1} - 4Z_{2m} + 6Z_{2m+1} - 4\hat{Z}_{2m+2} + 2\hat{Z}_{2m+3}\} \\ &= \frac{1}{16} \{B^2 - 8B + 7\} Z_{2m+1} = \frac{1}{16} (7 - B) (1 - B) Z_{2m+1}. \end{aligned} \quad (10.91)$$

$$\begin{aligned} \hat{s}_{2m} &= \frac{1}{16} \{B^2 - 4B + 7 - 4B^{-1}\} Z_{2m} = \frac{1}{16} (-B + 3 - 4B^{-1}) (1 - B) Z_{2m}. \\ \hat{a}_{2m} &= \frac{1}{16} \{-B^2 + 4B + 9 + 4B^{-1}\} Z_{2m} = \frac{1}{16} (-B + 5 + 4B^{-1}) (1 + B) Z_{2m}, \\ \hat{a}_{2m+1} &= \frac{1}{16} \{-B^2 + 8B + 9\} Z_{2m+1} = \frac{1}{16} (9 - B) (1 + B) Z_{2m+1}. \\ \hat{p}_{2m} &= \frac{1}{16} \{B^2 + 4B + 7 + 4B^{-1}\} Z_{2m} = \frac{1}{16} (B + 3 + 4B^{-1}) (1 + B) Z_{2m}. \\ \hat{p}_{2m+1} &= \frac{1}{16} \{B^2 + 8B + 7\} Z_{2m+1} = \frac{1}{16} (7 + B) (1 + B) Z_{2m+1}. \end{aligned} \quad (10.92)$$

Since  $(1 - B^2) = (1 - B)(1 + B)$ , the factored formulas on the right for both types of filters show that the trend and seasonal differencing operator factors of an asymmetric filter are of lower degree than those of the symmetric filters. For example, whereas the factorization  $\beta_s(B) = B^{-2}(1 - B)^4$  of (10.88) shows that  $\beta_s(B)$  can annihilate a cubic trend to estimate the seasonal  $s_t$ , the concurrent filters of  $\hat{s}_{2m}$  and  $\hat{s}_{2m+1}$  involving forecasts and backcasts can only annihilate a constant mean.

## 10.16 Differencing Operators of General ARIMA Filters

In the nonstationary case, with  $\delta(B)$  denoting the differencing operator of the ARIMA model of  $Z_t$ , the denominator factors  $|\delta(e^{i2\pi\lambda})|^2$  in (10.43) give rise to factors of  $\delta(B)\delta(B^{-1})$  in the filters (10.78). For example, if  $\delta(B) = (1 - B)(1 + B) = \delta_p(B)\delta_s(B)$  as in the SRW<sub>2</sub>, then  $\beta_s(B)$  and  $\beta_u(B)$  will contain the factor  $(1 - B)(1 - B^{-1}) = B^{-2}(1 - B)^2$ , which will annihilate a linear trend  $a + bt$  because differencing lowers the degree of a polynomial by one, e.g.,  $(1 - B)t^2 = t^2 - (t - 1)^2 = 2t - 1$ . Consequently, the trend filter  $\beta_p(B) = 1 - \beta_s(B) - \beta_u(B)$  will preserve such a trend without change.

The SRW<sub>2</sub> asymmetric filter only has a single  $1 - B$  factor, as the filter formulas of Section 10.15 show. Therefore only a constant level term is annihilated by  $\beta_s(B)$  and  $\beta_u(B)$  and preserved by  $\beta_p(B)$ . Similarly, in the asymmetric case, only the first power of  $(1 + B)$  occurs in  $\beta_p(B)$ . The filter  $\beta_u(B)$  annihilates a period  $r = 2$  deterministic seasonal component  $a(-1)^t$  which is preserved by  $\beta_s(B)$ .

### 10.16.1 What Seasonal Decomposition Filters Annihilate or Preserve

The tables of Bell (2012) and Bell (2015) cover the annihilation and preservation properties of more general differencing operators and also several generations of symmetric and asymmetric X-11 filters. Table 2 of Bell (2012) summarizes the main results for the practical case of asymmetric filters.

## 10.17 Canonical Decomposition and Smoothing Trade-Offs\*

White noise has neither seasonal features nor the smooth properties expected of a trend or any other component of interest for cyclical analysis. Therefore specifying all components other than the irregular in a way that makes them white noise free (after differencing if needed), as the canonical decomposition does, is appropriate. Hillmer and Tiao (1982) shows that the ARIMA models of the canonical seasonal and the canonical trend have smaller one-step-ahead forecast error variances than all noncanonical models for these components. Thus the seasonal and trend components are more predictable one period ahead, a kind of increased smoothness – something that always has a cost. Corollary 3 of Maravall (1986) shows that each canonical non-irregular component has a larger revision variance than a noncanonical specification for the component. Mostly summarizing Maravall, this is the price paid for cleaning the signal of white noise. It is an instance of an important phenomenon mentioned in Section 10.10: greater smoothing is usually associated with some kind of increased statistical instability. Another example: the AMBSA trend is a smoothed version of the AMBSA seasonal adjustment, see Findley et al. (2016), and concurrent trend estimates generally have statistically larger revisions than concurrent seasonal adjustments. Maravall and Planas (1999) analyze features and costs of an interesting variety of conceptually less simple alternatives to the canonical p-sd specification.

## 10.18 Structural Models: A Trend Estimation Example

The models known as *Structural Time Series Models* offer a different approach to component estimation by directly specifying models of a simple form for the components and jointly estimating the parameters of their sum, which is the implied model for the data. See Harvey and Koopman (2000). This approach requires less modeling background and modeling effort than ARIMA modeling, possibly at the cost of reduced forecast performance and goodness of fit compared to what can be obtained from direct ARIMA modeling of the data. However, unless parameter estimation fails, this approach always yields an admissible p-sd decomposition.

We illustrate with the simplest structural trend model for nonstationary  $Z_t$ . This prescribes a random walk trend  $p_t^*$  and a white noise irregular  $N_t^*$ ,

$$\begin{aligned} Z_t &= p_t^* + N_t^* \\ (1 - B)p_t^* &= b_t^*, \end{aligned} \quad (10.93)$$

with mutually uncorrelated white noise  $N_t^*$  and  $b_t^*$ . The parameters to be estimated are  $\sigma_b^2$  and  $q = \sigma_N^2 / \sigma_b^2$ . For  $w_t = (1 - B)Z_t$ , the prescriptions result in  $w_t = b_t^* + (1 - B)N_t^*$  being a stationary process with  $\gamma_0 = \sigma_b^2 + 2\sigma_N^2$ ,  $\gamma_{\pm 1} = -\sigma_N^2$  and  $\gamma_k = 0$ , for  $|k| > 1$ . Thus  $w_t$  is an MA(1) process,  $w_t = a_t - \theta a_{t-1}$ , with a negative lag one autocorrelation  $\rho_1 = \gamma_1 / \gamma_0 = -\sigma_N^2 (2\sigma_N^2 + \sigma_b^2)^{-1}$ . Since also  $\rho_1 = -\theta / (1 + \theta^2)$ , necessarily  $\theta > 0$ . Thus a differently parameterized constrained IMA(1,1) model with the constraint  $\theta > 0$  is prescribed. Its trend model (10.93) differs from that of the canonical decomposition (10.56).

This ARIMA model representation is the *reduced form* of the Structural Model. Structural Models always have a parameter-constrained ARIMA reduced form whose parameters can be determined from the autocovariances. See Section 10.19 and, for this example, also Subsection 10.19.1. The reduced form is not needed to derive the likelihood function and is not usually of interest.

Unfortunately, Structural Models tend to have parameter estimation problems, see Bell (1993) for example. Eurostat does not recommend the use of Structural Models for seasonal adjustment.

## 10.19 Spectral Factorization

A nonzero function of the form

$$Q(\lambda) = \gamma_0 + \sum_{j=1}^q \gamma_j \left( e^{i2\pi j\lambda} + e^{-i2\pi j\lambda} \right), \quad q \geq 1 \quad (10.94)$$

that is nonnegative for all  $\lambda$  is the spectral density of an MA( $q$ ) process with autocovariances  $\gamma_j$ ,  $0 \leq j \leq q$ . More specifically,

$$Q(\lambda) = \sigma^2 \left| 1 - \sum_{j=1}^q \theta_j e^{i2\pi j\lambda} \right|^2, \quad (10.95)$$

with MA( $q$ ) polynomial  $\theta(z) = 1 - \sum_{j=1}^q \theta_j z^j$  such that  $\theta(z) \neq 0$  for  $|z| < 1$ , a property that uniquely determines the coefficients  $\theta_j$  and  $\sigma^2$ , see Findley (2012). The determination of  $\theta(z)$  from (10.94) is known as the *spectral factorization of  $Q(\lambda)$* . It can be accomplished by constructing a polynomial of degree  $q$ , scaled to have  $\theta(0) = 1$  at  $z = 0$ , whose zeroes are the zeroes of  $\gamma_0 + \sum_{j=1}^q \gamma_j (z^j + z^{-j})$  having  $|z| \geq 1$  (usually found by numerical methods if  $q > 1$ ). Some AMBSA software does not use spectral factorization to calculate component estimates, but uses it for other purposes, e.g., to express the ARIMA or ARMA models of the



bi-infinite seasonal, trend and irregular estimators in a familiar form for diagnostic purposes. The efficient algorithm derived in Appendix A of Maravall and Mathis (1994) is used. (In formula (A.4) of this reference,  $z_j$  should be  $z_j^{-1}$ .)

### 10.19.1 The MA(1) Case

When  $q = 1$ , the quadratic formula provides (10.95): For an MA(1)  $Z_t = a_t - \theta a_{t-1}$  with  $|\theta| \leq 1$ , the coefficient  $\theta$  and the first order autocorrelation  $\rho_1 = \gamma_1/\gamma_0 = -\theta/(1 + \theta^2)$  are connected by the properties that  $\theta = 0$  if and only if  $\rho_1 = 0$  and, for  $\rho_1 \neq 0$ ,  $\theta^2 + \rho_1^{-1}\theta + 1 = 0$ . Thus  $\theta$  satisfies

$$\theta = \frac{-\rho_1^{-1} \pm \sqrt{\rho_1^{-2} - 4}}{2}.$$

For  $\theta(z) = 1 - \theta z$ , the requirement that  $\theta(z) \neq 0$  for  $|z| < 1$  is equivalent to  $|\theta| \leq 1$ , which determines the  $\pm$  choice. Finally  $\gamma_0 = (1 + \theta^2) \sigma_a^2$  yields  $\sigma_a^2 = (1 + \theta^2)^{-1} \gamma_0$ .

## 10.20 Regarding AMBSA Software Model Choices and Decomposition Components

**Terminology.** An ARIMA model is *balanced* if the numerator and denominator functions on the right in (10.43) have the same degree, ( $\deg \vartheta = \deg \delta + \deg \varphi$ ), *bottom heavy* if the denominator has larger degree, ( $\deg \delta + \deg \varphi > \deg \vartheta$ ), and *top heavy* if the numerator has larger degree ( $\deg \vartheta > \deg \delta + \deg \varphi$ ).

The automatic ARMA or ARIMA modeling procedures of AMBSA software are biased toward balanced models because these more often have p-sds with admissible decompositions. With top heavy p-sds, the partial fraction decomposition used to obtain their components always yields an additive moving average component in addition to a balanced component, see Wikipedia Contributors (2011). Among the p-sd examples of Section 10.7, (10.45) is bottom heavy, and (10.47) and (10.49) are balanced. Model (6.3) of Hillmer and Tiao (1982) is a top heavy model whose canonical p-sd decomposition is shown by the authors to be admissible for a range of parameter values. With balanced and bottom heavy models, the nature of the p-sd decomposition is determined by a factorization of the total autoregressive polynomial  $\varphi(B) = \delta(B) \phi(B) \Phi(B^r)$  and a corresponding decomposition of the p-sd  $\varphi(z)$ .

## 10.21 Additional Components of Some AMBSA Software Decompositions

For ARIMA seasonal time series, the differencing operator usually has the form

$$\delta(B) = (1 - B)^d (1 - B^r) = (1 - B)^{d+1} U(B), d \geq 0,$$

with even-length seasonal period  $r \geq 2$  and  $U(z) = 1 + \sum_{j=1}^{r-1} z^j$ . Assuming  $d + D > 0$ , the zero  $z = 1$  of  $(1 - z)^{d+D}$  is the *trend unit root*. The  $r - 1$  zeroes of  $U(z)$  are the *seasonal unit roots*. These always include  $z_{r/2} = -1$  and for  $r > 2$ , also  $z_k = e^{i2\pi k/r}$ ,  $k = \pm 1, \dots, \pm(r/2 - 1), r/2$ .

The associated functions  $z_k(\lambda) = e^{i2\pi\lambda(k/r)}$ ,  $k = 0, \pm 1, \dots, r/2$  are periodic, repeating  $|k|$  times a year.

### 10.21.1 Stationary Components

When the ARIMA model has a stationary autoregressive polynomial  $\varphi(z) = \phi(z)\Phi(z^r)$  with zeroes close to the seasonal unit roots and/or to the trend root, and/or certain other unit roots, it is decomposed as  $\varphi(z) = \phi_{seas}(z)\phi_{trend}(z)\phi_{other}(z)$ , with any factor set equal to 1 when there are no zeroes that qualify for inclusion of the factor. The magnitude of "close" is defined by a software default, often a user-changeable magnitude. With these AR factors, the p-sd decomposition can include a seasonal component p-sd with denominator  $|\varphi_{seas}(e^{i2\pi\lambda})|^2$ , where  $\varphi_{seas}(e^{i2\pi\lambda}) = U(e^{i2\pi\lambda})\phi_{seas}(e^{i2\pi\lambda})$ , a trend component p-sd with denominator  $|\varphi_{trend}(e^{i2\pi\lambda})|^2$ , where  $\varphi_{trend}(e^{i2\pi\lambda}) = (1 - e^{i2\pi\lambda})^{d+1}\phi_{trend}(e^{i2\pi\lambda})$ , and possibly a spectral density for a stationary component not connected to trend or seasonal, with denominator  $|\phi_{other}(e^{i2\pi\lambda})|^2$ , which is generally called a *transitory component*, with  $\varphi_{trans}(e^{i2\pi\lambda}) = \phi_{other}(e^{i2\pi\lambda})$  in the total AR notation. However, in the monthly case, if  $\phi_{other}(z)$  is of degree 2 and has complex zeroes with arguments  $\lambda$  close in magnitude to the main trading frequency 0.348 cycles/month, i.e.  $|\lambda| \doteq \pm 0.348$ , then it is called a stochastic trading day component, with  $\varphi_{td}(e^{i2\pi\lambda}) = \phi_{trans}(e^{i2\pi\lambda})$ . If  $\phi_{other}(z)$  is of degree 3 and contains such a degree 2 factor, then it is factored as  $\phi_{other}(z) = \varphi_{td}(z)\varphi_{trans}(z)$  resulting in denominator factors  $|\varphi_{td}(e^{i2\pi\lambda})|^2$  and  $|\varphi_{trans}(e^{i2\pi\lambda})|^2$ . Then, with the irregular, there are five possible stationary decomposition components.

### 10.21.2 A Trend-Cycle Decomposition Option for Long Series

If a monthly ARIMA series has an estimated nonstationary trend component  $\hat{p}_t$  of length at least ten years, then by (changeable) default, some AMBSA software automatically applies a Hodrick-Prescott (HP) filter, see Wikipedia Contributors (2017c), here denoted  $H(B)$ , to the trend estimate  $\hat{p}_t$  extended by forecasts and backcasts. The result is an estimate  $\hat{C}_t = H(B)\hat{p}_t$  of a stationary cycle  $C_t$  and the estimate  $\hat{T}_t = \hat{p}_t - \hat{C}_t$  of the nonstationary long-term trend  $T_t$ . Expressed in terms of transfer functions, the trend filter transfer function  $g_p(e^{i2\pi\lambda})$  is decomposed as the sum of transfer functions  $g_C(e^{i2\pi\lambda}) = |H(e^{i2\pi\lambda})|^2 g_p(e^{i2\pi\lambda})$  and  $g_T(e^{i2\pi\lambda}) = g_p(e^{i2\pi\lambda}) - g_C(e^{i2\pi\lambda})$ .

Kaiser and Maravall (2001) and Maravall and Kaiser (2005) give further background, as do McElroy (2006) and McElroy and Holan (2012), who provide matrix formulas for MMSE finite-sample estimates and their mean square errors, and also results of simulation experiments. Trend-Cycle decompositions are also available with other observation frequencies, e.g. quarterly data, from some AMBSA software for sufficiently long series. Wikipedia Contributors (2017c) discusses limitations of HP filters for cycle extraction, but doesn't consider the situation in which, as here, an ARIMA model for the  $\hat{p}_t$  can be derived from which any needed forecasts and backcasts can be obtained.

Many users of seasonally adjusted data are interested in detecting cyclical movements in the adjusted data. But validation and interpretation of cycle estimates is not part of the discipline of seasonal adjustment. It belongs to a less developed discipline requiring substantial data knowledge and practical training as well as technical knowledge. It is not amenable to high volume production.

## 10.22 Model-Based SA versus X-11 Filter SA

Maravall and Pérez (2012) illustrates the application by experts of AMBSA to an important economic indicator at a time of economic instability. It was stimulated by preceding results obtained at a different central bank with different software using X-11 filter estimates. Its focus is not the comparison of estimation methods but rather the versatility of the tools available in the AMBSA software and the greater versatility of the model-based approach. It cannot be assumed *a priori* that model-based filters will provide a better seasonal adjustment than X-11 filters (for X-11 details see Ladiray and Quenneville (2001)). Although AMBSA has a wider range of filters, the results are often very similar, as the filters can also be, see Bell et al. (2012). What is clearly advantageous about the AMBSA approach with an admissible decomposition is that it specifies ARIMA models and other properties of the canonical seasonal and nonseasonal components and their estimates. This provides a context for the seasonally adjusted series that is rich in auxiliary information of interest, for example statistical precision, not only for seasonal adjustments (for X-11 adjustments, see Bell and Kramer (1999) and Scott et al. (2012)) but also for derived quantities, such as seasonally adjusted growth rates, and covariances of component estimates, from which quality diagnostics can be derived, see Maravall (1987) and Findley et al. (2005). The X-11 approach has no such rich and coherent context. In particular, its traditional adjustment quality diagnostics are *ad hoc* and difficult to validate.

Advantageous features of the X-11 filter method include the directness of its time-tested multiplicative decomposition procedure, which avoids the level bias of log-additive adjustment, and the conceptual simplicity of its filters and iterative procedure (if its complicated extreme value procedure is not considered, for which the outlier identification and adjustment procedure of AMBSA software is a possible substitute). This simplicity makes it easier to explain SA to non-experts and reduces the time series background and amount of training required for new users, compared to AMBSA.

Modern software makes it easy to obtain and compare AMBSA and X-11 method adjustments. When they are close, confidence in each adjustment is increased.

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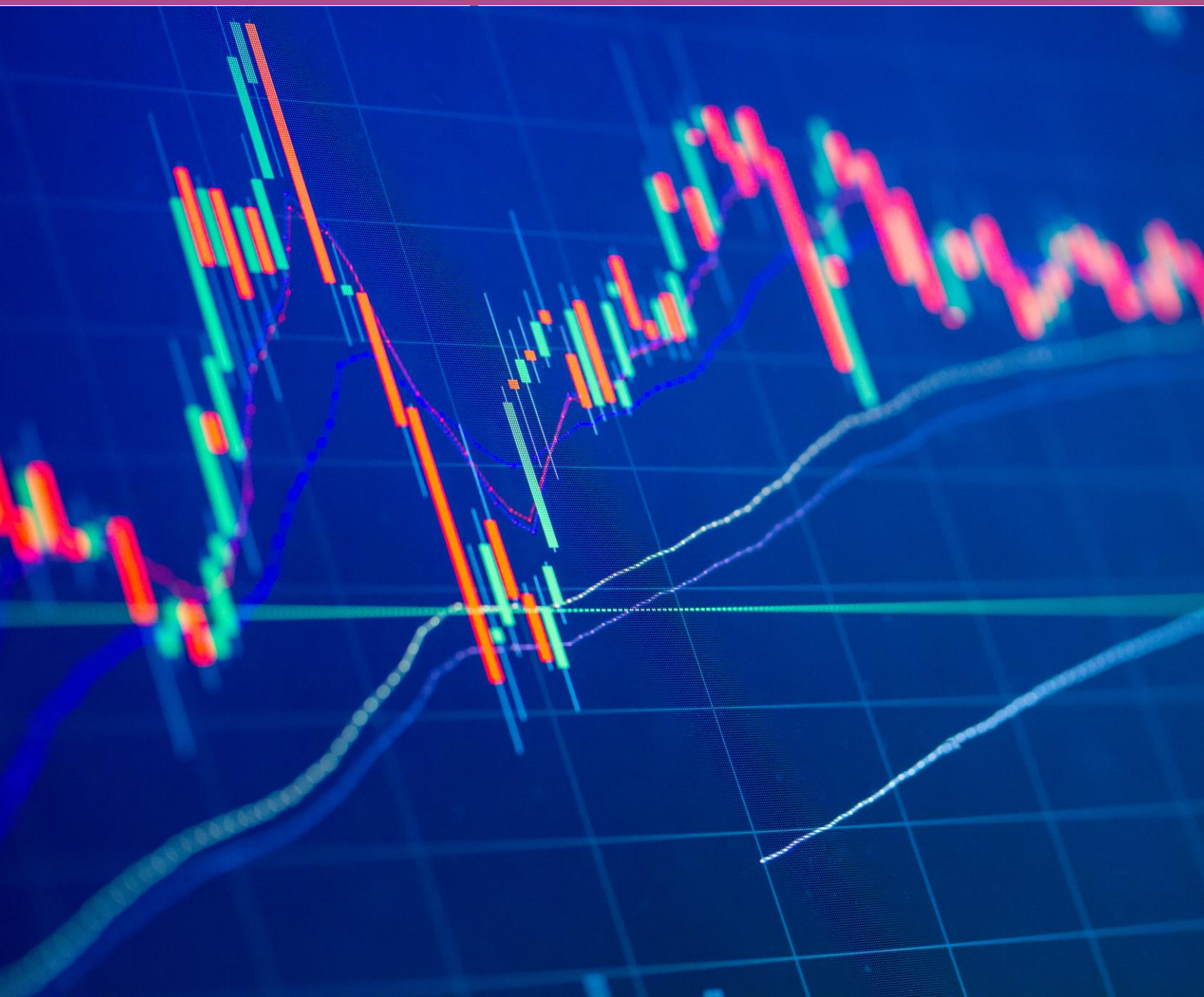
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# 11

## Seasonality and Unobserved Components Models: An Overview



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## Table of Contents

11.1	Introduction	257
11.2	Dummy variable and trigonometric seasonal models	257
11.2.1	Deterministic seasonality	257
11.2.2	Stochastic dummies	258
11.2.3	Trigonometric seasonality	259
11.2.4	Restricted seasonality and daily observations	259
11.3	Basic structural model and ARIMA models	260
11.3.1	Stationary form of the BSM	260
11.3.2	Seasonal ARIMA models	260
11.4	Trading day and calendar effects	261
11.4.1	Trading day variation	261
11.4.2	Moving festivals	262
11.5	Tests	263
11.5.1	Seasonal stationarity tests	263
11.5.2	Seasonality test	264
11.5.3	Seasonal unit root tests	265
11.5.4	Testing for trading day effects	266
11.6	Seasonal adjustment	266
11.7	Breaks in the seasonal pattern	267
11.8	Seasonal splines and weekly data	269
11.9	Seasonal specific models	271
11.9.1	Seasonal heteroscedasticity	271
11.9.2	Periodic models	272
11.9.3	Relationship between periodic models and trend plus seasonal models	272
11.9.4	Partly periodic models	274
11.10	Data irregularities and survey design	276
11.11	Robust seasonal adjustment	276
11.12	Conclusions	277

## 11.1 Introduction

Seasonal adjustment is an exercise in signal extraction and so an unobserved components model is a natural starting point. Models of this kind provide a way of weighting the data that is determined by the properties of the time series.

The starting point is the *basic structural model* (BSM)

$$y_t = \mu_t + \gamma_t + \varepsilon_t, \quad t = 1, \dots, T, \quad (11.1)$$

where  $\mu_t$  is a stochastic trend,  $\gamma_t$  is a stochastic seasonal component; see Akaike (1980) and Harvey (1989). The irregular component,  $\varepsilon_t$ , is assumed to be random, and the disturbances in all three components are taken to be mutually and serially uncorrelated. The model may be extended by including explanatory variables other stochastic components, such as cycles. Models of this kind can be used for forecasting and nowcasting as well as signal extraction; see Harvey (2006). The menu-driven STAMP package of Koopman (2009) enables many of the methods described here to be implemented.

The state space form offers a flexible and general way to carry out filtering, smoothing and parameter estimation. The observations are related to an  $m \times 1$  vector,  $\alpha_t$ , the *state vector*, through a *measurement equation*

$$y_t = \mathbf{z}'_t \alpha_t + d_t + \varepsilon_t, \quad t = 1, \dots, T \quad (11.2)$$

where  $\mathbf{z}_t$  is an  $m \times 1$  vector,  $d_t$  is nonstochastic and  $\varepsilon_t$  is serially uncorrelated disturbance with mean zero and variance  $\sigma_\varepsilon^2$ . The state vector is generated by a first-order Markov process, the *transition equation*,

$$\alpha_t = \mathbf{T}_t \alpha_{t-1} + \mathbf{c}_t + \eta_t, \quad t = 1, \dots, T \quad (11.3)$$

where  $\mathbf{T}_t$  is an  $m \times m$  matrix,  $\mathbf{c}_t$  is an  $m \times 1$  vector, and  $\eta_t$  is a  $g \times 1$  vector of serially uncorrelated disturbances with  $E(\eta_t) = \mathbf{0}$  and  $Var(\eta_t) = \mathbf{Q}_t$ . In the simpler models  $\mathbf{T}_t$ ,  $\mathbf{c}_t$ ,  $\mathbf{Q}_t$ ,  $\mathbf{z}_t$  and  $d_t$  are time-invariant, but the generality allowed by the fact that they can change over time is of considerable importance. When the model is cast in state space form, the optimal weights are implicitly determined at all points in the sample by the Kalman filter and associated smoother.

## 11.2 Dummy variable and trigonometric seasonal models

### 11.2.1 Deterministic seasonality

A basic requirement of a seasonal component is that when the seasonal effects are fixed, they should sum to zero over a year. Thus if  $\gamma_j$ ,  $j = 1, \dots, s$ , is the seasonal effect in season  $j$ ,

$$\sum_{j=1}^s \gamma_j = 0 \quad (11.4)$$

The restriction is easily imposed by defining the seasonal effect at time  $t$  as

$$\gamma_t = \sum_{j=1}^{s-1} \gamma_j z_{jt}, \quad t = 1, \dots, T$$

where for  $t = i, i + s, i + 2s, \dots$ , and  $i = 1, \dots, s - 1$ , the variable  $z_{jt}$  is one for  $j = i$  and zero for  $j \neq i$ , while for  $t = s, 2s, 3s, \dots$ ,  $z_{jt} = -1$  for  $j = 1, \dots, s - 1$ . In other words  $\gamma_t$  is equal to  $\gamma_j$  in season  $j$  and  $-\sum_{j=1}^{s-1} \gamma_j$  in

season  $s$ . The restriction also means that the seasonal effects over the past  $s$  periods sum to zero, that is

$$\sum_{j=0}^{s-1} \gamma_{t-j} = 0, \quad t = 1, \dots, T \quad (11.5)$$

Rather than using a set of dummy variables, a fixed seasonal pattern may be captured by a set of trigonometric terms at the seasonal frequencies,  $\lambda_j = 2\pi j/s$ ,  $j = 1, \dots, [s/2]$ . The seasonal effect at time  $t$  is then

$$\gamma_t = \sum_{j=1}^{[s/2]} (\alpha_j \cos \lambda_j t + \beta_j \sin \lambda_j t), \quad t = 1, \dots, T \quad (11.6)$$

When  $s$  is even, the sine term for  $j = s/2$  disappears and so the number of trigonometric parameters, the  $\alpha_j$ 's and  $\beta_j$ 's, is always  $s - 1$ , the same as the number of coefficients in the seasonal dummy formulation. By using standard trigonometric identities, it is straightforward to show that the seasonal effects over a year sum to zero, as in (11.5). Provided that the full set of trigonometric terms is included, (11.6) is equivalent to the dummy variable specification and the estimated seasonal patterns will be identical.

### 11.2.2 Stochastic dummies

By introducing a disturbance term into the right-hand side of (11.5), the seasonal effects can be allowed to change over time. Thus

$$\sum_{j=0}^{s-1} \gamma_{t-j} = \omega_t \quad \text{or} \quad \gamma_t = -\sum_{j=1}^{s-1} \gamma_{t-j} + \omega_t \quad (11.7)$$

where  $\omega_t$  is white noise with mean zero and variance  $\sigma_\omega^2$ . The bigger the value of  $\sigma_\omega^2$  relative to the variances of other disturbances in the model, the more rapidly the seasonal pattern changes over time and the more rapidly are past observations discounted in constructing a seasonal pattern for the forecast function. The forecasts satisfy the recursion

$$\tilde{\gamma}_{T+l|T} = -\sum_{j=1}^{s-1} \tilde{\gamma}_{T+l-j|T}, \quad l = 1, 2, \dots \quad (11.8)$$

where the starting values are given by the (smoothed) estimates of the seasonal effects,  $\gamma_T, \dots, \gamma_{T-s+2}$ , at time  $T$ . Thus the seasonal pattern projected into the future is fixed and the seasonal effects sum to zero over any period of one year.

An alternative way of allowing the seasonal dummy variables to change over time is to suppose that the effect of each season evolves as a random walk. Let  $\gamma_{jt}$  denote the effect of season  $j$  at time  $t$  and define  $\gamma_t = (\gamma_{1t}, \dots, \gamma_{st})'$ . Seasonals evolve as a multivariate random walk

$$\gamma_t = \gamma_{t-1} + \omega_t, \quad t = 1, \dots, T, \quad (11.9)$$

in which  $\omega_t = (\omega_{1t}, \dots, \omega_{st})'$  is a vector of zero mean, serially uncorrelated disturbances with

$$\text{Var}(\omega_t) = \sigma_\omega^2 (\mathbf{I} - s^{-1} \mathbf{ii}'), \quad t = 1, \dots, T, \quad (11.10)$$

where  $\sigma_\omega^2$  is a non-negative parameter and  $\mathbf{i}$  is an  $s \times 1$  vector of ones. Although all  $s$  seasonal components are continually evolving, only one affects the observation at any particular point in time, that is  $\gamma_t = \gamma_{jt}$  for  $t = 1, \dots, T$ , when season  $j$  is prevailing at time  $t$ . The requirement that the seasonal effects in the forecast function sum to zero over  $s$  consecutive time periods is enforced by the restriction that, at any particular point in time, the seasonal components, and hence the disturbances, sum to zero, that is

$$\sum_{j=1}^s \gamma_{jt} = 0 = \sum_{j=1}^s \omega_{jt}, \quad t = 1, \dots, T \quad (11.11)$$

This restriction is implemented by the structure in (11.10), where it can be seen that  $\text{Var}(\mathbf{i}'\omega_t) = 0$ , implying

$\mathbf{i}'\omega_t = 0$ . An initial covariance matrix proportional to (11.10) with  $E(\gamma_0) = \mathbf{0}$  will ensure that  $\mathbf{i}'\gamma_t = 0$  for  $t = 1, \dots, T$ . A noninformative prior is obtained by replacing  $\sigma_\omega^2$  by a scalar that is allowed to tend towards infinity.

The relationship between the two forms of dummy variable seasonality is examined in Proietti (2000). In practice, it is usually preferable to work with the *balanced dummy variable seasonal model* of (11.9) though the simplicity of the *single shock* model of (11.7) can be useful for pedagogic purposes. It can be shown that (11.9) can be written in the form of (11.7), but with  $\omega_t$  following an  $MA(s-2)$  process.

### 11.2.3 Trigonometric seasonality

A trigonometric seasonal pattern may be allowed to evolve over time by writing the component at each frequency as a recursion and adding disturbances. Thus

$$\gamma_t = \sum_{j=1}^{[s/2]} \gamma_{jt}, \quad t = 1, \dots, T \quad (11.12)$$

and

$$\left. \begin{aligned} \gamma_{jt} &= \gamma_{j,t-1} \cos \lambda_j + \gamma_{j,t-1}^* \sin \lambda_j + \omega_{jt} \\ \gamma_{jt}^* &= -\gamma_{j,t-1} \sin \lambda_j + \gamma_{j,t-1}^* \cos \lambda_j + \omega_{jt}^* \end{aligned} \right\}, \quad j = 1, \dots, [(s-1)/2] \quad (11.13)$$

where  $\omega_{jt}$  and  $\omega_{jt}^*$  are zero mean white-noise processes which are uncorrelated with each other and have a common variance  $\sigma_j^2$  for  $j = 1, \dots, [(s-1)/2]$ . The component  $\gamma_{jt}^*$  appears as a matter of construction, and its interpretation is not particularly important. When  $s$  is even,

$$\gamma_{s/2,t} = \gamma_{s/2,t-1} \cos \lambda_{s/2} + \omega_{s/2,t} = (-1)^t \gamma_{s/2,t-1} + \omega_{s/2,t}. \quad (11.14)$$

The estimators of the  $\gamma_{jT}$ 's and  $\gamma_{jT}^*$ 's provide starting values for a projection of the latest seasonal pattern into the future.

Assigning different variances to each harmonic allows them to evolve at different rates, as in Hindrayanto et al. (2013). However, it is usually the case that these variances is given by:

$$\text{Var}(\omega_{jt}) = \text{Var}(\omega_{jt}^*) = \sigma_j^2 = \bar{\sigma}_\omega^2, \quad j = 1, \dots, [(s-1)/2] \quad (11.15)$$

though for  $s$  even,

$$\text{Var}(\omega_{s/2,t}) = \bar{\sigma}_\omega^2/2, \quad t = 1, \dots, T \quad (11.16)$$

As a rule, very little is lost in terms of goodness of fit by imposing this restriction. Furthermore the model is identical to the balanced dummy variable seasonal model with  $\sigma_\omega^2 = 2\bar{\sigma}_\omega^2/s$  for  $s$  even and  $\sigma_\omega^2 = 2\bar{\sigma}_\omega^2/(s-1)$  for  $s$  odd; see Proietti (2000).

### 11.2.4 Restricted seasonality and daily observations

If some of the seasonal effects are assumed to be the same, the number of dummy variables can be reduced. This assumption is particularly relevant for modelling daily effects, so the term 'seasons' should be given a wide interpretation.

Let  $w$  be the number of different types of seasons and let  $k_j$  be the number of seasons of the  $j$ -th type for  $j = 1, \dots, w$ . Thus, for example, if, in a model for daily data, all weekdays are alike but both Saturdays and Sundays are different,  $w = 3$ ,  $k_1 = 5$  and  $k_2 = k_3 = 1$ . The  $\gamma_t$  vector in (11.9) is now  $w \times 1$ , as is  $\omega_t$ . The requirement that all  $s$  seasons sum to zero over the year is enforced by the covariance matrix

$$\text{Var}(\omega_t) = \sigma_\omega^2 (\mathbf{I} - \mathbf{k}^{-1} \mathbf{k} \mathbf{k}'), \quad t = 1, \dots, T, \quad (11.17)$$

where  $\mathbf{k} = (k_1, k_2, \dots, k_w)'$  and  $k = \sum_{j=1}^w k_j^2$ . Clearly  $Var(\mathbf{k}'\omega_t) = 0$  so  $\mathbf{k}'\omega_t = 0$ .

The number of parameters in the trigonometric form of seasonality can be straightforwardly reduced by cutting out some of the frequencies.

## 11.3 Basic structural model and ARIMA models

In order to compare the properties of UC models and hence compare them with ARIMA models, it is helpful to write down their stationary form.

### 11.3.1 Stationary form of the BSM

A stochastic trend component is made stationary by applying the first difference operator twice, that is  $\Delta^2\mu_t = \Delta\eta_t + \zeta_{t-1}$ . The seasonal component is made stationary by the seasonal summation operator. Thus the stationary form of the BSM, (11.1), is obtained by multiplying through by the first and seasonal difference operators and taking note of the identity  $\Delta_s = \Delta \cdot S(L)$ , where  $S(L) = 1 + L + \dots + L^{s-1}$ . The reduced form of the balanced dummy and trigonometric seasonal models has  $S(L)\gamma_t$  being an  $MA(s-2)$  process. The stationary form of the BSM is such that  $\Delta\Delta_s y_t$  is  $MA(s+1)$ .

When the slope is deterministic, equal to  $\beta$  in all time periods, the observations are rendered stationary by the seasonal difference operator. In this case  $\Delta_s y_t$  is  $MA(s+1)$ .

### 11.3.2 Seasonal ARIMA models

For modelling seasonal data, Box and Jenkins (1976) (ch. 9) proposed a class of multiplicative seasonal ARIMA models. The most important model within this class has subsequently become known as the 'airline model' since it was originally fitted to a monthly series on UK airline passenger totals. The model is written as

$$\Delta\Delta_s y_t = (1 + \theta L)(1 + \Theta L^s)\xi_t \quad (11.18)$$

where  $\Delta_s = 1 - L^s$  is the seasonal difference operator and  $\theta$  and  $\Theta$  are MA parameters which, if the model is to be invertible, must have modulus less than one. Box and Jenkins (1976) (pp. 305-6) gave a rationale for the airline model in terms of EWMA's at monthly and yearly intervals.

Maravall (1985), compares the autocorrelation functions of  $\Delta\Delta_s y_t$  for the BSM and airline model for some typical values of the parameters and finds them to be quite similar, particularly when the seasonal MA parameter,  $\Theta$ , is close to minus one. In fact in the limiting case when  $\Theta$  is equal to minus one, the airline model is equivalent to a BSM in which  $\sigma_\zeta^2$  and  $\sigma_\omega^2$  are both zero. It is straightforward to see that this is true because

$$\Delta\Delta_s y_t = \Delta_s \eta_t + \Delta\Delta_s \varepsilon_t = (1 - L^s)(\eta_t + \varepsilon_t - \varepsilon_{t-1})$$

and the last term in parentheses is an  $MA(1)$ . The airline model thus provides a good approximation to the reduced form when the slope and seasonal are close to being deterministic. If this is not the case the implicit link between the variability of the slope and that of the seasonal component may be limiting.

The plausibility of other multiplicative seasonal ARIMA models can, to a certain extent, be judged according to whether they allow a canonical decomposition into trend and seasonal components; see Hillmer and Tiao (1982). Although a number of models fall into this category the case for using them is not always convincing. It is hardly surprising that many procedures for ARIMA model-based seasonal adjustment are based on the airline model.



Pure AR models can be very poor at dealing with seasonality since seasonal patterns typically change rather slowly and this may necessitate the use of long seasonal lags. A slowly changing seasonal pattern shows up in the airline model when  $\Theta$  is close to minus one.

A model for aggregate consumption,  $c_t$ , provides an illustration of the way in which a simple parsimonious STM that satisfies economic considerations can be constructed. Using UK data from 1957q3 to 1992q2, Harvey and Scott (1994) show that a special case of the BSM consisting of a random walk plus drift,  $\beta$ , and a stochastic seasonal not only fits the data but yields a seasonal martingale difference that does little violence to the forward-looking theory of consumption. The unsatisfactory nature of an autoregression is illustrated in the paper by Osborn and Smith (1989) where sixteen lags are required to model seasonal differences. As regards ARIMA models, Osborn and Smith (1989) select a special case of the airline model in which  $\theta = 0$ . This contrasts with the reduced form for the structural model which has  $\Delta_s c_t$  following an  $MA(s-1)$  process (with non-zero mean). The seasonal ARIMA model approximates the sample ACF but does not yield forecasts satisfying a seasonal martingale difference, that is  $E[\Delta_s c_{t+s}] = s\beta$ .

## 11.4 Trading day and calendar effects

It is not unusual for the level of a monthly time series to be influenced by calendar effects. Such effects arise because of changes in the level of activity resulting from variations in the composition of the calendar between years. The two main sources of calendar effects are trading day variation and moving festivals. They may both be introduced into a time series model and estimated along with the other components in the model. Thus, for example, the BSM is extended so as to become

$$y_t = \mu_t + \gamma_t + \tau_t + \varphi_t + \varepsilon_t \quad (11.19)$$

where  $\tau_t$  is the trading day variation component and  $\varphi_t$  is the moving festival component.

Calendar effects should be modelled so as not to affect the level of the trend. Thus when the forecast function is constructed, they should cancel out under temporal aggregation in the same way as the seasonal component. Furthermore, because they represent what are basically artificial movements in the series, there is a clear case for removing them as part of the process of seasonal adjustment.

### 11.4.1 Trading day variation

Trading day variation occurs when the activity of an industry or business varies with the day of the week. Thus for a flow variable, or a time-averaged stock, the observation recorded for a particular month will depend on which days of the week occur five times.

The trading day component in month  $t$  is

$$\tau_t = \sum_{j=1}^7 n_{jt} \theta_{jt}, \quad t = 1, \dots, T, \quad (11.20)$$

where  $n_{jt}, j = 1, \dots, 7$  is the number of times day  $j$  occurs in month  $t$  and  $\theta_{jt}$  is an unknown parameter associated with it. The constraint

$$\sum_{j=1}^7 \theta_{jt} = 0 \quad (11.21)$$

ensures the trading day effects are not confounded with the trend. When the  $\theta_{jt}^s$  are deterministic, the sum of the trading day effects for each month over a period equal to a whole number of weeks is zero because  $\sum_t \tau_t = \sum_j \sum_t \theta_j n_{jt} = \sum_j \theta_j \sum_t n_{jt}$  and  $\sum_t n_{jt}$  is the same for all  $j$ . Over a year the sum of trading day effects will be almost, but not exactly, equal to zero, as 52 weeks is equal to 364 rather than 365 or 366 days.



When the trading day effects are stochastic, the  $\theta'_{jt}$ s evolve as random walks as in the balanced dummy variable seasonal model and the constraint in (11.21) is imposed by specifying the covariance matrix as in (11.10). When different days give rise to the same effect, a more parsimonious trading day model is obtained, namely

$$\tau_t = \sum_{j=1}^w n_{jt} \theta_{jt}, \quad t = 1, \dots, T, \quad (11.22)$$

where there are  $w$  different types of day and the covariance matrix of the disturbances driving the  $\theta'_{jt}$ s has the same form as (11.17) but with  $\mathbf{k}$  changing over time and defined as  $\mathbf{k}_t = (n_{1t}, n_{2t}, \dots, n_{wt})'$ .

An implementation of a stochastic trading day effects model can be found in Dagum et al. (1992). The deterministic version was used by Kitagawa and Gersch (1984) and Bell and Hillmer (1983).

### 11.4.2 Moving festivals

The month in which certain holidays and religious festivals fall can vary from year to year. A prime example is Easter. In connection with retail sales, Bell and Hillmer (1983) suggest modelling Easter as

$$\varphi_t = \alpha h_t \quad (11.23)$$

where  $h_t$  is the proportion of the time period  $H$  days before Easter that falls in month  $t$ . This model can be defined for any positive  $H$  and if  $H \leq 22$  the only months for which  $h_t$  will ever be non-zero are March and April. A similar model could be used for road and air traffic except that in this case the time period up to and including Easter Monday might be the relevant one. The value of  $H$  would probably be four or five.

As it stands, (11.23) does not have the property that the  $\varphi_t$ 's sum to zero over a year. Fortunately this is easily remedied. In addition, the form of the moving festival component may be generalised. Suppose that  $\varphi_t$  is modelling any moving festival effect, not necessarily Easter, and that  $h_t$  is now a weight given to month  $t$ . Let the sum of the  $h_t$ 's over any one year be unity. The pattern of the  $h_t$ 's depends on the location of the moving festival in question and its postulated effect on the surrounding days. Thus, for example, the weight pattern for a series on road accidents might be derived by assigning initial weights of  $\frac{1}{3}$ ,  $\frac{1}{6}$ ,  $\frac{1}{6}$  and  $\frac{1}{3}$  to the days from Good Friday to Easter Monday. If Easter Monday were 1 April in a particular year, this would imply that  $h_t$  for March would be  $\frac{2}{3}$ , while for April it would be  $\frac{1}{3}$ . A moving festival may now be formulated as

$$\varphi_t = \alpha(h_t - 1/s), \quad (11.24)$$

where  $s$  is twelve unless the timing interval is lunar months, in which case it is thirteen.

The parameter  $\alpha$  may be allowed to change over time by modelling it as a random walk. The weight function,  $h_t$ , then appears in the corresponding position in the  $\mathbf{z}_t$  vector in the measurement equation.

The handling of moving festivals becomes a major feature of models for weekly data. Harvey et al. (1997) discuss how Easter is treated in a model for the UK money supply.

## 11.5 Tests

### 11.5.1 Seasonal stationarity tests

The basic form of the LBI test against nonstationary stochastic seasonality is obtained for the model

$$y_t = \mu + \gamma_t + \varepsilon_t, \quad t = 1, \dots, T \quad (11.25)$$

where  $\mu$  is a constant. The test against the presence of a stochastic trigonometric component at any one of the seasonal frequencies,  $\lambda_j$ , apart from the one at  $\pi$ , is based on the statistic

$$\omega_j = 2T^{-2}\hat{\sigma}^{-2} \sum_{t=1}^T \left[ \left( \sum_{i=1}^t e_i \cos \lambda_j i \right)^2 + \left( \sum_{i=1}^t e_i \sin \lambda_j i \right)^2 \right], \quad j = 1, \dots, [(s-1)/2], \quad (11.26)$$

where  $\hat{\sigma}^2$  is the sample variance of the OLS residuals,  $e_t, t = 1, \dots, T$ , from a regression on the seasonal sines and cosines,  $z_t$ , and a constant. Following Canova and Hansen (1995), it can be shown that, under the null hypothesis, the asymptotic distribution of this statistic is generalized Cramér-von Mises with two degrees of freedom. If  $s$  is even, the statistic at frequency  $\pi$  is

$$\omega_{s/2} = T^{-2}\hat{\sigma}^{-2} \sum_{t=1}^T \left( \sum_{i=1}^t e_i (-1)^i \right)^2,$$

and this has an asymptotic distribution which is Cramér-von Mises with one degree of freedom. A joint test against the presence of stochastic trigonometric components at all seasonal frequencies is obtained by summing the individual test statistics, that is

$$\omega = \sum_{j=1}^{[s/2]} \omega_j \quad (11.27)$$

This statistic has an asymptotic distribution which is generalized Cramér-von Mises with  $s-1$  degrees of freedom, denoted  $CvM(s-1)$ . Canova and Hansen (1995) point out that the same test is obtained if the stochastic seasonal component is of the balanced dummy variable form.

Canova and Hansen show how the above tests can be generalized to handle serial correlation and heteroscedasticity by making a nonparametric correction. However, if the process generating the non-seasonal part of the model is taken as given, the LBI test against stochastic seasonality is constructed from a set of ‘smoothing errors’. As shown in Busetti and Harvey (2003) (Appendix B) the smoothing errors are, in general, serially correlated but the form of this serial correlation may be deduced from the specification of the model, thereby allowing the construction of a statistic that has a Cramér-von Mises distribution, asymptotically, under the null hypothesis. An alternative possibility is to use the  $T$  standardized one-step ahead prediction errors, the innovations, calculated by treating nonstationary and deterministic components as having fixed initial conditions. No correction is then needed; the statistic is of the form (11.26) and has the same asymptotic distribution. Calculating innovations under the assumption that the initial conditions are fixed requires that the initial conditions be estimated, but a backward smoothing recursions can be avoided simply by reversing the order of the observations and calculating a set of innovations starting from the filtered estimator of the state at the end of the sample. Actually, the forward and backward innovations are not the same and in neither case do the sums, weighted by  $\cos \lambda_j t$  and  $\sin \lambda_j t$ , equal zero, so statistics formed from forward and backward sums are different. Fortunately the asymptotic properties are unaffected. Smoothing errors do not suffer from these ambiguities.

For both the smoothing error and innovation forms of the test, nuisance parameters will normally have to be estimated. For stationarity tests, Leybourne and McCabe (1994) argue that this is best done under the

alternative using maximum likelihood. Proceeding in this way has the compensating advantage that since there will often be some doubt about a suitable model specification, estimation of the unrestricted model affords the opportunity to check its suitability by the usual diagnostics and goodness of fit tests. Once the nuisance parameters have been estimated, the test statistic is calculated from the innovations obtained with  $\sigma_{\omega}^2$  set to zero.

The parametric test may be applied in models which include a deterministic trend, a random walk with or without a drift, or a trend with a stochastic slope. In all these cases the asymptotic distribution of the test statistics is unaffected.

We will refer to these tests as *seasonal stationarity* tests. The nonparametric statistics will be denoted  $\omega(m)$ , where  $m$  is the number of lags in the estimator of the spectrum.

### 11.5.2 Seasonality test

Seasonal stationarity tests take the null to be deterministic seasonality. Sometimes we may wish to test whether there is any seasonality at all. Buseti and Harvey (2003) suggest two possible tests. Both can be implemented parametrically or nonparametrically.

- 1) A Wald test of the null hypothesis that the deterministic seasonal coefficients are zero.
- 2) A seasonal stationarity test without fitting seasonal dummies. Such a test will also have power against deterministic, as well as stochastic, seasonality. If the test statistic,  $\omega_0$ , is formed without fitting seasonal dummies, its asymptotic distribution under the null will be a function of Brownian motion rather than of a Brownian bridge, that is  $CvM_0(s-1)$ . The 5% critical value for three degrees of freedom, as is appropriate for a full test on quarterly data, is 3.46.

The logarithm of 3-month money market interest rate in Spain for the period 1977Q1-2001Q4 is depicted in the top panel of Figure 11.1; the source is the Bank of International Settlements (BIS) macroeconomic series database. It is difficult to detect a seasonal pattern from a casual glance at the graph and one would not normally expect one to be present in an interest rate series; however the functioning of the interbank loans market may imply some seasonality.

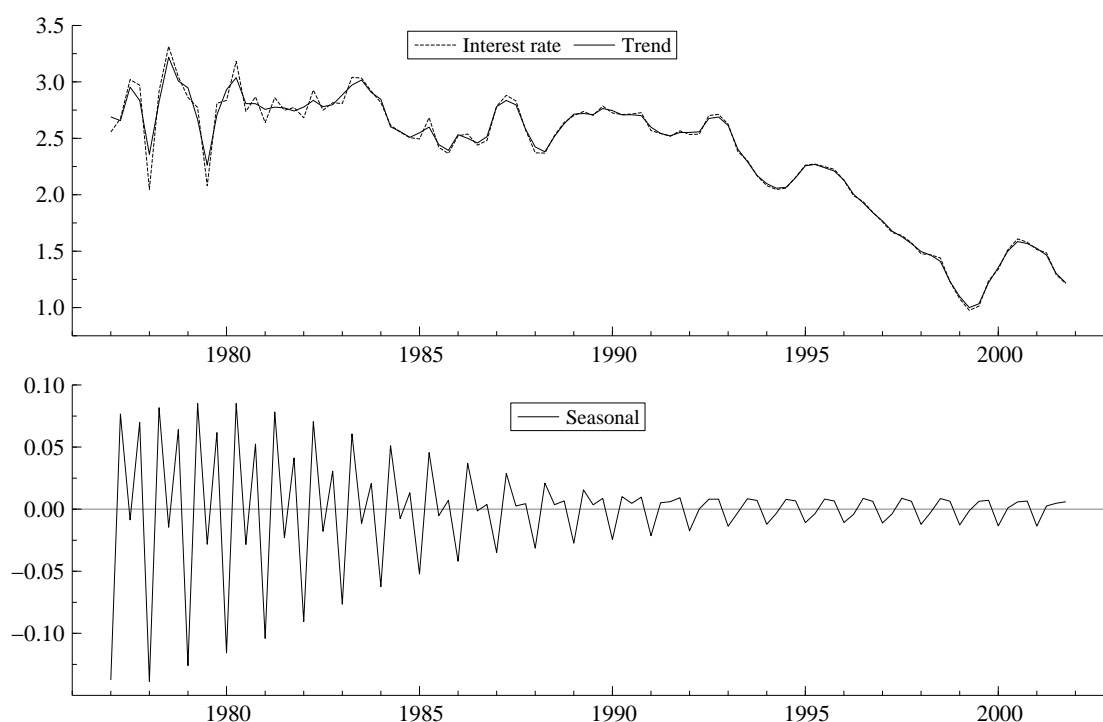
Fitting the BSM to the series gives a seasonal component as shown in the bottom panel of Figure 11.1; the slope variance is estimated to be zero and the estimate of the (fixed) slope is small and insignificant. We have used logarithms of the data only because the diagnostics are better; if the raw series is used, the resulting seasonal pattern is similar.

The chi-square statistic for the seasonals at the end of the series is only 0.09 which is clearly not significant as the 5% critical value for a  $\chi_3^2$  is 7.81. However the graph shows a fairly strong seasonal pattern until the mid-eighties. The question is whether the pattern as a whole is in any sense significant.

Setting the seasonal variance to zero and re-estimating the BSM gives a Wald statistic of 4.76, with a p-value of 0.19. This is still not significant. If the series is differenced and a nonparametric Wald test is computed using the Newey-West covariance matrix estimator with three lags a similar p-value, 0.17, is obtained. On the other hand, the spectral nonparametric seasonal stationarity test statistic,  $\omega_0(m)$ , computed using forward summations takes the values 3.83 and 3.01 for  $m = 3$  and 6 respectively, rising to 4.64 and 3.89 for  $\underline{\omega}_0^*(m)$ , the preferred form in which the spectrum is estimated after fitting seasonal regressors. As the 5% critical value is 3.46, this test provides a firm rejection of the hypothesis that there is no seasonality in the series.

Finally, for  $m = 3$  and 6 the seasonal stationarity test statistic,  $\omega(m)$ , takes the values 1.17 and 1.02 respectively (against a 5% critical value of 1.00), thereby confirming the presence of stochastic seasonality.

Figure 11.1: Spanish interest rates



### 11.5.3 Seasonal unit root tests

The test of Hylleberg et al. (1990) - HEGY- is testing the null of a nonstationary seasonal against the alternative of a stationary seasonal. Its relationship to the seasonal stationarity test is analogous to that of the relationship between the (augmented) Dickey-Fuller test and KPSS.

The UC seasonal unit root test can be set up by introducing a damping factor into (11.13) so that each trigonometric term in the seasonal component is modelled by

$$\begin{bmatrix} \gamma_{j,t} \\ \gamma_{j,t}^* \end{bmatrix} = \phi_j \begin{bmatrix} \cos \lambda_j & \sin \lambda_j \\ -\sin \lambda_j & \cos \lambda_j \end{bmatrix} \begin{bmatrix} \gamma_{j,t-1} \\ \gamma_{j,t-1}^* \end{bmatrix} + \begin{bmatrix} \omega_{j,t} \\ \omega_{j,t}^* \end{bmatrix}, \quad \begin{matrix} j = 1, \dots, [s/2], \\ t = 1, \dots, T. \end{matrix} \quad (11.28)$$

with  $\gamma_{s/2,t}^*$  dropping out for  $s$  even. The seasonal component, obtained by summing the  $\gamma_{j,t}^s$  is then embedded in a general UC model which contains deterministic seasonal trigonometric terms. However, since the forecasts would gradually die down to zero for  $\phi_j < 1$ , such a seasonal component is not capturing any (non-deterministic) persistent effects of seasonality. In any case the empirical evidence, for example in Canova and Hansen (1995), clearly points to seasonal unit roots as the norm. Nevertheless we may still wish to test the null hypothesis of seasonal unit roots against the alternative of stationary seasonality.

A parametric test of the null hypothesis that the component at a particular frequency is nonstationary against the alternative that it is stationary, that is  $H_0 : \phi_j = 1$  against  $H_1 : \phi_j < 1$ , can be constructed from the null hypothesis innovations as

$$\omega_j = 2T^{-2} \sum_{i=1}^T \left[ \left( \sum_{t=1}^i \tilde{\nu}_t \cos \lambda_j t \right)^2 + \left( \sum_{t=1}^i \tilde{\nu}_t \sin \lambda_j t \right)^2 \right] < c, \quad j = 1, \dots, [(s-1)/2]. \quad (11.29)$$

Under the null hypothesis the asymptotic distribution is  $CvM_0(2)$  since if the nonstationary seasonal operator,

$1 - 2\cos\lambda_j L + L^2$ , were to be applied it would remove the corresponding deterministic seasonal. For  $j = s/2$

$$\omega_{s/2} = T^{-2} \sum_{i=1}^T \left( \sum_{t=1}^i \tilde{\nu}_t \cos \pi t \right)^2 = T^{-2} \sum_{i=1}^T \left( \sum_{t=1}^i (-1)^t \tilde{\nu}_t \right)^2$$

and this has a  $CvM_0(1)$  asymptotic distribution under the null. The full seasonal test statistic is formed by summing the  $\omega'_j$ s and its asymptotic distribution under the null is  $CvM_0(s - 1)$ . With seasonal slopes the asymptotic distributions are  $CvM_1(\cdot)$ .

Seasonality tests based on an autoregressive model will tend to perform poorly in situations where an unobserved components model is appropriate. The simulation evidence in Hylleberg (1995) illustrates this point by looking at the results of using the HEGY test for moving average models, which, as Harvey and Scott (1994) note, typically arise as the reduced form of unobserved components models.

A rejection of the null hypothesis in a seasonal unit root test may be an indication of a deterministic seasonal component rather than a stationary seasonal component; see the evidence in Canova and Hansen (1995)(p 244). Following the argument in Harvey and Streibel (1998), it can be shown that the appropriate test of the null of deterministic seasonality against the alternative of near-persistent stationary seasonality, that is (11.28) with the  $\phi_j$  close to one, is, in fact, the seasonal stationarity test. Therefore we may only want to do a test against stationary seasonality if the hypothesis of deterministic seasonality has first been rejected by the seasonal stationarity test.

#### 11.5.4 Testing for trading day effects

Cleveland and Devlin (1980) showed that peaks at certain frequencies in the estimated spectra of monthly time series indicate the presence of trading day effects. Specifically there is a peak at a frequency of  $0.348 \times 2\pi$  radians, with the possibility of subsidiary peaks at  $0.432 \times 2\pi$  and  $0.304 \times 2\pi$  radians. An option in the output of the X-12-ARIMA program provides a comparison of the estimates of these frequencies with the adjacent frequencies; see Soukup and Findley (2000). However, there is no formal test. Buseti and Harvey (2003) suggest a seasonality test at the relevant frequency or a joint test at all three frequencies. Assuming that no (deterministic) trading day model has been fitted, the asymptotic distribution is  $CvM_0$ , as in sub-section 11.5.2, with the 5% critical value being 2.63 for a test at a single frequency and 5.68 for a test at all three frequencies.

As an example consider the irregular component, obtained from X12-ARIMA, of series s0b56ym, *U.S. Retail Sales of Children's, Family, and Miscellaneous Apparel*, as supplied by the Bureau of the Census. Since the process followed by this irregular component cannot be derived, it was decided to use the nonparametric test. The  $\omega(10)$  test statistic for the single main frequency was 7.03. For all three frequencies it was 8.21. Both give a clear rejection of the null hypothesis that there is no trading day effect.

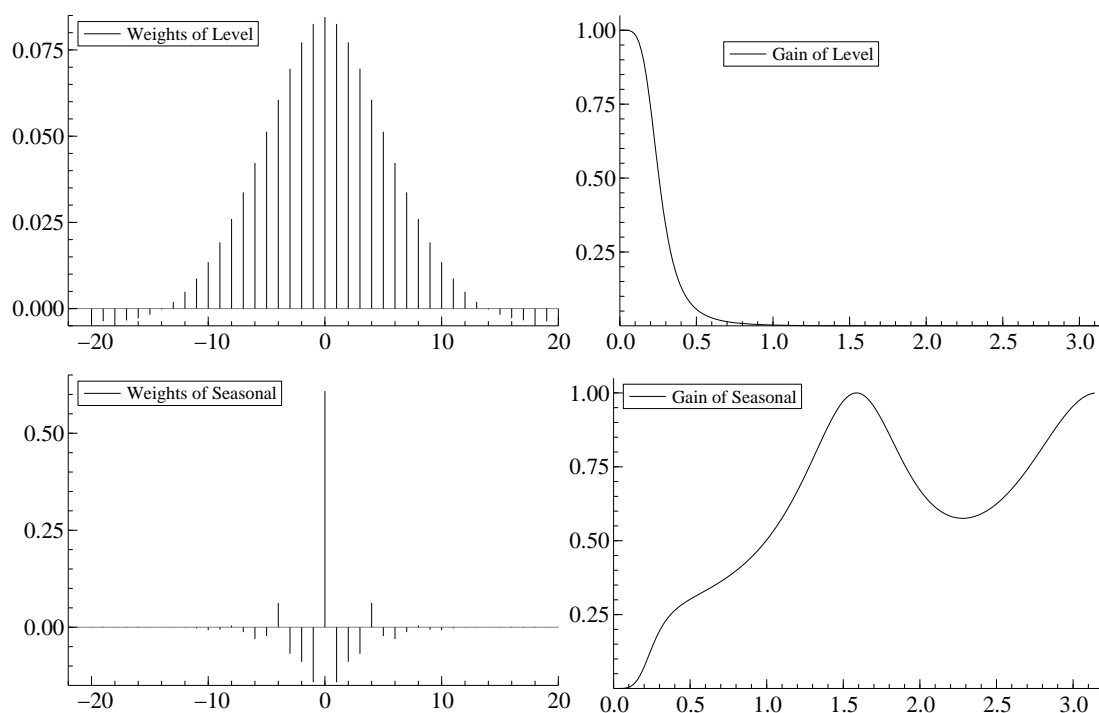
## 11.6 Seasonal adjustment

Once a STM has been fitted, the seasonally adjusted series is obtained by signal extraction using the Kalman filter and associated smoother (KFS). Efficient algorithms are described in Durbin and Koopman (2012) (chapter 4). The KFS automatically adjusts the weighting pattern to give optimal (MMSE) estimates at the ends of the series. The KFS is much easier to implement than the Wiener-Kolmogorov (WK) filter and is more general - for example it can be used with models that are not time invariant. Unlike WK, the weights are implicit, but they can be calculated by the algorithm of Koopman and Harvey (2003).

Figure 11.2 shows weights for the model fitted to the logarithms of gas consumption by 'Other final users in the UK' as displayed by STAMP. The weights for the seasonally adjusted series are  $1 - w_s(L)$  where  $w_s(L)$

is the polynomial of weights for extracting the seasonal. The gain shows the effect of the filter on a stationary series. The gain of the seasonal adjustment filter is one minus the gain of the seasonal filter. It is zero at the seasonal frequencies,  $\pi$  and  $\pi/2$ , in order to remove the non-stationary stochastic seasonal component: the pseudo-spectrum of the seasonal at these frequencies is infinity. Figure 11.3 shows the (asymmetric) weights at the end of the series.

**Figure 11.2:** Weights for quarterly gas series in the middle of the sample.



## 11.7 Breaks in the seasonal pattern

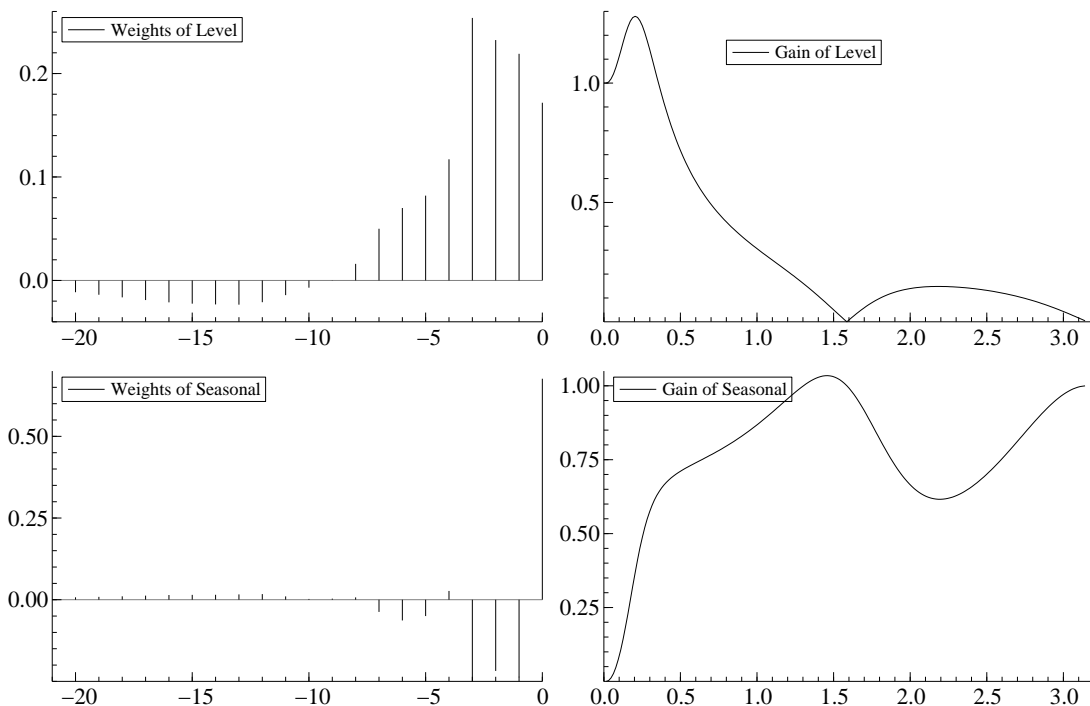
The seasonal pattern sometimes changes as the result of an intervention. Modelling an effect of this kind requires the introduction of  $s - 1$  dummy variables into the measurement equation, starting at time  $\tau$ . These dummies are constrained to sum to zero over  $s$  consecutive time periods. Alternatively pulse dummies can be added to the part of the state vector associated with the seasonal, that is  $\gamma_t$ .

Figure 11.4 shows the number of marriages in the UK every quarter. Estimating (11.1) with a random walk trend using the STAMP program gives

$$\tilde{\sigma}_\varepsilon = 0.00 \quad \tilde{\sigma}_\eta = 1.61 \quad \tilde{\sigma}_\omega = 2.69$$

with an equation standard error (the standard deviation of the innovations),  $\tilde{\sigma}$ , of 7.91. The lower panel in figure 11.4, the plot of individual seasons, displays a dramatic switch between the first and second quarters starting in 1969. Indeed the parametric seasonal stationarity test statistic, constructed from the Kalman filter innovations, is 6.96 which is a very decisive rejection of the null hypothesis of a constant seasonal pattern. The reason is that there was a change in the tax law. Up to the end of 1968 couples were allowed to claim the married persons tax allowance retrospectively for the entire year in which they married. As the tax year begins in April this arrangement provided an incentive to marry in the first quarter of the calendar year, rather

Figure 11.3: Weights for quarterly gas series at the end of the sample.



than in the spring. The abolition of this rule led to a marked decrease in the number of weddings in quarter one and a compensating rise in quarter two.

Adding a set of three seasonal break dummy variables, starting in the first quarter of 1969, to take account of a complete change in the seasonal pattern leads to the following estimates of the parameters:

$$\tilde{\sigma}_\varepsilon = 2.42 \quad \tilde{\sigma}_\eta = 1.59 \quad \tilde{\sigma}_\omega = 1.36$$

with

$$Q(9, 7) = 12.54 \quad \text{and} \quad \tilde{\sigma} = 5.66,$$

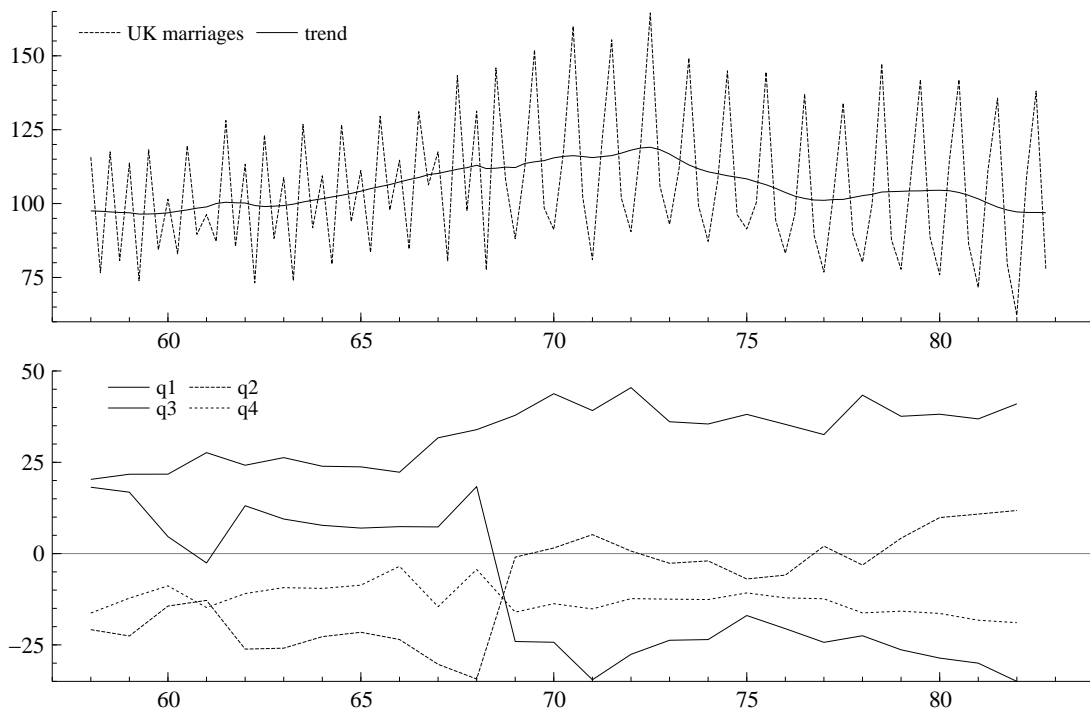
where  $Q(P, f)$  is the Box-Ljung statistic based on  $P$  residual autocorrelations but with  $f$  degrees of freedom. The  $t$ -statistics for the seasonal break dummies are -8.33, 7.58 and 2.09 respectively. There is a big reduction in the estimate of the seasonal parameter,  $\sigma_\omega$ , which no longer needs to be such as to allow the stochastic seasonal model to accommodate the change, and the equation standard error,  $\tilde{\sigma}$ , has fallen considerably.

When a full set of seasonal break dummy variables is included in the model the distribution of the seasonal stationarity test statistic is  $CvM(2s - 2)$ . The parametric test statistic, calculated from the Kalman filter innovations, is 2.42, giving a strong indication that there is still stochastic seasonality present. This is backed up by the fact that estimating the model with a fixed seasonal gives a significant Box-Ljung statistic of  $Q(9, 8) = 22.38$  while the fourth order residual autocorrelation,  $r(4)$ , is 0.33.

When the breakpoint is unknown, Busetti and Harvey (2003) show that running the seasonal break tests with an estimated breakpoint leads to an asymptotically valid procedure.



Figure 11.4: UK marriages and plot of seasonal for each quarter



## 11.8 Seasonal splines and weekly data

With quarterly or monthly observations it is rarely necessary or desirable to cut down on the number of terms used to model the seasonal pattern. On the other hand, when  $s$  is large, as with weekly data, there are both statistical and computational reasons for wanting to construct a more parsimonious seasonal model.

The dimensions of the dummy variable seasonal model can be reduced by assuming that certain seasons are the same, as in the restricted seasonal model of sub-section 11.2.4. However, although this may be suitable for daily observations, it is rarely appropriate for monthly or quarterly data. Similarly, the trigonometric seasonal model can be straightforwardly cut down by excluding pairs of sines and cosines at certain frequencies, usually the higher ones, but whereas this may be a sensible option for a slowly changing seasonal effect, such as temperature, it may not be satisfactory for modelling economic variables, where there are often sharp peaks at certain times of the year. What is needed is a model that is parsimonious but flexible enough to capture marked variations in a periodic pattern while retaining a reasonable degree of continuity. This leads to the notion of time-varying seasonal splines. [The term periodic splines is sometimes used but this risks confusion with the seasonal specific models described in section 10.] Models with seasonal splines are described, with applications to weekly and intra-daily observations, in Harvey et al. (1997) and Harvey and Koopman (1993).

Splines may be introduced in the context of regression. Suppose there are  $n$  pairs of observations  $(x_j, y_j)$ ,  $j = 1, \dots, n$ , and that we wish to set up a nonlinear regression model of the form

$$y_j = f(x_j) + \varepsilon_j, \quad j = 1, \dots, n, \quad (11.30)$$

where the  $\varepsilon_j$ 's are mutually uncorrelated disturbances with zero mean and constant variance,  $\sigma^2$ . In a cubic spline regression model,  $f(x_j)$  is constructed by putting together polynomials of degree at most three in such a way as to preserve continuity in second derivatives. The  $h$  individual cubics are joined at the co-ordinates  $(x_i^\dagger, \gamma_i^\dagger)$ ,  $i = 0, \dots, h$ . The set of  $x$  values,  $x_0^\dagger < x_1^\dagger < \dots < x_h^\dagger$ , is known as a *mesh*; the  $h + 1 \geq 3$  individual

points are called *knots*. The setup is completed by making assumptions about the spline at its end points.

Given the knots and the associated values of the ordinates,  $x_0^\dagger, \dots, x_h^\dagger$ , it can be shown that any point on the spline function is a linear combination of the  $\gamma_i^\dagger$ 's. Thus at the observation points, we can write

$$f(x_j) = \mathbf{w}'_j \boldsymbol{\gamma}^\dagger, \quad j = 1, \dots, n, \quad (11.31)$$

where  $\mathbf{w}_j$  is an  $(h+1) \times 1$  vector that depends on the position of the knots and the distance between them, as well as on the observed value  $x_j$ , and  $\boldsymbol{\gamma}^\dagger = (\gamma_0^\dagger, \gamma_1^\dagger, \dots, \gamma_n^\dagger)'$ . When  $x_j$  corresponds to a knot,  $x_j = x_i^\dagger$ , and all the elements in  $\mathbf{w}_j$  are zero, apart from the  $i$ -th which is unity. Thus  $f(x_j) = \gamma_i^\dagger$ .

Substituting (11.31) in (11.30) gives the cubic spline regression model

$$y_j = \mathbf{w}'_j \boldsymbol{\gamma}^\dagger + \varepsilon_j, \quad j = 1, \dots, n \quad (11.32)$$

Given the assumptions on  $\varepsilon_j$ ,  $\boldsymbol{\gamma}^\dagger$  can be estimated by ordinary least squares.

Now suppose that the explanatory variable is time and that there is a pattern repeated over a stretch of  $s$  observations, so that we have  $s$  seasonal effects  $\gamma_j, j = 1, \dots, s$ . These effects can be modelled by a spline of the form (11.31), in which  $n = s, x_j = j$  and continuity from one period to the next is preserved by the condition that  $\gamma_0^\dagger = \gamma_h^\dagger$  together with the conditions that the first and second derivatives at 0 and  $h$  are the same. This removes the need for further assumptions about the end conditions. The implications for the  $\mathbf{w}_j$  vectors, which are now  $h \times 1$ , corresponding to  $\gamma_1^\dagger, \dots, \gamma_h^\dagger$ , are easily worked out. Full details can be found in Poirier (1976)(pp. 43-47) and Harvey et al. (1997)(appendix). The seasonal spline is therefore

$$\gamma_j = \mathbf{w}'_j \boldsymbol{\gamma}^\dagger, \quad j = 1, \dots, s, \quad (11.33)$$

where  $\boldsymbol{\gamma}^\dagger$  is  $h \times 1$ .

As with any seasonal component, the effects should sum to zero over a complete period so as not to be confounded with the trend. Thus

$$\sum_{j=1}^s \gamma_j = \sum_{j=1}^s \mathbf{w}'_j \boldsymbol{\gamma}^\dagger = \mathbf{w}' \boldsymbol{\gamma}^\dagger = 0, \quad (11.34)$$

where  $\mathbf{w}$  is the  $h \times 1$  vector  $\mathbf{w} = \sum_{j=1}^s \mathbf{w}_j$ . When the seasonal effects evolve over time, (11.33) becomes

$$\gamma_t = \gamma_{jt} = \mathbf{w}'_j \boldsymbol{\gamma}_t^\dagger, \quad j = 1, \dots, s, \quad t = 1, \dots, T, \quad (11.35)$$

where  $\gamma_{jt}$  is the  $j$ -th seasonal effect at time  $t$  and

$$\boldsymbol{\gamma}_t^\dagger = \boldsymbol{\gamma}_{t-1}^\dagger + \boldsymbol{\omega}_t^\dagger, \quad (11.36)$$

with  $\boldsymbol{\omega}_t^\dagger$  denoting an  $h \times 1$  vector of serially uncorrelated random disturbances each with mean zero. The zero-sum constraint, (11.34), is enforced by specifying the covariance matrix

$$E(\boldsymbol{\omega}_t^\dagger \boldsymbol{\omega}_t^{\dagger'}) = \sigma_\omega^2 \left[ \mathbf{I} - \frac{1}{\mathbf{w}'\mathbf{w}} \mathbf{w}\mathbf{w}' \right], \quad t = 1, \dots, T, \quad (11.37)$$

together with corresponding initial conditions; compare the treatment of the restricted dummy variable model based on (11.17). The measurement equation for the state space form comes from (11.35) with the state vector,  $\boldsymbol{\gamma}_t^\dagger$ , obeying the transition equation of (11.36).

## 11.9 Seasonal specific models

Periodic models were originally introduced to deal with certain problems in environmental science, such as modelling river flows; see Hipel and McLeod (1994) (ch. 14). The key feature of such models is that separate stationary AR or ARMA model are constructed for each season. (Note that the term ‘periodic model’ is used in a different way from the way in which it can be used in the context of seasonal splines as described in section 11.8. Econometricians have developed periodic models further to allow for nonstationarity within each season and constraints across the parameters in different seasons; see the monograph by Franses and Papp (2004). These approaches are very much within the autoregressive/ARIMA paradigm. The structural framework offers a more general way of capturing periodic features by allowing periodic components to be combined with components common to all seasons. These common components may exhibit *seasonal heteroscedasticity*, that is have different values for the variance parameters in different seasons. Such models have a clear interpretation and make explicit the distinction between an evolving seasonal pattern of the kind typically used in a structural time series model and genuine periodic effects.

The first sub-section introduces seasonal heteroscedasticity and then moves on to define periodic models. The relationship between nonstationary periodic models and STMs is then examined. The fourth sub-section introduces *partly periodic* models, in which some or all of the components are periodic with respect only to groups of seasons. These may be handled within a state space framework and generalised further to include seasonal heteroscedasticity. Overall we have a class of what might be called *seasonal specific* models.

### 11.9.1 Seasonal heteroscedasticity

If the variance hyperparameters in a STM are different in different seasons, the model is said to exhibit *seasonal heteroscedasticity (SH)*. Such models are not time-invariant but this poses no problem if they are handled using the SSF.

*Irregular* - The simplest example of seasonal heteroscedasticity is when the irregular component has a different variance in each season, that is

$$\text{Var}(\varepsilon_t^{(j)}) = \sigma_{\varepsilon,j}^2, \quad t = 1, \dots, T, \quad j = 1, \dots, s. \quad (11.38)$$

Proietti (1998) gives an example involving monthly water usage. The trend is modelled as a random walk plus drift whereas the seasonal ends up being deterministic even though allowance was made for it being stochastic. The seasonal heteroscedasticity is not part of the seasonal component because it reflects the transitory effects which are more volatile at certain times of the year. For example, unusually hot weather in the summer can give rise to much higher water consumption than would normally be expected. Another example is the level of activity in the construction industry in countries where severe weather can lead to a much higher variance in the winter quarter.

*Trend* - It is possible that there is more scope for a permanent change in a series at certain times of the year. Effects of this kind could be allowed for by letting the level and slope variances be seasonal specific. They do not belong in the seasonal component since they do not give rise to a seasonal pattern in the forecast function.

*Seasonal* - The seasonal component itself may be subject to seasonal specific effects insofar as the way in which it evolves depends on the time of year. The time series of monthly Italian industrial production provides a good example. The series is typically very low in August due to holidays and it is also very variable. One possibility is to increase the variance of the irregular component in August. However, Proietti (1998) argues that the variability arises because the seasonal effect associated with August changes at a faster rate than the seasonal effect associated with other months. In order to capture this kind of phenomenon, he generalizes

the (balanced) dummy variable seasonal model. The covariance matrix of the disturbances becomes

$$\text{Var}(\omega_t) = \mathbf{W} - \frac{1}{\mathbf{i}'_s \mathbf{W} \mathbf{i}_s} \mathbf{W} \mathbf{i}_s \mathbf{i}'_s \mathbf{W}, \quad (11.39)$$

where  $\mathbf{W}$  is a diagonal matrix with  $\sigma_{\omega,j}^2, j = 1, \dots, s$ , as the  $j$ -th diagonal term. (Note that at least two variances must be non-zero if  $\mathbf{W}$  is to be non-null.) Since  $\text{Var}(\mathbf{i}'_s \omega_t) = 0$ , the seasonals sum to zero over a year. The above covariance matrix reduces to (11.10) if  $\sigma_{\omega,j}^2 = \sigma_{\omega}^2$  for all  $j = 1, \dots, s$ . In the case of Italian industrial production, Proietti sets all  $\sigma_{\omega,j}^2$ 's the same except for August.

### 11.9.2 Periodic models

A classic example of the need for a periodic model arises in hydrology when there are monthly observations on river flow. If melting snow is an important factor in river flow in the spring, the correlation between the flow in successive months may be negative whereas at other times of the year it is positive. These features show up in the periodic autocorrelation functions, as illustrated in Hipel and McLeod (1994) (p504-5). The autocorrelation function for period (season)  $j$  is

$$\rho_j(\tau) = \text{Cov}(y_t^{(j)}, y_{t-\tau}) / \sqrt{\text{Var}(y_t^{(j)}) \text{Var}(y_{t-\tau})}, \quad j = 1, \dots, s,$$

where the superscript in  $y_t^{(j)}$  indicates that the time  $t$  is in period  $j$ . It should, however, be noted that different acf's for different seasons are not necessarily evidence for a periodic model. They can, for example, be produced by seasonally heteroscedastic models; see Proietti (1998)(p 9).

The periodic AR(1) model is

$$y_t^{(j)} = \mu_j + \phi_j (y_{t-1} - \mu_{j-1}) + \xi_t^{(j)}, \quad j = 1, \dots, s$$

and this generalizes to higher order autoregressions. In theory, MA and ARMA can also be constructed, usually under the assumption that  $\xi_t^{(j)}$  is not periodic, that is  $\xi_t^{(j)} = \xi_t$  is serially uncorrelated with constant variance; see Franses and Papp (2004), pp 29-30).

For economic time series, the periodic model may need to be nonstationary. Osborn (1988) fits a first-order periodic model to UK consumption. She imposes the constraint  $\prod_{j=1}^s \phi_j = 1$ , so the model has  $s$  seasonal and nonseasonal unit roots with a reduced form such that  $\Lambda_s y_t \sim MA(s-1)$ . However, she reports that the model exhibits some residual serial correlation and so is not entirely satisfactory. Harvey and Scott (1994), p.1331-2, show that a similar fit can be obtained by a simple non-periodic model consisting of a random walk and a stochastic seasonal. This structural model also has a reduced form in which  $\Lambda_s y_t \sim MA(s-1)$ .

### 11.9.3 Relationship between periodic models and trend plus seasonal models

A periodic generalisation of the local level model is

$$y_t = \mu_t^{(j)} + \varepsilon_t^{(j)}, \quad t = 1, \dots, T, \quad (11.40)$$

where  $\text{Var}(\varepsilon_t^{(j)}) = \sigma_{\varepsilon,j}^2$ , as in (11.38), and

$$\mu_t^{(j)} = \mu_{t-1}^{(j)} + \eta_t^{(j)}, \quad \text{Var}(\eta_t^{(j)}) = \sigma_{\eta,j}^2, \quad j = 1, \dots, s, \quad (11.41)$$

where the  $\eta_t^{(j)'}_s$  are mutually as well as serially independent, as is the case with the  $\varepsilon_t^{(j)'}_s$ . (There is a slight difference with respect to conventional periodic models in that the levels in (11.41) are assumed to evolve in all time periods - as in the seasonal model of section 2 - not just in the one in which they directly affect the observations). The periodic local level model in (11.40) contains  $2s$  hyperparameters. This contrasts with a three parameter structural model consisting of a random walk trend, a seasonal and irregular.

A more general formulation allows the  $\eta_t^{(j)'}_s$  to be correlated with each other. In this case, the  $s \times 1$  vector of levels,  $\boldsymbol{\mu}_t$ , which has  $j$ -th element  $\mu_t^{(j)}$ ,  $j = 1, \dots, s$ , satisfies the multivariate random walk

$$\boldsymbol{\mu}_t = \boldsymbol{\mu}_{t-1} + \boldsymbol{\eta}_t, \quad \text{Var}(\boldsymbol{\eta}_t) = \mathbf{N}, \quad (11.42)$$

where  $\boldsymbol{\eta}_t = (\eta_t^{(1)}, \dots, \eta_t^{(s)})'$  and  $\mathbf{N}$  is a positive semi-definite matrix. A common level can be constructed as the average of the individual levels, that is

$$\bar{\mu}_t = (1/s)\mathbf{i}'\boldsymbol{\mu}_t, \quad (11.43)$$

with

$$\bar{\mu}_t = \bar{\mu}_{t-1} + \bar{\eta}_t. \quad (11.44)$$

The seasonals are then defined as deviations from this average, that is

$$\boldsymbol{\gamma}_t = \boldsymbol{\mu}_t - \mathbf{i}\bar{\mu}_t = \boldsymbol{\mu}_t - (1/s)\mathbf{ii}'\boldsymbol{\mu}_t, \quad (11.45)$$

and so

$$\boldsymbol{\gamma}_t = \boldsymbol{\gamma}_{t-1} + \boldsymbol{\omega}_t, \quad \boldsymbol{\omega}_t = \boldsymbol{\eta}_t - (1/s)\mathbf{ii}'\boldsymbol{\eta}_t = \boldsymbol{\eta}_t - \mathbf{i}\bar{\eta}_t. \quad (11.46)$$

Thus

$$y_t = \bar{\mu}_t + \boldsymbol{\gamma}_t + \varepsilon_t, \quad t = 1, \dots, T \quad (11.47)$$

where in a type  $j$  season,  $\boldsymbol{\gamma}_t = \boldsymbol{\gamma}_{jt}$  and  $\varepsilon_t = \varepsilon_t^{(j)}$ .

Now

$$\text{Var}(\boldsymbol{\omega}_t) = \mathbf{N} + s^{-2}\mathbf{ii}'\mathbf{N}\mathbf{ii}' - s^{-1}\mathbf{ii}'\mathbf{N} - s^{-1}\mathbf{N}\mathbf{ii}'. \quad (11.48)$$

As is evident from (11.48),  $\text{Var}(\mathbf{i}'\boldsymbol{\omega}_t) = 0$  because the disturbances sum to zero over a year by construction. The correlation between the level and seasonal disturbances is

$$\text{Var}(\eta_t \boldsymbol{\omega}_t') = \text{Var}(((1/s)\mathbf{i}'\boldsymbol{\eta}_t)(\boldsymbol{\eta}_t - (1/s)\mathbf{ii}'\boldsymbol{\eta}_t)') = s^{-1}\mathbf{i}'\mathbf{N} - s^{-2}\mathbf{i}'\mathbf{N}\mathbf{ii}'.$$

When  $\mathbf{H} = \sigma_\omega^2 \mathbf{I}$ , the trend and seasonal are orthogonal because  $\text{Var}(\eta_t \boldsymbol{\omega}_t') = 0$ . In this case  $\text{Var}(\boldsymbol{\omega}_t)$  is as in (11.10) and  $\text{Var}(\bar{\eta}_t) = \sigma_\omega^2/s$ , which is a restricted form of the BSM in that the variance of the level is tied to the seasonal variance. Tests on the rank of  $\mathbf{N}$  can be carried out as in Nyblom and Harvey (2000); see Proietti and Hillebrand (2017), section 3. Evidence that  $\mathbf{N}$  is of less than full rank may indicate that some levels are deterministic or co-integrated.

An alternative way to proceed is to introduce a disturbance common to all seasons at the outset so that (11.41) becomes

$$\mu_t^{(j)} = \mu_{t-1}^{(j)} + \eta_t^\dagger + \eta_t^{(j)}, \quad \text{Var}(\eta_t^\dagger) = \sigma_\eta^{\dagger 2}, \quad j = 1, \dots, s, \quad (11.49)$$

where  $\eta_t^\dagger$  and the  $\eta_t^{(j)'}_s$  are mutually independent. The vector of levels is then

$$\boldsymbol{\mu}_t = \boldsymbol{\mu}_{t-1} + \mathbf{i}\eta_t^\dagger + \boldsymbol{\eta}_t^*, \quad \text{Var}(\boldsymbol{\eta}_t^*) = \mathbf{N}^*, \quad (11.50)$$

where  $\mathbf{N}^*$  is usually diagonal. If the common level and the seasonal vector are defined as in (11.43) and (11.46), then

$$y_t = \bar{\mu}_t + \boldsymbol{\gamma}_t^{(j)} + \varepsilon_t^{(j)}, \quad t = 1, \dots, T \quad (11.51)$$

with  $\bar{\eta}_t = \eta_t^\dagger + (1/s)\mathbf{i}'\boldsymbol{\eta}_t$ . (When  $\text{Var}(\boldsymbol{\eta}_t)$  is diagonal,  $\text{Var}(\boldsymbol{\omega}_t)$  in (11.48) is slightly different from the one in

(11.39)). In general the seasonal and level are not orthogonal. Proietti (2004) shows how to construct a level as a weighted average,  $\mathbf{w}'\boldsymbol{\mu}_t$ , that is orthogonal to the seasonal components, but the price paid is that the seasonals do not normally sum to zero.

Setting  $Var(\boldsymbol{\eta}_t) = \sigma_\omega^2 \mathbf{I}$  makes the level variance equal to  $\sigma_\eta^{\dagger 2} + \sigma_\omega^2/s$ . Thus the BSM is not restricted.

The general model consisting of (11.40) and (11.49) therefore contains both the pure periodic and the BSM as special cases. In principle, an LR test that  $Var(\boldsymbol{\eta}_t)$  is a scalar matrix can be carried out quite easily, as can a test that the irregular has a constant variance. When  $\mathbf{N}^*$  is diagonal,  $2(s-1)$  restrictions are needed to give the time invariant structural model. A purely periodic model is obtained when  $\sigma_\eta^{\dagger 2} = 0$ . Because the alternative is one-sided, the LR test statistic for this hypothesis has a distribution that is an even mixture of chi-squares with zero and one degree of freedom.

In the BSM, the signal extraction filters are (in a doubly infinite sample) the same in all seasons. When periodic features are present this is no longer the case. This has implications for seasonal adjustment using standard procedures. On the other hand, if a model with periodic features is handled using the SSF there is no problem. The measurement equation for (11.47) is

$$y_t = \mathbf{z}'_t \boldsymbol{\mu}_t + \varepsilon_t^{(j)}, \quad t = 1, \dots, T,$$

with the  $s \times 1$  vector  $\mathbf{z}_t$  having zero elements everywhere except in position  $j$ . A decomposition into level and seasonals is then made from (11.43) and (11.45). If MSEs are required then the model is best set up as in (11.51) with  $\boldsymbol{\mu}_t$  and  $\boldsymbol{\gamma}_t$  in the state vector.

The model can easily be extended to include slopes in the trends. If the slope is the same in all seasons then (11.50) is replaced by

$$\begin{aligned} \boldsymbol{\mu}_t &= \boldsymbol{\mu}_{t-1} + \mathbf{i}\eta_t^\dagger + \mathbf{i}\beta_{t-1} + \boldsymbol{\eta}_t, \\ \beta_t &= \beta_{t-1} + \zeta_t \end{aligned}$$

Seasonal specific slopes are also possible, as in Proietti (1998).

Replacing the  $\varepsilon_t^{(j)}$ 's by stationary periodic processes, is also possible. Thus for an AR(1) periodic model,

$$\begin{aligned} y_t &= \mu_t^{(j)} + u_t^{(j)}, \quad t = 1, \dots, T, \\ u_t^{(j)} &= \phi_j u_{t-1} + \xi_t^{(j)}, \quad j = 1, \dots, s. \end{aligned}$$

Implementation in the SSF is straightforward. Proietti and Hillebrand (2017) (Table 4) report estimates of the  $\phi_j$ 's for a dataset of temperatures in central England. The values are higher in the winter and summer than in the autumn and spring. Further discussion of modeling and testing can be found in Trimbur and Bell (2012).

### 11.9.4 Partly periodic models

Having different trends in all seasons, as in (11.40), may be a little extreme and it certainly requires a large number of parameters to be estimated. This suggests a class of models which are partly periodic. There are two ways in which this can arise. Firstly only a subset of components may be periodic. Secondly, periodicity may apply to groups of seasons rather than to all seasons. The two ideas can be combined in that only particular components are periodic with respect to groups of seasons. The SSF allows such models to be handled because the measurement equation picks out the relevant components at any particular point in time.

When all the components are periodic we will refer to the model as being *purely partly periodic*. We begin by considering such models. Suppose there are just two groups for which separate models are to be constructed,

with  $s_1$  seasons in the first group and  $s_2$  in the second and  $s_1 + s_2 = s$ . Then

$$y_t = \mu_t^{(k)} + \gamma_t^{(k)} + \varepsilon_t^{(k)}, \quad k = 1, 2, \quad (11.52)$$

where the seasonal,  $\gamma_t^{(k)}$ , is modelled by a set of  $s_k, k = 1, 2$  time-varying dummies which embody the zero sum restriction over the group. If a group has only one season then the seasonal component is not needed. The trends can be assumed to have slopes for more generality so that

$$\begin{aligned} \mu_t^{(k)} &= \mu_{t-1}^{(k)} + \beta_{t-1}^{(k)} + \eta_t^{(k)}, & \text{Var}(\eta_t^{(k)}) &= \sigma_{\eta,k}^2, & k &= 1, 2. \\ \beta_t^{(k)} &= \beta_{t-1}^{(k)} + \zeta_t^{(k)}, & \text{Var}(\zeta_t^{(k)}) &= \sigma_{\zeta,k}^2. \end{aligned}$$

The overall trend is given by

$$\mu_t = (s_1/s)\mu_t^{(1)} + (s_2/s)\mu_t^{(2)}.$$

This may be useful in the context of smoothing and forecasting. A seasonal component for the model as a whole can be defined as

$$\gamma_t = \mu_t^{(k)} - \mu_t + \gamma_t^{(k)}, \quad k = 1, 2.$$

In a pure partly periodic model, the disturbances in the two groups are mutually independent. However, some correlation could be introduced between the disturbances in the trends. For example suppose  $\rho$  is the correlation between  $\eta_{1t}$  and  $\eta_{2t}$  and between  $\zeta_{1t}$  and  $\zeta_{2t}$ . If  $\rho = 1$  together with  $\sigma_{1\eta}^2 = \sigma_{2\eta}^2$  and  $\sigma_{1\zeta}^2 = \sigma_{2\zeta}^2$ , the model reduces to the seasonal heteroscedastic formulation of (11.38), while if  $\sigma_{1\omega}^2 = \sigma_{2\omega}^2$  as well, the BSM is obtained. (There is a constant difference between the two trends, which can be transferred to the seasonals).

Now suppose the group periodic effects only apply to certain components. At the simplest level this might mean letting the variance of the irregular in (11.52) be the same in all seasons. This restriction is easily enforced by the joint treatment of the two groups within the SSF; in fact there is one less parameter to estimate. If a common cycle were added the model would become

$$y_t = \mu_t^{(k)} + \gamma_t^{(k)} + \psi_t + \varepsilon_t, \quad k = 1, 2, \quad (11.53)$$

Further generalisation might involve the introduction of seasonal heteroscedasticity. This does not need to be connected in any way with the periodic groupings.

A test of seasonal stability within a group, that is  $\sigma_{k\omega}^2 = 0$ , can be carried out using a seasonal specific version of the seasonal stationarity test. To test that the seasonals in the first group are fixed, let  $\mathbf{A}$  be an  $s \times (s_1 - 1)$  full rank matrix with each of the first  $(s_1 - 1)$  columns containing a one and a minus one while the remaining elements are all zero. Then  $\mathbf{A}'\mathbf{y}_t$  is stationary under the null hypothesis that  $\sigma_{1\omega}^2 = 0$  and the asymptotic distribution of the test statistic is  $CvM(s_1 - 1)$ .

In the Italian industrial production series analysed by Proietti (1998), August behaves so differently from the other months that it is worth letting it have its own trend. Thus the model is as in (11.52) but with slopes in the trend components and no seasonal component, as such, for August. Other seasons may be deterministic, while August is not. Such a feature is not possible in the seasonally heteroscedastic model of (11.39). Note that the August trend is allowed to be correlated with the (common) trend in the other seasons.



## 11.10 Data irregularities and survey design

The most striking benefits of the structural approach to time series modelling and seasonal adjustment only become apparent when we start to consider more complex problems. The state space form offers considerable flexibility with regard to dealing with data irregularities, such as missing observations and observations at mixed frequencies, for both stock and flow variables; see Harvey (1989). Furthermore, as Pfeiffermann (1991) pointed out, the structure of survey sample designs may be built into the specification of the irregular component.

The study by Harvey and Chung (2000) on the measurement of British unemployment from the Labour Force Survey (LFS) provides an illustration. As well as the observations being at mixed frequencies, the data are obtained from a rotating sample in which households are interviewed for a period of three months. In the next quarter 80% of the households are retained and 20% are replaced by new households. As a result the irregular component follows a moving average process but its form and parameters are known.

## 11.11 Robust seasonal adjustment

Allowing the irregular component in a UC model to have a heavy-tailed distribution, such as Student's  $t$ , provides a robust method of dealing with outliers. An outlier is defined as an observation that is inconsistent with the model, but an observation that is deemed to be an outlier for a normal distribution may well be consistent with a heavy-tailed distribution. Removing an outlier with a dummy variable effectively says that it contains no useful information. This is rarely the case, except perhaps when an observation has been recorded incorrectly. Furthermore although outliers that have been identified as such can be dealt with *ex post* by dummy variables, only a robust model offers a viable solution to coping with them in the future.

Models with non-Gaussian irregular disturbances can be estimated by simulation techniques as described in Durbin and Koopman (2012). In small samples it may prove difficult to estimate the degrees of freedom of a  $t$ -distribution. A reasonable solution then is to impose a value, such as six, that is able to handle outliers. Other heavy tailed distributions may also be used. For example in their comparison of X-12-ARIMA and robust structural time series model, Bruce and Jurke (1996) adopt a mixture of normals.

Durbin and Koopman (2012) note that estimating a Gaussian BSM for quarterly UK gas consumption produces a rather unappealing wobble in the seasonal component at the time North Sea gas was introduced in 1970. They therefore allow the irregular to follow a  $t$ -distribution and estimate its degrees of freedom to be 13. The robust treatment of the atypical observations in 1970 produces a more satisfactory seasonal pattern around that time.

The use of simulation methods that are potentially computationally intensive can be avoided by adopting an observation-driven model. Such models can be estimated by maximum likelihood. The Dynamic Conditional Score (DCS) models recently developed by Harvey (2013) and Creal et al. (2013), who call them Generalized Autoregressive Score (GAS) models, provide estimates which are robust to outliers when the conditional distribution is assumed to be heavy-tailed. Caivano et al. (2016) apply these methods to seasonal adjustment. A DCS model has features in common with methods described in the robustness literature; see Maronna et al. (2006)(ch 8). Robust procedures for guarding against additive outliers typically respond to large observations by Winsorizing or trimming: with Winsorizing the response function converges to a positive (negative) constant for observations tending to plus (or minus) infinity whereas with trimming it goes to zero. In DCS models the location is dynamic and driven by the score of the conditional distribution. The score for a  $t$ -distribution converges to zero and so can be regarded as a parametric form of trimming. Likewise a parametric form of Winsorizing is given by the exponential generalized beta distribution of the second kind (EGB2) distribution.

The DCS model for trends and seasonals,

$$y_t = \mu_{t|t-1} + \gamma_{t|t-1} + v_t, \quad t = 1, \dots, T,$$

has a structure which is similar to that of the innovations form of the Kalman filter for the BSM. The dynamics depend on the score. The filter for the trend is

$$\begin{aligned} \mu_{t+1|t} &= \mu_{t|t-1} + \beta_{t|t-1} + \kappa_1 u_t \\ \beta_{t+1|t} &= \beta_{t|t-1} + \kappa_2 u_t, \end{aligned}$$

where

$$u_t = \left(1 + \nu^{-1} e^{-2\lambda} (y_t - \mu_{t|t-1})^2\right)^{-1} v_t, \quad t = 1, \dots, T,$$

when  $v_t$  has a  $t$ -distribution. The filter for the seasonal is

$$\gamma_{t|t-1} = \mathbf{z}'_t \gamma_{t|t-1}, \quad \gamma_{t+1|t} = \gamma_{t|t-1} + \boldsymbol{\kappa}_t u_t,$$

where the  $s \times 1$  vector  $\mathbf{z}_t$  picks out the current season from the vector  $\gamma_{t|t-1}$ . If  $\kappa_{jt}$ ,  $j = 1, \dots, s$ , denotes the  $j$ -th element of  $\boldsymbol{\kappa}_t$ , then in season  $j$  we set  $\kappa_{jt} = \kappa_s$ , where  $\kappa_s$  is a non-negative unknown parameter, whereas  $\kappa_{it} = -\kappa_s / (s - 1)$ ,  $i \neq j$ ,  $i = 1, \dots, s$ . The above filter may be regarded as a robust version of the well-known Holt-Winters filter, but with the important property that the seasonals sum to zero.

In contrast to the Gaussian BSM, the DCS model has no exact solution for smoothing. Caivano et al. (2016) propose an iterative procedure in which the observations are modified by the score to mitigate the effects of outliers. These pseudo-observations are then used to estimate the parameters in a BSM and the signal is estimated by smoothing. This signal is then used to construct new pseudo-observations. On convergence the smoothed seasonal component is extracted. This method is applied to a series on the number of tourists entering Spain from January 2000 to April 2014.

## 11.12 Conclusions

Unobserved components models offer a straightforward way of modelling seasonality and carrying out seasonal adjustment. Furthermore the use of the state space form and the associated filtering algorithms offers considerable flexibility and allows a variety of non-standard situations to be handled. For example complex sampling designs can be accommodated and dynamic splines can be used to parsimoniously capture the evolving seasonal pattern in a weekly series and, at the same time, make allowance for the changing location of moving festivals, such as Easter. Finally recent developments have shown how to model data robustly so that outliers do not distort the seasonal pattern or the seasonally adjusted series.

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# 12

## Moving Average Based Seasonal Adjustment



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## Table of Contents

12.1	Introduction	283
12.2	Definitions and useful tools	284
12.2.1	Definitions and example	285
12.2.2	Gain and phase shift functions	286
12.3	Desirable properties of a moving average	288
12.3.1	Trend preservation	288
12.3.2	Elimination of seasonality	289
12.3.3	Reduction of the irregular component	290
12.3.4	The Slutsky-Yule effect	290
12.3.5	An optimization problem	291
12.4	Usual moving averages	291
12.4.1	Simple moving averages	291
12.4.2	Composite simple moving averages	292
12.4.3	Spencer moving averages	292
12.4.4	Kendall and Stuart moving averages	293
12.4.5	Henderson moving averages	293
12.4.6	Musgrave asymmetric moving averages	295
12.4.7	Generalization of the Whittaker-Henderson approach	297
12.5	The X11 seasonal adjustment algorithm	297
12.5.1	Components and decomposition models	297
12.5.2	A simple seasonal adjustment algorithm	298
12.5.3	The basic algorithm of the X-11 method	299
12.5.4	Extreme observations and calendar effects	300
12.5.5	The iterative principle of X-11	302
12.5.6	From Census X-11 to X-11-ARIMA and X-12-ARIMA	303
12.6	The moving averages used in X-11	304
12.6.1	Composite simple moving averages	304
12.6.2	Estimation of trend: 2x4 and 2x12 averages	304
12.6.3	Estimation of Seasonality: $3 \times 3$ , $3 \times 5$ and $3 \times 9$ Averages	306
12.6.4	Henderson moving averages	307
12.6.5	Musgrave asymmetric moving averages	311
12.6.6	The X-11 moving average filter	315
12.7	Conclusions	318



## 12.1 Introduction

Even if, almost by nature, seasonal adjustment is a non-linear process, the most important seasonal adjustment methods are based on linear filters. TRAMO-SEATS ((Gomez and Maravall , 1997)), a parametric method based on the ARIMA modeling of the series, uses a Wiener-Kolmogorov filter to compute the components of the series that is optimally derived from the spectrum of the ARIMA model adjusted to the series. Following a non-parametric approach, the “X11 family”<sup>1</sup> methods use a set of predefined linear filters, the moving averages, to adjust the series; the choice of the filters finally used to compute the components being based on the characteristics of the series<sup>2</sup>.

Moving averages have a long but unfortunately not well-known history. They do not involve a priori the use of sophisticated concepts or model, are very simple in principle and especially flexible in their application: it is possible to construct a moving average that has good properties in terms of trend preservation, elimination of seasonality, noise reduction, and so on.

According to Klein (2009), the Bank of England’s data transformations to reveal patterns of change in the holding of bullion while masking actual values, led not only to the first publication of a scaled series of index numbers in 1797, but also to the first publication of a moving average series in 1832. Using an asymmetric moving average of order 3 allowed the Bank not only to hide the real values but also to take advantage of the delay introduced by the moving average to implement its monetary policy.

The majority of early publications on moving averages have appeared in American, English and Scottish actuarial journals starting with John Finlaison in 1829. Finlaison started in January 1823 preparing the mortality data that were to provide the first life table consisting of graduated observations at individual ages.<sup>3</sup> Finlaison (1829) wrote that in order “*to present in every case the uniform tendency of the apparent law of nature rather than the mere arithmetical probability resulting from the actual numbers in each particular instance, I have tried many methods of adjustment with more or less success...*” and came out with a symmetrical 9-terms moving average formula. This piecewise approach to smoothing was extended by the eminent Italian meteorologist Schiaparelli (1866) and the first writer to make a systematic investigation of such averages was the American mathematician E. L. De Forest (1873, 1875, 1876, 1877).

This idea of decomposing a time series appeared at the same period in the works of economists, some of whom did not hesitate to acknowledge that it came to them directly from astronomy or meteorology.<sup>4</sup> The aim of many studies was, then, to reveal “cycles”, the study and analysis of which might make it possible to explain and predict economic crises. In these conditions, short-term periodic components were of little interest and it was expedient to eliminate them:

*Every kind of periodic fluctuations, whether daily, weekly, quarterly, or yearly, must be detected and exhibited not only as a subject of study in itself, but because we must ascertain and eliminate such periodic variations before we can correctly exhibit those which are irregular or non-periodic and probably of more interest and importance.*

Jevons (1862)

In a paper read to the Edinburgh Mathematical Society in 1919, E. T. Whittaker suggested an alternative method of graduation, which has found much favor and is called nowadays the Whittaker-Henderson graduation. Commenting the works of Whittaker (1923), Henderson (1924) wrote: “*He arrives at an approximate solution of the difference equations resulting from assigning the sum of the squares of the third differences as*

<sup>1</sup> The X11 family contains all the methods based on X11: X11, X-11-ARIMA, X-12-ARIMA and X-13ARIMA-SEATS.

<sup>2</sup> By default, the choice of the moving averages is made among a limited number of predefined filters but done according to the so-called  $I/c$  and  $I/S$  ratios that depend on the characteristics of the series.

<sup>3</sup> In these times, an important issue was the estimation of life tables. The terminology “graduation by the summation method” would be replaced nowadays with “smoothing by moving averages”, especially in the time series domain.

<sup>4</sup> One can hardly fail to cite the works of the meteorologist Buys-Ballot who, in 1847, studied periodic temperature variations by modelling the trend by a polynomial, seasonality by indicators and implicitly relying on linear regression techniques to estimate the parameters. See Buys-Ballot (1847).

*measure of the irregularity and the sum of the squares of the differences between the graduated and ungraduated values of the functions as the measure of the departure from the observed facts. His difference equations are thus the conditions for a minimum value of  $\sum (\Delta^3 u_x)^2 + k \sum (u_x - u_x'')^2$ , where  $u_x$  is the graduated and  $u_x''$  the ungraduated value of the function.*" It has to be noted that Bohlmann (1899) proposed a very similar criterion, using first differences instead of third differences<sup>5</sup>.

The Whittaker-Henderson minimization problem, based on fidelity and smoothness, will be referred to extensively in this chapter as it is a global and very convenient framework to understand the properties of most moving averages proposed in the literature.

In the earlier twentieth century, a large part of time series studies were devoted to the estimation of the seasonal variations. Persons (1919) developed for example the "link relatives method", one of the first main methodologies for seasonal adjustment. The process of decomposition was refined by Frederic R. Macaulay (1931), of the National Bureau of Economic Research, who in the 20's introduced the "ratio-to-moving-average procedure", variations of which are widely used today. In 1954, the manual procedure was replaced by a computer program (Census I) which was modified and enlarged in 1955 and since then known as Census II. Since 1955, there have been several variants of Census II starting from X-1 to the currently used X-13ARIMA-SEATS.

Section 12.2 presents some definitions and useful tools like the gain and phase-shift functions that can be associated to any moving average. Section 12.3 details the desirable properties a moving average should have and introduces a formalism, the minimization of a quadratic form under a set of linear constraints, that will be used in the chapter for the construction of various moving averages. 12.4 presents an overview of the commonly used moving averages that still remain an important tool for the statistician who wants to perform smoothing or seasonal adjustment. Section 12.5 presents how the X11 seasonal adjustment methods estimates the various components of a time series, using an iterative principle and a set of symmetric and asymmetric moving averages. Section 12.6 details the different moving averages used by the X11 method and presents their characteristics. The global central filter of X11 is then derived for both monthly and quarterly time series. Section 12.7 briefly concludes the chapter.

## 12.2 Definitions and useful tools

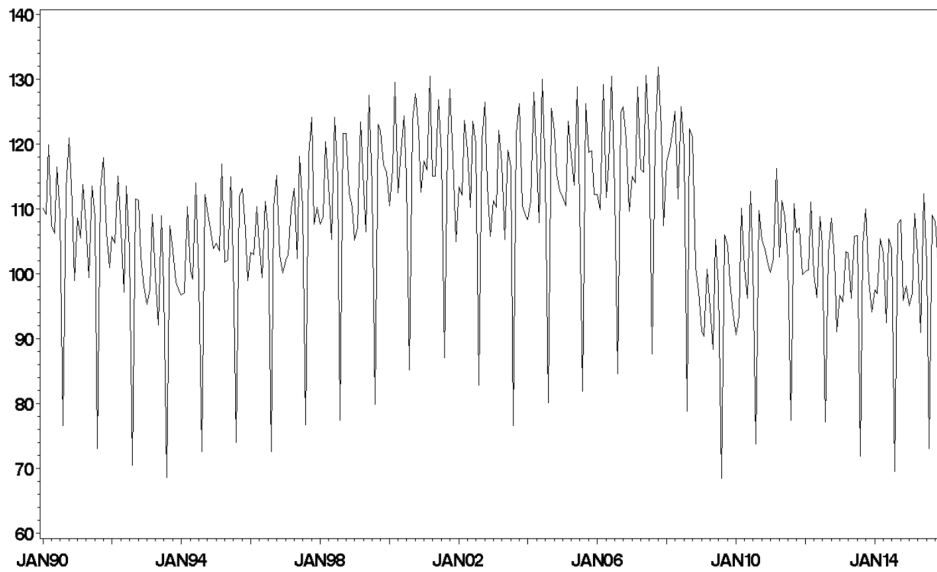
A time series may be considered from two standpoints: time, and frequency:

- In the time domain, the series  $\{X_t\}$  is regarded as a succession of  $T$  observed values at instants  $t$ ,  $t$  varying from 1 to  $T$ . This is how a time series is generally approached, and it is easy to show graphically, as in Figure 12.1, its evolution over time. Note that this series is characterized by strong seasonality expressing the drop in industrial activity in the month of August. The modeling of the series or its components, comparing the value at instant  $t$  to those of past instants, is especially easy to formalize. This is the case, for example, for the modelling of the series using a seasonal ARIMA model, the expression of a linear, exponential or even locally polynomial trend, or the modelling of the irregular component by a white noise.
- In contrast, in the frequency domain, one begins with the series  $\{X_t\}$  expressed as the sum of trigonometric functions.<sup>6</sup> The importance of each frequency in the composition of the series is measured: the

<sup>5</sup>Moreover, it is interesting to note that Bohlmann's criteria is a local version of the Hodrick-Prescott filter, see Hodrick and Prescott (1981), and that the Whittaker-Henderson criteria has been used both by Akaike and Ishiguro (1980) as a smoothness prior in a Bayesian model for seasonal adjustment, implemented in a computer program called BAYSEA and for spline smoothing, see Härdle (1990).

<sup>6</sup>In his *Théorie Analytique de la Chaleur*, published in 1807, Jean-Baptiste Fourier established that any mathematical function could be decomposed into a sum of sine and cosine functions. This theorem gave rise initially to harmonic analysis, and then, once generalized, to spectral analysis.

Figure 12.1: French Industrial Production Index, Manufacturing, Monthly figures



graph that associates with each frequency its importance in the series is called the spectrum of the series. Thus, Figure 12.2 shows the spectrum of the French industrial production index.

As can be seen, this spectrum reveals a strong contribution (called a spectral peak) of the multiple frequencies of  $\pi/6$  ( $30^\circ$ ,  $60^\circ$ ,  $90^\circ$ , etc.). The period associated with this frequency is  $\omega = \frac{2\pi}{f} = 2\pi / [\frac{\pi}{6}] = 12$  and we find the monthly seasonality observed in the previous chart.

The low frequencies correspond naturally to slowly changing components, for example trend and cycle, and the high frequencies to more quickly changing components, such as the irregular component.

These two approaches often prove complementary and we will subsequently use one or the other to show the qualities and defects of moving average filters.

### 12.2.1 Definitions and example

We call the moving average of coefficients  $\{\theta_i\}$  the operator written as  $M\{\theta_i\}$ , or simply  $M$ , defined by:

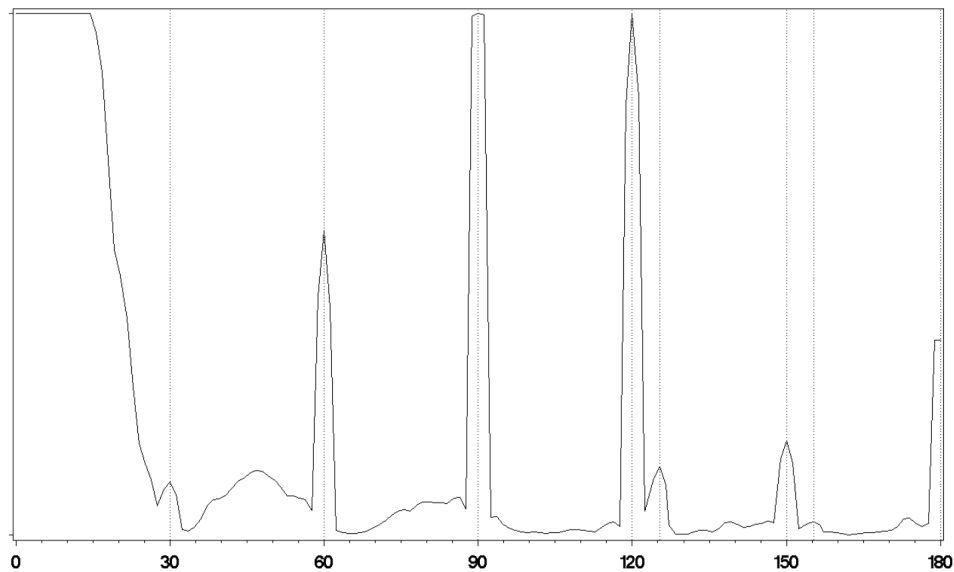
$$M(X_t) = \sum_{k=-p}^{+f} \theta_k X_{t+k}$$

The value at instant  $t$  of the unadjusted series is therefore replaced by a weighted average of  $p$  "past" values of the series, the current value, and  $f$  "future" values of the series.

- The quantity  $p + f + 1$  is called the **moving average order**.
- When  $p$  is equal to  $f$ , that is, when the number of points in the past is the same as the number of points in the future, the moving average is said to be **centered**.
- If, in addition, you have  $\theta_{-k} = \theta_k$  for any  $k$ , the moving average  $M$  is said to be **symmetric**. In this case, when listing the coefficients of the moving average it will suffice, following Kendall (1973), to specify the order of the moving average and the  $(k + 1)$  first coefficients:

$$\frac{1}{24}\{1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 1\} \text{ is written simply as } [13]; \frac{1}{24}\{1, 2, 2, 2, 2, 2, 2\}$$

Figure 12.2: Spectrum of the French industrial production index



Generally, with a moving average of order  $p + f + 1$  calculated for instant  $t$  with points  $p$  in the past and points  $f$  in the future, it will be impossible to smooth out the first  $p$  values and the last  $f$  values of the series.

In the X-11 method, symmetric moving averages play an important role; to avoid losing information at the series ends, they are supplemented by ad hoc asymmetric moving averages.

### 12.2.2 Gain and phase shift functions

Let us consider the series  $X_t = \sin\left[\frac{\pi}{3}t\right]$  and transform it using the asymmetric moving average defined by  $M(X_t) = \frac{1}{3}[X_{t-2} + X_{t-1} + X_t]$  which replaces the value at instant  $t$  with the simple average of the values at the present instant and the two previous instants.

Figure 12.3 expresses the result of the smoothing and reveals two phenomena:

- First of all, a reduction in the amplitude of the series, which effectively meets our objective of smoothing;
- But also a time lag, called a *phase shift*: the two series do not show turning points on the same dates.

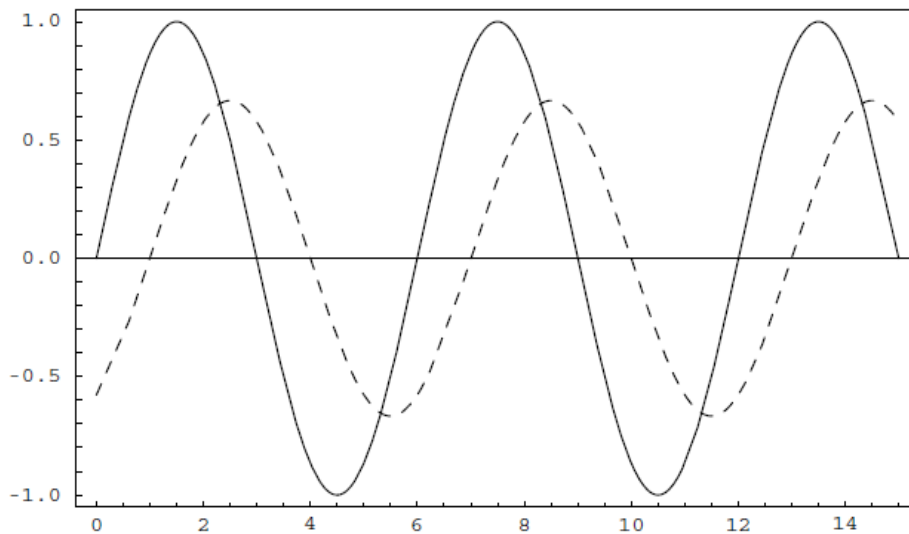
This phase shift phenomenon is disagreeable insofar as it transforms the series changes themselves. It can nevertheless be shown that symmetric moving averages do not result in a phase shift; see, for example, Gouriéroux, and Monfort (1997).

More generally, let  $X_t = R \sin(\omega t + \varphi)$  be a series of frequency  $\omega$  (or period  $\frac{2\pi}{\omega}$ ), of amplitude  $R$  and of phase  $\varphi$ . The transform of  $\{X_t\}$  by any moving average will also be a modified amplitude sine curve, presenting a phase shift in relation to the original series:

$$M(X_t) = M[\sin(\omega t + \varphi)] = G(\omega) \sin[\omega t + \varphi + \Gamma(\omega)]$$

- The function that associates  $|G(\omega)|$  with  $\omega$  is called the **gain function** of the moving average.
- The function that associates  $\Gamma(\omega)$  with  $\omega$  is called the **phase shift function** of the moving average. It is sometimes represented as  $\Gamma(\omega)/\omega$  which allows the phase shift to be measured in number of periods.

**Figure 12.3:** Smoothing of the series  $X_t = \sin \left[ \frac{\pi}{3}t \right]$  by the moving average  $\frac{1}{3}[X_{t-2} + X_{t-1} + X_t]$



In the case of the asymmetric 3-term moving average below, we have:

$$\begin{aligned}
 M(X_t) &= \frac{1}{3}(X_{t-2} + X_{t-1} + X_t) = \frac{1}{3}R[\sin(\omega t - 2\omega + \varphi) + \sin(\omega t - \omega + \varphi) + \sin(\omega t + \varphi)] \\
 &= \frac{1}{3}R(1 + 2 \cos(\omega)) \sin(\omega t + \varphi - \omega)
 \end{aligned}$$

and so :

- $G(\omega) = \frac{1+2 \cos(\omega)}{3}$
- $\Gamma(\omega) = -\omega$  or even  $\Gamma(\omega)/\omega = -1$

The gain function, represented in Figure 12.4, shows that the moving average cancels out the frequencies  $2\pi/3$ . It would be well suited to surveys done every 4 months (therefore, period 3) as it would thus eliminate seasonality while retaining the basic changes corresponding to low frequencies. In contrast, this average introduces a systematic phase shift of a period that would result in possible trend turning points being noticed too late.

The phase shift introduced by the moving average  $\frac{1}{3}[X_{t-2} + X_{t-1} + X_t]$  can also be seen by applying this asymmetric to a simple straight line  $X_t = at + b$ . We have :

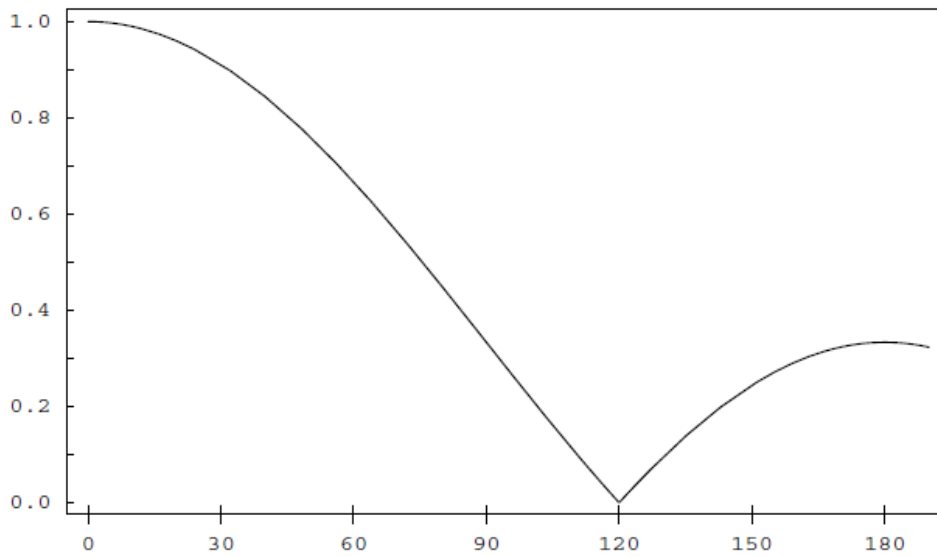
$$M(X_t) = \frac{1}{3}(X_{t-2} + X_{t-1} + X_t) = \frac{1}{3}[a(t-2) + b + a(t-1) + b + at + b] = a(t-1) + b = X_{t-1}$$

The gain function therefore shows frequencies eliminated or preserved by the moving average. The phase shift function shows the lags introduced by the use of asymmetric moving averages.

For smoothing, an “ideal” filter would be one that would leave low frequencies unchanged, such as, for example, the periodic functions of a period extending beyond the year (trend and cycle), but would eliminate all high frequencies corresponding to periodicities less than or equal to the year (seasonality and irregular). The “ideal” gain function of this filter, known as the “low-pass” filter, would therefore appear as follows:

$$G(\omega) = \begin{cases} 1 & \text{for } \omega \leq \omega_0 \\ 0 & \text{for } \omega > \omega_0 \end{cases}$$

**Figure 12.4:** Gain function of the moving average  $\frac{1}{3}[X_{t-2} + X_{t-1} + X_t]$



## 12.3 Desirable properties of a moving average

From now on, let  $(X_t)_{t \in \mathbb{Z}}$  denote a time series that can be usually considered as the sum of three components - the trend-cycle  $TC_t$ , the seasonality  $S_t$  and the irregular component  $I_t$  - with  $X_t = TC_t + S_t + I_t$ . Moving averages are the basic tool of the X12-ARIMA seasonal adjustment method and are used to estimate these three main components.

If we want, for example, to estimate the trend-cycle  $TC_t$  using a moving average  $M_\theta$ ,  $M_\theta$  should preserve the trend-cycle, remove the seasonality and reduce the irregular component as soon as possible.

### 12.3.1 Trend preservation

It is therefore desirable for a moving average to respect certain simple trends, particularly polynomials.

- For any moving average to respect the constant series  $X_t = a$ , it is necessary that :

$$M(X_t) = \sum_{k=-p}^{+f} \theta_k X_{t+k} = \sum_{k=-p}^{+f} \theta_k a = a \sum_{k=-p}^{+f} \theta_k = a$$

and therefore that the sum of the coefficients of the moving average  $\sum_{k=-p}^{+f} \theta_k$  be equal to 1.

- For any moving average to retain the straight lines, it is necessary, for any  $t$ , that:

$$M(X_t) = \sum_{k=-p}^{+f} \theta_k X_{t+k} = \sum_{k=-p}^{+f} \theta_k [a(t+k) + b] = at \sum_{k=-p}^{+f} k\theta_k + b \sum_{k=-p}^{+f} \theta_k = at + b$$

which result in

$$\sum_{k=-p}^{+f} \theta_k = 1 \text{ and } \sum_{k=-p}^{+f} k\theta_k = 0$$

- Generally, it would also be shown that for a moving average to retain a polynomial of degree  $d$ , it is

necessary and sufficient that its coefficients establish:

$$\sum_{k=-p}^{+f} \theta_k = 1 \quad \text{and} \quad \sum_{k=-p}^{+f} k^j \theta_k = 0 \quad \text{for } j = 1, \dots, d$$

The preservation of a polynomial trend of degree  $d$  introduces constraints on the moving average coefficients that can be expressed in matrix form:  $C\Theta = \alpha$ , where  $\Theta$  is the column vector of the  $(p + f + 1)$  coefficients of moving average  $M$ , and where  $C$  and  $\alpha$  are the matrix of dimensions  $(d + 1, p + f + 1)$  and  $(d + 1, 1)$  equal to:

$$C = \begin{bmatrix} 1 & 1 & \dots & \dots & 1 & 1 \\ -p & -p+1 & \dots & \dots & f-1 & f \\ (-p)^2 & (-p+1)^2 & \dots & \dots & (f-1)^2 & f^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ (-p)^{d-1} & (-p+1)^{d-1} & \dots & \dots & (f-1)^{d-1} & f^{d-1} \\ (-p)^d & (-p+1)^d & \dots & \dots & (f-1)^d & f^d \end{bmatrix} \quad \text{and} \quad \alpha = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

**Examples:**

Thus, for the asymmetric 3-term moving average defined above, we have:  $\sum_{k=-2}^0 \theta_k = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$  and  $\sum_{k=-2}^0 k\theta_k = -2 \times \frac{1}{3} - 1 \times \frac{1}{3} + 0 \times \frac{1}{3} = -1$ ; and this average, while it retains the constants, does not retain the straight lines.

It is easy, however, to establish that the following symmetric moving averages retain the straight lines:

$$M(X_t) = \frac{1}{3}(X_{t-2} + X_{t-1} + X_t) \quad \text{and} \quad M(X_t) = \frac{1}{8}(X_{t-2} + 2X_{t-1} + 2X_t + 2X_{t+1} + X_{t+2}).$$

### 12.3.2 Elimination of seasonality

As we saw when defining the gain function (paragraph 12.2.2), moving averages can eliminate certain frequencies and therefore certain seasonal components. Fixed seasonality can be “modeled” by periodic functions of period  $k$  (4 for a quarterly series, 12 for a monthly series, etc.). If it is also assumed that the sum of the seasonal coefficients is nil over one year, it is shown<sup>7</sup> that these functions then create a vector subspace of dimension  $(k - 1)$  for which a base can easily be presented. For example, in the quarterly case, we find the subspace created by the row vectors of the matrix:

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 1 & -1 & 0 & 0 & \dots \\ 1 & 0 & -1 & 0 & 1 & 0 & -1 & 0 & \dots \\ 1 & 0 & 0 & -1 & 1 & 0 & 0 & -1 & \dots \end{bmatrix}.$$

The cancellation of such series therefore introduces constraints on the moving average coefficients that are expressed in matrix form:  $C\Theta = \alpha$ , where  $\Theta$  is the column vector of the coefficients of moving average  $M$ ,  $C$  is the matrix of dimensions  $(k - 1, p + f + 1)$  whose lines are, for  $k = 4$ , the basic vectors above, and  $\alpha$  is the null matrix of dimensions  $(k - 1, 1)$ .

For example, if we consider a 4-term moving average, we should have:

$$C\Theta = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{which gives in this case} \quad \begin{cases} \theta_1 = \theta_2 \\ \theta_1 = \theta_3 \\ \theta_1 = \theta_4 \end{cases} \quad \text{and thus } \theta_1 = \theta_2 = \theta_3 = \theta_4$$

<sup>7</sup>see for example Gouriéroux, and Monfort (1997)



and this moving average eliminates the seasonalities of period 4. Consequently, its gain function will cancel out at frequency  $\pi/2$  ( $= 2\pi/4$ ).

Generally, a simple moving average of order- $k$  (and so of coefficients all equal to  $1/k$ ) cancels out the fixed seasonalities of period  $k$  and its gain function therefore cancels out in  $2\pi/k$ . It is also possible to design a moving average that cancels a seasonality which can evolve polynomially with time. Let us note  $S_t = (a_0 + a_1t_1 + a_2t_2 + \dots + a_d t_d)u_t$  where  $u_t$  is a periodical function with period  $\ell$ . And let us define  $n$  the integer satisfying  $p + f + 1 = n\ell$  where  $p + f + 1$  is the order of the moving average. It can be shown<sup>8</sup> that for a moving average to cancel a seasonality evolving like a polynomial of degree  $d$ , it is necessary and sufficient that its coefficients satisfy:

$$\begin{cases} \sum_{j=0}^{n-1} \theta_{k-j\ell} - \sum_{j=0}^{n-1} \theta_{f-j\ell} & = 0 \\ \sum_{j=0}^{n-1} (k-j\ell)\theta_{k-j\ell} - \sum_{j=0}^{n-1} (f-j\ell)\theta_{f-j\ell} & = 0 \\ \dots & \\ \sum_{j=0}^{n-1} (k-j\ell)^d \theta_{k-j\ell} - \sum_{j=0}^{n-1} (f-j\ell)^d \theta_{f-j\ell} & = 0 \end{cases}$$

### 12.3.3 Reduction of the irregular component

After trend and seasonality, it remains for us to see the effect of a moving average on the irregular component. The residual, in the decomposition of the unadjusted series, is often modeled in the form of a white noise, a sequence of random variables  $\epsilon_t$  with  $E(\epsilon_t) = 0$ ,  $\text{Var}(\epsilon_t) = \sigma^2$  and  $\text{Cov}(\epsilon_t, \epsilon_{t+h}) = 0$  for all  $h \neq 0$ . This white noise is transformed by a moving average  $M$  of order  $p + f + 1$  and coefficients  $\theta_i$  into a sequence of random variables,  $\epsilon_t^*$ , with the following properties:

$$\begin{aligned} \epsilon_t^* &= \sum_{k=-p}^{+f} \theta_k \epsilon_{t+k} \\ E(\epsilon_t^*) &= E[\sum_{k=-p}^{+f} \theta_k \epsilon_{t+k}] = \sum_{k=-p}^{+f} \theta_k E(\epsilon_{t+k}) = 0 \\ \text{Var}(\epsilon_t^*) &= \text{Var}[\sum_{k=-p}^{+f} \theta_k \epsilon_{t+k}] = \sum_{k=-p}^{+f} \theta_k^2 \text{Var}(\epsilon_{t+k}) = \sigma^2 \sum_{k=-p}^{+f} \theta_k^2, \end{aligned}$$

and

$$\text{Cov}(\epsilon_t^*, \epsilon_{t+h}^*) = \sum_{i=-p}^{+f} \sum_{j=-p}^{+f} \text{Cov}(\epsilon_{t+i}, \epsilon_{t+h+j}) = \sum_{i=-p}^{+f} \sum_{j=-p}^{+f} E(\epsilon_{t+i} \epsilon_{t+h+j}),$$

which gives:

$$\text{Cov}(\epsilon_t^*, \epsilon_{t+h}^*) = \begin{cases} \sigma^2 \sum_{j=-p}^{j=f-h} \theta_j \theta_{j+h} & \text{for } h \leq p + f \\ 0 & \text{otherwise} \end{cases}$$

Reducing the irregular component, and therefore its variance, amounts to reducing the quantity:  $\sum_{k=-p}^{+f} \theta_k^2$ .

### 12.3.4 The Slutsky-Yule effect

The existence of non-zero correlations in the process  $\epsilon_t^*$  introduces a spurious effect called “the Slutsky-Yule effect.”<sup>9</sup> The series  $\epsilon_t^*$  will present more or less regular oscillations which may suggest the presence of a periodic or quasi-periodic component. Slutsky showed that, if  $\epsilon_t$  is a Gaussian noise, the average length  $\tau$  of these spurious cycles is equal to:

<sup>8</sup>See Grun-Rehomme and Ladiray (1994) for a complete proof.

<sup>9</sup>In a very simple statistical experiment Slutsky (1927) - Yule (1926) developed a similar viewpoint - showed that random numbers subjected to basic statistical calculations like averages might form wavelike patterns looking like business cycles. The implication was that a similar stochastic process, “the summation of random causes”, might be at work in the actual economy, explaining the quite regular rises and falls of the activity. Without a doubt Slutsky’s 1927 paper made an enormous contribution to business cycle theory that forever changed the way economists view economic fluctuations.

$\tau = 2\pi / \arccos[\rho(1)]$ , where  $\rho(1) = \sum_{j=-p}^{j=f-1} \theta_j \theta_{j+1} / \sum_{j=-p}^{j=f} \theta_j^2$ .

It is not possible to avoid this Slutsky-Yule effect but one can note that these spurious oscillations become negligible when there is a significant reduction in the irregular amplitude so, when the ratio  $\sigma^{*2} / \sigma^2 = \sum_{k=-p}^{+f} \theta_k^2$  is small.

### 12.3.5 An optimization problem

We introduce here a formalism that will be used in the chapter to generate moving averages having desirable properties in terms of trend preservation, noise cancellation etc. Let us consider:

- $\theta$  the vector of dimension  $(p + f + 1, 1)$  whose elements are the unknown coefficients of the moving average  $M$ ;
- $\Omega$  a known, symmetric, definite and positive matrix of dimensions  $(p + f + 1, p + f + 1)$ ;
- $C$  a known matrix of full rank and of dimensions  $(k, p + f + 1)$ ;
- $w, \alpha$  two known vectors of dimensions  $(p + f + 1, 1)$  and  $(k, 1)$  respectively.

We will see that most moving averages can be seen as solutions of the following optimization problem:

$$\begin{cases} \min_{\theta} & (\theta - w)' \Omega (\theta - w) \\ \text{subject to} & C\theta = \alpha \end{cases} \quad (12.1)$$

Providing that  $k < (p + f + 1)$ , the unique solution to this well-known problem is:

$$\theta = \Omega^{-1} C' (C \Omega^{-1} C')^{-1} (\alpha - Cw) + w.$$

In the case  $k = (p + f + 1)$ ,  $C$  is invertible and the solution is directly given by  $\theta = C^{-1} \alpha$ .

## 12.4 Usual moving averages

### 12.4.1 Simple moving averages

Moving averages are used to smooth a series and it is natural to first look for a moving average reducing the irregular (i.e. minimizing  $\sum_{k=-p}^{+f} \theta_k^2$ ) when preserving constants (i.e. such that  $\sum_{k=-p}^{+f} \theta_k = 1$ ). Using the formalism introduced in equation 12.1, we have:

- $\sum_{k=-p}^{+f} \theta_k^2 = \theta' \theta = (\theta - 0)' I (\theta - 0)$ , where  $I$  is the identity matrix. Which implies that  $w = 0$  and  $\Omega = I$ ;
- $C = [111 \cdots 111]$  and  $\alpha = 1$ .

The solution is then  $\theta = C' / (p + f + 1)$ , i.e. the simple moving average of coefficients  $1 / (p + f + 1)$ .

As we saw in section 12.3.2, we could also look for a moving average of order  $k = (p + f + 1)$  that cancels

fixed seasonality of period  $k$  when preserving constants. In this case, we have:

$$C = \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 & 1 \\ 1 & -1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & -1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & 0 & \cdots & -1 & 0 \\ 1 & 0 & 0 & 0 & \cdots & 0 & -1 \end{bmatrix} \quad \text{and } \alpha = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

In this case,  $C$  is of full rank and the solution is directly given by the system of constraints on the coefficients which gives:  $\theta_1 = \theta_2, \theta_1 = \theta_3$ , etc. and  $\sum_{k=-p}^{+f} \theta_k = 1$ . And the solution is again the simple moving average of order  $k$ .

### 12.4.2 Composite simple moving averages

A so-called  $P \times Q$  moving average is obtained by composing a simple moving average of order  $P$ , with coefficients all equal to  $1/P$ , and a simple moving average of order  $Q$ , with coefficients all equal to  $1/Q$ . In concrete terms, this amounts to applying both simple moving averages to our series in succession.

These moving averages can easily be generated using our optimization problem when looking for a moving average of order  $(P + Q - 1)$  that cancels fixed seasonality of period  $P$  and  $Q$  when preserving constants. As shown in Grun-Rehomme and Ladiray (1994), the case of seasonalities moving polynomially with time can be easily handled in this context. For example, the composite simple moving average [4][4] of order 7 ( $= 4 + 4 - 1$ ) will cancel any seasonality of period 4 varying linearly with time. It is easy to verify that the coefficients of the moving average are:  $\frac{1}{16}(1, 2, 3, 4, 3, 2, 1)$ . Therefore, this moving average also preserves the straight lines.

### 12.4.3 Spencer moving averages

Spencer (1904) proposed moving averages having two very important qualities: (1) they should be easy to compute and (2) they should yield to a sufficient degree of smoothness in the resulted series. To achieve the first objective, he proposed to use composite simple averages, like for example a [4][4][5] whose smoothing properties did not seem to be sufficient.

Spencer looked then for a symmetric moving average of order 5 and coefficients  $(a, b, c, b, a)$  that, if applied after the [4][4][5], would result in a global moving average preserving local polynomial of order 3. In this case - see Grun-Rehomme and Ladiray (1994) for a similar demonstration -  $a, b$  and  $c$  should verify:

$$\begin{cases} 2a + 2b + c & = 1 \\ 34a + 22b + 9c & = 0 \end{cases}$$

So, the resulting moving average should have the following system of coefficients:  $\frac{1}{12}[2c-11, 17-8c, 12c, 17-8c, 2c-11]$ . Spencer's moving average of order 15 corresponds to  $c = 1$  and has therefore the following coefficients:

$$\frac{1}{320}[-3, -6, -5, +3, +21, +46, +67, +74, +67, +46, +21, +3, -5, -6, -3].$$

### 12.4.4 Kendall and Stuart moving averages

It would also be natural to look for moving averages preserving polynomials of degree  $d$ . Kendall and Stuart (1968) solved the problem using a “moving regression” technique. Let us suppose a series  $X_t$  with  $t = 1, 2, \dots, T$  and a given odd number  $2p + 1$ .

- On the first  $2p + 1$  points of the series, the time interval  $[1, 2p + 1]$ , a polynomial of degree  $d$  is adjusted by ordinary least squares;
- The adjusted value obtained for the center of the window, corresponding to date  $(p + 1)$ , will be the value of the smoothed curve for the corresponding data;
- Then, the window slides from one point and the regression is done on the interval  $[2, 2p + 2]$  which gives the second point of the smoothed series;
- The process is repeated until the end, which gives an estimate of the smoothed curve on the interval  $[p + 1, T - p]$ .

The authors show that this method amounts to applying to the initial series a moving average of order  $2p + 1$  whose coefficients depend only on the degree  $d$  of the selected polynomial and on the order  $2p + 1$ .

Grun-Rehomme and Ladiray (1994) show that this approach can be put in the framework of equation 12.1 where:

- The criterion to be minimized is  $\sum_{k=-p}^{+p} \theta_k^2$  which means that  $\Omega = I$ , and  $w = 0$ ;
- Matrices  $C$  and  $\alpha$  correspond to the preservation of a polynomial trend of degree  $d$ , see section 12.3.1:

$$C = \begin{bmatrix} 1 & 1 & \dots & \dots & 1 & 1 \\ -p & -p + 1 & \dots & \dots & p - 1 & p \\ (-p)^2 & (-p + 1)^2 & \dots & \dots & (p - 1)^2 & p^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ (-p)^{d-1} & (-p + 1)^{d-1} & \dots & \dots & (p - 1)^{d-1} & p^{d-1} \\ (-p)^d & (-p + 1)^d & \dots & \dots & (p - 1)^d & p^d \end{bmatrix} \text{ and } \alpha = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

Moreover, the ordinary least square approach of Kendall and Stuart shows that the “noise reduction” criterion can also be interpreted as a “Fidelity” criterion as in the Whittaker-Henderson framework.

Of course it would be possible, as proposed by Bongard (1962) to ask the moving average also to cancel out some seasonalities.

### 12.4.5 Henderson moving averages

Like Spencer, Henderson was preoccupied by the smoothing properties of moving averages. What criterion can be used to ensure a good smoothing of a series?

Let us consider the series

$$X_t = \begin{cases} 1 & \text{if } t = 0 \\ 0 & \text{if } t \neq 0 \end{cases}$$

Its transform by a centered moving average  $M$  of order  $2p + 1$  and with coefficients  $\{\theta_i\}$ , is rendered by:

$$MX_t = \begin{cases} 0 & \text{if } t < -p \\ \theta_t & \text{if } -p \leq t \leq p \\ 0 & \text{if } t > p \end{cases}$$

This transform will therefore be smooth if the coefficients curve of the moving average is smooth.

Henderson (1916) proposed using the quantity  $H = \sum (\nabla^3 \theta_i)^2$ , where  $\nabla$  represents the first difference operator<sup>10</sup>, to measure the “flexibility” of the coefficient curve. This quantity vanishes when the coefficients  $\{\theta_i\}$  are located along a parabola and, in the general case, it measures the difference between a parabola and the form of the coefficient curve. Henderson then looked for centered averages of order  $2p + 1$  that preserve quadratic polynomials and minimize quantity  $H$ .

The order  $2p+1$  Henderson moving average will therefore be the solution of the minimization program:

$$\begin{cases} \min \theta \sum (\nabla^3 \theta_i)^2 \\ \sum_{i=-p}^{+p} \theta_i = 1, \sum_{i=-p}^{+p} i \theta_i = 0, \text{ and } \sum_{i=-p}^{+p} i^2 \theta_i = 0 \end{cases}$$

This program is a special case of equation 12.1 with:

$$\bullet \Omega = \begin{bmatrix} 20 & -15 & 6 & -1 & 0 & \dots & 0 & 0 & 0 & 0 \\ -15 & 20 & -15 & 6 & -1 & \dots & 0 & 0 & 0 & 0 \\ 6 & -15 & 20 & -15 & 6 & \dots & \dots & \dots & \dots & \dots \\ -1 & 6 & -15 & 20 & -15 & \dots & \dots & \dots & \dots & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & 20 & -15 & 6 & -1 \\ 0 & 0 & 0 & 0 & 0 & \dots & -15 & 20 & -15 & 6 \\ 0 & 0 & 0 & 0 & 0 & \dots & 6 & -15 & 20 & -15 \\ 0 & 0 & 0 & 0 & 0 & \dots & -1 & 6 & -15 & 20 \end{bmatrix};$$

•  $w = 0$ ;

• Matrices  $C$  and  $\alpha$  correspond to the preservation of a polynomial trend of degree 3:

$$C = \begin{bmatrix} 1 & 1 & \dots & \dots & 1 & 1 \\ -p & -p+1 & \dots & \dots & p-1 & p \\ (-p)^2 & (-p+1)^2 & \dots & \dots & (p-1)^2 & p^2 \end{bmatrix} \text{ and } \alpha = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

The coefficients of these moving averages may also be calculated explicitly and, for an order  $2p + 1$  average, by positing  $n = p + 2$ , we have:

$$\theta_i = \frac{315[(n-1)^2 - i^2][n^2 - i^2][(n+1)^2 - i^2][3n^3 - 16 - 11i^2]}{8n(n^2 - 1)(4n^2 - 1)(4n^2 - 9)(4n^2 - 25)}$$

Using this formula, it is therefore possible to calculate, in rational form, the coefficients of the Henderson moving averages used in X-11. Therefore, for the sake of symmetry, presenting only the necessary coefficients,

<sup>10</sup> $\nabla X_t = X_t - X_{t-1}$

**Table 12.1:** Coefficients of the Henderson moving averages used in X-11, their variance reducing power,  $\sum \theta_i^2$ , and Henderson criterion,  $\sum (\nabla^3 \theta_i)^2$ .

i	5-term	7-term	9-term	13-term	23-term
-11	.	.	.	.	-0.00428
-10	.	.	.	.	-0.01092
-9	.	.	.	.	-0.01569
-8	.	.	.	.	-0.01453
-7	.	.	.	.	-0.00495
-6	.	.	.	-0.01935	0.01343
-5	.	.	.	-0.02786	0.03893
-4	.	.	-0.04072	0.00000	0.06830
-3	.	-0.05874	-0.00987	0.06549	0.09740
-2	-0.07343	0.05874	0.11847	0.14736	0.12195
-1	0.29371	0.29371	0.26656	0.21434	0.13832
0	0.55944	0.41259	0.33114	0.24006	0.14406
1	0.29371	0.29371	0.26656	0.21434	0.13832
2	-0.07343	0.05874	0.11847	0.14736	0.12195
3	.	-0.05874	-0.00987	0.06549	0.09740
4	.	.	-0.04072	0.00000	0.06830
5	.	.	.	-0.02786	0.03893
6	.	.	.	-0.01935	0.01343
7	.	.	.	.	-0.00495
8	.	.	.	.	-0.01453
9	.	.	.	.	-0.01569
10	.	.	.	.	-0.01092
11	.	.	.	.	-0.00428
$\sum \theta_i^2$	0.4963	0.3566	0.2833	0.2038	0.1217
$\sum (\nabla^3 \theta_i)^2$	1.4965	0.2629	0.0675	0.0083	0.0003

we have:

$$\begin{aligned}
 \text{5-term:} & \quad [5]; \frac{1}{286} \{-21, 84, 160\}, \\
 \text{7-term:} & \quad [7]; \frac{1}{715} \{-42, 42, 210, 295\}, \\
 \text{9-term:} & \quad [9]; \frac{1}{2431} \{-99, -24, 288, 648, 805\}, \\
 \text{13-term:} & \quad [13]; \frac{1}{16796} \{-325, -468, 0, 1100, 2475, 3600, 4032\}, \\
 \text{23-term:} & \quad [23]; \frac{1}{4032015} \{-17250, -44022, -63250, -58575, -19950, \\
 & \quad 54150, 156978, 275400, 392700, 491700, 557700, 580853\}.
 \end{aligned}$$

Table 12.1 provides, in decimal form, the coefficients of the Henderson moving averages used in X-11 as well as the related criteria, variance reduction and smoothness, values.

### 12.4.6 Musgrave asymmetric moving averages

By applying a centered moving average of order  $2p + 1$ , it is not possible to have, by construction, estimates of the smoothed series for the first and last instants  $p$  of the series, which is, at the very least, bothersome. One is therefore prompted in practice to use non-centered moving averages to perform these estimations.

Musgrave (1964a) studied this problem in the context of the X-11 method and proposed a set of asymmetric averages that supplement the Henderson moving averages.

Let us suppose that the series to be smoothed out ends in July 1999. If we use an 13-term symmetric moving average, the last point smoothed out using this average will be that of January 1999, it being necessary to estimate the last six months using asymmetric moving averages. Six months later, so in January 1999, it will be possible to calculate the smoothed value for July 1999 using the 13-term symmetric moving average. We will then have two different estimates of this value, and it would be desirable that they not be too different.

Musgrave's idea is precisely to construct asymmetric moving averages that minimize the revisions of estimates. To this end, he formulates the following assumptions:

- The series to be smoothed out can be modeled linearly in the form:  $X_t = a + bt + \varepsilon_t$  where  $a$  and  $b$  are constants, and  $\varepsilon_t$  are non-correlated random variables, of nil average and of variance  $\sigma^2$ .
- We have a series of weights  $\{w_1, w_2, \dots, w_N\}$  whose sum is equal to 1 (for example, a centered Henderson moving average) and are looking for a series of weights  $\{v_1, v_2, \dots, v_M\}$ , with  $M < N$ , whose sum is also equal to 1.
- This new moving average must also minimize the revisions of estimates; that is, for example, it must minimize the criterion:  $E(\sum_{i=1}^M v_i X_i - \sum_{i=1}^N w_i X_i)^2$ .

Under these assumptions, Doherty (2001) shows that the weights may be calculated explicitly as a function of the ratio  $D = \frac{b^2}{\sigma^2}$ :

$$v_j = w_j + \frac{1}{M} \sum_{i=M+1}^N w_i + \frac{[j - \frac{M+1}{2}] D}{1 + \frac{M(M-1)(M+1)}{12} D} \sum_{i=M+1}^N \left( i - \frac{M+1}{2} \right) w_i$$

Of course at this point the  $D$ -ratio value is unknown but we will see in Section 12.6.5 how it is specified in the X11 seasonal adjustment context. Gray and Thomson (2002) give also some ideas for an optimal estimation of the ratio.

Musgrave's approach can be generalized and put in our optimization approach. Let us suppose that the series  $X_t$  can be modeled on the last points as a polynomial of degree  $d$ . The model can be written:

$$X_i = b_0 + b_1 i + b_2 i^2 + \dots + b_d i^d + \epsilon_i \text{ for } i \in -p, -p+1, \dots, f-1, f.$$

Or in matrix form:  $X = A\beta + \Sigma$ , with:

$$C = \begin{bmatrix} 1 & -p & (-p)^2 & \dots & \dots & (-p)^d \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & f & f^2 & \dots & \dots & f^d \end{bmatrix}, \beta = \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_d \end{bmatrix} \text{ and } \Sigma = \begin{bmatrix} \epsilon_{-p} \\ \epsilon_{-p+1} \\ \vdots \\ \epsilon_f \end{bmatrix}$$

Let us also note  $w$  the vector of coefficients of the moving average used to smooth the central part of the series and  $\theta$  the vector of coefficients of the unknown asymmetric moving average expanded with 0 to be of dimensions  $(p + f + 1, 1)$ . Musgrave's criterion can then be written:

$$E(\sum_{i=-p}^{i=f} \theta_i X_i - \sum_{i=-p}^{i=f} w_i X_i)^2 = E[(\theta - w)' X X' (\theta - w)] = (\theta - w)' \Omega (\theta - w),$$

with  $\Omega = E(X X') = E[(A\beta + \Sigma)'(A\beta + \Sigma)] = (A\beta)'(A\beta) + \sigma^2 I$ , as  $E(\Sigma) = 0$  and  $\text{Var}(\Sigma) = \sigma^2 I$ .

$\Omega$  is a definite positive matrix of order  $(p + f + 1, p + f + 1)$  and for the  $C$  matrix of constraints, we have to include the fact that  $\sum_{i=-p}^{i=f} \theta_i$  is equal to 1 and that some  $\theta_i$  are equal to 0. The full rank matrix  $C$  is therefore equal to:



$$C = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 1 \end{bmatrix}$$

### 12.4.7 Generalization of the Whittaker-Henderson approach

As it has been demonstrated in the previous sections, most of the currently used moving averages can be considered from the optimization program 12.1 point of view. This framework allows also generating asymmetric moving averages for the ends of the series.

Starting from this optimization program, Grun-Rehomme and Ladiray (1994) and Gray and Thomson (1996) proposed a generalization of the Whittaker-Henderson approach based on a "Fidelity" plus "Smoothness" criterion. The basic idea is to consider a convex combination of the 2 criteria instead of a simple summation.

Symmetric and associated asymmetric moving averages are in this framework solutions of the same minimization program:

$$\begin{cases} \min_{\theta} & (\theta - w)' \frac{aF+bS}{a+b} (\theta - w) \\ \text{subject to} & C\theta = \alpha \end{cases} \quad (12.2)$$

Where  $F = \sum_{k=-p}^{+f} \theta_k^2$  is the noise reduction criterion, and  $S = \sum (\nabla^3 \theta_i)^2$  is the Henderson criterion.

## 12.5 The X11 seasonal adjustment algorithm

Most of the previous moving averages have been used in the seasonal adjustment context. The X-11 method for example is based on an iterative principle of estimation of the different components, this estimation being done at each step using appropriate moving averages.

### 12.5.1 Components and decomposition models

The X-11 method allows for the decomposition and seasonal adjustment of monthly and quarterly series. The components that may appear at one time or another of the decomposition are:

- The series trend, representing the long-term evolution of the series;
- The cycle, the smooth, almost periodic movement around the trend, revealing a succession of phases of growth and recession.

X-11 does not separate these two components: the series studied are generally too short for both components to be easily estimated. Consequently, hereafter we will speak of the trend-cycle component, written as  $TC_t$ .

- The seasonal component, written as  $S_t$ , representing intra-year fluctuations, monthly or quarterly, that are repeated more or less regularly year after year;
- A so-called "trading-days" component, written as  $D_t$ , that measures the impact on the series of the everyday composition of the month or quarter;
- A component measuring the effect of the Easter holiday, written as  $E_t$ ;

- And finally, the irregular component, written as  $I_t$ , combining all the other more or less erratic fluctuations not covered by the previous components.

We would point out that these definitions are qualitative and rather imprecise. They are still today the subject of controversy and various interpretations. For example, here are two quotations of eminent statisticians who apparently do not have the same objective:

- Kendall (1973): “*The essential idea of trend is that it shall be smooth.*”
- Harvey (1989): “*There is no fundamental reason, though, why a trend should be smooth.*”

In the X-11 method, the components are in fact defined implicitly by the tools used to estimate them.

The X-11 method considers three decomposition models to be possible:

- The additive model:  $X_t = TC_t + S_t + D_t + E_t + I_t$
- The multiplicative model:  $X_t = TC_t \times S_t \times D_t \times E_t \times I_t$
- The log additive model:  $X_t = \log(TC_t) + \log(S_t) + \log(D_t) + \log(E_t) + \log(I_t)$

In addition, X-12-ARIMA proposes a "pseudo-additive" model:  $X_t = TC_t(S_t + D_t + E_t + I_t - 1)$

### 12.5.2 A simple seasonal adjustment algorithm

Let a monthly unadjusted series be  $X_t$  that we will assume here decomposed into trend-cycle, seasonality and irregular portion according to an additive model:  $X_t = TC_t + S_t + I_t$ . A simple seasonal adjustment algorithm can be thought of in four steps:

#### 1. Estimation of the trend-cycle by moving average:

$$TC_t^{(1)} = M_0(X_t).$$

The moving average used here should therefore reproduce, at best, the trend-cycle component while eliminating the seasonal component and minimizing the irregular component.

#### 2. Estimation of the seasonal-irregular component:

$$(S_t + I_t)^{(1)} = X_t - TC_t^{(1)}.$$

#### 3. Estimation of the seasonal component by moving average over each month:

$$S_t^{(1)} = M_1 \left[ (S_t + I_t)^{(1)} \right]$$

and therefore also

$$I_t^{(1)} = (S_t + I_t)^{(1)} - S_t^{(1)}.$$

Here, it is a question of smoothing out the values of the seasonal-irregular component separately for each month to extract the evolution of the seasonal factor of the month concerned. The moving average used here must reproduce, as best as possible, the seasonal component of each month by minimizing the irregular component.

A normalizing constraint, for example that they sum to zero, may be imposed on the factors.

#### 4. Estimation of the seasonally adjusted series:

$$SA_t^{(1)} = (TC_t + I_t)^{(1)} = X_t - S_t^{(1)}.$$

The whole difficulty lies, then, in the choice of the moving averages used in steps 1 and 3.

### 12.5.3 The basic algorithm of the X-11 method

It is this simple algorithm that the X-11 method implements using judiciously chosen moving averages, and gradually refining, by iteration of the algorithm, the estimates of the components.

It is thus possible to define the basic algorithm of the X-11 method. It actually corresponds to using the previous simple algorithm twice, changing the moving averages each time.

**1. Estimation of the trend-cycle by a  $2 \times 12$  moving average:**

$$TC_t^{(1)} = M_{2 \times 12}(X_t).$$

The moving average used here is a so-called  $2 \times 12$  moving average, of coefficients

$$\frac{1}{24}\{1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 1\},$$

which preserves linear trends, eliminates order-12 constant seasonalities and minimizes the variance of the irregular component.

**2. Estimation of the seasonal-irregular component:**

$$(S_t + I_t)^{(1)} = X_t - TC_t^{(1)}.$$

**3. Estimation of the seasonal component by a  $3 \times 3$  moving average over each month:**

$$S_t^{(1)} = M_{3 \times 3} \left[ (S_t + I_t)^{(1)} \right].$$

The moving average used here is a so-called  $3 \times 3$  moving average over 5 terms, of coefficients  $\frac{1}{9}\{1, 2, 3, 2, 1\}$ , which preserves linear trends. The seasonal factors are then normalized so that their sum over each 12-month period is approximately zero.

$$\tilde{S}_t^{(1)} = S_t^{(1)} - M_{2 \times 12} \left( S_t^{(1)} \right).$$

**4. Estimation of the seasonally adjusted series:**

$$SA_t^{(1)} = (TC_t + I_t)^{(1)} = X_t - \tilde{S}_t^{(1)}.$$

This first estimate of the seasonally adjusted series must, by construction, contain less seasonality. The X-11 method again uses our simple algorithm, changing the moving averages to take this property into account.

**5. Estimation of the trend-cycle by a 13-term Henderson moving average:**

$$TC_t^{(2)} = H_{13} \left( SA_t^{(1)} \right).$$

Henderson moving averages, while they do not have special properties in terms of eliminating seasonality (limited or none at this stage), are very good smoothers and preserve a locally polynomial trend of degree 2<sup>11</sup>.

<sup>11</sup>Because the Henderson moving average is symmetric, it also preserves a locally polynomial trend of degree 3.

## 6. Estimation of the seasonal-irregular component:

$$(S_t + I_t)^{(2)} = X_t - TC_t^{(2)}.$$

7. Estimation of the seasonal component by a  $3 \times 5$  moving average over each month:

$$S_t^{(2)} = M_{3 \times 5} \left[ (S_t + I_t)^{(2)} \right].$$

The moving average used here is a so-called  $3 \times 5$  moving average over 7 terms, of coefficients  $\frac{1}{15} \{1, 2, 3, 3, 3, 2, 1\}$ , which preserves linear trends. The seasonal factors are then normalized so that their sum over each 12-month period is approximately zero.

$$\tilde{S}_t^{(2)} = S_t^{(2)} - M_{2 \times 12} \left( S_t^{(2)} \right).$$

## 8. Estimation of the seasonally adjusted series:

$$SA_t^{(2)} = (TC_t + I_t)^{(2)} = X_t - \tilde{S}_t^{(2)}.$$

This X-11 basic algorithm is further summarized in Table 12.2.

### 12.5.4 Extreme observations and calendar effects

As with any linear operator, moving averages respond dramatically to the presence of extreme observations. The X-11 method therefore incorporates a **tool for the detection and modification of extreme values** used to clean up the series prior to seasonal adjustment.

Also, effects other than seasonality may explain the variations observed in the series; the most common are effects related to the calendar: trading-day effect, Easter effect, and so on. These components are estimated using linear regression models, based on the irregular component<sup>12</sup>.

The X-11 basic algorithm, described in Table 12.2, produces 3 different estimates of the irregular component:

- At step 3 by subtracting the estimate of the seasonal component from the estimate of the seasonal-irregular component obtained in step 2:

$$I_t^{(1)} = (S_t + I_t)^{(1)} - \tilde{S}_t^{(1)}.$$

X-11 will use this estimate to detect and correct the extreme observations and obtain a better estimate of the seasonal component.

- At step 7, by subtracting the estimate of the seasonal component from the estimate of the seasonal-irregular component obtained in step 6:

$$I_t^{(2)} = (S_t + I_t)^{(2)} - \tilde{S}_t^{(2)}.$$

X-11 will use this estimate again to detect and correct the extreme observations and obtain a more reliable estimate of the seasonal component.

- At step 8, by subtracting from the estimate of the seasonally adjusted series the estimate of the trend-

<sup>12</sup>X-12-ARIMA has a "regARIMA" module by which it is possible to estimate directly these effects on the unadjusted series before performing the seasonal adjustment. This module will not be described here.

Table 12.2: X-11 basic algorithm.

Monthly unadjusted series:  $X_t = TC_t + S_t + I_t$

1. **Estimation of the trend-cycle by a  $2 \times 12$  moving average:**

$$TC_t^{(1)} = M_{2 \times 12}(X_t)$$

2. **Estimation of the seasonal-irregular component:**

$$(S_t + I_t)^{(1)} = X_t - TC_t^{(1)}$$

3. **Estimation of the seasonal component by  $3 \times 3$  moving average over each month:**

$$S_t^{(1)} = M_{3 \times 3} \left[ (S_t + I_t)^{(1)} \right]$$

and normalization

$$\tilde{S}_t^{(1)} = S_t^{(1)} - M_{2 \times 12} \left( S_t^{(1)} \right)$$

4. **Estimation of the seasonally adjusted series:**

$$SA_t^{(1)} = (TC_t + I_t)^{(1)} = X_t - \tilde{S}_t^{(1)}$$

5. **Estimation of the trend-cycle by a 13-term Henderson moving average:**

$$TC_t^{(2)} = H_{13} \left( SA_t^{(1)} \right)$$

6. **Estimation of the seasonal-irregular component:**

$$(S_t + I_t)^{(2)} = X_t - TC_t^{(2)}$$

7. **Estimation of the seasonal component by  $3 \times 5$  moving average over each month:**

$$S_t^{(2)} = M_{3 \times 5} \left[ (S_t + I_t)^{(2)} \right]$$

and normalization

$$\tilde{S}_t^{(2)} = S_t^{(2)} - M_{2 \times 12} \left( S_t^{(2)} \right)$$

8. **Estimation of the seasonally adjusted series:**

$$SA_t^{(2)} = (TC_t + I_t)^{(2)} = X_t - \tilde{S}_t^{(2)}$$

cycle component obtained in step 5:

$$I_t^{(3)} = SA_t^{(2)} - TC_t^{(2)}.$$

X-11 will use this estimate to evaluate, by linear regression, the trading-day component and to detect and correct extreme observations<sup>13</sup>.

### 12.5.5 The iterative principle of X-11

To evaluate the different components of the series, while taking into account the possible presence of extreme values, X-11 proceeds iteratively: estimation of components, search for disruptive effects in the irregular component, estimation of components from a corrected series, search for disruptive effects in the irregular component, and so on.

The Census X-11 program presents 4 processing stages (A, B, C, and D), plus 3 stages, E, F, and G, that present statistics and charts that are not part of the decomposition per se.

#### Part A: Pre-Adjustments

This part, which is optional, allows the user to correct *a priori* the series by introducing adjustment factors. The user can thus:

- introduce monthly (or quarterly) adjustments factors that will allow him to correct the effect of certain statutory holidays, change the series level (effect of a strike for example), etc.;
- for monthly series only, introduce 7 weights, one of each day of the week, to take into account the variations attributable to the trading-day composition of months.

Based on these data, the program calculates prior adjustment factors that are applied to the raw series. The series thus corrected, Table B1 of the printouts, then proceeds to Part B.

#### Part B: First Automatic Correction of the Series

This stage comprises a first estimation and downweighting of the extreme observations and, if requested, a first estimation of the trading-day effects. This stage is performed by applying the basic algorithm detailed in Section 12.5.3.

These operations lead to tables B19, evaluation of trading-day effects, and B20, adjustment values for extreme observations, that are used to correct the prior-adjusted series in Table B1, and result in the series shown in Table C1.

#### Part C: Second Automatic Correction of the Series

Still applying the basic algorithm, this part leads to a more precise estimation of trading-day effects (Table C19) and replacement values for the extreme observations (Table C20).

The series, finally “cleaned up,” is shown in Table D1 of the printouts.

<sup>13</sup>These various methods are presented in Ladiray and Quenneville (2001).

### Part D: Seasonal Adjustment

This part, at which the basic algorithm is applied for the last time, is that of the seasonal adjustment per se, as it leads to final estimates:

- of the seasonal component (Table D10),
- of the seasonally adjusted series (Table D11),
- of the trend-cycle component (Table D12),
- of the irregular component (Table D13).

### Parts E, F and G: Statistics and Charts

Parts E and F propose statistics for judging the quality of the seasonal adjustment.

Part G proposes graphics in character mode. It can be ignored, as today it can be replaced by the usual office-based graphics software.

A summary of the processing stages of the X-11 method is shown in Table 12.3.

**Table 12.3:** Simplified diagram of X-11 operation.

Part A:	<b>Prior adjustments</b> <ul style="list-style-type: none"> <li>• for significant known extremes</li> <li>• for trading-day</li> </ul>
Part B:	<b>First automatic correction of the series</b> Estimation of the irregular component Detection and automatic correction of extreme observations Correction of trading-day effects
Part C:	<b>Second automatic correction of the series</b> Estimation of the irregular component Detection and automatic correction of extreme observations Correction of trading-day effects
Part D:	<b>Seasonal adjustment</b> <ol style="list-style-type: none"> <li>1 Calculation of the temporary seasonally adjusted series (Tables D1 to D6)</li> <li>2 Smoothing out of the seasonally adjusted series by a Henderson moving average and new estimation of seasonal factors (Tables D7 to D10)</li> <li>3 Calculation of the final seasonally adjusted series (Table D11) and extraction of the trend-cycle (Table D12) and irregular component (Table D13)</li> </ol>
Part E:	<b>Components modified for large extreme values</b>
Part F:	<b>Seasonal adjustment quality measures</b>
Part G:	<b>Graphics</b>

### 12.5.6 From Census X-11 to X-11-ARIMA and X-12-ARIMA

The use of moving averages, as we will see in the next chapter, poses problems related to the series ends, notably with regard to the stability of the estimates. Thus, when you have an additional point and the series is



again seasonally adjusted using the Census X-11 software, it is not uncommon to note appreciable variations in the estimates for the most recent dates.

As early as 1975, Estella B. Dagum proposed largely resolving these problems using the ARIMA models that became popular some years earlier through the works of Box and Jenkins (1970). She thus showed that revisions were reduced appreciably by adapting an ARIMA model to the series, predicting the future values of the series using this model and applying the X-11 seasonal adjustment procedure to the series thus extended. This is the idea that underlies the X-11-ARIMA software<sup>14</sup>.

Unfortunately, the estimation of ARIMA models is made tricky by the presence of extreme observations, level changes, calendar effects, and so on. X-11-ARIMA is therefore based on the following structure:

### 1. First seasonal adjustment by the X-11 method

At this stage, it is possible to estimate the atypical values, the trading-days effects, as we have seen, but also the Easter effects using the estimate of the irregular component from Table D13.

### 2. ARIMA modelling of the series corrected for all these effects

### 3. Second seasonal adjustment by the X-11 method.

As described in Findley and al (1998), X-12-ARIMA proposes a very complete module, called Reg-ARIMA, that allows for the initial series to be corrected for all sorts of deterministic effects like outliers, ruptures and calendar effects. These effects are estimated using regression models with ARIMA errors and the corrected series is then decomposed using an enhanced X-11 algorithm. X-13ARIMA-SEATS follows the same principle illustrated in Figure 12.5.

## 12.6 The moving averages used in X-11

### 12.6.1 Composite simple moving averages

A so-called  **$P \times Q$  moving average** is obtained by composing a simple moving average of order  $P$ , whose coefficients are all equal to  $1/P$ , and a simple moving average of order  $Q$ , whose coefficients are all equal to  $1/Q$ . In concrete terms, this amounts to applying both simple moving averages in succession.

Thus, the  $3 \times 3$  moving average that results from the double application of the simple arithmetic 3-term moving average is a moving average of coefficients  $\{1, 2, 3, 2, 1\}/9$ . Generally, a  $P \times Q$  moving average is a symmetric moving average of order  $P + Q - 1$ .

When the order  $Q$  is even, for example equal to  $2q$ , there is a slight ambiguity in the definition, in that one can choose either  $q$  points in the past and  $q - 1$  points in the future, or  $q - 1$  points in the past and  $q$  points in the future. The problem is usually resolved by using a symmetric  $2 \times Q$  composite average that corresponds to the average of the two possible moving averages.

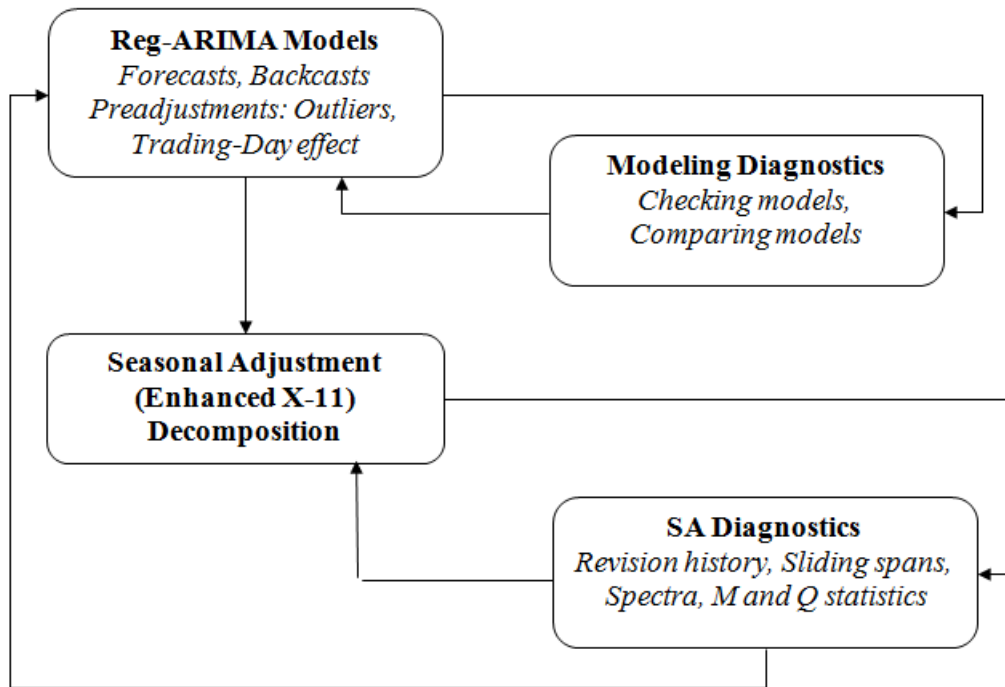
### 12.6.2 Estimation of trend: 2x4 and 2x12 averages

When X-11 performs an initial estimation of trend-cycle (Tables B2, C2 and D2), it uses a  $2 \times 4$  moving average in the quarterly case, and a  $2 \times 12$  moving average in the monthly case. At that point, the series to

<sup>14</sup>This idea was already implicitly expressed by Macaulay (1931):

*“However, graduation of the ends of almost any series is necessarily extremely hypothetical unless facts outside the range covered by the graduation are used in obtaining the graduation . . . . Though mathematically inelegant, the most desirable procedure in a majority of the cases of graduation is to graduate not only the actual data, but extrapolated data which sometimes may be extremely crude estimates.*

Figure 12.5: X-13ARIMA-SEATS principle.



be smoothed is composed of the trend-cycle, seasonal and irregular components. In the case of an additive decomposition model, it may be written:  $X_t = TC_t + S_t + I_t$ .

The  $2 \times 4$  average

This is a moving average of order 5, with coefficients  $\{1, 2, 2, 2, 1\}/8$ . The coefficients curve and the gain function, shown in Figure 12.6, reveal the properties of this moving average:

- It eliminates frequency  $90^\circ = 2\pi/4 = \pi/2$  corresponding to period 4 and is therefore well suited to quarterly series having a constant seasonality.
- The sum of its coefficients is equal to 1 and it is symmetric: it therefore preserves linear trends.
- The sum of the squares of its coefficients is equal to 0.250 and it therefore reduces the variance of white noise by 75%.

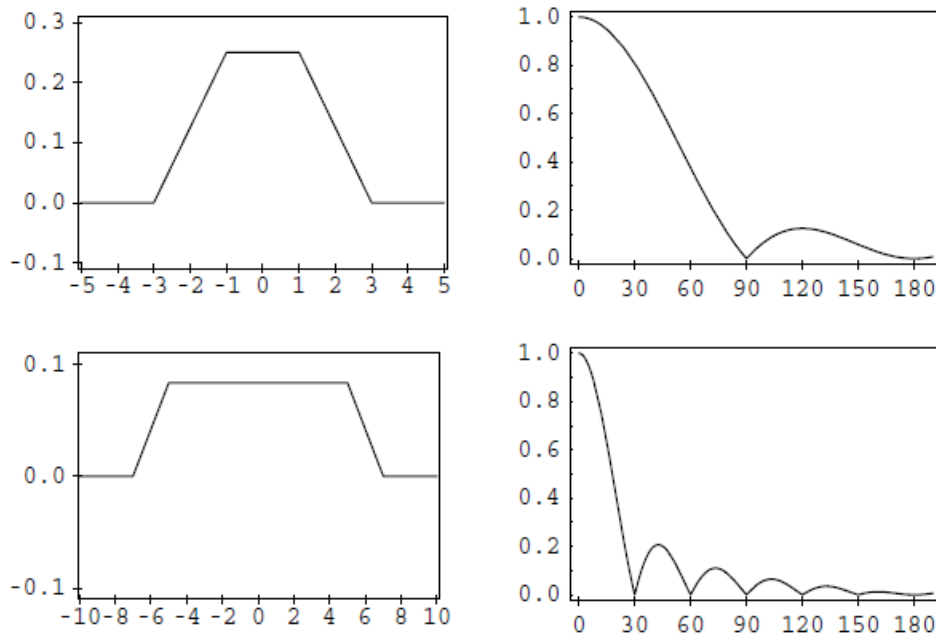
Using this moving average will work optimally when the trend-cycle of our series is linear, or locally linear, that the seasonal factors are constant or vary little over time, and that the irregular component has no structure and is of limited amplitude. In this case, we will have:

$$\begin{aligned}
 M_{2 \times 4}(X_t) &= M_{2 \times 4}(TC_t + S_t + I_t) \\
 &= M_{2 \times 4}(C_t) + M_{2 \times 4}(S_t) + M_{2 \times 4}(I_t) \\
 &\approx TC_t + 0 + \epsilon_t \\
 &\approx TC_t
 \end{aligned}$$

This moving average, however, restores rather poorly the low frequencies associated with periods greater than a year. Thus, 3-year periodic functions, which correspond, in the quarterly case, to frequencies of

$30^\circ = 2\pi/12 = \pi/6$ , are only about 80% restored.

**Figure 12.6:** Coefficient curves (on the left) and gain functions (on the right) of the composite moving averages used in X-11 trend-cycle estimation. The  $2 \times 4$  is displayed in the upper panel, and the  $2 \times 12$  in the lower panel.



### The $2 \times 12$ Average

This average is based on the same notions as those explained for the  $2 \times 4$  average and is used in the monthly case. Its coefficients are:

$$\{1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 1\}/24.$$

This average is also known as a centered 12-term moving average. It therefore preserves the straight lines and, as its gain function shows, eliminates annual seasonalities (which correspond to a frequency of  $30^\circ = 2\pi/12 = \pi/6$ ). Moreover, the sum of the squares of its coefficients being equal to  $23/288$ , it reduces the variance of white noise by more than 90%.

But here again, not all periodic series of periods less than a year are very well restored. Thus, a 3-year periodic function, which corresponds here to a frequency of  $10^\circ = 2\pi/36 = \pi/18$ , will be only 80% restored.

### 12.6.3 Estimation of Seasonality: $3 \times 3$ , $3 \times 5$ and $3 \times 9$ Averages

These averages<sup>15</sup> are used by X-11 to extract the seasonal component based on an estimation of the seasonal-irregular component. They are therefore used in the construction of Tables B4, B5, B9, B10, C5, C10, D5 and D10.

At that point, the series to be smoothed is composed of the seasonal and irregular components. In the case of an additive decomposition model, it may be written:  $SI_t = S_t + I_t$  and, unlike previously, the problem is strictly one of smoothing.

<sup>15</sup>X-12-ARIMA also allows for the use of a  $3 \times 15$  moving average and of a simple moving average of order 3 (a  $3 \times 1$ ).

**Table 12.4:** Coefficients of the composite moving averages used in X-11, their variance reducing power,  $\sum \theta_i^2$ , and Henderson criterion,  $\sum (\nabla^3 \theta_i)^2$ .

i	2 × 4	2 × 12	3 × 3	3 × 5	3 × 9
-6		1/24			
-5		1/12			1/27
-4		1/12			2/27
-3		1/12		1/15	3/27
-2	1/8	1/12	1/9	2/15	3/27
-1	1/4	1/12	2/9	3/15	3/27
0	1/4	1/12	3/9	3/15	3/27
1	1/4	1/12	2/9	3/15	3/27
2	1/8	1/12	1/9	2/15	3/27
3		1/12		1/15	3/27
4		1/12			2/27
5		1/12			1/27
6		1/24			
$\sum \theta_i^2$	0.2188	0.0799	0.2346	0.1644	0.1001
$\sum (\nabla^3 \theta_i)^2$	0.1250	0.0139	0.1481	0.0356	0.0110

The coefficients of the 3 × 3, 3 × 5 and 3 × 9 averages are as follows:

$$\begin{aligned}
 M_{3 \times 3} &: \{1, 2, 3, 2, 1\}/9 \\
 M_{3 \times 5} &: \{1, 2, 3, 3, 3, 2, 1\}/15 \\
 M_{3 \times 9} &: \{1, 2, 3, 3, 3, 3, 3, 3, 3, 2, 1\}/27.
 \end{aligned}$$

As can easily be established, each of these symmetric moving averages preserves straight lines. But the gain functions, shown in Figure 12.7, are fairly different from the ideal form associated with a low-pass filter. Here, it is not a hindrance insofar as the seasonal-irregular component is not supposed to present a cyclical component whose periodicity is of the order of 3 to 6 years.

Applying, for example, the 3 × 3 filter separately to each month of an estimate of the seasonal-irregular component amounts to applying to this selfsame component a 49-term moving average of coefficients:

$$\begin{aligned}
 &\{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \\
 &3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1\}/9.
 \end{aligned}$$

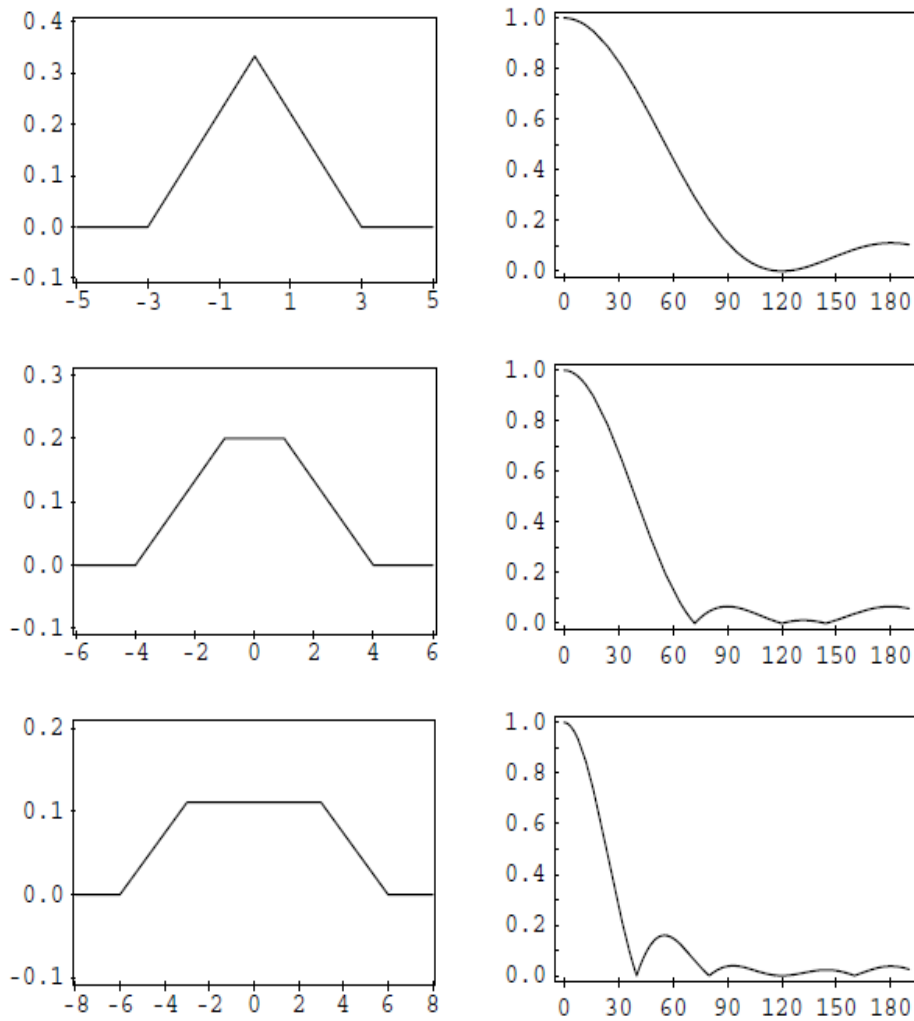
The same is true with respect to the 3 × 5 and 3 × 9 filters with which are associated 73-term and 121-term moving averages respectively! The gain functions of these new moving averages are shown in Figure 12.8. As can be seen, these moving averages preserve the annual seasonality since they exactly restore the multiple frequencies of  $30^\circ = 2\pi/12 = \pi/6$ .

### 12.6.4 Henderson moving averages

Henderson moving averages are used in X-11 to extract the trend from an estimate of the seasonally adjusted series. The coefficients of the Henderson moving averages used in X-11 as well as the related criteria, variance reduction and smoothing values are presented in Table 12.4<sup>16</sup>.

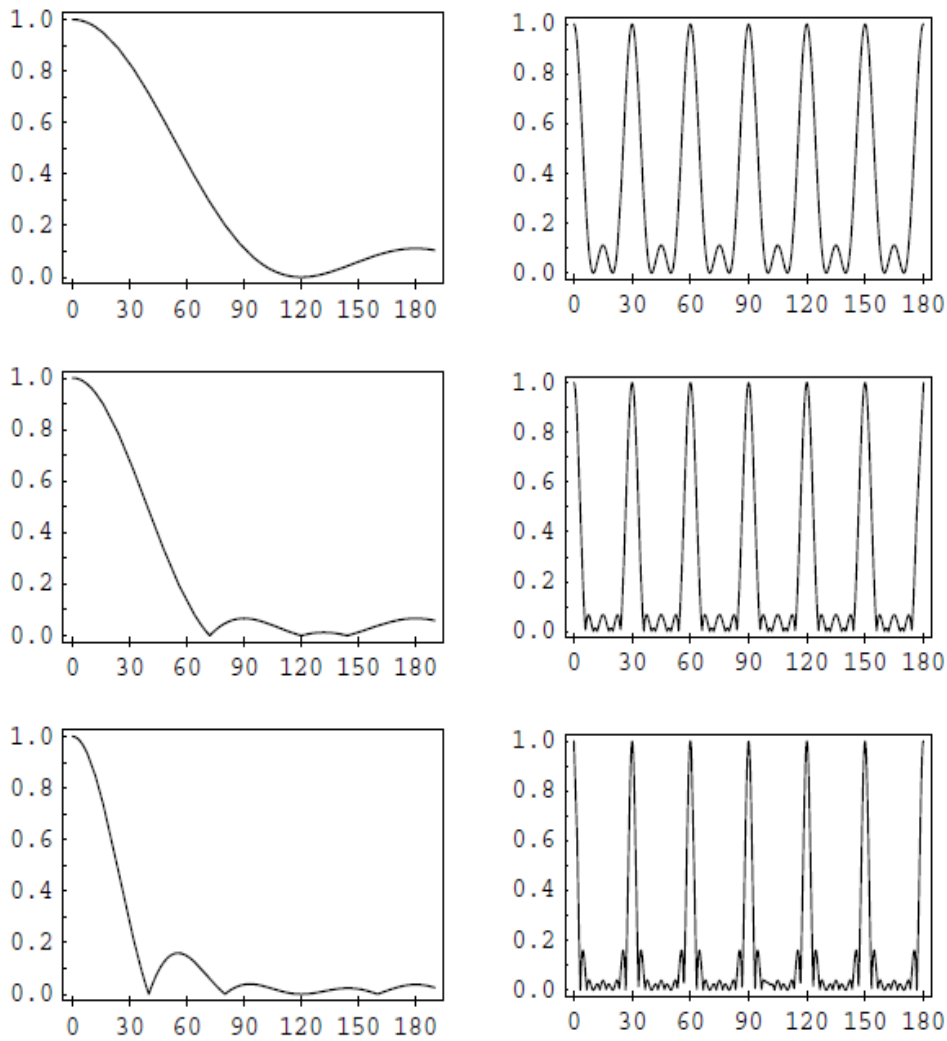
<sup>16</sup>With X-12-ARIMA, it is possible to use any odd Henderson moving average of order less than 101.

**Figure 12.7:** Coefficient curves (on the left) and gain functions (on the right) of the composite moving averages used in X-11 seasonal factor estimation. The  $3 \times 3$  is displayed in the upper panel, the  $3 \times 5$  in the middle panel and the  $3 \times 9$  in the lower panel.

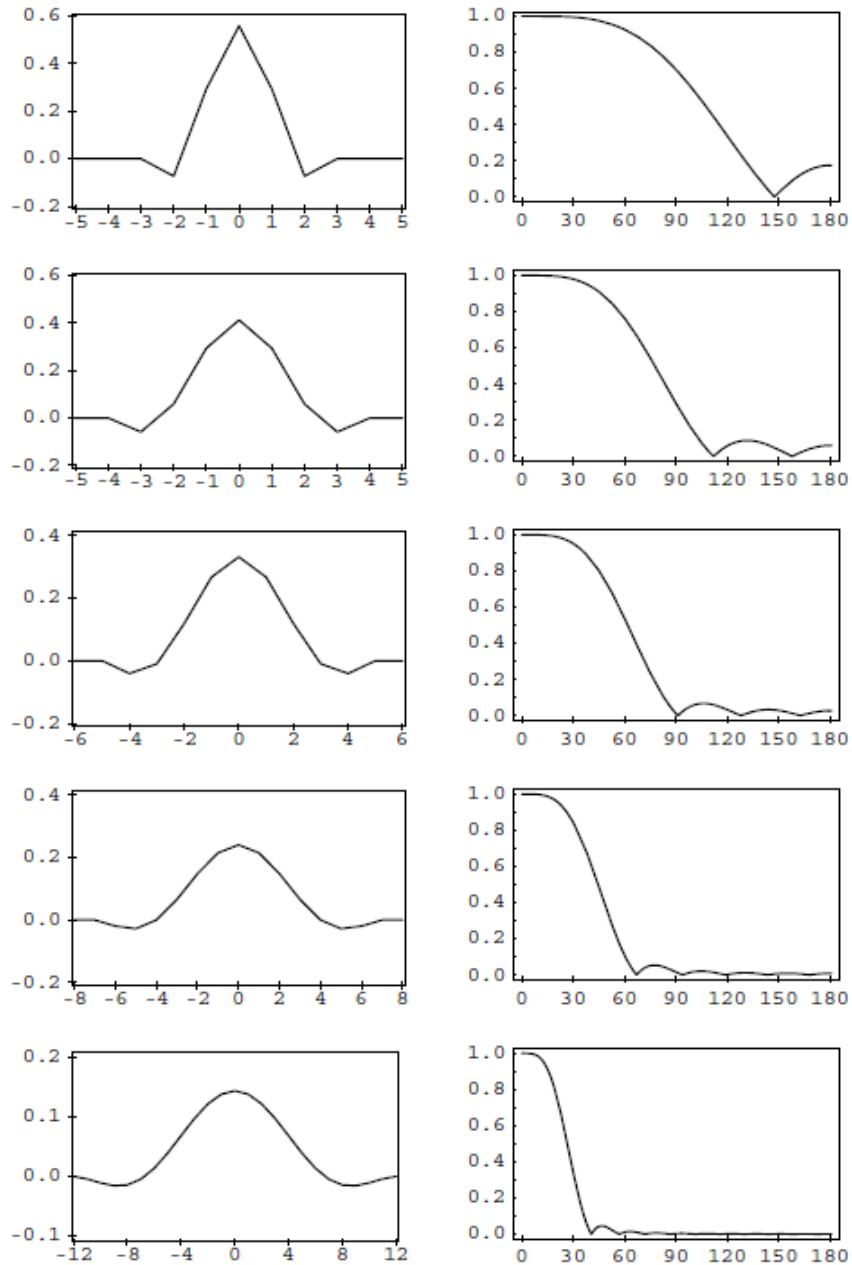


The coefficient curves, show in Figure 12.9, are smooth and the gain functions of these averages are closer to the ideal “low-pass” form than those of the composite moving averages seen earlier.

**Figure 12.8:** Gain functions of the  $3 \times 3$ ,  $3 \times 5$  and  $3 \times 9$  averages. The  $3 \times 3$  is displayed in the upper panel, the  $3 \times 5$  in the middle panel and the  $3 \times 9$  in the lower panel. The gain functions over each month are displayed on the left, and over the whole series on the right.

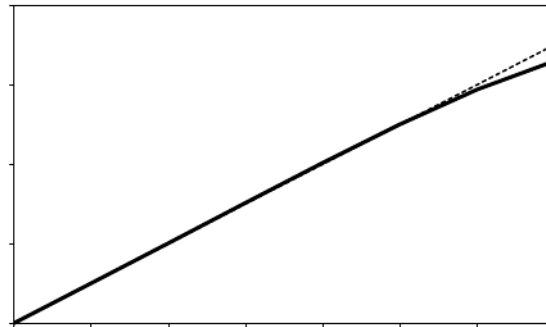


**Figure 12.9:** Coefficient curves and gain functions, from top to bottom, of the centered 5, 7, 9, 13 and 23-term Henderson moving averages used in X-11. Coefficient curves are on the left, gain functions on the right.





**Figure 12.10:** Smoothing of a straight line by a 13-term Henderson moving average supplemented by Musgrave asymmetric moving averages



### 12.6.5 Musgrave asymmetric moving averages

#### 12.6.5.1 Musgrave asymmetric moving averages associated with Henderson symmetric moving averages

The coefficients of Musgrave’s moving averages depend on a value  $D$  which is unknown. But Musgrave points out that the order of the Henderson moving averages in X-11 is chosen based on the value of the ratio  $R = \frac{\bar{I}}{\bar{C}}$  where  $\bar{I}$  designates the average of the absolute monthly variations in the irregular portion of the series and  $\bar{C}$  designates the average of the absolute monthly variations in the trend of the series. Assuming the normality of the irregular  $\epsilon_t$ , it is shown that:  $D = \frac{4}{\pi} \left(\frac{1}{R}\right)^2$ , which makes it possible to calculate the asymmetric moving averages numerically.

Tables 12.7 to 12.12 show the coefficients of these moving averages calculated based on the values of the ratio  $R = \bar{I}/\bar{C}$  provided in Table 12.5. It is these asymmetric moving averages that are used in X-12-ARIMA. In all tables, the notation  $H_{p-f}$  means that the moving average has order  $p + f + 1$  with  $p$  points in the past and  $f$  points in the future.

**Table 12.5:** X-12-ARIMA default values of the ratio  $R = \bar{I}/\bar{C}$  used in the computation of Musgrave asymmetric moving averages.

5-term Henderson	$R = .001$
7-term Henderson	$R = 4.5$
9-term Henderson	$R = 1$
13-term Henderson	$R = 3.5$
23-term Henderson	$R = 4.5$

#### 12.6.5.2 Comment about Musgrave moving averages

While the series to be smoothed out is assumed to follow an end-of-period linear model, Musgrave moving averages do not retain the straight lines, only the constants. In order for it do so, it would also have been necessary to impose on the coefficients the additional constraint

$$\sum_{i=-p}^{+f} i\theta_i = 0$$

As Figure 12.10 shows, Musgrave asymmetric averages provide “cautious” estimates of the smoothed series while reducing the observed increase in the last points of the series.

## 12.6.5.3 Asymmetric moving averages associated with composite moving averages

Curiously, while Musgrave's first work concerned the generation of asymmetric filters associated with the composite moving averages used to estimate the seasonal factors, his recommendations have not been applied in the X-11 method.

The asymmetric filters associated with the  $3 \times 3$ ,  $3 \times 5$  and  $3 \times 9$  averages are shown in Tables 12.13 to 12.15. We do not know the rationale behind these asymmetric filters, and we are not aware of any publication that explains the choice of the coefficients. The  $2 \times 4$  and  $2 \times 12$  filters are not supplemented by asymmetric averages. In all tables, the notation  $S_{p-f}$  means that the moving average has order  $p + f + 1$  with  $p$  points in the past and  $f$  points in the future.

**Table 12.6:** Coefficients of the asymmetric moving averages associated with the 5-term Henderson average (X-11-ARIMA).

i	H2_2	H2_1	H2_0
-2	-0.07343	-0.073	-0.073
-1	0.29371	0.294	0.403
0	0.55944	0.522	0.670
1	0.29371	0.257	0
2	-0.07343	0	0

**Table 12.7:** Coefficients of Musgrave asymmetric moving averages associated with the 5-term Henderson average (X-12-ARIMA),  $R = 0.001$ .

i	H2_2	H2_1	H2_0
-2	-0.07343	-0.03671	-0.18357
-1	0.29371	0.29371	0.36713
0	0.55944	0.52273	0.81643
1	0.29371	0.22028	0
2	-0.07343	0	0

**Table 12.8:** Coefficients of Musgrave asymmetric moving averages associated with the 7-term Henderson average (X-12-ARIMA),  $R = 4.5$ .

i	H3_3	H3_2	H3_1	H3_0
-3	-0.05874	-0.05314	-0.05421	-0.03379
-2	0.05874	0.05818	0.06101	0.11601
-1	0.29371	0.28699	0.29371	0.38329
0	0.41259	0.39972	0.41032	0.53449
1	0.29371	0.27468	0.28917	0
2	0.05874	0.03356	0	0
3	-0.05874	0	0	0

**Table 12.9:** Coefficients of Musgrave asymmetric moving averages associated with the 9-term Henderson average (X-12-ARIMA),  $R = 1$ .

i	H4_4	H4_3	H4_2	H4_1	H4_0
-4	-0.04072	-0.03082	-0.02262	-0.04941	-0.15554
-3	-0.00987	-0.00426	-0.00021	-0.01056	-0.03384
-2	0.11847	0.11980	0.11969	0.12578	0.18536
-1	0.26656	0.26361	0.25933	0.28187	0.42429
0	0.33114	0.32391	0.31547	0.35445	0.57972
1	0.26656	0.25504	0.24244	0.29786	0
2	0.11847	0.10267	0.08590	0	0
3	-0.00987	-0.02995	0	0	0
4	-0.04072	0	0	0	0

**Table 12.10:** Coefficients of Musgrave asymmetric moving averages associated with the 13-term Henderson average (X-12-ARIMA),  $R = 3.5$ .

i	H6_6	H6_5	H6_4	H6_3	H6_2	H6_1	H6_0
-6	-0.01935	-0.01643	-0.01099	-0.00813	-0.01603	-0.04271	-0.09186
-5	-0.02786	-0.02577	-0.02204	-0.02019	-0.02487	-0.03863	-0.05811
-4	0	0.00127	0.00330	0.00413	0.00267	0.00182	0.01202
-3	0.06549	0.06594	0.06626	0.06608	0.06784	0.07990	0.11977
-2	0.14736	0.14698	0.14559	0.14441	0.14939	0.17436	0.24390
-1	0.21434	0.21314	0.21004	0.20784	0.21605	0.25392	0.35315
0	0.24006	0.23803	0.23324	0.23002	0.24144	0.29223	0.42113
1	0.21434	0.21149	0.20498	0.20076	0.21540	0.27910	0
2	0.14736	0.14368	0.13547	0.13024	0.14810	0	0
3	0.06549	0.06099	0.05108	0.04483	0	0	0
4	0	-0.00532	-0.01694	0	0	0	0
5	-0.02786	-0.03401	0	0	0	0	0
6	-0.01935	0	0	0	0	0	0

**Table 12.11:** Coefficients of Musgrave asymmetric moving averages associated with the 23-term Henderson average (X-12-ARIMA),  $R = 4.5$ , Part A.

i	H11_11	H11_10	H11_9	H11_8	H11_7	H11_6
-11	-0.00428	-0.00390	-0.00282	-0.00103	0.00108	0.00268
-10	-0.01092	-0.01059	-0.00968	-0.00817	-0.00642	-0.00511
-9	-0.01569	-0.01542	-0.01467	-0.01344	-0.01205	-0.01103
-8	-0.01453	-0.01431	-0.01372	-0.01279	-0.01175	-0.01101
-7	-0.00495	-0.00479	-0.00436	-0.00372	-0.00303	-0.00258
-6	0.01343	0.01354	0.01380	0.01416	0.01448	0.01465
-5	0.03893	0.03898	0.03908	0.03916	0.03913	0.03900
-4	0.06830	0.06830	0.06823	0.06802	0.06764	0.06723
-3	0.09740	0.09734	0.09711	0.09661	0.09587	0.09517
-2	0.12195	0.12184	0.12144	0.12066	0.11956	0.11858
-1	0.13832	0.13815	0.13759	0.13652	0.13507	0.13380
0	0.14406	0.14384	0.14312	0.14176	0.13995	0.13839
1	0.13832	0.13804	0.13716	0.13551	0.13334	0.13150
2	0.12195	0.12162	0.12057	0.11864	0.11611	0.11399
3	0.09740	0.09701	0.09580	0.09358	0.09070	0.08829
4	0.06830	0.06786	0.06649	0.06398	0.06075	0.05805
5	0.03893	0.03844	0.03690	0.03411	0.03052	0.02753
6	0.01343	0.01288	0.01118	0.00810	0.00415	0.00088
7	-0.00495	-0.00555	-0.00742	-0.01078	-0.01509	0
8	-0.01453	-0.01519	-0.01721	-0.02087	0	0
9	-0.01569	-0.01640	-0.01859	0	0	0
10	-0.01092	-0.01169	0	0	0	0
11	-0.00428	0	0	0	0	0

**Table 12.12:** Coefficients of Musgrave asymmetric moving averages associated with the 23-term Henderson average (X-12-ARIMA),  $R = 4.5$ , Part B.

i	H11_5	H11_4	H11_3	H11_2	H11_1	H11_0
-11	0.00258	-0.00065	-0.00861	-0.02293	-0.04520	-0.07689
-10	-0.00519	-0.00776	-0.01396	-0.02486	-0.04130	-0.06385
-9	-0.01109	-0.01300	-0.01744	-0.02491	-0.03554	-0.04893
-8	-0.01106	-0.01230	-0.01500	-0.01904	-0.02385	-0.02808
-7	-0.00261	-0.00319	-0.00413	-0.00475	-0.00373	0.00119
-6	0.01464	0.01472	0.01554	0.01834	0.02518	0.03925
-5	0.03902	0.03976	0.04233	0.04856	0.06121	0.08444
-4	0.06726	0.06866	0.07299	0.08264	0.10112	0.13350
-3	0.09522	0.09729	0.10337	0.11644	0.14074	0.18228
-2	0.11865	0.12137	0.12921	0.14571	0.17583	0.22652
-1	0.13389	0.13728	0.14687	0.16679	0.20273	0.26258
0	0.13850	0.14255	0.15390	0.17724	0.21901	0.28801
1	0.13163	0.13634	0.14945	0.17622	0.22380	0
2	0.11413	0.11951	0.13437	0.16456	0	0
3	0.08845	0.09449	0.11111	0	0	0
4	0.05823	0.06493	0	0	0	0
5	0.02773	0	0	0	0	0
6	0	0	0	0	0	0
7	0	0	0	0	0	0
8	0	0	0	0	0	0
9	0	0	0	0	0	0
10	0	0	0	0	0	0
11	0	0	0	0	0	0

**Table 12.13:** Asymmetric moving averages associated with the  $3 \times 3$  symmetric moving average.

i	S2_2	S2_1	S2_0
-2	1/9	3/27	5/27
-1	2/9	7/27	11/27
0	3/9	10/27	11/27
1	2/9	7/27	0
2	1/9	0	0

**Table 12.14:** Asymmetric moving averages associated with the  $3 \times 5$  symmetric moving average.

i	S3_3	S3_2	S3_1	S3_0
-3	1/15	4/60	4/60	9/60
-2	2/15	8/60	11/60	17/60
-1	3/15	13/60	15/60	17/60
0	3/15	13/60	15/60	17/60
1	3/15	13/60	15/60	0
2	2/15	9/60	0	0
3	1/15	0	0	0

**Table 12.15:** Asymmetric moving averages associated with the  $3 \times 9$  symmetric moving average. In X-11-ARIMA and X-12-ARIMA, the asymmetric moving averages are coded in decimal form with 3 figures only after the decimal point. The fractional form given here is the one closest to this decimal expression. A good approximation can also be obtained using Musgrave's formula with  $D = 9.8$ .

i	S5_5	S5_4	S5_3	S5_2	S5_1	S5_0
-5	1/27	35/1026	35/1026	33/1026	29/1026	52/1026
-4	2/27	75/1026	77/1026	81/1026	94/1026	115/1026
-3	3/27	114/1026	116/1026	136/1026	148/1026	177/1026
-2	3/27	116/1026	120/1026	136/1026	164/1026	202/1026
-1	3/27	117/1026	126/1026	147/1026	181/1026	227/1026
0	3/27	119/1026	131/1026	158/1026	197/1026	252/1026
1	3/27	120/1026	135/1026	167/1026	213/1026	0
2	3/27	121/1026	141/1026	177/1026	0	0
3	3/27	123/1026	145/1026	0	0	0
4	2/27	86/1026	0	0	0	0
5	1/27	0	0	0	0	0

### 12.6.6 The X-11 moving average filter

If the procedure for the detection and correction of extreme values and the procedure for estimating calendar effects are not taken into account, the X-11 method may be seen as the application in succession of several moving averages. The operator that makes it possible to go from the unadjusted series to the seasonally adjusted series is therefore itself a moving average.

Thus, in the case of a monthly series, the basic algorithm described in 1.8 can amount to the application of a single moving average which may be calculated by means of a matrix<sup>17</sup>.

**1. Estimation of the trend-cycle by a  $2 \times 12$  moving average:**

$$TC^{(1)} = M_{2 \times 12} X \text{ with } M_{2 \times 12} : [13]; \frac{1}{24} \{1, 2, 2, 2, 2, 2, 2, 2\}.$$

**2. Estimation of the seasonal-irregular component:**

$$(S + I)^{(1)} = X - TC^{(1)} = [I_d - M_{2 \times 12}] X$$

where  $I_d$  represents the identity operator that transforms the series into itself. Here,  $I_d$  would be the moving average  $[13]; \{0, 0, 0, 0, 0, 0, 0, 1\}$ .

**3. Estimation of the seasonal component by a  $3 \times 3$  moving average over each month:**

Applying the  $3 \times 3$  moving average to the values of each month separately amounts to applying, to the series, the average  $M_3$  over 49 months, defined by:

$$M_3 : [49]; \frac{1}{9} \{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3\}.$$

Consequently,

$$\begin{aligned} S^{(1)} &= M_{3 \times 3} (S + I)^{(1)} \\ &= M_3 [I_d - M_{2 \times 12}] X. \end{aligned}$$

The factors are then normalized such that their sum over each consecutive 12-month period is approximately zero.

$$\begin{aligned} \tilde{S}^{(1)} &= S^{(1)} - M_{2 \times 12} S^{(1)} \\ &= [I_d - M_{2 \times 12}] M_3 [I_d - M_{2 \times 12}] X \\ &= M_3 [I_d - M_{2 \times 12}]^2 X. \end{aligned}$$

**4. Estimation of the seasonally adjusted series:**

$$\begin{aligned} SA^{(1)} &= X - \tilde{S}^{(1)} \\ &= X - M_3 [I_d - M_{2 \times 12}]^2 X \\ &= \left( I_d - M_3 [I_d - M_{2 \times 12}]^2 \right) X. \end{aligned}$$

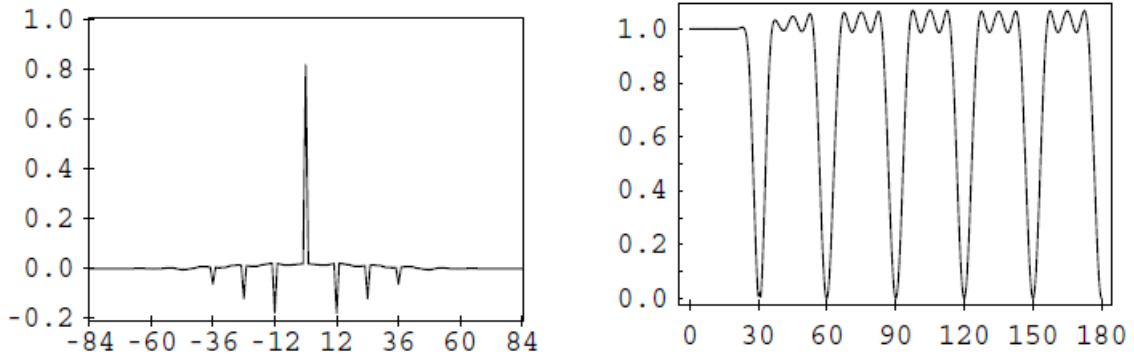
**5. Estimation of the trend-cycle by a 13-term Henderson moving average:**

$$\begin{aligned} TC^{(2)} &= H_{13}(SA^{(1)}) \\ &= H_{13} \left( I_d - M_3 [I_d - M_{2 \times 12}]^2 \right) X. \end{aligned}$$

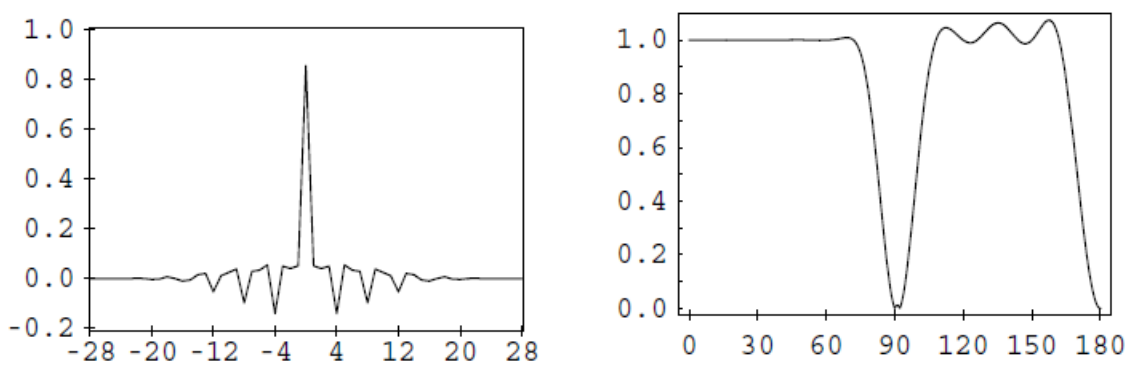
<sup>17</sup>See Gouriéroux, and Monfort (1997).



**Figure 12.11:** The asymmetric monthly moving average of X-11. The coefficient curve is on the left, the gain function on the right.



**Figure 12.12:** The symmetric quarterly moving average filter of X-11. The coefficient curve is on the left, the gain function on the right.





The central symmetric composite X-11 moving averages can be obtained by adjusting an ad hoc dummy variable with the X-11-ARIMA or X-12-ARIMA software's. For example, in the monthly case, where the global filter is a moving average of order 169 (when the 13-term Henderson, the 3x3 and the 3x5 moving averages are used), the moving average coefficients are obtained by seasonally adjusting a series of 193 (= 169 + 24) observations that are all equal to zero except the 97th observation that is set equal to one. The seasonal adjustment options use an additive decomposition model, specifying both the trend (for instance, Henderson 13) and the seasonal moving averages (for instance, the 3x3 and the 3x5) under study, and deactivating the automatic correction for outliers (for instance, the values for the two sigma limits can be set equal to 9.9). The resulting seasonally adjusted series in Table D11 is the series of coefficients corresponding to the central symmetric moving average. The extra 12 zero's at the beginning and the end can be deleted to get only the 169 coefficients. It is necessary to add those extras zero due to the way X11 applies the 2x12 moving averages in various Tables.

Another algorithm needs to be used to obtain the full matrix of asymmetric and symmetric moving averages corresponding to the X11 linear approximation applied to a given series of length N. In the example above, it first consists in seasonally adjusting each column of the Identity matrix of order 169 with the appropriate options. Next, each column of the Identity matrix is replaced by its corresponding seasonally adjusted series. The rows of the resulting matrix provide the 169 moving averages with the symmetric one being in row 85. This algorithm works for the following reasons.

Denote by  $X = (x_1, \dots, x_T)$  the series to be seasonally adjusted, by  $A = (a_1, \dots, a_T)$  its seasonally adjusted version, and by  $W$  the weight matrix such that  $A = WX$ . The goal is to find  $W$ .

The seasonally adjusted series at time  $t$  is  $a_t = \sum_{j=1}^T w_{t,j}x_j$ . Hence, if  $X = (1, 0, \dots, 0)$  is used, then  $a_t = w_{t,1}$ , and  $(w_{1,1}, w_{2,1}, \dots, w_{T,1})$  represents the first column of  $W$ . Similarly, if  $X = (0, 1, 0, \dots, 0)$  is used, then  $a_t = w_{t,2}$ , and  $(w_{1,2}, w_{2,2}, \dots, w_{T,2})$  represents the second column of  $W$ . This is continued until  $X = (0, 0, \dots, 0, 1)$  is used, to obtain  $a_t = w_{t,T}$ , and  $(w_{1,T}, w_{2,T}, \dots, w_{T,T})$  as the last column of  $W$ .

## 12.7 Conclusions

Moving averages have a very long tradition and have been used since the 19<sup>th</sup> century to smooth or seasonally adjusted series. They are non-parametric tools whose numerous properties are unfortunately not well-known. Thanks to an enhanced "Whittaker-Henderson" framework, that considers a moving average as the result of the minimization of a quadratic form under constraints, it is possible to construct a moving average that has good properties in terms of trend preservation, elimination of stable or moving seasonality, noise reduction, and so on.

Moving averages are the basis of the "X-11 family" of seasonal adjustment programs which are used for decades by numerous statistical agencies all around the world. One of the main problem of these linear filters, a problem shared by any model-based seasonal adjustment method, is the estimation of the components at the ends of the series: asymmetric moving averages must be used that tend to introduce a delay in the detection of turning-points. The "Whittaker-Henderson" framework can also be used to design moving averages minimizing this phase-shift as shown in Chapter 15.

Moreover, moving averages appear to be very well adapted for the analysis of high frequency data. As shown in Chapter 29, it is possible to adapt the X-11 algorithm to multiple and non-integer frequencies using a succession of moving averages specifically designed to deal with each seasonal frequency (hourly, daily, weekly etc.).

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# 13

## Quality of Seasonal Adjustment in the Model-Based Approach of TRAMO SEATS



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## Table of Contents

13.1	Introduction	325
13.2	Program TRAMO	325
13.3	Decomposition of the Series	326
13.3.1	Derivation of the models for the components	326
13.3.2	Estimation of the components	328
13.3.3	Example	330
13.4	Some Properties of the Estimators of the Components	331
13.4.1	Correlation between the estimators of the components	331
13.4.2	Non-linearity of the seasonally adjusted series	332
13.4.3	Non-invertibility	332
13.4.4	ARIMA modeling of SA series	332
13.5	Diagnostics	333
13.5.1	Quality Assessment and Diagnostics	333
13.5.2	Idempotency	339
13.6	Inference	340
13.6.1	Estimation error	340
13.6.2	Revision in SA series	340
13.7	Quality of Seasonal Adjustment	341
13.7.1	Good Model, Bad Seasonal	341
13.7.2	Bad Model, Good Seasonal	347
13.8	Conclusions	359



## 13.1 Introduction

We consider the decomposition of an observed series into a signal plus noise, and center on the case of seasonal adjustment, where the signal is the seasonal component and the noise is the seasonally adjusted (SA) series. The components are unobserved, and estimates have traditionally been obtained with fixed filters (perhaps with a few options). Given that no definition for the component is provided, it is difficult to assess the appropriateness of the estimator and a model would be needed. Still, unless an automatic procedure is employed, identification of the appropriate model for each series becomes prohibitive when the number of series is very large, as is often the case in seasonal adjustment. It has often been asserted that the dependence on an appropriate model is, in fact, the weak point of the model-based approach because inadequacy of the model would invalidate the decomposition. The ARIMA model-based (AMB) method of program TRAMO-SEATS (Gómez and Maravall, 1996), as enforced in program TSW (Caporello and Maravall, 2004), provides a reliable automatic model identification (AMI) procedure, so that inadequate models are seldom obtained. Besides, in practice, many of these inadequacies do not invalidate the decomposition. Also, based on the model, standard parametric inference and diagnostics can be derived, that complement ad-hoc checks and provide a more precise and complete assessment of the quality of seasonal adjustment.

## 13.2 Program TRAMO

Program SEATS evolved from the original approach of Box, Hillmer and Tiao (1978), Burman (1980), Hillmer and Tiao (1982), Maravall and Pierce (1987), and Maravall (1995). Program TRAMO evolved from the original approach of Hillmer, Bell and Tiao (1983), Tsay (1986), Chen and Liu (1993), and Gómez and Maravall (1994) among others. The two programs are described in Gómez and Maravall (2001a), Gómez and Maravall (2001b) and can be downloaded from [www.bde.es](http://www.bde.es) (entering “Services” and then “Statistical and Econometrics Software”). They are also available, slightly modified and together with X12ARIMA, as part of the US Bureau of the Census (USCB) X13-ARIMA-SEATS package and the JDEMETRA+ package of the European Commission (U.S. Census Bureau, 2016) and (European Commission, 2016).<sup>1</sup>

TRAMO stands for “Time series Regression with Arima noise, Missing observations, and Outliers.” If  $z_t$  denotes the observed series (perhaps log-transformed), TRAMO fits a model of the type:

$$z_t = y_t' \beta + x_t \quad (13.1)$$

where  $y_t' \beta$  is the regression term, and  $x_t$  follows an ARIMA model. The regression term is the preadjustment component. By default, it consists of automatic detection and correction of outliers (additive, transitory, or level shift ones) and of calendar effects (such as Trading Day and Easter). Simultaneously, TRAMO identifies the ARIMA model for  $x_t$ .

In Maravall, López and Pérez, D. (2015) and Maravall, López and Pérez, D. (2016), the reliability of the AMI procedure is discussed for monthly time series, first, on a set of 50000 simulated series from 50 different ARIMA models, to determine the performance of AMI when the series is indeed generated by an ARIMA model. It is found that, roughly, for 3 out of 4 series the exact model orders are obtained, and for 99% of the series the model obtained is a reasonable approximation. Second, on a set of 14.800 socio-economic series, with lengths between 60 and 360 observations, the performance on real time series is discussed. The preadjustment, in particular outlier adjustment, plays a relevant role: when no pre-adjustment is performed, the Normality (N) and the lack of autocorrelation (Q) tests applied to the residuals fail in 38% and 5.5% of the cases, respectively; when pre-adjustment is performed, the proportion of failures of the N and Q tests decreases to 8% and 1.3%, respectively. Further, preadjustment reduces the average number of ARIMA parameters from 2.6 to 2.2. Thus, for a series with 100 observations, with preadjustment, the average model

<sup>1</sup> See [www.seasonaladjustment.com/software](http://www.seasonaladjustment.com/software)



contains 1.4 ARIMA parameters, 1 outlier, and 0.5 calendar-effect parameters, a total of 2.9 parameters. With no preadjustment, this number is 1.7. Considering the improvement in the whiteness and Normality of the residuals, Calendar effect correction seems convenient, outlier correction seems necessary. For the set of 14.800 real series, the default run of TRAMO (i.e., with automatic model identification, including detection and correction for outliers and calendar effects) takes a few minutes and produces an acceptable model in 90.3% of the cases. Most of the failures are due to excess Kurtosis in the residuals. Although this failure may render a model unacceptable, it will often have little effect on point estimation, and the decomposition may still be acceptable. Thus the model that the TRAMO method provides is likely to be reliable when applied automatically by default.

## 13.3 Decomposition of the Series

### 13.3.1 Derivation of the models for the components

By default, no seasonal outliers are considered and hence all outliers are assigned to the SA series; calendar effects are assigned to the seasonal component. Both effects are deterministic, and I center next in the decomposition of the ARIMA model and estimation of the stochastic components as in program SEATS (“Signal Extraction in ARIMA Time Series”). Let the compact representation of the ARIMA model be

$$\varphi(B)x_t = \theta(B)a_t \quad (13.2)$$

where  $\varphi(B)$  and  $\theta(B)$  are polynomials in the lag operator  $B$  ( $B^j z_t = z_{t-j}$ ) and  $a_t$  is a white-noise innovation. For seasonal series the polynomials adopt the multiplicative specification

$$\phi_r(B)\phi_s(B^s)\nabla^D\nabla_s^{D_s}x_t = \theta(B)a_t \quad (13.3)$$

where  $\phi_r(B)$  and  $\phi_s(B^s)$  are the stationary regular and seasonal polynomials

$$\begin{aligned} \phi_r(B) &= 1 + \phi_{r,1}(B) + \dots + \phi_{r,P_r}(B^{P_r}) \\ \phi_s(B^s) &= 1 + \phi_{s,1}(B^s) + \dots + \phi_{s,P_s}(B^{sP_s}) \end{aligned}$$

of order  $P_r$  and  $P_s$ , ( $\nabla^D, \nabla_s^{D_s}$ ) is the stationary inducing transformation, where  $\nabla^D = (1 - B)^D$  and  $\nabla_s^{D_s} = (1 - B^s)^{D_s}$ , and  $\theta(B)$  is an invertible polynomial that may also factor into a regular polynomial in  $B$  of order  $Q_r$  and a polynomial in  $B^s$  of order  $Q_s$ .

In TRAMO-SEATS, the order of the polynomials are restricted to ( $0 \leq P$  and  $Q \leq 3$ ), ( $0 \leq D \leq 2$ ), ( $0 \leq D_s \leq 1$ ), ( $0 \leq P_s$  and  $Q_s \leq 1$ ), which allows for 384 possible combinations. Model 13.2 will be non-stationary when the AR polynomial  $\varphi(B)$  contains at least one unit root, that is, when  $D > 0$  and/or  $D_s > 0$ .

Factoring the stationary regular AR polynomial as

$$\phi_r(B) = \prod_{j=1}^{P_r} (1 - \lambda_j B)$$

where  $\lambda_j$  is a root for  $B^{-1}$ , each root is associated with a frequency  $\omega_j$  (in radians) and the Fourier transformation of the root will display a peak for that frequency. Thus, for example, if  $\lambda_j$  is a positive real number,  $\omega_j = 0$  and the root will be associated with a trend; also, for a monthly time series, a pair of complex conjugate roots for a frequency  $\omega_j = \pi j/6$ , ( $j = 1, 2, \dots, 5$ ) will be associated with the  $j$ -times-a-year seasonal frequency, and a real negative root with the 6-times-a-year frequency.

Thus it may happen that seasonal roots are solutions of the equation  $\phi_r(B) = 0$ , and hence the polynomial can be factored as

$$\phi_r(B) = \phi_{rn}(B)\phi_{rs}(B)$$

where  $\phi_{rs}(B)$  contains the seasonal roots and  $\phi_{rn}(B)$  the non-seasonal ones. As an example, the regular polynomial  $\phi_{rs}(B) = (1 + 0.92B + 0.85B^2)$  contains a pair of complex conjugate roots associated with the 4-times-a-year seasonal frequency ( $\omega = 2\pi/3$ ).

Likewise, the stationary seasonal autoregressive polynomial, in turn, may contain non-seasonal roots. This will be the case for the polynomial  $(1 + \phi_s B^s)$  when  $\phi_s > 0$  (not the usual case) because the spectral peaks it produces are not at seasonal frequencies (they are displayed towards the middle of the interval between seasonal frequencies). When  $\phi_s < 0$ , the polynomial factors as

$$(1 + \phi_s B^s) = (1 - \lambda B)(1 + \lambda B + \dots + \lambda^{s-1} B^{s-1}),$$

where  $\lambda = \phi_s^{1/s}$ . The first root will be associated with the trend and the polynomial

$$S_\lambda = 1 + \lambda B + \dots + \lambda^{s-1} B^{s-1}$$

contains the fundamental seasonal frequency as well as its harmonics. Therefore, the polynomial  $\phi_s(B)$  can also be factorized as

$$\phi_s(B^s) = \phi_{ss}(B)\phi_{sn}(B)$$

where  $\phi_{ss}(B)$  contains the seasonal roots and  $\phi_{sn}(B)$  the non-seasonal ones, the latter consisting of roots that are part of the trend-cycle and/or the transitory component.

The seasonal differencing operator also accepts a seasonal-nonseasonal factorization, namely

$$\nabla_s = \nabla S$$

where  $S = 1 + B + \dots + B^{s-1}$  includes unit roots for the seasonal frequencies. Therefore, grouping the seasonal roots and the non-seasonal ones in the two polynomials

$$\varphi_n(B) = \phi_{rn}(B)\phi_{sn}(B)\nabla^{D+D_s} \quad (13.4)$$

$$\varphi_s(B) = \phi_{rs}(B)\phi_{ss}(B)S^{D_s}, \quad (13.5)$$

the first contains only seasonal roots, the second only non-seasonal ones, and the ARIMA model (13.2) can be expressed as

$$\varphi_n(B)\varphi_s(B)x_t = \theta(B)a_t. \quad (13.6)$$

This representation will be useful in order to derive the seasonal adjustment algorithm described in the remaining of the section. The series  $x_t$  is to be decomposed as in

$$x_t = s_t + n_t \quad (13.7)$$

where  $s_t$  denotes the seasonal component and  $n_t$  the SA series. This additive decomposition is obtained from the partial fractions decomposition (PFD) of (13.6) as in

$$x_t = \frac{\theta(B)}{\varphi_n(B)\varphi_s(B)}a_t = \frac{\theta_s(B)}{\varphi_s(B)}a_{st} + \frac{\theta_n(B)}{\varphi_n(B)}a_{nt} \quad (13.8)$$

The first term in the PFD provides the model for the seasonal component; the second term, the model for the SA series, or

$$\varphi_s(B)s_t = \theta_s(B)a_{st} \text{ with } a_{st} \sim \text{WN}(0, V_s) \quad (13.9)$$

$$\varphi_n(B)n_t = \theta_n(B)a_{nt} \text{ with } a_{nt} \sim \text{WN}(0, V_n) \quad (13.10)$$

The moving-average (MA) polynomials  $\theta_s(B)$  and  $\theta_n(B)$ , as well as the components innovation variances  $V_s$  and  $V_n$ , are identified through three assumptions:

**Assumption A:** Let  $p, q, p_s, q_s, p_n$  and  $q_n$  denote the AR and MA orders of the models (13.2), (13.9), and (13.10), respectively. Then  $p_s = q_s$  and  $p_n = q_n$  if  $p \geq q$ , otherwise  $q_n = q - p_s$ .

**Assumption B:** The polynomial  $\theta_s(B)$  contains (at least) a unit root.

A unit MA root implies a zero in the spectrum of  $s_t$  and makes  $s_t$  a “canonical” component, that is, a component free of noise contamination.

**Assumption C:** The innovations  $a_{st}$  and  $a_{nt}$  are uncorrelated Normally distributed white noises.

From (13.8), the fundamental identity for the innovations is obtained:

$$\theta(B)a_t = \varphi_s(B)\theta_n(B)a_{nt} + \varphi_n(B)\theta_s(B)a_{st} \quad (13.11)$$

The parameters in  $\theta(B)$ ,  $\varphi_s(B)$  and  $\varphi_n(B)$  are determined from the ARIMA model. From (13.11) a system of  $(q+1)$  covariance equations is obtained with  $(q+2)$  unknowns, namely the parameters in  $\theta_s(B)$ ,  $\theta_n(B)$ , and the two variances  $V_s$  and  $V_n$ . The canonical assumption (Assumption B) reduces the number of unknowns to  $(q+1)$ , and the system provides a unique solution whereby all noise is assigned to the component  $n_t$ . (Notice that, while  $\phi(B) = \varphi_n(B)\varphi_s(B)$ ,  $\theta(B)$  is not equal to  $\theta_n(B)\theta_s(B)$ .)

Obviously, by exchanging  $n_t$  for  $s_t$  an alternative canonical decomposition is obtained with a noise-free  $n_t$  component.<sup>2</sup> In seasonal adjustment only what is strictly seasonality is wished to be removed and it is considered that the noise should remain in the adjusted series. This is the approach in SEATS, and a further decomposition of the SA series provides a noise free trend-cycle component.

For some ARIMA models and some values of the parameters not all component spectra can be non-negative for the range of frequencies  $\omega = [0, \pi]$ . The decomposition in those cases is termed non-admissible. SEATS will replace the model with a relatively close decomposable one, so that an admissible decomposition is always provided and tested; see Maravall, López and Pérez, D. (2015).<sup>3</sup>

## 13.3.2 Estimation of the components

### 13.3.2.1 Infinite realization $[x_{-\infty}, \dots, x_t, \dots, x_{+\infty}]$ and historical estimator

The MMSE estimator of  $s_t$  is given by

$$\hat{s}_t = \nu_s(B, F)x_t = \frac{\gamma_s(B, F)}{\gamma_x(B, F)}x_t,$$

where  $F$  denotes the forward operator ( $Fz_t = z_{t+1}$ ),  $\gamma_s(B, F)$  and  $\gamma_x(B, F)$  are the (pseudo) Auto Covariance Generating Functions (ACF) of  $s_t$  and  $x_t$ , and  $\nu_s(B, F)$  is the Wiener-Kolmogorov (WK) filter, extended to non-stationary series as in Bell (1984). (The estimator  $\hat{s}_t$  is also the conditional expectation  $\hat{s}_t = E(s_t|X)$ .) Considering that the ACF of the linear model  $z_t = A(B)a_t$  is  $\gamma_z(B, F) = V_a A(B)A(F)$ , after simplification:

$$\hat{s}_t = \frac{V_s}{V_a} \left[ \frac{\theta_s(B)\theta_s(F)\varphi_n(B)\varphi_n(F)}{\theta(B)\theta(F)} \right] x_t \quad (13.12)$$

So that the WK filter for  $\hat{s}_t$  is the ACF of the stationary ARMA model

$$\theta(B)z_t = [\theta_s(B)\varphi_n(B)]b_t, \text{ with } b_t \sim \text{WN}(0, V_s/V_a)$$

<sup>2</sup>Properties of the two alternative noise allocations are analyzed in citeCh13.MP1999.

<sup>3</sup>In the set of real series, 4.8% of the models that AMI yielded produced non-admissible decompositions.

and hence is centered, symmetric, and convergent. Thus it can be approximated by the finite filter

$$\hat{s}_t \cong \nu_0 x_t + \sum_{i=1}^k [\nu_i (x_{t-i} + x_{t+i})]$$

If, in (13.12),  $x_t$  is replaced by its ARIMA expression, an “ARIMA” model for  $\hat{s}_t$  in terms of  $a_t$  is obtained. Proceeding in a similar way with  $\hat{n}_t$ , after simplification, one gets:

$$\begin{aligned} \varphi_s(B)\hat{s}_t &= \frac{V_s}{V_a} \theta_s(B) \frac{\theta_s(F)\varphi_n(F)}{\theta(F)} a_t \\ \varphi_n(B)\hat{n}_t &= \frac{V_n}{V_a} \theta_n(B) \frac{\theta_n(F)\varphi_s(F)}{\theta(F)} a_t. \end{aligned} \quad (13.13)$$

In this way, the estimators can be expressed as ARIMA-type models with innovations those of the series  $x_t$ .

### 13.3.2.2 Finite realization $[x_1, x_2, \dots, x_T]$ and preliminary estimators

The MMSE estimator (and conditional expectation) of  $s_t$  is given by the WK filter applied to the series extended with forecasts and backcasts (as needed for convergence of the filter). If  $\hat{x}_{j|T}$  denotes the forecast ( $j > T$ ) or backcast ( $j < 1$ ) of  $x_j$  obtained at time  $T$ , the extended series is:

$$x_{t|T} = [\dots, \hat{x}_{-1|T}, \hat{x}_{0|T}, x_1, x_2, \dots, x_T, \hat{x}_{T+1|T}, \hat{x}_{T+2|T}, \dots]$$

and the preliminary estimator of  $s_t$  when  $x_T$  is the last observation is:

$$\hat{s}_{t|T} = \nu(B, F)x_{t|T}. \quad (13.14)$$

As shown in Burman (1980), the full filter effect can be captured with a finite number of backcasts and forecasts. Expression (13.14) also provides the MMSE forecasts of the components by further extending the series with additional forecasts. For an alternative matrix derivation see McElroy (2008).

The estimator  $\hat{s}_{t|T}$  is preliminary because, as  $x_{T+j}$  ( $j = 1, 2, \dots$ ) becomes available,  $\hat{s}_{t|T}$  will be revised to  $\hat{s}_{t|T+j}$ . The revision caused by the  $j$ -th new observations is equal to:

$$r_{T|T+j} = \hat{s}_{T|T+j} - \hat{s}_{T|T}, \quad (13.15)$$

and, as  $j \rightarrow \infty$ , the estimator  $\hat{s}_{t|T+j}$  converges to the historical estimator  $\hat{s}_t$ . Assuming a semi-infinite realization (in practice, for  $T > k$ ), to obtain the model for the preliminary estimator and for the revision, letting  $\xi_s(B, F) = \theta_s(B)\theta_s(F)\varphi_n(F)$ , a PFD of the filter in (13.13) yields:

$$\frac{\xi_s(B, F)}{\varphi_s(B)\theta(F)} = \frac{\alpha(B)}{\varphi_s(B)} + \frac{\beta(F)}{\theta(F)}, \quad (13.16)$$

where the first coefficient of  $\beta(F)$  is  $\beta_0 = 0$ . Letting  $k_s = V_s/V_a$ , after simplification, from (13.13) and (13.16) it is obtained that

$$\hat{s}_t = c_s \frac{\alpha_s(B)}{\varphi_s(B)} a_t + c_r \frac{\beta_s(F)}{\theta(F)} a_{t+1}, \quad (13.17)$$

where  $c_s = k_s \alpha_{s,0}$ ;  $\alpha_{s,j} = \alpha_j/\alpha_0$ , for  $j = 0, 1, 2, \dots, q_s$ ;  $c_r = k_s \beta_{s,1}$ ;  $\beta_{s,j} = \beta_j/\beta_1$ , for  $j = 0, 1, 2, \dots, h$ ; and  $h = q$  when  $p \geq q$ , or  $h = q_n + p_s$  when  $p < q$ .

Taking expectations at time  $t$ , the first term in (13.17) provides the model for the concurrent estimator, thus the model for the seasonal component is:

$$\varphi_s(B)\hat{s}_{t|t} = c_s \alpha_s(B) a_t, \quad (13.18)$$

and the second term provides the model for the total revision in the concurrent estimator, namely:

$$\theta(F)r_{t|\infty} = c_r\beta_s(F)a_{t+1}, \quad (13.19)$$

with MA representation  $r_{t|\infty} = \lambda(F)a_{t+1}$ , where

$$\lambda(F) = c_r \frac{\beta_s(F)}{\theta(F)} = \lambda_1 + \lambda_2 F + \lambda_3 F^2 + \dots \quad (13.20)$$

It follows that the model for the preliminary estimator of  $s_t$  when observations end at  $(t + j)$  is given by:

$$\varphi(B)\hat{s}_{t|t+j} = c_s\alpha(B)a_t + \lambda_1 a_{t+1} + \dots + \lambda_j a_{t+j}. \quad (13.21)$$

which is seen to depend on  $j$ . The revision the concurrent estimator  $\hat{s}_{t|t}$  will have undergone after  $j$  additional observations have become available is the MA( $j - 1$ ) model

$$r_{t|t+j} = \hat{s}_{t|t+j} - \hat{s}_{t|t} = \sum_{i=1}^j \lambda_i a_{t+i}, \quad (13.22)$$

see Pierce (1980) and Bell and Martin (2004).

Given that the component, its historical, and its preliminary estimators follow models that are different, their ACFs and spectra will also be different. Therefore, the SA series obtained when adjusting at time  $T$  a series  $[x_1, x_2, \dots, x_T]$  produced by an ARIMA model, being a mixture of historical, preliminary and concurrent estimators, will not be the output of a stable ARIMA model and will be non-linear with time-varying parameters and variance.

In SEATS, the SA series is further split into trend-cycle, irregular, and (possibly) transitory components. The trend-cycle, in turn, is decomposed into trend plus cycle. In essence, the trend-cycle captures the peak at frequency 0 in the series spectrum; the irregular is white noise, and the transitory component captures peaks for frequencies that are not zero, nor seasonal. The trend plus cycle decomposition follows Kaiser and Maravall (2005) and is based on ARIMA models for these two components that aggregate into the trend-cycle model, and whose MMSE estimators reproduce exactly the Hodrick-Prescott estimator applied to the extended trend-cycle. The full decomposition of the series is based on an internally consistent ARIMA model-based structure, a straightforward extension of the two-component case. Details of the SEATS procedure for some ARIMA models can be found in, for example, Findley, Lytras and Maravall (2016) and M elard, G. (2016).

### 13.3.3 Example

#### 13.3.3.1 Observed series $x_t$

$$\nabla\nabla_{12}x_t = (1 - 0.4B)(1 - 0.6B^{12})a_t \text{ and } \sigma_a = 1.$$

#### 13.3.3.2 Seasonally adjusted series $n_t$

- **Component:**

$$\nabla^2 n_t = (1 - 1.367B + 0.392B^2)a_{n,t} \text{ and } \sigma_n = 0.812, \quad (13.23)$$

with roots  $(1 - 0.409B)(1 - 0.958B)$ .

- **Concurrent estimator:**

$$\nabla^2 \hat{n}_{t|t} = 0.762(1 - 1.313B + 0.339B^2)a_t, \quad (13.24)$$

with roots  $(1 - 0.354B)(1 - 0.959B)$ .

- **Historical estimator:**

$$\begin{aligned} & (1 - 0.4F)(1 - 0.6F^{12})\nabla^2 \hat{n}_t \\ & = 0.659(1 - 1.367B + 0.392B^2)(1 - 1.367F + 0.392F^2)S(F)a_t, \end{aligned} \quad (13.25)$$

where  $S(F) = 1 + F + F^2 + \dots + F^{11}$ .

### 13.3.3.3 Seasonal component $s_t$

- **Component:**

$$Ss_t = (1 + 1.415B + 1.489B^2 + \dots - 0.414B^{11})a_{s,t} \text{ and } \sigma_n = 0.210, \quad (13.26)$$

where  $S=S(F)$ .

- **Concurrent estimator:**

$$S\hat{s}_t = 0.238(1 + 1.322B + 1.559B^2 + \dots - 0.077B^{11})a_t, \quad (13.27)$$

- **Historical estimator:**

$$\begin{aligned} & (1 - 0.4F)(1 - 0.6F^2)S\hat{s}_t = \\ & 0.044(1 + 1.415B + \dots - 0.414B^{11})(1 + 1.415F + \dots - 0.414F^{11})(1 - F^2)a_t. \end{aligned}$$

From the models, variances, auto- and cross-covariances, spectra, and more generally the joint distribution function of the theoretical components and their estimators (historical and preliminary) are easily obtained. This opens a vast field for parametric diagnostics and inference.

## 13.4 Some Properties of the Estimators of the Components

### 13.4.1 Correlation between the estimators of the components

The AMB method assumes uncorrelated components. Yet, sharing the same innovations  $(a_t)$ , their MMSE estimators will show non-zero cross-correlation. For two series with MA representation  $x_{1t} = \Psi_1(B)a_t$  and  $x_{2t} = \Psi_2(B)a_t$ , the Cross-Covariance generating function (CCF) is given by  $CCF_{(x_1, x_2)}(B, F) = \sigma_a^2 \Psi_1(B)\Psi_2(F)$  or, applied to (13.13), after simplification it is obtained that the CCF between the two estimators is equal to the ACF of the model

$$\theta(B)z_t = \theta_s(B)\theta_n(B)b_t, \text{ with } b_t \sim \text{WN}(0, \frac{V_s V_n}{V_a}), \quad (13.28)$$

a stationary ARMA model. Macroeconomic time series typically have non-stationary seasonality and/or non-stationary trend, so that the cross-correlations will have finite numerators and unbounded denominators; therefore, they will be close to zero. However, cross-correlation between the stationary transformations of the components will in general be non-zero. Write  $\varphi_s(B) = \Phi_s(B)\delta_s(B)$  and  $\varphi_n(B) = \Phi_n(B)\delta_n(B)$ , where

the  $\Phi$  and  $\delta$  polynomials contain the stationary and the unit (non-stationary) roots, respectively. Then, from (13.13),  $\delta_s(B)\hat{s}_t$  and  $\delta_n(B)\hat{n}_t$  have ARMA-type expressions with finite variances and covariances.

### 13.4.2 Non-linearity of the seasonally adjusted series

If the series  $[x_1, \dots, x_T]$ , output of an ARIMA model, is seasonally adjusted, the SA series  $[\hat{n}_{1|T}, \dots, \hat{n}_{T|T}]$  can be written as the sequence of estimators:  $\hat{n}_{T-j|T}$ , ( $j = T, T-1, \dots, 1, 0$ ). Given that their models depend on  $j$ , see (13.21), each one of these estimators is generated by a different ARIMA-type model. Therefore the underlying model has non-constant parameters. However, given that the changes in the model are moderate, the non-linear effect can be expected to be mild.

### 13.4.3 Non-invertibility

Being most often non-stationary, macroeconomic time series exhibit in the AR polynomials of their ARIMA models regular differences ( $\nabla = 1 - B$ ) and/or seasonal differences ( $\nabla_s = 1 - B^s = \nabla S$ ). (In the set of 50000 simulated ARIMA series, correct identification of unit AR roots is achieved in 96% of the cases. In the set of 14800 real series, about 90% of the series are found non-stationary; 84% require regular differencing and 72% require seasonal differencing.) According to (13.13), the AR polynomials in the models for  $s_t$  and  $n_t$  will show up in the MA polynomials of the models for  $\hat{n}_t$  and  $\hat{s}_t$ , respectively. As a consequence, when the original series has non-stationary seasonality, the SA series will be non-invertible, and the spectrum will display zeros for the seasonal frequencies. Therefore, a pure AR representation of the model for the historical estimator will not converge. Similarly, when the original series has a non-stationary trend, the seasonal component estimator will be non-invertible, with the root  $(1 - F)$  in the MA polynomial of the model.

Assume  $[\hat{n}_{1|T}, \dots, \hat{n}_{T|T}]$  is the SA series obtained at time  $T$ . Preliminary estimators will be used in the first and last years of the sample; unless the series is short, the central years will use historical estimators. It can be seen that, when the series has seasonal non-stationarity, the preliminary estimator remains invertible, although some of MA roots will be very close to unit roots (as seen in the example of Section 13.3.3), and hence a pure AR fit to the series  $[\hat{n}_{1|T}, \dots, \hat{n}_{T|T}]$  will be slow to converge. Given that the close-to-unit roots are seasonal, low-order ARs will miss this feature. In summary, point estimators of pure AR fits to SA series are of questionable interest

### 13.4.4 ARIMA modeling of SA series

Given that it is a standard procedure in applied econometrics to use SA series, it is of interest to see the performance of regression-ARIMA models with the (mildly) non-linear and close to non invertible SA series. If the series  $x_t$  is assumed to follow an ARIMA model, equation (13.13) shows that the ARIMA-type model of the estimator  $\hat{n}_t$  is more complex than the model for  $x_t$ . The AMI of TRAMO was applied to the 13500 SA series from the set of real series for which a seasonal component had been detected, and the results were compared to the ones for the original series. Again, the automatic default input was used in TRAMO and the main differences are shown in Table 1.

More parsimonious ARIMA models, with cleaner residuals and fewer outliers, are found for the unadjusted series. The decomposition of the SA series provides the trend-cycle component, the transitory component (if there is any) and the irregular component. The close-to-non invertibility property induces close to zero spectral values and causes a substantial increase of non-admissible decomposition models for the adjusted series. The adjusted series needs more regular parameters in an attempt to capture the more complex model structure. These additional parameters may capture spectral peaks at the non-zero and non-seasonal frequencies induced by the spectral zeros at the seasonal frequencies in the SA series. This effect explains the large increase in the number of transitory components in the SA series.



**Table 13.1: Seasonally Adjusted and Unadjusted Series: AMI results.**

(first five lines expressed as % and the last two as average of the total number of series)

	UNADJUSTED	ADJUSTED
Model produced by AMI is acceptable	90.3	83.0
Autocorrelation present in residuals	1.3	6.0
Residuals cannot be accepted as Normally distributed	7.9	5.7
Model does not accept an admissible decomposition	4.8	16.2
The decomposition includes a transitory component	27.0	43.4
Average number of regular parameters per ARMA model	1.3	1.8
Average number of outliers per series	1.6	2.0

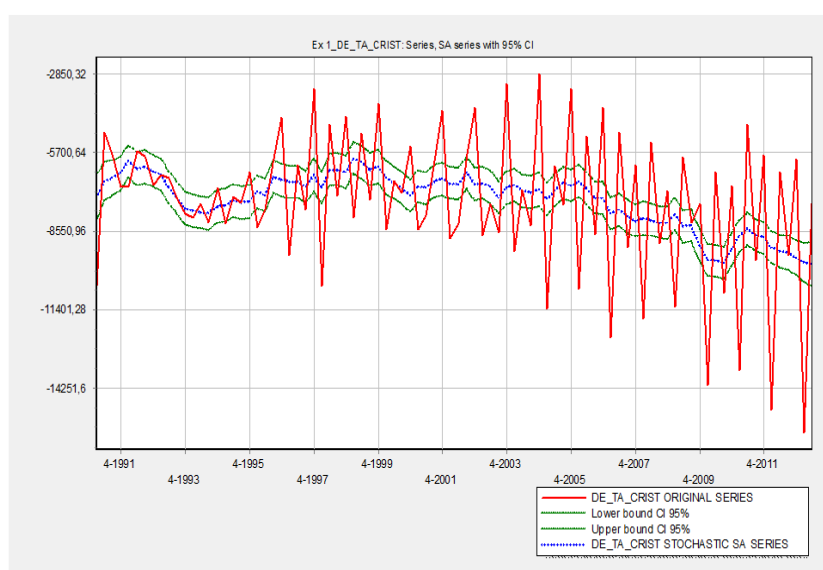
Although the adjusted series residuals perform better concerning the Normality assumption, the proportion of acceptable models is larger for the case of original (unadjusted) series. Still, the results for the SA series are far from disastrous.

## 13.5 Diagnostics

### 13.5.1 Quality Assessment and Diagnostics

In the X11 approach, lack of a definition of seasonality limits diagnostics to ad-hoc quality assessments on what is assumed desirable in a seasonal component. In X11 the so-called M-statistics play this role and, given that X11 will always produce a seasonal component, they are often used as criteria for rejecting a seasonal adjustment. Thus if, for example, the contribution of the irregular is very large or very small, or if the trend component has a very mild slope, or if moving seasonality dominates stable seasonality, seasonal adjustment may be rejected. The AMB approach provides the models for the canonical seasonal and non-seasonal components and, if the data supports the model, MMSE estimation of the seasonal component can be considered the “best” estimator. This estimator may have unpleasant features, but these are implied by the series dynamic structure and are thus to be respected (see Maravall, 1998).

**Figure 13.1: Example 1: German Transfer Accounts, Raw data, estimated SA series and 95% confidence interval.**



Example 1 illustrates the point. Figure 13.1 shows the quarterly series of German Transfer Accounts (90 observations that start in January 1991). TRAMO detects seasonality in the series and AMI selects the

seasonal random-walk model

$$\nabla_4 x_t = a_t, \quad (13.29)$$

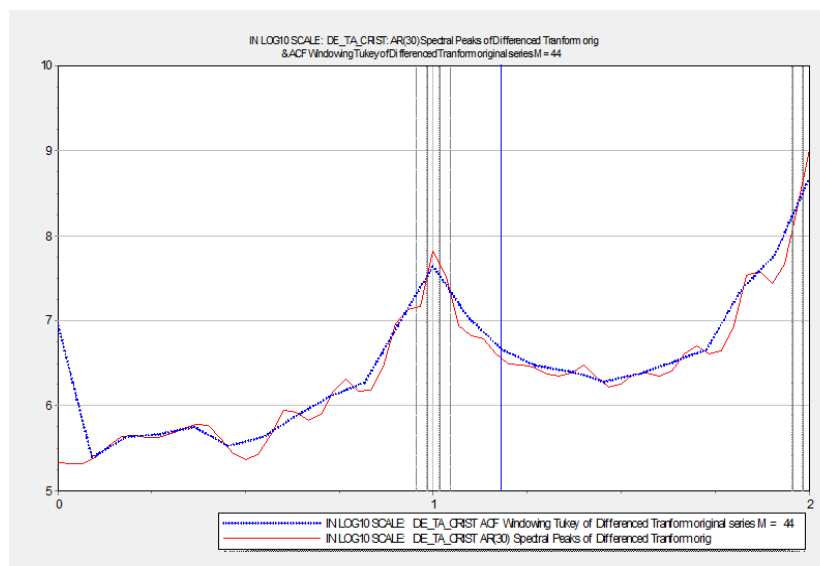
that comfortably passes all residual diagnostics. Figure 13.1 also shows the estimated SA series and the 95% confidence interval around these estimators. Figure 13.2 plots the spectral peaks for the seasonal frequencies of two types of spectra estimators: a non-parametric Tukey spectrum (Jenkins and Watts, 1968) and a parametric AR spectrum (in Findley et al., 1998), both applied to the differenced series. Figure 13.3 plots the pseudo-spectra of the models for the series and the components.

Figure 13.4 displays the trend component, and Figure 13.5 the seasonal and irregular components. The slope of the trend is mild, the irregular component is very small, and the seasonal components moves fast. Due to these features, when adjusting the series with X11 three M statistics fall out of the acceptance range. Yet it would make no sense to reject seasonal adjustment and conclude that the original series can be taken as the seasonally adjusted series. The seasonal random walk model is not close to the ARIMA models for which X11 may be appropriate, but AMB seasonal adjustment of the series causes no problem and provides a sensible decomposition.

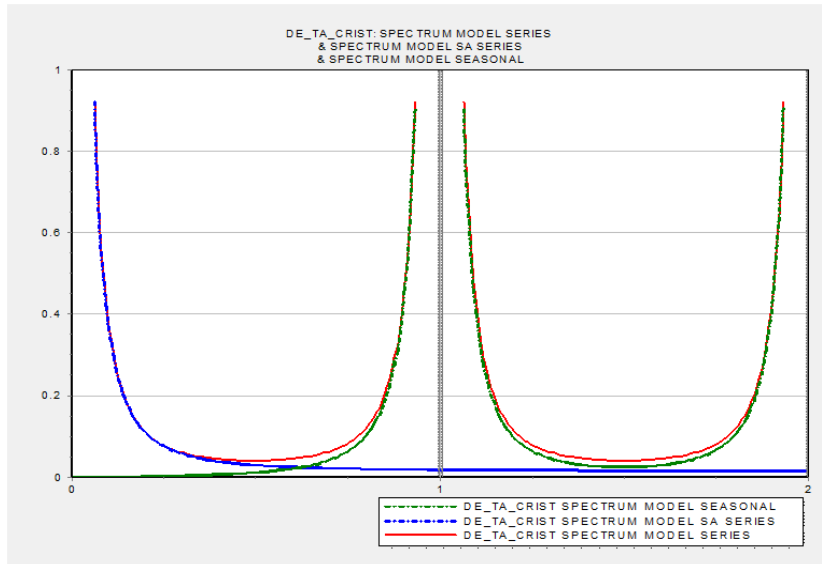
SEATS decomposes the SA series into a trend-cycle component plus orthogonal white noise, and the trend-cycle, in turn, is split into (long-term) trend plus business cycle following the AMB approach of Kaiser and Maravall (2005). This additional spectral decomposition is shown in Figure 13.6. The spectrum of the long-term trend (a narrow peak at the zero frequency) plus the cycle spectrum (a peak at the 10 year period and a mean at the 4.3 year period) add to the spectrum of the trend-cycle component. Figures 13.7 and 13.8 present the estimators of the first two components; their implied ARIMA models are also provided.

An important feature of the AMB adjustment is its flexibility. Figures 13.1 and 13.5 evidence that seasonality in the first years of the sample is markedly different than seasonality in the later years, so that one might be tempted to remove the early years in order to get a better estimate of seasonality in the last years. Removing the first 7 years, AML yields the same model (13.28). In fact, as seen in Figure 13.9, the seasonal factors for the shortened series are pretty much the same. This flexibility will also show up in the examples that follow. Thus the need to remove early years in order to obtain better estimates for recent periods may well be of little relevance.

**Figure 13.2:** Example 1: German Transfer Accounts, Spectral peaks in the Tukey and AR spectra.



**Figure 13.3:** Example 1: German Transfer Accounts, Pseudo-spectra of the models for the series and the components.



**Figure 13.4:** Example 1: German Transfer Accounts, Trend component.

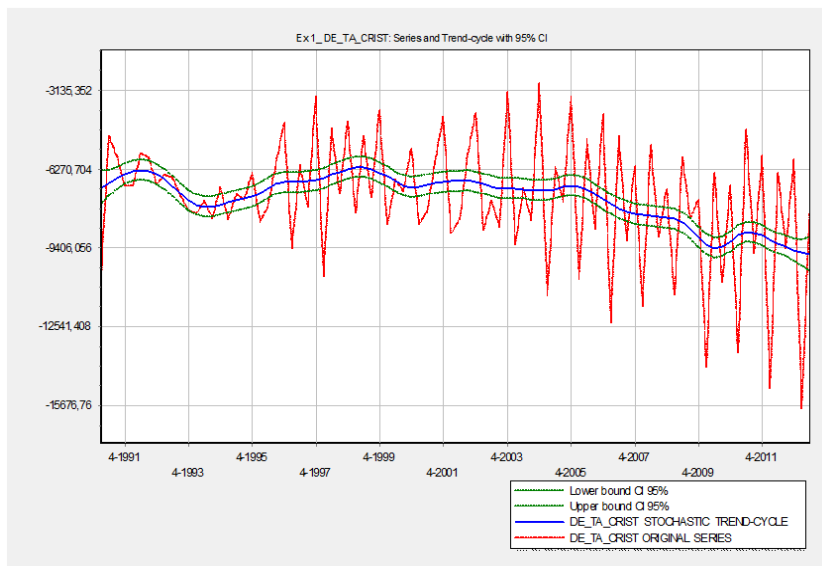


Figure 13.5: Example 1: German Transfer Accounts, Seasonal and irregular components.

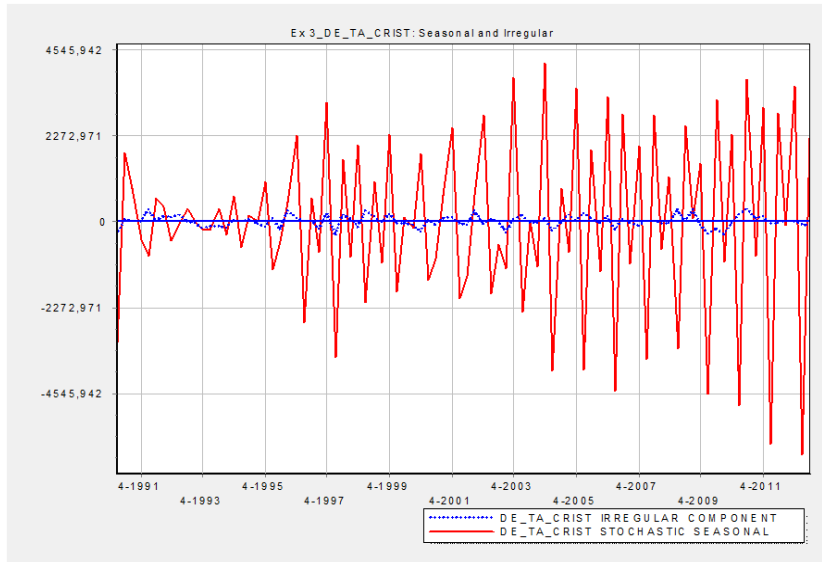
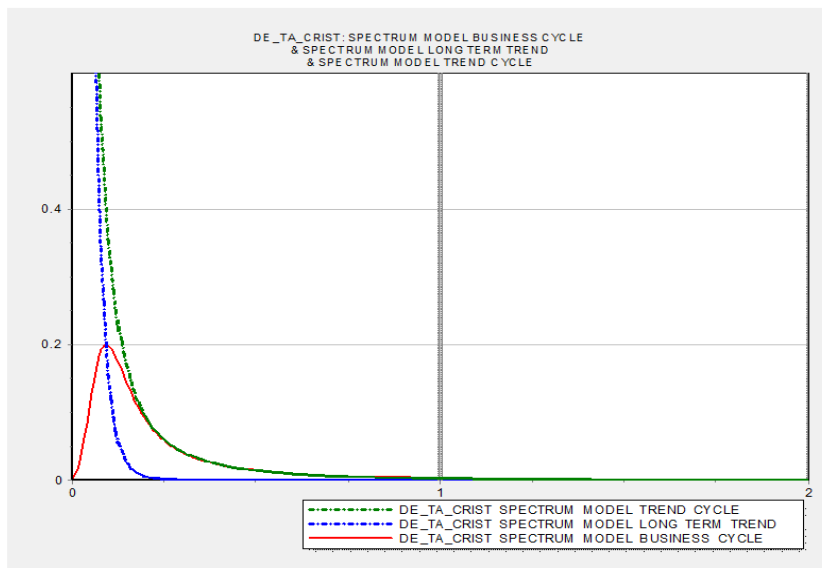


Figure 13.6: Example 1: German Transfer Accounts; Trend Cycle, Business-Cycle and Long Term Trend Model Spectra.



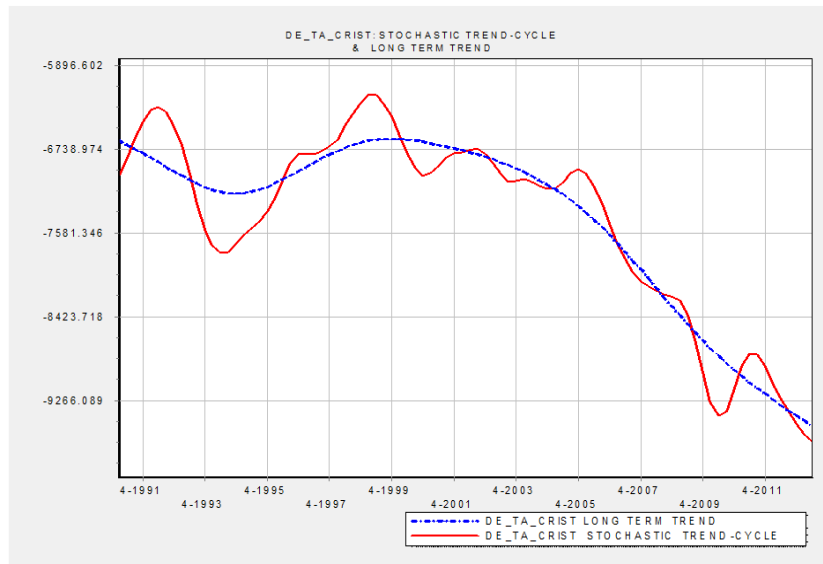
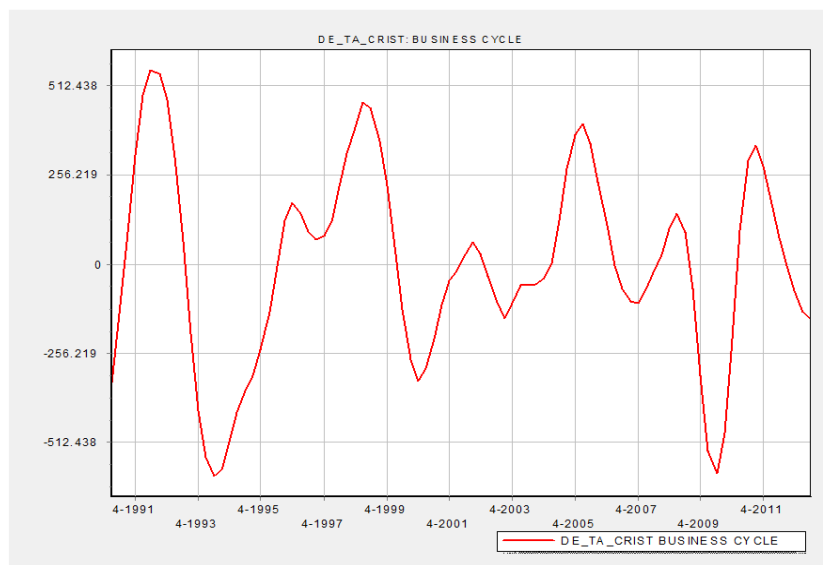
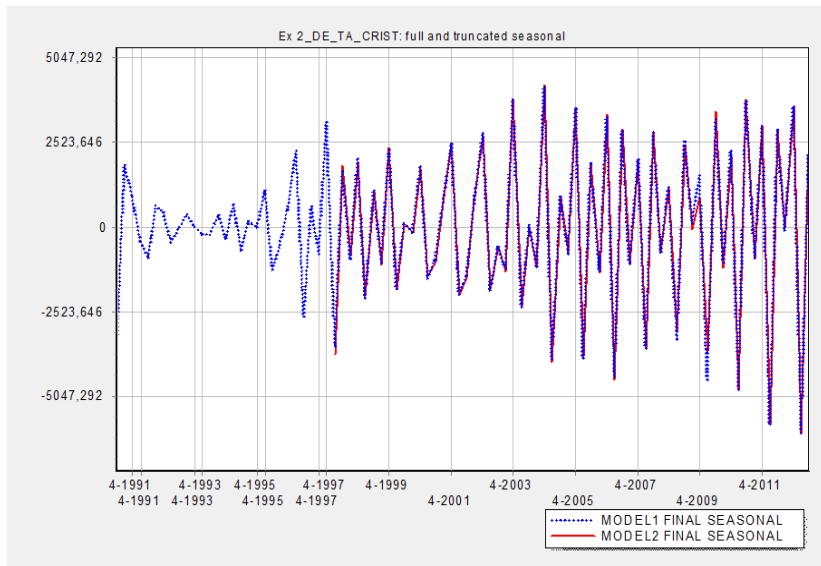
**Figure 13.7:** Example 1: German Transfer Accounts; Stochastic Trend Cycle and Long Term Trend.**Figure 13.8:** Example 1: German Transfer Accounts; Business-Cycle.

Figure 13.9: Example 1: German Transfer Accounts; Full and truncated seasonal.



As seen in Section 13.3, observed series, historical and preliminary estimators of the components, as well as the revision error in the latter, can be expressed as linear filters in the n.i.i.d. innovations  $[a_t]$ . Therefore, the joint distribution function for these statistics is straightforward to obtain, which allows for parametric inference and diagnosis. The theoretical ACF and CCF implied by the models for (the stationary transformation of) the estimators can be compared to the empirical ones obtained for the stationary transformation of the estimated components: are theoretical estimators and empirical estimates in agreement? A simple example from Maravall (1987) will illustrate the relevance of this information.

The series of the monthly Insurance Operations component of the Spanish monetary aggregate was seasonally adjusted with X11; the adjustment failed a quality assessment having to do with too much autocorrelation in the irregular component. The lag-1 autocorrelation in the X11 irregular was in fact  $\hat{\rho}_1(\hat{u}_t) = 0.42$ . Using the (rough) model approximation to X11 of Cleveland and Tiao (1976), namely

$$\nabla^2 \nabla_{12} x_t = (1 - 0.4B)(1 - 0.6B^{12})a_t, \quad (13.30)$$

the AMB estimator of the irregular component should have had  $\rho_1(\hat{u}_t) = -0.30$ . AMB estimation with this model yielded instead an estimate with  $\hat{\rho}_1(\hat{u}_t) = 0.40$  (SE=0.10), close to the X11 result and significantly different from the theoretical value implied by the model. This failure to agree with the data suggests a change of model. AMI applied to the series selects the model

$$\nabla \nabla_{12} x_t = (1 - 0.11B - 0.50B^2)(1 - 0.44B^{12})a_t, \quad (13.31)$$

which passes all TRAMO tests and implies an irregular component for which  $\rho_1(\hat{u}_t) = -0.82$ . TRAMO-SEATS outputs an irregular estimate for which  $\hat{\rho}_1(\hat{u}_t) = -0.83$ , so that the model and the data are now in agreement.

In essence, while X11 uses fixed acceptance regions for the statistics, the SEATS diagnostics are based on the statistical proximity of the statistics to the value it should have according to the underlying model. It is worth noticing that the models in the two previous examples are not in the range of models for which X11 yields results that are close to the AMB ones. In particular, the differencing operator is always  $\nabla \nabla_s$ , while in the two examples they are  $\nabla_s$  and  $\nabla^2 \nabla_s$ .

In AMB adjustment, the main diagnostic is ultimately the adequacy of the model for the series  $x_t$  and, in particular, its success in detecting and capturing seasonality. In the set of 50000 simulated series, that included non-seasonal as well as seasonal series, the percent of AMI failures in detecting seasonality or the lack thereof was 1 failure every 100 and 1 failure every 1000 series for the non-seasonal and seasonal series, respectively. There is some bias towards over-detection (mostly of stationary seasonality in non-seasonal series). As for presence of seasonality in the model residuals, it is detected in 1 every 1000 series.

### 13.5.2 Idempotency

An important property of a seasonal adjustment method should be idempotency, that is, seasonal adjustment of the seasonally adjusted series should leave the series untouched. Fixed filters will not exhibit this property and some additional variation will be further removed from the SA series. This should not be the case for a model-based adjustment. AMI applied to the SA series should provide a seasonality free model. As an example in the set of 14800 real series, 13500 required seasonal adjustment. If these adjusted series are further adjusted, seasonality is found in less than 1 every 600 series. Most diagnostics in TRAMO-SEATS consider the full span of data. Often, good fits provide resilient models (accordingly, standard seasonal adjustment routine procedures tend to check AMI once a year.) But it certainly happens that tests may yield significantly different results for different periods. For example, the idempotency test may say “no seasonality detected in the SA series” for the full period span, yet detect residual seasonality if only the last 8 years are considered (see Findley, Lytras and McElroy, 2017). When this happens, removing the first years from the series would likely be appropriate.



## 13.6 Inference

### 13.6.1 Estimation error

Given that short-term monitoring of macroeconomic time series is centered on concurrent estimators (and first revisions) of the SA series, it is important to have some measure of the precision of the estimator. For a long enough series, the error in the estimator of the component is

$$e_{t|T} = n_t - \hat{n}_{t|T} = h_t + r_{t|T}, \quad (13.32)$$

where  $h_t$  denotes the historical estimation error and  $r_{t|T}$  the revision the preliminary estimator of  $n_t$ , obtained with data up to period  $T$ , will suffer until it becomes historical.

The model for the revision error  $r_{t|T}$  was already derived in Section 13.3.2. From the model, the SD of the error  $r_{t|T}$  is easily obtained. As shown in Pierce (1979), the two errors  $h_t$  and  $r_{t|T}$  are orthogonal, and the ACF of  $h_t$  is the ACF of the ARMA model

$$\theta(B)z_t = \theta_s(B)\theta_n(B)b_t, \text{ with } b_t \sim \text{WN}\left(0, \frac{V_s V_n}{V_a}\right), \quad (13.33)$$

hence the full SD of  $(e_{t|T})$  is readily obtained. The procedure also provides SD of the forecast when  $t > T$ .

It is worth noticing that expressions (13.28) and (13.33) show that the CCF between the final estimators  $\hat{s}_t$  and  $\hat{n}_t$  is the same as the ACF of the historical estimation error  $h_t$ .

### 13.6.2 Revision in SA series

Equation (13.19) shows that the MA polynomial of the model for the series  $x_t$ ,  $\theta(B)$ , becomes the AR polynomial  $\theta(F)$  in the model for the series of total revisions. Convergence of the revision in the concurrent estimator is thus determined by the root with the largest modulus of the polynomial  $\theta(B)$ . For the majority of seasonal series,  $\theta(B)$  contains the factor  $(1 + \theta_s B^s)$  with  $0 \geq \theta_s > -1$  so that the revision in the SA series will also be seasonal.

The model for  $x_t$  is likely to contain the seasonal differencing  $\nabla_s$  in its AR polynomial (among the set of real series with seasonality, 85.3% of them required seasonal differencing.) For series that have a structure of the type  $(\bullet)\nabla_s x_t = (\bullet)(1 + \theta_s B^s)$  the following “compensation” principle applies: When  $\theta_s$  is close to  $-1$ , seasonality is very stable (in the limit it becomes deterministic.) The revision will be small but will take long to converge. When  $\theta_s$  is close to 0 and seasonality is moving, the revision will be large but will converge fast.

## 13.7 Quality of Seasonal Adjustment

### 13.7.1 Good Model, Bad Seasonal

In AMB seasonal adjustment there are two types of quality measures. One is the quality of the approximation to the dynamic structure of the data, as captured by the identified reg-ARIMA model. When the diagnostics of the fitted model are good, quality will be high: the captured seasonality will be the MMSE estimator of the seasonal component implied by the series model.

Nevertheless, it may happen that the seasonal component implied by the reg-ARIMA model shows features that make seasonal adjustment of questionable interest. For example, if the model for a monthly series is

$$x_t = 0.25x_{t-12} + a_t, \text{ or } x_t = a_t + 0.25a_{t-12},$$

unless the series is short, significant seasonal correlation will likely be detected, as well as peaks in the spectrum for the seasonal frequencies. But the seasonal component is erratic and moves fast, so that seasonality is very short lived and possibly not worth removing. However, if the model parameter is 0.9 instead of 0.25, the seasonal effect will be more persistent and hence worth removing. The lack of persistence will be mostly associated with highly stationary seasonal components. Still, where is the cutting point between worth and not-worth adjusting? It is difficult to give a precise answer, as the following examples illustrate (all are monthly series.)

Example 2 is an US employment series (MT134758) with 228 observations that start in January 1990. The detection-of-seasonality tests indicate presence of seasonality: for example, the seasonal autocorrelation test yields  $Q_s = 15.2$  (to be compared with a  $\chi^2_2$  distribution) and Figure 13.10 displays the two spectral estimators used in SEATS that evidence some moderate peaks for the seasonal frequencies (both are logged and applied to the regular difference of the series.) AMI obtains the model

$$(1 - 0.18B^{12})\nabla \log(x_t) = c + a_t,$$

with 4 outliers, and all model diagnostics are comfortably passed ( $c$  denotes a constant.) Figure 13.11 shows that the original and SA series are very close, and Figure 13.12 that the seasonal factors are not significant; moreover, as Figure 13.13 shows, for any given month, their sign is not persistent. Seasonal adjustment for this series may not be needed.

**Figure 13.10:** Example 2: US employment series (MT134758); Tukey and AR spectra on differenced series.

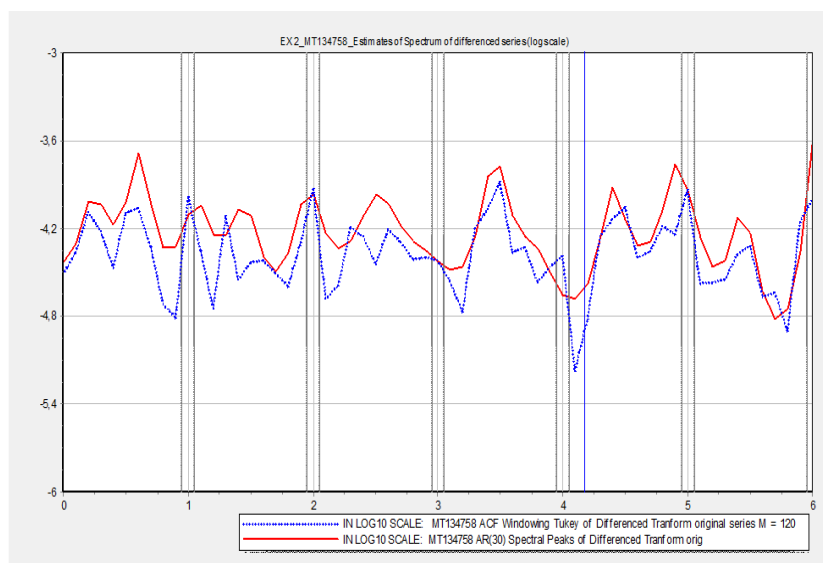


Figure 13.11: Example 2: US employment series (MT134758); Raw and SA series.

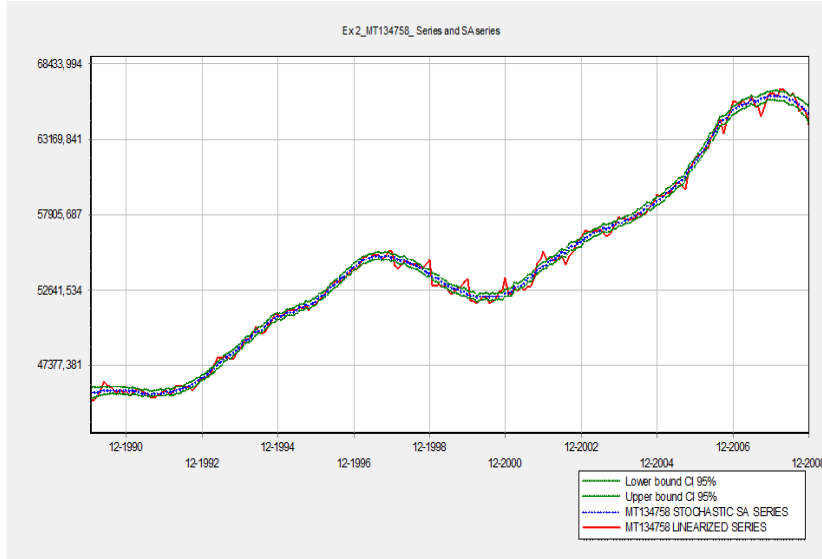
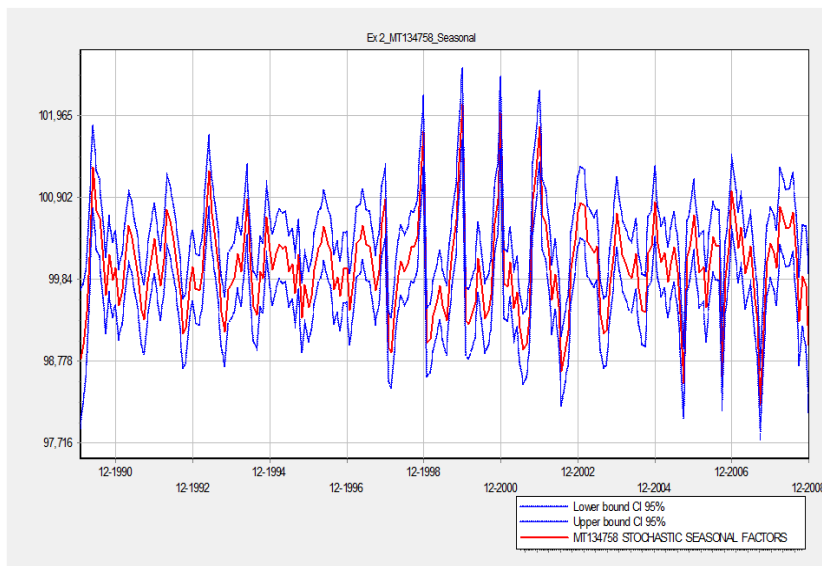
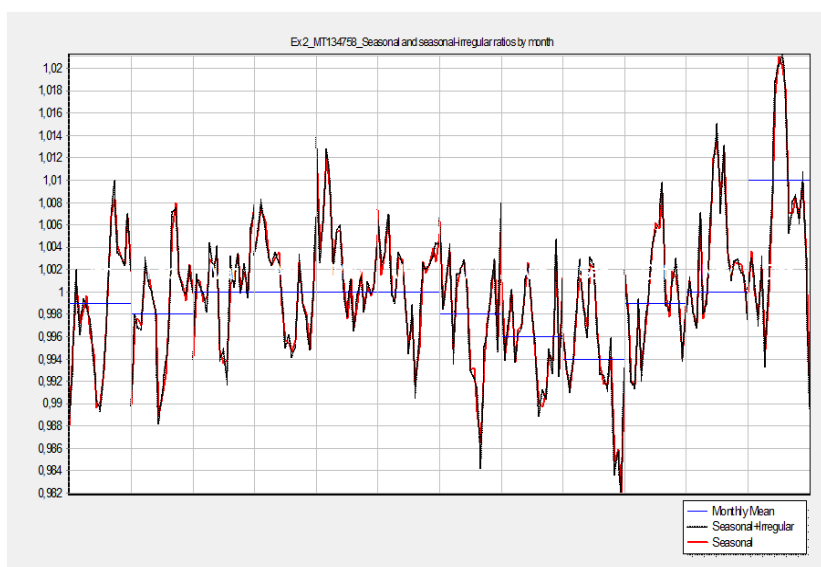


Figure 13.12: Example 2: US employment series (MT134758); Seasonal component.



**Figure 13.13:** Example 2: US employment series (MT134758); Seasonal factors and Seasonal-Irregular ratios by month.



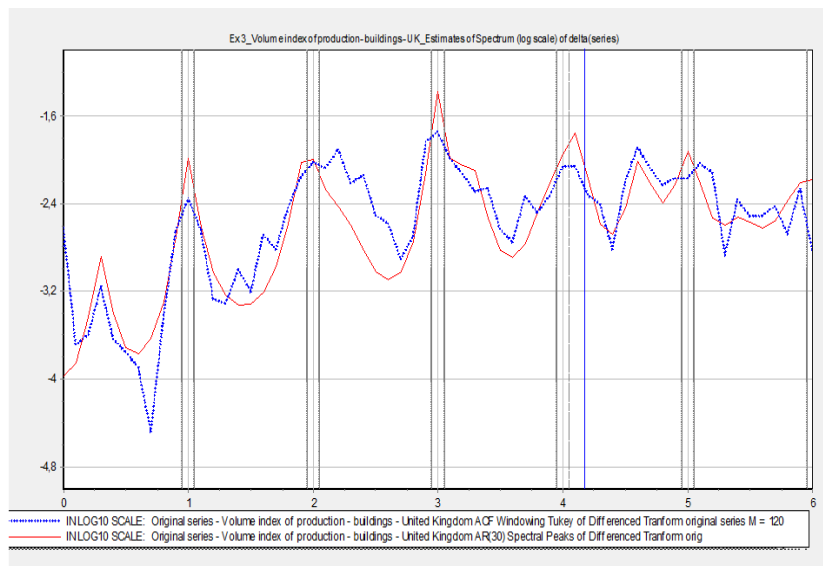
Consider now Example 3 (Volume Index of Industrial Production, Buildings, UK,) with 209 monthly observations that start in January 1996. Seasonality is detected in the ACF ( $Q_S = 17.9$ ) and in the spectrum (Figure 13.14), and the model

$$(1 - 0.31B^{12})\nabla \log(x_t) = (1 - 0.50B - 0.23B^2)a_t,$$

with 2 outliers, is produced. The model diagnostics are all satisfactory.

Figures 13.15 and 13.16 show that the series is unstable and that seasonality moves fast. This leads SEATS to classify the series as questionable. However, the last 3.5 years exhibit a considerably strong and more stable seasonality. If, for example, forecasts are desired, one may opt for extrapolating the stronger seasonality. But the instability of the seasonal casts some doubts as to whether the sequence of seasonal factors of the final 3 years will persist. In the dilemma, SEATS takes a cautious attitude: the parameter  $\phi_{12} = -0.31$  damps the seasonal effect over a horizon of a few years (Figure 13.17).

**Figure 13.14:** Example 3: Volume Index of Industrial Production, Buildings, UK; Tukey and AR spectra on log-differenced series.



When the seasonal polynomial in the model is  $(1 + \phi_s B^s)$ , the closer  $\phi_s$  is to -1, the more stable the seasonal component will be. By default, in SEATS, when  $\phi_s < -0.2$ , seasonal adjustment is always performed. When  $0 > \phi_s > -0.2$  the seasonality implied is considered unacceptably unstable. When  $\phi_s > 0$ ,  $\phi_s$  is not associated with seasonal frequencies. In the last two cases, the AR term  $(1 + \phi_s B^s)$  generates a “transitory component”.

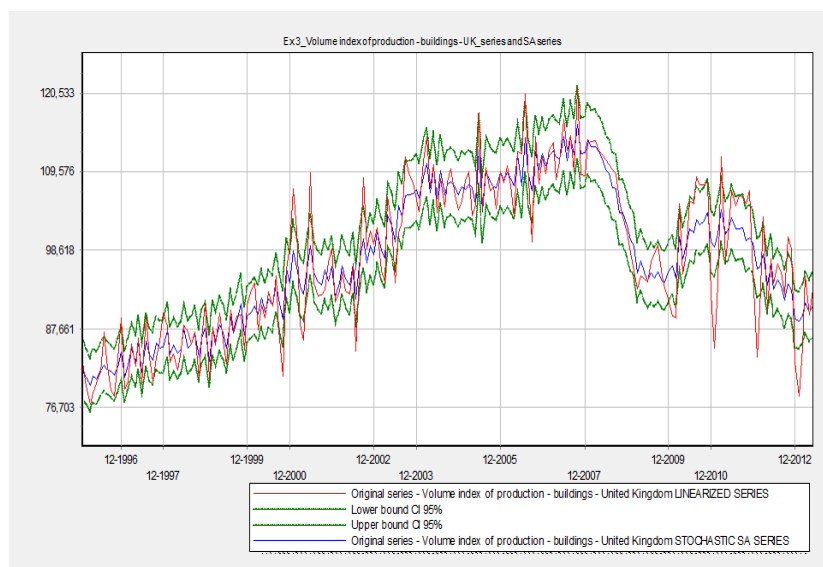
Seasonality may also show up in a model through the regular AR polynomial when the polynomial contains complex roots associated with seasonal frequencies. An example is Example 4 (series FT-106). Seasonality is detected:  $Q_S = 18.1$  and, as Figure 18 shows, the spectrum displays a large peak for the 4-times-a-year frequency. AMI provides the stationary model

$$(1 + 0.29B + 0.33B^2 - 0.58B^3)x_t = (1 + 0.11B + 0.07B^2 - 0.57B^3)a_t,$$

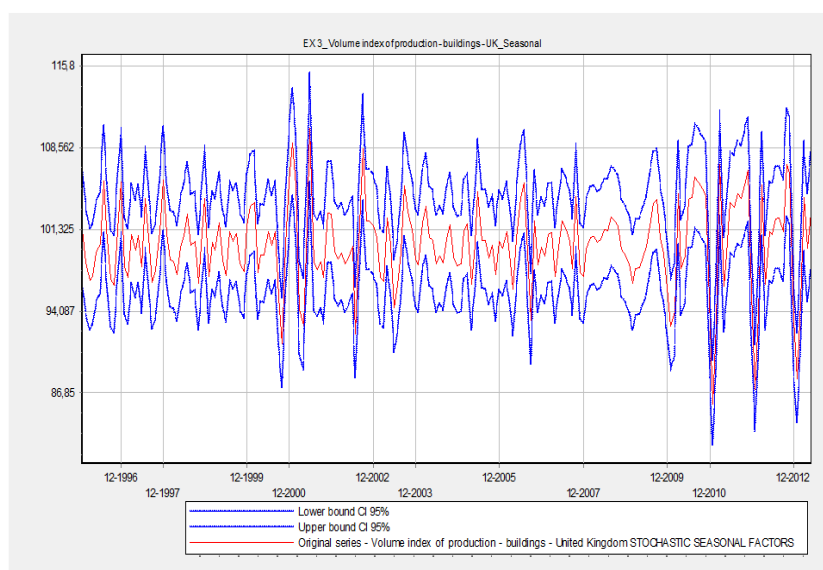
with no outliers. The AR(3) polynomial contains a complex conjugate pair of roots for the 4-times-a-year frequency with modulus of 0.96. Model diagnostics are acceptable, though SEATS classifies the series adjustment as questionable.

Figures 13.19 and 13.20 show that the SA series performs a slight smoothing of the series, and that seasonality is barely significant, although it displays a regular pattern with an amplitude that seems to follow a cyclical pattern. Figure 13.21 displays the 12 seasonal factors grouped by month. Looking at the monthly

**Figure 13.15:** Example 3: Volume Index of Industrial Production, Buildings, UK; Original and SA series with confidence intervals.



**Figure 13.16:** Example 3: Volume Index of Industrial Production, Buildings, UK; Seasonal component and confidence interval.

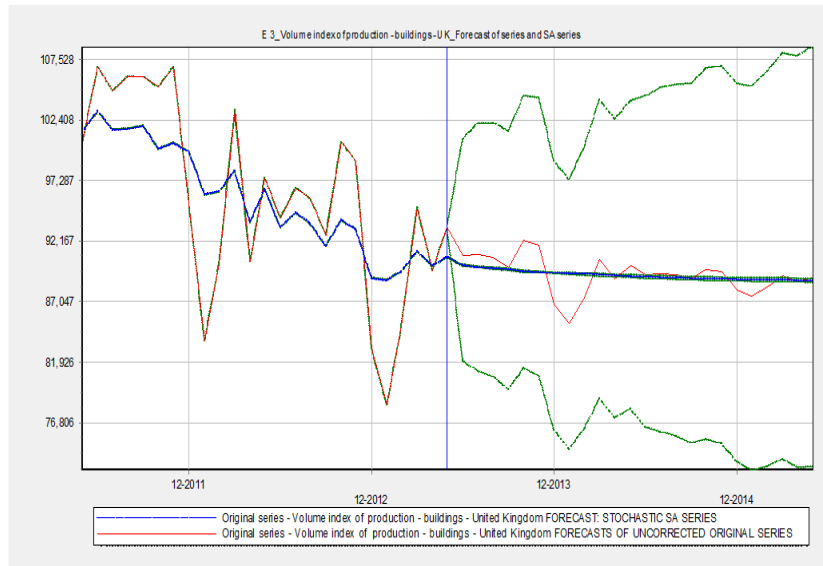


means, the 4-time-a-year pattern is clearly discernible and seasonal adjustment of the series would seem appropriate. Classifying a seasonal adjustment as questionable can be due to reasons other than a highly stationary seasonality. In SEATS the following controls are enforced.

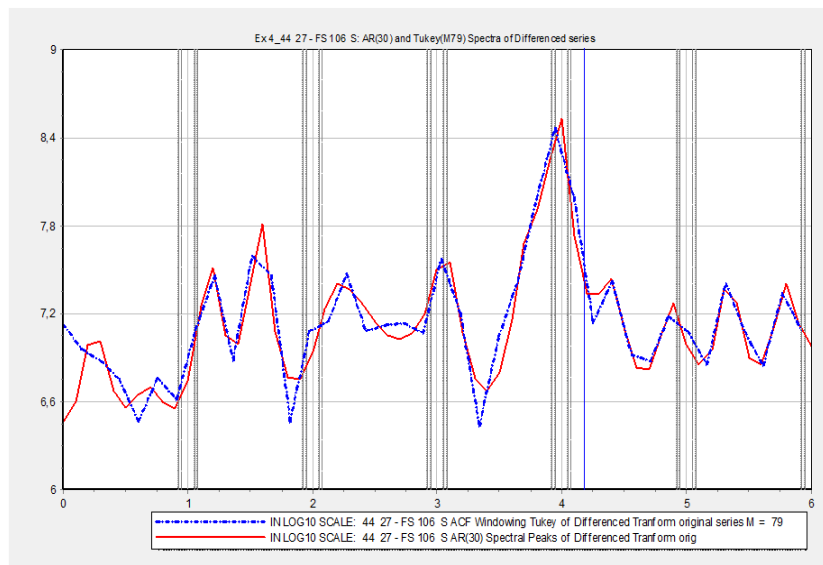
- The variance of the seasonal component innovation is too large;
- The variance of the estimation error of seasonality is too large;
- The variance of the revision the concurrent estimator will suffer is too large.
- The approximation to a model with no admissible decomposition is poor. When this happens, the seasonality estimator may on occasion be questionable.

These four controls are derived from the model underlying the series and its components; no data is needed. The first three consist of checking that some variance is not too close to the series innovation variance.

**Figure 13.17:** Example 3: Volume Index of Industrial Production, Buildings, UK; Forecasts of original and SA series.



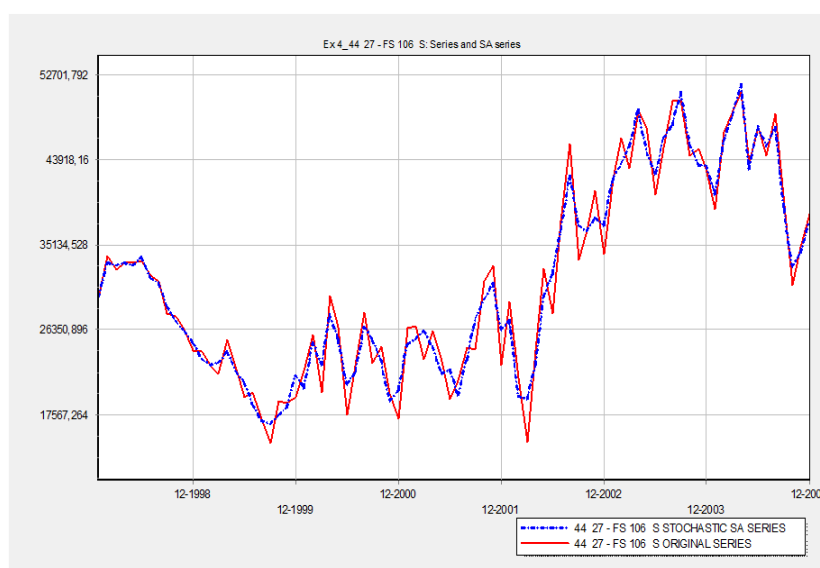
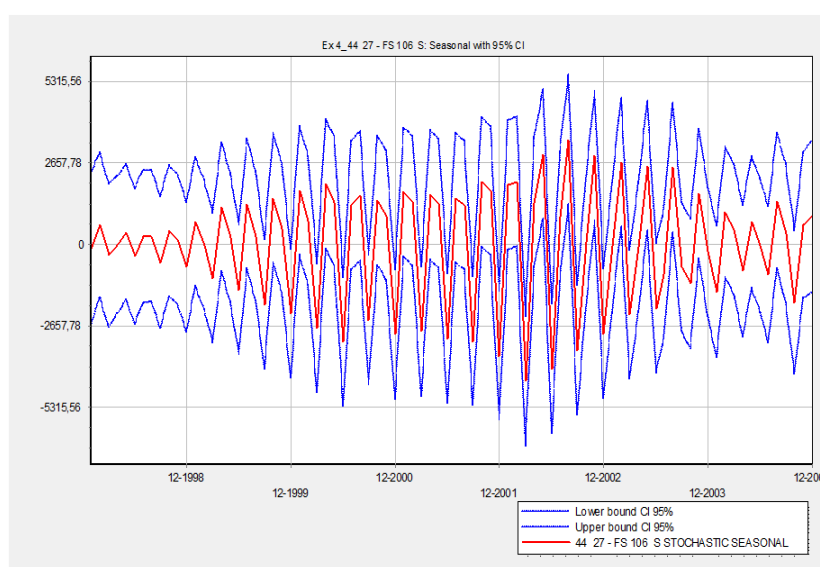
**Figure 13.18:** Example 4: series FT-106; Tukey and AR spectra of the differenced series.



Equations (9), (19), and (22) directly yield the variances of the seasonal component, its estimator, and its future revision. Three additional controls mix model and data.

- Lack of significance of the seasonal component estimates for the last year (often correlated with highly stationary seasonality).
- Lack of persistence in the sign of the seasonal component estimate for the same period of the year.
- Lack of agreement between the theoretical variances, auto-correlations, and cross-correlations of the component estimators (derived from equation 13.13) and the empirical estimates obtained from the data.

When one or more of this features is encountered, SEATS still adjusts the series but outputs the warning that seasonal adjustment is questionable.

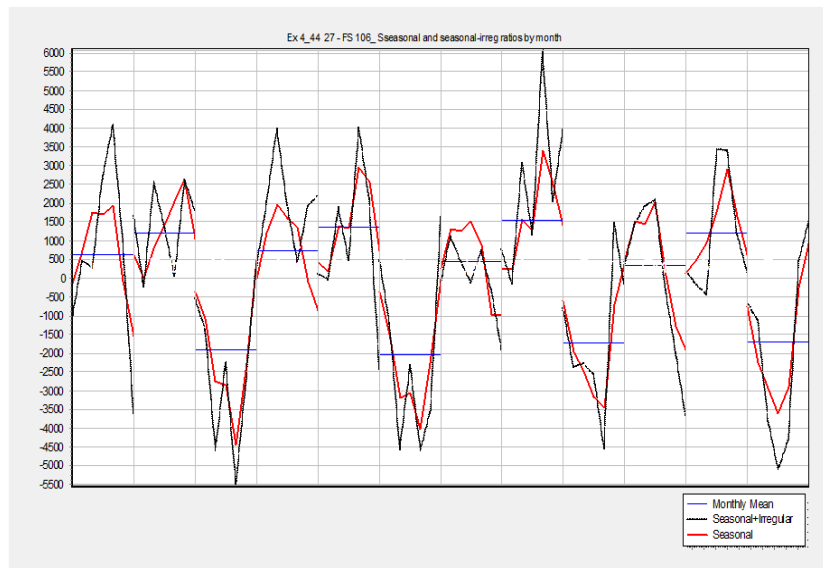
**Figure 13.19:** Example 4: series FT-106; Original and SA series.**Figure 13.20:** Example 4: series FT-106; Seasonal component and 95% confidence interval.

### 13.7.2 Bad Model, Good Seasonal

In the previous section, series with good models and poor seasonality were discussed. Now we turn to models that fail some residual diagnostics, yet yield sensible seasonality. By far, the most frequent cause of model failure is non-Normality, often due to excess kurtosis. Close to symmetric distribution of the residuals may yield acceptable results even when kurtosis is high.

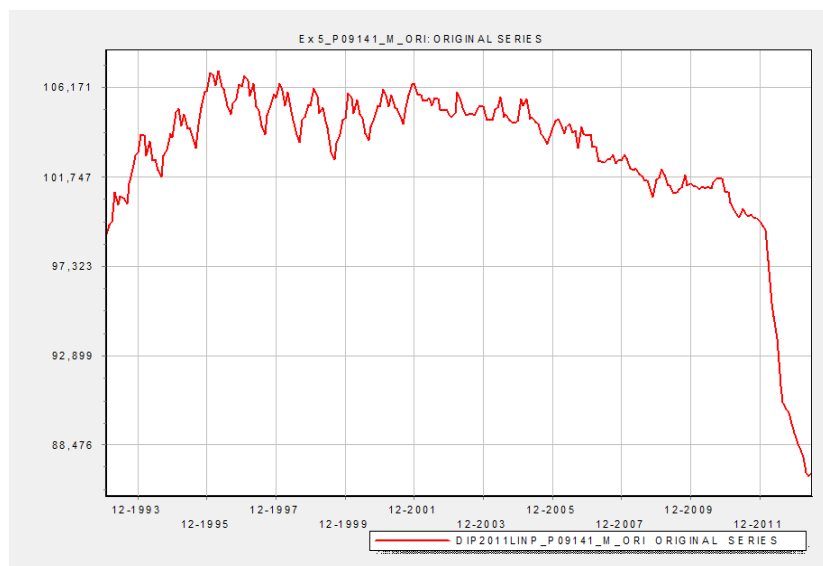
The following examples 5-8 are monthly components of the Spanish Consumer Price Index that evidence strong seasonality. In all cases TRAMO classifies as “poor” the model from AMI, yet the Quality controls of SEATS consider the seasonal adjustment acceptable. The four examples share excess kurtosis, a couple exhibit moderate excess skewness, and no other failed test is shared. The series in example 5, 6, and 7 contain 246 observations and start in January 1993; example 8 has 78 observations that start in January 2007. The four examples are shown in Figures 13.22 to 13.25, and none of them seems appropriate for a lineal model.



**Figure 13.21:** Example 4: series FT-106; Seasonal factors and Seasonal-Irregular ratios by month.

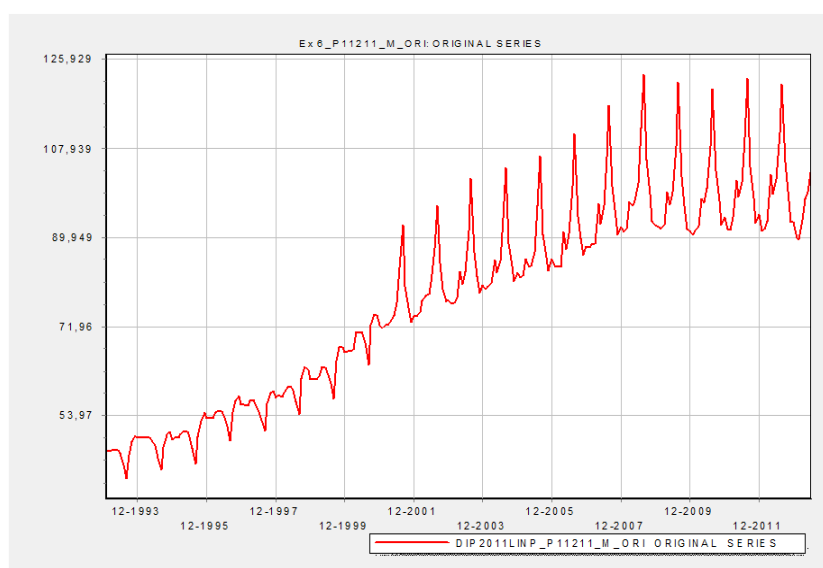
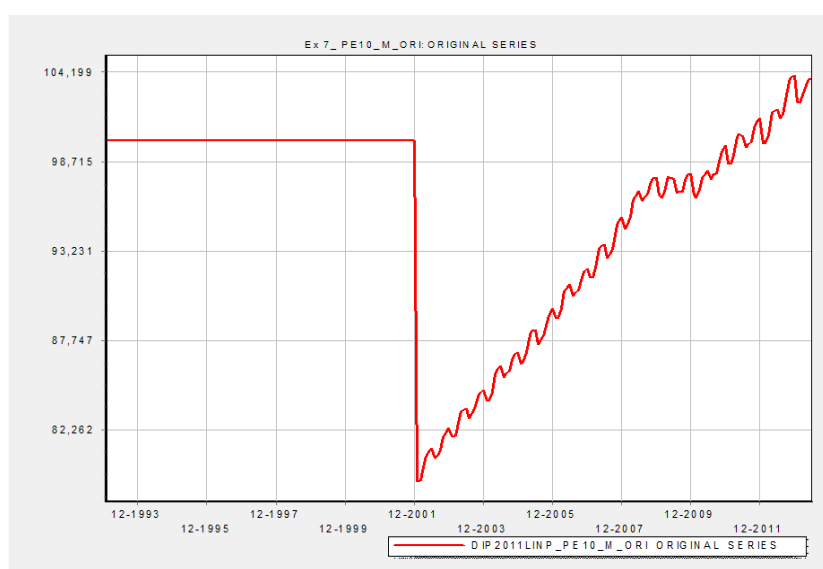
AMI yields the following ARIMA models:

- Ex. 5 :  $(1 - 0.22B^2)(1 - 0.68B^{12})\nabla \log(x_t) = a_t$  ; 6 outliers.  
 Ex. 6 :  $(1 - 0.26B^{12})\nabla\nabla_{12} \log(x_t) = a_t$  ; 5 outliers.  
 Ex. 7 :  $(1 + 0.19B^{12})\nabla\nabla_{12}x_t = (1 + 0.59B + 0.46B^2)a_t$  ; 2 outliers.  
 Ex. 8 :  $\nabla\nabla_{12} \log(x_t) = (1 + 0.95B^{12})a_t$  ; 8 outliers.

**Figure 13.22:** Example 5: Original series.

The residuals produced by the models are displayed in Figures 13.26 to 13.29; despite the diagnostics failure, the residuals look fairly random.

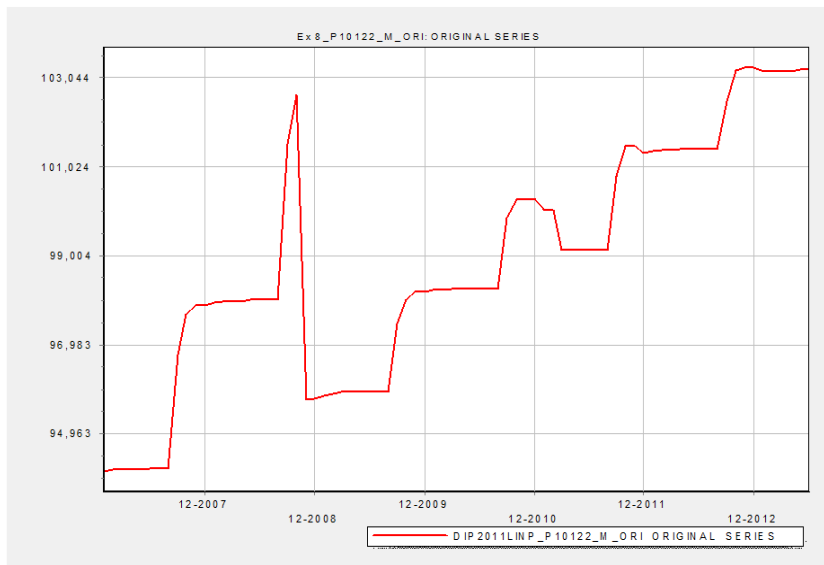
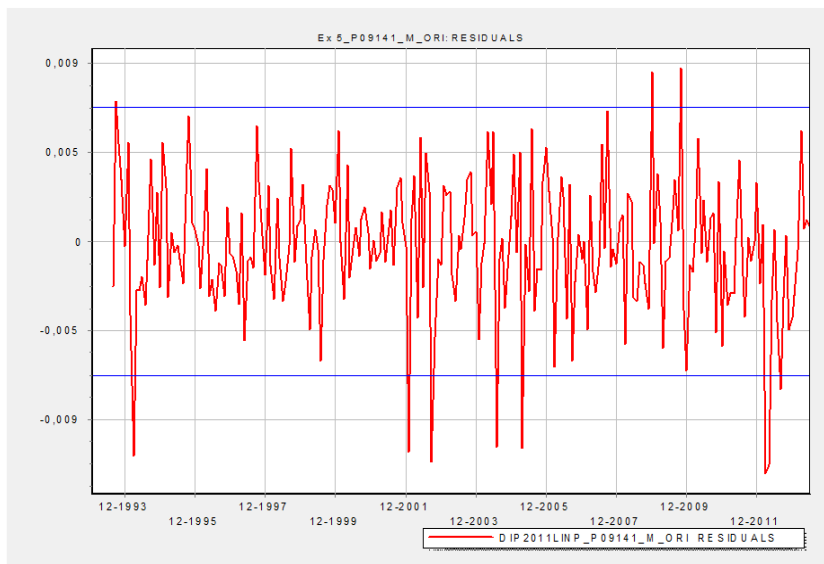
Figure 13.30 plots the original series of Example 5, the SA series, and the trend-cycle component. The series presents highly moving seasonality that becomes less relevant in the second half. In the last 18 months, the series experiences a spectacular drop and, as seen in Figure 13.31, seasonality is barely noticeable in the forecast function. Figure 13.32 exhibits the estimated seasonal and irregular factors. For the first 9 years seasonality is relatively stable and strong and becomes markedly unstable with reduced amplitude in the

**Figure 13.23:** Example 6: Original series.**Figure 13.24:** Example 7: Original series.

second half of the series. These two regimes (large and small amplitude) seem to affect also the irregular component, although the pattern is inverted. Altogether, the moving features, in particular the stable and unstable periods and the changes in amplitude of the seasonal are well captured by SEATS.

The original and SA series for Example 6 are given in Figure 13.33. The SA series is smooth and practically indistinguishable from the trend-cycle. The original series reveals two clear regimes, with the change being very sudden. Seasonality in the second half is more pronounced and the signs of the seasonal component have been inverted. Both regimes are stable and, as seen in Figure 13.34, the seasonal factors for a given month of the year are (roughly) symmetric with respect to the monthly mean. Figure 13.35 displays the seasonal and the (very small) irregular factors. The two seasonality regimes seem to have been accurately captured.

The original and the SA series for Example 7 are shown in Figure 13.36. For the first 9 years all observations are equal to 100. Then, the series drops to 79 and gradually increases subject to a seasonal perturbation. The seasonal and the very small irregular factors are displayed in Figure 13.37. Seasonal adjustment captures

**Figure 13.25:** Example 8: Original series.**Figure 13.26:** Example 5: Residuals.

well the deterministic first part, without any need to remove the first years from the series. The seasonal factors for each month of the year (Figure 13.38) show that the stochastic seasonality evolves in a stable manner and the first 6 and last 6 months of the year exhibit a roughly symmetric behavior around the midpoint of the graph.

The original and the SA series for Example 8 are plot in Figure 13.39. The series contains many short sequences of repeated numbers so that its appearance is not that of a stochastic series. Figure 13.40 displays the seasonal and irregular component that SEATS produces; the seasonal is stable (close to deterministic) and the irregular is negligible. Figure 13.41 displays the seasonal factors for a fixed month of the year.

The number of outliers is possibly too high, but if the series is part of a set of many series that cannot be individually checked and need automatic default treatment, the results for Example 8 would provide a reasonable SA series.

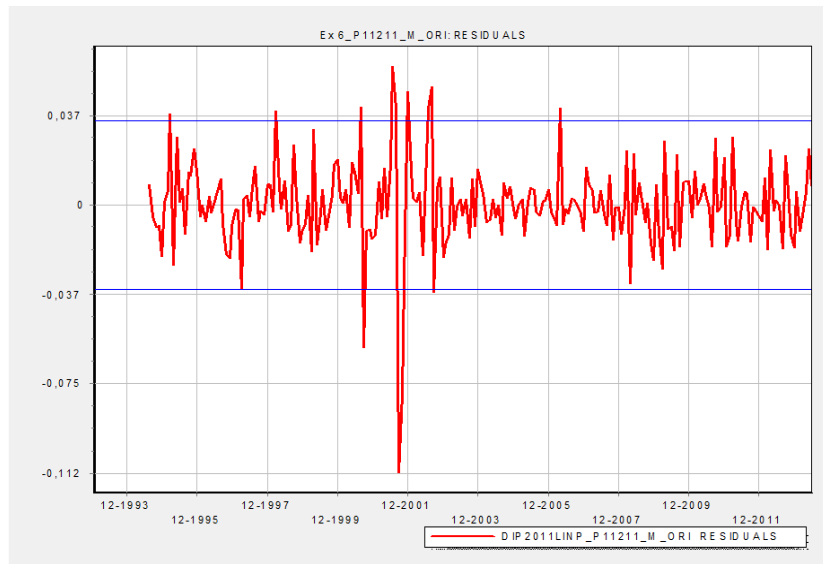
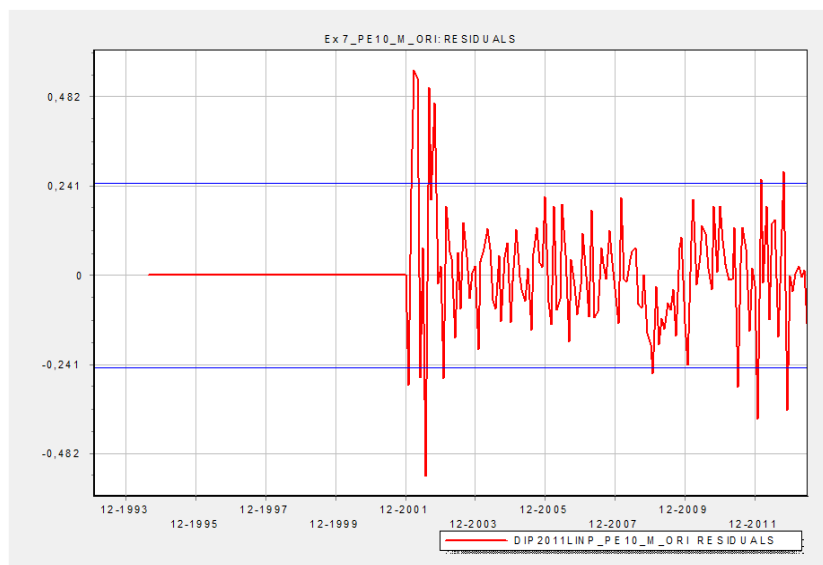
**Figure 13.27:** Example 6: Residuals.**Figure 13.28:** Example 7: Residuals.

Figure 13.29: Example 8: Residuals.

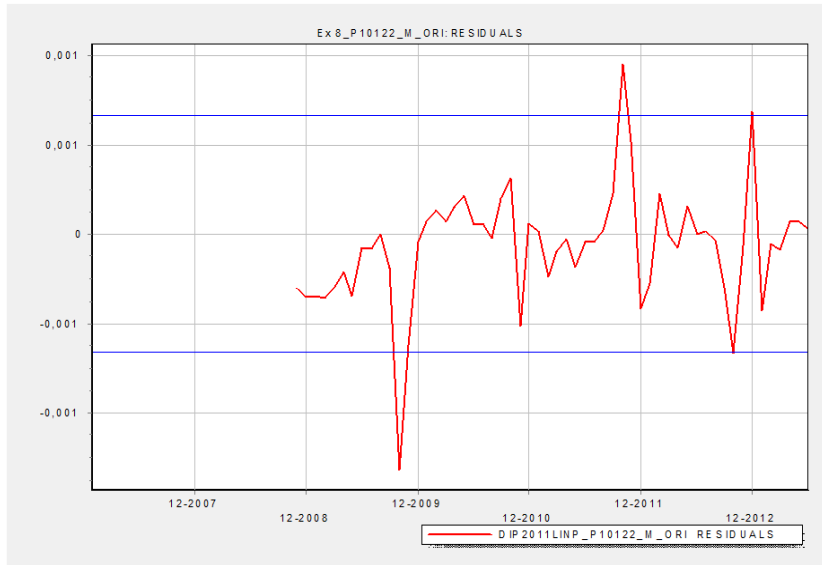
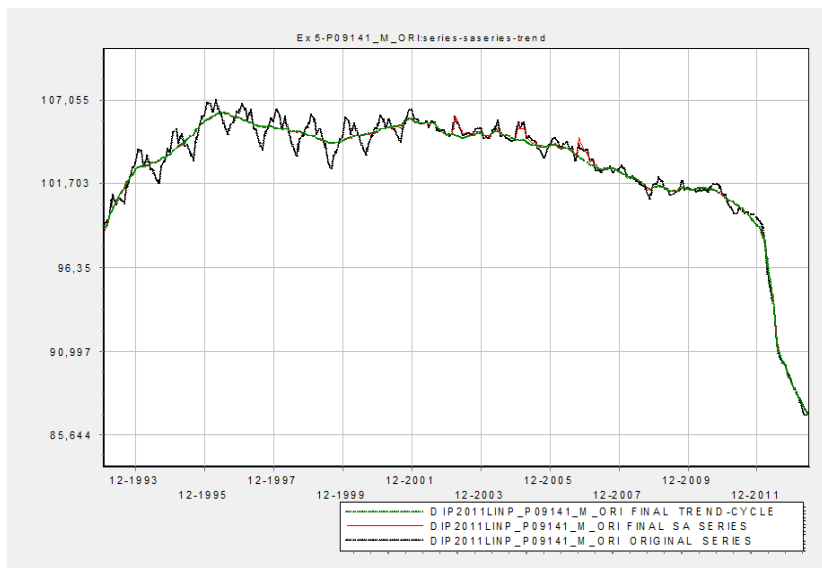


Figure 13.30: Example 5: Original, SA and Trend-cycle series.



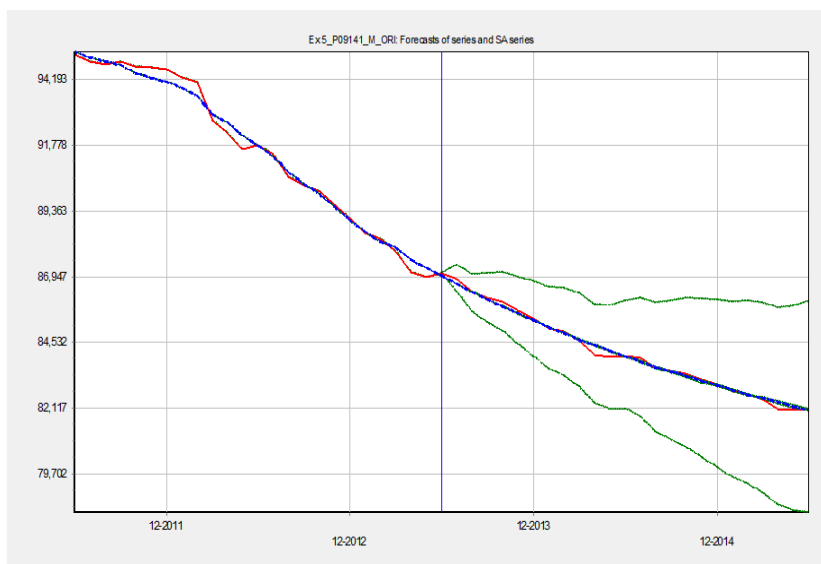
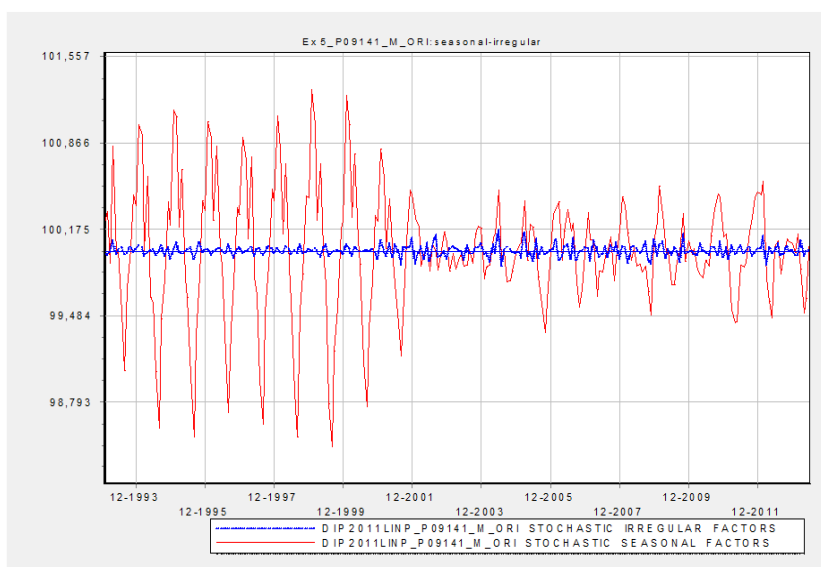
**Figure 13.31:** Example 5: Forecasts of original and SA series.**Figure 13.32:** Example 5: Seasonal and Irregular components.

Figure 13.33: Example 6: Original and SA series.

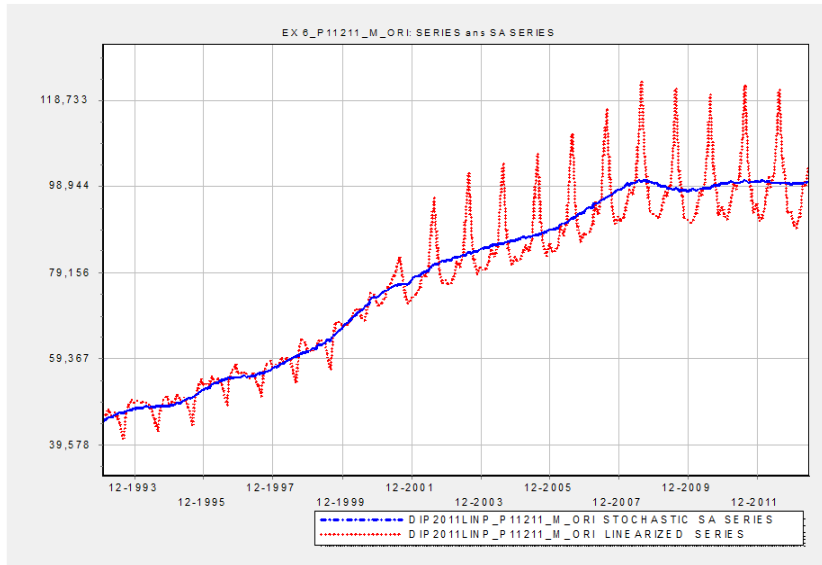


Figure 13.34: Example 6: Seasonal factors and Seasonal-Irregular ratios by month.

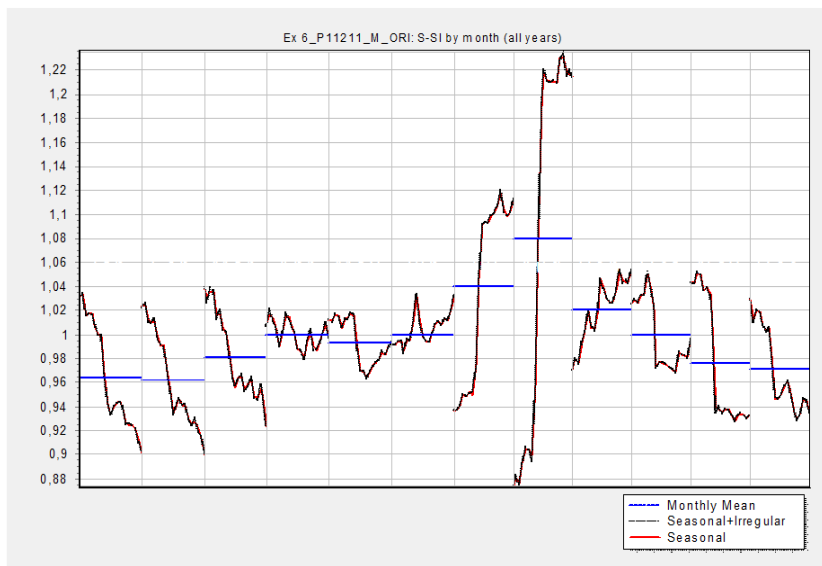


Figure 13.35: Example 6: Seasonal and Irregular components.

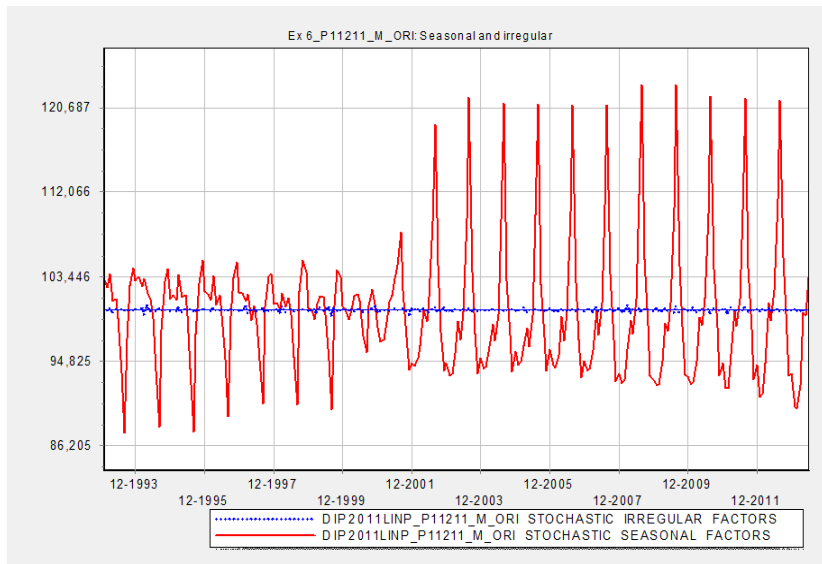


Figure 13.36: Example 7: Original and SA series.

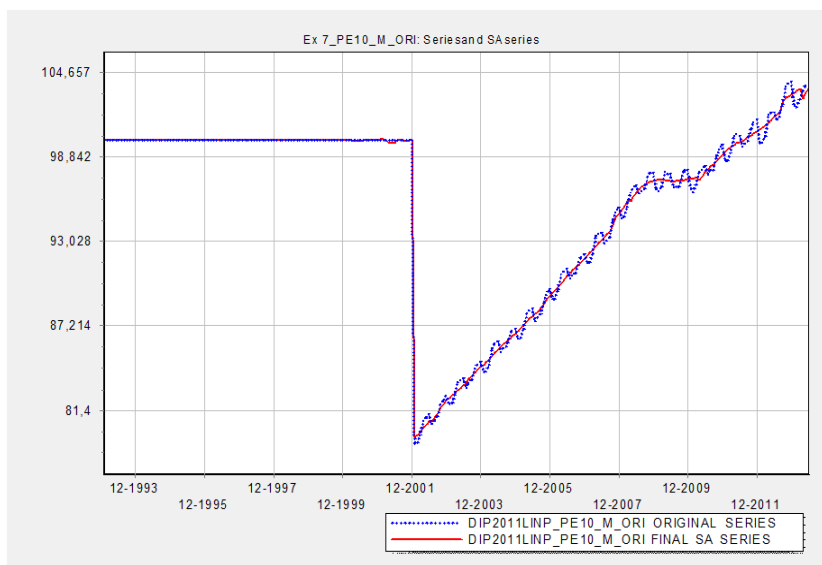




Figure 13.37: Example 7: Seasonal and Irregular components.

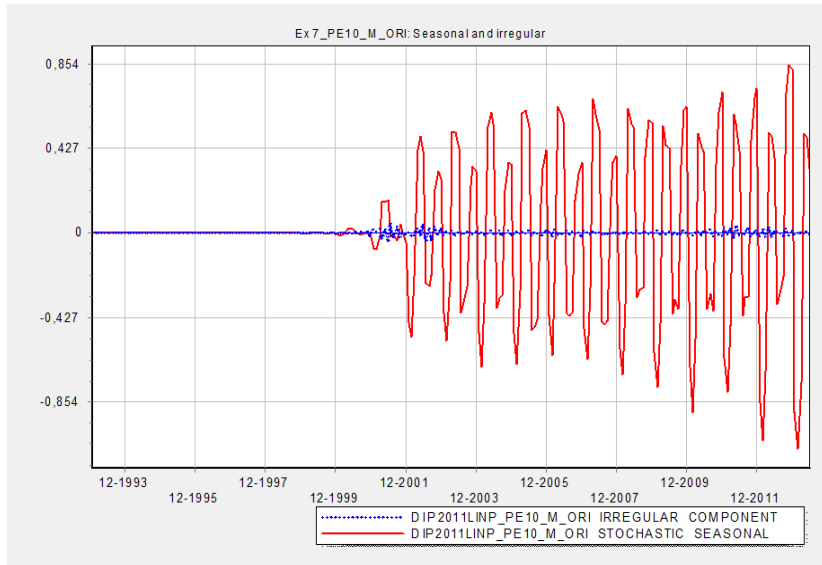


Figure 13.38: Example 7: Seasonal factors and Seasonal-Irregular ratios by month.

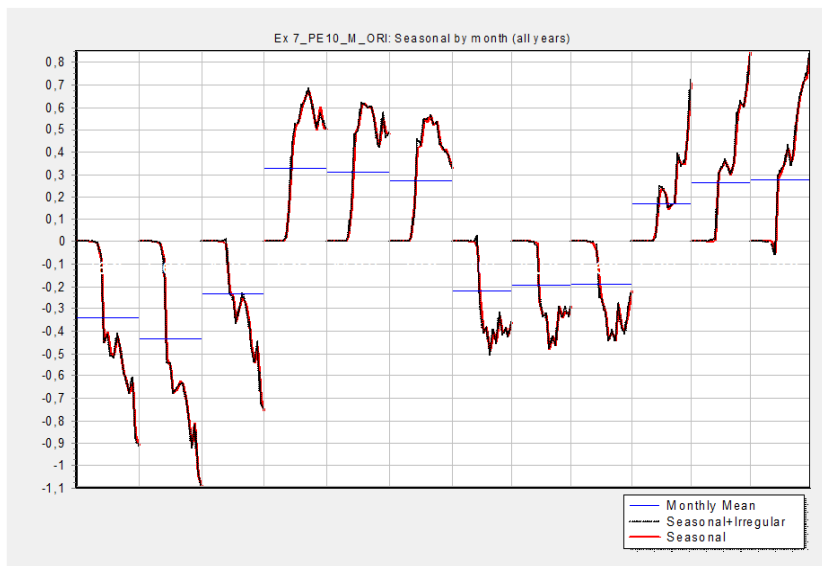


Figure 13.39: Example 8: Original and SA series.

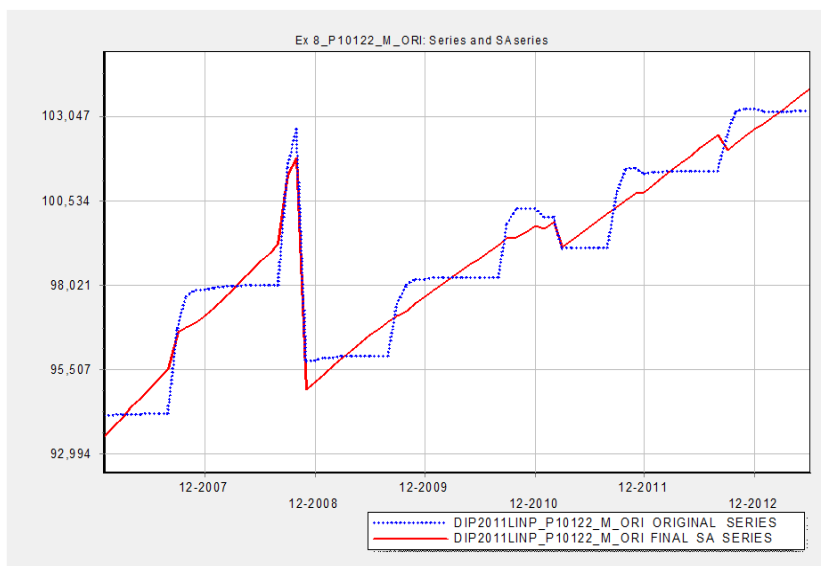


Figure 13.40: Example 8: Seasonal factors and Seasonal-Irregular ratios by month.

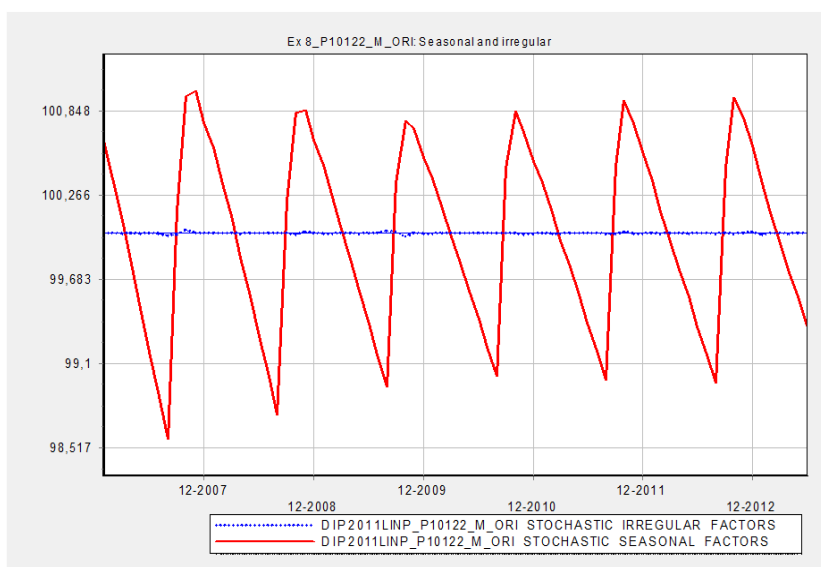
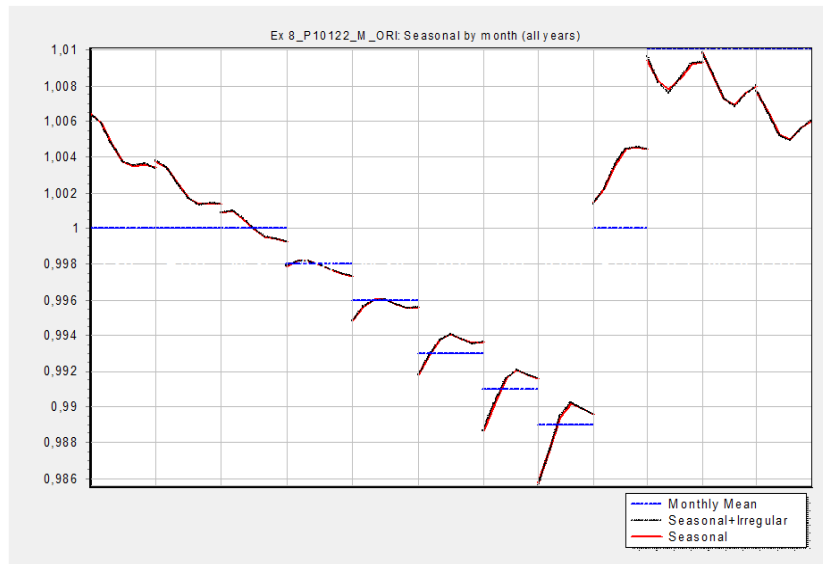


Figure 13.41: Example 8: Seasonal and Irregular components.



## 13.8 Conclusions

The ARIMA-Model-Based method of TRAMO-SEATS is used in its default automatic mode for seasonal adjustment. (This is the relevant mode when a large number of series need to be adjusted.) First, a brief review of the method is presented. A regression-ARIMA model is obtained for the observed series, and the model is decomposed into two models, one for the seasonal component, another one for the SA series. Next, the filters that yield the estimators of the two components are derived from the models as the MMSE linear estimators.

The filters are two-sided and hence, for the early and (more relevantly) the final periods of observation of the series, the estimator will be preliminary; for the last periods it will be revised as new observations become available. For the central periods of a long enough series, convergence of the filter yields the historical estimator (that will not be revised). Both types of estimators are seen to follow ARIMA-type models that share the innovations of the observed series model. The joint distribution of the series and the historical and preliminary estimators (and of the associated revision) is straightforward to compute.

From the models, some properties of the estimators are discussed. First, it is seen that the estimator of the SA series will be a non-linear series due to the combination of historical and preliminary estimators. Further, unless the series is stationary, the estimator will also be very close to non-invertibility thus estimates of the coefficients of pure AR models fit to SA series will be unstable. The underlying model being more complex, it is also of interest to check whether the fitting of simple ARIMA models to SA series proves unreliable. Using the AMI of TRAMO-SEATS on a set of monthly real series it is concluded that, although seasonal adjustment damages the fitting, this damage is not dramatic.

The model for the seasonal component given by (13.9) defines the seasonality present in the series under consideration, and the model-structure allows for parametric inference and diagnostics. Important inferences are MMSE forecasts of the components, as well as the standard error of the SA series estimator (preliminary, historical, and forecasts). As for diagnostics, two types need to be distinguished: one is the quality of the model fit to the series, the other is the quality of the seasonal adjustment. The first one consists of standard tests of the normally-independently-identically distributed assumption made on the residuals of the series model, complemented with out-of-sample forecast tests. Concerning quality of the adjustment, two basic checks are testing for residual seasonality in the SA series, and idempotency (seasonal adjustment of the SA series should replicate the SA series). While fixed filters cannot satisfy the idempotency requirement; in the AMB approach the model for the SA series would likely be identified as non-seasonal, and hence would not be adjusted.

Application of fixed filters, such as X11, to a large set of series produces seasonal components for all the series in the set. To minimize the lack of idempotency, and to avoid spurious seasonality in the estimator when the series contains none, ad-hoc quality criteria are employed that aim to reflect requirements of a “well-behaved” seasonal component. These requirements are independent of the stochastic structure of the series at hand, and although many standard series will respect them, for series with awkward structures they may be inadequate. Section 13.5.1 provides two examples of series for which the X11 M-statistics question the adjustment, yet TRAMO-SEATS provides a sensible one (both series follow ARIMA models that are not close to the ones for which X11 would be appropriate).

In SEATS, knowledge of the models for the estimators, ACF and CCF of their stationary transformation can be derived and compared to the empirical estimates obtained in the application. If the model for the observed series is appropriate and the previous comparisons are satisfactory, the components will have been correctly estimated. Therefore, an estimator that produces a seasonal component that moves perhaps too fast, should not cast doubts on the method; more likely, it would reflect the stochastic properties of the series. SEATS derives relevant properties such as the statistical significance of the seasonal component, the variance of the innovation in the seasonal component (if large, seasonality will move fast,) the variance of its estimation error (if large, the SA series may be of no interest,) the variance of the revision error in preliminary estimators (if large, the concurrent estimator may be useless). Hence it may happen that a good model fit provides a

decomposition of questionable interest. A series may evidence seasonality, its modeling may be satisfactory and seasonal adjustment may remove the evidence of seasonality, yet the seasonal component may be so unstable that adjustment of the series may be judged unjustified.

Setting up a cutting point between justified and unjustified adjustment is a delicate question. This point is illustrated with three examples of series that evidence seasonality, whose ARIMA modeling is satisfactory, for which adjustment effectively removes seasonality, yet produce unstable seasonal components. As is most frequently the case, this lack of stability is associated with a highly stationary seasonal component, which translates into a very low parameter in the seasonal AR polynomial. It is argued that, even when this parameter is low, it may be worth to adjust the series. After all, the decision not to adjust a series that evidences seasonality has an obvious drawback: Why not accept then adjustments with residual seasonality in the SA series? What is the point of testing for residual seasonality in the adjusted series? In SEATS all series with significant seasonality are adjusted and a message is sent when the seasonality is of questionable interest.

On the other hand, it has been often said that when the model fit is not good, AMB seasonal adjustment will be bad, and that dependence on the model is the big weakness of the method. This belief is partly unjustified. In the set of 14800 series, 9.7% of the models identified by the automatic run of TRAMO were not acceptable. In 80% of the cases, non-Normality was the main reason; kurtosis, specifically, accounted for 82% of them. As mentioned earlier, excess kurtosis does little damage to point estimation of the components, and the decomposition may still be sensible. Four examples illustrate this feature. They contain series with strong seasonality, very high kurtosis, and a drastic change in regime. Due to the flexibility of ARIMA models, helped by preadjustment, the decomposition is seen to capture seasonality well. There is no need, besides, to remove the first regime in order to homogenize the series. The SA series adapts fast to the different regimes, including deterministic ones. If the ceiling for kurtosis is removed, for more than 95% of the series TRAMO-SEATS in automatic mode provides an acceptable decomposition. The flexibility of the reg-ARIMA model to adapt to series that contain markedly different regimes is particularly relevant when adjusting sets of very many series for which appropriate truncation of each individual series would be difficult to enforce.

Altogether, the model-based signal extraction approach of TRAMO-SEATS provides a more accurate and complete description of reality than that provided by fixed filters. Ultimately, as Hawking and Mlodinow state, no test of reality can be model independent.

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# IV

## Improving End-Point Estimates for Seasonal Adjustment







# 14

## Trend-Cycle Estimation



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## Table of Contents

14.1	Introduction	367
14.2	Deterministic and stochastic global trend models	368
14.3	Local trend-cycle models	370
14.4	Local trend cycle estimation	372
14.4.1	Locally Weighted Regression Smoother (LOESS)	372
14.4.2	Henderson Smoothing Filters	372
14.4.3	Dagum's modified 13-term Henderson Filter (DMH)	375
14.5	Gaussian Kernel Smoother	377
14.6	Cubic Smoothing Spline	378
14.7	Theoretical properties of the symmetric and asymmetric linear trend-cycle filters	381
14.8	Illustrative Results	382

## 14.1 Introduction

Most information in social sciences, biology, and many other sciences occurs in the form of time series where their main property is that the observations are dependent and the nature of this dependence is of interest in itself. A time series is a finite realization of a stochastic process and often compiled for consecutive and equal period, such as weeks, months, quarters, and years. In time series decomposition, four types of movements have been traditionally distinguished, namely, the trend, the cycle, the seasonal variations (for sub annual data), and the irregular fluctuations. As a matter of statistical description, a given series can always be represented by one of these components or a sum of several of them. The four components are usually interrelated and for most series, they influence one another.

The trend corresponds to sustained and systematic variations over a long period of time. It is associated with the structural causes of the phenomenon in question, for example, population growth, technological progress, new ways of organization, or capital accumulation. For the majority of socioeconomic time series, the trend is very important because it dominates the total variation of the series. The identification of trend has always posed a serious statistical problem. The problem is not one of mathematical or analytical complexity but of conceptual complexity. This problem exists because the trend as well as the remaining components of a time series are latent (no directly observables) variables and, therefore, assumptions must be made on their behavioural pattern. The trend is generally thought of as a smooth and slow movement over a long term. The concept of “long” in this connection is relative and what is identified as trend for a given series span might well be part of a long cycle once the series is considerably augmented, such as the Kondratieff economic cycle. Kondratieff (1925) estimated the length of this cycle to be between 47 and 60 years. Often, a long cycle is treated as a trend because the length of the observed time series is shorter than one complete face of this type of cycle.

To avoid the complexity of the problem posed by a statistically vague definition, statisticians have resorted to two simple solutions: One consists of estimating trend and cyclical fluctuations together calling this combined movement trend-cycle; the other consists of defining the trend in terms of the series length, denoting it as the longest non periodic movement. The estimation of the time series trend-cycle can be done via a specified model applied to the whole data called the global trend-cycle, or by fitting a local polynomial function in such a way that, at any time point, its estimates depend on only the observations at that point and some specified neighbouring points.

Local polynomial fitting has a long history in the smoothing of noisy data. Henderson (1916), Whittaker and Robinson (1924) and Macauley (1931) are some of the earliest classical references. These authors were very much concerned with the smoothing properties of linear estimators, being Henderson (1916), the first to show that the smoothing power of a linear filter depends on the shape and values of its weighting system. On the other hand, more recent contributions (among others, Eubank (1988); Hardle (1990); Fan (1992) and Fan (1993); Green and Silverman (1994); Fan and Gijbels (1997); Wand and Jones (1995); Simonoff (1995)) concentrated on the asymptotic statistical properties of optimally estimated smoothing parameters. Optimality being defined in terms of minimizing a given loss function, usually, the mean square error or the prediction risk. In this chapter we will review some of the stochastic and deterministic trend-cycle models formulated for global and local estimation with special reference to the Henderson filters used in the Census X11 and its variants X11ARIMA and X12 ARIMA .

## 14.2 Deterministic and stochastic global trend models

Deterministic and stochastic global trend models are based on the assumption that the trend or nonstationary mean of a time series can be approximated closely by simple functions of time over the entire span of the series. The most common representation of a deterministic trend is by means of polynomials functions. The time series from which the trend is to be identified is assumed to be generated by a nonstationary process where the nonstationary property results from a deterministic trend. A classical model is the regression or error model (Anderson (1971)) where the observed series is treated as the sum of a systematic part or trend and a random part or irregular. This model can be written as

$$Z_t = Y_t + \mu_t \quad (14.1)$$

where  $\mu_t$  is a purely random process, that is,  $\mu_t \sim i.i.d.(0, \sigma_\mu^2)$  (independent and identically distributed with expected value 0 and variance  $\sigma_\mu^2$ ). In the case of a polynomial trend-cycle,

$$Y_t = a_0 + a_1t + a_2t^2 + \dots + a_nt^n \quad (14.2)$$

where generally  $n \leq 3$ . The trend is said to be of a deterministic character because it is not affected by random shocks which are assumed to be uncorrelated with the systematic part. Model (1) can be generalized by assuming that  $\mu_t$  is a second-order linear stationary stochastic process, that is, its mean and variance are constant and its autocovariance is finite and depends only on the time lag. Besides polynomials in time, other suitable mathematical functions are used to represent deterministic trends. Three of the most widely applied functions, known as growth curves, are the modified exponential, the Gompertz, and the logistic. The modified exponential trend can be written as

$$Y_t = a + bc^t, \quad a \text{ real}, \quad b \neq 0, \quad c > 0, \quad c \neq 1 \quad (14.3)$$

For  $a = 0$ , model (3) reduces to the unmodified exponential trend

$$Y_t = bc^t = Y_0e^{\alpha t}; \quad b = Y_0, \quad \alpha = \log c \quad (14.4)$$

when  $b > 0$  and  $c > 1$ , and so  $\alpha > 0$ , model (4) represents a trend that increases at a constant relative rate  $\alpha$ . For  $0 < c < 1$ , the trend decreases at the rate  $\alpha$ . Models (3) and (4) are solutions of the differential equation

$$dY_t/dt_t = \alpha(Y - a), \quad \alpha = \log c, \quad (14.5)$$

which specifies the simple assumption of no inhibited growth. Several economic variables during periods of sustained growth or of rapid inflation, as well as population growths measured in relative short periods of time, can be well approximated by trend models (3) and (4). But in the long run, socioeconomic and demographic time series are often subject to obstacles that slow their time path, and if there are no structural changes, their growth tend to a stationary state. Quetelet made this observation with respect to population growth and Verhulst (1838) seems to have been the first to formalize it by deducing the logistic model. Adding to eq. 14.5 an inhibit factor proportional to  $-Y^2$ , the result is

$$\begin{aligned} dY_t/dt &= \alpha Y - \beta Y^2 = \alpha Y(1 - Y/k), \\ k &= \alpha/\beta, \alpha, \beta > 0 \end{aligned} \quad (14.6)$$

which is a simple null form of the Riccati differential equation. Solving eq. (6), we obtain the logistic model,

$$Y_t = k(1 + ae^{-\alpha t})^{-1}, \quad (14.7)$$

where  $a > 0$  is a constant of integration.

Model (7) belongs to a family of S-shaped curves generated from the differential equation (see Dagum (1985):

$$dY_t/dt = Y\psi(t)\phi(Y_t/k), \quad \phi(1) = 0. \quad (14.8)$$

Solving eq. (8) for  $\psi = \log c$  and  $\phi = \log(Y_t/k)$ , we obtain the Gompertz curve used to fit mortality table data; that is,

$$Y_t = kb^{c^t}, \quad b > 0, b \neq 1, 0 < c < 1, \quad (14.9)$$

where  $b$  is a constant of integration.

It should be noted that differencing will remove polynomial trends and suitable mathematical transformations plus differencing will remove trends from nonlinear processes; e.g., for (7) using  $Z_t = \log[Y_t/(k - Y_t)]$  and then taking differences gives  $\Delta Z_t = \alpha$ . The second major class of global trend models is the one that assumes the trend to be a stochastic process, most commonly that the series from which the trend will be identified follows a homogeneous linear nonstationary stochastic process (Yaglom (1962)). Processes of this kind are nonstationary, but applying a homogeneous filter, usually the difference filter, we obtain a stationary process in the differences of a finite order. In empirical applications, the nonstationarity is often present in the level and/or slope of the series; hence, the order of the difference is low. An important class of homogeneous linear nonstationary processes are the ARIMA (autoregressive integrated moving average processes) which can be written as (Box and Jenkins (1970))

$$\begin{aligned} \phi_p(B)\Delta^d Y_t &= \theta_q(B)a_t, \\ a_t &\sim i.i.d.(0, \sigma_a^2) \end{aligned} \quad (14.10)$$

where  $B$  is the backshift operator such that  $B^n Y_t = Y_{t-n}$ ;  $\phi_p(B)$  and  $\theta_q(B)$  are polynomials in  $B$  of order  $p$  and  $q$ , respectively, and satisfy the conditions of stationarity and invertibility;  $\Delta_d = (1 - B)^d$  is the difference operator of order  $d$  and  $a_t$  is a purely random process. Model (10) is also known as an ARIMA process of order  $(p, d, q)$ . If  $p = 0$ , the process follows an *IMA model*. Three common stochastic trend models are the *IMA(0, 1, 1)*, *IMA(0, 2, 2)*, *ARMA(2, 1, 2)* which take the form, respectively,

*IMA(0, 1, 1)*

$$\begin{aligned} (1 - B)Y_t &= (1 - \theta B)a_t, \quad |\theta| < 1, \\ a_t &\sim i.i.d.(0, \sigma_a^2) \end{aligned} \quad (14.11)$$

or, equivalently,

$$Y_t = Y_{t-1} + a_t - \theta a_{t-1},$$

*IMA(0, 2, 2)*

$$\begin{aligned}(1 - B)^2 Y_t &= (1 - \theta_1 B - \theta_2 B^2) a_t, \\ \theta_2 + \theta_1 &< 1, \theta_2 - \theta_1 < 1, -1 < \theta_2 < 1 \\ a_t &\sim i.i.d.(0, \sigma_a^2)\end{aligned}\quad (14.12)$$

or equivalently,

$$Y_t = 2Y_{t-1} - Y_{t-2} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2}$$

, and

*ARMA*(2, 1, 2)

$$\begin{aligned}(1 - \phi_1 B - \phi_2 B^2)(1 - B)Y_t &= (1 - \theta_1 B - \theta_2 B^2)a_t, \\ \phi_2 + \phi_1 &< 1, \phi_2 - \phi_1 < 1, -1 < \phi_2 < 1 \\ \theta_2 + \theta_1 &< 1, \theta_2 - \theta_1 < 1, -1 < \theta_2 < 1 \\ a_t &\sim i.i.d.(0, \sigma_a^2)\end{aligned}\quad (14.13)$$

or equivalently

$$Y_t = (\phi_1 + 1)Y_{t-1} - (\phi_1 - \phi_2)Y_{t-2} - \phi_2 Y_{t-3} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2}. \quad (14.14)$$

The  $a$ 's may be regarded as a series of random shocks that drive the trend and  $\theta$  can be interpreted as measuring the extent to which the random shocks or "innovations" incorporate themselves into the subsequent history of the trend. For example, in model (11) the smaller the value of  $q$ , the more flexible the trend; the higher the value of  $\theta$ , the more rigid the trend (less sensitive to new innovations). For  $\theta = 1$ , model (11) reduces to one type of random walk model which has been used mainly for economic time series such as stock market price data (Granger and Morgenstern (1970)). In such models, as time increases the random variables tend to oscillate about their mean value with an ever increasing amplitude. The use of stochastic models in business and economic series has received considerable attention during recent years (see, for example, Burman (1980), Nelson and Plosser (1982), and Harvey (1985)).

## 14.3 Local trend-cycle models

Economists and statisticians are often interested in the short term trend of socio-economic time series. The short term trend generally includes cyclical fluctuations and is referred to as trend-cycle. In recent years, there has been an increased interest to use trend-cycle estimates or smoothed seasonally adjusted data to facilitate recession and recovery analysis. Among other reasons, this interest originated from major economic and financial changes of global nature which have introduced more variability in the data, and consequently, in the seasonally adjusted numbers. This makes very difficult to determine the direction of the short-term trend, particularly to assess the presence or the upcoming of a turning point. The local polynomial regression predictors developed by Henderson (1916) and LOESS due to Cleveland (1979) are the most widely applied to estimate the short-term trend of seasonally adjusted economic indicators. Particularly, the former is available in nonparametric seasonal adjustment software such as the U.S. Bureau of the Census X11 method (Shiskin et al. (1967)) and its variants the X11ARIMA (Dagum (1980) and Dagum (1988)) and X12ARIMA (Findley et al. (1998)), the latter, in STL (Cleveland et al. (1990)). The basic assumption is that the input series  $\{y_t, t = 1, 2, \dots, N\}$  can be decomposed into the sum of a systematic component called the signal (or nonstationary mean)  $g_t$ , plus an erratic component called the noise  $u_t$ , such that

$$y_t = g_t + u_t. \quad (14.15)$$

The noise component  $u_t$  is assumed to be either a white noise,  $WN(0, \sigma_u^2)$ , or, more generally, to follow a



stationary and invertible Autoregressive Moving Average (ARMA) process. Assuming that the input series  $\{y_t, t = 1, 2, \dots, N\}$  is seasonally adjusted or without seasonality, the signal  $g_t$  represents the trend and cyclical components, usually referred to as trend-cycle for they are estimated jointly. The trend-cycle can be represented locally by a polynomial of degree of the time distance  $j$ , between  $y_t$  and the neighboring observations  $y_{t+j}$ . Hence, given  $u_t$  for some time point  $t$ , it is possible to find a local polynomial trend estimator

$$g_t(j) = a_0 + a_1j + \dots + a_pj^p + \epsilon_t(j), \quad (14.16)$$

where  $a_0, a_1, \dots, a_p$  are real and  $\epsilon_t$  is assumed to be purely random and mutually uncorrelated with  $u_t$ . The coefficients  $a_0, a_1, \dots, a_p$  can be estimated by ordinary or weighted least squares or by summation formulae. The solution for  $\hat{a}_0$  provides the trend-cycle estimate  $\hat{g}_t(0)$ , which equivalently is a weighted average (Kendall et al. (1983)), applied in a moving average, such that

$$\hat{g}_t(0) = \hat{g}_t = \sum_{j=-m}^m w_j y_{t-j}, \quad (14.17)$$

where  $w_j, j < N$ , denotes the weights to be applied to the observations  $y_{t+j}$  to get the estimate  $\hat{g}_t$  for each point in time  $t = 1, 2, \dots, N$ . The weights depend on: (1) the degree of the fitted polynomial, (2) the amplitude of the neighborhood, and (3) the shape of the function used to average the observations in each neighborhood. Once a (symmetric) span  $2m + 1$  of the neighborhood has been selected, the  $w_j$  for the observations corresponding to points falling out of the neighborhood of any target point are null or approximately null, such that the estimates of the  $N - 2m$  central observations are obtained by applying  $2m + 1$  symmetric weights to the observations neighboring the target point. The missing estimates for the first and last  $m$  observations can be obtained by applying asymmetric moving averages of variable length to the first and last  $m$  observations, respectively. The length of the moving average or time invariant symmetric linear filter is  $2m + 1$ , whereas the asymmetric linear filters length is time varying.

Using the backshift operator  $B$ , such that  $B^n y_t = y_{tn}$ , equation 14.17 can be written as

$$\hat{g}_t = \sum_{j=-m}^m w_j B^j y_t = W(B) y_t \quad t = 1, 2, \dots, N \quad (14.18)$$

where  $W(B)$  is a linear nonparametric estimator. The nonparametric estimator  $W(B)$  is said to be a second order kernel if it satisfies the conditions

$$\sum_{j=-m}^m w_j = 1, \quad \sum_{j=-m}^m j w_j = 0 \quad (14.19)$$

Hence, it preserves a constant and a linear trend. On the other hand,  $W(B)$  is a higher order kernel if

$$\sum_{j=-m}^m w_j = 1, \quad \sum_{j=-m}^m j^i w_j = 0 \quad (14.20)$$

for some  $i = 1, 2, \dots, p > 2$ . In other words, it will reproduce a polynomial trend of degree  $p - 1$  without distortion.



## 14.4 Local trend cycle estimation

The nonparametric function estimators are based on different assumptions of smoothing. We shall next discuss : (1) the local polynomial regression predictor LOESS, (2) the Henderson linear filters , (3) the Dagum Modified Henderson filter, (4) the Gaussian kernel, and (5) the cubic smoothing spline. The first three estimators are often applied to estimate the short-term trend of seasonally adjusted economic indicators.

### 14.4.1 Locally Weighted Regression Smoother (LOESS)

The locally weighted regression smoother (loess) developed by Cleveland (1979) fits local polynomials of a degree  $d$  where the parameters are estimated either by ordinary or weighted least squares. Thus, it satisfies the property of best fit to the data in the sense of minimizing the mean square error.

Given a series of equally spaced observations and corresponding target points  $\{(y_j, t_j), j = 1, \dots, N\}$ ,  $t_1 < \dots < t_N$ , where  $t_j$  denotes the time the observation  $y_j$  is taken, Loess produces a smoothed estimate as follows

$$y_j = \mathbf{m}t^T \hat{\beta}_j \quad (14.21)$$

where  $\mathbf{m}t_j$  is a  $(d + 1)$ -dimensional vector of generic component  $t_j^p$ ,  $p = 0, \dots, d$ ,  $d = 0, 1, 2, \dots$  denotes the degree of the fitting polynomial, and  $\hat{\beta}_j$  is the  $(d + 1)$ -dimensional least squares estimate of a weighted regression computed over a neighborhood of  $t_j$  constituting a subset of the full span of the series. The weights of the regression depend on the distance between the target point  $t_{j^*}$  and any other point belonging to its neighborhood, through a weight function  $W(t)$ . The weighting function most often used is the tricube proposed by Cleveland et al. (1990), i.e.

$$W(t) = (1 - |t|^3)^3 I_{[-1,1]}(t) \quad (14.22)$$

In particular, at each point in the neighborhood of the target point  $t_{j^*}$ ,  $N(t_{j^*})$ , has assigned a weight

$$w(t_k) = W\left(\frac{|t_{j^*} - t_k|}{\Delta(t_{j^*})}\right) \quad \forall t_k \in N(t_{j^*}) \quad (14.23)$$

with  $D(t_{j^*})$  representing the distance of the furthest near-neighbor from  $t_{j^*}$ . Each neighborhood is made of the same number of points chosen to be nearest to  $t_{j^*}$ , and the ratio between the amplitude of the neighborhood,  $k$ , and the full span of the series,  $N$ , defines the bandwidth or smoothing parameter.

Cleveland (1979) derived the filters for the first and last observations by weighting the data belonging to an asymmetric neighborhood which contains the same number of data points of the symmetric one.

### 14.4.2 Henderson Smoothing Filters

The Henderson smoothing filters are derived from the graduation theory and used in Census X11 method and its variants X11ARIMA and X12ARIMA. The basic principle of the graduation theory is the combination of operations of differencing and summation in such a manner that, when differencing above a certain order is ignored, they will reproduce the functions operated on. The merit of these procedures is that the smoothed values thus obtained are functions of a large number of observations whose errors, to a considerable extent, cancel out. These smoothers have the properties that when fitted to second or third degree parabolas, their

output will fall exactly on those parabolas and when fitted to stochastic data, they will give smoother results than can be obtained from weights that give the middle point to a second-degree parabola fitted by ordinary least squares. Recognition of the fact that the smoothness of the resulting graduation depends directly on the smoothness of the weight diagram led Henderson (1916) to develop formula which makes the sum of squares of the third differences of the smoothed series a minimum for any number of terms. Henderson's starting point was the requirement that the filter should reproduce a cubic polynomial trend without distortion. Henderson showed that three alternative smoothing criteria lead to the same formula: (1) minimization of the variance of the third differences of the series defined by the application of the moving average, (2) minimization of the sum of squares of the third differences of the coefficients of the moving average formula, and (3) fitting a cubic polynomial by weighted least squares, where the weights are chosen as to minimize the sum of squares of their third differences. Kenny and Durbin (1982) and Gray and Thomson (1996) showed the equivalence of these three criteria. The problem is one of locally fitting a cubic trend by weighted least squares to the observations where the weights are chosen to minimize the sum of squares of their third differences (*smoothing criterion*). The objective function to be minimized is

$$\sum_{j=-m}^m W_j [y_{t+j} - a_0 - a_1j - a_2j^2 - a_3j^3]^2 \quad (14.24)$$

where the solution for the constant term  $\hat{a}_0$  is the smoothed observation  $\hat{g}_t$ ,  $W_j = W_{-j}$ , and the filter length is  $2m + 1$ . The solution is a local cubic smoother with weights

$$W_j \propto \{(m+1)^2 - j^2\} \{(m+2)^2 - j^2\} \{(m+3)^2 - j^2\} \quad (14.25)$$

and the weight diagram known as Henderson's ideal formula is obtained, for a filter length equal to  $2m - 3$ , by

$$w_j = \frac{315 \times [(m-1)^2 - j^2](m^2 - j^2)[(m+1)^2 - j^2](3m^2 - 16 - 11j^2)}{8m(m^2 - 1)(4m^2 - 1)(4m^2 - 9)(4m^2 - 25)} \quad (14.26)$$

Making  $m = 8$ , the  $w_j$  values are obtained for each  $m$  of the 13-term filter ( $m=6$  for the 9-th term Henderson filter, and  $m = 13$  for the 23 term Henderson). On the contrary, the weights of the usually known as "asymmetric Henderson filters" developed by Musgrave (1964) are based on the minimization of the mean squared revision between the final estimates and the preliminary estimates subject to the constraint that the sum of the weights is equal to one. The assumption made is that the most recent values of the series (where seasonality has been removed if present in the original observations) follow a constant linear trend plus an erratic component (see Laniel (1985) and Doherty (2001)). The equation used is

$$E[r_t^{(i,m)}]^2 = c_1^2 \left( t - \sum_{j=-i}^m h_{ij}(t-j) \right)^2 + \sigma^2 \sum_{j=-m}^m (h_{mj} - h_{ij})^2 \quad (14.27)$$

where  $h_{mj}$  and  $h_{ij}$  are the weights of the symmetric (central) filter and the asymmetric filters, respectively;  $h_{ij} = 0$  for  $j = -m, \dots, -i - 1$ ,  $c_1$  is the slope of the line and  $\sigma^2$  denotes the noise variance. There is a relation between  $c_1$  and  $\sigma^2$  such that the noise to signal ratio,  $I/C$  is given by

$$I/C = (4\sigma^2/\pi)^{1/2}/|c_1| \text{ or } \frac{c_1^2}{\sigma^2} = \frac{4}{\pi(1/c)^2} \quad (14.28)$$

The  $I/C$  noise to signal ratio (32) determines the length of the Henderson trend-cycle filter to be applied. Thus, setting  $t = 0$  and  $m = 6$  for the end weights of the 13-term Henderson, we have,

$$\frac{E[r_0^{(i,6)}]^2}{\sigma^2} = \frac{4}{\pi(I/c)^2} \left( \sum_{j=-i}^6 h_{ij} \right)^2 + \sum_{j=-6}^6 (h_{6j} - h_{ij})^2 \quad (14.29)$$

Making  $I/C = 3.5$  (the most noisy situation where the 13-term Henderson is applied), equation (33) gives the same set of end weights of Census X11 variant (Shiskin et al. (1967)). The end weights for the remaining monthly Henderson filters are calculated using  $I/C = .99$  for the 9-term filter and  $I/C = 4.5$  for the 23-term filter. The estimated final trend-cycle is obtained via cascade filtering resulting from the convolution of various linear trend and seasonal filters. In fact, if the output from the filtering operation  $H$  is the input to the filtering operation  $Q$ , the coefficients of the cascade filter  $C$  result from the convolution of  $H * Q$ . For symmetric filters  $H * Q = Q * H$  but this is not valid for asymmetric filters. Assuming an input series  $x_t, t = 1, 2, \dots, T$ , we can define a matrix  $H = [h_{kj}], k = 1, 2, \dots, m_h, j = 1, 2, \dots, 2m_h + 1$ , where each row is a filter and  $m_h$  is the half length of the symmetric filter.  $h_1$  denotes an asymmetric filter where the first  $m_h$  coefficients are zeroes and  $h_{m_h + 1}$  denotes the symmetric filter.

Given data up to time  $T$ , the  $m_h + 1$  most recent values of the output (filtered series) are given by

$$y_{T+1-k}^h = \sum_{j=m_h-k+2}^{2m_h+1} h_{kj} x_{T-k+m_h+2-j} \quad k = 1, 2, \dots, m_h + 1 \quad (14.30)$$

The 13-term Henderson filter can then be put in matrix form as follows:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -.092 & -.058 & .012 & .120 & .244 & .353 & .421 \\ 0 & 0 & 0 & 0 & 0 & -.043 & -.038 & .002 & .080 & .174 & .254 & .292 & .279 \\ 0 & 0 & 0 & 0 & -.016 & -.025 & .003 & .068 & .149 & .216 & .241 & .216 & .148 \\ 0 & 0 & 0 & -.009 & -.022 & .004 & .066 & .145 & .208 & .230 & .201 & .131 & .046 \\ 0 & 0 & -.011 & -.022 & .003 & .067 & .145 & .210 & .235 & .205 & .136 & .050 & -.018 \\ 0 & -.017 & -.025 & .001 & .066 & .147 & .213 & .238 & .212 & .144 & .061 & -.006 & -.034 \\ -.019 & -.028 & 0 & .066 & .147 & .214 & .240 & .214 & .147 & .066 & 0 & -.028 & -.019 \end{bmatrix} \quad (14.31)$$

Recently, Dagum and Bianconcini (2006) and Dagum and Bianconcini (2008) have found Reproducing Kernels in Hilbert Spaces (KHS) for the Henderson and LOESS local polynomial regression predictors with particular emphasis on the asymmetric filters applied to most recent observations. These authors show that the asymmetric filters can be derived coherently with the corresponding symmetric weights or from a lower or higher order kernel within a hierarchy, if preferred. In the particular case of the currently applied asymmetric Henderson and LOESS filters, those obtained by means of the RKHS are shown to have superior properties relative to the classical ones from the view point of signal passing, noise suppression, and revisions.

Linear asymmetric filters were developed by Dagum and Bianconcini (2015) and Dagum and Bianconcini (2016) using the RKHS methodology. Given the length of the RKHS asymmetric filter, its properties strongly depend on the bandwidth parameter of the asymmetric kernel function from which the filter weights are derived. Since the  $m$  asymmetric filters corresponding to a  $2m+1$  symmetric filter are time varying, one for each specific point, these authors proposed local time-varying bandwidth parameters. They considered three main criteria for bandwidth selection in order to determine an optimal smoother. An optimal filter is defined as the one that minimizes revisions and time lag to detect the upcoming of a true turning point. The three main criteria of bandwidth parameter selection are minimization of the following: (1) the distance between the gain functions of asymmetric and symmetric filters, (2) the distance between the transfer functions of asymmetric and symmetric filters, and (3) the phase shift function over the domain of the signal. Dagum and Bianconcini showed theoretically that any of the three criteria produces asymmetric trend-cycle filters to be preferred to those developed by Musgrave concerning both size of revisions and time delay to detect the upcoming of

true turning points. To highlight how the proposed filters perform when applied to series that are impacted differently by the trend-cycle, they look at the revision path of the corresponding estimates. In this regard, they compared the performance of the filters on three composite indicators, namely, leading, coincident and lagging. The composite index of ten leading indicators presents a deep turning point on May 2009, whereas shallow turning points are shown by the coincident and lagging composite indicators on August 2009 and May 2010, respectively. The real time trend-cycle filter calculated with the bandwidth parameter that minimizes the distance between the asymmetric and symmetric filters gain functions is to be preferred. This last point trend-cycle filter reduces around one half the size of the total revisions as well as the time delay to detect a true turning point with respect to the Musgrave filter. The new set of asymmetric kernel filters can be applied in many fields, such as economics, finance, health, hydrology, meteorology, criminology, physics, labor markets, utilities and so on, in fact, in any time series where the impact of trend plus cyclical variations is of relevance.

### 14.4.3 Dagum's modified 13-term Henderson Filter (DMH)

The modified 13-term Henderson filter developed by Dagum (1996) is nonlinear and basically consists of : (a) extending the seasonally adjusted series with ARIMA extrapolated values, and (b) applying the 13-term Henderson filter to the extended series where extreme values have been modified using very strict sigma limits.

To facilitate the identification and fitting of simple ARIMA models, Dagum (1996) recommends, at step (a), to modify the input series for the presence of extreme values using the standard  $\pm 2.5\sigma$  limits of X11ARIMA (versions 1980, 1988 or 2000) and X12ARIMA (Findley et al. (1998)). These computer programs are those that can be used to implement DMH. In this way, a simple and very parsimonious ARIMA model, the (0,1,1) is often found to fit well a large number of series. Concerning step (b), it is recommended to use very strict sigma limits, such as  $\pm 0.7\sigma$  and  $\pm 1.0\sigma$ . The extension of the series is performed with the purpose of reducing the size of the revisions of the most recent estimates of the trend-cycle. On the other hand, the use of stricter sigma limits for the identification and replacement of the extreme values has the purpose of reducing the number of unwanted ripples (false turning points) created by the classical 13-term Henderson filter.

The DMH can be formally described in matrix notation as follows.

Let  $y \in R^N$  be the N-dimensional time series to be smoothed, which consists of a nonstationary trend  $\tilde{y}$  plus an erratic component  $e$ , that is

$$y = \tilde{y} + e \quad (14.32)$$

It is assumed that the trend is smooth and can be well estimated by means of the 13-term Henderson filter applied to  $y$ , hence,

$$\tilde{y} = Hy \quad (14.33)$$

where  $H$  is the  $N \times N$  matrix (canonically) associated to the 13-term Henderson filter as given in (35). Replacing  $\tilde{y}$  in eq. (36) by eq. (37), we have

$$y = Hy + e \quad (14.34)$$

or,

$$e = (I_N - H)y \quad (14.35)$$

where  $I_N$  is the  $N \times N$  identity operator on  $R^N$ . Assigning now a weight to the residuals in such a way that if the observation  $y_j$  is recognised to be an extreme value (with respect to  $\pm 2.5\sigma$  limits, where  $\sigma$  is a 5 year

moving standard deviation) then, the corresponding residual  $e_j$  is zero weighted (i.e. the extreme value is replaced by  $\tilde{y}_j$  which is a preliminary estimate of the trend). If  $y_j$  is not an extreme value then the weight for  $e_j$  is one (i.e. the value  $y_j$  is not modified). In symbols,

$$W_0 e = W_0 (I_N - H) y \quad (14.36)$$

where  $W_0$  is a zero-one diagonal matrix, being the diagonal element  $w_{ii}$  equal to zero when the corresponding element  $y_j$  of the vector  $y$  is identified as an outlier. For instance, if in the series  $y$  the only extreme value is  $y_2$  then the weight matrix for the residuals will be

$$\mathbf{m}W_0 = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix} \quad (14.37)$$

Denoting by,

$$e_0 = W_0 e \quad (14.38)$$

the vectors of the modified residuals, then the series modified by extreme values with zero weights becomes

$$y_0 = \tilde{y} + e_0 \quad (14.39)$$

which can be written as

$$y_0 = H y + W_0 (I_N - H) y = [H + W_0 (I_N - H)] y. \quad (14.40)$$

Using (14.40), one year of ARIMA extrapolations are obtained in order to extend the series modified by extreme values. Denoting with  $y_0^E$  the extended series, that is the  $N + 12$  vector whose first  $N$  elements are given by  $y_0$  while the last 12 are the extrapolated ones, in block-matrix notation we have

$$y_0^E = \begin{bmatrix} [H + W_0 (I_N - H)] y \\ y^{12} \end{bmatrix} \quad (14.41)$$

where  $y^{12}$  is the  $12 \times 1$  block of extrapolated values. Setting

$$[H + W_0 (I - H)]^{+12} = \begin{bmatrix} [H + W_0 (I_N - H)] & O_{N \times 12} \\ O_{12 \times N} & I_{12} \end{bmatrix} \quad (14.42)$$

and

$$y^{+12} := \begin{bmatrix} y \\ y^{12} \end{bmatrix} \quad (14.43)$$

$y_0^E$  becomes

$$y_0^E = [H + W_0 (I - H)]^{+12} y^{+12}. \quad (14.44)$$

This concludes the operations involved in step (a) of the DMH filter. Step (b) follows. The procedure for obtaining  $y_0$  on the series  $y_0^E$  is repeated but with stricter sigma limits (such as  $\pm 0.7\sigma$  and  $\pm 1.0\sigma$ ) and with different weights assigned to the residuals for the replacement of the extreme values. The estimates  $y^E$  computed over the series  $y_0^E$  are

$$y^E = [H + W(I - H)]^E y_0^E. \quad (14.45)$$

The  $N + 12 \times N + 12$  matrix  $[H + W\{I - H\}]^E$  is analogue to  $[H + W_0(I - H)]^{12}$  except for the matrix  $W$  that is also diagonal but with generic diagonal element  $w_{ii}$  such that,  $w_{ii} = 0$  if the corresponding value  $y_i$  falls out of the upper bound selected limits, say,  $\pm 1.0\sigma$ ;  $w_{ii} = 1$  if the corresponding  $y_i$  falls within the lower bound selected limits, say,  $\pm 0.7\sigma$  and  $w_{ii}$  decreases linearly (angular coefficient equal to  $-1$ ) from 1 to 0 in the range from  $\pm 0.7\sigma$  to  $\pm 1.0\sigma$ . Under the assumption of normality, these sigma limits imply that 48% of the values will be modified (replaced by the preliminary smoothed trend): 32% will be zero weighted while the remaining 16% will get increasing weights from zero to one. Note that  $y^E$  can also be written as

$$y^E = [H + W(I - H)]^E [H + W_0(I - H)]^{+12} y^{+12}. \quad (14.46)$$

Finally, the DMH estimates are given by applying a 13-term Henderson filter to eq.14.46, that is

$$\hat{y}^{+12} = [H + W(I - H)]^E [H + W_0(I - H)]^{+12} y^{+12} \quad (14.47)$$

$$\hat{y}^{+12} = \begin{bmatrix} \hat{y} \\ \hat{y}^{12} \end{bmatrix}$$

where  $\hat{y}$  is the  $N$ -dimensional vector of smooth estimates of  $y$ . It is apparent that the Dagum (1996) method reduces drastically the effects of extreme values by repeatedly smoothing the input data via down weighting points with large residuals. Furthermore, the ARIMA extension enables the use of the symmetric weights of the 13-term Henderson filter for the last six observations and, thus, reduces the size of the revision of the last estimates. Further studies on the Dagum (1996) trend-cycle filter were done by Chhab et al. (1999) and Dagum and Luati (2000). These later authors, in Dagum and Luati (2009) also developed an excellent linear approximation of the Dagum (1996) filter to facilitate applications.

## 14.5 Gaussian Kernel Smoother

Kernel type smoothers are locally weighted averages. Given a series of observations and corresponding target points  $\{y_j, t_j\}$ ,  $j = 1, \dots, N$ , a kernel smoothing gives, at time  $th^*$ ,  $1 \leq h \leq N$ , the smoothed estimate

$$\hat{y}_b = \sum_{j=1}^N w_{bj} y_j \quad (14.48)$$

where

$$w_{bj} = \frac{K\left(\frac{t_h^* - t_j}{b}\right)}{\sum_{j=1}^N K\left(\frac{t_h^* - t_j}{b}\right)} \quad (14.49)$$

are the weights from a parametric kernel which is a nonnegative function that integrated over its domain gives unity,  $b \neq 0$  is the smoothing parameter and  $K_b(x) = K_b(-x)$ . Kernel smoothers are local functions since their weighting systems are local. In fact, for any observed value  $y_h$ , a weighted average is computed. Each weight is obtained as a function of the distance between the target point  $t_{h^*}$  and the  $t_j$ 's close to  $t_{h^*}$  that belong to an interval whose amplitude is established by the smoothing parameter  $b$ , otherwise said bandwidth parameter. Increasing the distance between  $t_{h^*}$  and  $t_j$ ,  $j = 1, \dots, N$ , the weights assigned to the corresponding observations decrease till a certain point when they become zero. Such a point depends on the bandwidth parameter that, in practice, determines the lengths of the smoother, i.e. the number of observations near  $y_h$  that have non null weight.

In matrix form the action of kernel smoother can be represented by the relation

$$\hat{y} = S_b y \quad (14.50)$$

where  $S_b \in R^N$  is the smoothing matrix of generic element  $w_{hj}$  as defined in (55) for  $h, j = 1, \dots, N$ . Once the smoothing parameter has been selected, the  $w_{hi}$  corresponding to the target points falling out of the bandwidth of any  $t_{h^*}$  turn out to be null, and where the number of no null weights, depends on both the value of the bandwidth parameter and on the number of decimals chosen for each weight. Note that as long as the kernel is a symmetric function, the number of no null weights turns out to be odd. For instance, for a standard Gaussian kernel function

$$K\left(\frac{t_h^* - t_j}{b}\right) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{t_h^* - t_j}{b}\right)^2\right\} \quad (14.51)$$

with  $b = 5$  and weights  $w_{hj}$  taken with three decimals, the smoothing matrix of no null weights is of size 31x31. For further details we refer the reader to Dagum and Luati (2001). We have here chosen the standard Gaussian kernel, which is the most often applied, that is

$$K_{Gauss}(x) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}x^2\right\} \quad (14.52)$$

In kernel smoothing the bandwidth parameter is of great importance. Similar to Loess, increasing the smoothing parameter from the extreme case of interpolation ( $b \rightarrow 0$ ) to that of oversmoothing ( $b \rightarrow \infty$ ) produces an increase in bias and a decrease in variance.

## 14.6 Cubic Smoothing Spline

The current literature on spline functions, particularly on smoothing splines, is very extense and we refer the reader to Wahba (1987) for an excellent summary of the most important contributions on this topic. The name is due to the resemblance with the curves obtained by draftsmen using a mechanical spline, that is a thin and flexible rod with weights or "ducks" used to position the rod at points through which it was desired to draw a smooth interpolating curve. The problem of smoothing via spline functions is closely related to that of smoothing priors and signal extraction in time series, where these later are approached from a parametric point of view (see, among others, Akaike (1980) and Akaike (1980); and Kitagawa and Gersch (1984) and Kitagawa and Gersch (1996)). Similar to the Henderson filters, the original work on smoothing spline functions was based on the theory of graduation. The first two seminal works related to smoothing splines are due to Whittaker (1923) and Whittaker and Robinson (1924) who proposed a new graduation method that basically consisted of a trade-off between fitting fidelity and smoothing. The problem was that of estimating

an unknown "smooth" function  $f$ , observed with errors assumed to be white noise. That is, given a set of observations  $y_t$   $t = 1, 2, \dots, T$  such that,

$$y_t = f_t + \epsilon_t \quad \epsilon_i \sim IID(0, \sigma^2) \quad (14.53)$$

we want to minimize

$$\sum_{t=1}^T (y_t - f_t)^2 + \mu^2 \sum_{t=k+1}^T (\Delta^k f_t)^2 \quad (14.54)$$

where  $\Delta^k f_t$  denotes the  $k$ -th order difference of  $f_t$ , e. g.  $\Delta f_t = f_t - f_{t-1}$ ,  $\Delta^2 f_t = \Delta(\Delta f_t)$ , and so on. The smoothing trade-off parameter  $\mu$  must be appropriately chosen. Following this direction, Schoenberg (1964) extended Whittaker smoothing method to the fitting of a continuous function to observed data, not necessarily evenly spaced. In this case, the model is

$$y_t = f(x_t) + \epsilon_t \quad \epsilon_i \sim IID(0, \sigma^2) \quad (14.55)$$

where the unobserved function  $f$  is assumed to be "smooth" on the interval  $[a, b]$  and the observations are at the  $n$  points  $x_1, x_2, \dots, x_n$ . The problem is to find

$$f_\lambda = \min_{f \in C^m} \frac{1}{n} \sum_{t=1}^n (y_t - f(x_t))^2 + \lambda \int_a^b [f^{(m)}]^2 dx \quad (14.56)$$

where  $C^m$  is the class of functions with  $m$  continuous derivatives and  $\lambda > 0$ . The solution to (14.56) known as a smoothing spline is unique and given by a univariate natural polynomial (unp) or piecewise polynomial function spline of degree  $2m - 1$  with knots at the data points  $x_1, x_2, \dots, x_n$ . The smoothing trade-off parameter  $\lambda$  controls the balance between the fit to the data as measured by the residual sum of squares and the smoothness as measured by the integrated squared  $m$ -th derivative of the function. When  $m = 2$ , which is the case of a cubic smoothing spline then the integral of the squared second order derivative  $f^{(2)}$  is curvature and a small value for the integral corresponds visually to a smooth curve. As  $\lambda \rightarrow 0$  the solution  $f_\lambda$  tends to the unp spline which interpolates the data, and as  $\lambda \rightarrow \infty$ , the solution tends to the polynomial of degree  $m$  best fitting the data in the least squares sense. The smoothing trade-off parameter  $\lambda$  is known as hyperparameter in the Bayesian terminology and it has the interpretation of a noise to signal ratio, the larger the  $\lambda$  the smoother the trend-cycle. The estimation of  $\lambda$  was first done using ordinary cross-validation (OCV). OCV consisted of deleting one observation and solving the optimization problem with a trial value of  $\lambda$ , computing the difference between the predicted value and the deleted observation, accumulating the sums of squares of these differences as one runs through each of the data points in turn, and finally choosing the  $\lambda$  for which the accumulated sum is the smallest. This procedure was improved by Craven and Wahba (1979) who developed the generalized cross validation (GCV) method currently available in most computer packages. GCV can be obtained from OCV by rotating the system to a standard coordinate system, doing OCV, and rotating back. The GCV estimate of  $\lambda$  is obtained by minimizing

$$V(\lambda) = \frac{(1/n)|I - A(\lambda)\mathbf{m}y|^2}{[(1/n)Tr(I - A(\lambda))]^2} \quad (14.57)$$

where  $A(\lambda)$  is the influence matrix associated with  $f$ , that is,  $A(\lambda)$  satisfies



$$(\lambda) = \begin{pmatrix} f\lambda(x_1) \\ \vdots \\ f\lambda(x_n) \end{pmatrix} = A(\lambda) \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} \quad (14.58)$$

Trace  $A(\lambda)$  can be viewed as the "degrees of freedom for the signal" and so, (63) can be interpreted as minimizing the standardized sum of squares of the residuals.

Cross-validated smoothing splines has also given good results for the estimation of derivatives or other local features of maxima and minima and will be the ones used in this study. Since  $\lambda$  is usually estimated by cross-validation, cubic smoothing splines are considered nonlinear smoothers.

The cubic smoothing spline estimate is given by

$$\hat{y}_j = f(t_j) \quad (14.59)$$

with  $f(t_j)$  being a smooth function on the interval  $[a, b]$ ,  $a \leq t_1 \leq \dots \leq t_N \leq b$ . In matrix form, the estimated values  $\hat{y}_t$  are related to the observed values  $y_t$  by means of the following relation

$$\hat{y} = S(\lambda)y \quad (14.60)$$

where  $\mathbf{m}S(\lambda)$  is the influential matrix, depending on  $\lambda$ ; see Wahba (1990). Dagum and Capitanio (1999) show that if  $\lambda$  is fixed, i.e.  $\lambda = \lambda_0$  then  $\mathbf{m}S(\lambda_0) = \mathbf{m}S_{\lambda_0}$  where  $\mathbf{m}S_{\lambda_0}$  is the smoothing matrix associated to the linear transformation  $f_{\lambda_0}$

$$\mathbf{m}S_{\lambda_0} = \left[ I_N - \mathbf{m}D^T \left( \frac{1}{\lambda_0} \mathbf{m}B + \mathbf{m}D D^T \right)^{-1} \mathbf{m}D \right] \quad (14.61)$$

where  $\mathbf{m}B \in R^{(N-2) \times (N-2)}$  and  $\mathbf{m}D \in R^{(N-2) \times N}$  are as follows,

$$\mathbf{m}B = \begin{pmatrix} \frac{1}{3}(t_3 - t_1) & \frac{1}{6}(t_3 - t_2) & 0 & \dots & 0 \\ \frac{1}{6}(t_3 - t_2) & \frac{1}{3}(t_4 - t_2) & \frac{1}{6}(t_4 - t_3) & \dots & 0 \\ 0 & \ddots & \ddots & \ddots & \dots \\ \vdots & \ddots & \ddots & \ddots & \dots \\ 0 & \dots & \dots & \frac{1}{6}(t_N - t_{N-1}) & \frac{1}{3}(t_N - t_{N-2}) \end{pmatrix} \quad (14.62)$$

and

$$\mathbf{m}D = \begin{pmatrix} \frac{1}{t_2 - t_1} & -\frac{1}{t_2 - t_1} - \frac{1}{t_3 - t_2} & \frac{1}{t_3 - t_2} & \dots & 0 \\ 0 & \frac{1}{t_3 - t_2} & -\frac{1}{t_3 - t_2} - \frac{1}{t_4 - t_3} & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \dots \\ \vdots & \ddots & \ddots & \ddots & \dots \\ 0 & \dots & \frac{1}{t_{N-1} - t_{N-2}} & -\frac{1}{t_{N-1} - t_{N-2}} - \frac{1}{t_N - t_{N-1}} & \frac{1}{t_N - t_{N-2}} \end{pmatrix} \quad (14.63)$$

Making  $t_j = j$ ,  $j = 1, \dots, N$ , (14.62) and (14.63) become respectively,

$$\mathbf{m}B = \begin{pmatrix} \frac{2}{3} & \frac{1}{6} & 0 & \dots & 0 \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{6} & \dots & 0 \\ 0 & \ddots & \ddots & \ddots & \dots \\ \vdots & \ddots & \ddots & \ddots & \dots \\ 0 & \dots & \dots & \frac{1}{6} & \frac{1}{3} \end{pmatrix} \quad (14.64)$$

and

$$\mathbf{m}D = \begin{pmatrix} 1 & -2 & 1 & \dots & 0 \\ 0 & 1 & -2 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \dots \\ \vdots & \ddots & \ddots & \ddots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix} \quad (14.65)$$

In recent studies Dagum and Bianconcini (2009) and Dagum and Bianconcini (2013) derived a reproducing kernel in Hilbert space that gives an excellent approximation for the cubic spline when applied to short and medium spans (smaller than 23 weight terms).

## 14.7 Theoretical properties of the symmetric and asymmetric linear trend-cycle filters

The theoretical properties of the symmetric and asymmetric linear filters can be studied by analyzing their frequency response functions. The frequency response function is defined by

$$H(\omega) = \sum_{j=-m}^m \alpha_j e^{-i\omega j} \quad 0 \leq \omega \leq 1/2 \quad (14.66)$$

where  $\alpha_j$  are the weights of the filter and  $\omega$  is the frequency in cycles per unit of time. In general, the frequency response functions can be expressed in polar form as follow,

$$H(\omega) = A(\omega) + iB(\omega) = G(\omega)e^{i\phi(\omega)} \quad (14.67)$$

where  $G(\omega) = [A^2(\omega) + B^2(\omega)]^{\frac{1}{2}}$  is called the gain of the filter and  $\phi(\omega) = \tan^{-1}(-B(\omega)/A(\omega))$  is called the phase shift of the filter and is usually expressed in radians. The expression (37) shows that if the input function is a sinusoidal variation of unit amplitude and constant phase shift  $\psi(\omega)$ , the output function will also be sinusoidal but of amplitude  $G(\omega)$  and phase shift  $\psi(\omega) + \phi(\omega)$ . The gain and phase shift vary with  $\omega$ . For symmetric filters the phase shift is 0 or  $\pm\pi$ , and for asymmetric filters take values between  $\pm\pi$  at those frequencies where the gain function is zero. For a better interpretation the phase shifts will be here given in months instead of radians (the phase shift in months is given by  $\phi(\omega)/2\pi\omega$  for  $\omega \neq 0$ ).

The gain function shown should be interpreted as relating the spectrum of the original series to the spectrum of the output obtained with a linear time-invariant filter. For example, let  $y_t^{(0)}$  be the estimated seasonally adjusted observations for the current period based on data  $x_t$   $t = 1, 2, \dots, T$ , then the time series  $\{y_t\}^{(0)}$  is obtained from  $\{x_t\}$  by application of the concurrent linear time-invariant filter  $h^{(0)}(B)$ . Thus, the gain function shown in the below figures relates the spectrum of  $\{x_t\}$  to the spectrum of  $\{y_t\}^{(0)}$  and not

to the spectrum of the complete seasonally adjusted series produced at time  $t$  (which includes  $y_t^{(0)}$ , a first revision of time  $t - 1$ , a second revision at time  $t - 2$ , and so on). Dagum et al. (1996) derived the gain functions of standard, short and long convolutions corresponding to the 13-term (H-13), 9-term (H-9) and 23-term (H-23) symmetric Henderson filters, respectively. These authors showed how cycles of 9 and 10 months periodicity (in the 0.08- 0.18 frequency band) are not suppressed by any of the cascade filters, particularly, those using H-13 and H-9 filters. In fact, about 90%, 72% and 21% of the power of these short cycles are left in the output by the 9, 13, and 23-term Henderson filters, respectively. Figure 14.1 shows the gain functions of four symmetric trend-cycle filters of 13-term each. In the context of trend-cycle estimation, it is useful to divide the total range of  $\omega \in [0, 0.50]$  in two major intervals, one, for the signal, and another for the noise. There are no-fixed rules on defining the cutoff frequency but for monthly data the intervals are usually given by : (1)  $0 \leq \omega \leq 0.06$  associated with cycles of 16 months and longer attributed to the trend-cycle of the series, and (2)  $0.06 < \omega \leq 0.50$  corresponding to seasonality, if present, and noise. It is apparent that the 13-term Henderson and Loess are very close to each other and pass well the power in the frequency band  $0 \leq \omega \leq 0.06$  reproducing well the cycles associated with the trendcycle and suppressing a large amount of noise. But they have the limitation of passing too much power at  $\omega = 0.10$ , that will produce a large number of 10-month cycles in the output, also known as unwanted ripples. On the other hand, the Gaussian Kernel will not pass too much power at  $\omega = 0.10$  but will suppress also the power attributed to the trend-cycle. In other words, will produce a very smooth trend-cycle whereas the 13-term Cubic spline will do the opposite that is why in empirical applications the length of this filter is selected according to other criteria. Figures 14.2 and 14.3 exhibit the gain and phaseshifts of the last point of the corresponding asymmetric filters. The gains of the asymmetric filters suppress less noise and pass more power related to the trend-cycle ( particularly, H13 and Loess amplify the power ) relative to the symmetric filters. All the filters introduce a time lag to detect a true turning point as shown by their phase shift functions. In a recent study Bianconcini and Quenneville (2010) has considered the problem of estimating the trend of a time series in real time by means of reproducing kernel filters associated to symmetric Henderson averages. These authors showed that these filters share similar properties with the Musgrave surrogates adopted by X11 based seasonal adjustment procedures, that are known to minimize revisions for a certain class of time series. However, the asymmetric Musgrave filters are derived following a different optimization criteria with respect to the symmetric Henderson filter, with the consequence that the asymmetric filters do not converge monotonically to the symmetric one. At this regard, Dagum and Bianconcini (2006) and Dagum and Bianconcini (2008) provided a different characterization of the Henderson weights within the Reproducing Kernel Hilbert Space (RKHS) methodology. According to this approach, a continuous kernel representation of the Henderson filter is obtained, and the same kernel function is used to derive both symmetric and asymmetric weights. The density function (i.e. a second order kernel) embedded on the linear filter is firstly determined. This is the starting point for obtaining higher order kernels, which are based on the product of the density and its orthonormal polynomials. For each kernel order, the asymmetric filters can be derived coherently with the corresponding symmetric weights. The asymmetric filters are derived by applying the same kernel functions adapted to the length of the filter. This approach has been introduced by Gasser and Muller (1979) (called "cut-and-normalized" method) to improve the properties of kernel estimators in the boundaries. Bianconcini and Quenneville (2010) showed that the corresponding asymmetric filters share similar properties to the Musgrave one in terms of polynomial reproduction. In particular, when the bandwidth parameters are all fixed to  $m + 1$ , the former just pass a constant, whereas the latter a linear trend with small bias. On the other hand, when the filter-specific bandwidth parameters are selected in order to optimize the spectral properties of the asymmetric filters, most of the reproducing kernel filters also pass a linear trend with small bias.

## 14.8 Illustrative Results

We now illustrate with real series the various trend-cycle values obtained with the various filters discussed before. To select the appropriate lengths of the Henderson, the Loess and the Gaussian Kernel filters we use the irregular- trend cycle ratio  $I/C$ , whereas for the cubic spline we look at the smoothing parameter ( $Spar$ )

Figure 14.1: Gain functions of several symmetric 13-term filters

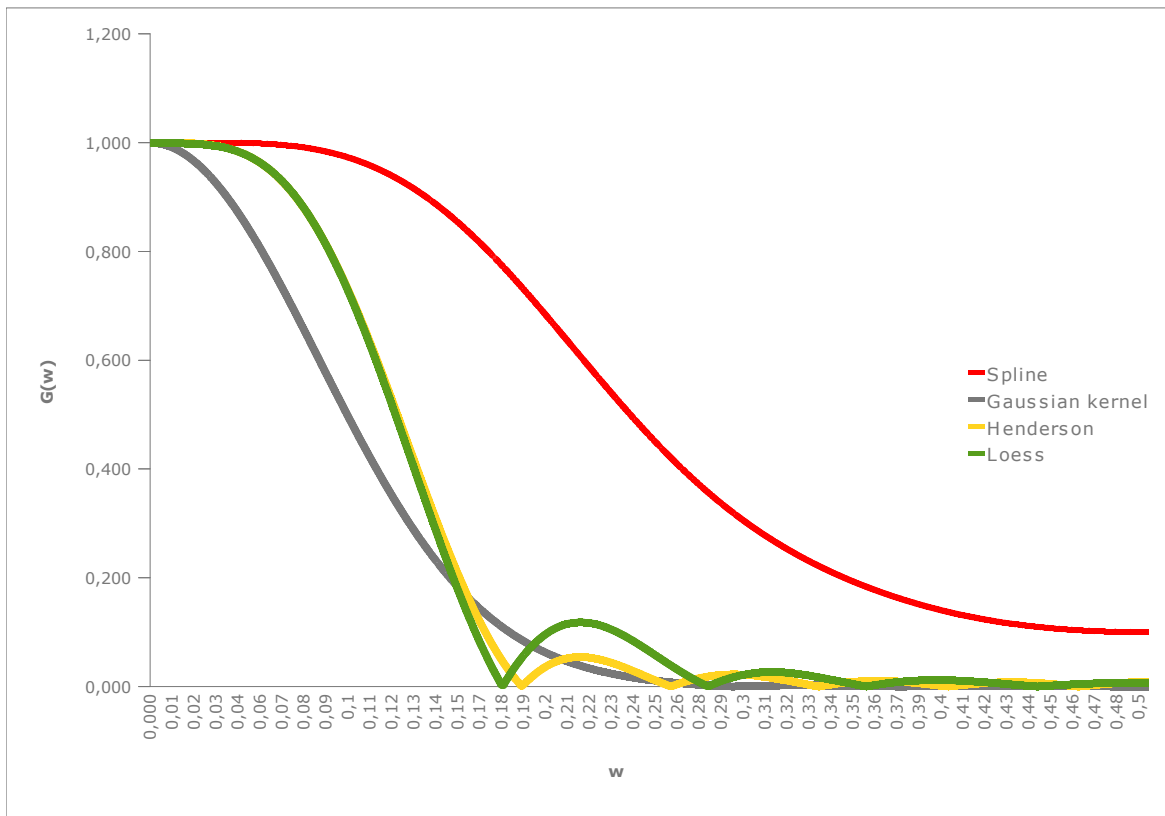


Figure 14.2: Gain functions of several symmetric 13-term filters

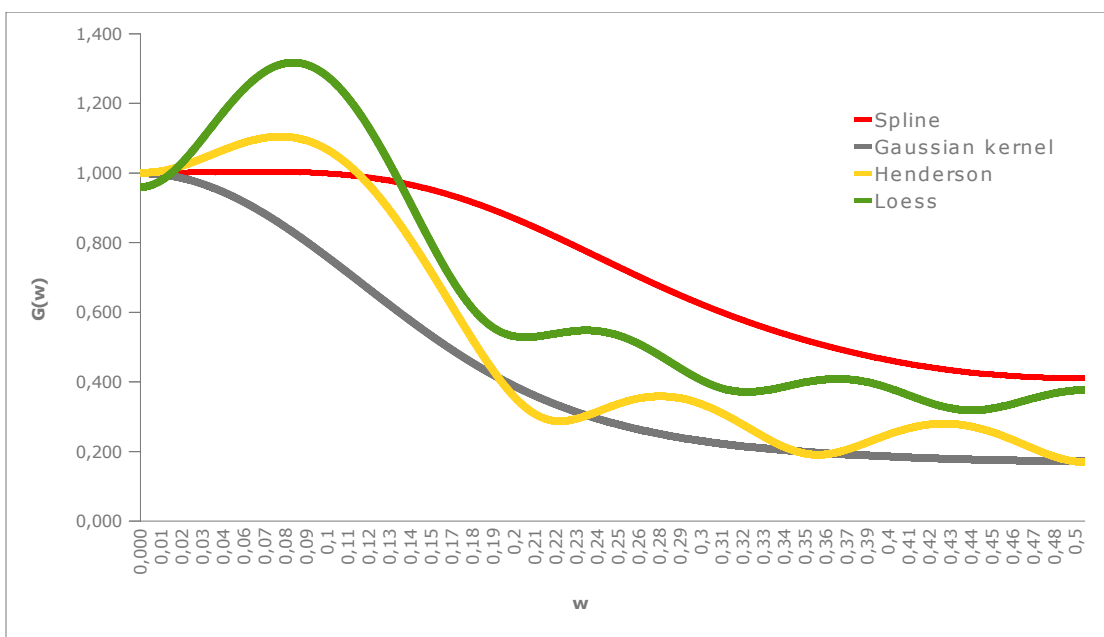
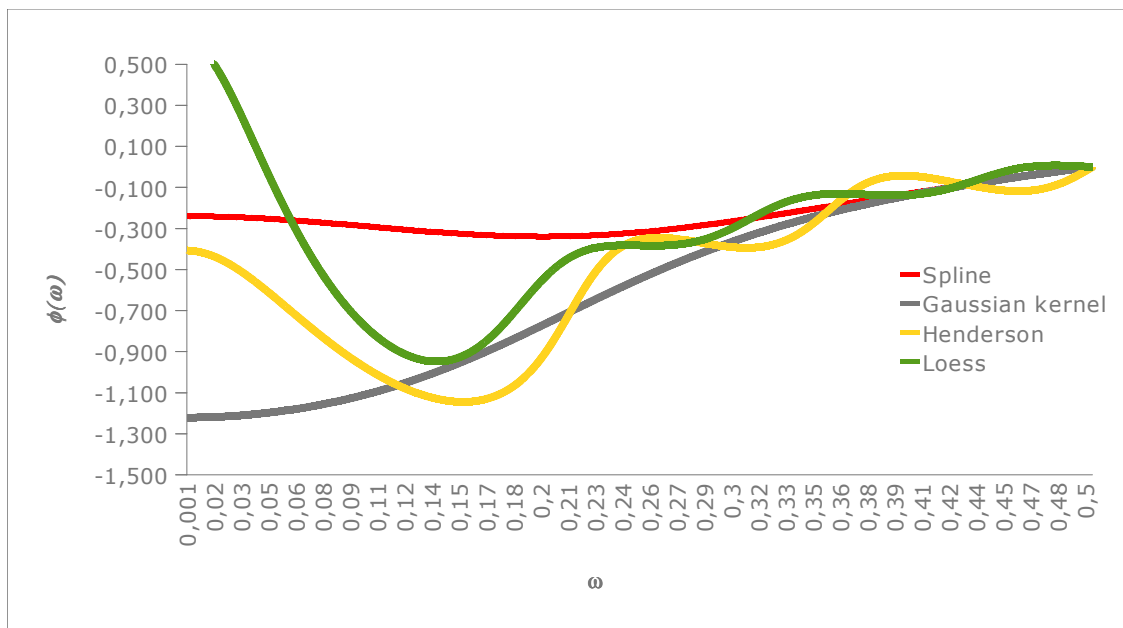


Figure 14.3: Phase shift functions of last point asymmetric filters



estimated by means of the generalized cross validation criterion (GCV). We show two series, the European Index of Industrial Production, from January 1990 till June 2010, and the European Unemployment rate since January 1995 till October 2010. Figure 14.4 exhibits the various trend-cycle estimates for the European Index of Industrial Production where 13-term filters have been chosen for the Henderson, the Loess and the Gaussian Kernel and the smoothing parameter (Spar) for the cubic spline is 3.165559. The results show that the Loess and Henderson filter give values close to each other whereas the Gaussian kernel produces the smoothest trend and the cubic spline gives the most volatile. The difference are more noticeable at cyclical turning points where the Gaussian kernel cuts them more whereas the Cubic Spline cuts them the least. For the European Unemployment rate the results are similar except that his series is less noisy, the I/C ratio indicate a 9-term filter for the Henderson, Loess and Gaussian Kernel whereas for the cubic spline, the Spar is 0.04986617 with 134.7177 degrees of freedom and the GVC is 0.001774807. The estimates for all the trend-cycles are very close to each other except at turning points where the previous observations apply.

Figure 14.4: Index of Industrial Production Trend-cycle estimates , Loess and Hender 13-term filters

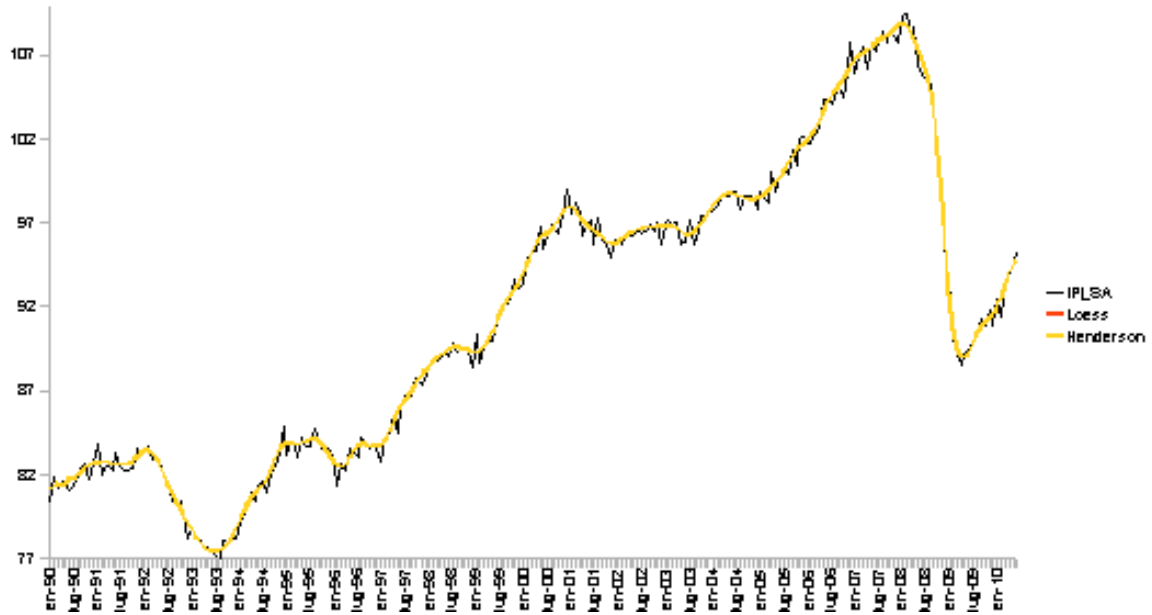


Figure 14.5: Index of Industrial Production Trend-cycle estimates with Gaussian kernel

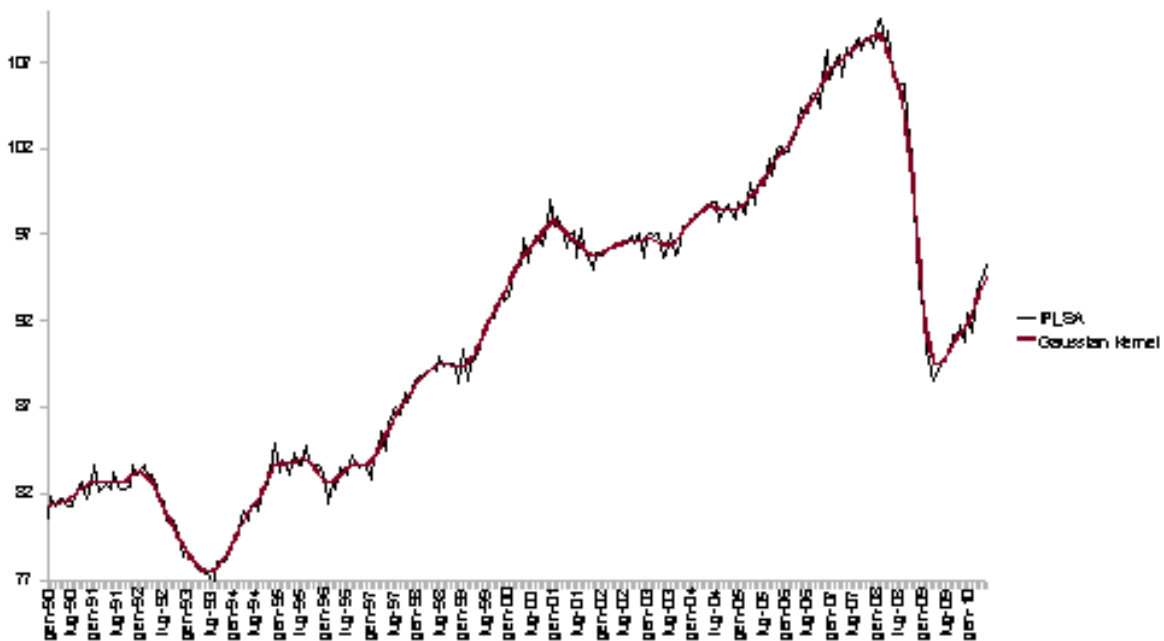


Figure 14.6: Index of Industrial Production Trend-cycle estimates with Cubic spline

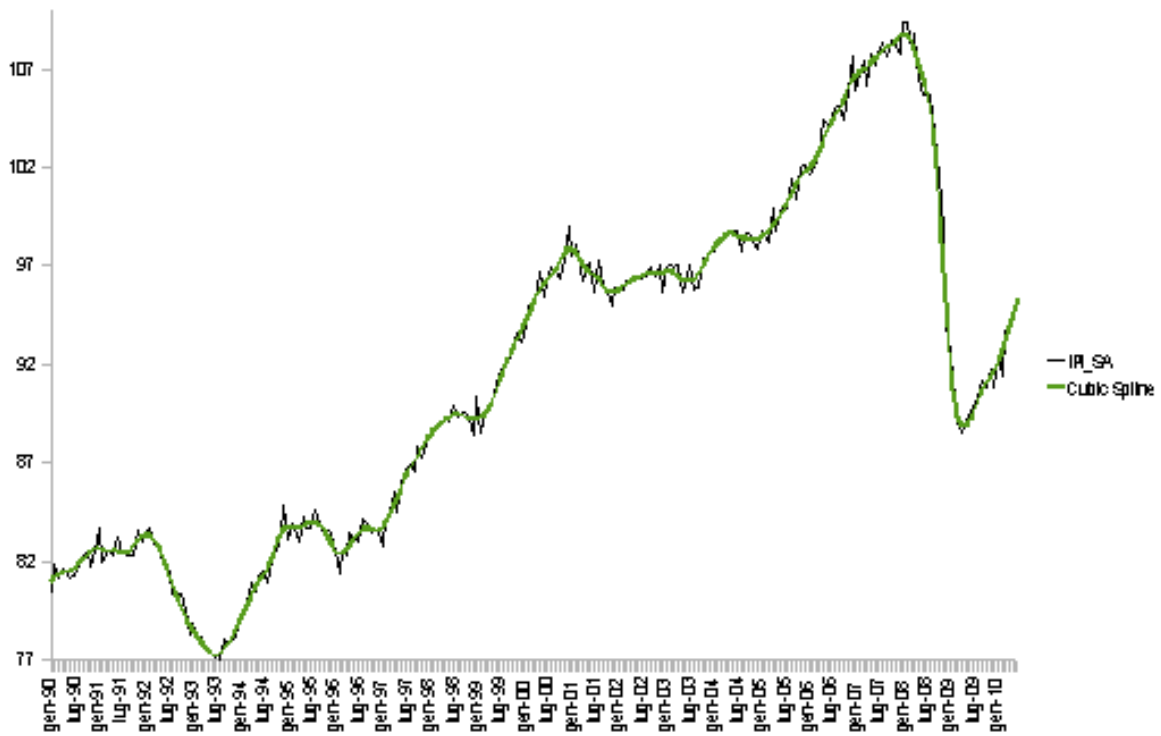


Figure 14.7: Index of European Consumer Price Trend-cycle estimates , Loess and Hender13-term filters.

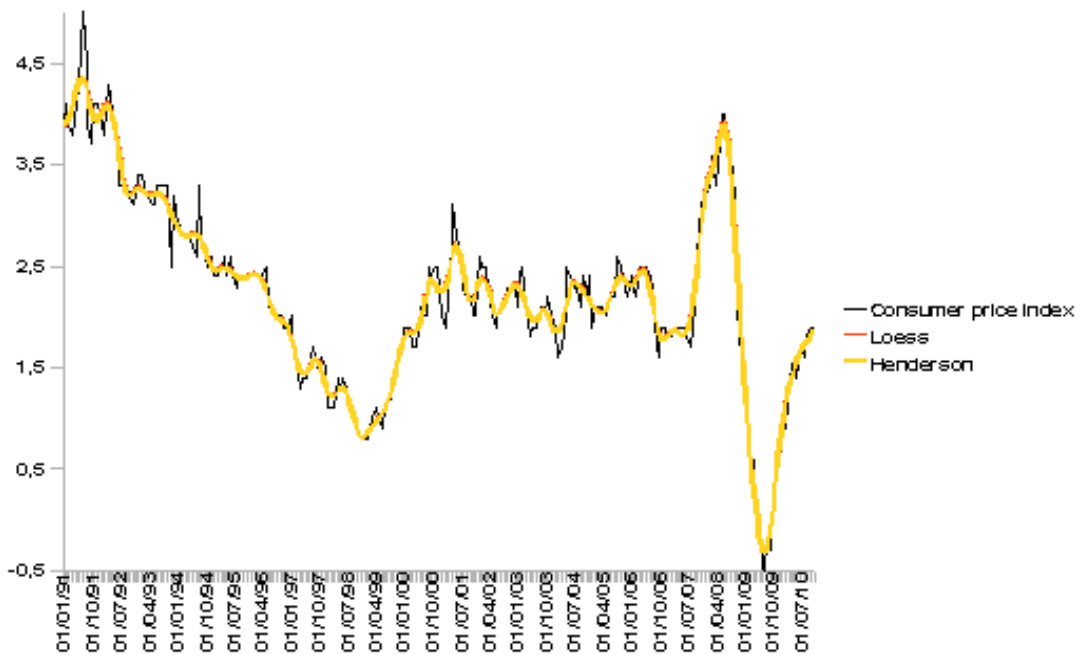


Figure 14.8: Index of European Consumer Price Trend-cycle estimates with Gaussian kernel

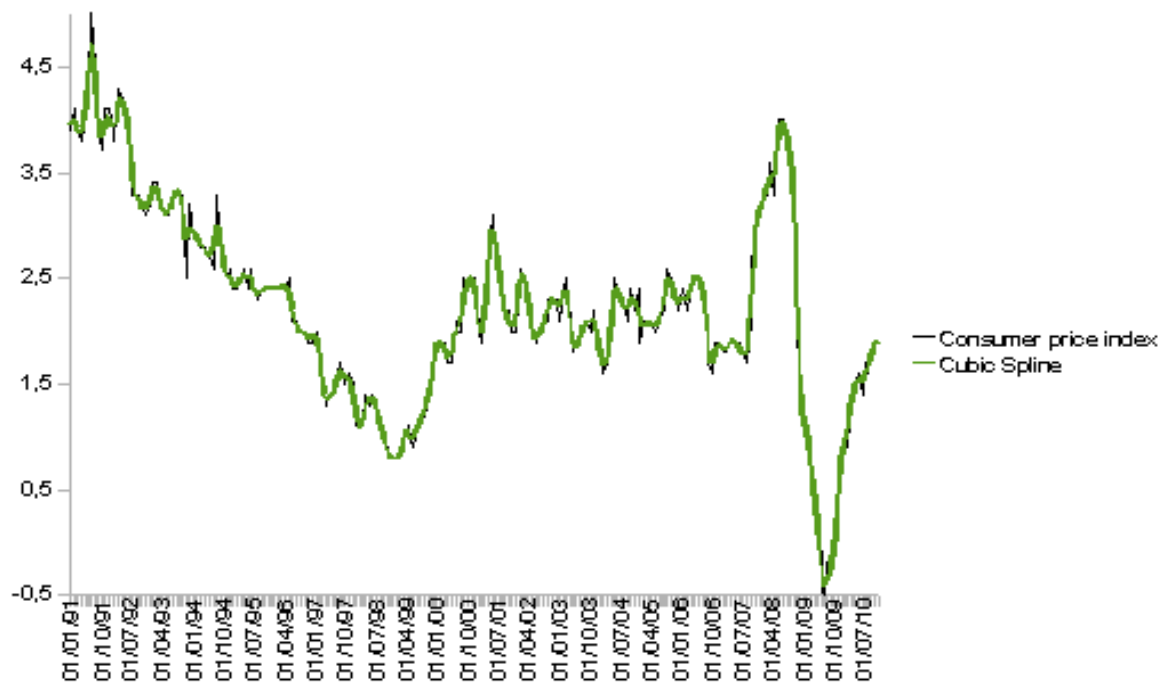
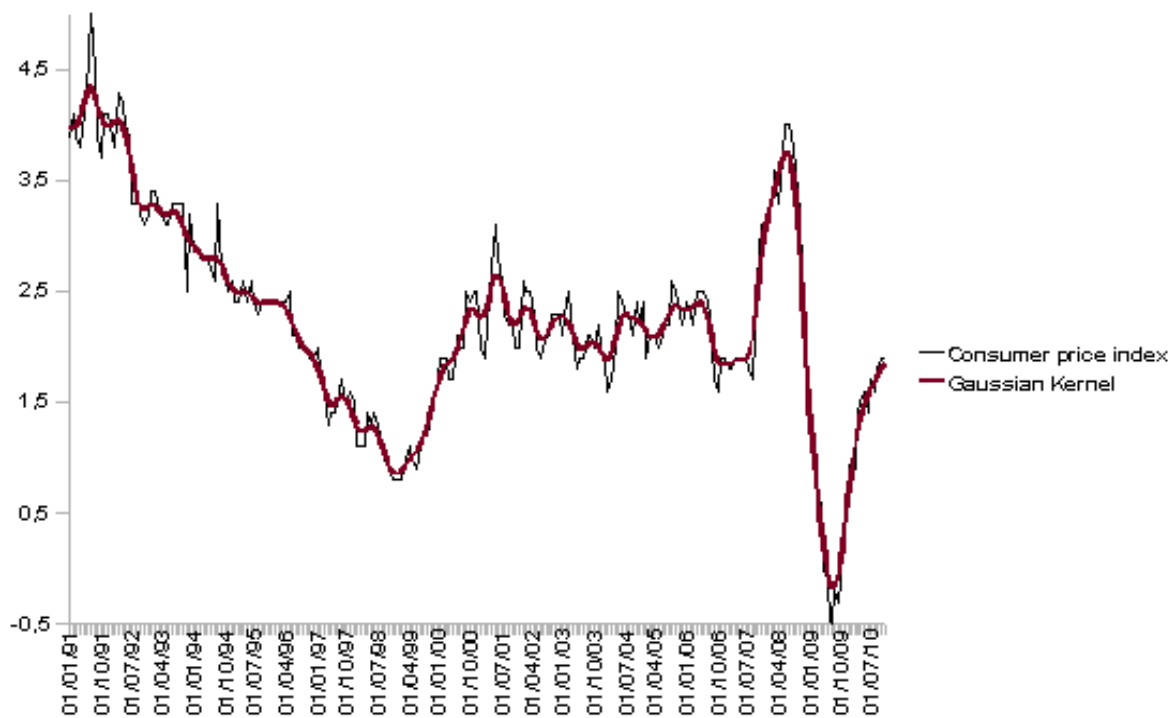


Figure 14.9: Index of European Consumer Price Trend-cycle estimates with Cubic spline





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# 15

## Asymmetric Moving Averages Minimizing Phase Shift



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## Table of Contents

15.1	Introduction	393
15.2	Moving averages: definitions and design	395
15.2.1	Definitions	395
15.2.2	Preserving trends and removing seasonalities	395
15.2.3	Gain and phase shift effects	396
15.3	Optimal moving averages based on revision criteria	397
15.3.1	Musgrave's approach	397
15.3.2	Towards the Generalized Direct Filter Approach	398
15.4	A general unifying framework to derive linear filters	399
15.4.1	From the revision criteria to a very general optimization problem	399
15.4.2	Classical particular cases	400
15.5	The Timeliness criterion	401
15.5.1	Minimizing the phase shift	401
15.5.2	The choice of a convenient penalty function	402
15.5.3	Variation on the Timeliness criterion	404
15.6	Operational procedure and applications	405
15.6.1	A mixed criterion	405
15.6.2	Application	406
15.7	Conclusions	409
15.8	Annex: Proofs	410

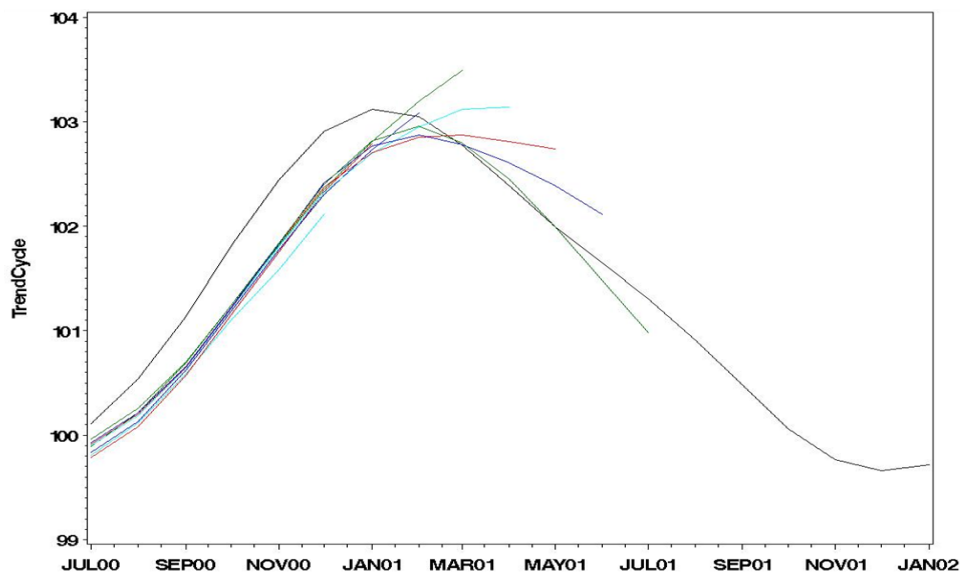


## 15.1 Introduction

Moving averages<sup>1</sup> or linear filters are ubiquitous in seasonal adjustment and business cycle extraction methods. For example, the commonly used software package X12-ARIMA uses Henderson's and composite moving averages to estimate the main components of a time series, when TRAMO-SEATS uses Wiener-Kolmogorov filters. Symmetric filters are applied to the center of the series, but when it comes to the estimation of the most recent points, all these methods must rely on asymmetric filters. X12-ARIMA or TRAMO-SEATS apply symmetric averages on forecasts obtained from an ARIMA modeling of the series. As forecasted values are linear combinations of past values, it turns out that these methods use asymmetric moving averages at the end of the series.

If these asymmetric moving averages have good properties regarding the size of future revisions induced by the smoothing process, see for example Pierce (1980), they also induce phase shifts that usually impact the real-time estimation of turning points. Figure 15.1 gives an illustration of the problem. The thick black line represents the trend-cycle of the French industrial production index estimated in June 2010 with X12-ARIMA. A turning point can be observed in January 2001<sup>2</sup>. The colored lines show the successive estimates of the trend-cycle obtained with X12-ARIMA using data up to March 2001, April 2001, ... and October 2001. It is only in June 2001, and in fact 4 or 5 months after the real date, that a turning point can be noticed .... in February 2001; it will even take more time to get the correct date of January 2001.

**Figure 15.1:** Real-time estimation with X12-ARIMA.



The thick black line represents the "true value" of the trend-cycle of the French industrial production index estimated in June 2010 with X12-ARIMA. The colored lines show the successive estimates of the trend-cycle obtained with X12-ARIMA using data up to March 2001, April 2001 .... October 2001.

The derivation and properties of symmetric and asymmetric moving averages have been studied, in particular in Macaulay (1931), Musgrave (1964), Dagum (1982), Laniel (1985), Grun-Rehomme and Ladiray (1994), Gray and Thomson (1996) and Gray and Thomson (2002). Major studies have been done on trend-cycle estimation during the last 20 years in several directions:

- Making changes to the Henderson filters, to improve the early detection of true turning points, Dagum (1996) introduced a nonlinear trend-cycle estimator also known as Nonlinear Dagum Filter (NLDF).

<sup>1</sup>This chapter precises and develops results presented at the Joint Statistical Meetings in Vancouver, Canada, July 31 - August 5, 2010 and in San Diego, USA, July 28 - August 3, 2012; see Guggemos et al (2012).

<sup>2</sup>The revision span of X12-ARIMA estimates is usually around 5 years and we can consider that we have here the "true/final values" for years before 2005.

Dagum, Luati (2009) proposed a Cascade Linear Filter (CLF) that closely approximates the NLDF and Dagum, Bianconcini (2008) and Dagum, Bianconcini (2015) developed an approximation to the Henderson filter via the Reproducing Kernel Hilbert Space (RKHS) methodology;

- Considering the general problem of estimating the trend-cycle in real time by means of local polynomial filters, Proietti and Luati (2008) proposed a general family of asymmetric filters that minimize the mean square revision error subject to polynomial reproduction constraints; in the case of the Henderson filter it nests the well known Musgrave's surrogate filters associated to Henderson's filters in X12-ARIMA.
- Other strategies have been proposed to estimate the end points: Kyung-Joon and Schucany (1998) use a nonparametric kernel regression, Vasyechko and Grun-Rehomme (2014) use the Epanechnikov's kernel with the Henderson criteria etc.

Whittaker (1923), Whittaker and Robinson (1924) and Henderson (1924) developed a strategy, known today as the Whittaker-Henderson method, to derive moving averages by minimizing an objective function based on Fidelity (the sum of the squares of the deviations of the data from the smoothed curve) and Smoothness (the sum of the squares of the third differences of the smoothed curve). In this chapter, we propose to address directly the phase shift problem adding a Timeliness criterion to the Whittaker-Henderson framework. It allows us to compute moving averages having good properties in terms of Fidelity and Smoothness but also minimizing the phase shift. By drawing up a general unifying framework, we get a theoretical link between this way of designing moving averages, the filters based on minimized revisions purposes and the Generalized Direct Filter Approach, a data-driven procedure proposed by Wildi (1998)<sup>3</sup>. As a consequence, even if we focus here on moving averages whose coefficients do not depend on the characteristics of the series, the results can be easily extended to any kind of moving averages.

Section 15.2 gives some definitions, identifies mathematically both main effects (gain and phase shift) of linear filters and shows how moving averages preserving specific trends or cancelling some seasonalities can be designed. Section 15.3 presents the revision criterion introduced by Musgrave (1964) and draws a parallel with the Wildi's Direct Filter Approach, the generalization of which emphasizes the necessity of minimizing the phase shift. Section 15.4 then shows that the construction of such linear filters can be seen as particular cases of a same very general unifying framework. By varying some modeling assumptions, it provides a large class of linear filters, data independent as well as data dependent, including not only those based on revisions, but also many well-known moving averages like Henderson's ones. The generalized Direct Filter Approach then yields an interesting point of comparison to the new Timeliness criterion introduced in Section 15.5, that permits to minimize the phase shift effects. Finally, in Section 15.6, we present the global procedure to construct asymmetric moving averages that induce a phase shift as small as possible, followed by some illustrations.

<sup>3</sup>See also Wildi (2008) and Wildi (2018).

## 15.2 Moving averages: definitions and design

From now on, let  $(X_t)_{t \in \mathbb{Z}}$  denote a time series that can be usually considered as the sum of three components - the trend-cycle  $TC_t$ , the seasonality  $S_t$  and the irregular component  $I_t$  - with  $X_t = TC_t + S_t + I_t$ . In case of a multiplicative model, the logarithmic transformation of  $(X_t)_{t \in \mathbb{Z}}$  should be rather considered. Moving averages are the basic tool of the X12-ARIMA seasonal adjustment method and are used to estimate these three main components<sup>4</sup>.

### 15.2.1 Definitions

Let  $p$  and  $f$  be two non negative integers and  $\theta = (\theta_{-p}, \dots, \theta_{+f})'$  a vector of  $p + f + 1$  real numbers. The moving average  $L_\theta$  is then defined as the linear endomorphism of the vector space of time series which associates to every series  $(X_t)_{t \in \mathbb{Z}}$ , also called the input signal, the series  $(Y_t)_{t \in \mathbb{Z}} = (L_\theta X_t)_{t \in \mathbb{Z}}$ , also called the output signal, such that:

$$\forall t \in \mathbb{Z}, Y_t = L_\theta X_t = \sum_{k=-p}^{+f} \theta_k X_{t+k}.$$

The value at instant  $t$  of the unadjusted series is therefore replaced by a weighted average of  $p$  “past” values of the series, the current value, and  $f$  “future” values of the series.

- The quantity  $p + f + 1$  is called the moving average order.
- When  $p$  is equal to  $f$ , that is, when the number of points in the past is the same as the number of points in the future, the moving average is said to be centered; otherwise the moving average is said non-centered and therefore asymmetric.
- If, in addition,  $\theta_{-k} = \theta_k$  for any  $k$ , the moving average is said to be symmetric.

As the observations  $X_t$  are available only for a finite set of indexes  $t = 1, \dots, T$ , using with a moving average of order  $p + f + 1$ , it will be impossible to smooth out the first  $p$  values and the last  $f$  values of the series. In the X-11 method, symmetric moving averages play an important role; but, to avoid losing information at the ends of the series, they are supplemented with ad hoc asymmetric moving averages.

If we want for example to get an estimation of the trend-cycle from the decomposition model  $X_t = TC_t + S_t + I_t$  using the moving average  $L_\theta$ , it should be desirable that  $L_\theta$  removes the seasonality  $S_t$ , reduces the irregular component  $I_t$  as much as possible and preserves the trend-cycle  $TC_t$ .

### 15.2.2 Preserving trends and removing seasonalities

It is possible to design a moving average, i.e. determining the set of coefficients  $\theta_k$ , that preserves simple trends, particularly polynomials. For instance, for any moving average to preserve a constant series  $X_t = a$ , it is necessary that, as we have  $a = L_\theta X_t = \sum_k \theta_k X_{t+k} = a(\sum_k \theta_k)$ , the sum of the coefficients of the moving average,  $\sum_{k=-p}^{+f} \theta_k$ , be equal to 1. When looking at preservation of straight lines  $X_t = at + b$ , it is necessary that, as we have for any  $t$ ,  $at + b = L_\theta X_t = \sum_k \theta_k [a(t+k) + b] = (at + b)(\sum_k \theta_k) + a(\sum_k k\theta_k)$ ,  $\sum_{k=-p}^{+f} \theta_k = 1$  and  $\sum_{k=-p}^{+f} k\theta_k = 0$ . Generally, it can be shown that for a moving average to preserve a

<sup>4</sup>In X12-ARIMA and TRAMO-SEATS, the other components, namely the outliers and the trading-day effects, are detected and estimated before the seasonal adjustment procedure per se (see Ladiray and Quenneville (2001) for a detailed presentation of the X12-ARIMA algorithm)



polynomial of degree  $d$ , it is necessary and sufficient that its coefficients satisfy the following conditions:

$$\sum_{k=-p}^{+f} \theta_k = 1 \text{ and } \sum_{k=-p}^{+f} k^j \theta_k = 0, j = 1, \dots, d$$

It is also possible to design a moving average that cancels a seasonality which can evolve polynomially with time. Let us note  $S_t = (a_0 + a_1 t_1 + a_2 t_2 + \dots + a_d t_d) u_t$  where  $u_t$  is a periodical function with period  $\ell$ . And let us define  $n$  the integer satisfying  $p + f + 1 = n\ell$  where  $p + f + 1$  is the order of the moving average. It can be shown<sup>5</sup> that for a moving average to cancel a seasonality evolving like a polynomial of degree  $d$ , it is necessary and sufficient that its coefficients satisfy:

$$\begin{cases} \sum_{j=0}^{n-1} \theta_{k-j\ell} - \sum_{j=0}^{n-1} \theta_{f-j\ell} & = 0 \\ \sum_{j=0}^{n-1} (k-j\ell) \theta_{k-j\ell} - \sum_{j=0}^{n-1} (f-j\ell) \theta_{f-j\ell} & = 0 \\ \dots & \\ \sum_{j=0}^{n-1} (k-j\ell)^d \theta_{k-j\ell} - \sum_{j=0}^{n-1} (f-j\ell)^d \theta_{f-j\ell} & = 0 \end{cases}$$

Preserving trends and removing seasonalities imply therefore that the choice of coefficients is generally subject to some minimal linear constraints. More generally, the subset of admissible vectors  $\theta$  in  $\mathbb{R}^{p+f+1}$  - i.e. the subset of vectors satisfying every constraint - is supposed to be **convex** and will be denoted by  $\Theta$ .

### 15.2.3 Gain and phase shift effects

To identify both main effects of the moving averages, let us now consider an harmonic time series at frequency  $\omega$ ,  $X_t(\omega) = e^{i\omega t}$ . The transform of  $X_t$  by any moving average  $L_\theta$  will be:

$$Y_t = L_\theta X_t = \sum_{k=-p}^{+f} \theta_k X_{t+k} = \sum_{k=-p}^{+f} \theta_k e^{i\omega(t+k)} = \left( \sum_{k=-p}^{+f} \theta_k e^{i\omega k} \right) \cdot X_t.$$

Mathematically, the harmonic time series appear as the eigenvectors of operator  $L_\theta$ , whose eigenvalues are respectively the values taken by the transfer function of the filter,  $\widehat{L}_\theta(e^{i\omega}) = \sum_{k=-p}^{+f} \theta_k e^{i\omega k}$ .

If, for any frequency  $\omega$ ,  $\rho_\theta(\omega)$ ,  $\varphi_\theta(\omega)$  denote respectively the modulus and the argument in  $]-\pi; \pi]$  of this transfer function, then we have  $L_\theta e^{i\omega t} = \rho_\theta(\omega) e^{i[\omega t + \varphi_\theta(\omega)]}$ , or, to stay within real time series domain,

$$L_\theta \cos(\omega t) = \rho_\theta(\omega) \cos[\omega t + \varphi_\theta(\omega)].$$

Then, applying a moving average to an harmonic time series then transform it in two different ways : on the one hand, by multiplying it by an amplitude coefficient  $\rho_\theta(\omega)$ ; on the other hand, by 'shifting' it in time by  $\varphi_\theta(\omega) / \omega$ , which directly affects the detection of turning points. In signal theory,  $\rho_\theta(\omega)$  and  $\varphi_\theta(\omega)$  are simply the gain and the phase of the transfer function  $\widehat{L}_\theta(e^{i\omega})$ . For any stationary time series  $(X_t)_{t \in \mathbb{Z}}$ , its component at frequency  $\omega$  is then affected by these two distinct effects.

<sup>5</sup>see Grun-Rehomme and Ladiray (1994) for a complete proof

## 15.3 Optimal moving averages based on revision criteria

### 15.3.1 Musgrave's approach

The filters commonly used for seasonal adjustment are derived by tuning their characteristics to application purposes, like minimization of revisions. In particular, Doherty (2001) reviewed the method of Musgrave (1964) for calculating the X12-ARIMA trend-cycle asymmetric averages. A summary of this method is given hereinafter.

Let  $\{w_1, \dots, w_N\}$  with  $N = 2p + 1$  be the weights of the Henderson's moving average, the definition of which is recalled in section 15.4.2 Let also  $\theta_1, \dots, \theta_M$  - with  $p + 1 \leq M \leq 2p$  and  $\sum_{i=1}^M \theta_i = 1$  - be one of the  $p$  vectors of Musgrave's asymmetric averages. There are  $p$  such averages, one for each of the last  $p$  points. Assume that  $M$  is fixed and that the last data points of the time series follow a simple linear trend of the following form:

$$X_t = a + bt + \varepsilon_t,$$

where the  $\varepsilon_t$  are uncorrelated random variables with zero mean and variance  $\sigma^2$ . It should be understood that the parameters  $a, b$  and  $\sigma^2$  refer to those at the end of the time series, and that they can change with time as more data become available. Then the weights  $\{\theta_1, \dots, \theta_M\}$  that minimize the expected squared revision,

$$R(\theta) = E \left( \sum_{i=1}^M \theta_i X_i - \sum_{i=1}^N w_i X_i \right)^2$$

under the constraint  $\sum_{i=1}^M \theta_i = 1$ , are given by

$$\theta_i = w_i + \frac{1}{M} \sum_{j=M+1}^{2p+1} w_j + \frac{(i - \frac{M+1}{2}) D}{1 + \frac{M^3 - M}{12} D} \sum_{j=M+1}^{2p+1} \left( j - \frac{M+1}{2} \right) w_j,$$

where  $D = b^2/\sigma^2$ . Note here that for  $M$  fixed, the index  $i$  varies from 1 to  $M$ . In X12-ARIMA, as in X-11, the order of the Henderson's moving average used to obtain an estimate of the trend-cycle is based on a "I/C-ratio" which is computed as:

$$I/C = \frac{\sum |I_t - I_{t-1}|}{\sum |C_t - C_{t-1}|},$$

when the series  $X_t$  is expressed as the sum of a trend-cycle  $C_t$  and an irregular component  $I_t$ . Under the additional assumption that the  $\varepsilon_t$ 's have a normal distribution, the value of  $D$  and  $I/C$  are related by the formula  $D = 4/(\pi(I/C)^2)$  as shown for example by Doherty (2001). In X12-ARIMA,  $I/C = 1.0, 3.5$  and  $4.5$  are used with the 9, 13 and 23-term Henderson's averages respectively and for the Musgrave's surrogates.

Actually, the approach adopted by Musgrave can be easily generalized in different ways, especially by considering less specific models to describe the series and by choosing other kinds of benchmark symmetric weights  $w_i$ . Thus, suppose that the last  $2p + 1$  data points of the time series follow a polynomial trend of degree  $d$  and that we have, for  $t = -p, \dots, +p$ ,  $X_t = a_0 + a_1 t^1 + \dots + a_d t^d + \varepsilon_t$ , where  $(\varepsilon_t)_t$  is a random stationary process with zero mean. With matricial notations, these assumptions can be rewritten as follows:  $\mathbf{X} = \mathbf{A}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$ , with  $\mathbf{X} = [X_{-p}, X_{-p+1}, \dots, X_{+p}]'$ ,  $\boldsymbol{\beta} = [a_0, a_1, \dots, a_d]'$ ,  $\boldsymbol{\varepsilon} = [\varepsilon_{-p}, \varepsilon_{-p+1}, \dots, \varepsilon_{+p}]'$  and

$$\mathbf{A} = \begin{pmatrix} 1 & -p & (-p)^2 & \dots & (-p)^d \\ 1 & -p+1 & (-p+1)^2 & \dots & (-p+1)^d \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & +p & (+p)^2 & \dots & (+p)^d \end{pmatrix}.$$

If  $\mathbf{w} = (w_{-p}, \dots, w_{+p})'$  denotes the vector of the benchmark symmetric moving average coefficients and  $\boldsymbol{\theta} = (\theta_{-p}, \dots, \theta_{+p})'$  the vector of the asymmetric moving average (with its coefficients  $\theta_k$  equal to 0 if  $p+1 \leq k \leq f$ ), the revision criterion to be minimized can be then expressed as:

$$R(\boldsymbol{\theta}) = E \left[ \sum_{i=-p}^{+p} \theta_i X_i - \sum_{i=-p}^{+p} w_i X_i \right]^2 = (\boldsymbol{\theta} - \mathbf{w})' \mathbf{R} (\boldsymbol{\theta} - \mathbf{w})$$

where, as  $E(\varepsilon) = 0$  and  $\text{Var}(\varepsilon) = \boldsymbol{\Sigma}$ , matrix  $\mathbf{R}$  is given by :

$$\mathbf{R} = E(\mathbf{X}\mathbf{X}') = E[(\mathbf{A}\boldsymbol{\beta} + \varepsilon)(\mathbf{A}\boldsymbol{\beta} + \varepsilon)'] = \mathbf{A}\boldsymbol{\beta}\boldsymbol{\beta}'\mathbf{A}' + \boldsymbol{\Sigma}.$$

### 15.3.2 Towards the Generalized Direct Filter Approach

Heretofore, the way to derive moving averages regarding optimal revision purposes is guided by parametric modelings, which force to make more or less strong assumptions on the input time series. Conversely, a non-parametric approach can also be considered. By only assuming that the filter error, i.e. the difference between the benchmark symmetric filtered signal and the asymmetric filtered one, is a stationary process, the Direct Filter Approach (DFA) - see Wildi (1998), Wildi (2005) - consists first in “translating” the revision criterion to be minimized in the frequency domain. For stationary processes  $X_t$ , the mean squared filter error can indeed be expressed as:

$$E(L_{\boldsymbol{\theta}}X_t - L_{\mathbf{w}}X_t)^2 = \int_0^{2\pi} \left| \widehat{L}_{\boldsymbol{\theta}}(e^{i\omega}) - \widehat{L}_{\mathbf{w}}(e^{i\omega}) \right|^2 dH_X(\omega), \quad (15.1)$$

where  $H_X(\omega)$  is the unknown spectral density of  $X_t$ . The weights  $\boldsymbol{\theta}$  of the asymmetric filter can then be computed by minimizing an estimation of the right-hand term of equation 15.1, based on a discretization of the interval  $[0; 2\pi]$ . The case of non-stationary integrated processes can be also handled within this framework, insofar as cointegration between both filtered signals can be easily satisfied by imposing linear constraints on vector  $\boldsymbol{\theta}$ , like those presented in section 15.1.

The main interest of this procedure lies in the fact that it is now possible to split the revision criterion into two distinct effects, one related to the gain and one related to the phase of the transfer functions. Indeed, as noticed by Wildi, we have:

$$\begin{aligned} \left| \widehat{L}_{\boldsymbol{\theta}}(e^{i\omega}) - \widehat{L}_{\mathbf{w}}(e^{i\omega}) \right|^2 &= (\rho_{\boldsymbol{\theta}}(\omega) - \rho_{\mathbf{w}}(\omega))^2 + 2\rho_{\mathbf{w}}(\omega)\rho_{\boldsymbol{\theta}}(\omega)[1 - \cos(\varphi_{\boldsymbol{\theta}}(\omega))] \\ &= \underbrace{(\rho_{\boldsymbol{\theta}}(\omega) - \rho_{\mathbf{w}}(\omega))^2}_{\text{Gain effect}} + \underbrace{4\rho_{\mathbf{w}}(\omega)\rho_{\boldsymbol{\theta}}(\omega)\sin^2(\varphi_{\boldsymbol{\theta}}(\omega)/2)}_{\text{Phase shift effect}}. \end{aligned} \quad (15.2)$$

The previous equations are derived from the fact that the phase function of a symmetric filter is real, so that its corresponding phase  $\varphi_{\mathbf{w}}(\omega)$  is always equal to 0 or  $\pi$ . In our scope of interest, we naturally suppose the latter is actually null (i.e. the transfer function is positive), at least at low frequencies. Indeed, classical constraints imposed to the filters, like preserving constant trends, are in agreement with such an assumption, since, for instance, condition  $\sum_k w_k = 1$  implies that  $\widehat{L}_{\mathbf{w}}(0)$  be equal to 1 and, by continuity arguments,  $\varphi_{\mathbf{w}}(\omega)$  be null at low frequencies.

Thanks to this factorization of the revision criterion, we are now able to deal with the trade-off between reliability and timeliness aspects, by emphasizing more or less the phase shift effect and by attaching varying importance to the different frequency components, depending on whether they appear in the spectrum of the initial time series or not. Thus, Wildi first suggests to modify the revision criterion in the following way,

$$\int_0^{2\pi} \left| \widehat{L}_{\boldsymbol{\theta}}(e^{i\omega}) - \widehat{L}_{\mathbf{w}}(e^{i\omega}) \right|^2 W(\omega) dH_X(\omega) + 4\lambda \int_0^{2\pi} \rho_{\mathbf{w}}(\omega)\rho_{\boldsymbol{\theta}}(\omega)\sin^2(\varphi_{\boldsymbol{\theta}}(\omega)/2) W(\omega) dH_X(\omega),$$

where  $\lambda$  is a tuning factor, balancing the gain and phase shift components, and  $W(\omega)$  is a frequency weighting function. Increasing  $\lambda$  amounts to giving priority to the minimization of the phase error and so to the real-time detection of turning points - or more generally of trend changes.

As mentioned previously, the presence of phase shift effects is mainly due to the asymmetry of the filter, or equivalently to the fact that the transfer function is complex. Consequently, another way to reduce them consists in giving an higher weight to the imaginary part of the transfer function in the criterion to be minimized. To generalize the DFA, Wildi has then proposed recently to replace  $\widehat{L}_\theta(e^{i\omega})$  by  $\Re(\widehat{L}_\theta(e^{i\omega})) + i\sqrt{1 + 4\lambda\widehat{L}_w(e^{i\omega})} \cdot \Im(\widehat{L}_\theta(e^{i\omega}))$  in (15.1), where  $\Re$  and  $\Im$  denote the real part and the imaginary part respectively. Developing expression (15.1) by accounting for this surrogate and weighting frequencies leads him eventually to minimize a discretized version of the following criterion:

$$\int_0^{2\pi} \left| \widehat{L}_\theta(e^{i\omega}) - \widehat{L}_w(e^{i\omega}) \right|^2 W(\omega) dH_X(\omega) + 4\lambda \int_0^{2\pi} \rho_w(\omega) \rho_\theta^2(\omega) \sin^2(\varphi_\theta(\omega)) W(\omega) dH_X(\omega).$$

Unlike the previous criterion given in (15.3.2), the latter is a quadratic function of the filter coefficients, making it easier to solve the minimization problem from a computational point of view.

## 15.4 A general unifying framework to derive linear filters

### 15.4.1 From the revision criteria to a very general optimization problem

The revision criterion on which Musgrave’s approach and the DFA are based is nothing else but a distance measure between the output signal and what the latter would be if we were not confronted with the lack of observations in the future. Actually, the choice of the distance measure as well as the time series to which the output signal is compared can be widened. The ways to derive linear filters presented in section 15.3 are encompassed within the following more general optimization problem,

$$\begin{cases} \min_{\theta} E \left[ [\nabla^q (L_\theta X_t - u_t)]^2 \right] \\ \text{subject to } C\theta = \mathbf{a}, \end{cases} \tag{15.3}$$

where  $\nabla$  is the discrete differentiation operator on time series,  $\nabla X_t = X_t - X_{t-1}$ , and, for any integer

$$q \geq 1, \nabla^q = \nabla \circ \nabla^{q-1} = \underbrace{\nabla \circ \dots \circ \nabla}_{q \text{ factors}}$$

with  $\nabla^0$  denoting the identity function. The filter coefficients are chosen by minimizing the  $q$ -th difference of the discrepancy between the output signal and any benchmark time series  $(u_t)_{t \in \mathbb{Z}}$ . Moreover, they must satisfy some given linear constraints, for which the equality  $C\theta = \mathbf{a}$  provides a matricial notation. The number of rows of matrix  $C$  and vector  $\mathbf{a}$  is equal to the number of constraints. Because the latter are supposed to be linearly independent, we assume that  $C$  is a full row rank matrix.

The benchmark time series  $u_t$  is chosen to yield a rough estimate of the trend-cycle of the initial signal  $X_t$ . It can be either deterministic or stochastic and may depend itself on the filter coefficients. The order of differentiation and the linear constraints are chosen so that the filter satisfy expected properties like preserving trends and removing seasonalities. In this way, they can ensure that the process  $\nabla^q (L_\theta X_t - u_t)$  be stationary.

Moreover, beyond these considerations, the choice of modeling assumptions to apply to the initial time series is completely free. By varying these assumptions, we obtain a very large class of linear filters. Musgrave’s

moving averages and the DFA are obtained by choosing  $q = 0$  and  $u_t = L_w X_t$ . In the first case, a parametric model is used whereas, in the second case, the filter coefficients are computed thanks to a non-parametric estimation of the data spectral density. Actually, in the next subsection, we will see that more constraining assumptions can lead to well-known filters which depend neither on the data nor even on the date of estimation.

## 15.4.2 Classical particular cases

### 15.4.2.1 Fidelity

A first application of the general framework given in section 15.4.1 provides simple moving averages suggested by Bongard (1962). Indeed, Bongard's approach consists in choosing moving averages having a strong attenuation effect on the irregular component, once trends have been preserved and seasonalities removed. In the decomposition of the unadjusted series, the residual is often modeled in the form of a white noise  $\varepsilon_t$  with zero expectation and constant variance  $\sigma^2$ . This white noise is transformed by the moving average into a sequence of random variables,  $\varepsilon_t^*$ , with a constant variance equal to  $\sigma^2 \sum_{k=-p}^{+f} \theta_k^2$ . Reducing the irregular component, and therefore its variance, then amounts to reducing the criterion  $\sum_{k=-p}^{+f} \theta_k^2$ . The output signal is supposed to be "as close as possible" to the input signal where noise components are removed; that is why this criterion will be called here the "Fidelity" criterion, denoted by  $F(\theta)$ . Considering additive white noises, the "Fidelity" criterion is a very simple positive quadratic form,

$$F(\theta) = \sum_{k=-p}^{+f} \theta_k^2 = \theta' \mathbf{F} \theta,$$

with  $\mathbf{F}$  being the identity matrix of order  $p + f + 1$ . This way of designing linear filters can be directly obtained by considering the general problem (15.3), with  $q = 0$  and where  $u_t$  is the deterministic benchmark time series  $u_t = E(L_\theta X_t)$ .

### 15.4.2.2 Smoothness

X12-ARIMA uses Henderson's moving averages to extract the trend-cycle from an estimate of the seasonally adjusted series (Tables B7, C7, D7, D12)<sup>6</sup>. In the additive case, the model which is considered then becomes  $X_t = TC_t + I_t$ . Henderson suggests to use a moving average derived from a criterion which ensures a smooth estimation of the trend-cycle. Let us consider the Dirac time series  $\delta_t^{t_0}$ , equal to 1 at instant  $t = t_0$  and to 0 at any other instant. Its transform by a moving average  $L_\theta$  of order  $p + f + 1$  with coefficients  $\{\theta_k\}$ , is given by:

$$L_\theta \delta_t^{t_0} = \begin{cases} \theta_{t_0-t} & \text{if } t_0 - f \leq t \leq t_0 + p \\ 0 & \text{otherwise} \end{cases}.$$

This transform will therefore be smooth if the coefficient curve of the moving average is not too irregular. Noting that the vector space of time series is generated by the Dirac ones, since any time series  $X_t$  can be written as  $X_t = \sum_{t_0 \in \mathbb{Z}} X_{t_0} \delta_t^{t_0}$ , Henderson (1916)<sup>7</sup> proposes to use the quantity  $S = \sum_k (\nabla^3 \theta_k)^2$  to measure the 'flexibility' of the coefficient curve. The notation  $\nabla$ , introduced in section 15.4.1 as a linear operator on time series, is extended with a similar meaning onto the set of vector coefficients  $\theta$ , i.e.  $(\nabla \theta)_k = \nabla \theta_k = \theta_k - \theta_{k-1}$ . Quantity  $S$  vanishes when the coefficients  $\{\theta_k\}$  are located along a parabola. In the general case, it measures the difference between a parabola and the form of the coefficient curve. Henderson then looked for centered averages that preserve quadratic polynomials and minimize  $S$ .

<sup>6</sup>See Ladiray and Quenneville (2001)

<sup>7</sup>See also Henderson (1924).

Minimizing Henderson's criterion, or more generally the 'Smoothness criterion'

$$S(\boldsymbol{\theta}) = \sum_k (\nabla^q \theta_k)^2,$$

aims at getting quite regular output signals. Notice that  $S(\boldsymbol{\theta})$ , as the previous 'Fidelity' criterion, is also a positive quadratic form over  $\mathbb{R}^{p+f+1}$ . For instance, if we consider Henderson's case  $q = 3$ , we have  $S(\boldsymbol{\theta}) = \boldsymbol{\theta}'\mathbf{S}\boldsymbol{\theta}$ , where  $\mathbf{S}$  is the following symmetric Toeplitz matrix :

$$\mathbf{S} = \begin{pmatrix} 20 & -15 & 6 & -1 & 0 & \dots & 0 \\ -15 & 20 & -15 & 6 & \ddots & \ddots & \vdots \\ 6 & -15 & 20 & \ddots & \ddots & \ddots & 0 \\ -1 & 6 & \ddots & \ddots & \ddots & 6 & -1 \\ 0 & \ddots & \ddots & \ddots & 20 & -15 & 6 \\ \vdots & \ddots & \ddots & 6 & -15 & 20 & -15 \\ 0 & \dots & 0 & -1 & 6 & -15 & 20 \end{pmatrix}.$$

Actually, let us extend the notation of the filter coefficients by stating that  $\theta_k = 0$  for any integer  $k \notin [-p; +f]$ . We have then :  $\nabla^q (L_\theta X_t) = (-1)^q \sum_{k \in \mathbb{Z}} (\nabla^q \theta_k) X_{t+k-q}$ . Thus, if we consider similar assumptions on the irregular component to those made for the Fidelity criterion in section 15.4.2.1, Henderson's approach then appears to be a particular case of problem (15.3) where  $u_t$  is the deterministic benchmark time series  $u_t = \mathbb{E}(L_\theta X_t)$  again, but  $q$  is now a positive integer.

Note that other classical linear filters, such as the Hodrick-Prescott filter, can also be considered as particular cases of problem (15.3).

## 15.5 The Timeliness criterion

### 15.5.1 Minimizing the phase shift

As suggested in section 15.2.3, an approach to reduce the observed phase shift between any input signal and its corresponding output signal could consist in searching for moving averages which introduce a small phase shift when they are applied to harmonic signals  $(e^{i\omega t})_{t \in \mathbb{Z}}$ . In other words, we are going to search for vectors  $\boldsymbol{\theta}$  such that the phase  $\varphi_\theta(\omega)$  of filter  $L_\theta$  be close to 0. Such an approach should provide moving averages with *a priori* satisfying properties in terms of phase shift, whatever the input signal to which they are applied.

If a vector  $\boldsymbol{\theta}$  is such that  $\varphi_\theta(\omega_0) = 0$ , then it guarantees that both signals  $(e^{i\omega_0 t})_{t \in \mathbb{Z}}$  and  $(L_\theta e^{i\omega_0 t})_{t \in \mathbb{Z}}$  be in phase, but this property is no more satisfied for any pair of signals  $(e^{i\omega t})_{t \in \mathbb{Z}}$  and  $(L_\theta e^{i\omega t})_{t \in \mathbb{Z}}$  as soon as we consider a frequency  $\omega$  different from  $\omega_0$ . In fact, as the transfer function is regular and the phase is supposed to be within the range  $] -\pi; \pi ]$ , it can be only stated that the phase  $\varphi_\theta(\omega)$  is also a regular function and thus is close to 0 in a neighbourhood of  $\omega_0$ . As any stationary signal can be decomposed into linear combinations of harmonic signals characterized by different frequencies, the filter coefficients  $\boldsymbol{\theta}$  must be actually chosen among those for which the phase function is close to 0, whatever the frequency  $\omega$ . Practically, we will rather consider the range of frequencies  $] \omega_1; \omega_2 [$  which belong to the spectrum of stationary components of time series to which the moving average  $L_\theta$  will be applied, with  $0 \leq \omega_1 \leq \omega_2 \leq 2\pi$ .

Besides, when filtering, the harmonic component of the input signal at frequency  $\omega$  is multiplied by an amplitude factor equal to the modulus function. Therefore, it is natural to consider that the impact of the phase shift on the harmonic component at frequency  $\omega$  will be all the more significant so as the value  $\rho_\theta(\omega)$  taken by the modulus function is high.

As a consequence, in order to reduce the phase shift effects between input and output signals, we suggest the use of a moving average whose coefficients minimize the following criterion, called the “timeliness” criterion:

$$\int_{\omega_1}^{\omega_2} f[\rho_{\theta}(\omega); \varphi_{\theta}(\omega)] d\omega, \quad (15.4)$$

where the function  $f$ , defined over  $[0; +\infty[ \times ]-\pi; \pi]$  is chosen beforehand and satisfies the six following conditions:

1.  $f \geq 0$
2.  $f(\rho, 0) = 0$
3.  $f(0, \varphi) = 0$
4.  $f(\rho, \varphi) = f(\rho, -\varphi)$
5.  $\frac{\partial f}{\partial \rho} \geq 0$
6.  $\varphi \cdot \frac{\partial f}{\partial \varphi} \geq 0$

From now on, the function  $f$  is called the penalty function. Conditions 1 and 2 assure that this function is minimized when the phase is null. Conditions 4 and 6 mean that, in the criterion (15.4), the phase is penalized only in function of its distance from 0, all the more so as such a distance is high. Moreover, condition 5 means that, the more a frequency is amplified by the filter, the more the corresponding phase shift should be reduced. Eventually, in the limit case of a frequency cut off by the filter, condition 3 stipulates that it is unnecessary to give a non null weight to the corresponding phase in the criterion.

### 15.5.2 The choice of a convenient penalty function

Now, the penalty function has to be chosen. It would be desirable to find a function satisfying the six previous conditions such that the problem consisting in minimizing (15.4) can be solved analytically or, if it is not possible, by using numerical algorithms. In the latter case, we must then be able to prove the existence of both a theoretical solution and a numerical algorithm converging quickly to this solution. A penalty function  $f$  providing a convex criterion would be suitable since minimizing the quantity given in (15.4) would then become a classical convex optimization problem for which some converging numerical algorithms are well-known (gradient methods,...).

**Proposition 1 (A first family):** *For any  $k > 0$  and  $\ell > 0$ , the function  $(\rho, \varphi) \mapsto \rho^k |\varphi|^\ell$  satisfies the six conditions imposed on the penalty function appearing in criterion (15.4).*

However, it seems better to force the second parameter  $\ell$  to be greater than or equal to 1. Otherwise, the partial derivative  $|\partial f / \partial \varphi|$  would tend to  $+\infty$  when  $\varphi$  tends to 0; non-null phases close to 0 would be too much penalized.

**Proposition 2 (A second family):** *For any  $k > 0$  and  $\ell > 0$ , the function  $(\rho, \varphi) \mapsto \rho^k \left| \sin\left(\frac{\varphi}{2}\right) \right|^\ell$  satisfies the six conditions imposed on the penalty function appearing in criterion (15.4).*

Once again, we can recommend the use of a function with a parameter  $\ell$  greater than or equal to 1. Let us notice that functions of this second family are  $2\pi$ -periodic with respect to the variable  $\varphi$  (so that it would be no more necessary to force the phase to belong to  $] -\pi; \pi ]$ ).

Here, we suggest choosing the penalty function given in Proposition 2, with parameters  $k = 1$  et  $\ell = 2$ . In

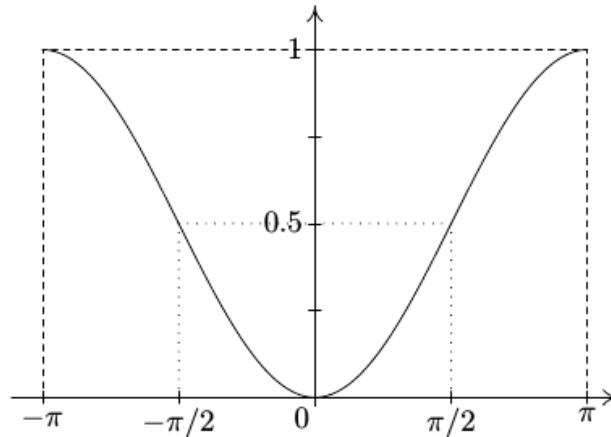


other words,

$$f[\rho_\theta(\omega); \varphi_\theta(\omega)] = \rho_\theta(\omega) \sin^2\left(\frac{\varphi_\theta(\omega)}{2}\right) = \frac{1}{2} \left[ \rho_\theta(\omega) - \Re\left(\widehat{L}_\theta(e^{i\omega})\right) \right] \quad (15.5)$$

As shown later, such a penalty function is worthwhile inasmuch as it makes the minimization of criterion (15.4) solvable, at least with classical numerical algorithms. But notice also that it corresponds exactly to the phase shift effect appearing in (15.2), which is then emphasized in the revision criterion (15.3.2) for the Direct Filter Approach. However, in this section, the filter is supposed to be independent from the data and its phase shift properties are analysed directly and not compared to those of a convenient symmetric filter.

**Figure 15.2:** Penalty function  $\varphi \mapsto f(1, \varphi) = \sin^2(\frac{\varphi}{2})$



Let now  $\mathbf{Co}(\omega)$  and  $\mathbf{Si}(\omega)$  denote the vectors defined by

$$[\cos(-p\omega), \dots, \cos(f\omega)]'$$

and

$$[\sin(-p\omega), \dots, \sin(f\omega)]'$$

respectively. Let also  $\mathbf{\Omega}(\omega)$  be the square matrix of order  $p + f + 1$  whose entry lying in the  $k$ -th row and the  $\ell$ -th column is given by  $\Omega_{k\ell} = \cos[(k - \ell)\omega]$  for any indices  $-p \leq k, \ell \leq +f$ . Note that we have then  $\Re(\widehat{L}_\theta(e^{i\omega})) = \mathbf{Co}(\omega)' \boldsymbol{\theta}$ ,  $\Im(\widehat{L}_\theta(e^{i\omega})) = \mathbf{Si}(\omega)' \boldsymbol{\theta}$  and  $\mathbf{\Omega}(\omega) = \mathbf{Co}(\omega) \mathbf{Co}(\omega)' + \mathbf{Si}(\omega) \mathbf{Si}(\omega)'$ , so that  $\rho_\theta^2(\omega) = \boldsymbol{\theta}' \mathbf{\Omega}(\omega) \boldsymbol{\theta}$ . Thus, matrix  $\mathbf{\Omega}(\omega)$  is symmetric, positive semi-definite but non-invertible (its rank is lesser than or equal to 2).

Considering the penalty function given in (15.5), criterion (15.4) can be rewritten in the following way:

$$2T_0(\boldsymbol{\theta}) = \int_{\omega_1}^{\omega_2} \sqrt{\boldsymbol{\theta}' \mathbf{\Omega}(\omega) \boldsymbol{\theta}} d\omega - \left( \int_{\omega_1}^{\omega_2} \mathbf{Co}(\omega) d\omega \right)' \boldsymbol{\theta} \quad (15.6)$$

**Proposition 3:** Criterion  $T_0(\boldsymbol{\theta})$  given in (15.6) is a convex function of the vector of coefficients  $\boldsymbol{\theta}$ . (Proof of the proposition in the annex)

The objective mentioned at the beginning of this section is achieved. Proposition 3 implies that the phase shift minimization problem for the penalty function given in (15.5), as a convex optimization problem, has a solution and is solvable by using for instance a gradient method. For this purpose, let us also give the expression of the gradient of criterion  $T_0(\boldsymbol{\theta})$ .



**Proposition 4:** The gradient of criterion  $T_0(\theta)$  given in (15.6) is given by:

$$2 \frac{\partial T_0(\theta)}{\partial \theta} = \left( \frac{1}{2} \int_{\omega_1}^{\omega_2} \frac{\Omega(\omega) \theta}{\sqrt{\theta' \Omega(\omega) \theta}} d\omega \right) - \left( \int_{\omega_1}^{\omega_2} \mathbf{Co}(\omega) d\omega \right)$$

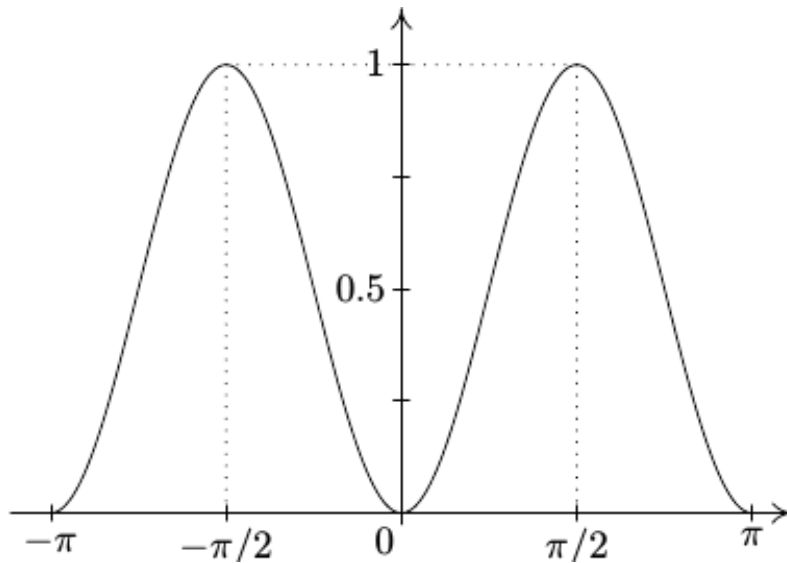
(Proof of the proposition in annex)

### 15.5.3 Variation on the Timeliness criterion

In this section, we propose releasing some constraints on the penalty function to get a more practical Timeliness criterion from a computational point of view. As already said in section 15.3.2, another way to restrict the phase shift effects consists in reducing the imaginary part of the transfer function. By doing so, this boils down to considering the phase function *modulo*  $\pi$  and consequently to tolerate anti-phases between input and output signals. Geometrically, this merely means that the distance from the transfer function to the real line in the complex plane is reduced as much as possible, whereas mathematically the penalty function  $f$  must now satisfy the six conditions given in section 15.5.1, but with a slight restriction. Indeed, the sixth condition has to be valid only for phases  $\varphi$  in the interval  $[-\pi/2; \pi/2]$  and is completed by a seventh one, stipulating that for any phase  $\varphi \in ]\pi/2; \pi]$ ,  $f(\rho, \varphi) = f(\rho, \pi - \varphi)$ . A family of penalty functions satisfying the seven conditions is given below:

**Proposition 5:** For any  $k > 0$  and  $\ell > 0$ , the function  $(\rho, \varphi) \mapsto \rho^k |\sin(\varphi)|^\ell$  satisfies the seven conditions imposed on the penalty function appearing in the problem where phase shift effects are minimized modulo  $\pi$ .

**Figure 15.3:** Penalty function  $\varphi \mapsto f(1, \varphi) = \sin^2(\varphi)$



Among these functions, the following one, obtained for  $k = \ell = 2$ ,

$$f[\rho_\theta(\omega); \varphi_\theta(\omega)] = \rho_\theta^2(\omega) \sin^2(\varphi_\theta(\omega)) = \left[ \Im \left( \widehat{L}_\theta(e^{i\omega}) \right) \right]^2, \quad (15.7)$$

is particularly worthwhile because it makes the minimization of the Timeliness criterion analytically solvable. Indeed, the latter, denoted by  $T(\theta)$ , is simply a quadratic form :

$$T(\theta) = \int_{\omega_1}^{\omega_2} \left[ \Im \left( \widehat{L}_\theta(e^{i\omega}) \right) \right]^2 d\omega = \theta' \mathbf{T} \theta, \quad (15.8)$$

where  $\mathbf{T} = \int_{\omega_1}^{\omega_2} \mathbf{Si}(\omega) \mathbf{Si}(\omega)' d\omega$  is a square matrix of order  $p + f + 1$ . After a few calculations, it can be shown that its entries are given for  $-p \leq k, \ell \leq f$  by:

$$\mathbf{T}_{kl} = \begin{cases} \frac{\sin((k-\ell)\omega_2) - \sin((k-\ell)\omega_1)}{2(k-\ell)} - \frac{\sin((k+\ell)\omega_2) - \sin((k+\ell)\omega_1)}{2(k+\ell)} & \text{if } |k| \neq |\ell| \text{ and } k\ell \neq 0 \\ \frac{\omega_2 - \omega_1}{2} - \frac{\sin(2k\omega_2) - \sin(2k\omega_1)}{4k} & \text{if } k = \ell \text{ and } k\ell \neq 0 \\ \frac{\sin(2k\omega_2) - \sin(2k\omega_1)}{4k} - \frac{\omega_2 - \omega_1}{2} & \text{if } k = -\ell \text{ and } k\ell \neq 0 \\ 0 & \text{if } k\ell = 0 \end{cases}$$

Not surprisingly, controlling the phase shift effects by the means of the imaginary part of the transfer function has led us to a criterion (15.7) which is exactly the non-data-driven equivalent of the Generalized Direct Filter Approach summed up in equation (15.3.2). Its main drawback lies in the lack of penalization of anti-phases. However, this is not really knotty. Indeed, for the same reasons as those invoked in section 15.3.2, the transfer function is close to 1 and therefore the phase function close to 0 at low frequencies. Thus, anti-phases can only occur at higher frequencies, but the constraints usually imposed to the filter coefficients, regarding trends or seasonalities, aim precisely at removing or at least strongly reducing the high frequency components of the input signals.

## 15.6 Operational procedure and applications

### 15.6.1 A mixed criterion

It is important to notice that there are always an endless number of solutions  $\theta$  to the minimization of the Timeliness criterion. For instance, criterion  $T_0$  is convex but not strictly convex, whereas criterion  $T$  is only a semi-definite positive quadratic form. For instance, the latter is null and so reaches its minimum exactly on the whole vector subspace of symmetric vectors, i.e. of vectors  $\theta$  such that  $\theta_k = \theta_{-k}$  for any  $|k| \leq \min(p, f)$  and  $\theta_k = 0$  otherwise.

Classical procedures to choose the coefficients of a moving average have been based on the minimization of the Fidelity and/or the Smoothness criteria. A quite natural idea could consist now in mixing them with the Timeliness criterion to account for the various biases induced by a moving average. Using the notations introduced in section 15.4.1, 15.4.2 and 15.5.3 to depict the linear constraints regarding trends and seasonalities and to compute the previously mentioned criteria, we suggest the following operational procedure to derive moving averages minimizing phase shift effects:

**Problem 1:** (The operational FST procedure)

$$\begin{cases} \min_{\theta} J(\theta) = \alpha \cdot F(\theta) + \beta \cdot S(\theta) + \gamma \cdot T(\theta) \\ \text{Subject to } \mathbf{C}\theta = \mathbf{a} \end{cases}$$

Where the 3 real numbers  $\alpha, \beta, \gamma$  are positive and satisfy the condition  $\alpha + \beta \neq 0$ . Such an optimization problem has a unique solution.

The first condition on real numbers  $\alpha, \beta, \gamma$  means that the mixed criterion  $J(\theta)$  is a convex linear combination of the Fidelity, Smoothness and Timeliness ones. The second condition guarantees that  $J(\theta)$  is a strictly convex function, that is why the solution to problem 1 is unique. Incidentally, the latter is given by  $\theta = \mathbf{J}^{-1} \mathbf{C}' (\mathbf{C} \mathbf{J}^{-1} \mathbf{C}')^{-1} \mathbf{a}$ , where  $\mathbf{J} = \mathbf{F} + \mathbf{S} + \mathbf{T}$ . Obviously, Timeliness criterion  $T(\theta)$  could be replaced by the more accurate one  $T_0(\theta)$  presented in formula (15.6), but resolution of problem 1 would be then no more analytical.

The problem of the optimal choice of the coefficients  $\alpha, \beta$ , and  $\gamma$  as well as the design of the matrix of linear constraints is still an open issue and we only give some examples in the next section.

## 15.6.2 Application

### 15.6.2.1 Surrogate Henderson's filters

Asymmetric Musgrave's filters (see Section 15.3.1) are used in X12-ARIMA in association with Henderson's filter to estimate the trend-cycle. The FST criterion allows looking for alternative sets of asymmetric averages. In this example, we focus on the 13-term symmetric Henderson's filter, 6 points in the past and 6 points in the future (H6\_6), and derive the following moving averages for the estimation of the last 6 points of the series:

- Musgrave's filters of decreasing order, with always 6 points in the past and 5, 4, 3, 2, 1 and 0 points in the future (M6\_5 to M6\_0). These filters preserve local linear trends and minimize the expected revisions.
- Henderson's asymmetric filters, derived using only the smoothness criterion ( $\alpha = 0$  and  $\gamma = 0$ ). They preserve local polynomials of order 2 and are all of order 13 with increasing number of points in the past and decreasing number of points in the future (H12\_0 to H7\_5);
- Henderson's asymmetric filters minimizing the phase shift. In this example, we put a large weight on the timeliness criterion ( $\alpha = 0$ ,  $\beta = 1$  and  $\gamma = 1000$ ). Like the previous filters, they preserve local polynomials of order 2 and are all of order 13 with increasing number of points in the past and decreasing number of points in the future (Hps12\_0 to Hps7\_5).

**Table 15.1:** Performances of various symmetric and asymmetric Henderson's moving averages.

Moving Average	Criteria	Number of terms in the future						Symmetric Henderson
		0	1	2	3	4	5	H_6_6
Musgrave	Fidelity	0.388	0.268	0.201	0.181	0.188	0.199	0.204
Musgrave	Smoothness	1.272	0.433	0.080	0.010	0.021	0.017	0.008
Musgrave	Timeliness	0.261	0.286	0.263	0.198	0.121	0.039	0
Henderson	Fidelity	0.985	0.494	0.258	0.176	0.173	0.193	0.204
Henderson	Smoothness	0.169	0.071	0.023	0.005	0.003	0.007	0.008
Henderson	Timeliness	0.116	0.015	0.003	0.005	0.007	0.003	0
No Phase-Shift	Fidelity	1.047	0.416	0.407	0.354	0.272	0.230	0.204
No Phase-Shift	Smoothness	2.403	0.229	0.166	0.134	0.053	0.018	0.008
No Phase-Shift	Timeliness	19E-6	0	1E-6	0	0	0	0

Table 15.1 summarizes the properties of the 3 sets of moving averages in terms of Fidelity, Smoothness and Timeliness. All sets complement the symmetric 13-term Henderson's noted H6\_6 in the table, which is a "benchmark" in this exercise with a Fidelity criterion equals to 0.204, a Smoothness criterion of 0.008 and a Timeliness criterion equal to 0.

- As expected, Musgrave's averages perform well in terms of Fidelity, except perhaps for the two last points (M6\_1 and M6\_0) and quite poorly in terms of Smoothness and Timeliness;
- Due to the choice of the FST weights, the asymmetric Henderson's moving averages minimizing phase-shift perform poorly as regards the Fidelity and Smoothness criteria and introduce very little phase-shift if any;
- The surprise comes perhaps from the usual asymmetric Henderson's moving averages, constructed from the Smoothness criterion only. The asymmetric average H9\_3 performs better than the symmetric Henderson on the Fidelity and Smoothness criteria, respectively 0.176 and 0.05, and still well on the Timeliness criterion (0.005).

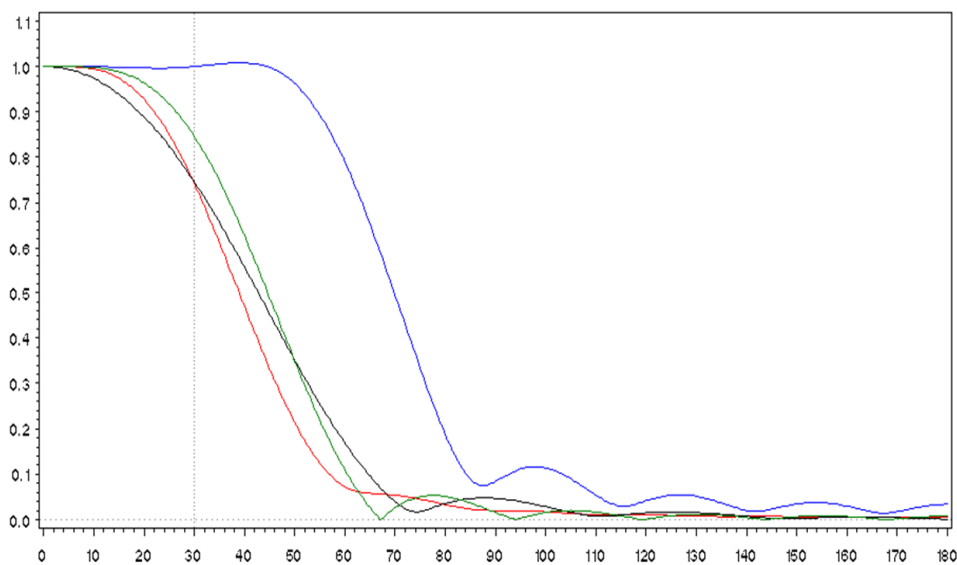
### 15.6.2.2 More on the asymmetric filters with 3 terms in the future.

Figures 15.4 and 15.5 compare the gain functions and the phase-shift functions of the preceding asymmetric filters with 3 points in the future. The black line is the asymmetric Musgrave's moving average (M6\_3). The red line is the asymmetric Henderson's moving average (H9\_3) and the blue line is the asymmetric Henderson's moving average minimizing the phase-shift (Hps9\_3). The green line corresponds to the symmetric Henderson's moving average of order 13 (H6\_6).

In terms of gain, the H9\_3 filter performs well and looks better than the Musgrave's filter. Moreover, this filter does not present any ripple as one can note on the H6\_6 and M6\_3 gain functions. On contrary, the Hps9\_3 filter preserves too many frequencies and does not focus enough on the trend-cycle low frequencies.

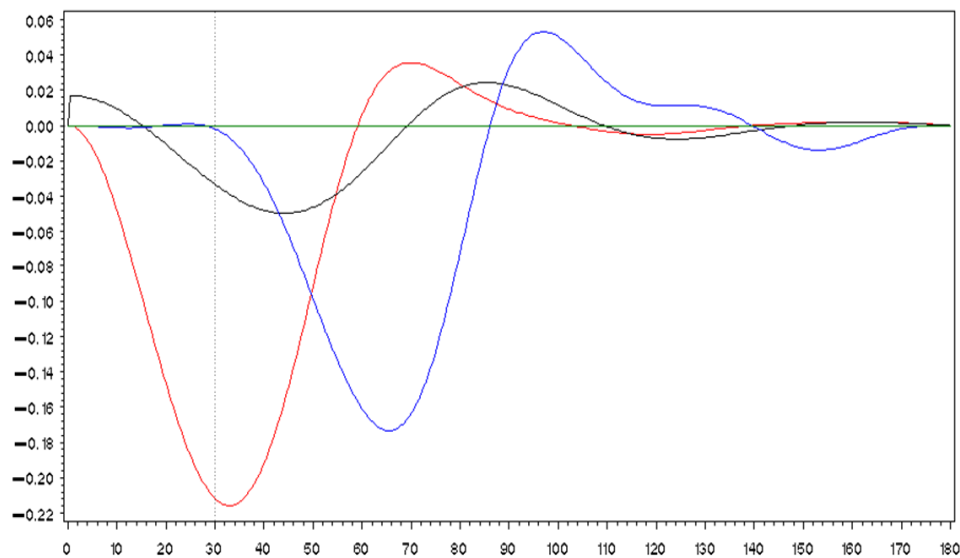
In terms of phase-shift and as expected, the Hps9\_3 filter does not induce any delay on the low frequencies and the Musgrave's filter seems to perform better than the asymmetric Henderson H9\_3.

**Figure 15.4:** Gain functions of various symmetric and asymmetric Henderson's moving averages.



The green line is the symmetric Henderson's moving average of order 13 (H6\_6). The black line is the asymmetric moving average M6\_3. The red line is the asymmetric Henderson's moving average H9\_3 and the blue line is the asymmetric Henderson's moving average minimizing the phase-shift H9\_3.

**Figure 15.5:** Phase shift functions of various symmetric and asymmetric Henderson's moving averages.



The green line is the symmetric Henderson's moving average of order 13 (H6\_6). The black line is the asymmetric Musgrave's moving average M6\_3. The red line is the asymmetric Henderson's moving average H9\_3 and the blue line is the asymmetric Henderson's moving average minimizing the phase-shift H9\_3.

## 15.7 Conclusions

In this chapter, we propose a new “FST optimization criterion” that generalizes the Whittaker-Henderson methodology to construct specific moving-averages and models the “Fidelity-Smoothness-Timeliness” dilemma. The construction of a moving average is seen as the solution of the minimization of a quadratic form under constraints. The constraints represent the properties of the moving average in terms of preservation of trends, cancellation of seasonalities and reducing of the noise. The criterion assures some balancing of the moving average properties in terms of accuracy, revisions and timeliness and allows constructing asymmetric moving averages that present almost no phase-shift.

This new methodology can generate many of well-known symmetric moving averages as well as their asymmetric complements: simple moving averages, Spencer’s moving averages, Henderson’s filters, Musgrave’s filter, local versions of the Hodrick-Prescott filter etc.

This generality, if it is clearly a nice feature, is also a drawback. The problem of the optimal choice of the coefficients  $\alpha$ ,  $\beta$ , and  $\gamma$ , which measure the importance given to the Fidelity, Smoothness and Timeliness criteria, as well as the design of the matrix of linear constraints is still an open issue.

## 15.8 Annex: Proofs

### Proof of proposition 3:

As the function  $\theta \mapsto \left( \int_{\omega_1}^{\omega_2} \mathbf{C}(\omega) d\omega \right)' \theta$  is linear, it is sufficient to prove that the following function,  $K : \theta \mapsto \int_{\omega_1}^{\omega_2} \sqrt{\theta' \Omega(\omega) \theta} d\omega$ , is convex. This results immediately from the well-known Schwarz inequality.

Let us consider two vectors  $\theta_1$  and  $\theta_2$ , a real number  $t$  in the interval  $[0; 1]$  and let  $\theta$  denote the convex combination  $\theta = t\theta_1 + (1-t)\theta_2$ . As matrix  $\Omega(\omega)$  is positive semi-definite, the Schwarz inequality yields  $|\theta_1' \Omega(\omega) \theta_2| \leq \sqrt{[\theta_1' \Omega(\omega) \theta_1][\theta_2' \Omega(\omega) \theta_2]}$ . Thus, we have:

$$\begin{aligned} \theta' \Omega(\omega) \theta &= t^2 [\theta_1' \Omega(\omega) \theta_1] + 2t(1-t) [\theta_1' \Omega(\omega) \theta_2] + (1-t)^2 [\theta_2' \Omega(\omega) \theta_2] \\ &\leq t^2 [\theta_1' \Omega(\omega) \theta_1] + 2t(1-t) \sqrt{[\theta_1' \Omega(\omega) \theta_1][\theta_2' \Omega(\omega) \theta_2]} \\ &\quad + (1-t)^2 [\theta_2' \Omega(\omega) \theta_2] \\ &\leq \left[ t \sqrt{\theta_1' \Omega(\omega) \theta_1} + (1-t) \sqrt{\theta_2' \Omega(\omega) \theta_2} \right]^2 \end{aligned}$$

and eventually:

$$\begin{aligned} K(\theta) &\leq \int_{\omega_1}^{\omega_2} \left[ t \sqrt{\theta_1' \Omega(\omega) \theta_1} + (1-t) \sqrt{\theta_2' \Omega(\omega) \theta_2} \right] d\omega \\ &\leq t \cdot K(\theta_1) + (1-t) \cdot K(\theta_2) \end{aligned}$$

So the function  $K$  is convex and proposition 3 is proved.  $\square$

### Proof of proposition 4:

Using the notations introduced for the proof of proposition 3, we have:

$$\frac{\partial T_0(\theta)}{\partial \theta} = \frac{\partial K(\theta)}{\partial \theta} - \left( \int_{\omega_1}^{\omega_2} \mathbf{C}_0(\omega) d\omega \right) \quad (15.9)$$

Note that the function  $\omega \mapsto \sqrt{\theta' \Omega(\omega) \theta}$  is summable over  $]\omega_1; \omega_2[$  for all vectors  $\theta$ . Moreover,

$$\frac{\partial}{\partial \theta} \sqrt{\theta' \Omega(\omega) \theta} = \frac{1}{2} \frac{\Omega(\omega) \theta}{\sqrt{\theta' \Omega(\omega) \theta}},$$

and the set  $\{\omega / \theta' \Omega(\omega) \theta = 0\}$  is negligible for Lebesgue's measure (except if  $\theta = \mathbf{0}$ ). Let us consider the following complex function,  $\Gamma : \theta \mapsto \mathbf{C}_0(\omega) + i \cdot \mathbf{S}_i(\omega)$ , with  $i$  being such that  $i^2 = -1$ . It is a regular function (of class  $\mathcal{C}^\infty$ ) such that  $\Omega(\omega) = \mathcal{R}e \left[ \overline{\Gamma(\omega)} \Gamma(\omega)' \right]$ . We then have:

$$\left\| 2 \cdot \frac{\partial}{\partial \theta} \sqrt{\theta' \Omega(\omega) \theta} \right\| = \left\| \frac{\mathcal{R}e \left[ \overline{\Gamma(\omega)} \Gamma(\omega)' \theta \right]}{|\Gamma(\omega)' \theta|} \right\| \leq \left\| \overline{\Gamma(\omega)} \right\| = \|\Gamma(\omega)\|$$

where the function  $\omega \mapsto \|\Gamma(\omega)\|$  does not depend on the vector  $\theta$  and is summable over  $]\omega_1; \omega_2[$ . According to the Lebesgue's dominated convergence theorem, the gradient of  $K$  can be derived as follows,

$$\frac{\partial K(\theta)}{\partial \theta} = \int_{\omega_1}^{\omega_2} \frac{\partial}{\partial \theta} \sqrt{\theta' \Omega(\omega) \theta} d\omega = \frac{1}{2} \int_{\omega_1}^{\omega_2} \frac{\Omega(\omega) \theta}{\sqrt{\theta' \Omega(\omega) \theta}} d\omega,$$

and replacing this expression of the derivative of  $K$  in formula (15.9) completes the proof of proposition 4.  $\square$

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# 16

## Real Time Trend Extraction and Seasonal Adjustment



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## Table of Contents

16.1	Introduction	417
16.2	Frequency Domain and Filter Effect	417
16.3	Data-Dependent Filters	419
16.3.1	Mean-Square Error Criterion	419
16.3.2	Controlling Speed and Reliability	420
16.3.3	New Generalized Criterion	421
16.4	Empirical Section	423
16.4.1	Introduction	423
16.4.2	Data and Modeling	424
16.4.3	T-DFA: Replication of Model-Based Mean-Square Performances (Airline Model)	425
16.4.4	Customization: Trend Applications	429
16.4.5	Customization beyond the Model Based Perspective	437
16.5	Conclusions	448

## 16.1 Introduction

Signal extraction concerns the definition, the analysis and the extraction of systematic patterns in time series. We here rely on linear filters and propose a new phenomenological approach which emphasizes filter effects. In this perspective, optimal designs are derived by tuning filter characteristics to application purposes. In essence, our approach aligns optimization criteria on problem structures and user priorities. We stress the importance of an agnostic approach whose scope generalizes classical filters (for example Hodrick-Prescott, Christiano-Fitzgerald or Henderson) as well as traditional model-based approaches (TRAMO-SEATS/STAMP/X-12-ARIMA). In particular, we propose customized criteria for minimizing revisions and for emphasizing timeliness and/or reliability of early (real-time) estimates.

The key towards our customized approach is a thorough analysis of filter effects in the frequency domain. We identify filters with transfer functions and decompose the filter effect into amplitude and phase errors. In a real-time perspective, the resulting decomposition of the mean-square filter error enables to track simultaneously reliability/accuracy issues (noise suppression) as well as timeliness (time-shift/delay) aspects. The resulting optimality concept blends with the structure of the estimation problem and the intention of the analyst, as well.

We here propose a new generalized optimization criterion which bridges the gap between the original Direct Filter Approach (DFA), as proposed in Wildi (Wildi (1998), Wildi (2005), Wildi (2008), Wildi (2010), McElroy Wildi (2016), McElroy Wildi (2018)), and a numerically fast linear approximation I-DFA of the former. Unlike I-DFA, the resulting new estimation method is able to replicate the original DFA perfectly and it is almost as fast, in computational terms, as I-DFA.

The paper is organized as follows. Section 16.2 reviews basic structural elements of the frequency domain and derives a descriptive approach for measuring and formalizing the ‘filter effect’. Section 16.3 presents the DFA and proposes our new criterion. The latter generalizes the DFA, the I-DFA as well as traditional model-based approaches in a unifying methodological framework. As a result, a formal bridge linking both philosophies is obtained. Section 16.4 presents empirical results based on the European industrial production index (IPI). A strict real-time perspective is emphasized based on asymmetric concurrent filters exclusively. It is shown that the new criterion replicates model-based performances perfectly. A series of illustrative customizations, obtained in various levels of abstraction and in any combination of attributes, then takes the traditional model-based design successively up to ‘full detachment’ from maximum likelihood principles. In the latter incarnation, solutions are obtained which conciliate conflicting requirements - speed/reliability (bias/variance) dilemma - in a way inaccessible to traditional model-based approaches. Finally, section 16.5 concludes by summarizing our main findings.

## 16.2 Frequency Domain and Filter Effect

Let  $X_t$ ,  $t = 1, \dots, T$  be a finite sample of observations and define  $Y_{T-r}$ ,  $r = 0, \dots, T - 1$  as the output of a filter with real coefficients  $\gamma_{kr}$ :

$$Y_{T-r} = \sum_{k=-r}^{T-r-1} \gamma_{kr} X_{T-r-k}$$

For  $r = 0$  a real-time or causal filter is obtained, namely a linear combination of present and past observations. For  $r > 0$ ,  $Y_{T-r}$  relies on ‘future’ observations  $X_{t-r+1}, \dots, X_T$  (smoothing). In order to derive the important

filter effect we assume a particular (complex) input series  $X_t := \exp(i\omega t)$ ,  $t \in \mathbb{Z}$ . The output signal is thus

$$Y_{T-r} = \sum_{k=-r}^{T-r-1} \gamma_{kr} \exp(i\omega(T-r-k)) \quad (16.1)$$

$$= \exp(i\omega(T-r)) \sum_{k=-r}^{T-r-1} \gamma_{kr} \exp(-i\omega k) \quad (16.2)$$

$$= \exp(i\omega(T-r)) \Gamma_r(\omega) \quad (16.3)$$

The (generally complex) function

$$\Gamma_r(\omega) := \sum_{k=-r}^{T-r-1} \gamma_{kr} \exp(-i\omega k) \quad (16.4)$$

is called the transfer function of the filter. We can represent the complex number  $\Gamma_r(\omega)$  in polar coordinates according to

$$\Gamma_r(\omega) = A_r(\omega) \exp(-i\Phi_r(\omega)) \quad (16.5)$$

where  $A_r(\omega) = |\Gamma_r(\omega)|$  is called the amplitude of the filter and  $\Phi_r(\omega)$  is its phase.

We deduce from 16.1 that  $X_t, t \in \mathbb{Z}$  is a periodic eigensignal of the filter with eigenvalue  $\Gamma_r(\omega)$ . Linearity of the filter implies that real and imaginary parts of  $X_t$  are mapped into real and imaginary parts of  $Y_t$  and therefore

$$\begin{aligned} \cos(t\omega) &\rightarrow A_r(\omega) [\cos(t\omega) \cos(-\Phi_r(\omega)) - \sin(t\omega) \sin(-\Phi_r(\omega))] \\ &= A_r(\omega) \cos(t\omega - \Phi_r(\omega)) \\ &= A_r(\omega) \cos(\omega(t - \Phi_r(\omega)/\omega)) \end{aligned} \quad (16.6)$$

The amplitude function  $A_r(\omega)$  can be interpreted as the weight (damping if  $A_r(\omega) < 1$ , amplification if  $A_r(\omega) > 1$ ) attributed by the filter to a sinusoidal input signal with frequency  $\omega$ . The function

$$\phi_r(\omega) := \Phi_r(\omega)/\omega \quad (16.7)$$

can be interpreted as the time shift function of the filter in  $\omega^1$ . As we shall see in section 3, real-time signal extraction, i.e. the case  $r = 0$ , aims at optimal simultaneous amplitude and time shift matchings or 'fits'. Amplitude and time-shift functions describe comprehensively the effect of the filter when applied to a simple trigonometric signal of frequency  $\omega$ . In order to extend the scope of the analysis and to found the validity of our approach we can rely on a well-known result stating that any sequence of numbers  $X_t, t = 1, \dots, T$ , sampled on an equidistant time-grid, can be decomposed uniquely into a weighted sum of mutually orthogonal complex exponential terms

$$X_t = \frac{1}{\sqrt{2\pi T}} \sum_{k=-T/2}^{T/2} DFT(\omega_k) \exp(-it\omega_k)$$

where  $DFT(\omega_k)$  is the discrete Fourier transform of  $X_t$  and  $\omega_k = \frac{k2\pi}{T}, k = -T/2, \dots, 0, \dots, T/2$  is a discrete frequency-grid in the interval  $[-\pi, \pi]$ . Linearity of the filter can then be invoked to extend the description of the filter effect in terms of amplitude and time-shift functions to arbitrary sequences of numbers  $X_t, t = 1, \dots, T$ . Note that this decomposition is a finite sample version of the fundamental spectral representation theorem and that it is fully compatible with the latter, asymptotically. However, the validity of the discrete finite-sample decomposition extends to any sequence of numbers, including realizations of non-stationary processes.

<sup>1</sup>The singularity in  $\omega = 0$  is resolved by noting that  $\Phi_r(0) = 0$  for filters satisfying  $\Gamma_r(0) > 0$  which will be the case for all applications in section 16.4. As a result  $\phi_r(0) := \Phi_r(0)$ .



## 16.3 Data-Dependent Filters

We here propose optimization criteria which emphasize optimal properties of asymmetric filters. For this purpose, we assume that a particular signal or, equivalently, a symmetric filter has been defined by the user. The signal could be a trend, a cycle or a seasonally adjusted component and the definition could be either ad hoc or 'model-based'. In order to simplify notations we here emphasize the practically relevant real-time or concurrent filter which approximates the signal at the end  $t = T$  of the sample. We propose criteria which emphasize the revision error as well as speed (timeliness) and reliability (noise suppression) issues. Criteria in the first group are able to replicate traditional model-based filters (X-12-ARIMA, TRAMO, Stamp) perfectly, see McElroy Wildi (2016) and McElroy Wildi (2018). Criteria in the second group are able to account for more complex real-time inferences (for example the detection of turning-points) and for more sophisticated user priorities (for example different levels of risk aversion).

### 16.3.1 Mean-Square Error Criterion

In order to introduce the relevant topics let  $\Gamma(\cdot)$  and  $\hat{\Gamma}(\cdot)$  denote the transfer functions of the symmetric and of the real-time (one-sided) filters with outputs  $Y_t$  and  $\hat{Y}_t$  respectively. For stationary processes  $X_t$ , the mean-square filter error can be expressed as

$$\int_{-\pi}^{\pi} |\Gamma(\omega) - \hat{\Gamma}(\omega)|^2 dH(\omega) = E[(Y_t - \hat{Y}_t)^2] \quad (16.8)$$

where  $H(\omega)$  is the unknown spectral distribution of  $X_t$ . Consider now the following finite sample approximation of the above integral

$$\frac{2\pi}{T} \sum_{k=-[T/2]}^{[T/2]} w_k |\Gamma(\omega_k) - \hat{\Gamma}(\omega_k)|^2 S(\omega_k) \quad (16.9)$$

where  $\omega_k = k2\pi/T$ ,  $[T/2]$  is the greatest integer<sup>2</sup> smaller or equal to  $T/2$  and the weights  $w_k$  are defined by

$$w_k = \begin{cases} 1 & , |k| \neq T/2 \\ 1/2 & , \text{otherwise} \end{cases}$$

In this expression,  $S(\omega_k)$  can be interpreted as an estimate of the unknown spectral density of the process. Consistency of this estimate is not necessary because we are not interested in estimating the (unknown) spectral density but the filter mean-square error instead. In this perspective, we may take benefit of the smoothing effect provided by the summation operator in 16.9. So for example Wildi (1998), Wildi (2005), Wildi (2008) and Wildi (2010) propose to plug the periodogram into the above expression:

$$S(\omega_k) := I_{TX}(\omega_k) = \frac{1}{2\pi T} \left| \sum_{t=1}^T X_t \exp(-it\omega_k) \right|^2$$

Formal efficiency results applying to the resulting Direct Filter Approach (DFA) are presented in Wildi (2008). Real-world true out-of-sample performances are extensively documented and discussed in Wildi (2008).

We propose to extend the original DFA by considering alternative spectral estimates  $S(\omega_k)$  derived from models of the Data Generating Process (DGP). It is understood that the term 'model' refers to explicit representations of the DGP by X-12-ARIMA, TRAMO or STAMP, for example, as well as to 'ad hoc' implicit

<sup>2</sup>In order to simplify the exposition we now assume that  $T$  is even. In our applications, the sample length is generally a multiple of 4 (quarterly data) or 12 (monthly data) in order that the important seasonal frequencies can be matched by  $\omega_k$ .



DGP-assumptions underlying classical filters, such as HP, CF or Henderson, for example. This way, a formal link between the original DFA and traditional model-based approaches is established which allows to transpose the powerful customization principle of the latter to the former. Applications to model-based seasonal adjustment and trend extraction are presented in section 16.4.

The case of non-stationary integrated processes can be handled very easily by noting that 16.9 addresses the filter error  $Y_t - \hat{Y}_t$ , not the data  $X_t$ . The former is generally stationary even if the latter isn't<sup>3</sup> and therefore all spectral decomposition results are still valid in a formal mathematical perspective. In the case of integrated processes, stationarity of the filter error is obtained by imposing cointegration between the signal  $Y_t$  and the real-time estimate  $\hat{Y}_t$ . Formally, this amounts to impose suitable real-time filter constraints. A comprehensive treatment of the topic is given in Wildi (2008) and Wildi (2010).

### 16.3.2 Controlling Speed and Reliability

Wildi Wildi (1998), Wildi (2005), Wildi (2008), Wildi (2010) and McElroy Wildi McElroy Wildi (2018) propose a decomposition of the mean-square filter error into distinct components attributable to the amplitude and the phase functions of the real-time filter. We here briefly review this decomposition and derive customized criteria which emphasize explicitly speed and/or reliability aspects subject to particular user priorities, such as, for example, various degrees of risk aversion.

The following identity holds for general transfer functions  $\Gamma$  and  $\hat{\Gamma}$ :

$$\begin{aligned} |\Gamma(\omega) - \hat{\Gamma}(\omega)|^2 &= A(\omega)^2 + \hat{A}(\omega)^2 - 2A(\omega)\hat{A}(\omega) \cos(\hat{\Phi}(\omega) - \Phi(\omega)) \\ &= (A(\omega) - \hat{A}(\omega))^2 \\ &\quad + 2A(\omega)\hat{A}(\omega) \left[ 1 - \cos(\hat{\Phi}(\omega) - \Phi(\omega)) \right] \end{aligned} \quad (16.10)$$

If we assume that  $\Gamma$  is symmetric and positive, then  $\Phi(\omega) \equiv 0$ . Inserting 16.10 into 16.9 and using  $1 - \cos(\hat{\Phi}(\omega)) = 2 \sin(\hat{\Phi}(\omega)/2)^2$  then leads to

$$\frac{2\pi}{T} \sum_{k=-T/2}^{T/2} w_k (A(\omega_k) - \hat{A}(\omega_k))^2 S(\omega_k) \quad (16.11)$$

$$+ \frac{2\pi}{T} \sum_{k=-T/2}^{T/2} w_k 4A(\omega_k)\hat{A}(\omega_k) \sin(\hat{\Phi}(\omega_k)/2)^2 S(\omega_k) \quad (16.12)$$

The first summand 16.11 is the distinctive part of the total mean-square filter error which is attributable to the amplitude function of the real-time filter (the MS-amplitude error). The second summand 16.12 measures the distinctive contribution of the phase or time-shift to the total mean-square error (the MS-time-shift error). The term  $A(\omega_k)\hat{A}(\omega_k)$  in 16.12 is a scaling factor which accounts for the fact that the phase function does not convey level information.

<sup>3</sup>Optimal real-time signal extraction aims precisely at a smallest possible mean-square filter error i.e. the filter error is neither trending nor unbounded in variance.

Now consider the following generalized version of the original mean-square criterion:<sup>4</sup>

$$\begin{aligned}
& \frac{2\pi}{T} \sum_{k=-T/2}^{T/2} w_k (A(\omega_k) - \hat{A}(\omega_k))^2 W(\omega_k) S(\omega_k) \\
& + (1 + \lambda) \frac{2\pi}{T} \sum_{k=-T/2}^{T/2} w_k 4A(\omega_k) \hat{A}(\omega_k) \sin(\hat{\Phi}(\omega_k)/2)^2 W(\omega_k) S(\omega_k) \\
= & \frac{2\pi}{T} \sum_{k=-[T/2]}^{[T/2]} w_k |\Gamma(\omega_k) - \hat{\Gamma}(\omega_k)|^2 W(\omega_k) S(\omega_k) \\
& + 4\lambda \frac{2\pi}{T} \sum_{k=-T/2}^{T/2} w_k A(\omega_k) \hat{A}(\omega_k) \sin(\hat{\Phi}(\omega_k)/2)^2 W(\omega_k) S(\omega_k) \rightarrow \min \quad (16.13)
\end{aligned}$$

where  $W(\cdot) := W(\omega_k, \eta, cutoff)$  is a two-parametric family of weighting functions

$$W(\omega_k, \eta, cutoff) = \begin{cases} 1, & \text{if } |\omega_k| < cutoff \\ (1 + |\omega_k| - cutoff)^\eta, & \text{otherwise} \end{cases} \quad (16.14)$$

The parameter *cutoff* marks the transition between pass- and stop-bands ( $cutoff = \pi/7$  in the empirical applications below) and positive values of the parameter  $\eta$  emphasize high-frequency components. Classical mean-square optimization is obtained for  $\lambda = \eta = 0$ : the revision error is addressed. For  $\lambda > 0$  the user can emphasize the contribution of the MS-time-shift error. As a result, corresponding real-time filters (typically low-pass trend or cycle extraction) will convey less delayed signals: turning-points can be detected earlier. Note that the weighting  $A(\omega_k)\hat{A}(\omega_k)$  in this expression implies that  $\lambda$  acts on the *pass-band* frequencies exclusively and that  $\eta$  does not alter the time-shift error. The latter parameter emphasizes the MS-amplitude error by magnifying ‘noisy’ high-frequency components in the *stop-band*. As a result, ‘noise’ is suppressed more effectively and the reliability of real-time estimates will improve accordingly.

It is generally admitted that reliability and timeliness (speed of detection) of real-time estimates are to some extent mutually exclusive requirements. It is not our intention to contradict this fundamental uncertainty principle, of course, but it seems obvious that the user can attempt to improve performances in both dimensions simultaneously by increasing  $\lambda$  as well as  $\eta$ . By doing so, control on the amplitude function is lost in the pass-band: mean-square performances are affected adversely. Risk-averse users can operationalize their preference by matching  $\eta$  to their specific needs. On the other hand, fast ‘early’ estimates are obtained by prioritizing the phase error over the amplitude error.

In the following section we propose a new criterion which generalizes 16.13 and which addresses numerical/computational aspects also.

### 16.3.3 New Generalized Criterion

The mean-square error criterion is a quadratic function of the filter parameters and therefore the solution can be obtained analytically. The expression 16.13, however, is more tricky when  $\lambda > 0$  because it involves non-linear functions of the filter parameters. Therefore, we here propose a new criterion which generalizes 16.13 and which opens the way to analytical approximations as well as to efficient numerical computations of the

<sup>4</sup>For notational simplicity it is assumed that  $\Gamma(\omega) > 0$  for all  $\omega$  such that  $\Gamma(\omega) = A(\omega)$ .

latter. Consider the following expression:

$$\frac{2\pi}{T} \sum_{k=-[T/2]}^{[T/2]} w_k \left| \Gamma(\omega_k) - \left\{ \Re(\hat{\Gamma}(\omega_k)) + i\sqrt{1 + 4\lambda\Gamma(\omega_k)f(\omega_k)}\Im(\hat{\Gamma}(\omega_k)) \right\} \right|^2 W(\omega_k)S(\omega_k) \rightarrow \min \quad (16.15)$$

where  $\Re(\cdot)$  and  $\Im(\cdot)$  denote real and imaginary parts and  $i^2 = -1$  is the imaginary unit. Assume, first, that  $f(\omega_k) = \text{Id}$ . Obviously, then, the above expression is quadratic in the filter coefficients. In analogy to 16.13, the weighting function  $W(\omega_k)$  emphasizes the fit in the stop band. The term  $\lambda\Gamma(\omega_k)$  affects the imaginary part of the real-time filter in the pass band: for  $\lambda > 0$  the imaginary part is artificially inflated and therefore the phase is affected in 'some manner'. Unfortunately, the clear-cut disentanglement of time-shift and amplitude errors, formalized in the decomposition 16.13, does no more apply because  $\lambda$  mixes-up both effects, to some extent. Therefore a proper operationalization of the important speed/reliability dimensions becomes ambiguous. In order to overcome this problem, we briefly analyze the 'discrepancy' between 16.13 and 16.15 and derive an effective numerical optimization rule from this analysis. In the following,  $f(\omega_k) = \text{Id}$  is still imposed. We then obtain

$$\begin{aligned} & \frac{2\pi}{T} \sum_{k=-[T/2]}^{[T/2]} w_k \left| \Gamma(\omega_k) - \left( \Re(\hat{\Gamma}(\omega_k)) + i\sqrt{1 + 4\lambda\Gamma(\omega_k)}\Im(\hat{\Gamma}(\omega_k)) \right) \right|^2 W(\omega_k)S(\omega_k) \quad (16.16) \\ &= \frac{2\pi}{T} \sum_{k=-[T/2]}^{[T/2]} w_k \left\{ \left( \Gamma(\omega_k) - \Re(\hat{\Gamma}(\omega_k)) \right)^2 + \Im(\hat{\Gamma}(\omega_k))^2 \right\} W(\omega_k)S(\omega_k) \\ &+ 4\lambda \frac{2\pi}{T} \sum_{k=-[T/2]}^{[T/2]} w_k \Gamma(\omega_k) \Im(\hat{\Gamma}(\omega_k))^2 W(\omega_k)S(\omega_k) \\ &= \frac{2\pi}{T} \sum_{k=-[T/2]}^{[T/2]} w_k |\Gamma(\omega_k) - \hat{\Gamma}(\omega_k)|^2 W(\omega_k)S(\omega_k) \\ &+ 4\lambda \frac{2\pi}{T} \sum_{k=-[T/2]}^{[T/2]} w_k A(\omega_k) \hat{A}(\omega_k)^2 \sin(\hat{\Phi}(\omega_k))^2 W(\omega_k)S(\omega_k) \quad (16.17) \end{aligned}$$

A direct comparison of 16.13 and 16.17 reveals the 'distortion' mechanisms:  $\hat{\Phi}(\omega_k)/2$  is replaced by  $\hat{\Phi}(\omega_k)$  and a supernumerary weighting-term  $\hat{A}(\omega_k)$  appears in the latter expression. In order to obtain the perfect disentanglement of phase and amplitude errors in 16.13 one has to transform - undistord - the last summand in 16.17 according to

$$4\lambda \frac{2\pi}{T} \sum_{k=-[T/2]}^{[T/2]} w_k f(\omega_k) A(\omega_k) \hat{A}(\omega_k)^2 \sin(\hat{\Phi}(\omega_k))^2 W(\omega_k)S(\omega_k)$$

where

$$f(\omega_k) := \begin{cases} \frac{\sin(\hat{\Phi}(\omega_k)/2)^2}{\hat{A}(\omega_k) \sin(\hat{\Phi}(\omega_k))^2}, & \text{if } \hat{A}(\omega_k) \sin(\hat{\Phi}(\omega_k))^2 > 0 \\ \frac{1}{4\hat{A}(\omega_k)}, & \text{if } \sin(\hat{\Phi}(\omega_k)/2)^2 = 0 \text{ and } \hat{A}(\omega_k) > 0 \\ 0, & \text{otherwise} \end{cases} \quad (16.18)$$

Note that  $f(\omega_k)$  could be set to any arbitrary value in the last case ( $\hat{A}(\omega_k) = 0$ ) because  $f(\omega_k)\hat{A}(\omega_k)^2$  will vanish anyway. Formally we can re-formulate 16.13 in terms of 16.15 by setting in expression 16.18 for  $f(\omega_k)$ .

For any fixed  $f(\omega_k)$ , not depending on the filter parameters, expression 16.15 would be a quadratic function of the parameters. If we replicate 16.13 then, of course,  $f(\omega_k)$  isn't fixed but a fast iterative optimization procedure can be derived in this case from 16.15 according to the following scheme:

- In step 1 the solution of 16.13 is approximated by the linear criterion 16.15, where  $f_1(\omega_k) = \text{Id}$ .
- In step  $j + 1$  the function  $f_{j+1}(\omega_k)$  in 16.18 is set-up by plugging amplitude and time-shift functions obtained in the previous step  $j$ . Conditional on this setting, 16.15 is quadratic in the filter parameters and therefore estimates in step  $j+1$  can be obtained analytically.
- The numerical optimization is stopped as soon as some finite convergence criterion is achieved.

The (fast) convergence of this algorithm is illustrated in section 16.4, where  $f_j(\omega_k)$  is plotted as a function of  $j$ . In comparison to the original DFA proposed in Wildi (2008) the new sequential optimization of 16.15 is much faster because it relies on a good initial setting obtained by imposing  $f_1(\omega_k) = \text{Id}$  in the first step and by relying on analytically tractable up-dates. In comparison to the linear approximation I-DFA ( $f(\omega_k) = \text{Id}$ ) of the original DFA, the sequential minimization of the generalized criterion 16.15 - where 16.18 is used - offers a perfect disentanglement of amplitude and time-shift error components. Therefore, the user can operationalize priorities straightforwardly by selecting suitable  $\lambda$  and  $\eta$  parameters which match his particular research interests. In comparison to classical filter designs (HP, CF, BK, Henderson) the above criteria invoke optimality by relying on suitable expressions for  $S(\omega_k)$ . Whereas the original DFA relies on the periodogram  $S(\omega_k) = I_{TX}(\omega_k)$ , we also propose to plug-in model-based spectral estimates in the following section. By doing so, we are able to transpose the powerful customization principle to the model-based paradigm. Therefore, proponents of the latter can operationalize various practically relevant priorities explicitly, without resigning to give-up a 'pure' maximum likelihood framework.

## 16.4 Empirical Section

### 16.4.1 Introduction

The generality and the flexibility of the new generalized criterion 16.15 is illustrated by emphasizing a series of important application fields. The following list is ordered according to the strength of the link connecting 16.15 to the model-based approach, from simple replication to full detachment.

- To start, it is demonstrated how model-based real-time performances can be replicated by 16.15. For this purpose we set  $\lambda = \eta = 0$  and we plug the model-based signal into  $\Gamma(\omega_k)$  as well as the model-based (pseudo) spectral density into  $S(\omega_k)$ . We analyze real-time mean-square (RT-MS) trend extraction as well as seasonal adjustment.
- We then propose to enhance model-based real-time trend estimates by emphasizing speed (phase artifacts) through  $\lambda$  and/or reliability (noise suppression) through  $\eta$ . As we shall see, the resulting real-time filters combine both smoothness- and speed-attributes in a way inaccessible to traditional approaches although the framework is still purely model-based.
- As a further step away from the model-based paradigm we propose to analyze a customized trend signal, the ideal trend, which is likely to support reliability aspects more comprehensively. In summary, we still rely on the model-based pseudo-spectral density but we propose to modify/customize the target signal. As a result, the optimization is still fully embedded in the model-based approach.
- Seasonal adjustment filters are, by definition, permeable to short-term high-frequency components ('noise'). Therefore, the previous trend-customization, namely the effective combination of speed and smoothness dimensions, fails since smoothness is not felt to be a relevant quality. We therefore address

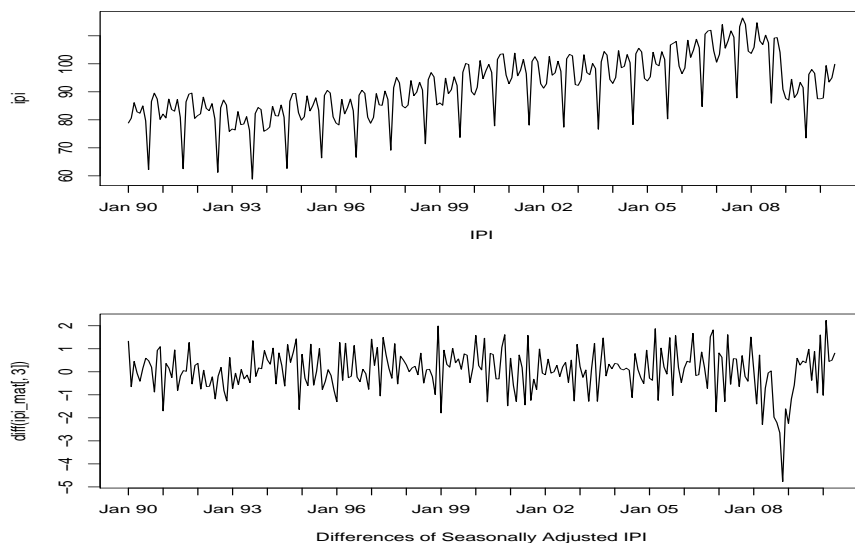
issues which are specific to wide (partitioned) pass-band filters, as is typically the case for SA-filters. In particular we emphasize under- and over-adjustments in seasonal frequencies as well as insufficiently damped calendar effects (for example residual trading days). These issues affect trend-filters less than SA-filters because the former damp undesirable high-frequency components anyway. We also investigate the practically relevant case of first differences of SA-series: seasonally adjusted first differences differ from first differences of seasonally adjusted data, in particular when considered in a real-time perspective. We analyze the latter case because it is widespread<sup>5</sup> and because it is trivially relevant when the unadjusted series is not released. All aspects are considered in a strict real-time perspective. In this setting we consider model-based signals but we substitute the periodogram to the model-based pseudo-spectral density: the optimization leaves the model-based paradigm.

- Finally, we propose to detach entirely from the model-based paradigm by combining customized signals and fully customized optimization. In particular we address trend estimation by emphasizing the slope in (noisy) differences of original data. This application is of particular interest in the context of leading indicators because turning-points of the original time series are anticipated by turning-points of the slope (anticipative effect of the difference filter).

### 16.4.2 Data and Modeling

We rely on the European Industrial Production Index, observed from Jan1990 to Jun2010 ( $T = 246$  observations), see fig.16.1. All model-based computations are obtained by TRAMO as implemented in Demetra+.

**Figure 16.1:** European IPI (top) and differences of seasonally adjusted series (bottom)



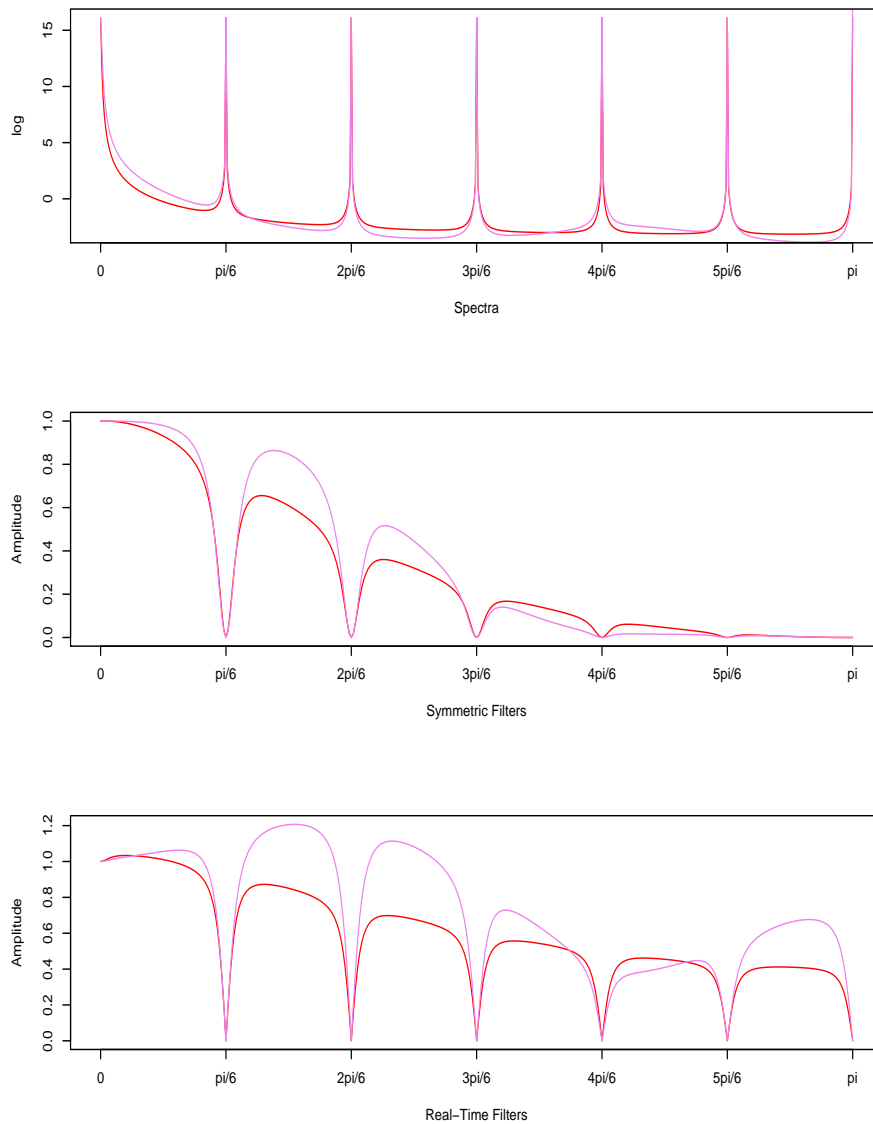
version 16. July 2010. Both full-automatic modeling (RSA5 option in Demetra+) as well as the airline model (RSA2) were considered. As can be seen in fig.16.2 the more complex model leads to real-time filters with undesirable characteristics<sup>6</sup> and therefore all results are based on the sole airline model. The estimated model-equation is

$$(1 - B)(1 - B^{12})X_t = (1 - 0.2963B)(1 - 0.6026B^{12})\epsilon_t \quad (16.19)$$

<sup>5</sup>Users interested in growth-dynamics typically rely on differences of SA-data: SA is performed first (for example by the data-provider) and differences apply 'after the fact'.

<sup>6</sup>The amplitude function of the real-time trend extraction filter in the bottom panel (violet line) is subject to severe leakage and high-frequency noise suppression is even worse than by the seasonal adjustment filter which is rather 'paradoxical', to say at least.

**Figure 16.2:** Pseudo-spectra (top), symmetric filters (middle) and real-time filters (bottom): airline model (red) vs. automatic model identification (violet). All series were generated by Demetra+.



The resulting pseudo-spectral density is plotted in the top panel of fig.16.2 (red line): this will be plugged into  $S(\omega_k)$ .

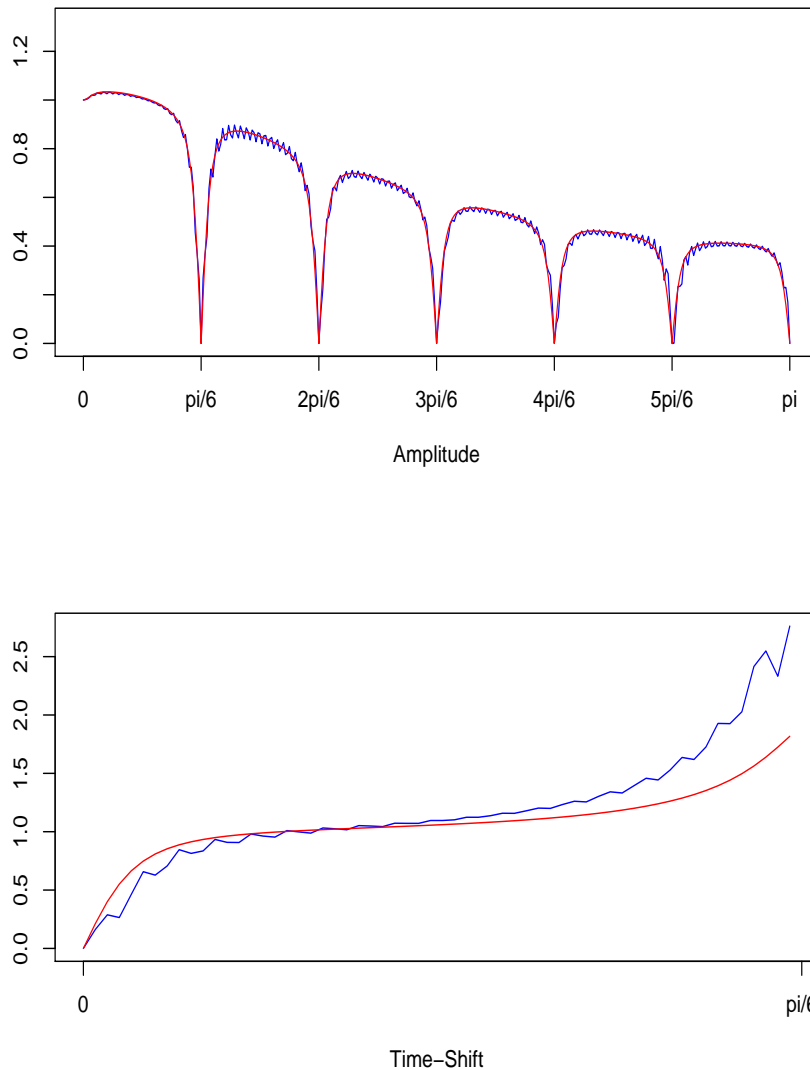
### 16.4.3 T-DFA: Replication of Model-Based Mean-Square Performances (Airline Model)

The transfer function of the symmetric trend filter, as defined by the airline model, is plotted in fig.16.2 (red line, mid-panel). Since Demetra+ computes so-called 'squared-gains', all plotted (model-based) amplitude functions were previously square-root transformed. Replication of model-based filters is obtained very easily in the framework of criterion 16.15 by inserting the corresponding pseudo-spectral density into the weighting function  $S(\omega_k)$  and by plugging the model-based signal into the target signal  $\Gamma(\omega_k)$ . These natural interfaces will allow for powerful later customizations.

## 16.4.3.1 Trend

Amplitude and time-shift functions of real-time filters obtained by TRAMO and by criterion 16.15 are compared in fig.16.3. The double unit-root of the airline model imposes real-time filter restrictions in frequency

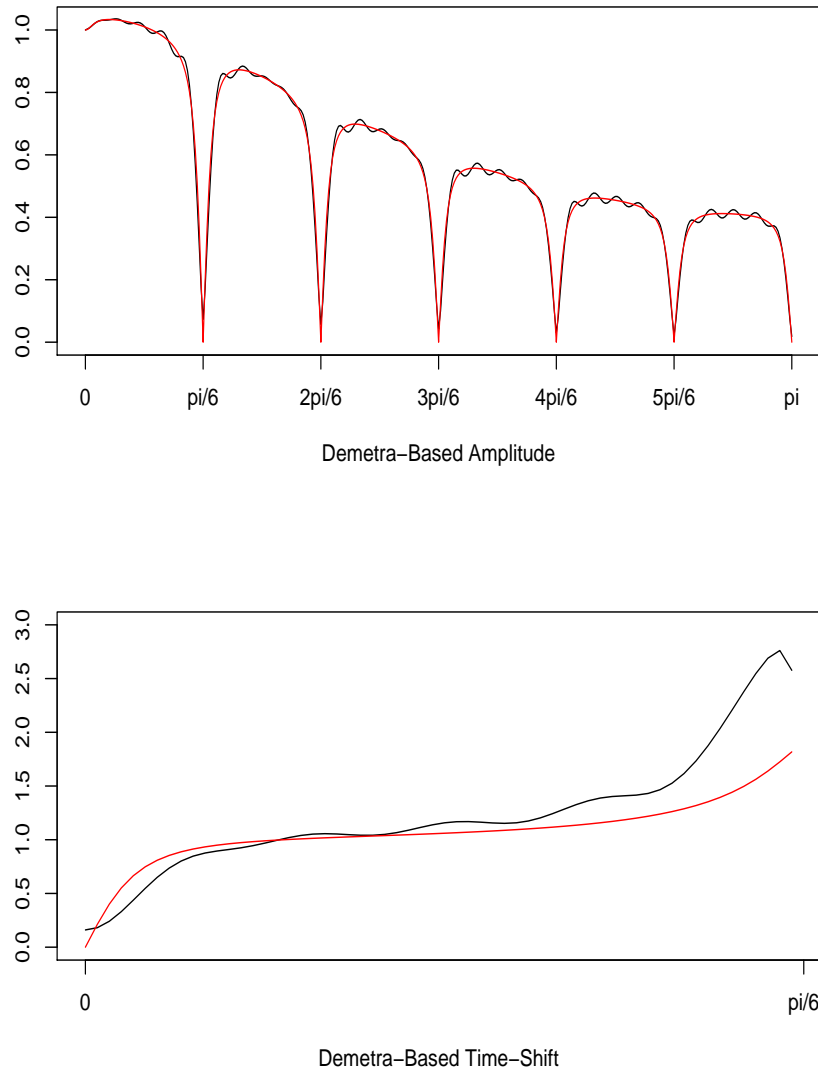
**Figure 16.3:** Comparison of amplitude (top) and time-shift functions (bottom) of model-based (red) and DFA-based (blue) real-time mean-square trend extraction filters



zero: amplitude and time shift functions satisfy  $\hat{A}(0) = 1$  and  $\hat{\phi}(0) = 0$ , as expected, see Wildi (2008), chapter 6 for details. The amplitude function obtained by criterion 16.15 appears 'rippled' because the filter is of finite length (MA(246)) and because the resolution is artificially increased<sup>7</sup>. Ignoring finite-sample issues, the replication of the amplitude function is obtained. The time-shifts, however, appear different. In order to cross-check results we computed amplitude and time-shift functions of the model-based filter by relying on its filter coefficients as calculated by Demetra+, see fig.16.4. Same picture: the amplitude functions match but

<sup>7</sup>Demetra+ computes amplitude and time-shift functions of the infinite-length filter on a (unnecessarily) tighter frequency-grid: 360 data points whereas  $T/2 = 123$  would be sufficient.

**Figure 16.4:** Comparison of amplitude (top) and time-shift (bottom) functions as calculated by Demetra+ (red) and as obtained by the filter coefficients calculated by Demetra+ (black)



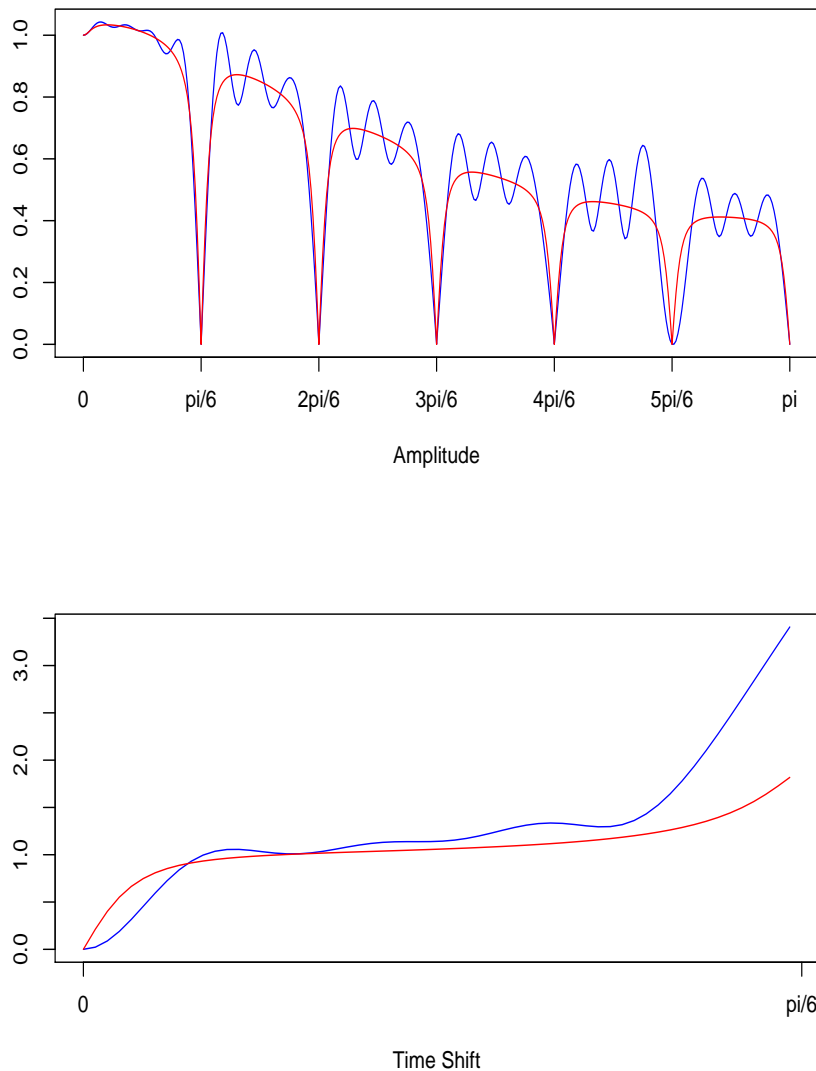
the time-shifts appear different in much the same way as in the previous figure<sup>8</sup>. We conclude that time-shifts as computed by Demetra+ should be subject to caution.

In order to compare and to analyze performances of various real-time designs, filter-lengths are shortened: lengths  $L = 36$  and  $L = 48$  are selected (multiples of the seasonal length) such that  $246-36=210$  and  $246-48=198$  filtered data points are available for comparisons. As can be seen in fig.16.5, short and long filters share the same attributes. Therefore, filter outputs are almost indistinguishable and turning-points are identical.

<sup>8</sup>Note that Demetra+ does not give access to the full filter sequence (only 60 coefficients were available) which explains the occurrence of larger ripples of the truncated filter. This may explain, also, that the corresponding time-shift does not vanish in frequency zero (black line bottom graph in the figure).



**Figure 16.5:** Amplitude (top) and time-shift (bottom) functions of mean-square real-time trend filters: short MA(48) (blue) and Demetra (red). Recall that time-shifts computed by Demetra+ are subject to caution (red line bottom plot).



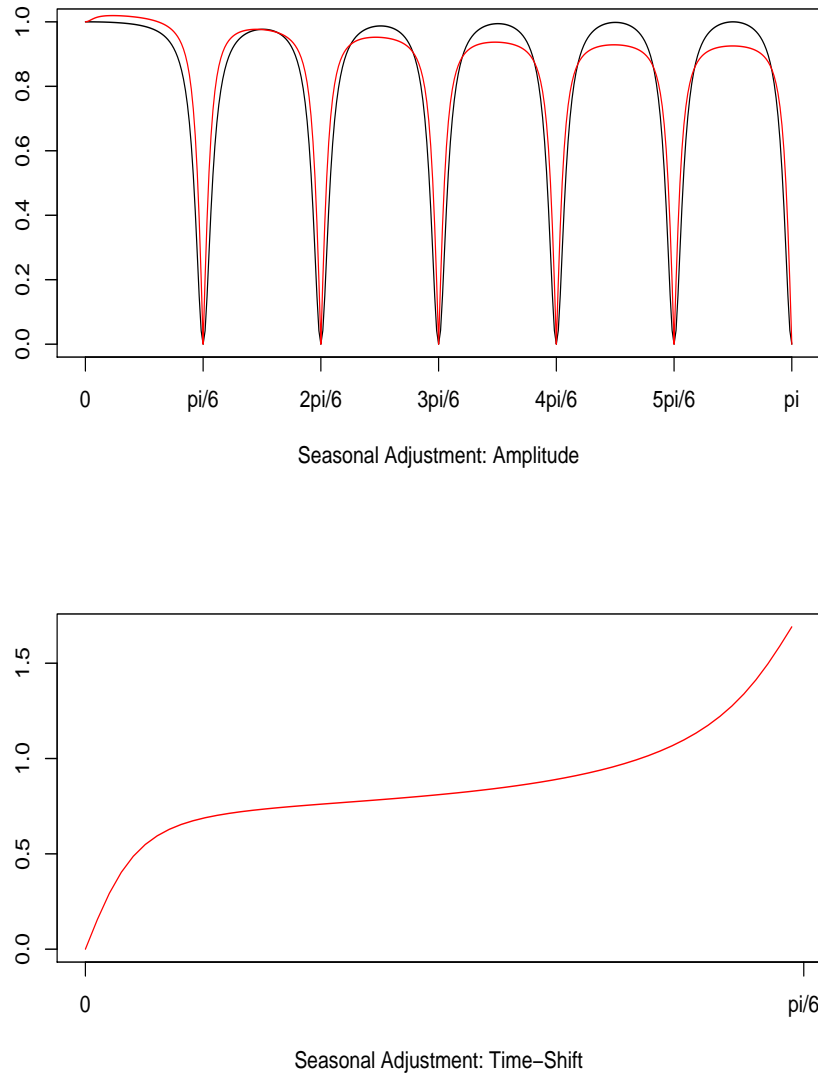
#### 16.4.3.2 Seasonal Adjustment

Amplitude of symmetric (black/upper graph) and asymmetric (red/upper) as well as the time-shift of the latter (red/bottom) are plotted in fig.16.6. Real-time filters obtained by TRAMO (above) and by the generalized criterion 16.15 are compared in fig.16.7. Amplitude functions match up to negligible sampling effects<sup>9</sup> but time-shifts differ, again.

After replication of classic model-based approaches by criterion 16.15, we address user-priorities such as

<sup>9</sup>Criterion 16.15 senses the filter on a discrete grid of frequency points  $\omega_k$ : tightening the grid would result in increased resolution and therefore in a perfect (convergent) match. This would require a higher resolution of the signal as well as of the pseudo-spectral density which, unfortunately, are not available in Demetra+. Also, criterion 16.15 computes real-world *finite-sample* filters whose amplitude functions may deviate slightly from the amplitudes of fictive *infinite-length* filters as computed by Demetra+. This phenomenon is likely to affect SA-filters more than trend filters because of the sharp seasonal dips which may afford larger sample-lengths to converge.

**Figure 16.6:** Amplitude (top) and time-shift (bottom) functions of mean-square seasonal-adjustment by Demetra. Symmetric filter in black, real-time in red.



timeliness and reliability of real-time estimates.

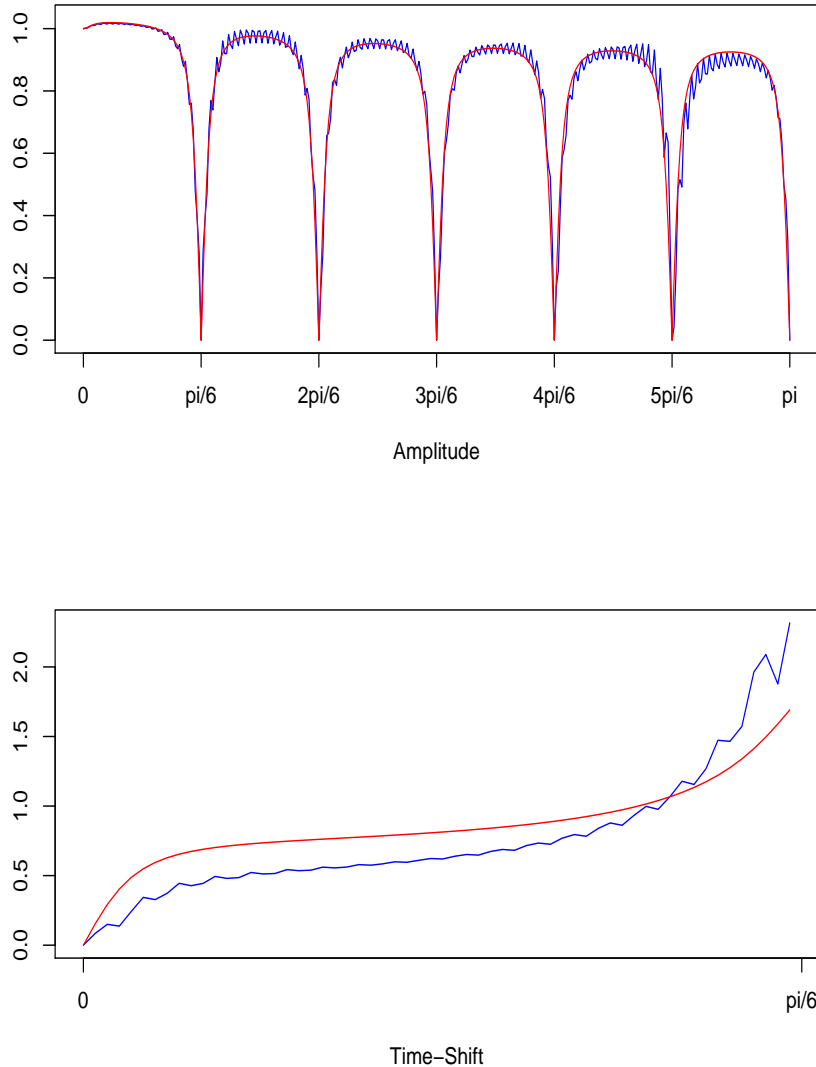
## 16.4.4 Customization: Trend Applications

### 16.4.4.1 introduction

Traditional concurrent MSE (mean-square error) trend filters are affected by time-shifts (delays) as well as by leakage (insufficient noise suppression). Criterion 16.15 can address both issues simultaneously, see Wildi (2008) and McElroy Wildi (2018). For illustration we address estimation of the model-based trend (see previous section) as well as of the so-called 'ideal' trend.

$$\Gamma(\omega, cutoff) = \begin{cases} 1, & |\omega| < cutoff \\ 0, & \text{otherwise} \end{cases} \quad (16.20)$$

**Figure 16.7:** Replication of model-based real-time filter: amplitudes (top) and time-shift (bottom); Demetra+ (red) and criterion 16.15 (blue).

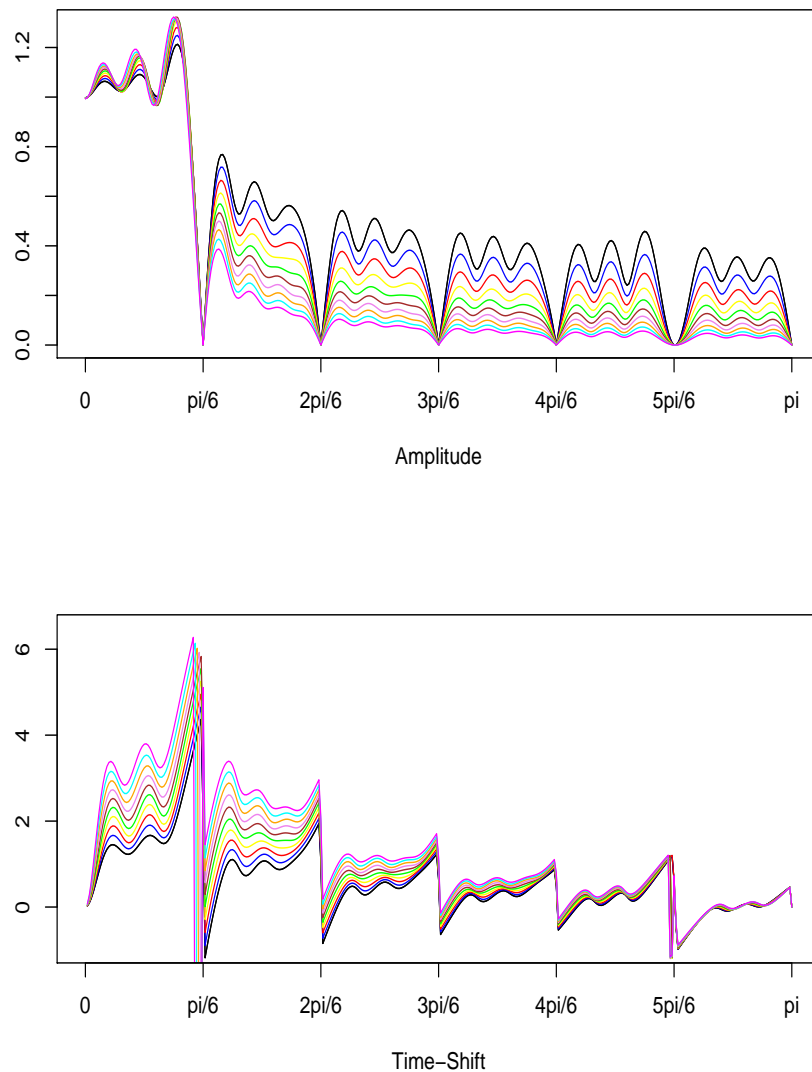


where  $cutoff = \pi/7$ . We report initial solutions based on  $f(\cdot) = Id$  as well as the final (converged) solution 16.18 based on the last iteration step proposed in section 16.3.3.

#### 16.4.4.2 Visualization of the Customization Effect

We here briefly assess the effects of the parameters  $\lambda$  and  $\eta$  entering criterion 16.15 on amplitude and time-shift functions of the resulting optimized concurrent filters. Fig.16.8 was obtained by fixing  $\lambda = 0$  and by increasing  $\eta$  incrementally from zero to 1 by steps of  $1/10^{10}$ . With increasing  $\eta$  the amplitude-fit gets tightened in the stop-band, as expected. Accordingly, the time-shift (delay) in the pass-band increases. Fig.16.9 illustrates the effect induced by  $\lambda$  for fixed  $\eta = 1.2$  (this setting will be used below): the parameter is increased

<sup>10</sup>For this particular experiment we selected  $L = 48$  (see above for a motivation of the filter-length),  $f(\omega_k) = Id$  (since  $\lambda = 0$  we can rely on the linearized solution), the target signal is the 'ideal' trend and the spectrum is the model-based estimate, see previous section.

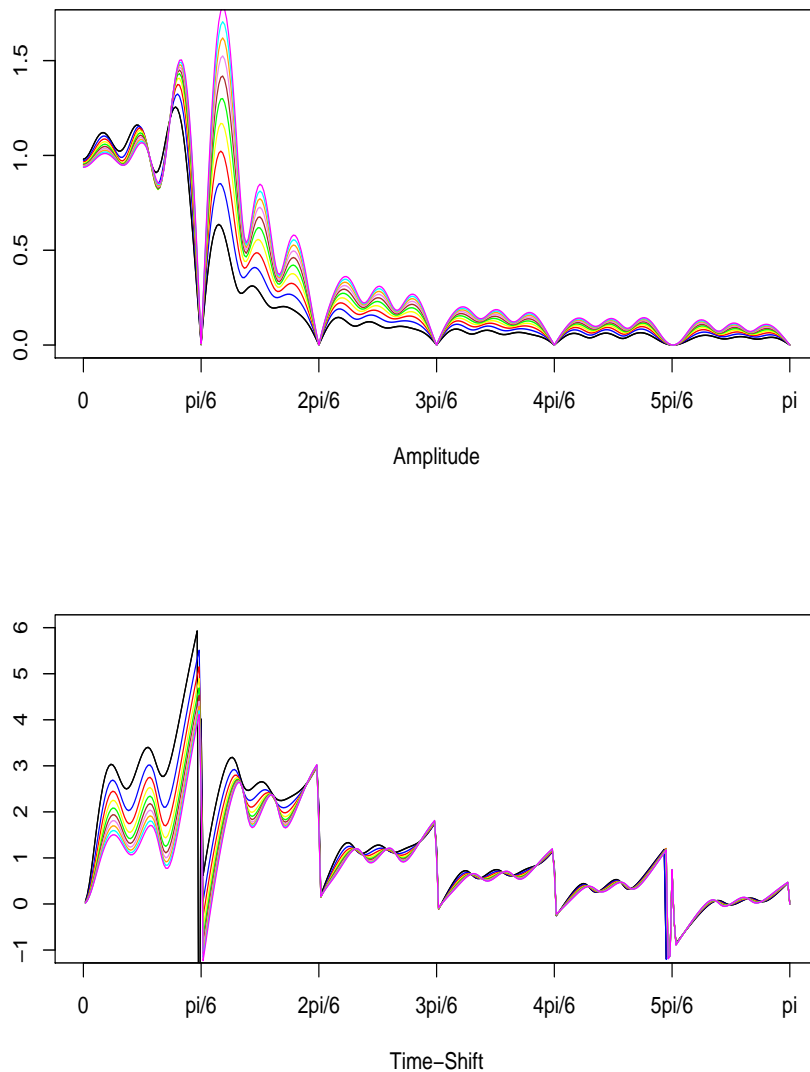
**Figure 16.8:** Effect of  $\eta$  (reliability) on real-time trend filter

incrementally from zero to 100 by steps of ten. The decreasing time-shift in the pass-band is compensated by a degradation of stop-band properties (noise-suppression) by the real-time filter. Finally, fig.16.10 illustrates convergence of the non-linear optimization criterion 16.13 based on the new generalized criterion 16.15 for  $\lambda = 50$  and  $\eta = 1.2$ .

#### 16.4.4.3 Model-Based Customization of Real-Time Model-Based Trend

Amplitude and time-shifts of customized (C-filter<sup>11</sup>) and mean-square (MS-)filter are compared in fig.16.11. Fig.16.11 suggests that the C-filter barely loses in terms of time-shift in the  $[0, \pi/6]$  pass-band and that it clearly wins in terms of noise suppression in the  $[\pi/6, \pi]$  stop-band. This is confirmed by real-time estimates in fig.16.12: for ease of visual inspection we focus on a tighter window centered around the financial crisis.

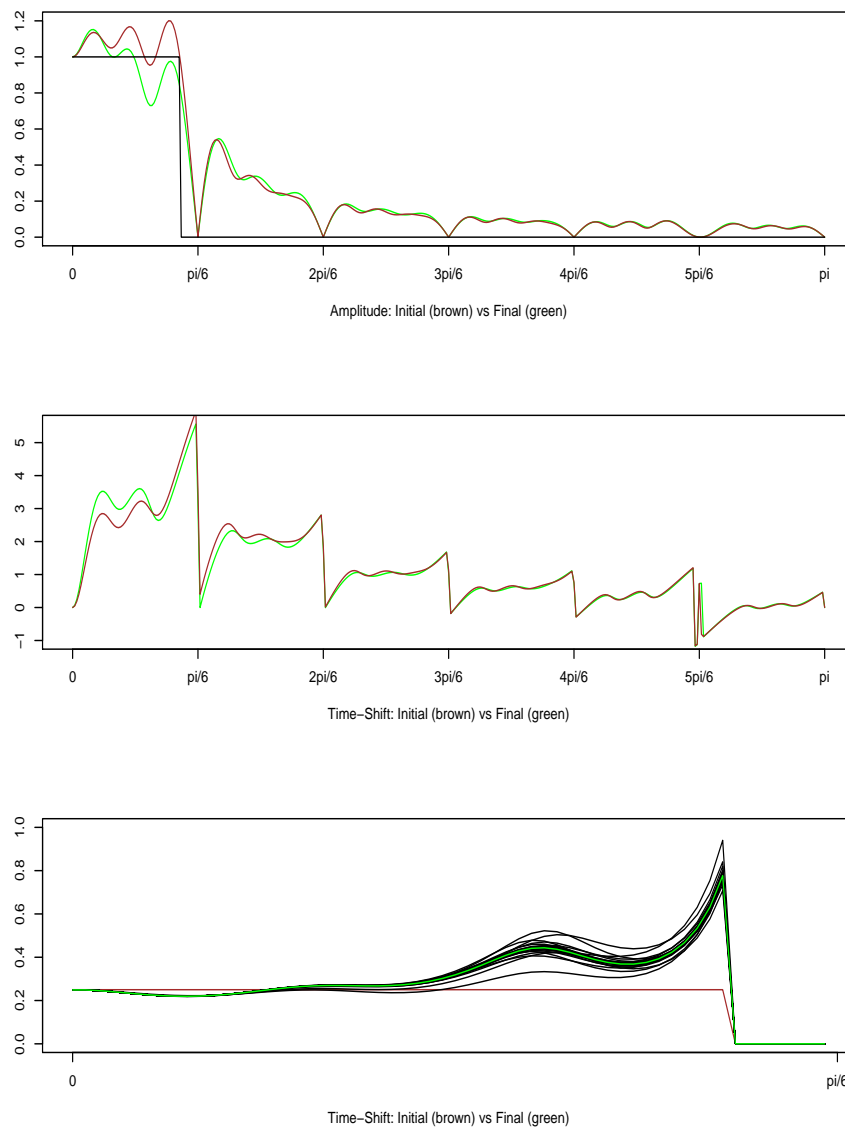
<sup>11</sup>C-parameters are  $\lambda = 200$  and  $\eta = 1.2$ .

Figure 16.9: Effect of  $\lambda$  (speed) on real-time trend filter

Squared second order differences in the lower panel illustrate smoothness (curvature) issues<sup>12</sup>: obviously, the C-filter is much smoother than the MS-filter without losing in terms of ‘speed’. The (final converged non-linear) solution of 16.15 is compared to the initial solution in fig.16.13. Again, convergence of the iterative solution is achieved rapidly (less than ten iterations) as illustrated by fig.16.14. As a result, numerical computations are fast despite the relatively large number of freely determined parameters ( $L = 48$ ). Even more interestingly, overfitting seems absent. There is a simple explanation for this welcome attribute:  $\eta$  emphasizes a tighter amplitude fit in the stop-band and  $\lambda$  favors a ‘typical spectral shape’ in the passband. Therefore, overfitting-impulses are inherently restrained by the very customization principle.

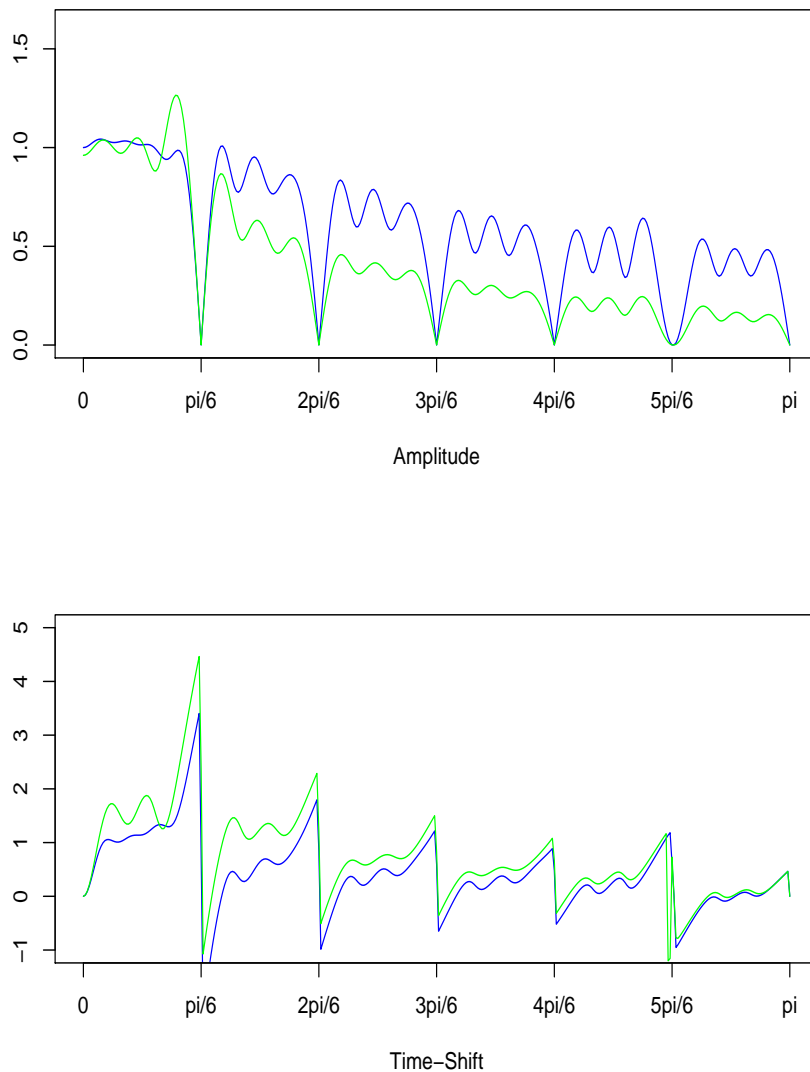
<sup>12</sup>The mean-square second order differences of the C-filter are 0.00455 compared to the much larger mean-square of 0.01753 for the MS-filter (these numbers refer to the standardized filtered series).

**Figure 16.10:** Amplitudes and Ideal Trend (top), Time-Shifts (middle) and  $f(\omega_k)\Gamma(\omega_k)$ : Initial (brown) vs. Final (green)



#### 16.4.4.4 Model-Based Customization of Real-Time Ideal Trend

We here substitute the ideal trend 16.20 to the previous model-based trend target and obtain a new concurrent real-time filter, called CI-filter, whose characteristics are compared to the previous C-filter in fig.16.15. As expected, the amplitude function of the CI-filter is closer to zero towards the higher frequencies. Interestingly, its time-shift is (almost) identical to the previous C-filter in the pass-band and therefore smoothness/reliability could be improved without sacrificing timeliness. Outputs of both customized designs are compared in fig.16.16 (for ease of visual inspection, the episode before and during the financial crisis has been emphasized). The new CI-filter detects turning points as fast as the previous C-filter and its squared second order differences (mean curvature) are even smaller: mean-square second order differences (mean curvature) are 0.00295 (CI) and 0.00455 (C-filter). As a consequence, it generates less false alarms without sacrificing timeliness.

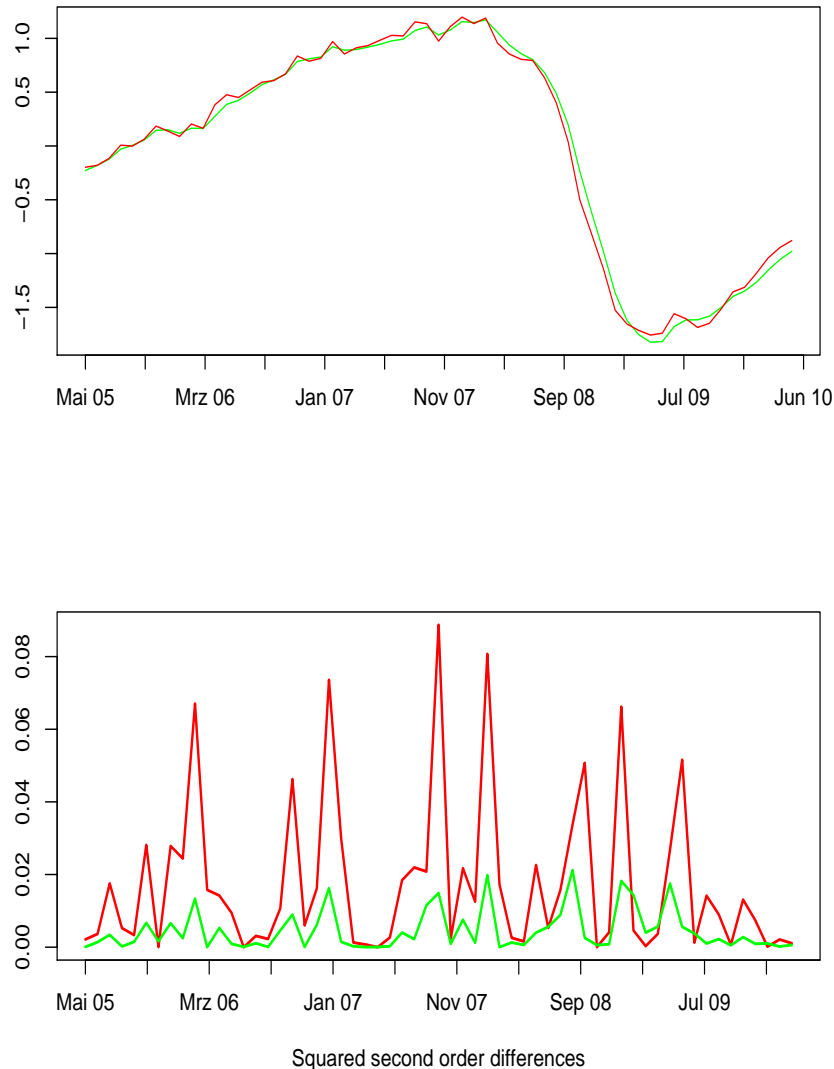
**Figure 16.11:** Customized (green) vs. mean-square (blue) real-time trend filters

#### 16.4.4.5 T-DFA Customization of Seasonal Adjustment Filters

Outputs of the (real-time) model-based SA-filter and of the CI-filter are compared in fig.16.17. The CI-filter is both 'smooth' and 'fast' which facilitates considerably the detection of turning-points as well as the identification of recessive phases, when compared to corresponding abilities of the SA-filter (or any of the model-based real-time MS-filters). Addressing speed- and reliability virtues simultaneously/explicitly is a formal strength of the generalized criterion 16.15. Unfortunately, seasonal adjustment does not fit well into this framework because the smoothness dimension is abandoned, to a large extent: 'noisy' outputs of real-time SA-filters differ not markedly from similarly 'noisy' outputs of symmetric filters, not least because high-frequency components passing the filters are likely to mask potential differences. Therefore, the types of customizations proposed in this section address issues specific to wide partitioned pass-band filters, as is the case for SA.

Researchers often rely on seasonally adjusted series for analyzing the growth in first differences of a series, as illustrated in the bottom plot of fig.16.1. By eliminating a unit root, the process is affected and therefore

**Figure 16.12:** Filter outputs of C- (green) and MS- (red) real-time trend filters: levels (upper) and squared second order differences (bottom)



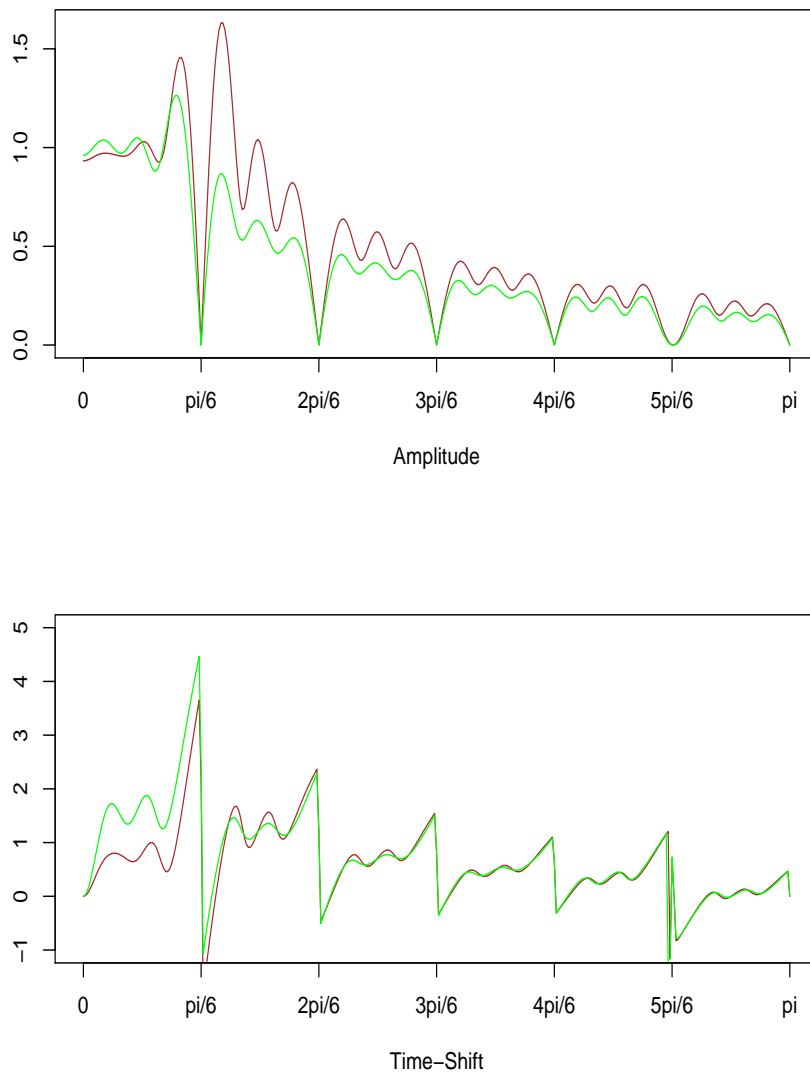
the model as well as the seasonal adjustment processing are affected too. The resulting time series are not invariant to the ordering of the transformations, particularly in a real-time perspective, because the difference filter magnifies high-frequency noise which is addressed differently in levels. We here replicate the typical ordering - difference of previously adjusted series -, reveal potential problems of this proceeding in a real-time perspective and propose dedicated customizations based on the formal framework provided by criterion 16.15.

The amplitude function of the real-time model-based SA(96)-filter ( $L = 96$ ), the periodogram of the first differences of the SA(96)-series and of the first differences of the official SA-IPi release (bottom graph in fig.16.1) are plotted in fig.16.18. Let us shortly comment this outcome:

- In contrast to trend extraction, real-time SA-filters and symmetric SA-filters have similar effects as can be seen by comparing red (real-time) and black (official release) periodograms.
- Shortening the support ( $L = 96$ ) does not seem to affect performances substantially.



**Figure 16.13:** Customized: final non-linear (green) vs. initial (brown) real-time trend filters

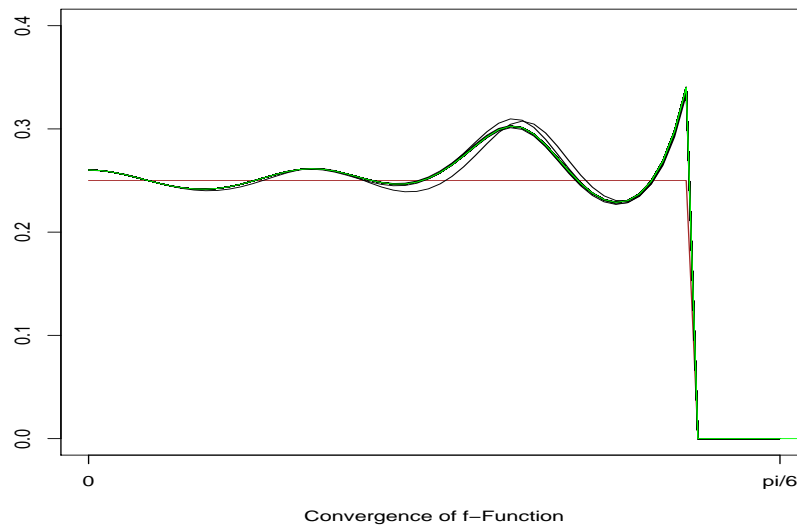


- Both adjusted series show evidence of residual trading day effects<sup>13</sup> (the largest peak slightly on the right of  $4\pi/6$ ).
- The real-time filter is subject to slight under-adjustment in  $4\pi/6$  and  $\pi$  where damping of the seasonal components seems less effective.
- The difference filter magnifies high-frequency components and therefore it magnifies issues/problems which would be negligible in the original levels. The undesirable effects are magnified further by taking a real-time perspective. The practical relevance of these problems is related to the fact that users frequently rely on differences of (official releases of) seasonally adjusted data in order to perform real-time analysis. The case is pertinent, in particular, when the original (non-adjusted) data is not published<sup>14</sup>.

The listed deficiencies can be addressed very easily by taking advantage of the flexible user-interface provided by criterion 16.15: a customized signal  $\Gamma(\cdot)$  is proposed in fig.16.19 (top graph) and the periodogram (red line)

<sup>13</sup>The selected RSA2-setting in Demetra should allow for removal, in principle.

<sup>14</sup>As is the case for the USRI, see <http://www.idp.zhaw.ch/usri>.

**Figure 16.14:** Convergence of f-function 16.18: initial (brown) vs. final (green)

is plugged into  $S(\omega_k)$ . The amplitude function of the customized (real-time) MA(96), denoted by C(96) can be seen in the mid-panel (blue line). The resulting effect can be observed in the periodogram of the filtered series (blue line, bottom panel). As expected, the undesirable under-adjustments in the seasonal harmonics  $4\pi/6$  and  $\pi$  have been tackled and the residual trading-day peak has been damped successfully without affecting other components. Also, the time-shift (not shown) is near zero and therefore turning-points are invariant to this transformation. ‘Undesirable’ components are not eliminated completely by C(96) because its amplitude function does not vanish in the corresponding frequencies. A complete elimination of isolated components could be obtained very easily by imposing a zero of  $\Gamma(\omega_k)$  and by assigning a large (infinite) value to the frequency-weighting function  $S(\omega_k)$ , the periodogram, in the corresponding frequencies. Some caution is advisable, however, when attempting to realize tight ‘surgical’ filter effects because narrowing dips or peaks in the transfer function requires correspondingly large filter orders: in the example we selected  $L = 96$  (instead of  $L = 48$  for the preceding trend filters). Fortunately, as noted earlier on and confirmed, once again, by the above example, the customized criterion 16.15 is pretty much immune against overfitting<sup>15</sup> and therefore ‘large’ filter orders do not conflict with real-time performances, as would typically be the case for model-based approaches.

Taken together, the above examples suggest that the appealing user-interface provided by criterion 16.15 allows for straightforward user-specific tailoring of the filter-design in a practically relevant real-time perspective. We now briefly develop this topic which takes us away from the original model-based design.

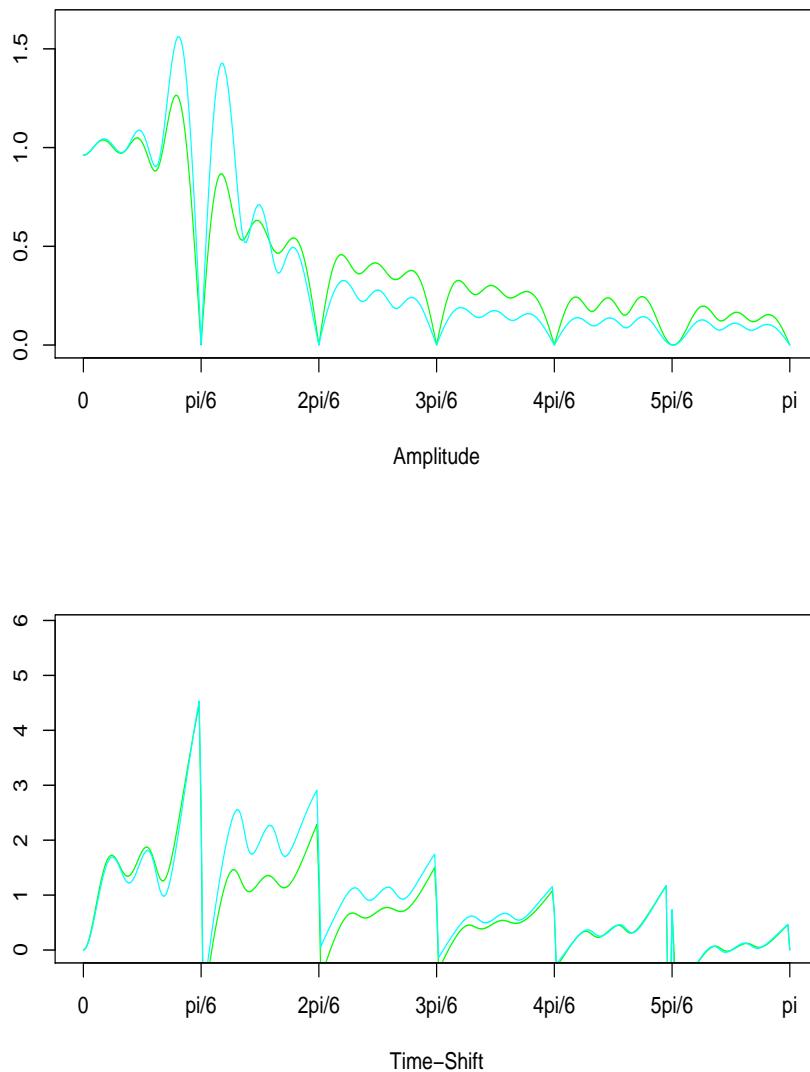
## 16.4.5 Customization beyond the Model Based Perspective

### 16.4.5.1 Seasonal Adjustment

A fully customized real-time seasonal adjustment of first differences of the original (unadjusted) IPI-series is addressed in this section: note that the ordering of the transformations is inverted (differences prior to SA). We deliberately decide to rely on a very simple ‘plain-vanilla’ design without sophisticated user interaction. The signal  $\Gamma(\cdot)$  relies on a straightforward extension of the ideal trend as can be seen in fig.16.20, top panel (black line). The bi-infinite incarnation of this filter removes seasonal components completely. As can be seen, dips of

<sup>15</sup>These issues are treated in chapters 7 and 8 in Wildi (2008).

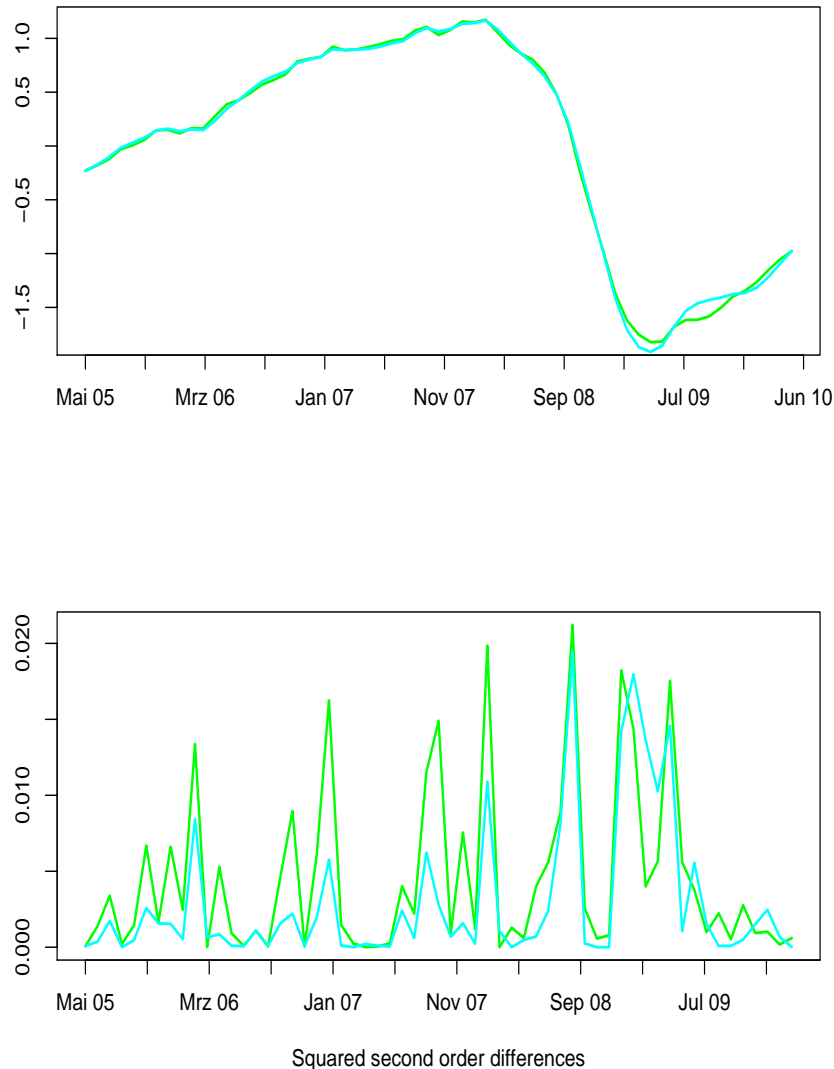
**Figure 16.15:** Customized: CI-filter ideal trend signal (cyan) vs. C-filter model-based trend signal



the signal match seasonal components as measured by the log-periodogram (red line). In contrast to model-based approaches, location, width and deepness of dips can be specified directly, by the user<sup>16</sup>. As shown in Wildi (2008), the periodogram is a very natural choice for the frequency weighting  $S(\omega_k)$ . We set  $\lambda = \eta = 0$  (MS-estimation) and  $L = 96$ , as in the previous section. This ‘plain-vanilla’ design is benchmarked with the first differences of the official SA-release. Please note that the latter series is revised: it is based on filters sensing the full sample information in each time point  $t = T$  (including future data  $t + 1, t + 2, \dots, T$ ) whereas our ‘toy’-design is a real-time MA(96), which we denote by CSA(96) in the following. The amplitude function of the filter is plotted in the bottom panel of fig.16.20 (blue line). The amplitude function is ‘rippled’ which is due, in part to finite sample effects as well as to the periodogram weighting. The time-shift of the filter (not shown) is almost vanishing and therefore turning points remain unaffected, as can be seen by gauging both adjusted series in the bottom plot of fig.16.21. Periodograms of CSA(96)-output (blue) and of benchmark (black) are compared in the top-panel. As can be seen, the resemblance is striking (the correlation coefficient between the differenced series in the bottom panel is 0.88) in particular when put into perspective with the design:

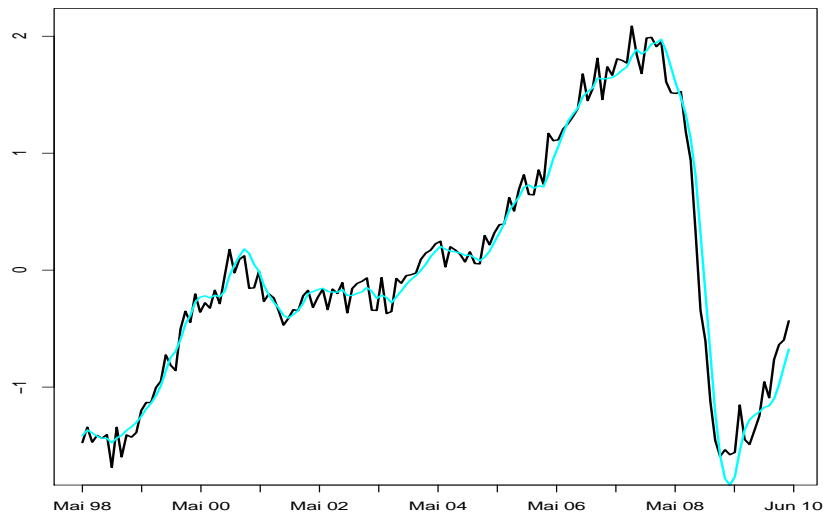
<sup>16</sup>‘Unloaded’ seasonal frequencies could be simply ‘passed-over’ and therefore typical over-adjustment, as induced by the ubiquitous  $1 - B^{12}$  seasonal difference filter, would be avoided elegantly.

**Figure 16.16:** Filter outputs: CI-filter (cyan) vs. C-filter (green). Levels (upper) and squared second order differences (bottom)



the ordering of the transformations (SA/diffs) is inverted, CSA(96) is a much shorter filter, it is a real-time (unrevised) design and it does not rely on any automatic/deterministic adjustment<sup>17</sup> (outliers, shifts, calendar effects such as Eastern, holidays, trading-days). Instead it is based on a fairly trivial target signal as well as on a century-old non-parametric and inconsistent spectral estimate. At this stage, various customizations (for example removing trading day effects) could be entertained by referring to the natural user-interface provided by criterion 16.15. Instead, we finalize our empirical investigation by analyzing fully customized real-time trend estimation for the above first differences of IPI.

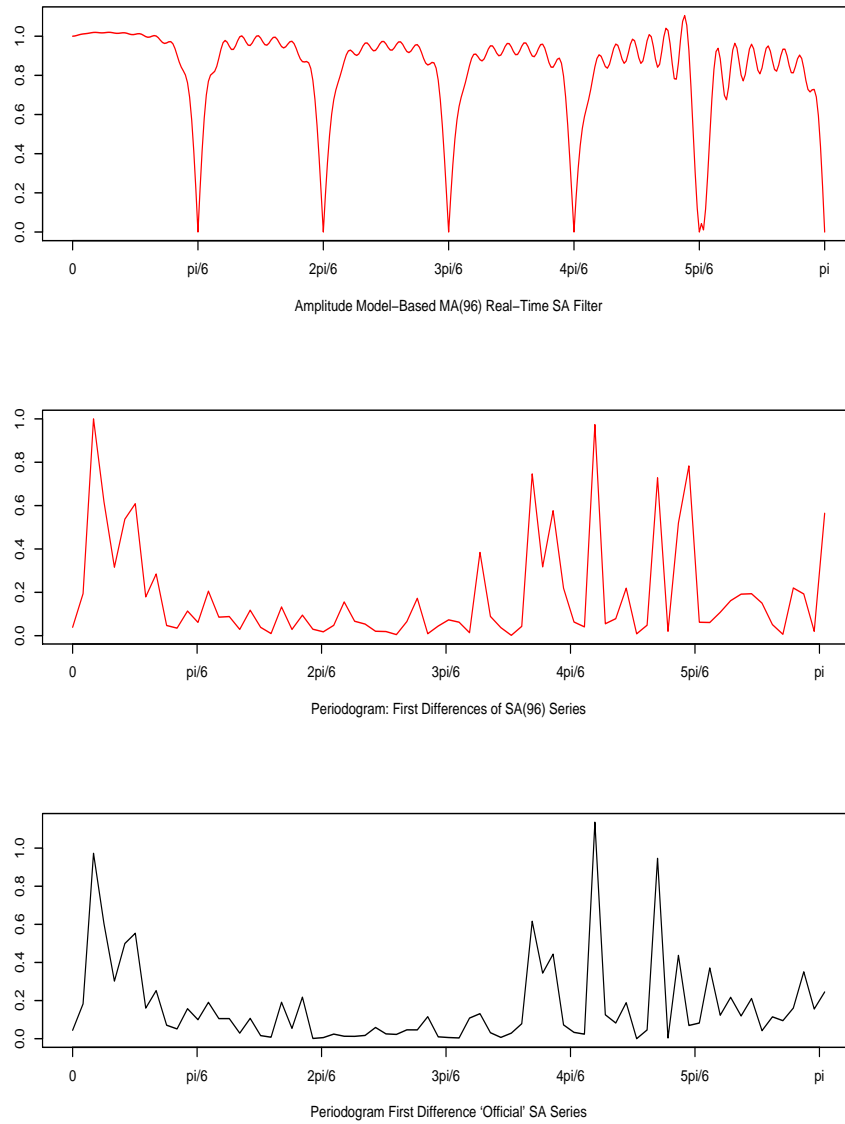
<sup>17</sup>TRAMO proposes adjustments at the begin of 2010 which may explain the observed differences between black (official release) and blue (CSA(96)) lines in the top-graph of fig.16.21.

**Figure 16.17:** Filter outputs: CI-filter (cyan) vs. classical SA model-based (black)

### 16.4.5.2 Trend

Analyzing the trend-growth in first differences can convey early information about the occurrence of turning-points in the original data. In contrast to the preceding seasonal adjustment application, real-time trend estimation makes fully use of the powerful speed-reliability customization provided by  $\lambda$  and  $\eta$ . For our last design we decided to detach completely from the model-based paradigm:  $\Gamma(\cdot)$  is the ideal trend (with cutoff  $\pi/7$ ),  $S(\omega_k)$  is the periodogram,  $\lambda = 40$  and  $\eta = 1.4$ . Smoothness has been emphasized explicitly because the difference filter magnifies noisy high-frequency components. On the other hand, the time-shift error is inflated in order to conciliate reliability with speed.

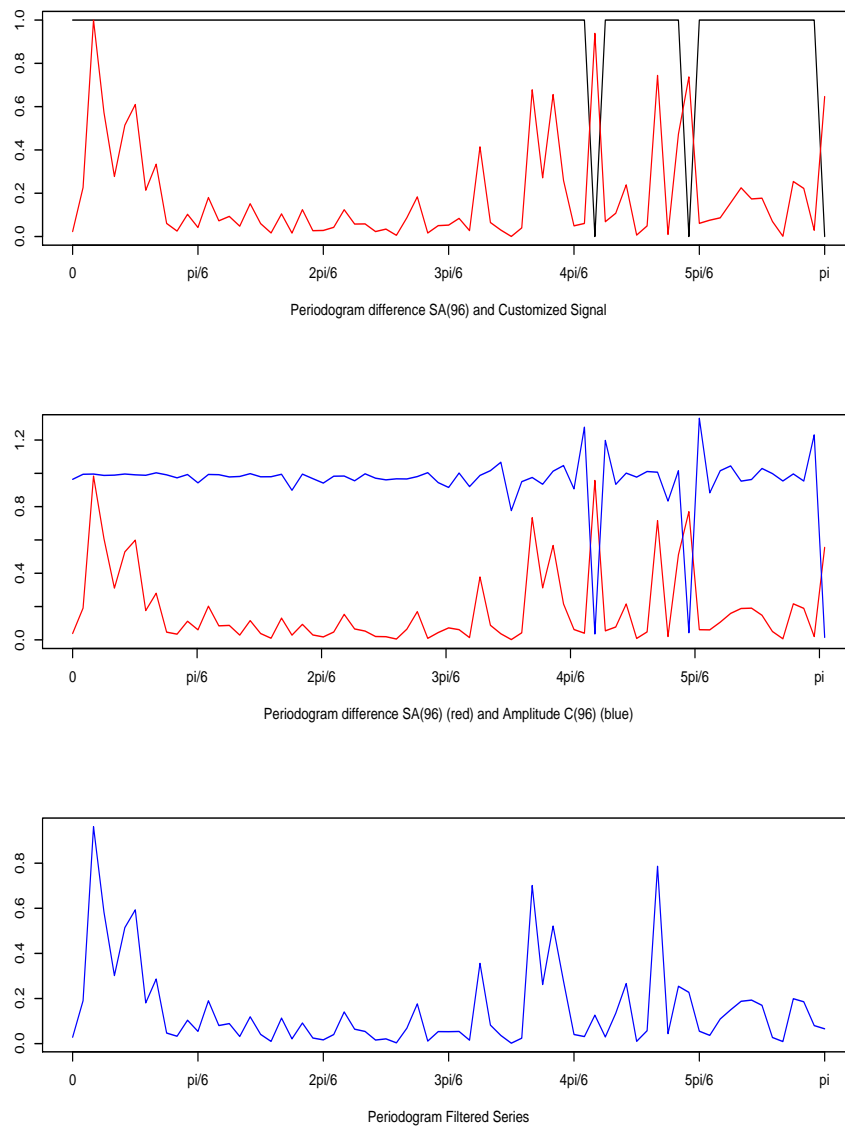
The amplitude function of the real-time MA(48) trend filter - denoted CT(48) - is plotted in fig.16.22 together with the (scaled) log-periodogram and the time-shift. The fast convergence of the non-linear optimization can be verified in fig.16.23. The rather unusual small amplitude values are due to a particular combination of circumstances: first-differences of IPI are not trending (which invalidates, incidentally, the double unit-root of the airline-model) and therefore the first order restriction  $\hat{A}(0) = 1$  can be relaxed. Moreover, the strong customization imposed by  $\eta$  in the stop-band pulls the whole amplitude function towards zero in the pass-band. Finally, the spectral mass in the passband is too weak to pull the amplitude function back to the unity-level referenced by the target signal. As a result, mean-square performances of the filter are (very) poor. However, the main ingredients for detecting turning-points are met: the time-shift is decently small in the pass-band and high-frequency noise is suppressed effectively (relative to components in the passband). Fig.16.24 compares the previous 'best' design, the CI-filter of section 16.4.4.4, with CT(48): the left graph compares CT(48) with differences of CI (recall that the latter is optimized in original levels) and the right graph compares the cumulated sum of CT(48) with CI. For ease of visual inspection the episode around the financial crisis (grey-shaded) is selected and series are scaled. Fig.16.25 compares CT(48) and SA(48) obtained in the previous section 16.4.5.1 filter. The real-time filter CT(48) dates begin and end of the recession in January 2008 and July 2009 in 'pseudo' real-time (data is based on the June-2010 release of IPI) and the recovery-path is steady. Although faster filters could be obtained (not shown here), we conjecture that the proposed CT(48)-design conciliates conflicting requirements in a way sensible to the analyst interested in extracting growth-dynamics of the industrial production series. The non-linear optimization based on the perfect disentanglement of phase and amplitude errors in 16.13 sustains a systematic building-up of best possible compromises in the stress-field spanned by reliability and speed dimensions. The resulting customization stabilizes richly parameterized

**Figure 16.18:** Amplitude real-time SA-filter (upper) and periodogram of adjusted series

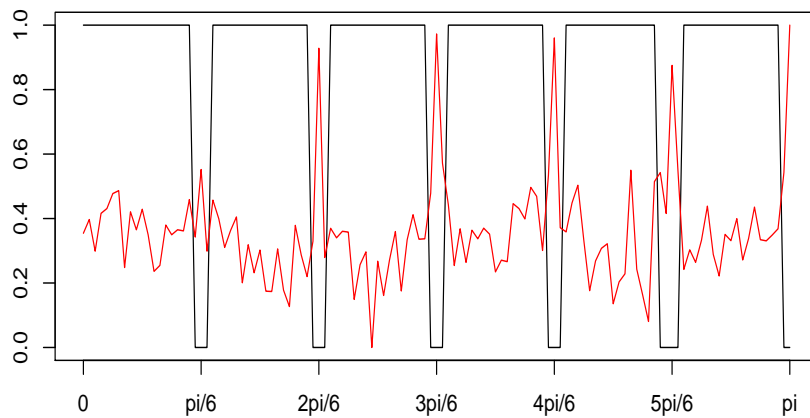
designs and therefore overfitting is effectively avoided<sup>18</sup>. Obviously, such a design is inaccessible to model-based approaches.

<sup>18</sup>Overfitting issues are addressed in Wildi (2008), chapters 7 and 8

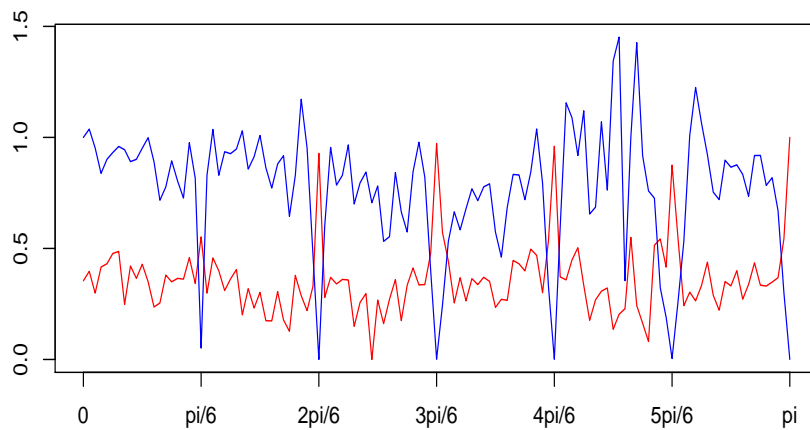
**Figure 16.19:** Customized signal and periodogram of differenced SA(96)-series (top), amplitude of customized C(96) real-time filter (blue/mid-panel) and periodogram of filtered series (blue/bottom)



**Figure 16.20:** Customized signal (black), log-periodogram of 1. differences of IPI (red) and amplitude CSA(96)



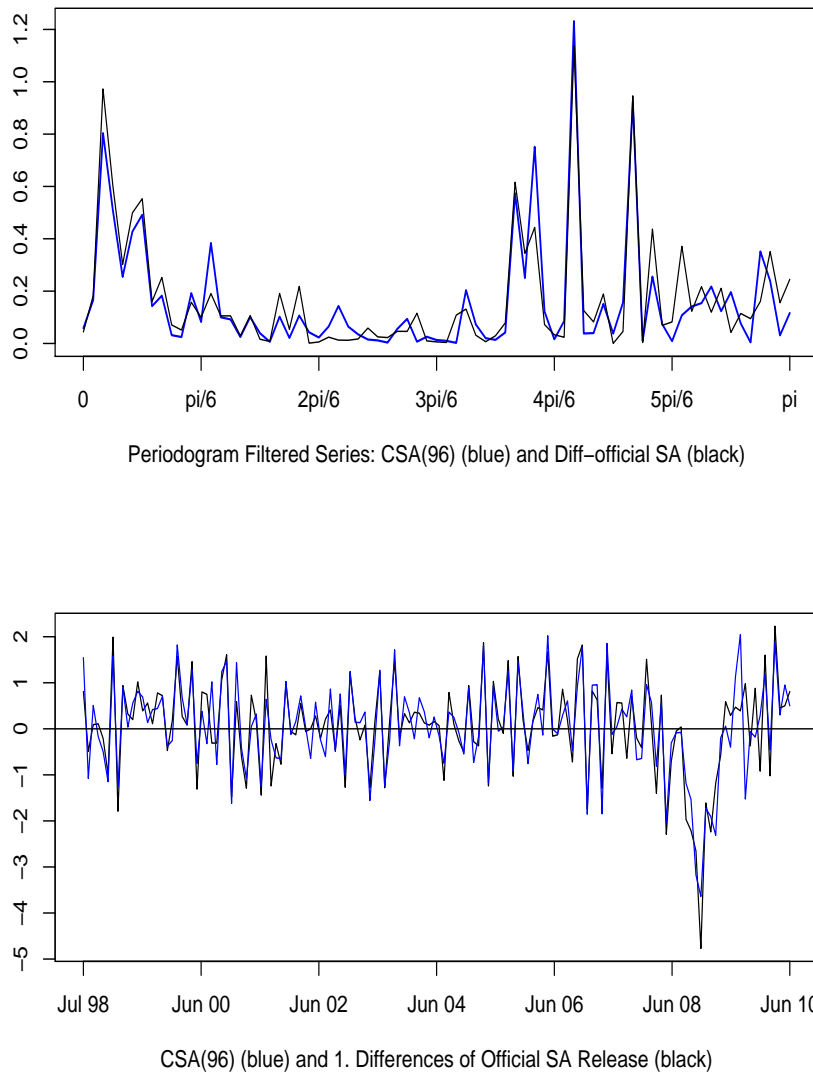
Periodogram and Customized Signal



Amplitude CSA(96) (blue) and Log-Periodogram Diff-IPI (red)



**Figure 16.21:** Periodograms (top) and filtered series (bottom) of CSA(96) (blue) and first differences official SA (black)



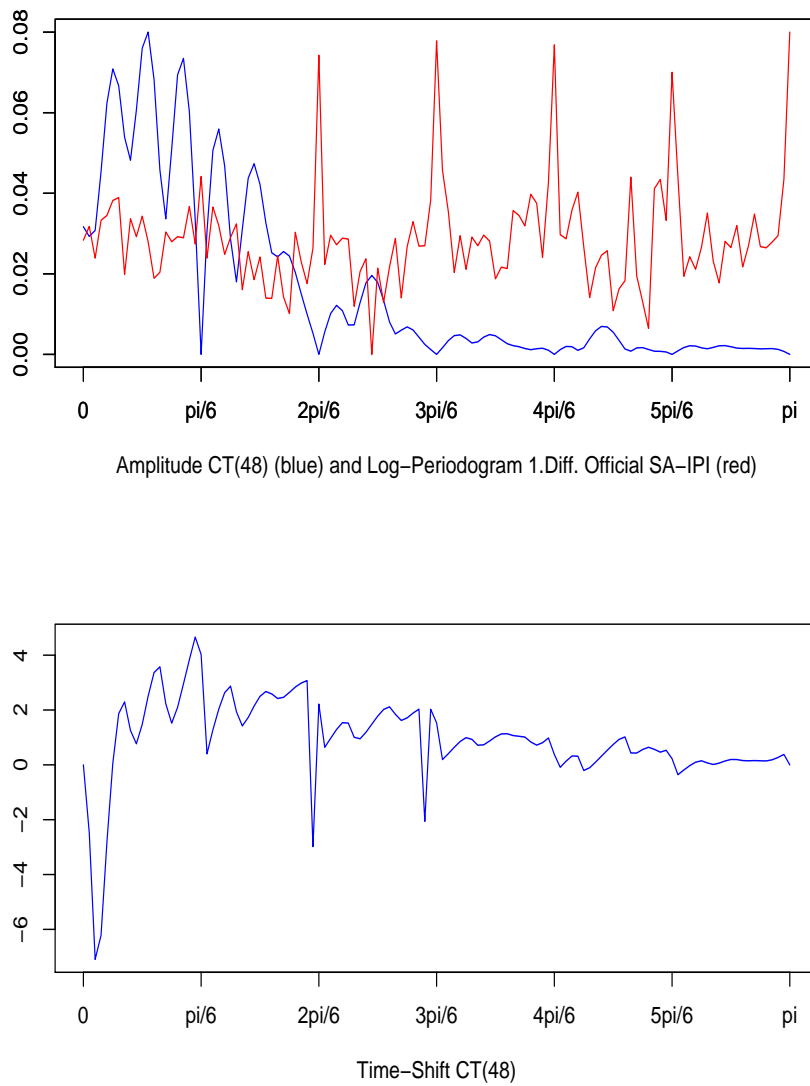
**Figure 16.22:** Periodogram (red/top), amplitude CT(48) (blue/top) and time shift (blue/bottom)

Figure 16.23: Convergence of  $f(\omega_k)$ -function 16.18: initial (brown) vs. final (blue)

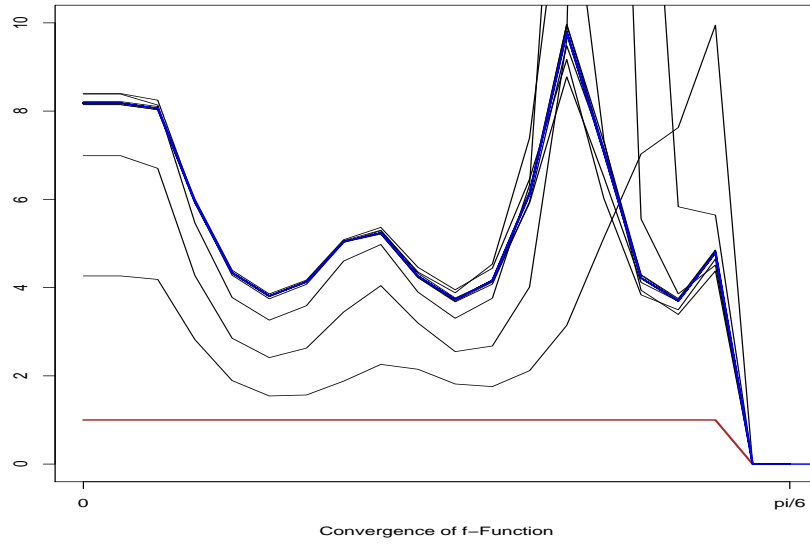


Figure 16.24: CT(48) (blue/left) and diff-CI (cyan/left). Cumsum CT(48) (blue/right) and CI (cyan/right)

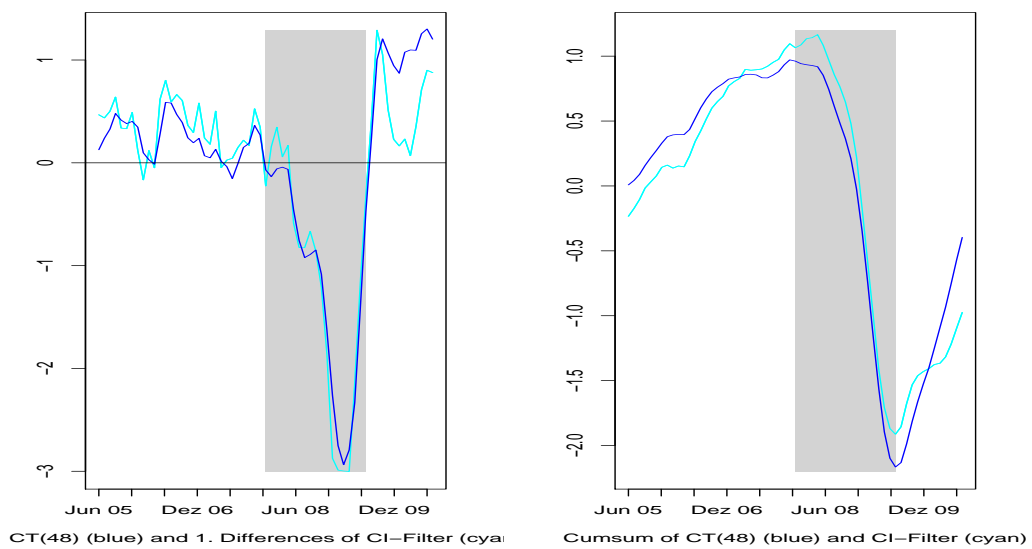
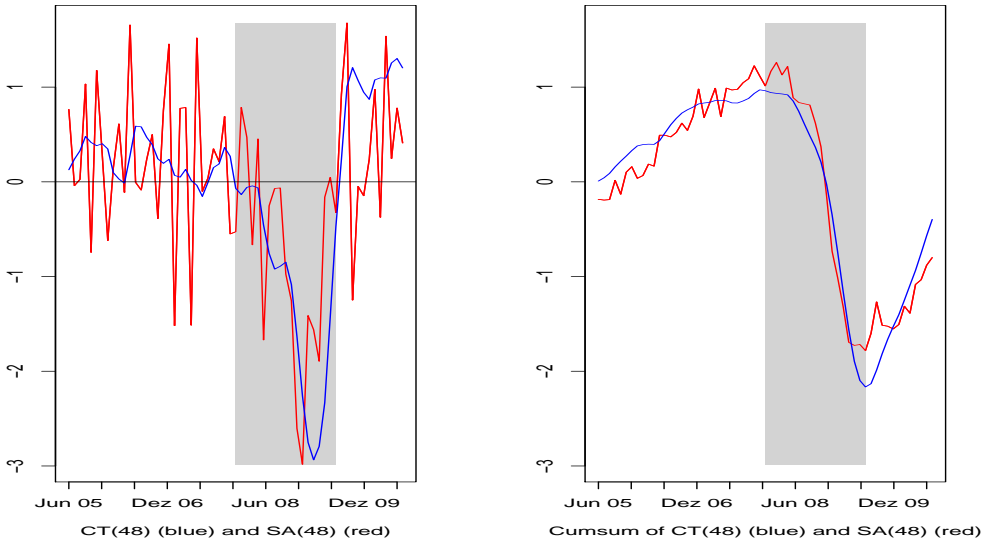


Figure 16.25: CT(48) (blue/left) and CSA(48) (red/left). Cumsum CT(48) (blue/right) and CSA(48) (red/right)



## 16.5 Conclusions

The new optimization criterion 16.15 generalizes traditional model-based approaches as well as the traditional DFA. Therefore, a formal bridge is obtained which links both 'philosophies'. In particular, the powerful customization aspect of the DFA can be transposed into a pure model-based perspective. The flexible and intuitively appealing user-interface provided by  $\Gamma(\omega_k)$  and  $S(\omega_k)$  allows the user to tailor (sculpture) the filter to particular research priorities. Alternatively, existing designs could be enhanced and/or corrected for apparent failures. A set of relevant frequency-domain statistics assists the user in this perspective: amplitude and time-shift functions of filters as well as periodograms of original and filtered series. Also,  $\lambda$  and  $\eta$  account comprehensively for the famous speed/reliability - bias/variance - dilemma in the context of real-time trend extraction. Criterion 16.15 emphasizes explicitly the practically relevant real time perspective (estimation in  $t = T$ ) though any time point  $t < T$  in the sample could be addressed, in principle.

Customization of model-based approaches has been obtained to various degrees and in various levels of abstraction (signal, frequency-weighting, speed/reliability dilemma) and in any combination, reaching from purely model-based designs up to 'full detachment'. In the latter case, solutions are achieved which conciliate conflicting requirements (speed/reliability dilemma) in a way totally inaccessible to model-based approaches. It is crucial, in this framework, that criterion 16.15 sustains a perfect disentanglement of phase and amplitude errors. In comparison to the classical DFA (Wildi (2008)) the new criterion is numerically fast and therefore user-friendly. A final remarkable property of the proposed approach is its immanent robustness against overfitting: instead of estimating parameters with respect to short-term one-step ahead mean-square performances, criterion 16.15 addresses directly the relevant real-time filtering problem as well as user priorities in a unified customized perspective.

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# V

## Seasonal Adjustment and Aggregation







# 17

## Benchmarking and Temporal Consistency



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## Table of Contents

17.1	Introduction	455
17.2	X-12-ARIMA FORCE spec	456
17.2.1	Benchmarking formulae	457
17.2.2	The type argument	457
17.3	Example	457
17.4	Alternative Benchmarking Software	460
17.5	Model-Based Methods	460
17.6	Conclusions	461

## 17.1 Introduction

The 2009 ESS Guidelines on Seasonal Adjustment (Eurostat (2009)) states there are no theoretical justification to force the sum (or average) of the seasonally adjusted data over each year to equal the sum (or the average) of the raw data but it can be of particular interest when benchmarking figures officially exist such as in the National Accounts, Balance of Payments, External Trade, where users' needs for time consistency are stronger. In such cases, benchmarking methods should be chosen according to their properties, such as movement preservations, absence of residual seasonality or calendar effects.

Prior to benchmarking, it may be advisable to perform a compatibility test (Quenneville and Gagné (2011)) to determine if the sub-annual series and the benchmarks can be considered suitable for benchmarking. If not, benchmarking should not be applied and a list of possible conceptual, operational and methodological differences to investigate is provided in Brisebois and Yung (2007). Temporal benchmarking should be applied when discrepancies are small and not characterized by systematic behavior. Any large or systematic temporal discrepancy should be investigated, most likely pointing to the presence of extreme observations or the need to revise the seasonal adjustment options.

When needed, temporal consistency is usually handled by benchmarking the sub-annual series to the annual benchmarks using Denton (1971) benchmarking method. Helfand et al. (1977), Cholette (1979), and Cholette (1984) modified Denton's method to correct the initial condition which sometimes introduces spurious movements in the benchmarked series and we will henceforth refer to the modified versions as Denton method.

Denton's method is well described and explained in Dagum and Cholette (2006). A specific variant of the solution is implemented at Statistics Canada as an in-house SAS procedure called PROC BENCHMARKING (see Latendresse et al. (2007)), which also includes the regression method of Cholette and Dagum (1994) and a few extensions described in Bloem et al. (2001) (their Chapter 6). The method and its links to the Denton method are discussed in Fortier and Quenneville (2007).

Assuming annual benchmarks have been imposed on the raw series, then for many seasonal adjustment methods, the annual totals of the seasonally adjusted (SA) series may not agree with those of the raw series when the seasonal pattern changes with time or when there are trading-day (TD) variations in the series. The TD effect over a year is variable and different from zero. Each day of the week occurs 52 times in a year except for one extra day in a non-leap year, and two extra days in leap years.

In X-12-ARIMA (Findley et al. (1998); U.S. Census Bureau (2009)), the SA series may be benchmarked to control totals derived from the raw series using the FORCE spec. This spec extends and includes the method that was implemented in X-11-ARIMA (Dagum (1981), Dagum (1988)). The FORCE spec comes with various arguments to parameterize the benchmarking method. Section 17.2 presents the numerical optimization justification and covers a few the arguments of the FORCE spec. Section 17.3 illustrates the available methods with a real example. Section 17.4 discusses alternative benchmarking software. Section 17.5 proposes a few extensions when benchmarking the seasonally adjusted series produced by model based seasonal adjustment software such as TRAMO-SEATS (Gómez and Maravall (1996)) or STAMP (Koopman et al. (2000)). Section 17.6 provides concluding remarks.

## 17.2 X-12-ARIMA FORCE spec

The X-12-ARIMA FORCE spec comes with various arguments to parameterize the benchmarking method. The *target* argument specifies which series is used as the target for forcing the totals of the seasonally adjusted series. The choices of target are the original series; the calendar adjusted series; the original series adjusted for permanent prior adjustment factors; and, the original series adjusted for calendar and permanent prior adjustment factors. For each of them, the option *usefcst* determines if forecasts are appended to the target series in order to achieved better timeliness for the observations in the current year with annual benchmark not yet available. This is because, under Denton's method, the last correction of the last complete year with a benchmark (an annual total) is applied to the incomplete current year without a benchmark.

This may entail large revisions in the current benchmarked SA values when the next annual benchmark becomes available, see for example Ferland et al. (2014). A similar problem exists in the elaboration of quarterly national accounts. For this case, Bloem et al. (2001) provide an enhanced proportional Denton method, in which the unknown annual total for the current year is forecasted. Their method assumes that the annual benchmark comes from an independent source available some time after the last quarterly estimate. The topic is also discussed in Di Fonzo and Marini (2012).

In the X-12-ARIMA FORCE spec, the annual total comes from the target series and it can be forecasted with increasing accuracy as the year-end approaches. The target series should be selected as the original series when external temporal benchmarks have been applied to the raw series prior to seasonal adjustment; the calendar adjusted should be selected if the annual totals in the calendar adjusted raw series become the annual benchmarks to be used in the production process that follow seasonal adjustment.

By default, the FORCE spec implies that the calendar year totals in the seasonally adjusted series will be made equal to the calendar year totals of the target series. An alternative starting period for the annual total can be specified with the *start* argument; consequently, annual totals between the benchmarked seasonally adjusted series and the target series that start at any other period other than that specified by the *start* argument may not be equal.

Let  $s = (s_1, \dots, s_T)$  represents the SA series,  $x = (x_1, \dots, x_T)'$  the target series and  $a = (a_1, \dots, a_L)$  the benchmarks. The dates of  $s$  and  $x$  are assumed to be mapped into the set of integers  $t = 1, \dots, T$ . Each benchmark  $a_l$  is therefore associated with its coverage period defined by a starting date  $t_{1,l}$  and an ending date  $t_{2,l}$  with  $t_{2,l} - t_{1,l} + 1 = P$  and where  $P$  is the seasonal period in the series ( $P = 4$  for quarterly series and  $P = 12$  for monthly series); obviously, the sub-index 1 in  $t_{1,l}$  refers to a month or quarter specified by the *start* argument. To simplify the notation, let  $t \in l$  mean that  $t_{1,l} \leq t \leq t_{2,l}$ . Let  $J$  be the matrix of dimensions  $L \times T$  where, for  $l = 1, \dots, L$  and  $t = 1, \dots, T$ :  $j_{l,t} = 1$  when  $t \in l$  and 0 otherwise. The matrix  $J$  is called the *temporal sum operator* and a typical row takes the form  $(0, \dots, 0, 1_P, 0, \dots, 0)$  where  $1_P$  is the unit vector of length  $P$ . The vector of benchmarks is  $a = Jx$ .

The benchmarked SA series  $\theta = (\theta_1, \dots, \theta_T)'$  is obtained as the solution of the following minimization problem: given parameters  $\rho$  and  $\lambda$  provided by the X-12-ARIMA FORCE arguments rho and lambda, find the values  $\hat{\theta}_t$  that minimize the following function of  $\theta_t, t = 1, \dots, T$ ,

$$f(\theta_1, \dots, \theta_T) = (1 - \rho^2) + \left( \frac{s_1 - \theta_1}{|s_1|^\lambda} \right)^2 + \sum_{t=2}^T \left[ \left( \frac{s_t - \theta_t}{|s_t|^\lambda} \right) - \rho \left( \frac{s_{t-1} - \theta_{t-1}}{|s_{t-1}|^\lambda} \right) \right]^2 \quad (17.1)$$

under the constraints

$$J\theta = Jx = a. \quad (17.2)$$

Parameter  $0 \leq \rho \leq 1$  controls movement preservation as it links one observation to its preceding value. Parameter  $\lambda$  quantifies how the discrepancies between the original series  $s$  and the desired series  $\theta$  contribute to the objective function and is used to select specific variants. The modified Denton's Proportional First Difference method (Cholette (1984)) uses  $\rho = 1$  and  $\lambda = 1$ ; the non-proportional method uses  $\lambda = 0$ . Simple temporal pro-rating uses  $\rho = 0$  and  $\lambda = 0.5$ .

### 17.2.1 Benchmarking formulae

Define  $C$  as the  $T \times T$  matrix with  $|s_t|^\lambda$  as the  $t$ -th element of the main diagonal and 0 elsewhere. For  $\rho < 1$ , let  $\Omega_e$  be the  $T \times T$  matrix defined by  $\Omega_e = ((\rho^{|i-j|}))$ ,  $i, j = 1, \dots, T$ . For  $\rho = 0$ , the matrix  $\Omega_e$  is the Identity matrix. Let  $V_e = C\Omega_e C$  and  $V_d = J V_e J'$ . The minimization of 17.1 under the constraints 17.2 entails minimizing the function  $(s - \theta)'(C\Omega_e C)^{-1}(s - \theta) + 2\nu'(J\theta - a)$  with respect to the elements of  $\theta$  and  $\nu$ , where  $2\nu$  is the vector of Lagrange multipliers. It is shown in Quenneville and Fortier (2011) that the benchmarked series is

$$\hat{\theta} = s + V_e J' V_d^{-1} (a - J s). \quad (17.3)$$

For  $\rho = 1$ , the original derivation can be found in Cholette (1984). Let  $\Delta$  be the  $T - 1 \times T$  first difference matrix with 1 at index  $(i, i)$ , 1 at index  $(i, i + 1)$ ,  $i = 1, \dots, T - 1$ , and 0 elsewhere and let  $I_L$  be the  $L \times L$  Identity matrix. Minimization of 17.1 under the constraints 17.2 entails minimizing the function  $(s - \theta)' C^{-1} \Delta' \Delta C^{-1} (s - \theta) + 2\nu'(J\theta - a)$  with respect to the elements of  $\theta$  and  $\nu$ , where  $2\nu$  is the vector of Lagrange multipliers. It is shown in Quenneville and Fortier (2011) that the benchmarked series is

$$\hat{\theta} = s + W(a - J s) \quad (17.4)$$

where  $W$  is the  $T \times L$  upper-right corner matrix from the following matrix product:

$$\begin{bmatrix} C^{-1} \Delta' \Delta C^{-1} & J' \\ J & 0 \end{bmatrix}^{-1} \begin{bmatrix} C^{-1} \Delta' \Delta C^{-1} & 0 \\ J & I_L \end{bmatrix} = \begin{bmatrix} I_T & W \\ 0 & W_v \end{bmatrix}$$

The  $L \times L$  matrix  $W_v$  is associated with the Lagrange multipliers.

### 17.2.2 The type argument

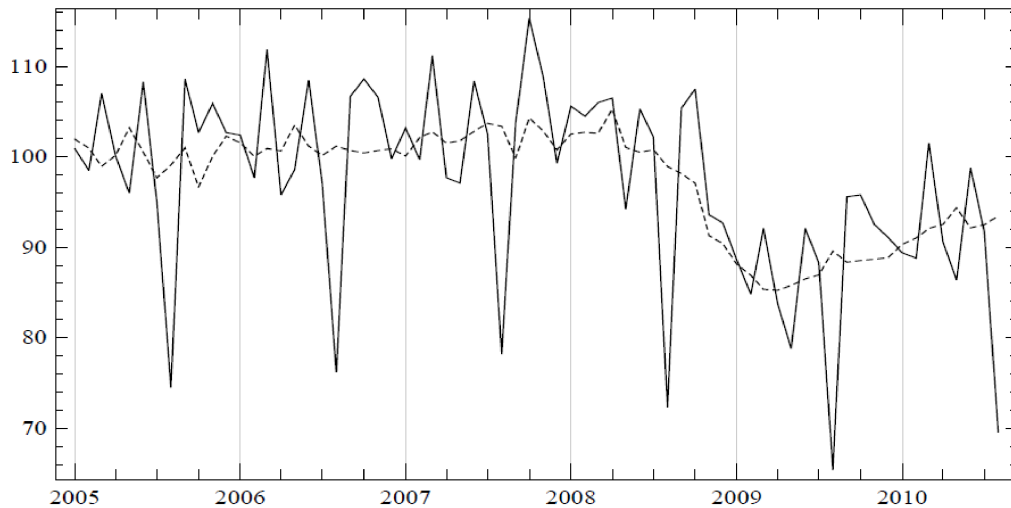
The option *type=regress* implements the computations described by Equations 17.3 and 17.4. The option *type=denton* uses the method implemented in X-11-ARIMA (Dagum (1981), Dagum (1988)). It corresponds to using *type=regress* with arguments *rho* = 1, *lambda* = 0 but applied in a 5-year moving interval fashion. Full computational details are provided in Ladiray and Quenneville (2001).

## 17.3 Example

The monthly index of industrial production in France — ID 001562714 — from January 1990 to August 2010 is used for illustration purpose. The raw data and a SA version of it are provided in Figure 17.1. The seasonally adjusted series and the benchmarked SA series using the regression method with *rho* = 0.9, *lambda* = 0 and *fcst* = *yes* are shown in Figure 17.2. It can be observed that there are some years where the annual levels are slightly different. When required, the benchmarked SA series has the added properties of having the

same annual averages as the raw. The benchmarked SA series with  $\rho = 0.9$ ,  $\lambda = 1$  and  $fcst = yes$  is graphically undistinguishable from that obtained with  $\lambda = 0$ .

**Figure 17.1:** Original (solid line) and seasonally adjusted (tiny dashed line) series, Monthly Index of the Industrial Production in France, January 2005 to August 2010



**Figure 17.2:** Seasonally adjusted (tiny dashed line) and benchmarked seasonally adjusted (solid line) series with  $\rho = 0.9$ ,  $\lambda = 0$  and  $fcst = yes$

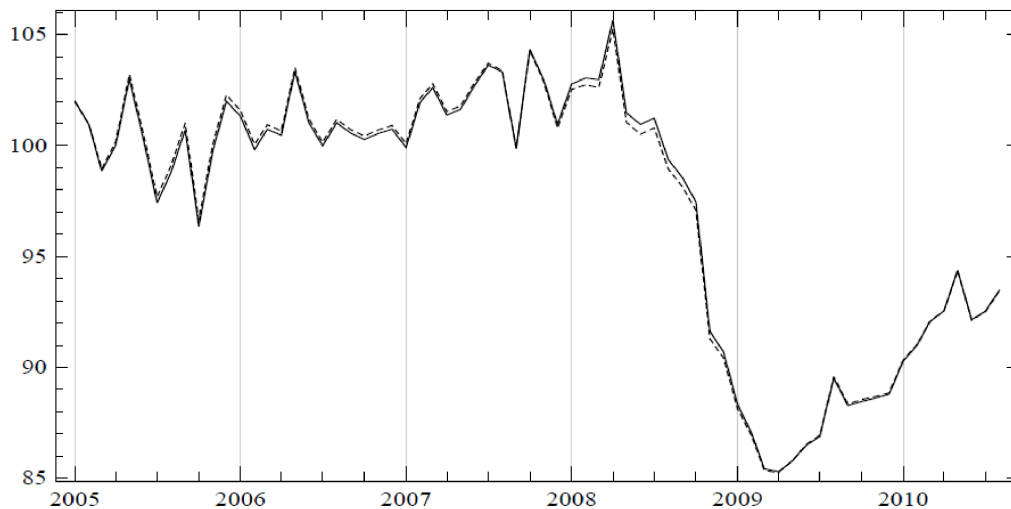
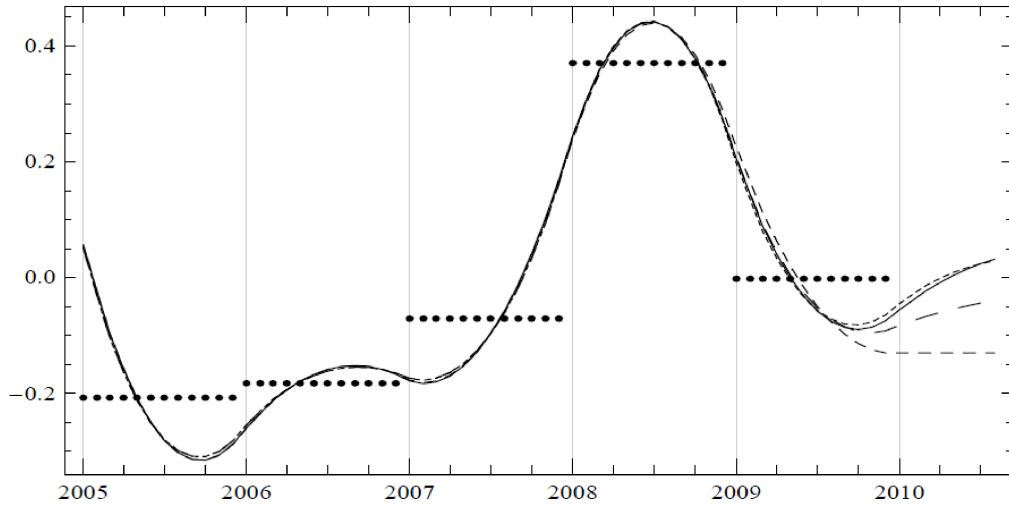


Figure 17.3 shows how the average annual additive discrepancies have been distributed over the months using  $\rho = 0.9$  or  $1.0$  with and without using the  $fcst$  option. Denton's corrections obtained with  $\rho = 1$ ,  $\lambda = 0$  and  $fcst = no$  are constant for January to August 2010. Using  $fcst = yes$ , produce 2010 corrections that average to the forecasted discrepancy. Using either  $\rho = 0.9$  or  $\rho = 1.0$  with  $fcst = yes$  produce similar adjustment values, and hence, it will be much faster to use  $\rho = 0.9$  in production. This is because computing  $W$  from Equation 17.4 requires the inversion of a matrix of dimensions  $(T + L) \times (T + L)$  whereas  $V_d$  in Equation 17.3 is of dimensions  $L \times L$ . Finally, it is only for January 2010 to August 2010 that we see differences between additive corrections for the various options. Figure 17.3 also confirms that for this example, the regression method with  $\rho = 0.9$  without forecasts may well approximate Denton ( $\rho = 1$ ) with forecasts. Figure 17.4 shows the proportional corrections obtained with  $\lambda = 1$ . The same conclusions hold; hence, for this series, and for all practical purposes,  $type=regress$ , with its default's X-12-ARIMA options  $\rho = 0.9$ ,  $\lambda = 0$  and  $fcst = yes$  can be used to smooth out the average annual discrepancies between the annual total in the raw and the SA series to obtain the benchmarked SA series.

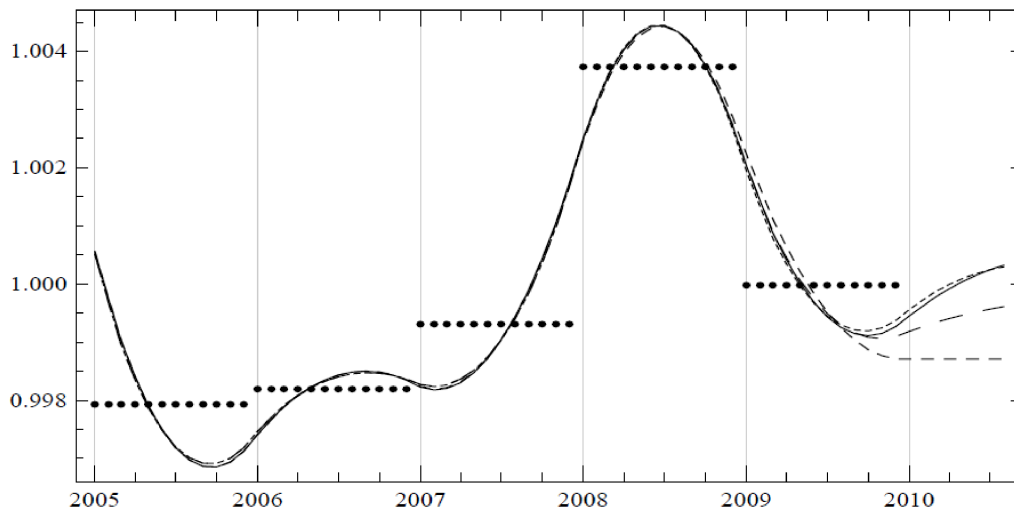


**Figure 17.3:** Additive corrections for various methods. Solid line:  $\rho = 1, \lambda = 0, fcst = yes$ ; tiny dashed:  $\rho = 0.9, \lambda = 0, fcst = yes$ ; small dashed:  $\rho = 1, \lambda = 0, fcst = no$ ; medium dashed:  $\rho = 0.9, \lambda = 0, fcst = no$



It will speed up the computations and likely produce the smallest revisions when the December value will be published.

**Figure 17.4:** Proportional corrections for various methods. Solid line:  $\rho = 1, \lambda = 1, fcst = yes$ ; tiny dashed:  $\rho = 0.9, \lambda = 1, fcst = yes$ ; small dashed:  $\rho = 1, \lambda = 1, fcst = no$ ; medium dashed:  $\rho = 0.9, \lambda = 1, fcst = no$





## 17.4 Alternative Benchmarking Software

Ferland et al. (2014) show that it is better to use the option  $fcst = yes$  when benchmarking the seasonally adjusted series to the annual totals. The previous example shows that with the option  $fcst = yes$ , either  $type = denton$  or  $type = regress$  can be used since they yield similar adjustment factors. With the option  $fcst = no$ , it is better to use  $type = regress$ . If an external benchmarking software is used, it should have a method based on movement preservation principle. Moreover, the selected method should produce benchmarking adjustment factors that converge to the expected discrepancy.

## 17.5 Model-Based Methods

Consider the case where a model-based seasonal adjustment procedure such as TRAMO-SEATS (Gómez and Maravall (1996)) or STAMP (Koopman et al. (2000)) has been used to compute the seasonally adjusted series,  $s$ , and its error covariance matrix,  $V_e$ . Suppose now that  $s$  is viewed as a preliminary estimate of the benchmarked seasonally adjusted series  $\theta$ , and that the next step consists in computing the estimates that satisfy the benchmarking constraints 17.2. This translates into the following equations:

$$s = \theta + e; E(e) = 0, Cov(e) = V_e, \quad (17.5)$$

$$a = J\theta \quad (17.6)$$

It follows from the two-stage benchmarking approach in Durbin and Quenneville (1997) and Hillmer and Trabelsi (1987) that  $\hat{\theta}$  from Equation 17.3 is the optimal estimator of the benchmarked seasonally adjusted series with error covariance matrix

$$V_{(\hat{\theta}-\theta)} = V_e - V_e J' (J V_e J')^{-1} J V_e. \quad (17.7)$$

Benchmarking with optimal estimation of  $\theta$  using the information on the measurement errors instead of using a movement preservation criteria is also discussed in Quenneville and Fortier (2011). Let  $C$  be the diagonal matrix with the standard deviation of the measurement errors of the series  $s_t$  and  $\Omega_e$  the correlation matrix of the measurement errors so that  $V_e = C\Omega_e C$  becomes the covariance matrix of the measurement error vector. Furthermore, assume the benchmarks are subjected to measurement errors uncorrelated with those in  $s_t$ , and let  $V_d = J V_e J' + V_\epsilon$  where  $V_\epsilon$  is the covariance matrix of the measurement errors in the benchmarks.  $\hat{\theta}$  is then the optimal estimator of  $\theta$ . It is optimal in the sense that  $E(\hat{\theta} - \theta)^2$  is minimized. When  $V_\epsilon > 0$ , the benchmarks are not satisfied since  $\hat{a} = J\hat{\theta} \neq a$ ; however, the estimator  $\tilde{\theta} = s + V_e J' (J V_e J')^{-1} (a - J s)$  will be such that  $J\tilde{\theta} = a$ . For the calculation of the benchmarking prediction error covariance matrices, the measurement error model  $s = \theta + e$  and  $a = J\theta + \epsilon$  with  $e$  and  $\epsilon$  uncorrelated will hold. Hence  $\hat{\theta} - \theta = s - \theta + V_e J' V_d^{-1} (a - J s) = e + V_e J' V_d^{-1} (\epsilon - J e)$  permits deriving the benchmarking prediction error covariance matrix of  $\hat{\theta}$  easily since  $e$  and  $\epsilon$  are uncorrelated. If  $V_\epsilon = 0$ , the error covariance matrix is provided by (17.7). The above formulation also permits to use the standard deviations of the estimation errors produced by the model-based seasonal adjustment method to define the matrix  $C$  and to either recompute  $\Omega_e$  or use a working-error model to approximate it.

## 17.6 Conclusions

In this chapter, the available solutions in X-12-ARIMA to modify a seasonally adjusted series so that its yearly totals match those derived from the raw series have been described and illustrated with an example. The use of an alternative benchmarking software was also discussed. Finally, possible extensions of benchmarking model-based seasonally adjusted series were presented.

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# 18

## Seasonal Adjustment of European Aggregates: Direct vs. Indirect Approach



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## Table of Contents

18.1	Introduction	467
18.2	The direct versus indirect Problem	468
18.2.1	Direct, Indirect and Mixed Indirect Seasonal Adjustments	468
18.2.2	Could Direct and Indirect Approaches Coincide ?	469
18.2.3	A Priori Advantages and Drawbacks	470
18.3	Methodology	471
18.3.1	Various Causes of Revisions	471
18.3.2	Quality Measures	472
18.3.3	Software, Methods and Parameters	474
18.4	Gross Domestic Product, Geographical Aggregation	474
18.4.1	The Data	474
18.4.2	A first comparison between seasonally adjusted series	474
18.4.3	Concordance analysis of growth rates	479
18.4.4	Quality measures of seasonal adjustments	480
18.4.5	Roughness measures	480
18.4.6	Revision analysis	484
18.4.7	Analysis of the residuals	484
18.5	Gross Value Added, Aggregation by Sector	485
18.5.1	The Data	485
18.5.2	A first comparison between seasonally adjusted series	486
18.5.3	Concordance analysis of growth rates	489
18.5.4	Quality measures of seasonal adjustments	490
18.5.5	Roughness measures	490
18.5.6	Revision analysis	490
18.5.7	Analysis of the residuals	490
18.6	Conclusions	495



## 18.1 Introduction

Most of the European and euro area economic short term indicators are computed either through "horizontal" aggregation, e.g. by country, or through "vertical" aggregation, e.g. by sector, branch or product<sup>1</sup>. To obtain seasonally adjusted figures at the European level, three main strategies can be used:

- The **direct approach**: the European indicator is first computed by aggregation of the raw data and then seasonally adjusted;
- The **indirect approach**: the raw data (for example the data by country) are first seasonally adjusted, all of them **with the same method and software**, and the European seasonally adjusted series is then derived as the aggregation of the seasonally adjusted national series;
- The **"mixed" indirect approach**: each Member State seasonally adjusts its series, **with its own method and strategy**, and the European seasonally adjusted series is then derived as the aggregation of the adjusted national series.

Unfortunately, these strategies could produce quite different results. In an important paper, Lovell (1963) demonstrated that there is no perfect solution as:

*"... there exists no non-trivial technique for seasonal adjustment that preserves both sums and products, an adjustment procedure that preserves the definitional relationship between employment, unemployment, and the size of the labor force, cannot be expected to yield an adjusted unemployment rate equal to the ratio of adjusted unemployment to adjusted labor force."*

As a consequence, you will also never have a perfect coherence between seasonally adjusted volumes, values and prices. The choice between the direct and indirect approaches has been the subject of articles and discussions for decades and there is still no consensus on the best method to use.

For example, Geweke (1978) shows that, under a minimum mean square error (MMSE) criterion, within the class of linear methods, and assuming the joint distribution of the components is known, indirect adjustment is preferable. In practice, seasonal adjustment is often based on a non-linear procedure: outliers can be different in the aggregate and in the sub-components. Ghysels (1997) and Ghysels and Osborn (2001) relax the assumption of complete knowledge of the full distribution and show that direct adjustment must be advisable. Maravall (2006) advocates for the use of direct adjustment<sup>2</sup>. On the opposite, some agreement appears in the literature on the fact that the decision could be made case by case following some empirical rules and criteria (Dagum (1979), European Central Bank (3), Lothian and Morry (1977), Pfefferman et al. (1984), Planas and Campolongo (2000), Scott and Zadrozny (1999), Scott (1984) etc.). The "mixed" indirect approach is often used and, as it cannot be derived from a simple adjustment, is rarely compared to the others.

The choice between the methods cannot be based on accuracy and statistical considerations only. To publish timely estimates, Eurostat must often work with an incomplete set of national data. The indirect approaches imply estimation of missing raw and seasonally adjusted data and therefore different models have to be estimated, checked and updated. The direct approach is obviously easier to implement and should be preferred except if there is a strong evidence an indirect approach is better or a strong requirement for an indirect approach, as stated in Eurostat (2015)

Section 18.2 is devoted to a detailed presentation of the direct versus indirect problem, its implications and some non statistical guidelines for the choice of a strategy. Section 18.3 presents the methodology used in the

<sup>1</sup> If the chapter focuses on European indicators, most of the content and results can be applied to National statistics. For example, the geographical level might be the region, the province or simply not relevant and many National economic indicator are constructed through aggregation by sector, branch or product.

<sup>2</sup> Maravall (2006): "The dilemma direct versus indirect adjustment seems to have a clear answer. At any level of aggregation, direct adjustment should be used."



applications and the various quality measures that can be used to make the right choice. These applications focus on the Quarterly National Accounts: the geographical aggregation problem is studied in section 18.4 and the sectoral aggregation problem in section 18.5. TRAMO-SEATS and X-12-ARIMA were used for the decompositions; the same quality measures have been computed for both methods and the mixed indirect approach has been compared to the other approaches.

## 18.2 The direct versus indirect Problem

Nowadays it is common to decompose an observed time series  $X_t$  into several components, themselves unobserved, according to for example an additive model:

$$X_t = TC_t + S_t + TD_t + MH_t + I_t \quad (18.1)$$

where  $TC_t$ ,  $S_t$ ,  $TD_t$ ,  $MH_t$  and  $I_t$  designate, respectively, the **trend-cycle**, the **seasonality**, the **trading-day effect**, the **moving holiday effect** (Easter for example) and the **irregular** components.

The seasonality  $S_t$  and the calendar component ( $TD_t + MH_t$ ) are removed from the observed time series to obtain the seasonally adjusted series  $A_t = TC_t + I_t$ .

We will suppose from now on that  $X_t$  is an European indicator computed by linear aggregation of  $N$  national indicators ( $N = 28$  for the European Union or  $N = 19$  for the euro area); in this aggregation, each Member State  $n$  has a weight  $\omega_n$ . Therefore we have:

$$X_{n,t} = TC_{n,t} + S_{n,t} + TD_{n,t} + MH_{n,t} + I_{n,t} \quad (18.2)$$

and:

$$X_t = \sum_{n=1}^N \omega_n X_{n,t} \quad (18.3)$$

Note that the weights can be positive and sum up to 1, as in the IPI case, or can be all equal to 1, as in the GDP case.

### 18.2.1 Direct, Indirect and Mixed Indirect Seasonal Adjustments

The seasonally adjusted series  $A_t$  of the European aggregate  $X_t$  can be derived from at least three different strategies:

- The **direct approach** consists in adjusting directly the aggregate. The direct seasonally adjusted series is noted  $A_t^D$ ;
- In the **indirect approach**, all the national indicators  $X_{n,t}$  are seasonally adjusted, **with the same method and software**, and the European seasonally adjusted series is then derived as the aggregation of the seasonally adjusted national series. Thus we have:

$$A_t^I = \sum_{n=1}^N \omega_n A_{n,t} \quad (18.4)$$

- In the **"mixed" indirect approach**: each Member State seasonally adjusts its series, **with its own method and strategy**, and the European seasonally adjusted series is then derived as the aggregation of the adjusted national series. The mixed indirect seasonally adjusted series is noted  $A_t^S$ .

The multivariate approach, which permits to derive simultaneously the seasonally adjusted series for the aggregate and the components, must be mentioned at this stage as an alternative. This method has been proposed for many years (Geweke (1978)) but given its computational complexity, the limitations of existing software and its lack of optimality with respect to revision errors, it is rarely used in practice and the univariate approaches are generally preferred.

The mixed indirect approach is a quite popular strategy but, as it does not result from a simple seasonal adjustment process, it is scarcely studied by itself. The national seasonal adjustment policies can substantially differ:

- The methods are often different: some countries use a model based approach (TRAMO-SEATS, STAMP) or a non parametric approach (X-11 family);
- The software, or the release, that implements the method can itself differ: X-11-ARIMA, X-12-ARIMA and X13-ARIMA-SEATS are currently in use in European countries, sometimes in the same institute;
- The revision policies can vary and Member States may use current, controlled current, partial concurrent or concurrent seasonal adjustments;
- Member States can perform themselves direct or indirect seasonal adjustments;
- The strategy for the correction of calendar effects is usually not the same, the seasonal adjustments are not performed on the same span of time, the treatment of outliers differ etc.

All these differences show how it is difficult to compare, from the theoretical point of view, the mixed indirect seasonal adjustment to the direct and indirect ones. Furthermore, these three strategies are not the only possible ones and we can imagine for example a mixed approach: a subset of the basic series can be first aggregated in one new component, this component and the remaining sub-series can then be adjusted and the adjusted aggregate derived by implication. In fact this procedure is frequently used since many of what are considered the basic components are themselves aggregates of other components, although the latter are not always observed separately (Pfefferman et al. (1984)).

### 18.2.2 Could Direct and Indirect Approaches Coincide ?

As far as the aggregate is a linear combination of the components and the seasonal adjustment is a non-linear process, the answer is generally no, except under some very restrictive conditions (see for example Pfefferman et al. (1984)). Thus, if the aggregate is an algebraic sum (GDP for example), if the decomposition model is purely additive (equation 18.1), if there is no outlier in the series and if the global filter used in the seasonal adjustment process is the same for all series, the two approaches are equivalent. If the decomposition model is multiplicative, it could be shown that the equivalence of the two approaches requires for instance that there is no irregular, that the sub-series have identical seasonality patterns (or that the sub-series have identical or proportional trend-cycles) and that the filter used is the same for all sub-series. On the other hand, if the aggregate is a rate, any seasonal adjustment method that produces unbiased estimates will give different results (Dagum (1979)).

Clearly the reality is much more complex:

- For the majority of economic series the additive model is not the most appropriate and the series are adjusted using the multiplicative option. Such series may be converted to an additive model by a logarithmic transformation, as in TRAMO-SEATS, but this will not ensure the equivalence since the logarithm of a sum is not the sum of the logarithms.
- Outliers are frequently present in economic time series as the result of structural changes, anomalous conditions, external shocks etc.

- For model-based approaches, such as TRAMO-SEATS, the filter used for the seasonal adjustment is optimally derived from the characteristics of the series. It means that a different filter is associated to each series. In that case, direct and indirect adjustments would never coincide.

On the other hand, it may occur that sub-series are not very noisy, show a very similar trend-cycle or seasonal pattern and are affected more or less by the same external shocks. In these conditions, the direct and indirect approaches could not be too different. These considerations advocate for measures of the differences between the various adjusted series; such indicators will be presented in section 18.3.2.

### 18.2.3 A Priori Advantages and Drawbacks

The choice between direct, indirect and mixed indirect adjustment can be guided by some statistical or non statistical considerations or by some *a priori* desirable properties.

#### 1. The additivity constraint and the indirect approaches

Sometimes the additivity constraint plays an important role in some domains (quarterly national accounts, balance of payments) or has to be assured as the consequence of a legal act (external trade). Then the indirect or the mixed indirect approaches appear to be the relevant ones. Furthermore, the mixed indirect approach seems to imply there is no discrepancy between national figures and European data. Nevertheless, some points must be precised:

- Even with a direct approach it is always possible to assure ex-post the additivity constraint by distributing the discrepancies. Various univariate or multivariate statistical techniques exist to compute these adjustment factors.
- The additivity constraint is verified on the levels of the series. But users are more generally concerned with growth rates. Of course, as we have:

$$\frac{A_{t+1}^I - A_t^I}{A_t^I} = \frac{1}{\sum_{n=1}^N \omega_n A_{n,t}} \sum_{n=1}^N \left( \omega_n A_{n,t} \frac{A_{n,t+1} - A_{n,t}}{A_{n,t}} \right)$$

the indirect growth rate is a weighted average of the sub-series growth rates, with weights that sum up to 1. Therefore, the indirect growth rate is always between the smaller and the larger sub-series growth rate and in that sense, the indirect approach is consistent. But you can have a majority of sub-series increasing while the global growth rate decreases: see some examples in Sections 18.4 and 18.5.

- The additivity constraint does not hold anymore on chain-linked indicators.
  - The additivity constraint has nothing to do with either the time consistency problem, requiring for example that quarterly and annual figures have to be coherent, or the consistency between raw and adjusted data annual totals.
2. **Some "statistical" considerations** Most of the statisticians involved in seasonal adjustment agree on one point: if the sub-components do not have similar characteristics or if the relative importance of the sub-series (in terms of weight) is changing very fast, indirect adjustment should be preferred. From the opposite point of view (Dagum (1979)), if the sub-series have a similar seasonal and more or less the same timing in their peaks and troughs, the direct approach have to be used. The aggregation will produce a smoother series with no loss of information on the seasonal pattern.

On one hand, as some studies show a synchronization of the cycles in the European Union (see for example Blake et al. (2000)), the direct approach should be preferred. On the other hand, some countries have strong specificities and this point is of great importance with respect to the enlargement of the Union.

A combined two-step approach must therefore be seriously studied, a first direct approach for groups of similar series and then an indirect approach for the estimation of the final seasonally adjusted aggregate. For example, Ladiray et al. (2015) use a hierarchical cluster analysis on the seasonal components to determine the optimal number of groups of similar seasonalities. Finally, one must note that following this idea of similarity of the sub-series, the direct approach should be more adapted to "horizontal" aggregation, e.g. by country, and the indirect approach to "vertical" aggregation, e.g. by sector, branch or product.

The mixed indirect approach poses serious methodological problems for further statistical analysis of the aggregate. As Member States use in general different methods, they also implicitly use different definitions of the trend-cycle and the seasonality. As seasonal adjusted series are unfortunately often used in econometric modeling, this could generate artifacts and spurious relationships which could spoil the quality of the estimations and mislead the interpretation of the results. Of course, these undesirable effects are more evident for the end points of the series on which asymmetric filters are used. This can have negative implications for the construction of flash estimates, nowcasting and coincident or leading indicators.

### 3. Production consequences

Eurostat calculates European and euro area aggregates from national data. As national indicators are not produced at the same time by all Member States, Eurostat has to impute missing information using some modeling of the concerned series, in order to publish timely figures. As an example, Eurostat must publish the European IPI 45 days after the end of the month but at that date not all national results are usually available. A truly mixed indirect approach is impossible regarding the delays; an indirect approach implies to estimate and seasonally adjust missing national IPI; a direct approach requires much less work as it implies only to estimate and seasonally adjust the aggregate.

Once more the usual trade-off between accuracy and timeliness has to be taken into account in the choice of the "optimal" method.

## 18.3 Methodology

To assess empirically the quality of the different approaches, some problems have to be solved. The first one is that the effect of the aggregation strategy must be isolated from the other numerous sources of variation. The second problem resides in the definition of quality indicators that should be computed for the different approaches. And the last one is a computational problem as the two main methods currently used, TRAMO-SEATS and X-12-ARIMA, do not provide the user with a common set of quality statistics.

### 18.3.1 Various Causes of Revisions

Seasonally adjusted figures are usually subject to revisions that can be the consequences of numerous causes. For example, at the European level, we can underline:

- The imputation of national data. When more national data become available, the preliminary estimates calculated by Eurostat must be updated and the raw and adjusted series have to be revised.
- The usual sources of revisions due to the seasonal adjustment process: adjustment policy (current or concurrent adjustment), modification of the adjustment parameters, treatment of outliers, estimation of calendar effects, impact of new data etc.
- The revisions due to the seasonal adjustment process at the national level.

In order to study the direct versus indirect problem, it would be preferable to isolate the variations only due to this problem and therefore to work with quite stable time series and under stable assumptions. For instance, the series could be first cleaned from any calendar effect or outlier, the decomposition model could be fixed etc. Unfortunately, it is quite difficult to measure the relative importance of each source of revisions.

### 18.3.2 Quality Measures

There is no real consensus on the measures to assess the quality of a seasonal adjustment, and that explains the large number of criteria one can find in the literature. Several aspects of the seasonal adjustment can be addressed and, for each of them, some criteria are defined.

#### 1. How different the various approaches really are ?

The results of the three approaches are compared to see how important the direct vs indirect problem is.

We compute, for example for the direct and indirect seasonally adjusted series, the two following statistics:

- Mean Absolute Percentage Deviation:  $\frac{100}{N} \sum_{n=1}^N \left| \frac{A_t^D - A_t^I}{A_t^I} \right|$
- Max Absolute Percentage Deviation:  $100 * \text{Max} \left| \frac{A_t^D - A_t^I}{A_t^I} \right|$

These statistics can be calculated: for each couple of possible approach (Direct, Indirect), (Direct, Mixed indirect) and (Indirect, Mixed indirect); for the seasonally adjusted series, the trend-cycle estimates and the seasonal components; for the two softwares TRAMO-SEATS and X-12-ARIMA; on the complete series and on the last three years.

Users pay a lot of attention to the growth rate of the seasonally adjusted series. The mean and the range of the series of the growth rate differences must therefore be computed and checked.

#### 2. Inconsistencies

Moreover, the various seasonally adjusted series should deliver more or less the same message and their growth rates should have the same sign. To measure the degree of consistency in growth rates, two kinds of statistics are computed:

- The first measures the global percentage of concordance between the direct and indirect series;
- The second measures the percentage of concordance between the seasonally adjusted series and the national adjusted series. An inconsistency in the growth rates is detected when the aggregate does not evolve as the majority, in terms of weight, of adjusted sub-series.

#### 3. Quality of the seasonal adjustment

X-12-ARIMA proposes a set of M and Q-statistics to assess the quality of the seasonal adjustment<sup>3</sup>. These statistics have been adapted when possible to the TRAMO-SEATS estimates<sup>4</sup>. Approximated components linked to the mixed seasonally adjusted series have been computed in order to calculate these statistics.

#### 4. Roughness of the components

Dagum (1979) proposed two measures of roughness of the seasonally adjusted aggregates<sup>5</sup>. The first one is the  $L_2$ -norm of the differenced series:  $R1 = \sum_{t=2}^T (A_t - A_{t1})^2 = \sum_{t=2}^T (\nabla A_t)^2$ . The

<sup>3</sup>For a precise definition and the interpretation of these statistics, one can refer to (Ladiray et al. (2015)).

<sup>4</sup>For example, the M6 statistic cannot be computed as it refers specifically to the use of a  $3 \times 5$  moving average in the seasonal adjustment procedure.

<sup>5</sup>In the following definitions, B is the lag operator defined by  $BX_t = X_{t-1}$  and  $\nabla = I - B$  is the first difference operator.

second one is based on the 13-term Henderson filter: the adjusted series is smoothed with the Henderson filter and  $R2$  is defined as the  $L_2$ -norm of the residuals:  $R2 = \sum_{t=1}^T (A_t - H_{13}A_t)^2 = \sum_{t=1}^T [(I - H_{13})A_t]^2$ .

The rationale of these measures of roughness is that the involved filters (the first difference operator and  $I - H_{13}$ ) are high-pass filters that remove most of the low frequencies components that correspond to the trend-cycle variations. In other words, these statistics measure the size of the deviations to a smooth trend, e.g. the size of an "irregular component". This is why Pfefferman et al. (1984) suggested a "natural" third measure, a measure of similarity between seasonally adjusted data and trend:  $R3 = \sum_{t=1}^T (A_t - TC_t)^2$ .

Indeed, there is no fundamental reason why a seasonally adjusted series should be smooth as the irregular component, a characteristic of the series, is a part of the seasonally adjusted series. Gómez (2000) prefer to focus the quality measures on the other components, the trend-cycle and the seasonality. For the seasonality, they use the criteria  $Mar(S) = \sum_{t=1}^T [(1 + B + \dots + B^{11}) A_t]^2$  where  $1 + B + \dots + B^{11}$  is the annual aggregation operator. The smoothness of the trend-cycle is measured by the  $L_2$ -norm of the first and the second differences:  $Mar1(TC) = \sum_{t=1}^T (\nabla A_t)^2$  and  $Mar2(TC) = \sum_{t=1}^T (\nabla^2 A_t)^2$

## 5. Idempotency

A seasonal (and trading-day and holiday) adjustment that leaves detectable residual seasonal and calendar effects in the adjusted series is usually regarded as unsatisfactory. X-12-ARIMA and TRAMO-SEATS are used on the three seasonally adjusted series and the usual tests proposed by these softwares are used to check the idempotency.

## 6. Stability of the seasonally adjusted series

Even if no residual effects are detected, the adjustment will be unsatisfactory if the adjusted values undergo large revisions when they are recalculated as future time series values become available. Frequent and substantial revisions cause data users to lose confidence in the usefulness of adjusted data. Such instabilities can be the unavoidable result of the presence of highly variable seasonal or trend movements in the series being adjusted. But, in any case, they have to be measured and checked. X-12-ARIMA includes two types of stability diagnostics: sliding spans and revision histories (see Findley et al. (1998), U.S. Census Bureau (23)). Some of these diagnostics are used here:

- The mean and standard deviation of the absolute revisions after  $k$  periods;
- The two most important sliding spans  $A(\%)$ , percentage of dates with unstable adjustments, and  $MM(\%)$ , percentage of dates with unstable month-to-month percent changes.

Unfortunately, it is quite difficult to compute these statistics for the mixed indirect approach.

## 7. Impact on the business cycle estimation

Seasonally adjusted series are often used for business cycle analysis and to date the turning points. Cycles are extracted from the three adjusted series with a Baxter-King filter and compared with respect to the timing of the turning points and their amplitude.

## 8. Characteristics of the irregular component

The irregular component should not present any structure or residual seasonality. The irregulars derived from the various approaches are analyzed both with the Tramo automatic modeling module and the X-12-ARIMA software. The usual tests proposed by these softwares are used to check the randomness of the irregular components.

### 18.3.3 Software, Methods and Parameters

The two methods TRAMO-SEATS and X-12-ARIMA<sup>6</sup> have been used in the applications. They are implemented in the seasonal adjustment software JDemetra+, whose version 2.1.0 was used for the computations. A dedicated SAS program manages the outputs of the two methods and calculates all the quality statistics for the direct and indirect approach and most of them for the mixed indirect approach. Some specific features have been implemented in order to simulate different adjustment policies. For example, the series can be cleaned from calendar effects or outliers before any adjustment is performed. The decomposition model can be, or not, fixed by the user etc.

Nevertheless, in the applications, we use a default strategy for seasonal adjustment:

- The trading-day and Easter effects are tested and corrected only if they are significant.
- The decomposition model is fixed and common to each sub-series.
- In the revision and sliding span analysis, the ARIMA model and the decomposition model are fixed and equal to the ones found for the complete series.

## 18.4 Gross Domestic Product, Geographical Aggregation

### 18.4.1 The Data

We use here the Quarterly Gross Domestic Product in volume for the whole economy of the following countries: Belgium, Germany, Spain, Finland, France, Italy and Netherlands. The sample used ranges from 1996Q1 to 2015Q3, for a total of 79 observations. All these countries publish both raw and seasonally adjusted figures. The “benchmark” series (the so-called mixed series) is therefore available on the same period.

### 18.4.2 A first comparison between seasonally adjusted series

As it has already been mentioned, Member States use different seasonal adjustment techniques to produce Quarterly National Account figures. Moreover, seasonal adjustment policies concerning working day corrections, revisions and so on, are also quite disparate. Table 18.1 illustrates the variety of the seasonal adjustment procedures used by euro area Member States for Quarterly National Accounts<sup>7</sup>.

According to this document, there is no consensus neither on the software nor on working correction policy: 5 Member States are currently using X11, with or without Arima modeling (Belgium and Finland in our selection), 6 are using X-12-ARIMA (Germany, France and Netherlands in our selection) and 14 TRAMO-SEATS (Spain and Italy in our selection); 16 Member States provide trading-day seasonally adjusted figures (7 in our selection).

Figure 18.1 presents the mixed adjustment and the TRAMO-SEATS direct and indirect seasonally adjusted series. It is very difficult to detect a real difference between the three series (and it will be the same with the X-12-ARIMA estimates). This similarity is confirmed by the numerical indicators displayed in Table 18.2. The mean absolute percentage difference (Mean APD) between the two estimates is quite small: much less than

<sup>6</sup>These methods are recommended in the ESS guidelines for seasonal adjustment (Eurostat (2015)).

<sup>7</sup>This list, probably outdated, can be found at the following link <http://ec.europa.eu/eurostat/documents/24987/4253464/Season-adj-work-day-correction.pdf/b35f34af-1e7a-4c5a-b51f-b0337ecdaa18>.

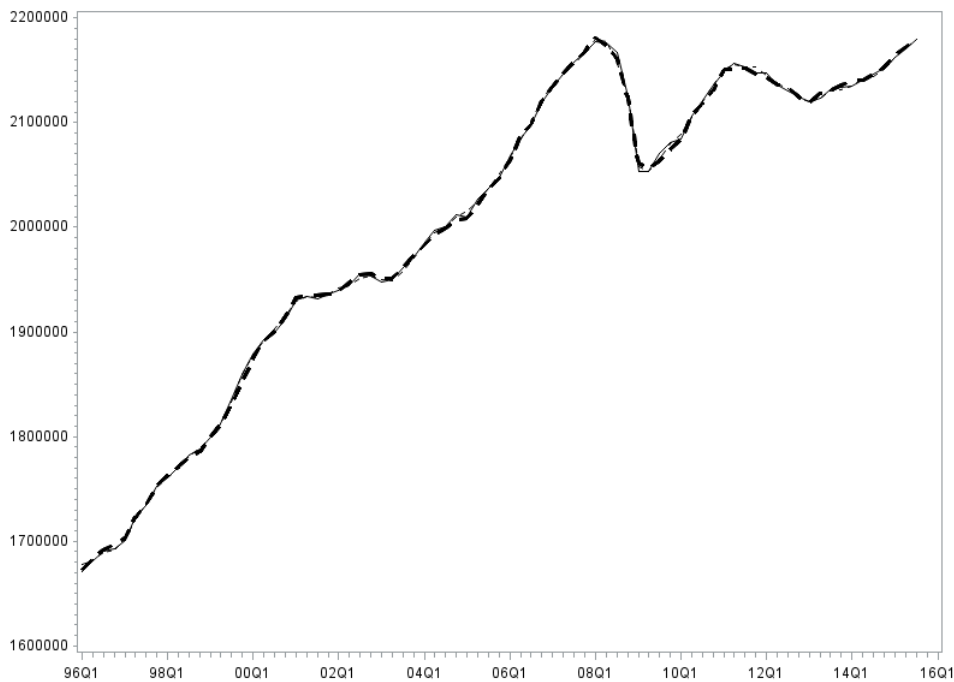


**Table 18.1:** Seasonal Adjustment methods and practices in European Union Member States for Quarterly National Accounts.

Country	Seasonal Adjustment Method	Trading/Working day Correction
European Union	Mixed	Yes
Austria	TRAMO-SEATS	Yes
Belgium	X-11	Yes
Bulgaria	No seasonal adjustment applied yet	
Cyprus	TRAMO-SEATS	No
Czech Republic	TRAMO-SEATS	Yes
Denmark	X-11	No
Estonia	X-12-ARIMA	No
Finland	TRAMO-SEATS	Yes
France	X-12-ARIMA	Yes
Germany	X-12-ARIMA and BV.4	Yes
Greece	X-12-ARIMA	No
Hungary	TRAMO-SEATS	Yes
Ireland	X-11	No
Italy	TRAMO-SEATS	Yes
Latvia	TRAMO-SEATS	No
Lithuania	TRAMO-SEATS	Yes
Luxembourg	X-12-ARIMA	No
Malta	TRAMO-SEATS	Yes
Netherlands	X-12-ARIMA	Yes
Poland	TRAMO-SEATS	Yes
Portugal	X-11	No
Romania	No seasonal adjustment applied yet	
Slovakia	TRAMO-SEATS	No
Slovenia	TRAMO-SEATS	Yes
Spain	TRAMO-SEATS	Yes
Sweden	TRAMO-SEATS	Yes
United Kingdom	X-11	Yes



**Figure 18.1:** Mixed (thick dashed line) and TRAMO-SEATS Direct (thin dashed line) and Indirect (solid line) Adjustments; GDP geographical aggregation.



0.1% between direct and indirect and, on this example, smaller for the X-12-ARIMA method. The difference between the mixed and the other estimates is also small, close to 0.12%.

The growth rates, which are in this case of greater interest, are displayed in figures 18.2 and 18.3. Finally, figures 18.4 and 18.5 show the relative difference of the mixed benchmark series with respect to the direct and indirect seasonal adjusted series obtained with TRAMO-SEATS and X-12-ARIMA.

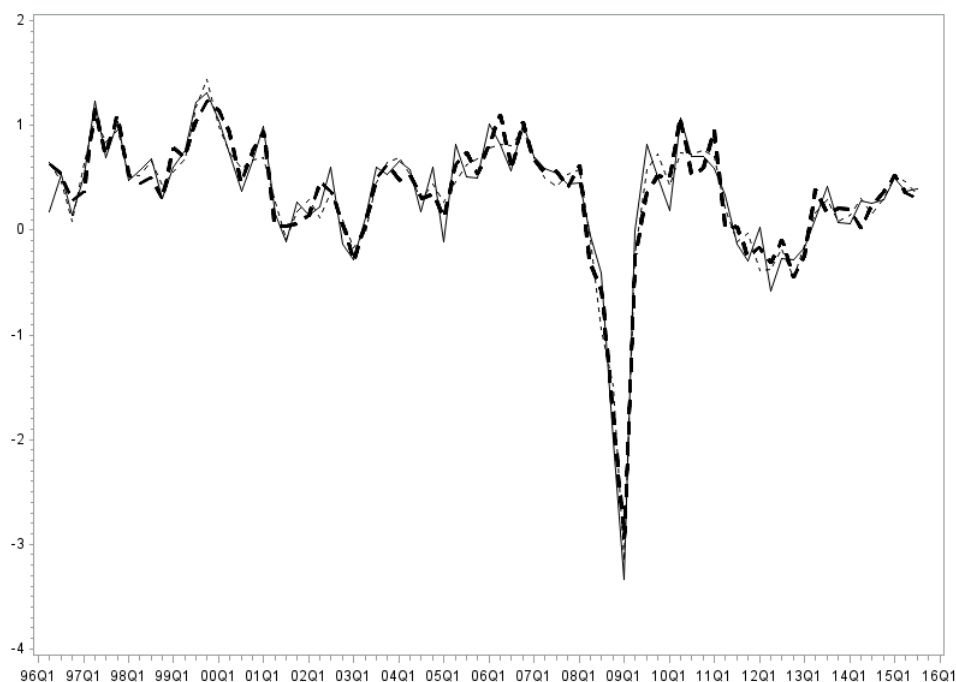
The key elements emerging from this set of pictures can be synthesized as follows:

- Direct and indirect adjusted series have a very similar behavior, regardless to the software used (X-12-ARIMA or TRAMO-SEATS);
- Largest differences are observed from 2010 onwards, after the 2008-2009 crisis;
- 2009Q1 appears to be a very special date where direct and indirect estimates show a much more

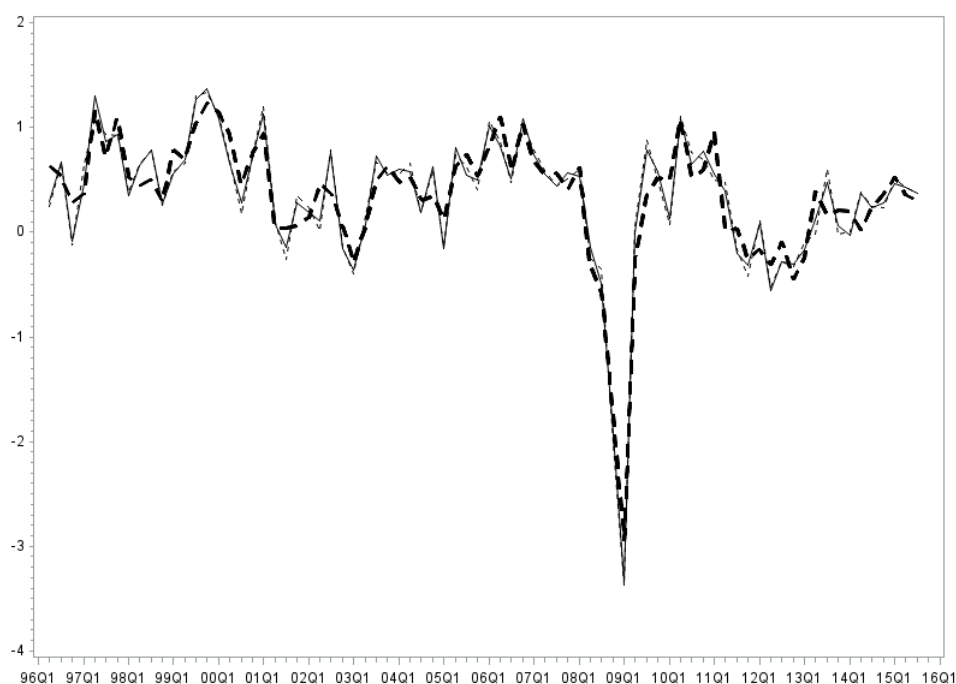
**Table 18.2:** Absolute Percentage Deviation Indicators (GDP geographical aggregation).

Indicator	Ind. vs Dir.		Best	Ind. vs Mixed		Dir. vs Mixed	
	T-S	X-12		T-S	X-12	T-S	X-12
Mean APD (SA)	0.089	0.032	X12ar	0.120	0.127	0.118	0.141
Max APD (SA)	0.487	0.136	X12ar	0.392	0.385	0.336	0.443
Mean APD (SA), Last 3 years	0.046	0.029	X12ar	0.083	0.093	0.098	0.119
Max APD (SA), Last 3 years	0.086	0.077	X12ar	0.250	0.310	0.237	0.328
Mean APD (TC)	0.072	0.047	X12ar	0.120	0.116	0.127	0.131
Max APD (TC)	0.650	1.566	Seats	1.367	0.767	1.323	0.811
Mean APD (TC), Last 3 years	0.036	0.034	X12ar	0.066	0.073	0.089	0.094
Max APD (TC), Last 3 years	0.087	0.080	X12ar	0.155	0.125	0.183	0.177

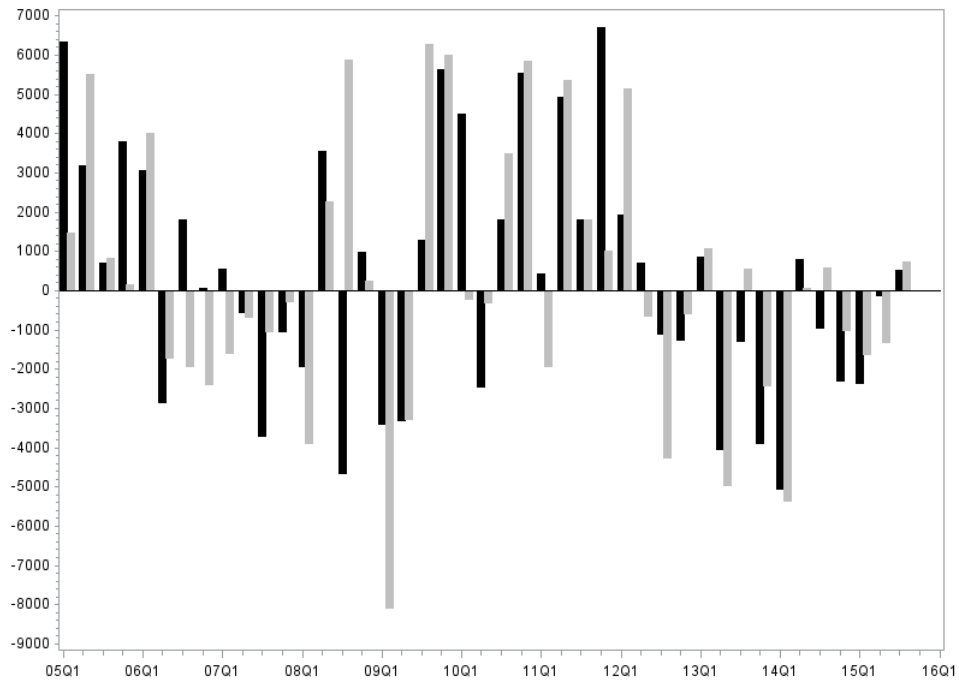
**Figure 18.2:** Growth Rates of Mixed (thick dashed line), TRAMO-SEATS Direct (thin dashed line) and Indirect (solid line) Adjustments; GDP geographical aggregation.



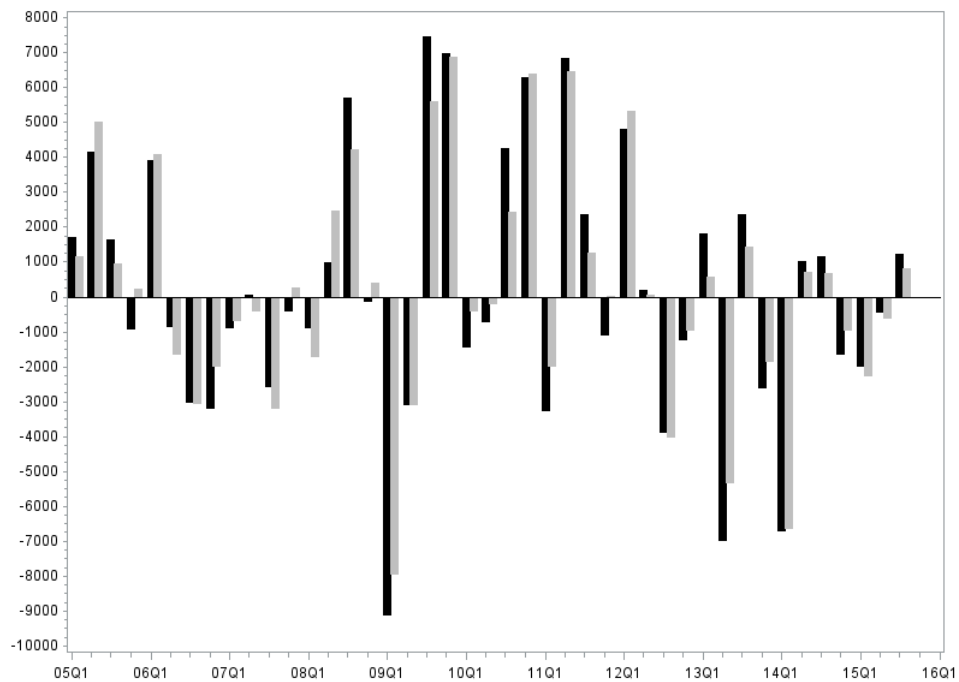
**Figure 18.3:** Growth Rates of Mixed (thick dashed line), X-12-ARIMA Direct (thin dashed line) and Indirect (solid line) Adjustments; GDP geographical aggregation.



**Figure 18.4:** Relative Differences of Mixed Aggregate versus Direct (in black) and Indirect (in grey) Adjustments, GDP geographical aggregation. TRAMO-SEATS.



**Figure 18.5:** Relative Differences of Mixed Aggregate versus Direct (in black) and Indirect (in grey) Adjustments, GDP geographical aggregation. X-12-ARIMA.



**Table 18.3:** Differences in growth rates between the three approaches; GDP geographical aggregation.

Indicator	Dir. vs Ind.		Mixed vs Ind.		Mixed vs Dir.	
	T-S	X-12	T-S	X-12	T-S	X-12
Mean	0.005	-0.000	0.004	0.003	-0.001	0.003
Minimum	-0.540	-0.215	-0.466	-0.424	-0.276	-0.514
Maximum	0.512	0.180	0.457	0.393	0.375	0.449
Variance	0.034	0.005	0.033	0.042	0.027	0.054
Range	1.053	0.396	0.923	0.817	0.651	0.964

important increase in the GDP than mixed approach.

- The X-12-ARIMA estimates are closer to the mixed estimate than those obtained with TRAMO-SEATS. This is certainly a direct consequence of the use of the Census filter in the large majority of Member States (five out of seven in our selection).

To complete this comparison, table 18.3 presents some descriptive statistics on the differences between growth rates of the various approaches. Differences close to zero with little variance indicate a good global agreement between the different estimates. This is the case as all average differences are very small. One must note that, as far as variances and ranges are concerned, the direct and indirect approaches are not very close to the mixed approach estimate and the range of the differences can reach 0.9%.

### 18.4.3 Concordance analysis of growth rates

Short-term analysts are mainly interested in the evolution of macroeconomic aggregates and therefore in growth rates of seasonally adjusted data. It is important that different seasonal adjusted data do not provide users with inconsistent messages.

Tables 18.4 and 18.5 detail the cases of discrepancies between the various aggregates and with the sub-components, according to the software.

Direct and indirect estimates only disagree, e.g. give an opposite evolution of the indicator, in four cases out of 78 observations for both X-12-ARIMA and TRAMO-SEATS. Some of these differences concern growth rates close to zero but others appear significant. In 2005Q1, TRAMO-SEATS estimates give very different messages, with an increase of 0.28% for the direct approach and a decrease of -0.11 for the indirect approach. This leads to a quite good concordance rate (94.9%) between the two approaches.

The discrepancies are more numerous when comparing with the mixed approach even if the global concordance rate remains good. 2005Q1 shows a quite strong inconsistency for the indirect approach and both software: the mixed series increases (0.123%) when indirect approaches decrease (-0.1% for TRAMO-SEATS and -0.154% for X-12-ARIMA). The other most important inconsistencies are observed for X-12-ARIMA in 1996Q4 for both approaches. TRAMO-SEATS gets better results in terms of consistency with the mixed series, especially with the direct approach.

The inconsistencies between the evolution of an aggregate and the majority of its components are presented, for the different approaches and softwares, in table 18.5.

TRAMO-SEATS presents a notable inconsistency in 2005Q1: 3 countries representing 57% of the euro area GDP where decreasing when the direct estimate was increasing (0.274%). X-12-ARIMA estimates also present some inconsistencies: 2 are recorded for the indirect approach and 6 for the direct one. At least, the mixed approach presents also 5 inconsistencies, including a notable one in 2014Q1 when 3 countries representing 57% of the euro area GDP where increasing when the direct estimate was increasing (0.196%).

Table 18.6 presents the concordance rates between the various approaches, a statistic that summarizes the previous elements. Direct and indirect approaches present concordance rates that are quite similar to the mixed approach ones. And the TRAMO-SEATS estimates are the more coherent with the national sub-series.

Unfortunately, some inconsistencies are observed in the last periods (since 2012) and the approaches do not always give consistent messages for the last years.

#### 18.4.4 Quality measures of seasonal adjustments

The various approaches were also seasonally adjusted using TRAMO-SEATS: it is expected that these seasonally adjusted series do not present any seasonal component. A comparison between the various adjustments can be also made with respect to the so-called quality measures proposed by X-12-ARIMA and here extended, where possible, to TRAMO-SEATS estimates and to the mixed approach. The results are presented in tables 18.7 and 18.8.

The seasonal adjustment of the mixed, direct and indirect adjustments do not reveal any residual seasonal or trading-day components: non-seasonal ARIMA models are found for all series and the seasonality tests are negative.

The M and Q-statistics are commonly analyzed by statisticians in order to have a synthetic view of the performance of their adjustment: a value greater than one, for any of these statistics, indicates a possible problem in the seasonal adjustment. All estimates present a problem:

- The M7 statistics is expected to be larger than 1 as no seasonal component should be present in the series. This is true for all series except the TRAMO-SEATS direct estimate;
- The indirect adjustments may present some autocorrelation in the irregular, when the direct and mixed approach do not show any structure;

#### 18.4.5 Roughness measures

Roughness measures are presented in Table 18.9 and refer to the seasonally adjusted series, the trend-cycle and the seasonal component. The smoothness of seasonally adjusted data is often considered as one of the most important criteria from the users point of view. Nevertheless, it must be stressed, as mentioned in section 18.4, that this criterion has to be used carefully: the irregular component is an integral part of the series and it seems quite strange to prefer the series that presents the smallest irregular!

A first very simple conclusion we can draw from this table is that there is no clear evidence in favor of one of the approaches.

- TRAMO-SEATS direct approach gives, on the whole period, the smoother seasonally adjusted series and the smoother trend, and as a consequence the less smooth seasonal component;
- X-12-ARIMA gets better results with the indirect approach (in 11 cases out of 12) and on the opposite, TRAMO-SEATS prefers the direct adjustment in 8 cases out of 12;
- As far as the smoothness of the seasonal component is concerned, the mixed adjustment X-12-ARIMA gives the best results, followed by the indirect approaches. Direct adjustments give the less smooth estimates.

**Table 18.4:** Inconsistencies in growth rates between the three approaches; GDP geographical aggregation.

direct versus indirect					
TRAMO-SEATS			X-12-ARIMA		
Date	Direct	Indirect	Date	Direct	Indirect
02Q4	0.096	-0.132	09Q2	0.055	-0.003
03Q2	-0.023	0.123	13Q2	-0.018	0.118
05Q1	0.274	-0.111	13Q4	-0.021	0.059
12Q1	-0.382	0.031	14Q1	0.004	-0.028

Mixed vs Direct					
TRAMO-SEATS			X-12-ARIMA		
Date	Mixed	Direct	Date	Mixed	Direct
01Q3	0.036	-0.091	96Q4	0.280	-0.120
03Q2	0.044	-0.023	01Q3	0.036	-0.257
11Q3	0.037	-0.107	02Q4	0.058	-0.145
			05Q1	0.123	-0.129
			09Q2	-0.238	0.055
			11Q3	0.037	-0.170
			12Q1	-0.161	0.112
			13Q2	0.396	-0.018
			13Q4	0.213	-0.021

Mixed vs Indirect					
TRAMO-SEATS			X-12-ARIMA		
Date	Mixed	Indirect	Date	Mixed	Indirect
01Q3	0.036	-0.111	96Q4	0.280	-0.085
02Q4	0.058	-0.132	01Q3	0.036	-0.148
05Q1	0.123	-0.111	02Q4	0.058	-0.159
11Q3	0.037	-0.128	05Q1	0.123	-0.154
12Q1	-0.161	0.031	11Q3	0.037	-0.205
			12Q1	-0.161	0.085
			14Q1	0.196	-0.028

**Table 18.5:** Inconsistencies in growth rates between Aggregate and Components; GDP geographical aggregation.

*If Weights is greater than 0.5 (less than 0.5), the main part of the national GDP increases (decreases).*

Direct or Indirect, TRAMO-SEATS										
Date	Direct	Indirect	Weights	BE	DE	ES	FI	FR	IT	NL
02Q4		-0.132	0.571	0.41	-0.59	0.86	0.79	-0.28	0.04	-0.37
03Q2	-0.023		0.571	0.05	0.40	0.76	1.38	-0.12	-0.32	-0.38
05Q1	0.274		0.429	0.25	-0.47	1.03	-0.52	0.28	-0.75	-0.13
08Q2	-0.083	-0.051	0.714	0.76	0.03	0.12	0.50	-0.27	-0.42	0.31
12Q1		0.031	0.429	0.21	0.57	-0.73	-0.34	0.26	-0.59	-0.12

Direct or Indirect, X-12-ARIMA										
Date	Direct	Indirect	Weights	BE	DE	ES	FI	FR	IT	NL
02Q4	-0.145	-0.159	0.571	0.35	-0.69	1.06	0.77	-0.36	0.08	-0.47
05Q1	-0.129	-0.154	0.571	0.23	-0.55	1.22	-0.23	0.31	-1.10	0.04
09Q2	0.055		0.286	-0.22	0.47	-0.50	-0.50	0.34	-0.48	-0.57
13Q2	-0.018		0.571	0.35	0.28	-0.35	0.61	0.35	-0.18	-0.12
13Q4	-0.021		0.571	0.42	0.05	0.19	-0.44	-0.05	-0.14	0.66
14Q1	0.004		0.429	0.33	0.48	0.34	-0.36	-0.52	-0.12	-1.09

Mixed Approach										
Date	Mixed		Weights	BE	DE	ES	FI	FR	IT	NL
01Q4	0.071		0.429	-0.05	0.17	0.70	-0.29	-0.19	-0.11	0.14
11Q2	0.004		0.429	0.21	0.17	-0.48	-0.20	-0.08	0.16	-0.09
12Q1	-0.161		0.571	0.28	0.39	-0.82	0.04	0.05	-1.02	-0.21
14Q1	0.196		0.429	0.42	0.72	0.37	-0.48	-0.14	-0.11	-0.39
14Q2	0.024		0.429	0.25	-0.06	0.49	-0.15	-0.10	-0.20	0.50

**Table 18.6:** Concordance Rates (in %); GDP geographical aggregation.

	TRAMO-SEATS	X-12-ARIMA
Direct and Indirect	94.87	94.87
Direct and Components	96.15	92.31
Indirect and Components	96.15	97.44
Mixed and Direct	96.15	88.46
Mixed and Indirect	93.59	91.03
Mixed and Components		93.59

**Table 18.7:** Seasonal adjustment of the Mixed, Direct and Indirect Series; GDP geographical aggregation.

Series	Model	Mixed, Direct and Indirect							Season?
		Leap Year	Easter	NbTD	AO	LS	TC	QS (Pvalue)	
Mixed	(1,1,0)(0,0,0)	No	No	0	0	1	1	0.45	No
Seats Indirect	(1,1,0)(0,0,0)	No	No	0	0	1	1	0.47	No
X-12 Indirect	(0,1,1)(0,0,0)	No	No	0	0	2	0	0.60	No
Seats Direct	(2,1,1)(0,0,0)	No	No	0	0	2	0	0.77	No
X-12 Direct	(0,1,1)(0,0,0)	No	No	0	0	2	0	0.55	No

**Table 18.8:** Quality measures; GDP geographical aggregation.

Indicator	TRAMO-SEATS		X-12-ARIMA		Mixed
	Direct	Indirect	Direct	Indirect	adjustment
M1	0.004	3.000	0.055	3.000	0.507
M2	0.001	3.000	0.326	3.000	0.124
M3	0.000	1.838	0.000	1.666	0.000
M4	0.733	2.514	0.838	2.514	0.733
M5	0.200	2.000	0.200	1.975	0.200
M7	0.447	2.742	2.853	2.359	3.000
M8	1.848	0.416	1.418	0.299	1.764
M9	0.150	0.142	0.139	0.162	0.789
M10	2.119	0.552	1.325	0.452	1.632
M11	0.493	0.552	0.694	0.452	0.674
Qstat	0.451	2.013	0.926	1.898	1.062
Mfailed	2	6	3	6	3

**Table 18.9:** Roughness measures; GDP geographical aggregation.

	TRAMO-SEATS		X-12-ARIMA		Mixed	SEATS	X-12
	Direct	Indirect	Direct	Indirect	approach		
R1 (SA)	13951.087	14629.068	15043.192	14853.979	13978.243	D	I
R1 (SA), Last 3 years	6339.939	6457.444	7136.898	6748.062	6857.561	D	I
R2 (SA)	1818.423	2168.231	2440.510	2340.712	2034.078	D	I
R2 (SA), Last 3 years	854.905	1165.457	1639.328	1200.229	1418.054	D	I
R3 (SA)	648.513	1808.709	5816.095	2603.910	3718.361	D	I
R3 (SA), Last 3 years	420.965	957.627	1819.880	1284.963	1706.645	D	I
Mar (TC,1)	13857.921	14250.905	17388.028	14868.373	15087.045	D	I
Mar (TC,1), Last 3 years	6250.083	5850.033	6273.495	6105.345	5857.905	I	I
Mar (TC,2)	9821.836	11175.456	20853.710	13177.752	14939.680	D	I
Mar (TC,2), Last 3 years	3016.534	2440.040	2731.729	2772.390	2840.508	I	D
Mar (S)	1435.020	258.488	972.938	159.957	19.378	I	I
Mar (S), Last 3 years	1412.584	208.399	1075.098	152.380	17.272	I	I



**Table 18.10:** Absolute Revisions (mean and standard deviation in %) and Sliding Spans analysis; GDP geographical aggregation.

Indicator	TRAMO-SEATS		X-12-ARIMA		direct versus indirect	
	Direct	Indirect	Direct	Indirect	Seats	X-12
Mean AR 1 qtr	0.045	0.016	0.016	0.019	I	D
Mean AR 2 qtrs	0.045	0.016	0.035	0.033	I	I
Mean AR 3 qtrs	0.050	0.033	0.041	0.038	I	I
Mean AR 4 qtrs	0.090	0.019	0.064	0.055	I	I
Mean AR 5 qtrs	0.071	0.013	0.076	0.056	I	I
Std AR 1 qtr	0.092	0.035	0.020	0.033	I	D
Std AR 2 qtrs	0.101	0.018	0.053	0.076	I	D
Std AR 3 qtrs	0.110	0.052	0.062	0.075	I	D
Std AR 4 qtrs	0.196	0.030	0.101	0.121	I	D
Std AR 5 qtrs	0.143	0.013	0.094	0.112	I	D
Sliding Spans						
A(%)	0.000	0.000	0.000	0.000	=	=
MM(%)	0.000	0.000	0.000	0.000	=	=

### 18.4.6 Revision analysis

Users very much prefer to deal with seasonally adjusted times series which are revised as few as possible. Non revised seasonally adjusted figures could be obtain with purely asymmetric filters, with the Dainties method for example, but with a possible serious counterpart: a phase shift in turning point detection.

Table 18.10 presents the revisions of direct and indirect estimates obtained with the two programs. It is important to note that this revision analysis concentrates on revisions induced by the seasonal adjustment process:

- For the simulations, the ARIMA model and the decomposition model have been fixed;
- Revisions of raw data which occur regularly, as new information became available have not been taken into account.

The results are very clear. For TRAMO-SEATS the indirect approach performs better both in terms of mean and variance of the revisions, whereas for X-12-ARIMA the direct approach appears better for the short-term revisions and in terms of variance. If we compare now the two programs, the X-12-ARIMA direct approach appears better than the TRAMO-SEATS direct approach. This is quite a surprising result, as from the theoretical point of view, TRAMO-SEATS is supposed to perform better as it is based on ARIMA models.

The Sliding-Spans analysis does not reveal any stability problem for the various estimates which are therefore equivalent from this point of view.

It is also useful to point out that in the case of indirect approach we are working with some linear combination of possibly different filters so that it is difficult to talk about the “revision properties of the filter” in this specific case. The situation is much more clear in the case of direct approach, where a unique filter is applied.

### 18.4.7 Analysis of the residuals

The various estimated irregular components, even if they contain possible outliers, are supposed to present no specific structure and are often modelised as  $N(0, \sigma^2)$  i.i.d processes. These estimates have been analysed with Tramo that provides us with an automatic identification of seasonal Arima model  $(p, d, q)(P, D, Q)$  and

**Table 18.11:** Analysis of the Mixed, Direct and Indirect Irregular Components; GDP geographical aggregation.

Series	Model	Mixed, Direct and Indirect							Season?
		Leap Year	Easter	NbTD	AO	LS	TC	QS (Pvalue)	
SEATS Indirect	(1,0,0)(0,0,0)	Yes	No	1	0	0	0	1	No
X-12 Indirect	(0,0,1)(0,0,0)	No	No	0	1	0	0	1	No
SEATS Direct	(0,0,0)(0,0,0)	No	No	0	0	0	0		No
X-12 Direct	(3,1,1)(0,0,1)	No	No	0	6	0	0	1	No

with the associated whiteness tests to assess the absence of any significant autocorrelation structure and the presence of residual seasonality.

Table 18.11 presents the results of this analysis. No residual seasonality is found in the four irregulars but some minor points can be noted:

- The TRAMO-SEATS direct estimates is modeled as non seasonal  $AR(1)$ , which is not a good sign since the AR part of the stochastic process generally represents its inertia;
- The X-12-ARIMA direct estimate is modeled as seasonal Arima;
- The TRAMO-SEATS direct estimate present an unexpected residual calendar effect.

## 18.5 Gross Value Added, Aggregation by Sector

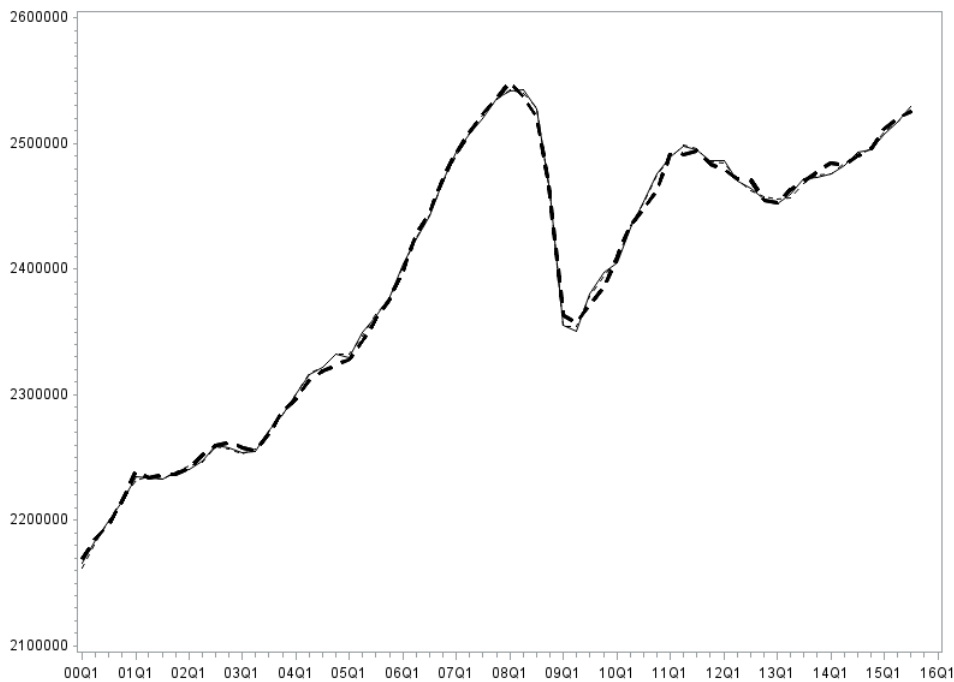
In this application, we consider that the euro area Gross Value Added at current prices can also be seen as the sum of the added value of 10 main industries (according to the NACE Rev 2 classification).

### 18.5.1 The Data

The seven components are therefore:

1. (A) Agriculture, forestry and fishing
2. (B-E) Industry (except construction)
3. (C) Manufacturing
4. (F) Construction
5. (G-I) Wholesale and retail trade, transport, accommodation and food service activities
6. (J) Information and communication
7. (K) Financial and insurance activities
8. (L) Real estate activities
9. M-N) Professional, scientific and technical activities; administrative and support service activities
10. (O-Q) Public administration, defense, education, human health and social work activities
11. (R-U) Arts, entertainment and recreation) other service activities; activities of household and extra-territorial organizations and bodies

**Figure 18.6:** Mixed (thick dashed line), TRAMO-SEATS Direct (thin dashed line) and Indirect (solid line) Adjustments; GVA aggregation by sector.



The exercise is done using data for the euro area with 12 countries. The data cover the period 2000Q1-2015Q3 and are available from Eurostat web site.

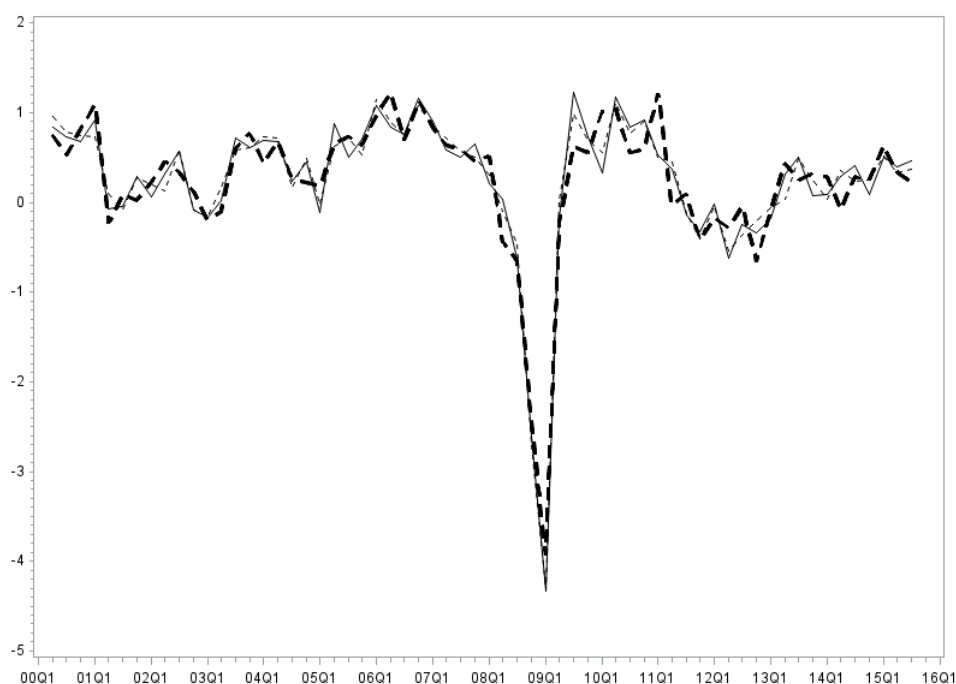
### 18.5.2 A first comparison between seasonally adjusted series

Figure 18.6 presents the mixed adjustment and the TRAMO-SEATS direct and indirect seasonally adjusted series. It is, like for the geographical aggregation case, quite difficult to detect a real difference between the three series (and it will be the same with the X-12-ARIMA estimates), even if we can note some discrepancies around the turning-points. This similarity is confirmed by the numerical indicators displayed in Table 18.12. The mean absolute percentage difference between two estimates is very small, and of the same order than for the geographical aggregation case: less than 0.1% between direct and indirect, and close to 0.15% between the mixed and the other estimates. The direct versus indirect problem is therefore not very important.

**Table 18.12:** Absolute Percentage Deviation Indicators (GVA aggregation by sector).

Indicator	Ind. vs Dir.		Best	Ind. vs Mixed		Dir. vs Mixed	
	T-S	X-12		T-S	X-12	T-S	X-12
Mean APD (SA)	0.063	0.094	Seats	0.155	0.133	0.153	0.159
Max APD (SA)	0.169	0.469	Seats	0.552	0.497	0.519	0.615
Mean APD (SA), Last 3 years	0.072	0.046	X12ar	0.118	0.112	0.119	0.120
Max APD (SA), Last 3 years	0.163	0.103	X12ar	0.369	0.347	0.372	0.359
Mean APD (TC)	0.037	0.082	Seats	0.126	0.130	0.123	0.147
Max APD (TC)	0.168	0.602	Seats	0.345	0.302	0.326	0.678
Mean APD (TC), Last 3 years	0.030	0.048	Seats	0.093	0.094	0.088	0.126
Max APD (TC), Last 3 years	0.077	0.088	Seats	0.247	0.227	0.228	0.258

**Figure 18.7:** Growth Rates of Mixed (thick dashed line) and TRAMO-SEATS Direct (thin dashed line) and Indirect (solid line) Adjustments; GVA aggregation by sector.



**Table 18.13:** Differences in growth rates between the three approaches; GVA aggregation by sector.

Indicator	Dir. vs Ind.		Mixed vs Ind.		Mixed vs Dir.	
	T-S	X-12	T-S	X-12	T-S	X-12
Mean	0.001	0.004	-0.005	-0.004	-0.006	-0.007
Minimum	-0.272	-0.730	-0.605	-0.502	-0.486	-1.013
Maximum	0.233	0.635	0.690	0.603	0.705	0.888
Variance	0.014	0.040	0.065	0.055	0.062	0.074
Range	0.505	1.365	1.295	1.105	1.191	1.901

The growth rates are displayed in figures 18.7 and 18.8. Finally, figures 18.9 and 18.10 show the relative difference of the mixed benchmark series with respect to the direct and indirect seasonal adjusted series obtained with TRAMO-SEATS and X-12-ARIMA.

The main conclusions we can draw from this set of pictures are the followings:

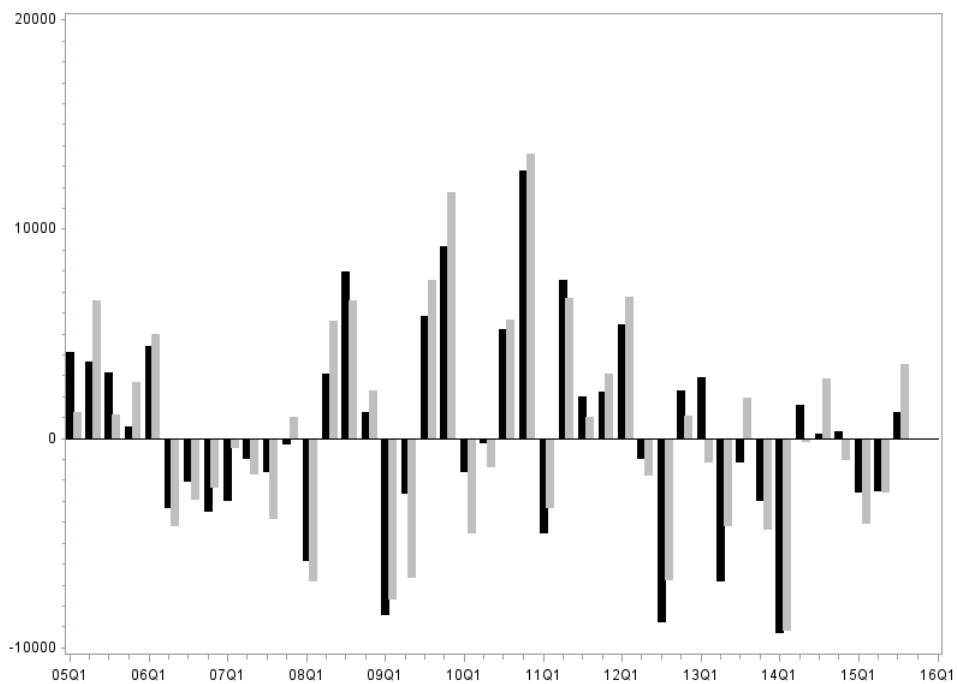
- Even if direct and indirect adjusted series show a quite similar behavior, regardless to the software used (X-12-ARIMA or TRAMO-SEATS), many discrepancies can be noted before and after the 2008-2009 crisis;
- Year 2009 appears to be a very special period where direct, indirect and mixed estimates show different behavior. As an example, in 2009Q3 the mixed approach shows an increase of 0.6% when all the other approaches were close or above 1%;

Table 18.13 statistics confirm the quite low agreement between the different estimates: if average differences are quite small, the ranges are quite important, above 1.1% for the differences with the mixed approach. Only the differences between the TRAMO-SEATS direct and indirect estimates are quite small.

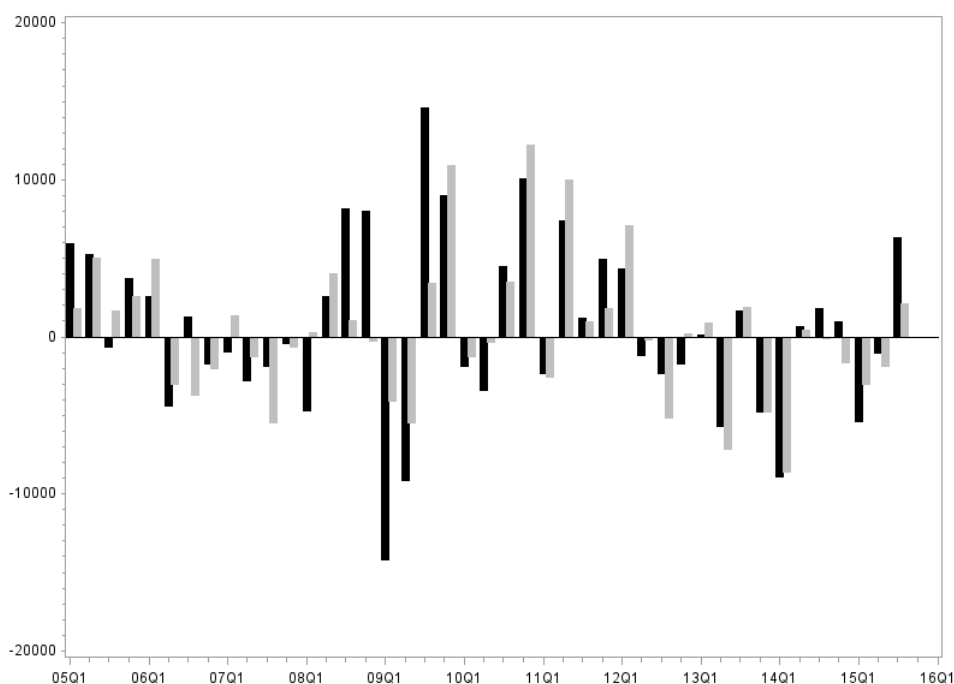
**Figure 18.8:** Growth Rates of Mixed (thick dashed line) and X-12-ARIMA Direct (thin dashed line) and Indirect (solid line) Adjustments; GVA aggregation by sector.



**Figure 18.9:** Relative Differences of Mixed Aggregate versus Direct (in black) and Indirect (in grey) Adjustments; GVA aggregation by sector. TRAMO-SEATS.



**Figure 18.10:** Relative Differences of Mixed Aggregate versus Direct (in black) and Indirect (in grey) Adjustments; GVA aggregation by sector. X-12-ARIMA.



### 18.5.3 Concordance analysis of growth rates

Tables 18.14 and 18.15 detail the cases of discrepancies between the various aggregates and with the sub-components, according to the software.

Direct and indirect estimates only disagree, e.g. give an opposite evolution of the indicator, for both software, in three cases out of 62 observations. These discrepancies concern growth rates close to zero except for X-12-ARIMA and quarter 2005Q1 for which the direct estimate gives an increase of 0.298% when the indirect one shows a decrease of -0.139%.

The discrepancies are more numerous when comparing with the mixed approach and some large inconsistencies can be observed. In particular, 2014Q2 shows a notable difference between the mixed approach (-0.079%) and the direct and indirect of both methods that show an increase close or above 0.3%. 2011Q2 shows the same disagreement between the mixed approach and the other approaches.

The inconsistencies between the evolution of an aggregate and the majority of its components are presented, for the different approaches and softwares, in table 18.15.

TRAMO-SEATS presents seven inconsistencies for the indirect estimate and eight for the direct estimate. 2010Q1 is perhaps the most important as seven branches representing 64% of the euro area GDP were decreasing when the direct and indirect estimates were quite strongly increasing (0.56% and 0.33%). X-12-ARIMA presents nine inconsistencies for the indirect estimate and eight for the direct estimate with strong discrepancies in 2008Q and 2010Q1. The mixed approach presents six inconsistencies: in each case, the GDP of the majority of sectors was increasing when the mixed estimates was decreasing.

Table 18.16 presents the concordance rates between the various approaches, a statistic that summarizes the previous elements. Direct and indirect approaches present concordance rates with the components smaller than 90%, that is a quite disappointing result.

One must also note that the different estimates do not give consistent messages for the last years as diver-

gences are observed in 2012, 2013 and 2014.

### 18.5.4 Quality measures of seasonal adjustments

The seasonal adjustment of the various estimates (see table 18.17) does not show any residual seasonal or trading-day component. The M and Q-statistics results are presented in Table 18.18. In general, all the adjustments seem correct according to these statistics: all the Q-statistics are smaller than 1. As expected, the M7 statistic is higher than 1 for all adjustments. The direct adjustments do not present any problem and therefore seem preferable. On the contrary, the indirect estimates present high values of the M1, M2 and M4 statistics: for these adjustments, the dynamics of the irregular component might present some problems.

### 18.5.5 Roughness measures

Roughness measures are presented in Table 18.19 and refer to the seasonally adjusted series, the trend-cycle and the seasonal component.

A first very simple conclusion we can draw from this table is that, once more, there is no clear evidence in favor of one of the approaches.

- The indirect approaches seem to give smoother seasonal components and, on the reverse, more volatile seasonally adjusted series;
- TRAMO-SEATS gets better results with the direct approach (except the smoothness of the seasonal component).

### 18.5.6 Revision analysis

Table 18.20 presents the revisions of direct and indirect estimates obtained with the two programs. For X-12-ARIMA the indirect approach performs better both in terms of mean and variance of the revisions, whereas for TRAMO-SEATS the direct approach appears better as far as mean absolute revisions are concerned. If we compare now the two programs, the X-12-ARIMA direct approach, with an average absolute revision close to 0.01%, is the best approach but TRAMO-SEATS direct and indirect approaches perform also very well.

The Sliding-Spans analysis does not reveal any stability problem for the various estimates which are therefore equivalent from this point of view.

### 18.5.7 Analysis of the residuals

The various irregular component estimates have been analyzed with Tramo.

Table 18.21 presents the results of this analysis.

- All the irregular estimates are modeled as non-seasonal ARIMA and no residual seasonality can be found in any of the four estimates;
- No irregular estimate presents a residual calendar effect;
- X-12-ARIMA irregulars and TRAMO-SEATS indirect irregular present AR parts in the model which is not a good sign as this part generally represents its inertia.

**Table 18.14:** Inconsistencies in growth rates between the three approaches; GVA aggregation by sector.

direct versus indirect					
TRAMO-SEATS			X-12-ARIMA		
Date	Direct	Indirect	Date	Direct	Indirect
01Q2	0.104	-0.075	03Q2	0.045	-0.027
08Q2	-0.096	0.040	05Q1	0.298	-0.139
09Q2	0.021	-0.179	12Q1	-0.194	0.041

Mixed vs Direct					
TRAMO-SEATS			X-12-ARIMA		
Date	Mixed	Direct	Date	Mixed	Direct
01Q2	-0.227	0.104	01Q2	-0.227	0.074
01Q3	0.090	-0.093	01Q3	0.090	-0.155
02Q4	0.118	-0.092	02Q4	0.118	-0.126
03Q2	-0.097	0.171	03Q2	-0.097	0.045
05Q1	0.185	-0.025	11Q2	-0.033	0.358
09Q2	-0.224	0.021	11Q3	0.095	-0.155
11Q2	-0.033	0.453	14Q2	-0.079	0.308
11Q3	0.095	-0.128			
14Q2	-0.079	0.357			

Mixed vs Indirect					
TRAMO-SEATS			X-12-ARIMA		
Date	Mixed	Indirect	Date	Mixed	Indirect
01Q3	0.090	-0.046	01Q2	-0.227	0.003
02Q4	0.118	-0.092	01Q3	0.090	-0.143
03Q2	-0.097	0.022	02Q4	0.118	-0.218
05Q1	0.185	-0.115	05Q1	0.185	-0.139
08Q2	-0.445	0.040	11Q2	-0.033	0.469
11Q2	-0.033	0.370	11Q3	0.095	-0.264
11Q3	0.095	-0.134	12Q1	-0.172	0.041
14Q2	-0.079	0.284	14Q2	-0.079	0.285



**Table 18.15:** Inconsistencies in growth rates between Aggregate and Components, GVA aggregation by sector.

*If Weights is greater than 0.5 (less than 0.5), the main part of the national GDP increases (decreases).*

Direct or Indirect, TRAMO-SEATS														
Date	Direct	Indirect	Weights	A	B-E	C	F	G-I	J	K	L	M-N	O-Q	R-U
01Q2		-0.075	0.545	-0.30	-0.66	-0.53	0.34	-0.04	2.56	1.06	0.31	-1.17	0.38	0.01
01Q3	-0.093	-0.046	0.636	3.16	-0.79	-0.70	-0.09	-0.17	2.27	0.04	0.57	0.18	0.24	0.00
02Q1	0.200	0.058	0.364	-1.13	0.14	-0.12	-0.28	-0.04	1.08	-0.03	0.50	-0.69	0.48	-0.24
03Q1	-0.159	-0.168	0.545	-0.42	-0.66	0.04	0.12	-0.04	-4.26	-1.37	0.39	0.82	0.19	0.23
03Q2	0.171	0.022	0.455	-1.48	-0.02	-0.01	0.28	0.28	1.16	-0.78	0.45	-1.19	0.29	-0.01
05Q1	-0.025	-0.115	0.545	-4.37	-0.51	0.32	-1.31	-0.37	-3.27	0.35	0.77	1.42	0.16	0.17
08Q2	-0.096		0.545	-0.05	-0.36	-0.58	-1.63	0.13	1.24	-0.34	0.54	0.81	0.48	0.80
09Q2	0.021		0.455	0.90	0.21	-0.07	-2.31	-0.60	1.26	-0.29	0.46	-1.34	0.37	-0.48
10Q1	0.558	0.326	0.364	-3.61	2.11	2.14	-2.23	-0.72	-1.36	0.38	-0.07	-0.06	0.22	-0.04

Direct or Indirect, X-12-ARIMA														
Date	Direct	Indirect	Weights	A	B-E	C	F	G-I	J	K	L	M-N	O-Q	R-U
01Q3	-0.155	-0.143	0.545	2.51	-1.11	-0.70	-0.22	-0.31	2.16	0.35	0.66	-0.16	0.27	0.13
03Q1	-0.225	-0.128	0.545	0.55	-0.59	0.83	0.06	-0.06	-4.44	-1.76	0.36	0.52	0.07	-0.35
03Q2	.	-0.027	0.545	-2.01	-0.07	-0.65	0.41	0.26	1.36	-0.94	0.44	-0.55	0.24	0.22
05Q1	0.298	.	0.364	-4.33	-0.35	-0.06	-1.48	-0.45	-2.99	0.23	0.80	1.87	0.08	-0.15
08Q2	-0.161	-0.296	0.545	0.11	-0.52	-1.68	-1.59	-0.21	0.73	-0.67	0.59	0.05	0.54	0.62
08Q3	-0.433	-0.773	0.545	0.42	-1.44	-2.66	-2.07	-1.06	0.88	0.38	0.26	-0.20	0.50	0.09
10Q1	0.552	0.495	0.455	-2.98	2.18	2.70	-2.34	-0.26	-1.29	0.42	-0.12	0.17	0.16	-0.12
12Q1	.	0.041	0.364	-1.35	1.02	-1.05	-2.54	1.34	-0.04	0.15	-0.59	0.08	-0.09	-0.07
13Q1	-0.013	-0.062	0.545	1.70	0.41	0.63	-2.12	-0.72	0.59	-0.94	0.43	0.32	-0.41	-0.55
13Q4	0.062	0.053	0.455	2.41	0.24	-0.01	-1.16	-0.14	0.03	-0.28	0.37	-0.31	0.35	-0.10

Mixed Approach														
Date	Mixed		Weights	A	B-E	C	F	G-I	J	K	L	M-N	O-Q	R-U
01Q2	-0.227		0.545	-0.98	-0.92	-1.22	0.11	-0.01	1.97	1.08	0.30	-0.81	0.33	0.12
03Q2	-0.097		0.545	-1.69	-0.77	-0.73	0.46	0.27	0.61	-0.47	0.56	-0.35	0.42	0.16
11Q2	-0.033		0.545	0.66	-0.25	-0.08	-1.48	-0.03	0.57	0.76	0.07	0.11	0.15	-0.38
12Q3	-0.048		0.545	-0.98	0.14	0.32	-1.64	-0.29	0.72	-0.45	0.13	-0.14	0.01	0.15
13Q1	-0.089		0.545	2.12	0.38	0.44	-1.96	-0.88	0.65	-1.07	0.42	0.09	-0.11	-0.79
14Q2	-0.079		0.545	-0.14	-0.16	-0.44	-1.63	0.02	0.12	0.04	0.45	0.39	-0.06	0.02

**Table 18.16:** Concordance Rates (in %), GVA aggregation by sector.

	TRAMO-SEATS	X-12-ARIMA
Direct and Indirect	95.16	95.16
Direct and Components	87.10	87.10
Indirect and Components	88.71	85.48
Mixed and Direct	85.48	88.71
Mixed and Indirect	87.10	87.10
Mixed and Components		90.32

**Table 18.17:** Seasonal Analysis of the Mixed, Direct and Indirect Series; GVA aggregation by sector.

Mixed, Direct and Indirect									
Series	Model	Leap Year	Easter	NbTD	AO	LS	TC	QS (Pvalue)	Season?
Mixed	(1,1,0)(0,0,0)	No	No	0	0	1	1	0.93	No
SEATS Indirect	(2,1,0)(0,0,0)	No	No	0	1	1	1	0.76	No
X-12 Indirect	(1,1,2)(0,0,0)	No	No	0	0	1	1	0.96	No
SEATS Direct	(1,1,2)(0,0,0)	No	No	1	1	1	1	0.83	No
X-12 Direct	(1,1,0)(0,0,0)	No	No	0	1	1	1	0.49	No

**Table 18.18:** Quality measures.

Indicator	TRAMO-SEATS		X-12-ARIMA		Mixed
	Direct	Indirect	Direct	Indirect	adjustment
M1	0.016	3.000	0.008	3.000	0.235
M2	0.030	3.000	0.011	3.000	0.096
M3	0.000	1.811	0.000	1.571	0.000
M4	0.431	2.196	0.431	2.196	0.549
M5	0.200	1.973	0.200	1.911	0.200
M7	3.000	3.000	3.000	2.686	3.000
M8	2.100	0.580	2.340	0.417	2.742
M9	0.370	0.302	0.312	0.277	0.561
M10	1.963	0.760	2.237	0.578	2.412
M11	1.037	0.760	0.677	0.578	1.742
Qstat	0.994	2.074	1.001	1.946	1.153
Mfailed	4	6	3	6	4

**Table 18.19:** Roughness measures; GVA aggregation by sector.

	TRAMO-SEATS		X-12-ARIMA		Mixed	SEATS	X-12
	Direct	Indirect	Direct	Indirect	approach		
R1 (SA)	20718.069	20921.372	21797.334	20456.253	20024.703	D	I
R1 (SA), Last 3 years	7728.292	8470.144	8472.591	8286.539	8298.096	D	I
R2 (SA)	3155.033	3163.142	3565.579	3152.686	3181.495	D	I
R2 (SA), Last 3 years	1533.810	1819.963	1679.571	1836.739	1640.263	D	D
R3 (SA)	1452.566	2096.612	1611.011	2351.928	2101.578	D	D
R3 (SA), Last 3 years	1040.423	1653.452	1545.149	1703.439	1671.810	D	D
Mar (TC,1)	20562.545	20638.153	21726.433	19950.909	19519.278	D	I
Mar (TC,1), Last 3 years	7185.515	7249.032	7648.346	7536.648	7447.219	D	I
Mar (TC,2)	17868.802	18146.759	20200.081	16430.882	15861.485	D	I
Mar (TC,2), Last 3 years	3459.518	3593.363	3483.736	3661.075	4419.451	D	D
Mar (S)	1431.422	389.812	1517.580	221.369	66.147	I	I
Mar (S), Last 3 years	1560.054	288.134	1467.622	189.828	47.714	I	I

**Table 18.20:** Absolute Revisions (mean and standard deviation in %) and Sliding Spans analysis, GVA aggregation by sector.

Indicator	TRAMO-SEATS		X-12-ARIMA		direct versus indirect	
	Direct	Indirect	Direct	Indirect	SEATS	X-12
Mean AR 1 qtr	0.022	0.031	0.178	0.009	D	I
Mean AR 2 qtrs	0.025	0.031	0.168	0.012	D	I
Mean AR 3 qtrs	0.025	0.035	0.051	0.013	D	I
Mean AR 4 qtrs	0.038	0.028	0.207	0.014	I	I
Mean AR 5 qtrs	0.055	0.042	0.236	0.015	I	I
Std AR 1 qtr	0.065	0.059	0.341	0.010	I	I
Std AR 2 qtrs	0.067	0.044	0.358	0.013	I	I
Std AR 3 qtrs	0.067	0.042	0.109	0.013	I	I
Std AR 4 qtrs	0.075	0.043	0.362	0.014	I	I
Std AR 5 qtrs	0.096	0.059	0.391	0.012	I	I
Sliding Spans						
A(%)	0.000	0.000	0.000	0.000	=	=
MM(%)	0.000	0.000	0.000	0.000	=	=

**Table 18.21:** Analysis of the Mixed, Direct and Indirect Series and of their Irregular Components; GVA aggregation by sector.

Series	Model	Irregular Components							Season?
		Leap Year	Easter	NbTD	AO	LS	TC	QS (Pvalue)	
SEATS Indirect	(1,0,0)(0,0,0)	No	No	1	0	0	0	1	No
X-12 Indirect	(0,0,1)(0,0,0)	No	No	0	0	0	1	0.69	No
SEATS Direct	(2,0,0)(0,0,0)	No	No	1	0	0	0	1	No
X-12 Direct	(1,0,0)(0,0,0)	No	No	0	2	0	0	0.28	No

## 18.6 Conclusions

The comparison presented in this paper shows some interesting results. The first one is that the figures currently published by Eurostat using the mixed indirect approach are quite different with respect to those obtained by using either the direct or the indirect approaches, independently of the seasonal adjustment program. This can be viewed as a consequence of the different seasonal adjustment policies and options adopted by Member States to compile their aggregates. Moreover the mixed seasonal adjustment appears to be the less satisfactory in terms of the different quality measures that have been analysed in this paper. On the basis of these considerations, we may conclude that the seasonal adjustment strategy currently used by Eurostat in the field of Quarterly National Accounts needs to be amended in order to supply users with higher quality and more transparent results. The second result is that there are no evident and significant differences between the direct and the indirect approach, both using TRAMO-SEATS and X-12-ARIMA. In some cases the proposed quality measures seem to privilege the direct approach, in other cases the indirect. Moreover for the same group of criteria, like for roughness measures for example, the subjective appreciation can lead to prefer one approach to the other or vice versa. These elements apply both to the geographical and to the sectoral aggregation. Even if the definition of a general Eurostat strategy to compute seasonal adjusted QNA figures is outside the scope of this paper, a reasonable seasonal adjustment strategy emerging from the results can be summarized as follows:

- use of a direct approach on each aggregate for the geographical aggregation;
- use of the indirect approach for the sectoral aggregation.

The indirect seasonal adjusted may be less useful for the geographical aggregation because it would be impossible for Eurostat to publish different seasonal adjusted figures with respect to those published by Member States. Under this constraint, the additivity of the seasonal adjusted data that one would obtain by using an indirect approach cannot be used in practice. Considering also the similarity of the results obtained with the direct/indirect methods, it seems then natural to prefer the direct approach since it is easier to implement in practice. The use of the indirect approach for sectoral aggregation could be replaced by the use of a direct approach with redistribution of discrepancies in the case of sectoral aggregation. Nevertheless this alternative solution is only feasible under the condition that the discrepancy between the sum of the seasonally adjusted components and the seasonally adjusted aggregate series are small. The results presented in the paper suggest, anyway, that this condition is fulfilled.

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# 19

## Multivariate Benchmarking: The Direct vs. Indirect Problem





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## Table of Contents

19.1	Introduction	499
19.2	Seasonal adjustment of variables linked by aggregation constraints	499
19.2.1	Direct and indirect seasonal adjustment	500
19.2.2	Advantages and disadvantages	500
19.2.3	Guidelines	501
19.2.4	The two-way classification	501
19.2.5	SA of a two-way table	501
19.3	Techniques to force the respect of aggregation constraints of seasonally adjusted data when a direct adjustment is performed	502
19.3.1	Pro-rating	502
19.3.2	Generalization of pro-rating	503
19.3.3	Generalization of pro-rating to higher dimensional tables	503
19.3.4	Simple numerical examples	504
19.3.5	Specifying the alterability coefficients	504
19.4	Movement preservation principle based benchmarking techniques	505
19.4.1	The 2-step based methods	505
19.4.2	The simultaneous method of Di Fonzo and Marini (2010)	506
19.5	Example	506
19.5.1	Bottom-up	506
19.5.2	Top-bottom	506
19.5.3	Middle-up middle-down	513
19.6	Conclusions	513

## 19.1 Introduction

The vast majority of time series data produced by a statistical agency are part of a system of time series classified by attributes that must satisfy cross-sectional or contemporaneous aggregation constraints. This requires that the values of the component elementary series add up to marginal totals for each period of time. In some cases, each series must also add up to temporal benchmarks and thus must satisfy temporal aggregation constraints.

Most univariate seasonal adjustment (SA) methods are non-linear. When they are applied to each component series, marginal totals or grand total directly, the linear relationships of the system are likely to be destroyed. Similarly, the temporal aggregation constraints may not hold anymore. To have a set of seasonally adjusted series that respect the aggregation constraints, temporal benchmarking and reconciliation processes have to be applied. Alternatively, multivariate seasonal adjustment approaches, which permits to derive simultaneously the seasonally adjusted series for the marginal totals and the components could be considered. Given their computational complexity and the limitations of existing programs, they are rarely used in practice and the univariate seasonal adjustment approaches are generally preferred.

Chapter 17 covers benchmarking and temporal consistency when a single time series is seasonally adjusted. This chapter covers the topics of indirect and direct seasonal adjustment with multivariate benchmarking methods also known as raking, reconciliation or balancing.

The book by Dagum and Cholette (2006) covers benchmarking and reconciliation of time series with relevant historical references on the subject and appropriate discussions. Some of their solutions for reconciliation are presented here along with those from Quenneville and Fortier (2012) and Di Fonzo and Marini (2011).

This chapter is organized as follows: Section 19.2 introduces the general problem of seasonal adjustment of variables linked by aggregation constraints. Section 19.3 and 19.4 discuss techniques to force the respect of aggregation constraints. Section 19.5 provides an example with the unemployment series at the European level by age-sex. Section 19.6 concludes the chapter.

## 19.2 Seasonal adjustment of variables linked by aggregation constraints

The general set-up of the chapter is to consider the seasonal adjustment of a two-way table of time series  $X_{m,n}$ ,  $m = 1, \dots, M$  and  $n = 1, \dots, N$ , including the  $M$  marginal totals  $X_{m,\bullet}$ , the  $N$  marginal totals  $X_{\bullet,n}$  and the grand total  $X_{\bullet,\bullet}$ . In order to simplify the presentation and to review the existing methods, first consider only the one-way table that consist of a set of  $M + 1$  series made of the  $M$  component series and their weighted total

$$X_{\bullet,\bullet} = w_{1,\bullet}X_{1,\bullet} + \dots + w_{M,\bullet}X_{M,\bullet} \quad (19.1)$$

where  $w_{m,\bullet}$  is the weight given to the component  $X_{m,\bullet}$ . The weights can be positive and sum to 1 as in the case where  $X_{\bullet,\bullet}$  is an index or can be all equal to 1 in case of a flow series. The weight can also be negative such as in the balance equation of exports minus imports.

A direct or indirect adjustment can be used to seasonally adjust the total  $X_{\bullet,\bullet}$  with X-12-ARIMA (Findley et al. (1998); U.S. Census Bureau (2009)) or X-11-ARIMA (Dagum (1981), Dagum (1988)). Both permit weighting the components and using the addition, subtraction, multiplication and division operators to specify how a component is incorporated in the composite series. Non-linear relationships involving cases where a component series is incorporate in the composite series via multiplication is outside the scope of this chapter;



furthermore, for the rest of this chapter, we will consider the case where the weights are all equal to 1 but the general interpretation should be borne in mind.

### 19.2.1 Direct and indirect seasonal adjustment

The direct seasonal adjustment is obtained when the total series is seasonally adjusted on its own using the total of the raw series. Let  $A_{\bullet,\bullet}$  denotes the direct seasonally adjusted series of  $X_{\bullet,\bullet}$ . The indirect seasonal adjustment of the total series is obtained by summing the seasonal adjustment of the components or the breakdown series. Let  $A_{\bullet,\bullet}^I$  denotes the indirect seasonally adjusted series. By definition

$$A_{\bullet,\bullet}^I = \sum_{m=1}^M A_{m,\bullet} \quad (19.2)$$

where  $A_{m,\bullet}$  denotes the direct seasonal adjustment of  $X_{m,\bullet}$ .

### 19.2.2 Advantages and disadvantages

As stated in the 2009 ESS Guidelines on Seasonal Adjustment (Eurostat (2009)), the advantages of the direct approach are that  $A_{\bullet,\bullet}$  does not usually contain residual seasonality and can be easily reproduced by external users with publicly available seasonal adjustment options; however, the disadvantage is that the sum of the  $M$  seasonally adjusted components does not add to the directly seasonally adjusted total, because seasonal adjustment is in general a nonlinear process, i.e.,

$$\sum_{m=1}^M A_{m,\bullet} = A_{\bullet,\bullet}^I \neq A_{\bullet,\bullet}, \quad (19.3)$$

and consequently there will be a discrepancy  $A_{\bullet,\bullet} - A_{\bullet,\bullet}^I$ . To cope with the discrepancies at each of the time point, the choices are to publish them or if they are small enough, to apply an appropriate reconciliation procedures to ensure additivity. Obviously, any large or systematic discrepancy should be investigated, most likely pointing to either errors in the raw series, the presence of extreme observations, or potentially misspecified seasonal adjustment options.

The advantages of the indirect approach are that the additivity is respected; moreover, the period to period change or the growth rate in  $A_{\bullet,\bullet}^I$  could be explained by that in some of the major components since

$$A_{\bullet,\bullet,t}^I - A_{\bullet,\bullet,t-1}^I = \sum_{m=1}^M (A_{m,\bullet,t} - A_{m,\bullet,t-1}) \quad (19.4)$$

and

$$\frac{A_{\bullet,\bullet,t}^I - A_{\bullet,\bullet,t-1}^I}{A_{\bullet,\bullet,t-1}^I} = \sum_{m=1}^M \left( \frac{A_{m,\bullet,t-1}}{\sum_{m=1}^M A_{m,\bullet,t-1}} \right) \frac{A_{m,\bullet,t} - A_{m,\bullet,t-1}}{A_{m,\bullet,t-1}} \quad (19.5)$$

The disadvantage is that the indirect seasonally adjusted series may contain residual seasonality.

### 19.2.3 Guidelines

The 2009 ESS Guidelines on Seasonal Adjustment (Eurostat (2009)) states that users should carefully consider the application of either direct or indirect and make an informed choice relating to all known requirements. The direct approach is preferred for transparency and accuracy, especially when component series show similar seasonal patterns. The indirect approach is preferred when components series show seasonal patterns differing in a significant way. The presence of residual seasonality should always be checked in all of the indirectly seasonally adjusted aggregates.

There are situations where the direct approach is preferred and the discrepancies have to be removed especially when there are strong user requirements for consistency between the lower and higher level aggregates. In such a case, reconciliation must be used, i.e., the seasonally adjusted components and the total series must be corrected to satisfy the constraints.

### 19.2.4 The two-way classification

Consider now the two-way table where there are two classifications for the total series, i.e,

$$X_{\bullet,\bullet} = \sum_{m=1}^M X_{m,\bullet} = \sum_{n=1}^N X_{\bullet,n} \quad (19.6)$$

It is very likely that

$$A_{\bullet,\bullet} \neq \sum_{m=1}^M A_{m,\bullet} \neq \sum_{n=1}^N A_{\bullet,n} \quad (19.7)$$

and consequently there will be more than just one set of discrepancies.

### 19.2.5 SA of a two-way table

Bottom-up: Obviously, one simple solution to the reconciliation problem is to seasonally adjust at the lowest level of the cross-classification, say  $A_{m,n}$ , and to obtain all the marginal totals indirectly; however, this level may be too detailed and the impact of the irregular components at that level too dominant for proper seasonal adjustment.

#### Top-bottom

An alternative solution is to adopt a top-bottom approach where  $A_{\bullet,\bullet}$  is first obtained. Then  $A_{m,\bullet}, m = 1, \dots, M$  and  $A_{\bullet,n}, n = 1, \dots, N$  are raked to agree with  $A_{\bullet,\bullet}$ . Then the lower component series  $A_{m,n}, m = 1, \dots, M; n = 1, \dots, N$  are raked to agree with the raked values of  $A_{m,\bullet}, m = 1, \dots, M$  and  $A_{\bullet,n}, n = 1, \dots, N$ .

#### Middle-down, middle-up:

Obviously, a middle approach can be used. For example,  $A_{m,\bullet}, m = 1, \dots, M$  can be used to obtain the indirect SA of  $X_{\bullet,\bullet}$ . Next,  $A_{\bullet,n}, n = 1, \dots, N$  are raked to agree with  $A_{\bullet,\bullet}^I$ . Then the lower component series are raked to those marginal totals.

## 19.3 Techniques to force the respect of aggregation constraints of seasonally adjusted data when a direct adjustment is performed

For now on, raking, reconciliation and balancing are synonyms used to describe the process of restoring cross-sectional aggregation constraints in time series systems. Optionally, temporal constraints can also be preserved. Some general concepts and consideration follow.

Binding and non-binding totals: Raking occurs when the total series and the components series were independently adjusted. Then the component series are raked to the total series. A given total is defined as binding if it must not be altered by the raking process. In that case, all the discrepancies are allocated to the component series. It is defined as a non-binding total if it can be altered i.e. some of the discrepancies can be allocated to the total itself.

Raking could be done in one, two or more dimensions with or without additional temporal constraints. Here are a few typical cases that can be handle with Statistics Canada in-house SAS Proc TSRaking procedure (Bérubé and Fortier (2009); Quenneville and Fortier (2012)).<sup>1</sup>

- One-way raking: Reconcile  $A_{m,\bullet}, m = 1, \dots, M$  to ensure they add up to the grand total  $A_{\bullet,\bullet}$ .
- One-way raking with annual totals: One-way raking with the added constraints that for each series, the annual total will be preserved. This case applies when the series have to satisfy annual benchmarks.
- Two-way raking: Reconcile  $A_{m,n}, m = 1, \dots, M; n = 1, \dots, N$  to agree with the raked values of  $A_{m,\bullet}, m = 1, \dots, M$  and  $A_{\bullet,n}, n = 1, \dots, N$ .
- Two-way raking with annual totals: Two-way raking with the added constraints that for each series, the annual total will be preserved

### 19.3.1 Pro-rating

To illustrate the methods, consider the one-way classification where  $M$  components  $A_{m,\bullet} > 0$  for  $m = 1, \dots, M$  must add up to the grand total  $A_{\bullet,\bullet} > 0$ . With pro-rating, the reconciled estimates, denoted  $\hat{A}_{m,\bullet}$ , are:

$$\hat{A}_{m,\bullet} = A_{\bullet,\bullet} \frac{A_{m,\bullet}}{\sum_m A_{m,\bullet}} = A_{m,\bullet} + \left( A_{\bullet,\bullet} - \sum_m A_{m,\bullet} \right) \frac{A_{m,\bullet}}{\sum_m A_{m,\bullet}}, \quad (19.8)$$

showing that the discrepancy  $(A_{\bullet,\bullet} - \sum_m A_{m,\bullet})$  is allocated to the components  $A_{m,\bullet}$  in proportion of the contribution of  $A_{m,\bullet}$  to the total  $\sum_m A_{m,\bullet}$ ; obviously,  $\sum_m \hat{A}_{m,\bullet} = A_{\bullet,\bullet}$ . The reconciled estimates are also obtained as the solution to the following constrained minimization problem with respect to the unknown parameters  $\theta_m$ :

$$\min \sum_{m=1}^M \frac{(A_{m,\bullet} - \theta_m)^2}{A_{m,\bullet}} \quad (19.9)$$

subject to

<sup>1</sup>SAS is a registered trademark of SAS Institute Inc.

$$\sum_{m=1}^M \theta_m = A_{\bullet,\bullet} \quad (19.10)$$

The solution is simply  $\hat{\theta}_m = \hat{A}_{m,\bullet}$  from 19.8.

### 19.3.2 Generalization of pro-rating

It is shown in Quenneville and Fortier (2012) that pro-rating can be easily generalized to handle a non-binding total and specifying prior alterability coefficients. For example, consider the minimization problem with respect to the unknown parameters  $\theta_m$ :

$$\min \sum_{m=1}^M \frac{(A_{m,\bullet} - \theta_m)^2}{|c_m A_{m,\bullet}|} + \frac{(A_{\bullet,\bullet} - \sum_{m=1}^M \theta_m)^2}{|c_g A_{\bullet,\bullet}|} \quad (19.11)$$

where  $c_m$  and  $c_g$  are pre-specified alterability coefficients associated with each estimates. The solution may be written as:

$$\hat{\theta}_m = \hat{A}_{m,\bullet} = A_{m,\bullet} + (A_{\bullet,\bullet} - \sum_m A_{m,\bullet}) \frac{|c_m A_{m,\bullet}|}{|c_g A_{\bullet,\bullet}| + \sum_m |c_m A_{m,\bullet}|} \quad (19.12)$$

which permits  $c_m \geq 0$  and  $c_g \geq 0$  as long as they are not all equal to 0.

### 19.3.3 Generalization of pro-rating to higher dimensional tables

To make the transition from the one-way classification to higher dimensional cases, let  $y = (s', g)'$  where  $s = (A_{1,\bullet}, \dots, A_{M,\bullet})'$  and  $g = A_{\bullet,\bullet}$ . Let  $X = [I_M, G']'$  where  $I_M$  is the Identity Matrix of order  $M$  and  $G = 1'_M$  the unit row vector of length  $M$ . Let  $V_e$  be a diagonal matrix with  $(|c_1 A_{1,\bullet}|, \dots, |c_M A_{M,\bullet}|)$  on the main diagonal,  $V_e = (|c_g A_{\bullet,\bullet}|)$  and  $V_\mu = \text{block}(V_e, V_e)$ . Then  $\hat{\theta}$  from (19.12) is computed as

$$\begin{aligned} \hat{\theta} &= s + V_e G' (G V_e G' + V_e)^{-1} (g - G s) \\ &= [V_e^{-1} + G' V_e^{-1} G]^{-1} (V_e^{-1} s + G' V_e^{-1} g) \\ &= (X' V_\mu^{-1} X)^{-1} X' V_\mu^{-1} y. \end{aligned} \quad (19.13)$$

$\hat{\theta}$  from 19.13 is the solution to the minimization of the following quadratic form

$$(y - X\theta)' V_\mu^{-1} (y - X\theta) \quad (19.14)$$

with respect to  $\theta$ . The solution 19.13 can therefore be easily generalized to handle redundant constraints using the Moore-Penrose inverse. It will also handle higher dimensional problems, including temporal benchmarks, by properly defining the components of the observation vector  $y$ , the matrix  $G$  containing the linear constraints between the components and the matrix  $V_\mu$ . See Dagum and Cholette (2006), Quenneville and Fortier (2012) and Di Fonzo and Marini (2011) for more details.

### 19.3.4 Simple numerical examples

One-way classification: Let  $s = (18, 14, 13)$  and  $g = 50$ . As  $s_1 + s_2 + s_3 = 45$ , there is a discrepancy of  $5 = 50 - 45$  to allocate to the components. Suppose  $c_s = (c_1, c_2, c_3) = (1, 0, 1)$  so that  $s_2 = 14$  remains unchanged and that  $c_g = 0$  to have a binding total. Define  $V_e, V_\epsilon$  and  $V_\mu$  as in the previous section. Let  $X = [I_3, G']'$  where  $G = 1'_3$ . The solution is  $\hat{\theta} = (20.9, 14, 15.1)$ . Note that  $\theta_2 = 14 = s_2$  because  $c_2 = 0$ .

Two-way classification: Consider having to reconcile the following inner table to the given marginal total:

18	14	13	50
20	12	15	45
43	20	32	

Note that the marginal totals are coherent as  $43 + 20 + 32 = 95 = 50 + 45$ . Let  $s = (18, 14, 13, 20, 12, 15)$  represents the components of the inner table,  $g = (50, 45, 43, 20, 32)$  the alterability constraints,  $c_s = 1'_6$  and  $c_g = 0'_5$ . Let  $V_e, V_\epsilon$  and  $V_\mu$  defined as in the previous section. Let

$$G = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

and  $X = [I_6, G']'$ .

The solution is

22.03	11.9	16.07	50
20.97	08.1	15.93	45
43.00	20.0	32.00	

### 19.3.5 Specifying the alterability coefficients

The alterability coefficients are used to specify the variance of the measurement error associated with  $s$  and  $g$ . For example,  $(c_m s_m)^{1/2}$  is the standard error of the measurement error associated with  $s_m$ . Possible choices for the alterability coefficients are as follow:

- $c_m = 1/s_m$ , the standard error is 1 or constant and the discrepancies will be allocated uniformly to all the components assuming  $c_g = 0$ . For example, if  $s = (25, 5)$  and  $g = 40$ , the discrepancy 10 is split equally and the reconciled values are (30, 10).
- $c_m = 1$ , the variance of the measurement is proportional to  $s_m$ ; consequently, this generalizes pro-rating and this preserves the relative contributions of the components. In the simple example, the reconciled values are (33.33, 6.67).
- $c_m = s_m^2$ , the variance of the error is proportional to  $s_m^2$ ; consequently, it reduces the changes to the smallest components by allocating more of the discrepancies to the largest components. In the simple example, the reconciled values are (34.62, 5.38).
- $c_m = 0$ , then one must have  $c_g \neq 0$ ; the reconciled values are unmodified and the new total is  $25 + 5 = 30$ .

## 19.4 Movement preservation principle based benchmarking techniques

### 19.4.1 The 2-step based methods

Raking requires that, in the end, the contemporaneous and the temporal constraints are respected. This should be done without undue impact on the basic characteristics of the original component series, such as the period to period movement. This is usually achieved by specifying the matrix  $V_\mu$  in some optimal way.

Like in Dagum and Cholette (2006), and as described in Quenneville and Fortier (2012), the last line of the least squares solution( 19.13) comes from assuming the linear model:

$$y = X\theta + \mu, \quad E(\mu) = 0, \quad E(\mu\mu') = V_\mu \quad (19.15)$$

where

$$y = \begin{bmatrix} s \\ g \end{bmatrix}, \quad X = \begin{bmatrix} I_K \\ G \end{bmatrix}, \quad \mu = \begin{bmatrix} e \\ \epsilon \end{bmatrix}, \quad V_\mu = \begin{bmatrix} V_e & Cov(e, \epsilon) \\ Cov(\epsilon, e) & V_\epsilon \end{bmatrix} \quad (19.16)$$

and  $I_K$  is the  $K \times K$  identity matrix.  $K$  depends on the problem at hand, for example,  $K = 3$  and  $K = 6$  in the simple numerical examples of Section 19.3.4. For a reconciliation model, one has to properly describe the constraints coverage matrix  $G$  and most importantly the variance matrix  $V_\mu$  made of the variance matrix of  $e$ , of the variance matrix of  $\epsilon$ , and of the covariance matrix between  $e$  and  $\epsilon$ . In general, the size of the matrices involved can become quite large when cross-correlation is allowed between and within the components series. Dagum and Cholette (2006), Section 11.2 provides a convenient expression of the solution when  $Cov(e, \epsilon) = 0$ , that is, the first line of 19.13.

Still, even by assuming  $Cov(e, \epsilon) = 0$ , the size of  $V_e$  can be quite large. Dagum and Cholette (2006) propose a 2-step procedure. In the first step, benchmark each component series to its temporal constraints and compute the resulting error covariance matrices for each benchmarked series; and in the second step, reconcile the contemporaneous constraints using the error covariance matrices computed in the first step. With this solution, the matrix  $V_e$  in the second step becomes a block diagonal matrix where each diagonal element is itself the error covariance matrix associated with the  $m$ th series from the first step. Even with this simplification, the size of the matrices involved in the second step can be large, this usually leads to an extremely computer intensive solution.

Quenneville and Rancourt (2005) simplified further the 2-step approach. In the first step, benchmark each component series to its temporal constraints without computing any kind of error covariance matrices; and for the second step, reconcile the contemporaneous constraints with a simple weighted (and not generalized) least squares approach; in other words, the covariance matrix  $V_\mu$  in the second step may have terms on the diagonal but no covariance terms: neither between or within the benchmarking prediction error vectors of the component series. When there is an annual benchmark, the second step only require to jointly use the observations and constraints within the range of an annual benchmark; hence, the method is applied year by year. For contemporaneous constraints alone, processing for the second step is done by period.

The main advantage of this approach is its simplicity. The univariate benchmarking in the first step may be achieved with any software package that implement temporal benchmarking (see Chapter 17). In the reconciliation step, the year by year application also significantly reduces the size of the matrices involved and decreases the processing time. Details and proof of the optimality of this 2-step method are provided in Quenneville and Fortier (2012).

## 19.4.2 The simultaneous method of Di Fonzo and Marini (2010)

In the simultaneous method of Di Fonzo and Marini (2011), the reconciliation of systems of time series subject to both temporal and contemporaneous constraints is solved in such a way that the temporal profiles of the original series are preserved according to a pre-defined optimal way, usually a Denton's type criteria such as the one discussed in Chapter 17. They show that the constrained minimization problem can be solved by exploiting the sparsity of the linear system to be solved. They show that the 2-step procedure of Quenneville and Rancourt (2005) might be more suitable in case of large systems and compare the results of applying both methods to two examples. Their main contribution is to have shown the feasibility of the simultaneous reconciliation of a large system of series using an efficient algorithm to inverse sparse matrices.

## 19.5 Example

Let  $X_{m,n}$  be the Monthly Unemployment(1000 persons) by Sex ( $m = 1, 2$  for Males and Females) and Age groups ( $n = 1, 2$  for Minus 25 years old, Plus 25 years old) at the European Union (27 countries) level according to the ILO definition from January 2000 to September 2010. The 4 data series including the 4 marginal totals and the grand total are available from the Eurostat web site. The 9 data series form a two-way table of time series, which, apart from a few rounding errors, are balanced for the 129 time points in the range from January 2000 to September 2010. For the purpose of this example, the 9 series were seasonally adjusted by X-12-ARIMA as implemented in SAS Proc X12 with only the X11 specifications with default options.

### 19.5.1 Bottom-up

The bottom-up approach consists in seasonally adjusting the 4 series  $X_{m,n}$ ,  $m = 1, 2$  and  $n = 1, 2$  and to obtain all the seasonally adjusted marginal totals indirectly, ie,

$$\begin{aligned} A_{m,\bullet}^I &= A_{m,1} + A_{m,2}, m = 1, 2 \\ A_{\bullet,n}^I &= A_{1,n} + A_{2,n}, n = 1, 2 \\ A_{\bullet,\bullet}^I &= A_{1,1} + A_{1,2} + A_{2,1} + A_{2,2}. \end{aligned} \quad (19.17)$$

That is pretty obvious and it will not be discussed further.

### 19.5.2 Top-bottom

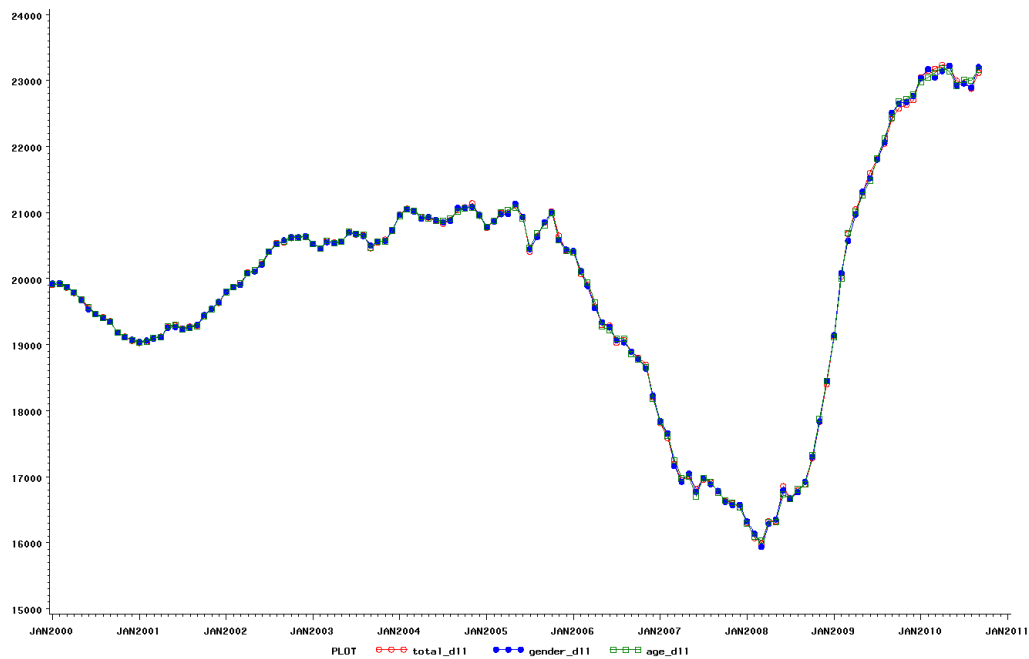
The purpose of this chapter is mostly to illustrate the top-bottom approach with reconciliation. It consists in seasonally adjusting the 9 series  $X_{\bullet,\bullet}$ ,  $X_{m,\bullet}$ ,  $m = 1, 2$ ,  $X_{\bullet,n}$ ,  $n = 1, 2$ , and  $X_{m,n}$ ,  $m = 1, 2$ ;  $n = 1, 2$  and to restore the additivity constraints in an hierarchical way.

First, the grand total and the marginal totals are seasonally adjusted. Next, the indirect seasonally adjusted totals from the Sex (Gender) and Age classifications are computed and compared against the direct seasonally adjusted series. The series are provided in Figure 19.1. The discrepancies are provided in Figures 19.2 and 19.3.

The discrepancies expressed in percentage are very small, less than 1%. The raked series for the Males and Females series showed in Figures 19.4 and 19.5 are obtained by simply allocating the discrepancies from Figure 19.2 at pro-rata to each time point.

Figures 19.6 and 19.7 show the raked Minus 25 years and Plus 25 years series obtained in a similar way using the discrepancies from Figure 19.3. That is,  $A_{1,\bullet,t}$  and  $A_{2,\bullet,t}$  are raked to  $A_{\bullet,\bullet,t}$  for all time points  $t$ ;

**Figure 19.1:** Monthly SA Unemployment (1000 persons) at the European Union level from January 2000 to September 2010: Direct SA total and indirect totals from the Gender and Age classifications



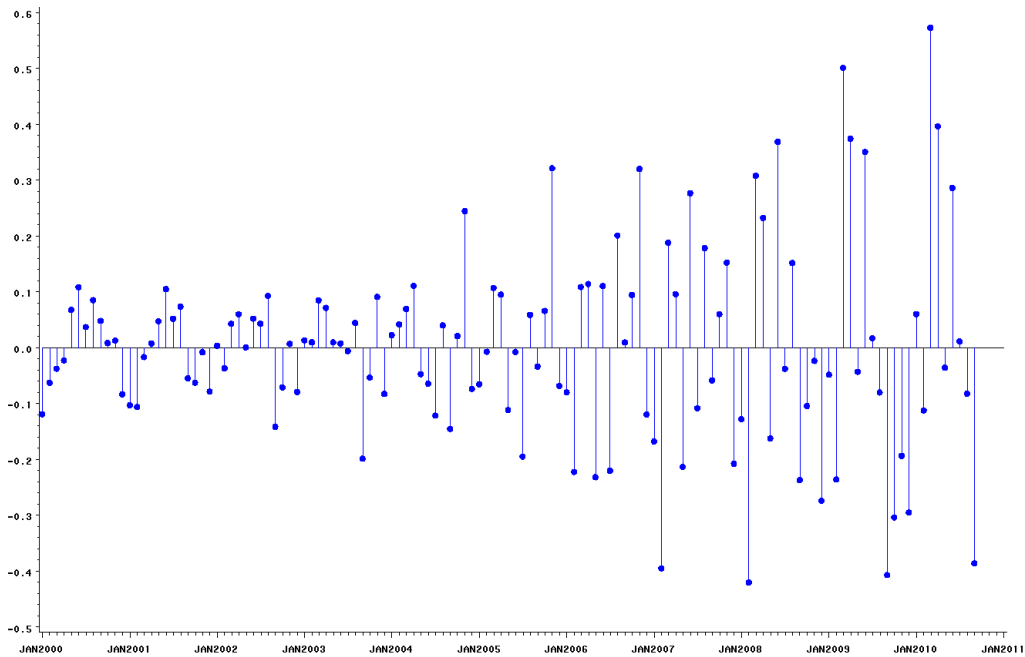
and  $A_{\bullet,1,t}$  and  $A_{\bullet,2,t}$  are raked to  $A_{\bullet,\bullet,t}$  for all time points  $t$ . This involves two one-way reconciliations for each of the 129 time points.

Next, the inner tables of the cross-classified series  $A_{m,n,t}$  are raked to the raked marginal totals for all time points  $t$ . This involves a two-way reconciliation for each of the 129 time point. This can be done with Proc TSRaking; alternatively, iterative proportional fitting can be used if available, and both provide the same results. The 4 raked series are displayed in Figures 19.8, 19.9, 19.10 and 19.11. Again, it can be observed that the initial and raked series are very close to each other.

For this example, reconciliation was a simple mathematical reallocation of the discrepancies using generalized pro-rating. There were no temporal benchmarking constraints; so, the 2-step method reduces to only the second step using either the one-way and two-way raking options of Statistics Canada in-house SAS Proc TSRaking procedure with alterability coefficients set to 1 for the components and 0 for the constraints.



**Figure 19.2:** Discrepancies in percentage between the direct SA total and indirect total from the Gender classification



**Figure 19.3:** Discrepancies in percentage between the direct SA total and indirect total from the Age classification

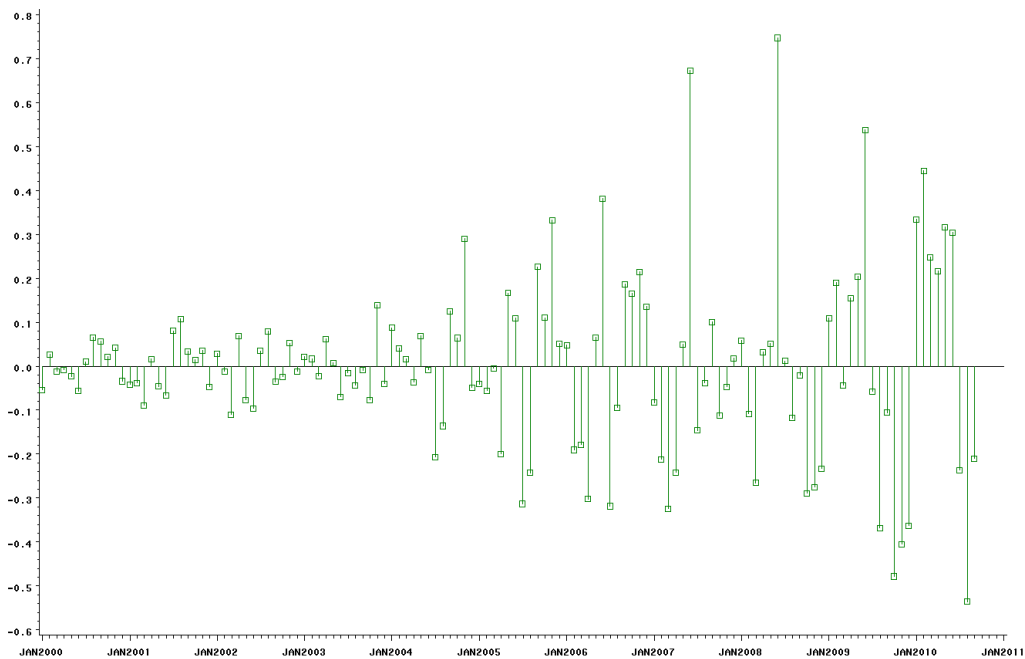


Figure 19.4: Direct and raked SA series for Males

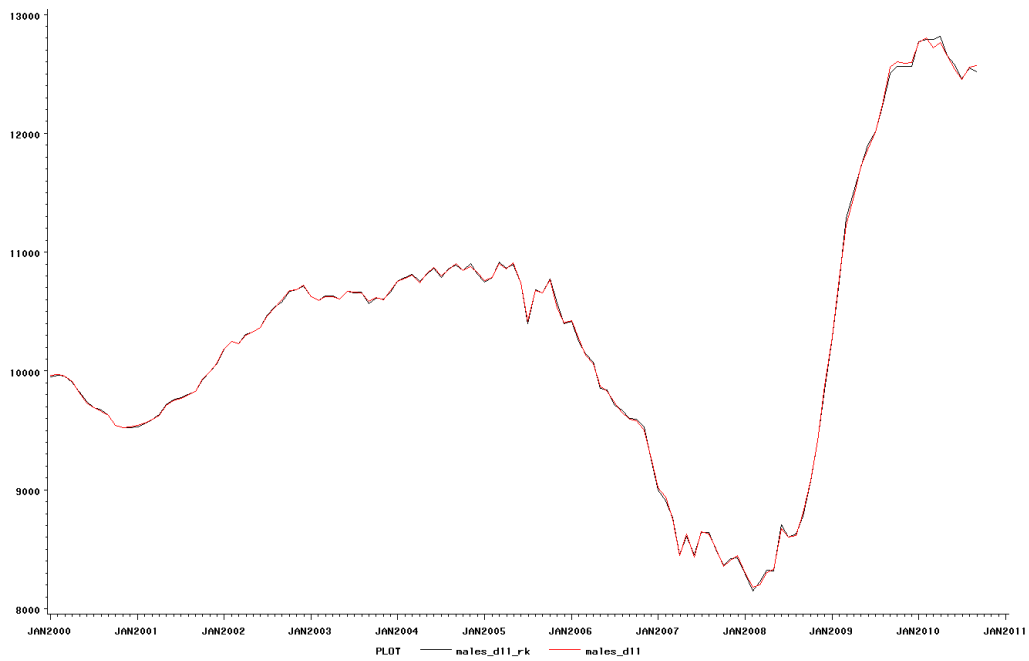


Figure 19.5: Direct and raked SA series for Females



Figure 19.6: Direct and raked SA series for Minus 25 years

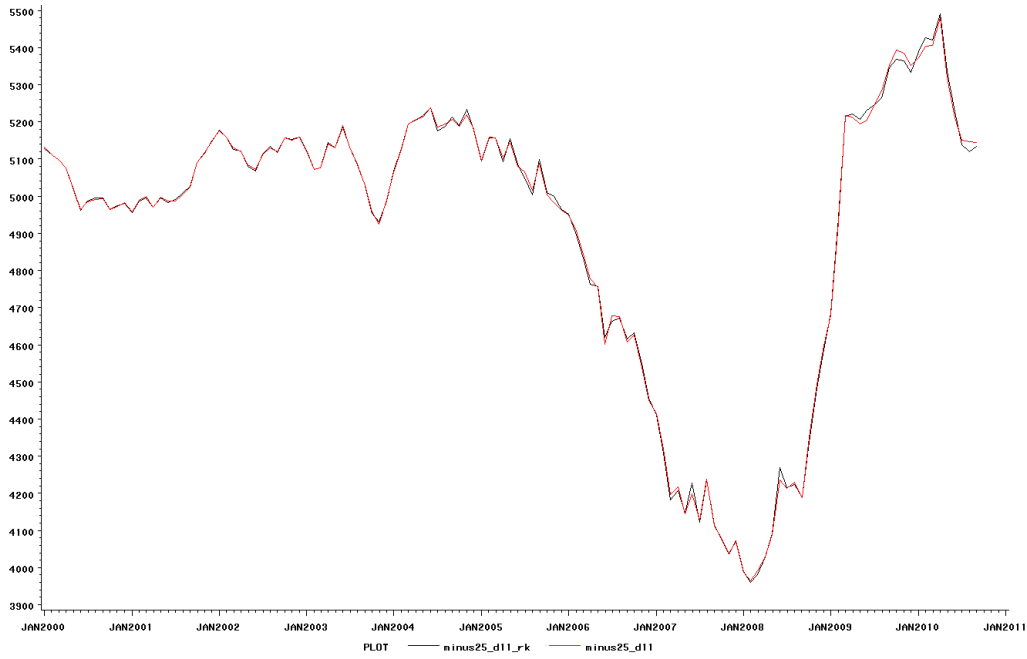
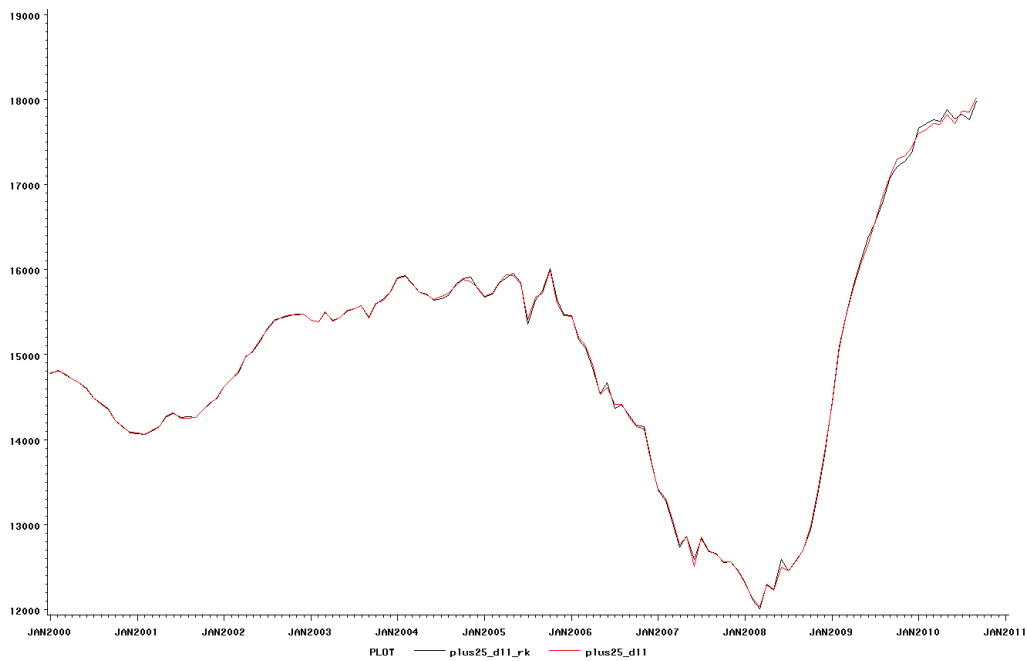


Figure 19.7: Direct and raked SA series for Plus 25 years



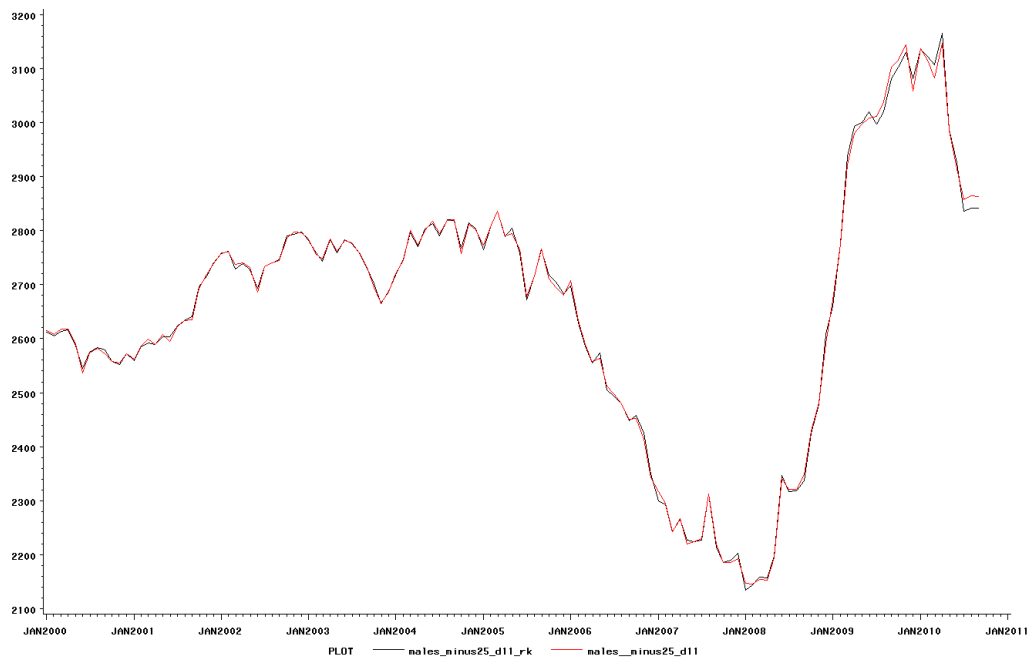
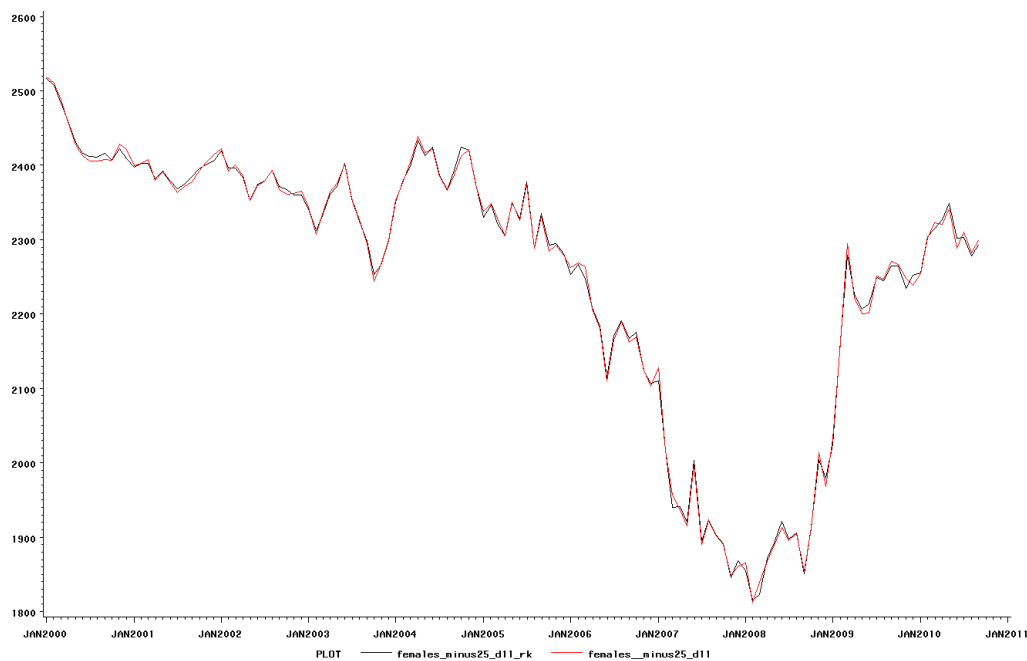
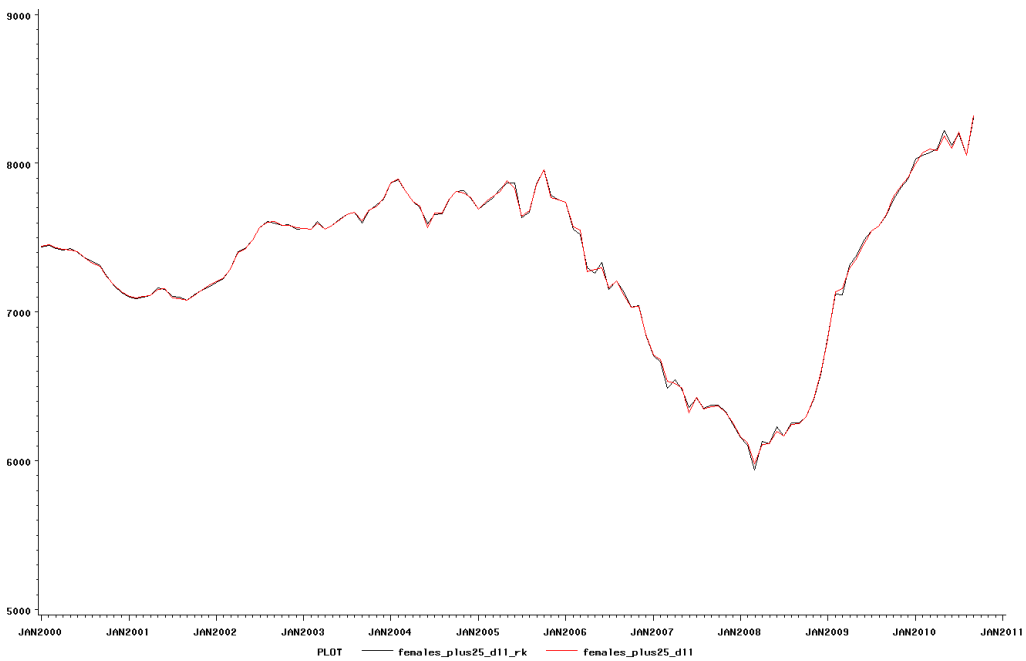
**Figure 19.8:** Direct and raked SA series for Minus 25 years**Figure 19.9:** Direct and raked SA series for Females Minus 25 years

Figure 19.10: Direct and raked SA series for Males Plus 25 years



Figure 19.11: Direct and raked SA series for Females Plus 25 years



### 19.5.3 Middle-up middle-down

For a middle-up and middle-down approach, the previous top-bottom approach can be applied to each of the 9 series from the 27 countries. Next, the various higher up aggregates are computed indirectly using the bottom-up approach.

## 19.6 Conclusions

This chapter reviewed some concepts and methods for the seasonal adjustment of variables linked by aggregation constraints. In particular, it discussed simple techniques to force the respect of aggregation constraints when necessary. The reconciliation was illustrated with the series of Monthly Unemployment by Sex and Age groups at the European Union (27 countries) level according to the ILO definition from January 2000 to September 2010 using Statistics Canada in-house SAS Proc TSRAking procedure.

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# VI

## Revision and Communication







# 20 Revisions



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## Table of Contents

20.1	Introduction	519
20.2	Defining revisions	519
20.3	Causes of revisions	520
20.3.1	Causes of revisions in observed data	521
20.3.2	Causes of revisions in seasonal adjustment outputs	522
20.4	To revise or not to revise?	528
20.5	Revision policies	530
20.5.1	Revision policies for methodological changes	531
20.5.2	Revision policies for original estimates	531
20.5.3	Revision policies for seasonally adjusted estimates	532
20.5.4	Revision policies for specific outputs and methods	533
20.6	Presenting revisions	533
20.6.1	Revision triangles	533
20.6.2	Graphical approaches	534
20.7	Analysis of revisions	542
20.7.1	Revision diagnostics	542
20.7.2	Decomposing revisions	544
20.7.3	Using X-13-ARIMA-SEATS to assess revisions	548
20.8	Dealing with revisions in practice	554
20.8.1	Case study: Argentine currency in circulation	554
20.8.2	Case study: Real time databases	560
20.8.3	Case study: Revision analysis using History	560
20.9	Discussion	561

## 20.1 Introduction

Statistical data are generally revised to incorporate new and updated information. Therefore, updates of estimates are inevitable whenever statistical data are produced that report promptly on economic developments, despite the fact that some relevant information is still outstanding. From both the user and producer perspective, changes to published information are something of a double-edged sword. On one hand, changes enhance the information available and are welcome. On the other hand, changes to previously published economic data may result in a different assessment of the state of the economy. In practice, frequent and/or major revisions can damage the credibility of statistical data. Hence, it is an important question whether statistical results should be revised or not. Section 20.3 deals with this question after defining revisions in section 20.2.

The causes of revisions are described in section 20.4. Apart from the reasons for revising unadjusted data, there are additional sources which can yield changes in the seasonally adjusted results. Especially within a filter based seasonal adjustment approach, the changeover from using fully asymmetric filters at the end of a time series to more and more symmetric filters in the middle of a time series affects the adjusted results.

In order to deal with the changes in the data, different revision policies can be employed (20.5). For the sake of transparency, the applied approach should be communicated to users. Hence, presentation issues are discussed in section 20.6.

There are many possible ways for analysing revisions. Different diagnostics shall be used depending on the purpose of the investigation (see section 20.7). The next subsection tries to decompose revisions of seasonally adjusted results into corrections of unadjusted data and revisions stemming from seasonal adjustment. This is not an easy task because some of the changes in the seasonal component are the result of modified unadjusted values and cannot be explained with the changeover from asymmetric to symmetric filters. Section 20.7 furthermore deals with the assessment of revisions using the X-13-ARIMA-SEATS program (or variants). Examples are given in section 20.8.

Finally, revisions are discussed in the context of the ESS Guidelines on Seasonal Adjustment.

## 20.2 Defining revisions

Revisions to published information are the changes between two publications that describe the same phenomenon. In the context of published time series estimates, revisions are usually the change in level or the change in growth rate of a time series for a particular time point. For example, if a growth rate for a seasonally adjusted index of production, measuring the rate of growth from January 2009 to January 2010 is published as +1.2% in February 2010, but is revised the following publication in March 2010 to +1.3%, then there is a revision of +0.1%.

If  $Y_t$  is the value of a time series at time  $t$ , then  $Y_{t|k,k+l}$  is the value of a time series at time  $t$ , given that it is part of a time series from time point  $k$  to time point  $k+l$ . For example,  $Y_{Jan.2010|Jan.2000,Jan.2010}$  is the value of a time series at January 2010 given time points for the series are available from January 2000 up to January 2010. Whereas,  $Y_{Jan.2010|Jan.2000,Feb.2010}$  is the value of a time series at January 2010 given time points for the series are available from January 2000 to February 2010.

When discussing revisions in the context of seasonal adjustment, the value of  $k$  may be of less interest as the first point in a time series, generally, does not change often. Whereas there is a new end point every period, which is one source of revisions to seasonally adjusted and trend estimates. Therefore this notation is shortened to  $Y_{t|\tau}$  where  $\tau = l + k$ . A revision to a time series at point  $t$  given  $m$  additional data points is

$$R_{t|\tau+m} = Y_{t|\tau+m} - Y_{t|\tau} \quad (20.1)$$

Revisions could also be presented as a relative revision, percentage difference, absolute revision, or the absolute relative revision.

$$\begin{aligned}
 \rho_{t|\tau+m} &= (Y_{t|\tau+m} - Y_{t|\tau})/Y_{t|\tau} \\
 P_{t|\tau+m} &= (Y_{t|\tau+m} - Y_{t|\tau})/Y_{t|\tau} \times 100 \\
 \delta_{t|\tau+m}^x &= |Y_{t|\tau+m} - Y_{t|\tau}| \\
 K_{t|\tau+m}^x &= (|Y_{t|\tau+m} - Y_{t|\tau}|)/Y_{t|\tau}
 \end{aligned}
 \tag{20.2}$$

Most often in published official statistics  $Y_{t|\tau}$  will be a level or growth rate of the original, seasonally adjusted or trend estimate. In this chapter the following notation will be used.

$O_{t|\tau}$  = level of an unadjusted time series estimate at time  $t$ , given time points to time  $\tau$

$A_{t|\tau}$  = level of a seasonally adjusted estimate at time  $t$ , given time points to time  $\tau$

$S_{t|\tau}$  = level of an estimated seasonal factor at time  $t$ , given time points to time  $\tau$

$T_{t|\tau}$  = level of trend estimate at time  $t$ , given time points to time  $\tau$

$I_{t|\tau}$  = level of the irregular component at time  $t$ , given time points to time  $\tau$

$r_{t,n|\tau}^x$  = rate of growth of time series  $x$  from time  $t - n$  to time  $t$ , given time points to time  $\tau$

$R_{t|\tau+m}^x$  = revision to time series  $x$  at point  $t$  given  $m$  additional data points from time  $\tau$

$\rho_{t|\tau+m}^x$  = relative revision to time series  $x$  at point  $t$  given  $m$  additional data points from time  $\tau$

$P_{t|\tau+m}^x$  = percent revision to time series  $x$  at point  $t$  given  $m$  additional data points from time  $\tau$

$\delta_{t|\tau+m}^x$  = absolute revision to time series  $x$  at point  $t$  given  $m$  additional data points from time  $\tau$

$K_{t|\tau+m}^x$  = absolute relative revision to time series  $x$  at point  $t$  given  $m$  additional data points from time  $\tau$

Whilst this general notation fully explains all information that could be required it is overly detailed for many purposes. In many instances it is simplified by dropping subscripts where it is not required.

## 20.3 Causes of revisions

In official statistics, revisions to published estimates occur for a number of reasons: from the on-going incorporation of late survey data, to the less regular update of weights used to combine lower level survey estimates, as well as methodological processes used and planned changes in international classifications and standards. Often, there is no single reason to explain why time series estimates are revised, but rather a combination of reasons.

Revisions will occur due to changes to the original estimates, methodological processes and seasonal adjustment methods. In practice, there are many different ways for classifying revisions and there is a range of frameworks available which categorise different revisions. One example is a comprehensive framework which was developed by a joint OECD and Eurostat taskforce (OECD/Eurostat taskforce (2008)):

Some of these categories are discussed in more detail in the following sections with a focus on how they impact on the original and seasonally adjusted estimates. It is important to understand the causes of revisions in order to appropriately communicate them to users and to understand what aspects to revise and when revisions should be incorporated into the statistical process.

## 20.3.1 Causes of revisions in observed data

### 20.3.1.1 Returns from survey respondents

**Table 20.1:** Framework

Data revisions	Incorporation of “late” data Replacement of data by expert judgement or by statistical methods Incorporation of data more closely linked to the concept being measured, e.g. benchmarking Correction of data or compilation errors
Time series adjustment revisions	From use of concurrent seasonal adjustment From reassessment of seasonal adjustment From changes to the time series model
Changes in concepts, definitions and classifications	Changes in classifications Rebasing Re-referencing
Methodological improvements	Improvements to estimation methods Revisions arising from changes in surveys Introduction of new data sources, e.g. survey or administrative

Late data returns from survey respondents are the main reason why the original estimates are revised. Usually, not all the data that is needed to produce the statistics are available for the most recent time point and missing data, such as survey responses, may need to be estimated. Since it is generally the case that the estimated values do not correspond precisely to the late incoming data, the original figures have to be revised once new information becomes available.

In exceptional cases, respondents may even provide the wrong information. For example, returned data from a respondent may include the wrong coverage such as for a bigger region than needed, or include taxes when the estimates should not include them, or place items in the wrong classification. Once the correct data is received this means that the original estimates for that particular respondent will need to be revised to ensure that comparable estimates are produced over time.

### 20.3.1.2 Benchmarking data

The use of benchmarking also gives rise to revisions. The process of benchmarking has already been discussed in more detail in this Handbook (see chapters 17 and 19). In general, the process of benchmarking means adjusting (generally) higher frequent data such as monthly or quarterly estimates to take account of more complete lower frequent results, which become available only later such as annual data. Benchmarking techniques are also used in other areas of the statistical process. Benchmarking adjustments are commonplace in the compilation of the national accounts; for example, they are used when incorporating the results of the turnover tax statistics (which may become available some 12 months later to the reference period) or those of multi-annual surveys. For example, in Germany, the Federal Statistical Office brings the monthly production indices into line with the results of the more complete quarterly production statistics.



### 20.3.1.3 Methodological and classification changes

Methodological changes can occur for a large number of reasons. A full list is not attempted here, but any change to the statistical value change, from question changes on the survey instrument, to changes in the sample design, to changes in data sources such as use of administrative data instead of survey estimates, editing and imputation procedures, methods of estimation, or index number construction, could all lead to revisions in the original estimates.

Revisions will be caused by changes to the base year, or re-referencing, which together with new weighting systems such as moving from fixed based weights to chain-linking, can also alter the paths of previously published estimates.

The calculation of constant price estimates will be impacted by the level and availability of deflator information. For example, if the deflation method is changed to a different product level this will cause revisions to the constant price estimates even though the original current price estimates may have remained unchanged.

Changes in classifications will cause revisions, but these type of events will occur on a less regular basis, for example, every five years or so. This change can cause items to be redefined, meaning that the results including the old and new items are different, if they are even comparable at all. It can also include items for the first time. However, even if the definitions of certain sections did not change at all, data in any newly-defined subgroups might be assigned a different weighting meaning that, in practice, the result for the aggregated item (though identically defined) differed from that obtained previously. Revisions to previously published back data are needed to ensure that the published estimates are comparable over time.

## 20.3.2 Causes of revisions in seasonal adjustment outputs

### 20.3.2.1 Changes to the original estimates

A change, or revision, to the original estimates will ultimately have an effect on the seasonal adjustment outputs. Even if the seasonal and calendar-related components remain with a constant underlying pattern over time, seasonally adjusted results will still change because of revisions to the original estimates as this additional information allows new estimates for the seasonal related components to be calculated.

### 20.3.2.2 Seasonal adjustment processes

Certain reasons for revisions are inherent to the process of seasonal adjustment itself and would occur irrespective of revisions to the original data. For example, when using a seasonal adjustment package, revisions have to be made to the value at time  $t$  when new values for  $t + 1$ ;  $t + 2$ , ... become available.

This is because, in seasonal adjustment, only the past and present information can be used to determine the value at time  $t$ . By contrast, the seasonal pattern can only be estimated reliably if the future values are also available, since they provide the information to assess whether the changes in seasonal fluctuation observed in the past will continue, or evolve, or even change to a different seasonal pattern. However, these upcoming figures are not yet available. The seasonal adjustment procedures can therefore only take present and past experience into account when estimating the current seasonal fluctuations and therefore the seasonally adjusted estimates. However, once new data points are available, they are also used when calculating the adjusted value at time  $t$  which means that the seasonally adjusted value at time  $t$  can change with each item of incoming data. New information could cause the seasonal adjustment parameters to change.

Depending on the program used for seasonal adjustment, the results will change due to any modified parameters. In practice, the variables used for calendar adjustment are key to both procedures. Among other things, extra observations enable new calendar effects to be detected. If this culminates in the use of new explanatory

variables, then data that have only been adjusted for calendar variation, or seasonal and calendar variation, will be revised, sometimes significantly.

There is a user choice to be made on how and when the seasonal adjustment options and parameters are updated. For example, the model and parameters used could be recalculated on a regular basis, say monthly or quarterly, or once a year. The ESS guidelines recommend that one acceptable option is that the model, filters, past outliers and calendar regressors are re-estimated only once a year but the parameters and factors are re-estimated every time new or revised data becomes available.

### 20.3.2.3 Use of filters within seasonal adjustment

In practice, economic time series typically contain seasonality that evolves over time (i.e., moving or stochastic seasonality). Hence a filter based seasonal adjustment approach, which allows for evolving seasonality, are suitable tools to estimate and remove the evolving seasonality.

A common filter based approach to seasonal adjustment are the X-11 related packages, e.g. X-13-ARIMA-SEATS. Under that approach, it is only possible to use asymmetrical filters (that take into account data up to the most recent time period) to derive the most recent seasonally adjusted value. The use of forecasts, through the use of ARIMA models for example, can reduce the impact of asymmetric filters. In practice, as each new original value changes, the formula used to seasonally adjust the value at time  $t$  changes as well. This is because of the use of increasingly symmetrical filters which take into account data from the subsequent periods. In the middle of a suitably long time series, it is possible to calculate the seasonally adjusted value using fully symmetrical filters. In this case, the original estimates that are the same distance from the relevant time period are equally weighted, although the weights decrease the further the figures are from the time period in question. The other advantage of using symmetrical filters is that this results in no systematic phase distortion in the historical data.

One component of revisions to the seasonally adjusted data arise as a result of the transition from asymmetrical to symmetrical filters. In practice the aim is to reduce the number of technically-induced revisions by producing optimum forecasts of future values. This is discussed further in chapter 10 in this Handbook.

These concepts can be illustrated as follows.

Let  $x_t$  denote the preadjusted time series:  $[\dots, x_{t-1}, x_t, x_{t+1}, \dots]$ .

Assume that the series  $x_t$  contains unobserved seasonality, as in

$$x_t = n_t + s_t, \quad (20.3)$$

where  $s_t$  represents the seasonal component, and  $n_t$  represents the seasonally adjusted series.

For seasonal adjustment, the aim is to estimate and remove  $s_t$  from  $x_t$ . Because seasonality can evolve over time (e.g. moving seasonality), the estimator can be obtained by means of a filter after the series has been pre-adjusted. The pre-adjusted series is assumed to follow an ARIMA model with normal independently and identically distributed innovations *n iid*, i.e., a linear series. Accordingly, linear filters can be used. Because the seasonally adjusted series needs to be in phase with the original series, the filter should be two-sided, centered, and symmetric. Assume the filter has  $2k + 1$  coefficients denoted by  $v_i$  (or weights), note that the finite filter assumption simplifies expositions but the following discussion can be extended in a straightforward manner, to infinite convergent filters. Then, we can express our estimator of the seasonally adjusted series as

$$\hat{n}_t = v_0 x_t + v_1 (x_{t+1} + x_{t-1}) + v_2 (x_{t+2} + x_{t-2}) + \dots + v_k (x_{t+k} + x_{t-k}). \quad (20.4)$$

Let  $B$  denote the backward shift operator, such that  $B^j x_t = x_{t-j}$  ( $j = 1, 2, \dots$ ) and  $F$  denote the forward shift operator, such that  $F^j x_t = x_{t+j}$  ( $j = 1, 2, \dots$ )



Equation 20.4 can be rewritten as

$$\hat{n}_t = [v_0 + v_1(B + F) + \dots + v_k(B^k + F^k)]x_t, \quad (20.5)$$

or

$$\hat{n}_t = v(B, F)x_t, \quad (20.6)$$

so that the seasonal adjustment filter can be expressed as

$$v(B, F)x_t = v_0 + \sum_{j=1}^k v_j(B^j + F^j). \quad (20.7)$$

When the observed series starts before period  $t - k + 1$  and extends beyond period  $t + k - 1$ , equation (20.4) provides the final or historical estimator of  $n_t$ . This estimator requires a long enough realization. Equation 20.4 yields the final or historical estimator of the SA series for time period  $t$ . To compute it, observations for later periods are needed. At time  $t$ , these future values have not been observed yet, and the filter cannot be completed.

To obtain preliminary estimators of the SA series for the end points of the period considered, there are several ways of circumventing the problem of the lack of observations at both ends.

- a Weights, e.g.  $v_i$ , can be designed that only apply to the available observations.
- b The observations which are not yet available,  $x_{t+j}$ ,  $j > 0$ , can be replaced with forecasts. The initial observations needed are estimated by backcasting the series. This is in fact equivalent to applying the Kalman Filter.
- c X-13-ARIMA-SEATS uses by default one year of ARIMA forecasts. Given that  $k$  will extend well beyond one year, the filters can be seen as a mixture of options a) and b) above. TRAMO-SEATS uses the number of ARIMA forecasts needed to complete the filter where this number is determined by the chosen model. In the Structural Time Series Model approach, the Kalman filter algorithm implicitly uses forecasts and backcasts that are then used for smoothing.

Given that it is likely that observations prior to the first observed period will never be observed, for a well specified model and a long enough series, backcasts will not generate significant or noticeable revisions to the seasonally adjusted estimates. The revisions will be generated by replacing and updating forecasts as new observations become available.

For completeness, let  $\hat{n}_{t|T}$  denote the estimator of the SA series  $n_t$  obtained when  $x_T$  is the most recent observation. According to (20.4), the estimator  $\hat{n}_t$  obtained with the complete filter, e.g. the "final" or "historical" estimator, requires  $k$  observations posterior to  $x_t$ . Thus, the preliminary estimator  $\hat{n}_{t|T}$ , e.g. the "concurrent" estimator, requires  $k$  forecasts of the series.

Starting with the concurrent estimator

$$\hat{n}_{t|t} = \sum_{j=-k}^0 v_j x_{t-j} + v_1 \hat{x}_{t+1|t} + v_2 \hat{x}_{t+2|t} + v_3 \hat{x}_{t+3|t} + \dots + v_k \hat{x}_{t+k|t}, \quad (20.8)$$

when the observation for period  $(t + 1)$  becomes available, the forecast  $\hat{x}_{t+1|t}$  is replaced by  $x_{t+1}$ , and the forecast  $\hat{x}_{t+i|t}$  for  $i > 0$  is updated to  $\hat{x}_{t+i|t+1}$ . This yields the first-revised estimator

$$\hat{n}_{t|t+1} = \sum_{j=-k}^0 v_j x_{t-j} + v_1 x_{t+1} + v_2 \hat{x}_{t+2|t+1} + v_3 \hat{x}_{t+3|t+1} + \dots + v_k \hat{x}_{t+k|t+1}, \quad (20.9)$$

The second revision replaces  $\hat{x}_{t+2|t+1}$  by  $x_{t+2}$  and updates  $\hat{x}_{t+i|t+1}$  to  $\hat{x}_{t+i|t+2}$   $i > 2$  yielding:

$$\hat{n}_{t|t+2} = \sum_{j=-k}^0 v_j x_{t-j} + v_1 x_{t+1} + v_2 x_{t+2} + v_3 \hat{x}_{t+3|t+2} + \dots + v_k \hat{x}_{t+k|t+2}, \quad (20.10)$$

Eventually, the historical estimator of the SA series at period  $t$ , given by (20.4) is obtained when  $x_{t+k}$  is observed. In brief, a preliminary estimator can be expressed in a manner similar to (20.6), as

$$\hat{n}_{t|T} = v(B, F)x_{t|T}^e, \quad (20.11)$$

where  $x_{t|T}^e$  is the observed series extended with forecast, i.e.,

$$x_{t|T}^e = [\dots, x_{t-1}, x_t, x_{t+1}, \dots, x_T, \hat{x}_{T+1|T}, \dots, \hat{x}_{t+k|T}]. \quad (20.12)$$

and, in general, the preliminary estimator of the SA series  $\hat{n}_{t|T}$  will be revised over the next  $(T - t)$  periods (as long as  $T - t \leq k$ ). By setting  $t = T + h$  in (20.11), an expression for the  $h$ -period ahead forecast of the SA series is obtained,

$$\hat{n}_{T+h|T} = v(B, F)x_{T+h|T}^e, \quad (20.13)$$

where  $x_{T+h|T}^e$  is the observed series extended with  $h$  additional forecasts, and centered at period  $T + h$ :

$$x_{T+h|T}^e = [\dots, x_{T-1}, x_T, x_{t+1}, \dots, \hat{x}_{T+h|T}, \dots, \hat{x}_{T+h+k|T}]. \quad (20.14)$$

Thus the minimal mean squared error forecasting of the unobserved component  $\hat{n}_{T+h|T}$  can be seen as a straightforward extension of the preliminary estimation.

The linear filters used in practice have specific properties.

- a. Revisions for the concurrent estimate can be illustrated as follows. The concurrent estimator (20.11) can be expressed as

$$\hat{n}_{t|t} = v_0 x_t + \sum_{j=1}^k v_j (x_{t-j} + \hat{x}_{t+j|t}). \quad (20.15)$$

Denote  $e_{t+j|t} = x_{t+j} - \hat{x}_{t+j|t}$  which is the error in the forecast  $\hat{x}_{t+j|t}$ , and subtract (20.15) from (20.4). This gives the revision of the concurrent estimator as

$$r_{t|t} = \hat{n}_t - \hat{n}_{t|t} = \sum_{j=1}^k e_{t+j|t}, \quad (20.16)$$

which shows that the revision is a linear combination of the observed series forecast errors. The coefficients of the linear combination are the seasonal adjustment filter weights.

Given that the series ARIMA forecast error  $e_{t+j|t}$  is the moving average

$$e_{t+j|t} = \sum_{i=1}^j \psi_i a_{t+i}, \quad (20.17)$$

where the  $\psi_i$  weights are those of the MA representation of the ARIMA model, and  $a_{t+1}, \dots, a_{t+j}$  are the series innovations, from (20.16) and (20.17),

$$r_{t|t} = \sum_{i=1}^k \alpha_i a_{t+i}, \quad (20.18)$$

where the  $\alpha_i$  coefficients are obtained from the convolution of the appropriate  $v$  and  $\psi$  filters, and can be easily computed. Strictly speaking, the variables in (20.17) and (20.18) are the sequence of 1-period-ahead forecast errors of the series made at periods  $t + 1, \dots, t + k$ . If the model is correct, they will be the series *n iid* residuals, also called innovations.

- b. Mean and variance. Equation (20.18) provides an easy way to derive properties of the revisions, used be linear filters. For example,

$$E(r_{t|t}) = 0; \text{Var}(r_{t|t}) = V_\alpha \sum_{i=1}^k \alpha_i^2, \quad (20.19)$$

where  $V_\alpha$  is the variance of  $a_t$ . This shows that the revisions implied by 2-sided seasonal adjustment filters are unbiased and, knowing the filter weights and the ARIMA model for the series, their variances can be computed.

This knowledge implies that the uncertainty in the preliminary estimator implied by revisions is straightforward to quantify concurrently. As a consequence, confidence intervals associated with future revisions can be built for the concurrent figure. If, after revisions are completed, the ones obtained empirically depart substantially from the concurrently computed measures, the departure would shed doubts on the adequacy of the series ARIMA model (because of misspecification or perhaps because some recent structural change in the time series). Departures can be detected because, for example, too many historical estimators lay outside the confidence interval built around the concurrent estimator; or the variance of the empirical revisions is markedly different from the concurrent measure.

- c. Extension to other estimators. The analysis performed on the revision in the concurrent SA series estimators extends easily to any preliminary estimator. For example, by letting  $n_t$  denote the trend-cycle and  $v(B, F)$  the filter used for its estimation, a similar analysis can be carried out for the trend-cycle (or any other unobserved component). The trend-cycle removes the irregular component (e.g. high transitory variations) from the SA and can be thus seen as a smooth SA series. As a general rule, the concurrent trend-cycle estimator is subject to (moderately) larger revisions during the first year, although the convergence to the final estimator tends to be considerably faster.
- d. Speed of convergence to final estimate. From (20.19), for the preliminary estimator  $\hat{x}_{t|T}$ , the variance of the revision will be equal to

$$\text{Var}(r_{t|T}) = V_\alpha \sum_{i=T-t}^k \alpha_i^2, \quad (20.20)$$

Therefore, letting  $T$  approach  $t + k$ , the speed of convergence of the preliminary to the historical estimator can be obtained by computing the variance  $\text{Var}(r_{t|T})$  as a function of  $T$  (with  $t$  fixed). This can

answer questions such as: for how many periods should the concurrent estimator be revised? If in 3 years, for example, the variance of the revision has been reduced by 95%, 3 years of revision would seem long enough.

- e. Revisions, asymmetric filters, and phase delays. Writing the series forecasts  $\hat{x}_{t+j|j}$  as a linear combination of observations up to and including  $x_T$ , the preliminary estimator (20.11) can also be expressed as a linear combination of these observations, that is

$$\hat{n}_{t|T} = \dots + \eta_{-1}x_{t-1} + \eta_0x_t + \eta_1x_{t+1} + \dots + \eta_{T-t}x_T. \quad (T - t > k) \quad (20.21)$$

The filter that yields the preliminary estimator is thus asymmetric (around  $t$ ). As an example, consider the filter (20.7) applied to a quarterly series that follows the model  $x_t = \phi x_{t-4} + \alpha_t$ . Letting  $j = 1, 2, 3, 4$  and  $i = 1, 2, 3, \dots$ , the forecast of the series made at period  $t$  can be written as

$$\hat{x}_{t+j+4(i-1)} = \phi^i x_{t+j-4}, \quad (20.22)$$

so that, for  $i = 1$ ,  $\hat{x}_{t+j|t} = \phi x_{t+j-4}$ ; for  $i = 2$ ,  $\hat{x}_{t+4+j|t} = \phi^2 x_{t+j-4}$ , and so on. Replacing the forecast in expression (20.15) using (20.22), the concurrent estimator of the SA series is found to be equal to the expression  $\hat{n}_{t|t} = \eta_0x_t + \eta_{-1}x_{t-1} + \eta_{-2}x_{t-2} + \eta_{-3}x_{t-3} + \sum_{j=-4}^{-k} \eta_j x_{t-j}$ , where  $\eta_j = v_j$ ,  $j = 4, \dots, k$ , and the coefficients for the last four quarters are given by  $\eta_j = v_j + \sum_{i=1}^{I_j} v_{(4i-j)} \phi^i$ ,  $j = 1, 2, 3$ , where  $I^j$  is the largest positive integer  $i$  such that  $\{k - (4i - j)\} \geq 0$ . The filter for  $\hat{n}_{t|t}$  is asymmetric.

Frequency domain analysis of asymmetric filters reveals that they induce a phase delay in the preliminary estimator of the SA series. This phase effect shifts the timing of the series ups and downs and, most importantly, of the turning points. Among the preliminary estimators of the seasonally adjusted series,  $n_{t|T}$  with  $t \leq T$ , the phase effect is largest for the concurrent estimator and vanishes as revisions are completed.

#### 20.3.2.4 Use of models within seasonal adjustment

Filters can be derived from models. For example, within TRAMO-SEATS filters are derived which represent specific models. The majority of issues mentioned in the previous section are relevant in the case of TRAMO-SEATS model-based filters.

Separately, a change in model parameters, or regression matrices, will result in a revision to the seasonal adjustment outputs. Model parameters may change as new original values become available which, in turn, will result in different seasonal filters, or ARIMA models used, and therefore modified seasonally adjusted estimates. As new information becomes available, it is also conceivable that values previously identified automatically as outliers are no longer classed as such or that past values are identified as outliers for the first time. Both bring about notable revisions to the seasonally adjusted estimates. Ultimately, new information can cause the previous model to be rejected and another model to be used which is liable to result in particularly large changes.

#### 20.3.2.5 Use of concurrent or current seasonal adjustment

There are two approaches to estimate the seasonal factors. Current seasonal adjustment (sometimes referred to as forward factor) is an annual analysis of the original estimates to calculate the seasonal factors for the next year ahead. This means that the seasonal factors would only be revised on a yearly basis. Under this approach, revisions to the seasonal factors are effectively hidden from the user until the seasonal factors are re-estimated each year. By setting the seasonal adjustment parameters a year ahead, this means that this approach may not detect subtle changes or evolution in the seasonal pattern. Technically, this approach is

achieved by calculating a seasonal adjustment for a time series that contains unadjusted figures up to time  $t$ , using this opportunity to estimate the seasonal and calendar factors for future points in time, e.g. from  $t + 1$  to  $t + 12$  for a monthly time series. These estimated factors are then saved in a database and are not changed until time  $t + 12$ , where one year additional information is available for a monthly time series. If, for example, there is a new unadjusted value for  $t + 1$ , it is adjusted (outside of the seasonal adjustment program) with the seasonal and calendar factors for  $t + 1$  that were calculated and saved at time  $t$ . Thus, under this approach, there are no revisions due to seasonal adjustment until  $t + 12$ . Only changes to the unadjusted data have an impact on the seasonally adjusted figures, as the latter are calculated using the unadjusted data and the unchanged seasonal and calendar factors. However, not revising the seasonal adjustment factors comes at a price. It is impossible to draw upon any relevant new information when seasonally adjusting the values from  $t + 1$  to  $t + 12$ , and, as with any projection, errors can occur. In the worst case scenario, the seasonally adjusted results do not show the true economic developments, leading to economic misjudgements. This is likely to occur when there is rapidly evolving seasonality in the series.

Concurrent seasonal adjustment means that the seasonal adjustment parameters are re-estimated at each time period. This approach provides the most up-to-date estimates of the seasonal factors, but will give regular revisions to previous published estimates.

The Australian Bureau of Statistics has demonstrated that the use of concurrent seasonal adjustment provides reduced revisions when compared to the use of current factor seasonal adjustment (Australian Bureau of Statistics (1999)). The Table below is reproduced from Australian Bureau of Statistics (2003) and gives the average revision of the initial seasonally adjusted estimates compared for concurrent adjustment and current factor method for three Australian housing finance series.

**Table 20.2: Average revision between concurrent and current factor estimation — Australian Bureau of Statistics (2003)**

Housing Finance series	Average revision (%)		Percentage improvement $100 * \frac{(cf - cc)}{cf}$
	Concurrent (cc)	Current Factor (cf)	
Banks for Established - Number	2.33	2.83	17.67%
Banks for Construction - Number	2.44	2.98	18.12%
Refinancing, All Lenders - Number	1.69	2.14	21.03%

## 20.4 To revise or not to revise?

The compilation, production and calculation of time series estimates is a complicated process. Users of official statistics need to be aware that the compilation and production complexities of the statistical process directly impact on the published estimates. The process to calculate time series estimates will often involve incomplete data sources, late data sources and even changes in methods (discussed in detail in section 20.3). These complexities mean that in practice, time series estimates will be revised from one period to the next. This is the case for the original estimates and the analytical output derived from them such as the seasonally adjusted and trend estimates.

Official statistics face close scrutiny from policy makers and the media. Cook (2004) notes the importance of revisions to official statistics and highlights that revisions are a necessary consequence and part of normal statistical processes. Revisions ensure that the published data is the most up to date and as relevant as possible. It is important that policy makers and users, such as media commentators, acknowledge this as revisions do not necessarily mean that there were non-statistical errors, rather it allows for the best information at the time of publication to be used. For example, a quote that captures the impact of revisions on users is:

*“Only the uninitiated treat them [the estimates] as though they were handed down by God, etched on tablets of stone.”*

Ross Gittins, Economic commentator, Sydney Morning Herald, March 1 2003, commenting on Australian National Accounts estimates. Web link: <http://www.smh.com.au/articles/2003/02/28/1046407753162.html>

In practice, when compiling and deriving estimates, there is a tension between the timeliness and reliability of published estimates. Revisions should be expected because of this tradeoff.

For example, users of official statistics demand information to be made available as soon as possible, but this can mean that the process for deriving the estimates may not be complete and may contain incomplete data sources. If the estimates cannot be compiled until a full set of data sources are available, the estimates may not be relevant any more. There needs to be a balance between how long it takes to produce and disseminate an estimate, referred to as timeliness, and also the reliability of the estimate.

An example of the impact of timeliness is a comparison of short term indicators such as monthly and quarterly estimates compared with annual estimates, e.g. when annual data is used to benchmark sub-annual estimates. In practice, source data for short term monthly and quarterly estimates are usually obtained from a smaller regular sample survey, whereas annual data can often be obtained from a larger sample collected over a longer time frame. The smaller regular sample surveys provide an indication of the most recent activity of the variable of interest, but once greater detail is made available either in subsequent periods or from a larger sample, then this may cause the earlier estimates to be revised.

The timeliness of the release of estimates needs to also be balanced against the need to avoid frequent revisions. Users need to have confidence in the statistical process and also the estimates that are produced. When estimates are continually revised this can impact on the confidence of the statistical process and the outputs. This is of particular relevance for high profile estimates produced by National Statistics Institutes where important policy decisions are made based on those estimates. For example, if estimates for Gross Domestic Product are showing a rise in one period, and this is revised in the next quarter to a fall, then there is the potential to mislead key users such as policy makers who may make the wrong judgements based on the earlier information.

Table 20.3 gives a real example of how the interpretation of economic events can change based on revisions to previously published estimates. The United Kingdom produces monthly estimates of the quarterly GDP number. This means that there are revisions each month for a given quarter's estimate, and also revisions to previously published quarterly estimates.

**Table 20.3:** Published monthly quarter to quarter growth estimates of quarterly GDP for the United Kingdom: [www.ons.gov.uk](http://www.ons.gov.uk)

	2009 Q1	2009 Q2	2009 Q3	2009 Q4	2010 Q1
Oct-09	-2.5	-0.6	-0.4		
Nov-09	-2.5	-0.6	-0.3		
Dec-09	-2.5	-0.7	-0.2		
Jan-10	-2.5	-0.7	-0.2	0.1	
Feb-10	-2.6	-0.6	-0.3	0.3	
Mar-10	-2.6	-0.7	-0.3	0.4	
Apr-10	-2.6	-0.7	-0.3	0.4	0.2
May-10	-2.6	-0.7	-0.3	0.4	0.3
Jun-10	-2.3	-0.7	-0.3	0.4	0.3

Table 20.3 shows how the quarter to quarter growth estimates have been revised at each monthly publication. For example, the first estimate for the 2009 Q4 growth was 0.1%, which was then revised the next month to 0.3% and then to 0.4%. In addition, the preceding quarters growth estimates were revised down. For example, in February 2010 the 2009 Q1 growth estimate was revised from -2.5% to -2.6% and 2009 Q3 estimate was

revised from -0.2% to -0.3%. While this may seem small, the issue for users was that the historical revisions also changed the interpretation of the estimates because it impacted on the length and depth of the recession and revised it to lower than initially estimated.

This example highlights that National Statistics Institutes need the ability to update and revise previously published estimates as and when required to provide the most recent information. This allows revisions to be taken into account in the production process and also ensures that the most relevant and up to date data is available for all users. It is important to note that having low revisions does not necessarily mean that there is accurate measurement, it may mean that an active revision policy is in place (Chapter 20.5).

If published estimates are not revised, or revised on a semi-regular or ad-hoc basis, this can lead to estimates not being as up-to-date as possible and hence potentially providing misleading information. In practice, any revisions need to be communicated and explained to users on a regular basis. This ensures that revisions are placed in an appropriate context. It is important to explain revisions and the causes of revisions for the following reasons:

**Transparency:** This ensures that the reasons why the revisions have occurred are relevant and related to the statistical process and not to change the historical direction of series for non-statistical purposes. Revisions of estimates contribute to the promotion of transparency and independence of the production process by ensuring the estimates are using the most up-to-date information.

**Interpretability:** Understanding what data sources have been used and changed in the compilation of estimates is important to ensure the estimates are placed in the correct context. For example, an aggregate estimate may be revised due to a change in a single sub-component which allows any revisions to be placed in context.

**Coherence:** If a set of estimates are not revised in a coherent way this can also lead to issues in interpretation. For example, revising the original estimates and not the seasonally adjusted and trend estimates may result in lack of coherence between the datasets. It is important for users to understand which estimates have been revised to ensure transparency of the production process.

Revisions are a necessary and normal part of the statistical compilation and estimation process. It is also important to recognise that the even the revised estimate, calculated once all the relevant data are available, will still retain a component of statistical uncertainty.

## 20.5 Revision policies

The ESS Guidelines on Seasonal Adjustment Eurostat (2015), note that “Revisions to seasonally adjusted data are published in accordance with a coherent, transparent and officially published revision policy and release calendar, which is aligned with the revision policy and the release calendar for the unadjusted data. Revised seasonally adjusted data should not be released more often than unadjusted data. The public is informed about the size, direction and volatility of past revisions of important seasonally adjusted macroeconomic variables.” (item 4.1, best alternative).

Similar and related issues to revision policies and practices for European indicators are also covered in Mazzi and Cannata (2008). In practice, revision policies are typically used to manage the timing of the release of revisions to published estimates while taking into account user interests and the availability of the data. Different revision policies can be applied to the original, seasonally adjusted or trend estimates. The revision policy should cover regular revisions due to the statistical process, but also how to deal with unexpected revisions.

Revision policies can be different between countries, and even within countries for different types of statistical outputs. For example, when estimates are combined at a higher aggregate level, e.g. member countries of the European Union, then new estimates become available on a regular basis. This occurs because the



aggregates are derived from country data which are published and revised according to differing schedules for each country. Similarly, National Accounts estimates may have different revision policies related to the balancing and constraining processes whereas other estimates not used within National Accounts could choose an open revisions policy as they are not subject to the same conceptual constraints.

Revision policies should be developed by the individual organisation. They should be published and transparent to users. In practice, the application of revision policies should be consistent with the publically available policy. If the application of revisions is not consistent with the publically available policy, then users of the data need to be made aware of the reasons why the change occurred. For example, the United Kingdom Statistics Authority in their Code of Practice for Official Statistics, United Kingdom Statistics Authority (2009), specifically mentions to “Publish a Revisions Policy for those outputs that are subject to scheduled revisions. Provide a statement explaining the nature and extent of revisions at the same time that they are released.”

### 20.5.1 Revision policies for methodological changes

Methodology, conceptual or classification, changes also lead to revisions and policies should be developed to manage the introduction of these planned changes. These types of changes can lead to revisions after the source data have been finalised. It is common for these large types of changes to be introduced in a coordinated way across an organisation or across different countries. For example, the use of the European System of Accounts, Standard Industry Classification changes, and methods like annual chain-linking.

It is difficult to introduce major developments and improvements without making revisions to historical data. This is because the historical data often needs to be on the same methodological, conceptual or classification basis, so this data may need to be revised in line with the new approaches.

Changes in methods should be accompanied by a communication plan, which might consist of a series of articles and seminars on concepts, methodology and probable effects. In addition, significant changes to earlier periods could be released shortly before the main data release to allow users to become accustomed to the changes and renew their models and check their processes for obtaining data. In the published data, revisions to the main data could be published in a table, and supplementary information made available which provides information to enable users to put the revisions into an historical context.

### 20.5.2 Revision policies for original estimates

Many different revision policies are available for use in practice. Some examples of alternative revisions policies for the original estimates are:

1. Regularly update for each time period: The original estimates, and all analytical outputs (e.g. seasonally adjusted estimates), are continually updated each period to reflect new and updated incoming data relating to the past estimates. This ensures the best possible information at all times but does require continuous revisions. Issues with the credibility of the statistics may arise, especially if frequent revisions are coupled with significant changes in published estimates.
2. Update on an annual basis: Only new original estimates for the current reporting period are published as they become available. Late incoming data and revisions to previous data are first collected and then revisions for all twelve months of the previous year for a monthly time series are published simultaneously in a single annual revision. This approach tries to reduce the impact of lots of revisions to previous estimates. Where changes in the data are only minor, the economic conclusions drawn from it are unaltered and the analyst does not incur the cost of constantly updating past data. On the other hand, smaller revisions should not be concealed. They could all be published at once after an initial delay. A practical example related to this approach would be based on supplying estimates from countries to form higher level aggregates (e.g. at a European level). The idea of this argument is that each national



statistical agency should decide on its revision policy and time table, but considering harmonised principles where needed. Since revisions to country data may only have a small effect on the aggregate, it would be possible to dispense with the practice of carrying out permanently low revisions to aggregate estimates in order to maintain consistency with the aggregation of country data.

3. Combinations between the two different options are possible, where data may be updated on a semi-annual basis.

A broad approach for revisions to the original estimates of all statistical outputs is beyond the scope of this publication, but certain principles may help guide the development of specific revisions policies, and basically revolve around the notions of transparency and magnitude. Having a transparent and publically stated revisions policy can help users understand the necessity and importance of revisions. This ensures that revisions add value to the interpretation of the data and may help maintain confidence in the statistical process. Revisions policies for the original estimates of any output will necessarily have to consider a number of issues, such as the burden on users, the costs to the producers, and the relative impact of a revision on the published estimates.

### 20.5.3 Revision policies for seasonally adjusted estimates

When setting a revision policy for seasonally adjusted (or trend) estimates, it is advisable to take into account the revision method used for the original estimates and also the methods used to produce the estimates. Some alternative revisions policies for the seasonally adjusted estimates include:

1. Regularly update each time period: To ensure that the latest data are always processed, a new seasonal adjustment is carried out and updated seasonal factors for all time periods are estimated. This means new seasonally adjusted estimates are calculated for the past data as well, even if the original values have not changed. The revisions can be small provided the seasonal pattern has changed only gradually. If, on the other hand, new seasonal adjustments mean that a value is sometimes automatically identified as being extreme and sometimes not, then the seasonally adjusted figures change substantially.
2. Update on an annual basis: The philosophy of collecting smaller, less significant revisions and publishing them only during a single annual revision involves working with forecasted seasonal and calendar factors. In this connection, wide-ranging studies into seasonal and calendar effects are carried out only once a year. At the same time, seasonal and calendar factors for the following year are forecasted. Provided that the seasonal components do indeed change only very slightly, this process is sufficient.
3. Combinations between the two different options are possible, where data may be updated on a semi-annual basis. Several variations of these approaches are available. The options used with the seasonal adjustment package can also provide alternatives. For example, when using TRAMO-SEATS, it is sometimes recommended that the model parameters be re-estimated continually while the model itself should only be determined on an annual basis. In theory, it is possible to create other hybrid forms, depending on which options are modified monthly, multi-monthly or annually.

The ESS Guidelines on Seasonal Adjustment, Eurostat (2015) (section 4) provide a further discussion of some of the issues surrounding revisions policies, including a specific discussion of concurrent versus current adjustment (section 4.2) and the revision horizon (section 4.3). In terms of general recommendations for revision policies for seasonally adjusted estimates, the guidelines suggest:

*“Revisions to seasonally adjusted data are published in accordance with a coherent, transparent and officially published revision policy and release calendar, which is aligned with the revision policy and the release calendar for the unadjusted data. Revised seasonally adjusted data should not be released more often than unadjusted data. The public is informed about the size, direction and volatility of past revisions of important seasonally adjusted macroeconomic variables.”*

As important, is the recommendation on a policy (explicit or otherwise) that is to be avoided:

*“No revision of seasonally adjusted data, absence of a clear and public revision policy, absence of a public release calendar, or policies leading to the publication of misleading information especially for the current period.”*

## 20.5.4 Revision policies for specific outputs and methods

As will be clear from the preceding discussion of revision policies, it is not possible to recommend explicit revision policies outright as all outputs will necessarily have issues particular to them that should be addressed. Two brief examples are given below that highlight why it is important to consider the issues associated with specific output and methods. An example of a method that may determine to some extent the revision policy for an output or group of outputs is the use of concurrent or current seasonal adjustment. This will impact on the type of revision policy that is implemented. For example, the use of concurrent seasonal adjustment means that the seasonal factors are recalculated each time period with the addition of updated data, but it may be the case that the seasonally adjusted estimates are not automatically updated as revision policies may need to be followed.

The use of current (or sometimes referred to as forward factor) seasonal adjustment means, by definition, that the seasonal factors are fixed for a given number of time periods, typically one year ahead. When the seasonal factors are recalculated on an annual basis, or longer, this leads to revisions. The revisions can then be limited based on the revisions policy. The issue of concurrent versus current adjustment is discussed further in section 4.2 of the ESS Guidelines on Seasonal Adjustment, Eurostat (2015, p. 33).

Different outputs may require different revision policies. For example, the compilation of the National Accounts is a complex data process, with the use of annual, quarterly and monthly data. Within the production of the National Accounts different time periods are open for revision at each publication as incorporating revisions from a single source is not an easy process. The addition of the supply and use balancing approach, and the requirement that the three measures of GDP are made equal, also constrains the practical aspect of managing revisions. Annual estimates tend to be incorporated into the National Accounts up to two years after the year they relate to and the approach of taking on new annual data will lead to revisions. Usually the data from the major sources will be revised when the next year's set of results are available, in order to incorporate any late returns from the previous year and to ensure consistency between consecutive years. At that point most of the data will have firmed up and only exceptional changes that lead to significant revisions will be considered in subsequent periods. The balancing process within the National Accounts is complicated and this means that it is typically done only once a year. At this point, revisions are then published for previous years data, which in effect means that revisions will have been stored up until this point. There is a balance between taking on revisions and revision of historical growth patterns.

## 20.6 Presenting revisions

### 20.6.1 Revision triangles

Statistical agencies and users of seasonally adjusted data alike are interested in revisions of real time data. Statistical agencies are concerned about the quality and usefulness of the statistics they publish and how to increase these. Users of seasonally adjusted real time data need to know the extent of revisions for economic analysis and forecasts and how to take them into account.

To achieve this, a revision triangle approach can be used. For example, Table 20.4 illustrates seasonally adjusted data on euro-area 12 GDP chain volume in million euro, the real time data vintages are arranged in

the typical triangle form in the table below. Column headers are reference quarters, row headers are dates of publication for which the first regular estimate was reported. This means that the time series published at the respective dates are printed in the particular rows.<sup>1</sup>

In table 20.4 the data is represented by bold = first estimate ( $t + 63$  days), bold and italic = quarterly update ( $t + 150$  days, the first estimate of the next quarter), bold and underlined = annual update (fourth quarter of the subsequent calendar year).

**Table 20.4:** Data structure of euro-area 12 GDP chain volume for revision analysis\* (in million euro)

Date of publication	Reference quarter				...
	2003Q1	2003Q2	2003Q3	2003Q4	
2003 June 05	<b>1,574,844</b>				
Sep 09	<b>1,576,201</b>	<b>1,574,970</b>			
Dec 03	1,575,869	<b>1,574,222</b>	<b>1,580,257</b>		
2004 Mar 04	1,575,306	1,573,925	<b>1,580,004</b>	<b>1,584,884</b>	
June 01	1,575,441	1,574,178	1,580,642	<b>1,586,504</b>	...
Sep 07	1,575,555	1,572,758	1,580,119	1,585,951	...
Dec 01	<b>1,578,067</b>	<b>1,575,042</b>	<b>1,583,513</b>	<b>1,589,728</b>	...
2005 Mar 02	1,578,921	1,576,387	1,585,225	1,591,973	...
⋮	⋮	⋮	⋮	⋮	⋮

In relation to this example, Eurostat publishes several estimates of euro-area GDP. Around 45 days after the end of the reference quarter (denominated as " $t + 45$  days") a so-called "flash estimate" of quarter-on-quarter seasonally adjusted volume growth is released. The flash estimate only refers to the most recent quarter and does not include revisions of back data. This is followed by two "regular" estimates, denoted as first and second estimates, which include detailed information, but may entail revisions to the GDP flash estimate or earlier data. The flash estimate provides timely information, but is generally based on less comprehensive source data than later estimates and may, therefore, be subject to revisions.

The first regular estimates of both unadjusted and seasonally adjusted data, which comprise data on GDP, its main expenditure components and the activity breakdown of value added, are published around two and a half weeks after the "flash" (around  $t + 63$  days). On the occasion of the second regular estimates, published around 100 days after the end of the reference quarter, the remaining details are provided. When first regular estimates of the next quarter become available, i.e. at around 150 days after the end of the reference quarter one quarter earlier, all quarterly source data can be assumed to have been incorporated in the data referring to one quarter earlier. Henceforth, this update is referred to as the "quarterly update". An estimate that may incorporate important new information from, in particular, annual data sources, referred to as the "annual update", is typically released in the fourth quarter of the subsequent calendar year.

The approach of using revision triangles, enables all the different vintages to be captured and analysed.

## 20.6.2 Graphical approaches

Graphs are powerful tools for analysing time series, condensing a lot of information into one visual aid and revisions to time series are no exception. Graphical analysis of revisions can assist in understanding the magnitude of revisions, the sources of revision, and the impact of revisions on components of a time series in

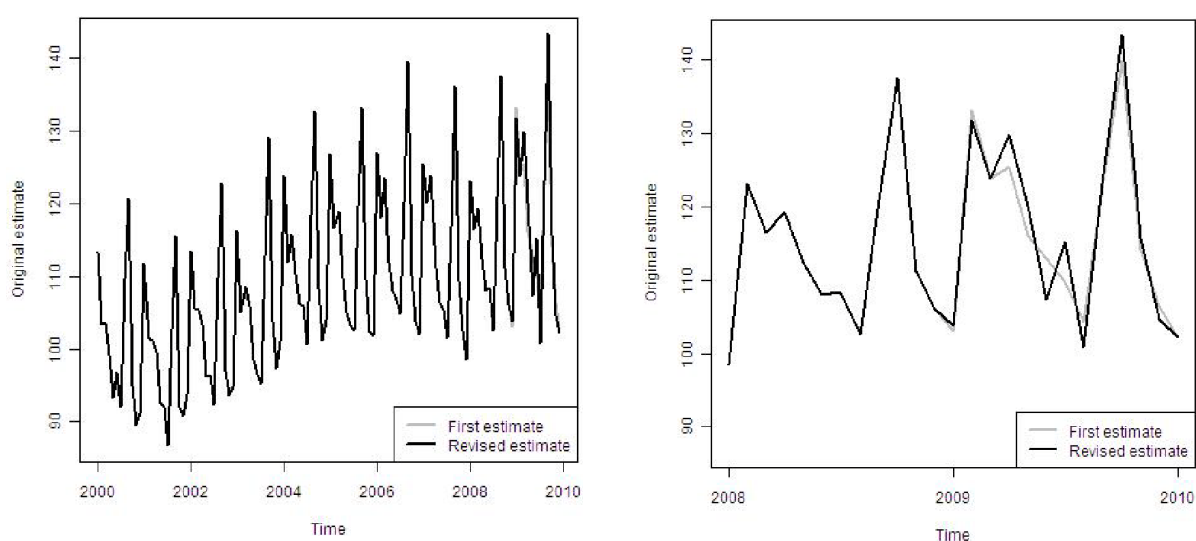
<sup>1</sup>The presentation of the revision matrix is not standardised in the literature. Depending on the authors, dates of publication are not arranged as row headers like here but as column headers. Accordingly, reference quarters switch from column headers to row headers. Technically speaking, the revisions matrix may be transposed. Hence, it is advisable to check the structure of the revision matrix before beginning the revision analysis itself.

a quick and efficient manner. They may also be used in such a way as to represent uncertainty in estimates as it is expected that revisions will occur but the magnitude and direction is unknown.

### 20.6.2.1 Magnitude of revisions

By examining plots of estimates and revised estimates of a time series, it is possible to obtain an idea of the magnitude of revisions, and the impact in terms of interpreting movements. Users and producers of statistics may be interested in revisions at a particular point in time or at a number of time points depending upon revisions policy and the purpose of the analysis. For example, in the monthly series shown in figure 20.1, revisions for the previous twelve months were stored and published in one go at the end of those twelve months. It can readily be seen that the revisions in the last twelve months of the time series have changed sign in some time periods.

**Figure 20.1:** Revisions to the original estimates



**Figure 20.2:** Revisions to UK Gross Domestic Product and magnitude of revisions

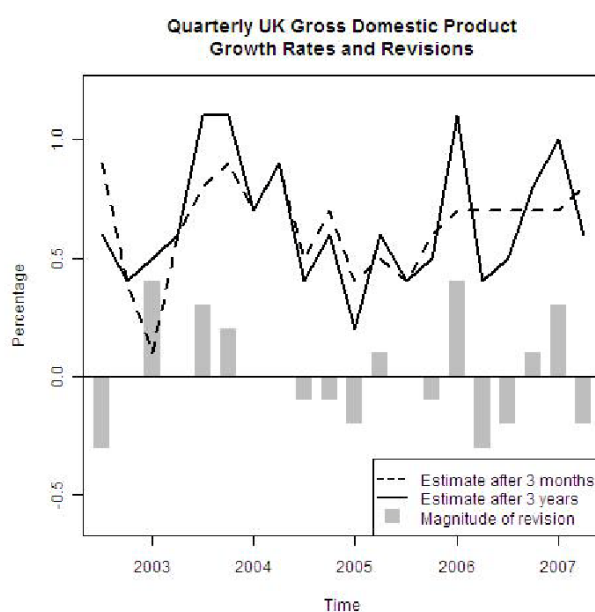
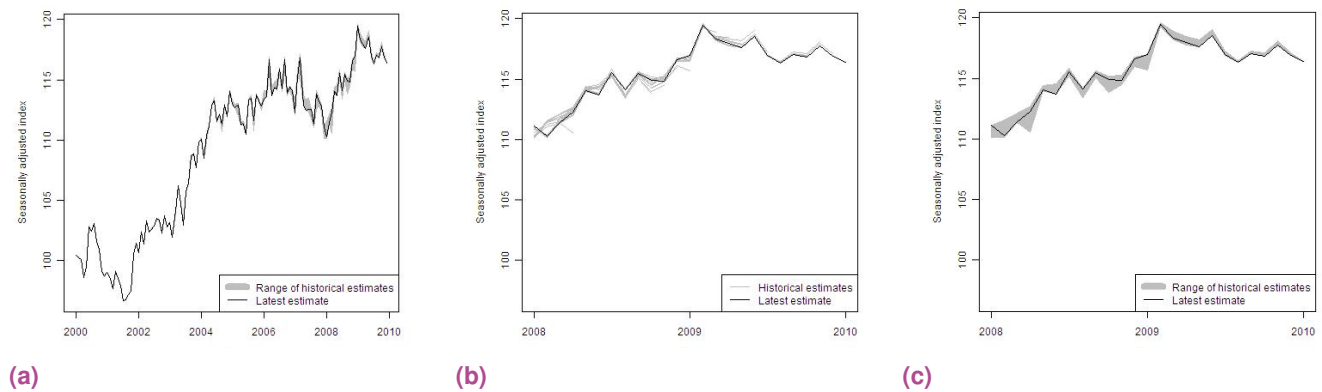


Figure 20.2 shows two estimates of the quarter on quarter growth rate of UK Gross Domestic Product (GDP)

from 2002 quarter 3 to 2007 quarter 2, and the magnitude of revisions. There are a number of different scheduled revisions to the estimates of the UK GDP. This chart shows the month three estimate and the three years later estimate of GDP growth and the magnitude of revision between these two estimates.

In the above examples, each time point for which revised estimates exist shows only one revised estimate. In practice estimates can be revised a number of times; and revision triangles are one method of presenting detailed historical revisions to time series estimates (see section 20.6.1). Similar information can also be presented graphically. For example, figure 20.3a shows the latest seasonally adjusted estimate and a number of historical estimates for the same monthly series, whilst Figure 20.3b shows the range of historical estimates as a shaded area on the chart.

**Figure 20.3: Range of historical estimates**



The charts with the range of historical estimates as a shaded area more clearly show the range in levels at each time point whilst those with a number of historical estimates more clearly show vintages of changes in level. Both sets of charts show the range of historical estimates are relatively small compared to the level and variability of the series. The above charts could be used to explain the nature of revisions for a published seasonally adjusted series. Analysts may also find such graphics useful when evaluating or re-evaluating methods for producing seasonal and trend estimates.

Further graphical analysis could include similar plots of revisions to the components of a time series. For example, Figures 20.3b to 20.4a and 20.4b to 20.5a below show the range of historical estimates and vintages of estimates for the trend, seasonal and irregular components of the above series for the last two years. Care must be taken when interpreting these charts especially when comparing revisions for different components as the y-axes are on different scales. Nevertheless, they do show the scale of revisions relative to the variability of each component, with revisions to the trend and irregular more noticeable than those for the seasonal component.

As described in section 20.3 there are a number of sources of revisions and plotting the magnitude of revisions only shows the impact of all sources of revision on particular estimates. In certain circumstances it can be useful to compare the magnitude of revisions holding particular sources of revision constant. For example, when comparing alternative methods of seasonal adjustment and holding all sources of revision to the original estimate constant. Or when methodological changes happen, it may help users to understand the impact by isolating the different changes. This can often be done by layering changes in stages as part of any revision analysis.

Revisions and graphical visualisation of revisions are only one type of diagnostic and should be used in conjunction with other diagnostics for evaluating the quality of alternative methods. When evaluating alternative methods of estimation for the seasonal or trend estimates many graphical comparisons of the magnitude of revisions can be made. Some of the most commonly used include the plots of time series, as above, as well as spectral plots, and auto-correlation and partial auto-correlation functions. These are easily produced by a

Figure 20.4: Revisions to different components of a time series

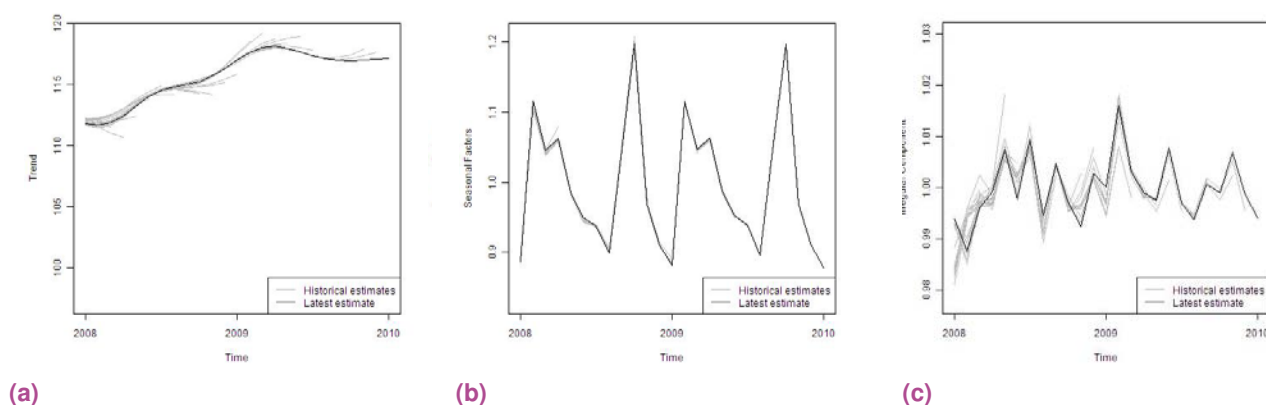
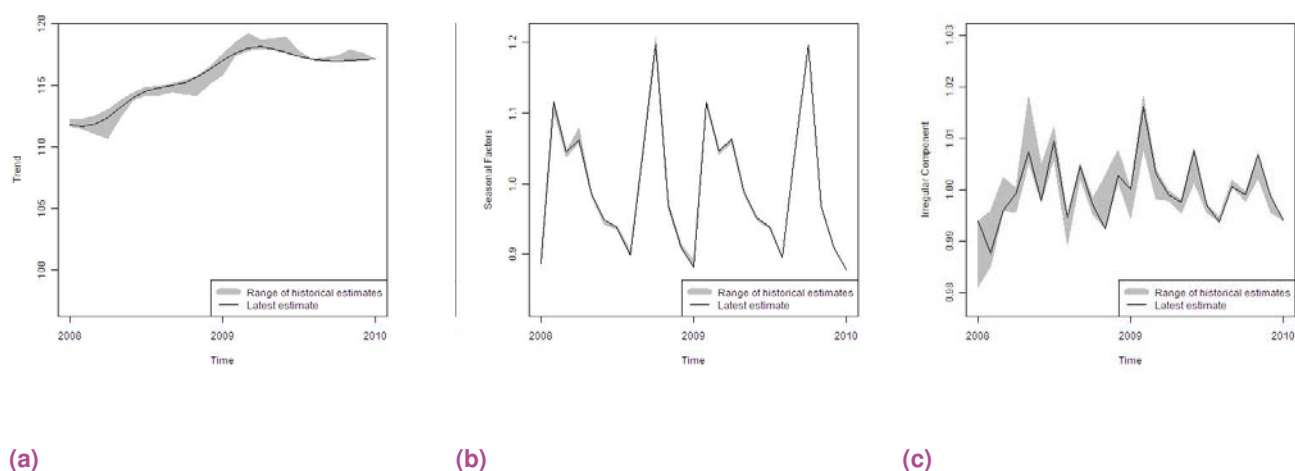


Figure 20.5: Revisions to different components of a time series (2)



range of software<sup>2</sup>.

### 20.6.2.2 Sources of revisions

Simple plots convey some information and can be useful presenting information on the magnitude of revisions to users or reviewing a number of different series quickly. For example, to identify series within an aggregation structure that have large revisions and may need further scrutiny. However, simple plots do not provide any information on the sources of revision.

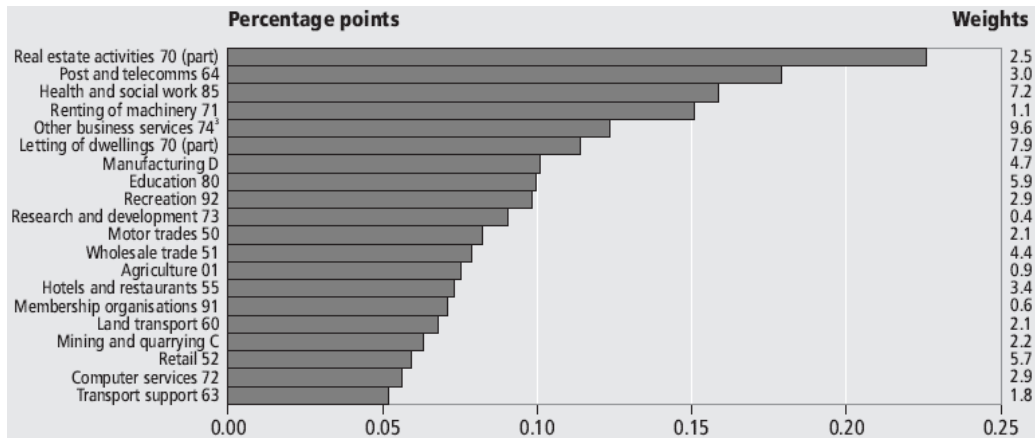
If sufficient information on the data is available then it may be possible to graphically represent sources of revision to the original estimate and/or seasonally adjusted estimate. An example of such an approach is provided by Mainwaring and Skipper (2007) who present a number of graphical tools for presenting revisions. For example, Figure 20.5c shows the contribution of various industries to revisions of the quarterly growth rate estimates of the UK output measure of GDP (GDP(O)) over the period 2005 Q1 to 2007 Q1<sup>3</sup>. The

<sup>2</sup>All charts in this section have been produced using R.

<sup>3</sup>Note that not all industries have been included in this example as the sum of weights in parentheses is 81.3%. The weighted sum of

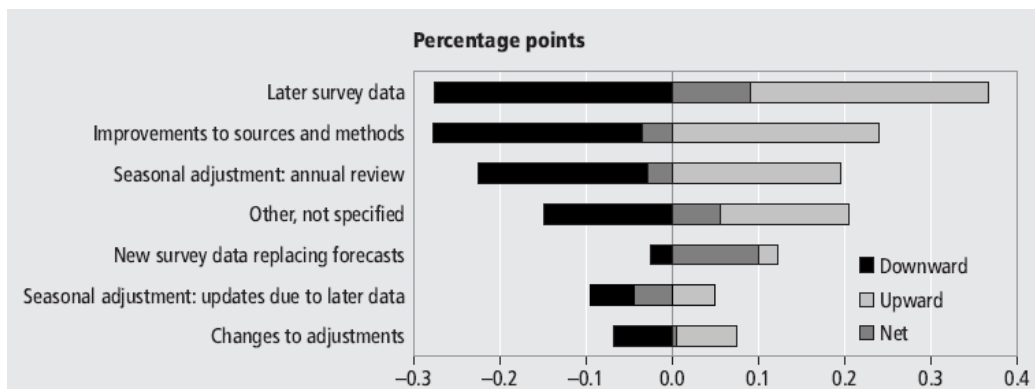
percentage weight contribution of each industry to GDP(O) is also given in parentheses. In this example real estate activities contributes most to revisions of the quarterly growth rate whilst it is by no means the largest contributor to GDP(O) at only 2.5%. A graphical analysis such as this can assist producers of statistics to see which components of an aggregate are contributing most to revisions at an aggregate level. This can be useful for detection of possible errors or problems in the compilation, or as a simple way to present information on revisions to users of the data.

**Figure 20.6:** GDP(O) data published June 2007: Absolute percentage points contributions to revisions to quarterly GDP(O) growth rates by industry (2005Q1-2007Q1). Example reproduced from Mainwaring and Skipper (2007)



As discussed in section 20.3 there are many sources of revisions. In order to understand revisions in more detail, it is possible to look at the contribution to revisions of various parts of the process used to compile the estimates. Mainwaring and Skipper (2007) also present a graph of percentage points contribution to revisions of quarterly GDP(O) by cause. This is shown in Figure 20.7 where net revision due to changes following the seasonal adjustment annual review (for example, ARIMA model, filter lengths, Easter, Trading Day and outlier adjustments) is small, whereas a relatively large positive net revision to the growth rate of GDP(O) is due to new survey data replacing forecasts. A further example discussed in their paper is a breakdown by SIC in terms of causes of revisions.

**Figure 20.7:** GDP(O) data published June 2007: Absolute percentage points contributions to revisions to quarterly GDP(O) growth, by cause (2005Q1-2007Q1). Example reproduced from Mainwaring and Skipper (2007)



absolute revisions is given for each industry.

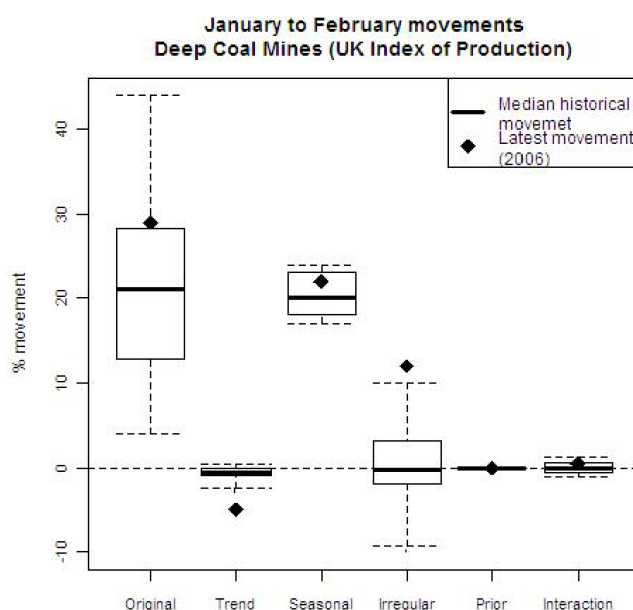


### 20.6.2.3 Decomposing revisions

Graphs showing revisions to levels of components of a time series are presented above. These show how the levels of components are revised. However, as suggested in the section on exploring sources of revision it may be of greater interest, especially to producers of data or analysts estimating seasonal or trend components, to have information on the contribution to revisions of an estimate from particular components. Boxplots can be used to provide concise information on such sources of revision.

A time series can be decomposed into a number of components such as trend, seasonal and irregular, as well as components for outlier type effects or non-seasonal cycles associated with moving holidays or the number of working days in a month. Algebraically, the growth rate of a time series can also be decomposed into the growth rates of components and potentially higher order interactions terms depending upon the nature of the decomposition (see section 20.7.2). Figure 20.8 shows boxplots of the historical range of January to February movements of a time series on Deep Coal Mines from the UK index of production. The diamond shaped points show the current growth rate of each component. Hence the growth from January to February 2006 in the original series of nearly 30% is principally due to a 20% movement in the seasonal component, although a significant proportion of the 30% movement is also due to a 10% movement in the irregular component. These positive movements are marginally offset by a negative movement of about minus 5% in the trend component.

**Figure 20.8: January to February movements Deep Coal Mines (UK Index of Production)**



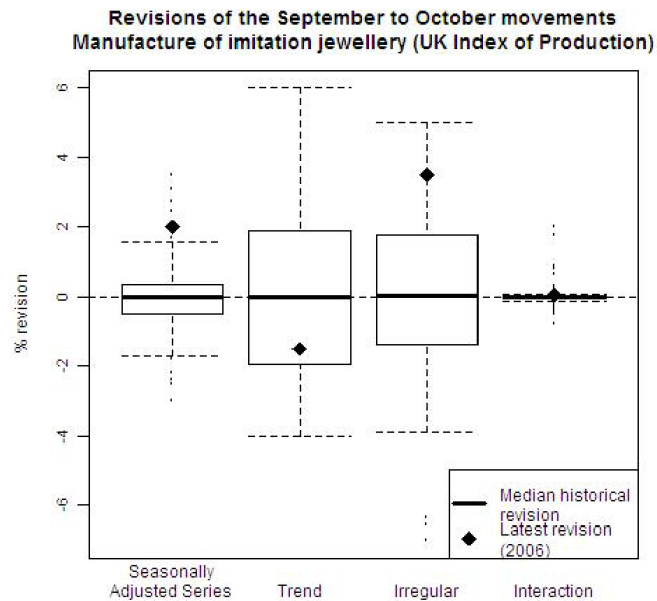
Whilst this does not show revisions it is useful graphical information to show how current growth rates compare to previous growth rates. A similar sort of plot can be used to present revisions to a series and its components. Figure 20.9 shows boxplots of the range of historical revisions and latest revision to growth rates of a seasonally adjusted estimate and its components, the growth rates of the trend, irregular and interaction term. The larger than usual positive revision in 2006 to the movement of the seasonally adjusted estimate from September to October is comprised of a large revision to the movement of the irregular component, slightly offset by a negative revision to the movement in the trend component. These charts may be of particular use when analysing the nature of an estimate or comparing different estimation methods for components of a time series.

### 20.6.2.4 Representing uncertainty

The final charts discussed are closely related to the issue of revisions, but are not plots of revisions or different vintages of time series. As discussed in this chapter, revisions are part of the process of producing time series



**Figure 20.9:** Revisions of the September to October movements Manufacture of imitation jewellery (UK Index of Production)

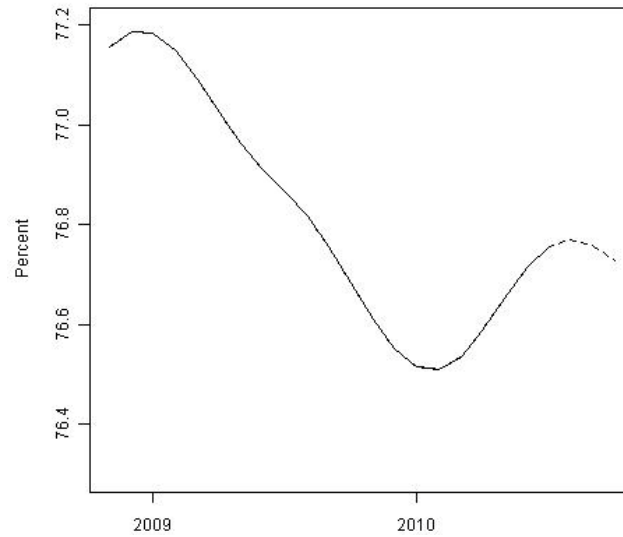
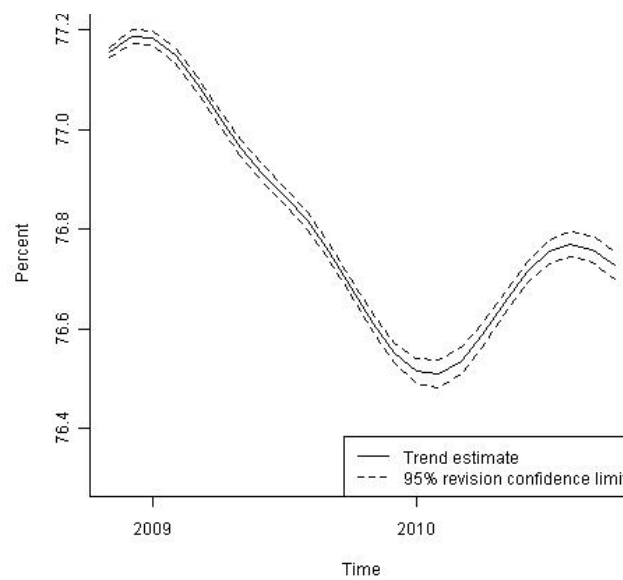


estimates to ensure that users have good quality estimates of the phenomenon of interest. What users may desire most is to know what revisions will happen in the future. Given that revisions are not wholly predictable it can be useful to represent the uncertainty of potential future revisions to an estimate, especially at the current end of a series. This can be particularly important for components subject to large revisions at the current end of a series where revision is predominantly due to additional time points.

Trend estimates at the current end can be prone to relatively large revisions, whereas seasonally adjusted estimates are less so, as revisions to the trend component are compensated for by revisions to the irregular component. Where trend estimates are published it may be useful to represent this uncertainty either as an interval or by changing the representation of the line used to indicate the trend at certain time points. The same could be done for other estimates but the focus in this section is on the publication of trend estimates.

Currently it is not common practice for National Statistics Institutes to publish graphical representations of the uncertainty of estimates. Headline graphs usually consist of the seasonally adjusted and/or trend estimates. Nevertheless, such graphics are easily obtainable within some seasonal adjustment packages, e.g. J-Demetra+ or SEATS, where revision standard errors can be estimated. If seasonally adjusted and trend estimates are obtained using X-13-ARIMA-SETAS then additional processing is required to obtain confidence limits for revisions of estimates (e.g. Kenny (2006)). Revisions tend to be larger at the current end of published time series. One very simple way of representing uncertainty at the current end of a series is simply to produce a line chart where the line for the final few points differs in its presentation to the rest of the series. For example, assuming that the final three time points of a trend estimate are subject to greater revisions than points earlier in the series, then the final three time points might be drawn as a dashed line whilst the remaining points are drawn as a solid line. Figure 20.10 shows an example representation of this with a trend estimate of the UK economic activity rate for those aged 16-64 years old.

An alternative way of representing uncertainty is by revision confidence limits. Figure 20.11 shows 95% revision confidence limits for the trend estimate of the UK economic activity rate following the method proposed by Kenny (2006). It should be noted that this takes no account of variability due to sampling and is based on the ARIMA model used for estimation. The revision confidence limits take no account of sampling variability as a source of uncertainty. The reported sampling variability for the level of the unadjusted estimate is  $\pm 0.3$  for the latest data point, which given the scale of the y-axis is large relative to the level of the trend estimate. Variances have not been estimated for the trend estimate. To continue the digression on representing uncertainty due

**Figure 20.10:** Trend estimate of economic activity rate UK aged 16-64**Figure 20.11:** Trend estimate of economic activity rate UK aged 16-64

to sampling error, methods exist for estimating variances of seasonally adjusted and trend estimates for the X-11 method of seasonal adjustment (e.g. Pfeiffermann (1994)). These are beyond the scope of this current chapter. Nevertheless no matter which method is used for estimating components, or the approach used to describe the uncertainty of estimates such a graphical representation can be of use to producers and users of the data as it assists in the interpretation of the time series. Whilst graphs are particularly powerful tools in time series analysis it is important that they are properly understood, as they can also be very misleading. This means that careful interpretation of the graphs must also be made. In any published output it is important that graphs are well labelled and that it is clear what they represent.

## 20.7 Analysis of revisions

### 20.7.1 Revision diagnostics

There are many possible ways of looking at data revisions. For example, describing revisions using some basic descriptive measures such as the average revision. Additionally, statistical tests can be performed, such as testing the statistical significance of a non-zero mean revision. Alternatively, one could try to model the data generation process that is at work behind the revisions with the help of econometric methods and try to answer the question of the rationality or predictability of revisions. All of these approaches are informative and give potential insight for assessing the reliability of the data analysed. When considering the adequacy of revision measures, it is helpful to ask what the results of the revisions analysis will be used for and what inference one wants to make from revisions diagnostics.

In what follows, some basic revision measures will be presented. Depending on the purpose of the exercise, it is recommended to complement these basic measures with more advanced ones or even with sophisticated econometric analyses.

#### 20.7.1.1 Terminology and notation for revision measures

Some terminology and notation is helpful for presenting revision measures. First of all, summary statistics calculated from data revisions usually refer to a specified revision interval. Given a particular time interval, a revision is defined as the change of some preliminary figure measured for the reference period  $t$  and a later figure measured for the same period. Usually, the difference is used to measure the change. This leads to the following definition of the revision  $R_t$  for the reference period  $t$ ,

$$R_t = L_t - P_t, \quad (20.23)$$

Here and throughout,  $L_t$  denotes the later estimate,  $P_t$  is the preliminary (or earlier) estimate,  $R_t = L_t - P_t$  is the revision and  $n$  is the number of observations.

#### 20.7.1.2 Different measures

A measure that gives an indication on whether revisions to first releases of different reporting periods have a tendency to cancel out over the revision interval under consideration is the mean revision. Thus, this measure gives an indication of the net effect of subsequent revisions on a time series. The Mean Revision (MR), sometimes also called average revision, is calculated according to the following formula of the arithmetic mean.

$$\bar{R} = \frac{1}{n} \sum_{t=1}^n (L_t - P_t) = \frac{1}{n} \sum_{t=1}^n R_t \quad (20.24)$$

This indicator answers the question

*“Is the average level of revision close to zero, or is there an indication that revisions are more in one direction than another, suggesting possible bias in the initial estimate?”*

A non-zero MR indicates that the first release might be systematically over or underestimated. In a second step, the mean revision might be tested for statistical significance.

In practice, given that the assumption that the revisions over time are identically independently distributed with an unknown variance and centred in zero, a t-test could be used. If these assumptions are violated, for example due to heteroscedasticity and/or autocorrelation in the revisions, an adjusted t-test should be used instead, e.g. the HAC (heteroskedasticity and autocorrelation consistent) *t*-test. See McKenzie and Gamba (2008), p. 8 for details. As the arithmetic average is affected by extreme observations, in some applications the median might be preferred as a robust alternative.

A useful measure of the size of the revisions is the Mean Absolute Revision (MAR). This avoids offsetting effects on the indicator from negative and positive revisions.

$$MAR = \frac{1}{n} \sum_{t=1}^n |(L_t - P_t)| = \frac{1}{n} \sum_{t=1}^n |R_t| \quad (20.25)$$

As in the case of the MR, the median absolute revision might be preferred as a robust alternative. Since the absolute revision can vary in proportion to the level of the indicator being analysed, it might be helpful to scale the MAR in terms of the size of the earlier estimates when doing comparisons over time or across different indicators. Such an adjusted MAR, which is named Relative Mean Absolute Revision (RMAR), can also be interpreted as a measure of robustness for interpreting revisions to first published estimates, as it gives the expected percentage of the first published estimate that will be revised over the revision interval being considered.

$$RMAR = \frac{\sum_{t=1}^n |(L_t - P_t)|}{\sum_{t=1}^n |L_t|} = \frac{\sum_{t=1}^n |R_t|}{\sum_{t=1}^n |L_t|} \quad (20.26)$$

A measure of the spread of revisions around their mean is the standard deviation of revisions,

$$SDR = \sqrt{\frac{1}{n-1} \sum_{t=1}^n (R_t - \bar{R}_t)^2} \quad (20.27)$$

However, as this measure is sensitive to outliers, it is not a good measure of dispersion for revisions with skewed distributions. Another informative measure is the Root Mean Square Revision, which combines the mean revision and the variance of the revision around its mean,

$$RMSR = \sqrt{\frac{1}{n} \sum_{t=1}^n R_t^2} \quad (20.28)$$

In order to put these measures in perspective, it might be informative to also consider the minimum revision,

the maximum revision and the range of revisions, which is the difference between the highest and the lowest revisions for all observation periods:

$$\text{Range} = \text{Maximum revision} - \text{Minimum revision} \quad (20.29)$$

Finally, in the case of rates of change, it might be helpful to know how often the later published rates are in the opposite direction to the earlier published rate. The non-parametric sign certainty indicator answers this question:

$$\%sign(\text{later}) = sign(\text{earlier}) \quad (20.30)$$

If this percentage is high, it indicates a good directional reliability. Nevertheless, when the values of the rates of change are close to zero, this indicator might give misleading signals, as the sign of the rate might not be statistically significant.

## 20.7.2 Decomposing revisions

Generally, revisions of seasonally adjusted real time data have two main sources which are inter-related. Firstly, the technical procedure of the method used for seasonal adjustment which can involve the release of new unadjusted data, where the old unadjusted data remains unchanged, and leads to a shift of the base period and a change in the weights of the smoothing filters. Secondly, the revision process of unadjusted data in real time, for example, on their first date of release the data contain estimates for missing values which will be updated by actual figures.

The following section describes a possible decomposition procedure as to how much each of the sources contributes to total revisions (see Eiglsperger et al. (2010) and Mehrhoff (2008)). The revisions are empirically quantified taking euro-area GDP volume growth as an example.

### 20.7.2.1 Decomposition methodology

Revisions of period-to-period movements of seasonally adjusted real time data can approximately be reformulated as the difference between the respective revisions of the growth rates of the unadjusted time series  $r_{t,\tau}^u$  and the seasonal component  $r_{t,\tau}^s$ , for which analogous definitions apply. Here,  $a_{t|t+\tau}$  denotes the estimate of the seasonally adjusted time series at time  $t$  based on unadjusted data up to time  $t + \tau$ .

$$\begin{aligned} r_{t,\tau}^a &:= (a_{t|t+\tau}/a_{t-1|t+\tau} - 1) - (a_{t|t}/a_{t-1|t} - 1) \\ &\approx \ln(a_{t|t+\tau}/a_{t-1|t+\tau}) - \ln(a_{t|t}/a_{t-1|t}) \\ &= \ln(a_{t|t+\tau}/a_{t|t}) - \ln(a_{t-1|t+\tau}/a_{t-1|t}) \\ &= [\ln(u_{t|t+\tau}/s_{t|t+\tau}) - \ln(u_{t|t}/s_{t|t})] - [\ln(u_{t-1|t+\tau}/s_{t-1|t+\tau}) - \ln(u_{t-1|t}/s_{t-1|t})] \\ &= [\ln(u_{t|t+\tau}/u_{t|t}) - \ln(u_{t-1|t+\tau}/u_{t-1|t})] - [\ln(s_{t|t+\tau}/s_{t|t}) - \ln(s_{t-1|t+\tau}/s_{t-1|t})] \\ &\approx r_{t,\tau}^u - r_{t,\tau}^s \end{aligned} \quad (20.31)$$

Before turning to the sources of revisions and their decomposition, it is worth revisiting four extreme cases and assuming that revisions of unadjusted real time data cannot be predicted, i.e. they are "new".

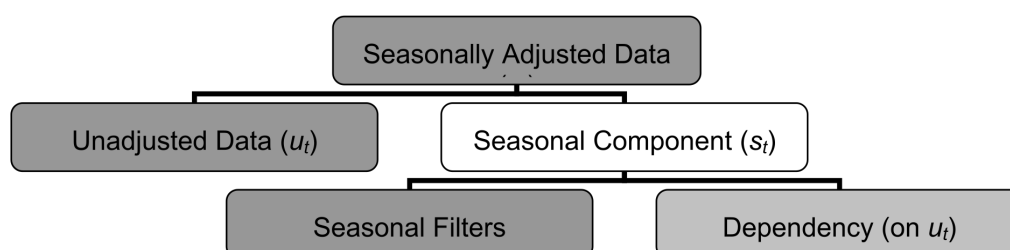
1. In the case of no seasonal fluctuations, i.e.  $s_t = 1 \forall t$ , the unadjusted time series is equal to the seasonally adjusted time series,  $u_t = c_t \cdot i_t = a_t$ . Then, revisions from seasonal adjustment are necessarily zero, i.e.  $r_t^s = 0$ , and thus  $r_t^a = r_t^u$ .
2. With the aid of regression-ARIMA modelling, a time series is extended beyond its end and symmetric or less asymmetric smoothing filters become applicable. The closer the forecast comes to the realised

value, the smaller are the revisions. In the case of perfect predictability of the seasonal pattern, revisions from seasonal adjustment are again zero.

3. If unadjusted data are not revised at all, i.e.  $r_t^u = 0 \forall t$ , revisions of seasonally adjusted data stem from seasonal adjustment alone, i.e.  $r_t^a = -r_t^s$ .
4. Revisions to unadjusted real time data may have a seasonal pattern, which, in turn, would show up in revisions of the seasonal component. If both revisions cancelled out during seasonal adjustment, i.e.  $r_t^u = r_t^s$ , the resulting revisions of seasonally adjusted real time data would be zero.

The above equation makes the interplay of the sources of revisions, unadjusted data on the one hand and the seasonal component on the other hand, evident. These are broken down in the figure below. In addition to revisions stemming from the seasonal filters, the seasonal component depends also on revisions of unadjusted data, constituting the "dependency", which can either attenuate or amplify total revisions.

### 20.7.2.2 Sources of Revisions



In order to separate the effects of revisions stemming from unadjusted data and from seasonal adjustment, Mehrhoff (2008) calculates a pure seasonal filter revision  $r_t^\omega$  in a first step. By construction, it is not related to the revisions of unadjusted data. Thus,  $r_t^\omega$  does not measure revisions of the seasonal component  $r_t^s$  in real time but those that result from the automatic history procedure known from X-13-ARIMA-SEATS (see section 20.7.3). This is similar to revisions of seasonally adjusted real time data with the difference that unadjusted data remain unchanged and the time series with data up to time  $t + \tau$  is truncated at time  $t$ . The following definition of  $r_t^\omega$  is comparable to  $r_t^a$  but now the subtrahend  $\Delta_{t|t+\tau}^a$  is defined as the corresponding element of the truncated time series.

$$r_{t,\tau}^\omega = \Delta_{t|t+\tau}^a - \text{trunc}\Delta_{t|t+\tau}^a \quad (20.32)$$

The dependency of the seasonal component on unadjusted data cannot be determined directly. Instead it will later be defined as the part of the seasonally adjusted real time data revisions that remains unexplained by revisions of unadjusted real time data and those of the seasonal filters.

### 20.7.2.3 Mehrhoff's model

The aim of Mehrhoff's model is to decompose revisions of seasonally adjusted real time data into an unadjusted real time data part and a seasonal adjustment part. Sources of information used are firstly the real time vintages of unadjusted data, and secondly the user setting of seasonal adjustment options which are held constant throughout all vintages. Using these settings, the vintages of unadjusted real time data are seasonally adjusted, i.e. historical published figures are not used. In doing so a third source of revisions is suppressed that would emerge if the user settings were changed. During the calculations, regression-ARIMA model parameters, namely those of calendar regressors, outliers and the ARMA parameters, are estimated with the full data span of the latest time series available. Therefore, the revisions derived mainly describe the properties of seasonal filters.

The hypothesis is that the variance of revisions of seasonally adjusted real time data is the result of revisions from unadjusted real time data and seasonal filters as well as the dependency of revisions of the seasonal component on those of unadjusted real time data. Basically, a causal relationship of the form  $r_{t,\tau}^a = f(r_{t,\tau}^u, \tau, r_{t,\tau}^\omega)$  is presumed. In particular, revisions are considered in a regression model which is formally stated below.

$$r_{t,\tau}^a = \beta_t^u \cdot r_{t,\tau}^u + \beta_\tau^\omega \cdot r_{t,\tau}^\omega + \epsilon_{t,\tau} \quad (20.33)$$

Estimated slope coefficients represent marginal effects, i.e. the change of revisions of seasonally adjusted real time data to a one percentage point change of either revisions of unadjusted real time data or revisions of seasonal filters.

Employing average absolute revisions,  $\bar{R}_\tau^a$ ,  $\bar{R}_\tau^u$  and  $\bar{R}_\tau^\omega$ , absolute contributions can be calculated as follows. For this, absolute values are used, since otherwise revisions would cancel out to a large extent. Note that the  $\beta_\tau$  are estimated according to the above equation and are plugged in here. This is done in order to calculate not only partial effects (the regression coefficients) but also average total effects of the respective (absolute) revisions, i.e. the typical influences of each of the sources on total revisions.

$$\begin{aligned} \bar{R}_\tau^a &:= T^{-1} \sum_{t=1}^T |r_{t,\tau}| \\ &= \beta_\tau^u \cdot \bar{R}_\tau^u + \beta_\tau^\omega \cdot \bar{R}_\tau^\omega + \iota_\tau \end{aligned} \quad (20.34)$$

The dependency  $\iota_\tau$  is defined as the unexplained revisions by the other two sources.

$$\iota_\tau = \bar{R}_\tau^a - \beta_\tau^u \cdot \bar{R}_\tau^u - \beta_\tau^\omega \cdot \bar{R}_\tau^\omega \quad (20.35)$$

For a convenient interpretation absolute contributions can be expressed as relative ones.

$$1 = \beta_\tau^u \cdot (\bar{R}_\tau^u / \bar{R}_\tau^a) + \beta_\tau^\omega \cdot (\bar{R}_\tau^\omega / \bar{R}_\tau^a) + (\iota_\tau / \bar{R}_\tau^a) \quad (20.36)$$

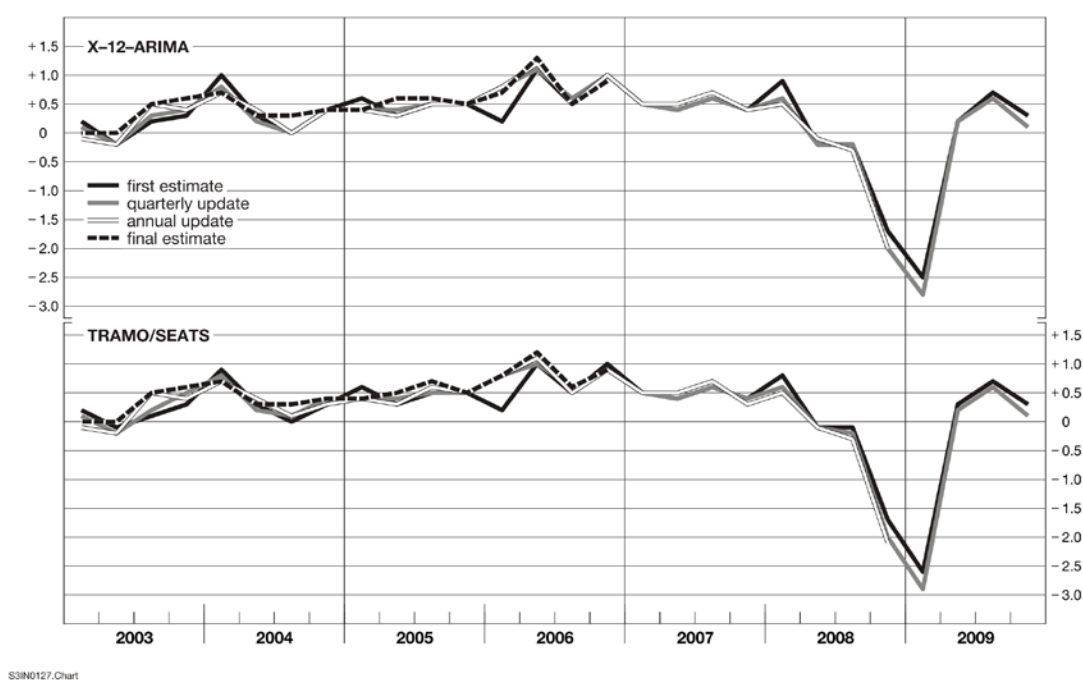
#### 20.7.2.4 Case study: Decomposition of revisions

Consider an example for euro-area GDP volume growth, where the period covered is from 2003 Q1 to 2010 Q1, with the data span starting at 1995 Q1. The last observation that is revised with a quarterly update is the one of 2009 Q4 ( $T = 28$ ), with an annual update (benchmark revision) it is the one of 2008 Q4 ( $T = 24$ ) (see section 20.6.1). Analysis of revisions of the final estimate is based on data up to 2006 Q4 ( $T = 16$ ). The respective estimates are considered almost final when the data were reported for 2010 Q1. A three-year period at the end is left in order to compare the results with almost final seasonally adjusted figures. Seasonal adjustment was performed with level shifts in 2008 Q4 and 2009 Q1 in order to deal with the abrupt strong movements caused by the financial and economic crisis at that time. Mehrhoff contrasts the X-12-ARIMA and TRAMO/SEATS methods using a version of X-13ARIMA-SEATS. The model was estimated and the results are presented in the table following the next figure, which shows the above mentioned vintages.

**Table 20.5: Regression results\***

Target	X-12-ARIMA			TRAMO-SEATS		
	$\beta_\tau^u$	$\beta_\tau^\omega$	$R^2$	$\beta_\tau^u$	$\beta_\tau^\omega$	$R^2$
Quarterly Update	.43***	.62**	.68	.40***	.64**	.70
Annual Update	.03***	.74***	.60	.28***	.81***	.63
Final Estimate	.015	1.09*	.36	.14	1.54**	.48

\* p-values (significance levels): \*\*\* = 1%, \*\* = 5%, \* = 10%.

**Figure 20.12:** Vintage graph of quarter-on-quarter seasonally adjusted volume growth in %

For the quarterly and annual updates, all estimated parameters are significant at least on the 5% significance level. For the final estimate the results are not that clear cut. With the increased availability of source data over time, marginal effects of unadjusted data revisions decrease while those of seasonal filter revisions increase. In combination with average absolute revisions stated in table below, the following picture emerges. Average absolute revisions stemming from the seasonal filters remain more or less the same. However, those of both seasonally adjusted and unadjusted real time data increase with the passing of time.

**Table 20.6:** Average absolute revisions (in percentage points)

Target	X-12-ARIMA			TRAMO-SEATS		
	$\bar{R}_\tau^a$	$\bar{R}_\tau^u$	$\bar{\omega}_\tau^a$	$\bar{R}_\tau^a$	$\bar{R}_\tau^u$	$\bar{\omega}_\tau^a$
Quarterly Update	.10	.17	.06	.09	.17	.06
Annual Update	.14	.27	.09	.13	.27	.09
Final Estimate	.18	.35	.06	.18	.35	.06

Using the final formula of Mehrhoff's model, contributions to total revisions reveal the following development from the quarterly update via the annual update to the final estimate: Seasonal adjustment, i.e. seasonal filters and the dependency, becomes more important when trying to understand the revision process in the long run. However, in the short run, revisions from unadjusted real time data are found to play the major role when explaining revisions, also reflecting the increasing availability of short-term "hard" data sources over subsequent releases.

The decomposition of sources of revisions revealed that a large influence on total revisions of euro-area GDP volume growth can be ascribed to revisions of unadjusted real time data in the short run. This corresponds with the increasing availability of short-term "hard" data sources over subsequent releases. In the longer run, unadjusted data tend to be revised less, and thus seasonal filters revisions gain more relevance. In the end, the influence of seasonal filters decreases as well and the dependency becomes more prominent. While unadjusted data are no more revised at some point in the future, seasonal adjustment generates revisions



**Table 20.7:** Contribution to total revisions (in %)

Target	X-12-ARIMA			TRAMO-SEATS		
	Unadj. Data	Seas. Filters	Depend- ency	Unadj. Data	Seas. Filters	Depend- ency
Quarterly Update	.10	.17	.06	.09	.17	.06
Annual Update	.14	.27	.09	.13	.27	.09
Final Estimate	.18	.35	.06	.18	.35	.06

long after that.

### 20.7.3 Using X-13-ARIMA-SEATS to assess revisions

There are tools within X-13-ARIMA-SEATS that allow some evaluation of revisions due to additional time points. For comprehensive details of X-13-ARIMA-SEATS capabilities readers are referred to the X-13-ARIMA-SEATS reference manual and in particular to Chapters 6 and 7. In this section examples are used to demonstrate the use of the “history spec” to assess revision histories and how they can be used for analysis.

#### 20.7.3.1 Revision histories

Revision histories, created using the “history spec” is a stability diagnostic available in X-13-ARIMA-SEATS. It considers the revision of continuous seasonal adjustment over a period of years and is a way of evaluating how a time series is affected when new data points are introduced. The basic revision calculated by X-13-ARIMA-SEATS is the ‘difference between the earliest adjustment of a month’s datum’ obtained when that month is the latest month in the series and ‘a later adjustment based on all future data available at the time of the diagnostic analysis’.

When a new data point becomes available for a series, more is learnt about the behaviour of that series. This is especially true of seasonal series. With that new data point, more is found out about the seasonal pattern, as well as the underlying trend. However, when the series is updated with the extra information, this creates revisions to the seasonally adjusted series, as discussed in section 20.3. Generally, users want to minimise the number and size of these revisions. How important this is depends on the user. Some users prefer to have the most up-to-date figures possible, regardless of the size of revisions. Other users prefer limited revisions, to make the data they work with more consistent. The choice of revision policy is also influenced by the nature of the data. The use of revision histories can help inform this decision (see section 20.5 for further information on revisions policies).

The revision history diagnostic is a way of seeing how a time series is affected when new data points are introduced. The diagnostic works by taking a period of existing data, and adding the data points one by one as if they were new observations. This creates revisions in the data, and it is these revisions that users are interested in.

#### 20.7.3.2 Producing revision histories

The process by which the revision histories are generated is:

1. The start date of the history diagnostic is chosen. This can either be the default choice, which is either 6, 8 or 12 years after the start of the series depending upon other specified parameters, or it is chosen by the user with the start argument.
2. The program seasonally adjusts the series, up to and including the start point of the revision history.

3. The program then seasonally adjusts the whole series again, but up to the observation after the revision history start date. The program repeats this process, including one extra data point each run, until all the data has been added and the entire series has been adjusted.
4. The final seasonal adjustment, which covers all of the data, is the most recent adjustment available, and represents the best estimate of the seasonal factors at that point in time.
5. The program works out the percentage difference between the first seasonal adjustment of the starting data point (worked out in step 2) and the final adjustment for the same month (as in step 4, though this can be changed by using the target argument). The program repeats this for every data point up to, but not including, the final data point in the series. The program also calculates any other histories specified by the user.
6. The program then produces the appropriate tables for the series.

### 20.7.3.3 Using revision histories

Revision histories are most often used when there are two or more competing methods for seasonally adjusting a series, all of which are acceptable in terms of other diagnostics. They can compare how prone different adjustments are to revisions by comparing the revisions produced over the test period. In general, smaller is better. An adjustment with small average annual revisions will be more reliable, than an adjustment with larger revisions. Unlike some other diagnostics, such as the Q-statistic and sliding spans (both available in X-13-ARIMA-SEATS), there is no absolute measure of what are acceptable revisions. A full explanation of the diagnostics and options available and when and how they could be used would be long and unwieldy. Therefore this section concentrates on three applications of the history diagnostic.

1. Using the revision history to compare two or more seasonally adjusted or trend estimates
2. Comparing revisions in direct and indirect seasonal adjustment
3. Using Akaike information criterion (AIC) histories to choose between two or more regression-ARIMA models

The revision history diagnostic, when used to help analyse a time series, tends to be used later in an analysis once one or more methods have been found acceptable using alternative diagnostics. "Method" is used in this case to describe the application of a particular set of seasonal adjustment parameters, e.g. choice of filters and regression-ARIMA model, to produce a seasonally adjusted or trend estimate. Therefore, the difference between two methods applied to the same time series might be for example a different regression-ARIMA model specification or different length seasonal filters within X-13-ARIMA-SEATS.

The history specification can be used with different sets of arguments and the choice of the arguments depends on the scope of the revision analysis. Generating a revision history for the seasonally adjusted estimate of a particular method is the simplest use of the revision history diagnostic. The following specification file for X-13-ARIMA-SEATS is used to create a revision history for the seasonally adjusted estimate of average weekly earnings excluding bonuses series (as specified by **estimattessadj** in the **history** specification). In this example, data is available from January 2000 to October 2010.

Running such a specification file produces three tables; R0, R1 and R1.S are all found at the end of the output. Table R0 shown below is simply a summary of the options selected in the history analysis. As can be seen the analysis is conducted for the final seasonally adjusted estimate, the default start date for the analysis is January 2008, percentage revisions are based on the difference between the historical estimates and final estimate based on all available data points, and revisions are produced for concurrent estimates and for lags of 1 and 2 months (the **sadjlags** argument used to produce this is discussed further below).

Table 20.15 is the percent revisions of the concurrent seasonal adjustment and shows the percent revision at each point of time in the history analysis between the first and final seasonally adjusted estimates. So

**Figure 20.13:** X-13-ARIMA-SEATS specification file for a revision history

```

series{
  title="Excluding Bonuses"
  start=2000.1
  period=12
  decimals=2
  name="Average weekly earnings excluding bonuses"
  file="awe_exc.dat"
}
transform{
  function=log
}
automdl{
}
forecast{maxlead=24
}
x11{
}
history{estimates=(sadj)
  sadjlags=(1,2)
}

```

**Figure 20.14:** X-13-ARIMA-SEATS example output for a revision history, R0

```

R 0 Summary of options selected for revisions history analysis.

History analysis performed for the following:
-----
- Final Seasonally Adjusted Series

Lags from Concurrent Analyzed for Seasonally Adjusted Series:
  1  2

Starting date for history analysis: 2008.Jan
Ending date for history analysis: 2010.Oct

Seasonal Adjustment Revisions Computed Using Final as Target.

regARIMA coefficient estimates are estimated during the history analysis.

```

what do the figures in Table R1 mean in practice? In this case, the data is monthly and the final adjustment (with all the data up to October 2010 available) is the target. The first column is the revision history of the initial adjustment. It is the percentage difference between the first vintage (including all the data up to the point in question in the adjustment) and the final vintage (including all the data in the series). For example, the value 0.42 for December 2009 has been calculated by working out the percentage difference between the seasonally adjusted estimate for December 2009 given all the data, and the value for December 2009 given the data up to December 2009. In other words, it is the percentage difference between the October 2010 vintage (the final vintage) and the December 2009 vintage (the first vintage of this data point and 24th vintage of the history output). These two vintages are plotted in Figure 20.16. The revision is the percentage difference between the levels of the two estimates at December 2009 as highlighted on the chart.

The next two columns are the revisions at lag 1 and lag 2 — this is stated in the output in R0, and is due to the **sadjlags** argument in the history spec. In this case, the value at lag 1 for December 2009 is 0.65 (column 2 in Table R1). This has been calculated by working out the percentage difference between the seasonally adjusted value for December 2009 given all the data, and the value for December 2009 given the data up to January 2010. This is the percentage difference between the final vintage and the January 2010 vintage (the second vintage) of December 2009 (or the 25th vintage of the output).

The final column is the revisions at lag 2. This is the percentage difference between the final vintage and the third vintage. In general, for a column representing the revision at lag  $n$ , the figure quoted is the percentage difference between the final vintage of the data and the  $(1 + n)$  vintage.

Why are the revisions at various lags of interest? They give some idea as to the size of revisions at each

Figure 20.15: X-13-ARIMA-SEATS example output for a revision history, R1

```

No outliers kept or deleted during this history analysis

R 1 Percent revisions of the concurrent seasonal adjustments
From 2008.Jan to 2010.Sep
Observations      33
Date              Conc - 1 later- 2 later-
                  Final      Final      Final
-----
2008
Jan               1.11      0.90      0.84
Feb               1.46      1.41      1.40
Mar              -0.04     -0.06      0.04
Apr              -0.16      0.00      0.14
May              -0.50     -0.37     -0.30
Jun              -0.51     -0.42     -0.36
Jul              -0.42     -0.33     -0.29
Aug              -0.62     -0.56     -0.55
Sep              -0.83     -0.82     -0.71
Oct              -0.97     -0.83     -0.75
Nov              -0.80     -0.72     -0.24
Dec              -0.50      0.08      0.56
2009
Jan              -0.65      0.56      0.40
Feb              -0.94     -1.02     -1.15
Mar              -0.44     -0.72     -0.76
Apr               1.32      1.24      1.19
May               0.18      0.13      0.16
Jun               0.11      0.15      0.16
Jul               0.08      0.09      0.06
Aug               0.00     -0.03     -0.06
Sep               0.01     -0.02     -0.01
Oct               0.20      0.21      0.18
Nov               0.24      0.21      0.39
Dec               0.42      0.65     -0.02
2010
Jan              -0.19     -0.18     -0.36
Feb               0.10     -0.47      0.05
Mar              -0.18      0.00     -0.02
Apr               0.02     -0.06      0.01
May              -0.03      0.05      0.03
Jun               0.08      0.06      0.04
Jul               0.08      0.06      0.00
Aug               0.06      0.01 *****
Sep               0.01 ***** *****

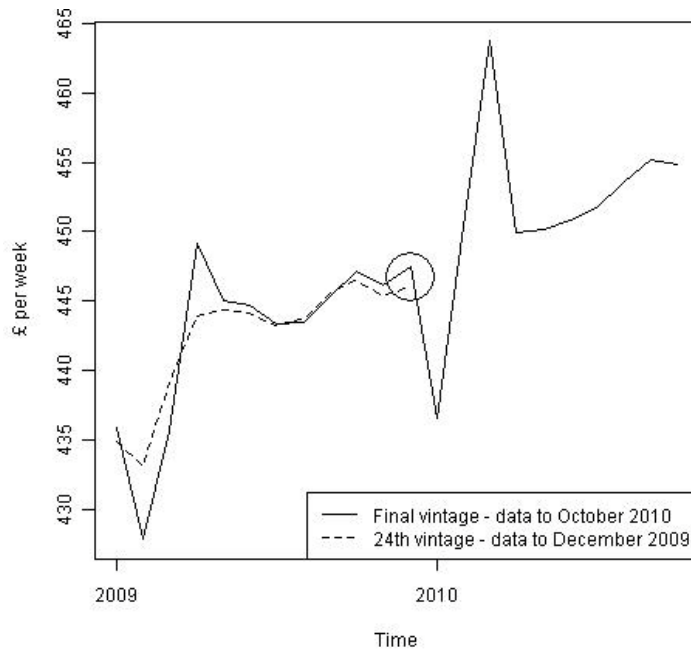
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lag, which can say a lot about the data. For example, the relative sizes of the revisions at each lag give you some information as to how quickly adjusted data converges to its final values. This information can be used to inform decisions on a revision policy for the series (see section 20.5). Up to five different lags at a time can be specified, using the **sadjlags** argument.

The final table in this example is the R1.S which is a summary of the revisions. It provides average absolute revisions by month, year and over the history analysis period, as shown above. The average absolute revision for the total period is 0.4 (Total: column 1). If two methods have similar average absolute revisions for the total history period then consideration could be made of the distribution of revisions over months or years. Whilst the total value of average absolute revisions, whether concurrent or at lag  $n$ , can be useful for comparing two methods of producing a seasonally adjusted estimate, it is important to consider any particularly large revisions at specific time points as well as the distribution of revisions.

Depending upon the options specified in the **history** specification a number of tables are produced. Revision histories can be estimated for seasonally adjusted estimates, month-on-month changes in seasonally adjusted estimates, trend estimates, month-on-month changes in trend estimates, seasonal factors, likelihood information for the regression-ARIMA model (AICC, corrected Akaike Information Criterion), and forecasts from the regression-ARIMA model.

**Figure 20.16:** Example: Revision history for seasonally adjusted estimate of UK average weekly earnings: Whole economy



**Figure 20.17:** Example: Table R 1.S

R 1.S Summary statistics : average absolute percent revisions of the seasonal adjustments

Date	Conc - Final	1 later- Final	2 later- Final
----	-----	-----	-----
Months:			
Jan	0.65	0.55	0.53
Feb	0.83	0.96	0.87
Mar	0.22	0.26	0.27
Apr	0.50	0.44	0.45
May	0.23	0.18	0.17
Jun	0.23	0.21	0.19
Jul	0.19	0.16	0.11
Aug	0.23	0.20	0.30
Sep	0.28	0.42	0.36
Oct	0.59	0.52	0.46
Nov	0.52	0.46	0.31
Dec	0.46	0.37	0.29
Years:			
2008	0.66	0.54	0.51
2009	0.38	0.42	0.38
2010	0.08	0.11	0.07
Total:			
	0.40	0.39	0.36
Hinge Values:			
Min	0.00	0.00	0.00
25%	0.08	0.06	0.05
Med	0.20	0.21	0.24
75%	0.62	0.69	0.55
Max	1.46	1.41	1.40

### 20.7.3.4 Comparing methods of estimation

Table 20.8 shows the revision history for one method of estimation and only considers the seasonally adjusted estimate. However, if two or more methods seem reasonable by other diagnostics, revision histories can be a useful tool for making comparisons, and it may be desirable to consider revision histories of other components or functions thereof. Table 20.8 shows the total average absolute revision for concurrent and lagged seasonally adjusted estimates and for concurrent and lagged month-on-month change for the trend estimate for three different methods. As can be seen the revisions for both the seasonally adjusted estimate and the month-on-month change in the trend estimate are generally lowest for Method 3. This may be one indication that Method 3 is preferable. However, other diagnostics should also be considered.

**Table 20.8:** Example: Revision history - method of estimation

Method	Seasonally adjusted estimate				Month-on-month change of the trend estimate			
	Concurrent	Lag 1	Lag 2	Lag 12	Concurrent	Lag 1	Lag 2	Lag 12
Method 1	0.38	0.41	0.40	0.16	0.23	0.18	0.13	0.09
Method 2	0.40	0.39	0.36	0.20	0.22	0.18	0.14	0.08
Method 3	0.33	0.29	0.24	0.22	0.22	0.17	0.13	0.08

### 20.7.3.5 Comparing direct and indirect estimation

Revision histories can be used to compare direct and indirect seasonal adjustment. Such output is obtained when a composite seasonal adjustment is performed in X-13-ARIMA-SEATS with the **history** specification. The revision history of the indirectly seasonally adjusted estimate is given in Table R3. Other than setting up the composite adjustment and including the revision specification in the individual component series, no other changes to the specification files are needed. As an example, seasonally adjusted estimates of total overseas visitors to the UK can be obtained by directly seasonally adjusting the unadjusted series for total overseas visitors to the UK or indirectly by seasonally adjusting series that form total overseas visitor (i.e. overseas visitors to the UK by country or region of departure).

The summary average absolute revisions for the direct estimate (as discussed above) are in Table R1.S whilst the summary average absolute revisions for the indirect estimate results are from Table R3.S, both are show in figure 20.18. The tables show that since 2009, the indirect adjustment has produced greater revisions than the direct adjustment. This might not be a problem, and the indirect adjustment may have other virtues, for example, it is more stable under sliding spans. While the direct adjustment is better than the indirect in terms of the size of revisions it suffers, the table shows how big the difference is and allows it to be set against any perceived advantages of indirect adjustment.

**Figure 20.18:** Example: Summary statistics for Table R 1.S and R 3.S

```

R 1.S Summary statistics : average absolute percent revisions of the
seasonal adjustments
Years:
2009 0.73
2010 0.50
Total: 0.65
R 3.S Summary statistics : average absolute percent revisions of the
concurrent indirect seasonal adjustments
Years:
2009 1.06
2010 0.67
Total: 0.92

```

Table R7 is a history of the model's AIC values over the period of the history. Unlike the tables discussed above, this is not a revision history. It is a history of the AIC values calculated when fitting the model. Lower AIC values indicate a better model fit. The AIC history is a useful and flexible way to choose between two

different models for seasonal adjustment. For example, Table 20.19 reports the last years worth of AICC values for three different models used for the UK average weekly earnings excluding bonuses series. As can be seen Model 2 has the lowest AICC values. Models 1 to 3 are the regression-ARIMA models used in the three different methods used in the section on comparing estimates. However, as seen above Model 2 (which corresponds to Method 2) does not provide the lowest revisions for the seasonally adjusted estimate or the month-on-month change in the trend estimate. If an analyst is thoroughly reviewing a series it is important to consider what is required and to use appropriate diagnostics.

**Figure 20.19:** Example: Table R7

Span End Date	AICC		
	Model 1	Model 2	Model 3
2009.Oct	641.286	625.167	683.326
2009.Nov	646.695	630.164	687.861
2009.Dec	652.108	635.2	693.252
2010.Jan	658.784	641.905	699.247
2010.Feb	671.153	662.217	727.132
2010.Mar	687.524	674.618	738.762
2010.Apr	696.992	686.878	752.768
2010.May	702.894	692.309	758.088
2010.Jun	708.459	697.791	764.99
2010.Jul	713.889	703.007	770.567
2010.Aug	719.384	708.268	775.53
2010.Sep	724.9	713.618	780.47
2010.Oct	730.299	718.813	785.583

The revision history diagnostic is most useful when comparing two or more methods of obtaining seasonally adjusted or trend estimates where all methods seem usable in terms of the X-13-ARIMA-SEATS M and Q statistics, sliding spans and so on. There is no absolute measure of what is an acceptable level of revisions, so the diagnostic is of more limited use on a single series, except when used to assist decisions of revisions policies.

## 20.8 Dealing with revisions in practice

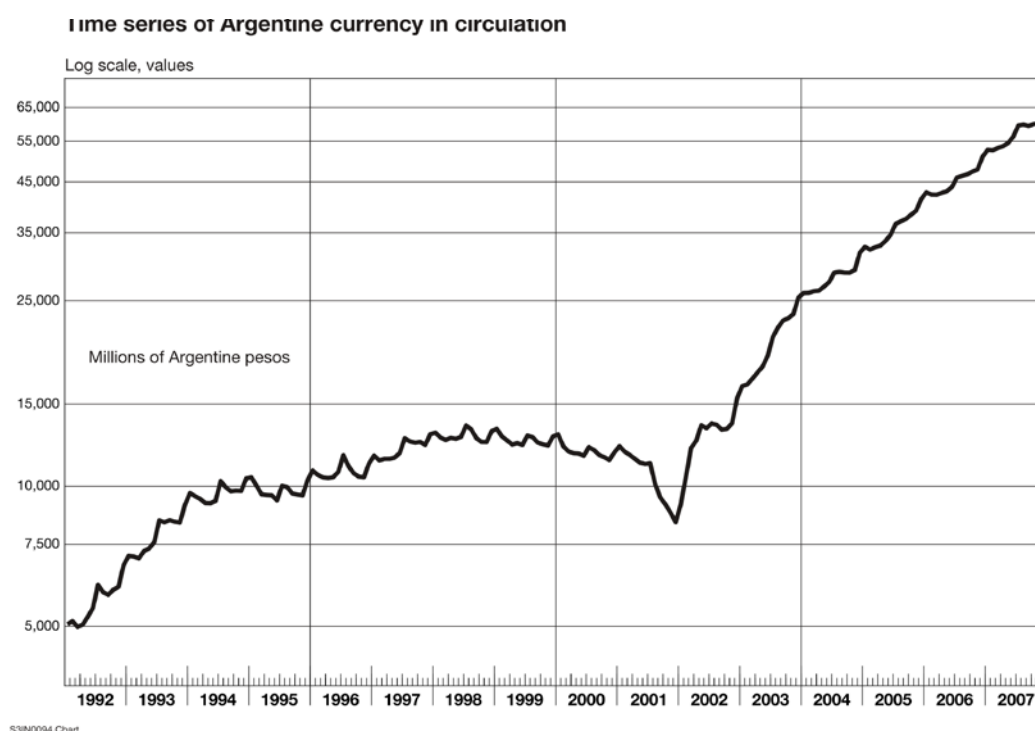
### 20.8.1 Case study: Argentine currency in circulation

The present subsection refers to the ESS Guidelines on Seasonal Adjustment and gives an example of how to use them in times of crisis. Taking as a basis the example of Argentine currency in circulation, a comparison is made between revisions from re-seasonally adjusting data every time a new figure is released (partial concurrent adjustment — with and without outlier modelling) and seasonally adjusting data with forecast seasonal and calendar factors (controlled current adjustment). See Mehrhoff (2010) for more details.

The empirical example chosen here is that of Argentine currency in circulation, which was affected by the 2001/2002 Argentine crisis. Three different structural periods can be identified in the time series. The first of these is the time before the Argentine crisis, in particular the period from January 1992 to November 2001. The height of the crisis covers the months from December 2001 to May 2002 — this view is supported by the

outlier identification of regARIMA modelling. Finally, from June 2002 to December 2007 a new development of the time series can be observed.

**Figure 20.20:** Time series of Argentine currency in circulation



A more detailed inspection of the unadjusted data reveals a certain pattern of seasonality in currency in circulation. There are peaks in December and January followed by troughs until July, when the value of currency in circulation peaks again before declining until December. Looking at reasons behind this pattern, it is the extra half-wage payments received in July, December and January which are the prime cause of the seasonality. As the time series is a monthly stock series of daily averages it shows working day effects. For the sake of parsimony and in order to avoid multicollinearity, two regressors are built. The "Monday" regressor counts the number of Mondays minus the number of Fridays. Tuesdays, Wednesdays and Thursdays are put together in the "weekday" regressor, with their number again being measured in deviation from the number of Fridays. Additionally, a special Christmas regressor is set up which counts the number of working days within 15 calendar days before Christmas. Results are given in Table 20.9.

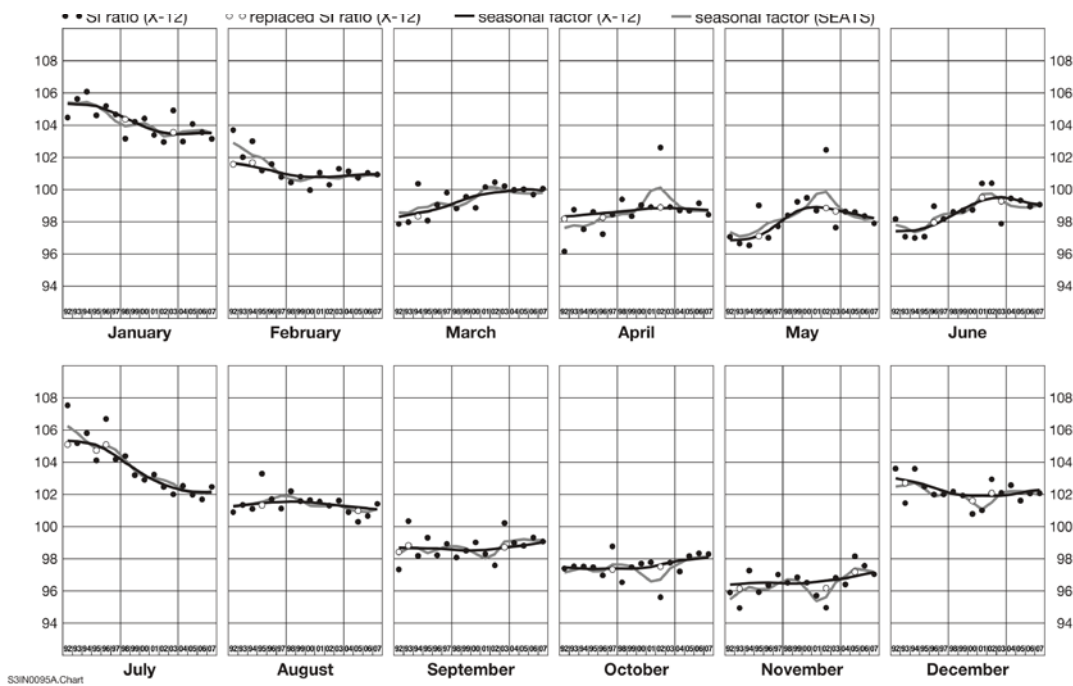
Prior to estimation, unadjusted data are transformed into natural logarithms. In addition to the aforementioned three calendar regressors, the baseline model (as of November 2001) consists of a constant term and a level shift for August 2001. The ARIMA model is seasonally and non-seasonally integrated with both a seasonal and non-seasonal AR term and a seasonal MA term, i.e.  $\ln(110)(111)_{12}$ . Following the Argentine practice, the monthly-specific seasonal smoothing filters in the seasonal adjustment core of X-12-ARIMA, the program is hereinafter referred to as X-12, are set to  $3 \times 9$ . By contrast, the decomposition with SEATS is based solely on signal extraction from TRAMO's regARIMA model. Estimation is performed with X-13ARIMA-SEATS.

**Table 20.9:** Estimated semi-elasticities of the calendar regressors

Variable	Parameter estimate	Standard error	t-value
"Monday" regressor	.17	.091	1.90
"Weekday" regressor	-.04	.029	-1.52
Christmas regressor	.30	.253	1.17



**Figure 20.21: X-12 SI ratios and X-12/SEATS seasonal factors by month**



### 20.8.1.1 Seasonal adjustment during the crisis

At the end of 2001, one of the most difficult political, economic and social crises in Argentina occurred. In order to avoid a bank run, the government de facto froze all bank accounts in December 2001. The most striking feature of currency in circulation is the change in the exchange rate regime beginning in January 2002. At the beginning of 2002, the national government declared that it would halt repayments on its national debt and strongly devalued the peso, adopting a managed float regime. How should one deal with such a situation in the context of seasonal adjustment? The ESS Guidelines suggest two viable alternatives for seasonally adjusting new data. Either partial concurrent adjustment where the extraordinary effects of the crisis are accounted for by the introduction of outlier variables or the use of forecast seasonal and calendar factors in combination with internal checks, i.e. controlled current adjustment.

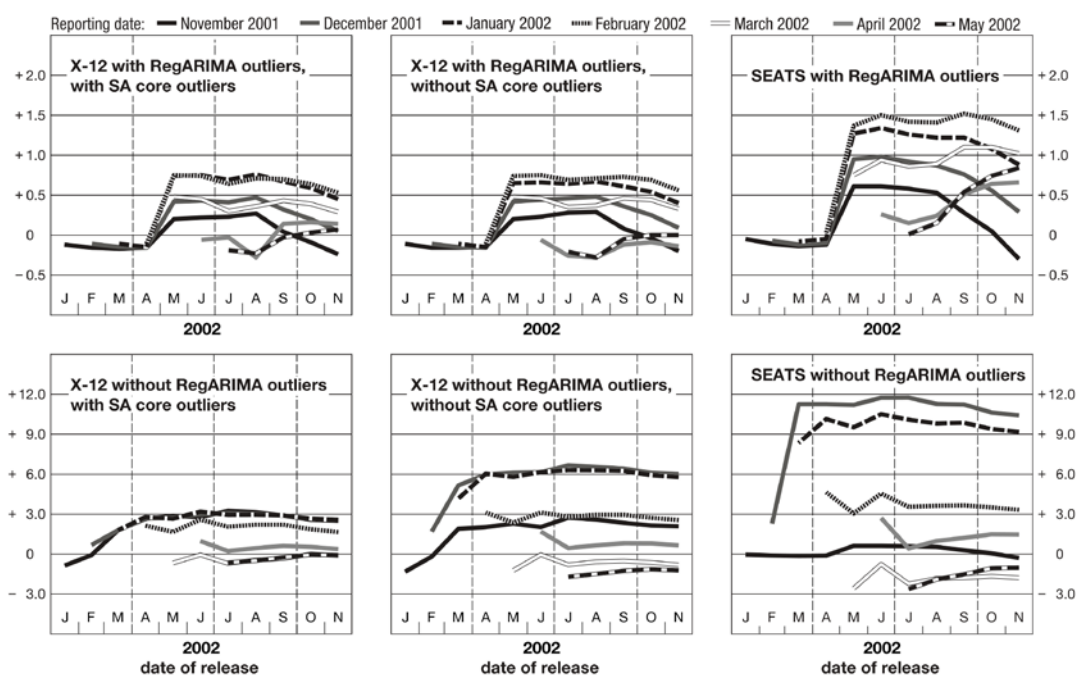
In the example below, revisions from partial concurrent adjustment are analysed with and without outliers in both regression-ARIMA modelling and — for X-12 only — the seasonal adjustment (SA) core. The model is identified based on November 2001 data. Provided regression-ARIMA outlier variables are introduced into the model, the following will be used which are identified both in real time and ex post (the corresponding t-values are at least about four in absolute terms throughout indicating that they are statistically significant events):

- a level shift in December 2001,
- an additive outlier in January 2002,
- an additive outlier along with a level shift in February 2002 and
- a level shift in May 2002.

While the model parameters are re-estimated, the model specification is kept constant. Revisions are calculated with the concurrent estimate as target, i.e.  $\text{revision} = \text{later estimate} / \text{first estimate} - 1$ , showing how much a given adjustment changes when adding more data. Old unadjusted data remain unchanged, i.e. revisions are not calculated in real time but much like the automatic history procedure from X-13-ARIMA-SEATS.

When outliers are correctly specified in regression-ARIMA modelling, the revisions from the X-12 method and the SEATS method are similar, although X-12 produces somewhat lower revisions than SEATS. Conversely,

Figure 20.22: Revision of the first seasonally adjusted estimates over time in %



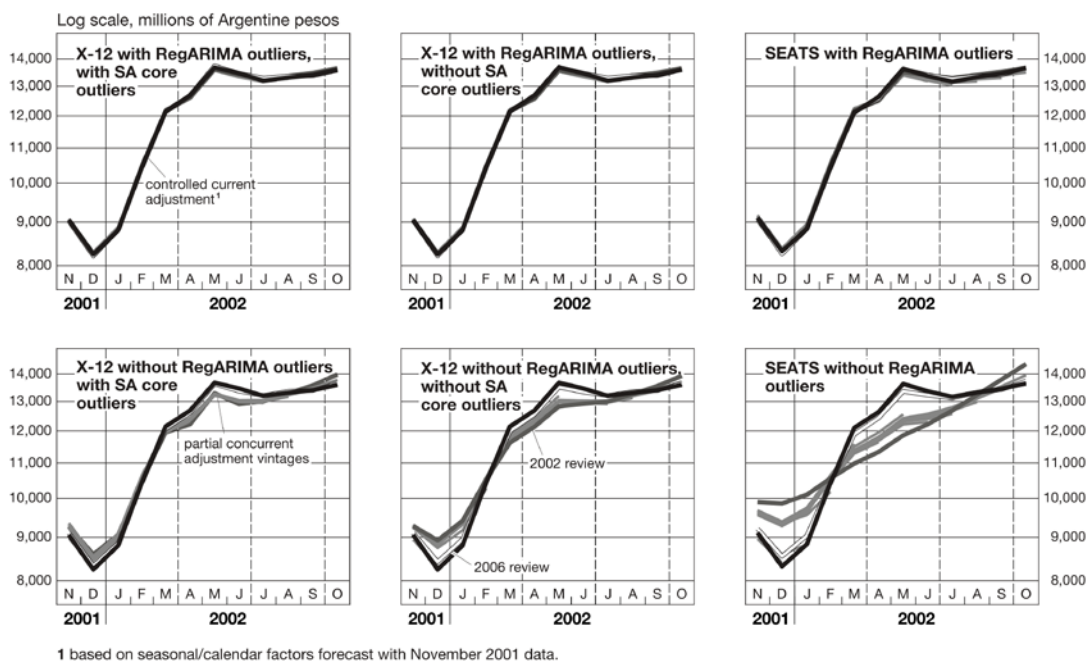
if regression-ARIMA outliers are neglected, revisions increase dramatically. The need for outlier modelling becomes even more obvious when considering SEATS without regression-ARIMA outliers: the estimated model is of little use, if at all, and thus SEATS fails to estimate the seasonal factor reliably. In some cases, SEATS is unable to admissibly decompose the model and has to replace it with a decomposable one. As regards to revisions, regression-ARIMA outlier modelling is more important than extreme value detection in X-13-ARIMA-SEATS seasonal adjustment core (the X-11 part). Nonetheless, the use of seasonal adjustment core outliers lowers revisions, especially if regression-ARIMA outliers are disregarded. Eventually, there is strong evidence that seasonality is present even in times of crisis.

### 20.8.1.2 Seasonal adjustment during the crisis revisited

After mid-2002 the first signs of a recovery of the Argentine economy became visible. The regression-ARIMA model is reviewed after 12 months with the first release of November 2002 data and again four years later with the first November 2006 data release. While no additional outliers are identified after the crisis, the model is changed in 2002 to  $\ln(012)(011)_{12}$  from  $\ln(110)(111)_{12}$ , i.e. the non-seasonal and seasonal AR terms, implying an  $MA(\infty)$  structure, are replaced by two non-seasonal MA terms. In 2006, the model is again revisited to  $\ln(111)(111)_{12}$  — a mixture of non-seasonal and seasonal AR and MA terms, now without a constant. The seasonal adjustment specification remains unchanged. However, if outliers are neglected right from the beginning, the models read  $\ln(012)(010)_{12}$  and  $\ln(110)(011)_{12}$ , respectively, both without the constant term.

While the partial concurrent adjustment data vintages that make use of regression-ARIMA outlier modelling closely resemble the development of controlled current adjustment figures, those estimated without regression-ARIMA outliers might send the wrong message during and shortly after the crisis. The gradients differ considerably between the vintages and to the forecast. This causes the above strong revisions. What is even more problematic is that, in addition, the direction of change is revised in several cases. However, a notable result is that, in the long run, the results from either adjustment are similar no matter whether outliers are modelled or not. In spite of this, seasonal adjustment without outlier modelling in the short run produces results that are remarkably different from the long run.

**Figure 20.23:** Partial concurrent adjustment and controlled current adjustment compared



S3N0099.Chart

There are at least two explanations for why the short-run results without outlier modelling are well apart from the final results, while those with outlier modelling are fairly close even in the short run. Firstly, regression-ARIMA estimates and forecasts are biased as regards the seasonality of the series if outliers are not modelled in the short run. This has a greater impact on SEATS than on X-12 because the decomposition of SEATS is based on regARIMA modelling alone. Secondly, in the short run, the effects of the crisis are partially allocated to the seasonal factors rather than to the trend-cycle or the irregular component, i.e. they do not remain fully visible in seasonally adjusted data. This misperception diminishes only in the long run when sufficient "normal" data before and after the crisis become available.

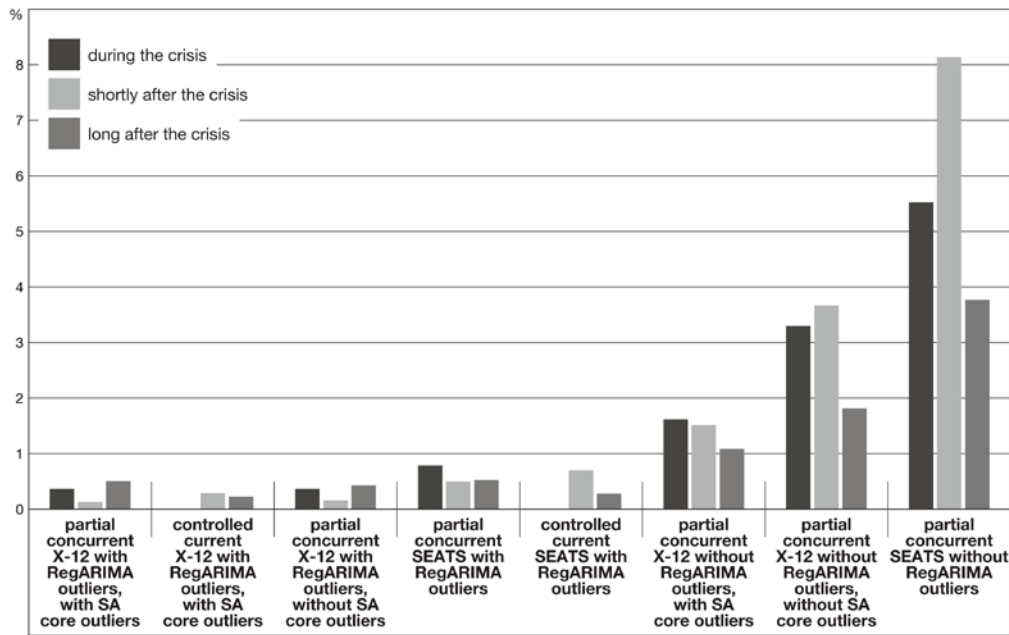
What is striking is that revisions of partial concurrent adjustment with regression-ARIMA outliers and of controlled current adjustment for both X-12 and SEATS remain low shortly and long after the crisis. Diametrically, revisions of X-12 and SEATS without regression-ARIMA outliers increase shortly after the crisis from an already high level and, though decreasing, remain high long after the crisis. This effect can be attenuated only if at least X-12's seasonal adjustment core outlier identification is utilised.

As the legislation governing the extra half-wage payments remained unchanged during the entire period under analysis, no significant changes should be observed in the seasonality pattern of the series. It will be examined whether the seasonality experienced changes after the crisis or whether, on the contrary, the seasonal pattern is similar to that observed before the crisis, meaning that seasonal adjustment should be carried out for the entire period. It turns out that seasonality extends beyond the crisis with the aforementioned peaks and troughs still visible after the crisis despite the strong growth of the series.

### 20.8.1.3 Implications for seasonal adjustment

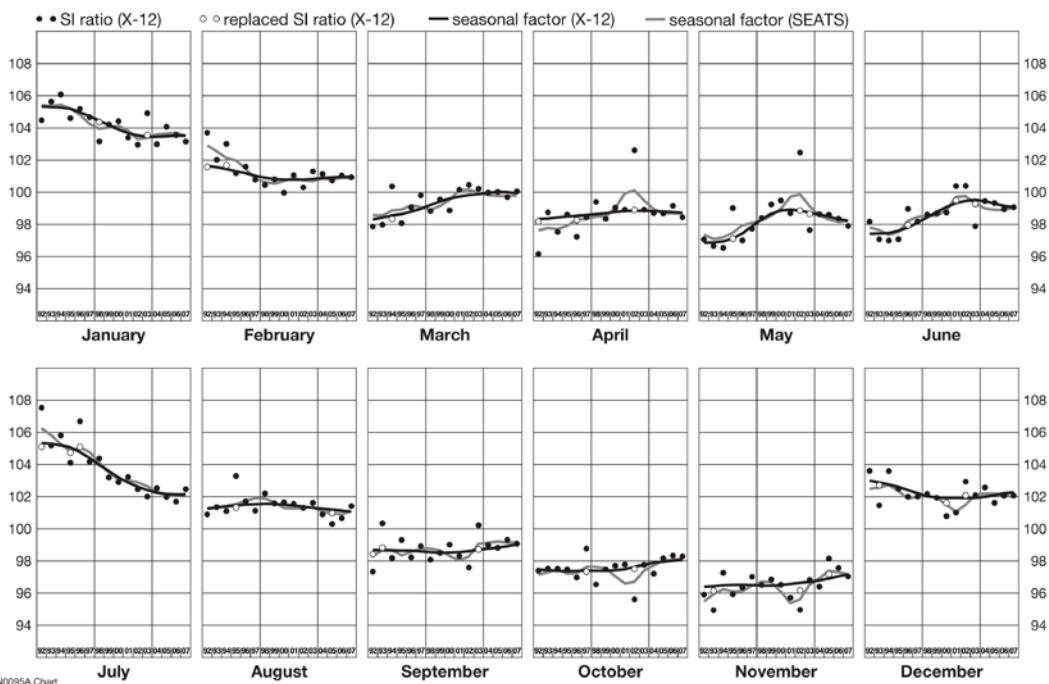
The results show that, as long as the reasons for seasonality continue to exist, seasonal adjustment makes sense. Furthermore, it is important to identify outliers, as otherwise the estimate of seasonal factors is distorted, resulting in major revisions. It should be emphasised that outlier modelling at the current end of the time series is a crucial issue. The results point to largely different estimates in the short run between seasonal adjustment with and without outliers. But, as time passes, these differences fade away and the results become more similar in the longer run. In the end, the preliminary results obtained with outlier modelling during the

Figure 20.24: Average absolute revisions of seasonally adjusted data from December 2001 to May 2002



S3IN0097.Chart

Figure 20.25: X-12 SI ratios and X-12/SEATS seasonal factors by month



S3IN0095A.Chart

crisis are close to the final results, no matter whether outliers are identified or not in the long run. Then again, seasonal adjustment in the short run without outlier treatment is remarkably distinct from the final results. This fact makes a good case for outlier modelling right from the beginning of a crisis. Revisions from both adjustment techniques are virtually the same if outliers are taken into account. Controlled current adjustment performs as well as partial concurrent adjustment. This justifies the classification of both methods as best alternatives in the ESS Guidelines. Hence, seasonal adjustment reveals important information for short-term business cycle analysis as it points out the news in the time series, even in times of strong economic changes.

### 20.8.2 Case study: Real time databases

Among other international and national institutions, both the OECD and the Deutsche Bundesbank operate a real time database which is publicly available.

- The real time database of the OECD is available at <http://stats.oecd.org/mei/default.asp?rev=1&lang=e>
- The real time database for the Deutsche Bundesbank is available at [https://www.bundesbank.de/Navigation/EN/Statistics/Time\\_series\\_databases/Real\\_Time\\_Data/realtime\\_zeitreihen\\_node.html](https://www.bundesbank.de/Navigation/EN/Statistics/Time_series_databases/Real_Time_Data/realtime_zeitreihen_node.html)

Real time databases save the vintages of time series in chronological order. Thus, they reflect the data actually available at that specific point in time. The importance of real time data becomes obvious when one tries to understand economic policy decisions that were made based on historical data and reconsiders these past situations in the light of more recent data. In addition to the vintages of the indicators, the OECD (to some extent) and the Deutsche Bundesbank offer accompanying metadata (sample methodology, estimation methods for missing values, timing of revisions, etc.). Without the use of metadata, the interpretation of results from revisions analysis can be highly misleading. A break in the compilation methodology will influence revision measures. Building expectation about future revisions based on such a measure will lead to erroneous conclusions. Thus, though often neglected in analysing revisions, metadata are of vital importance (Lorenz (2011)).

### 20.8.3 Case study: Revision analysis using History

When it comes to judgement of reliability of business cycles indicators, without any doubts, revisions of seasonally adjusted data are in the spotlight. As shown in Subsection 20.7.2, revisions of seasonally adjusted real time data are the result of two sources: revisions of unadjusted data in real time and revisions of seasonal (calendar) factors. Instead of the decomposition procedure — quantifying what part of total revisions is due to which source — presented there, one might be interested in analysing revisions stemming from the seasonal adjustment method itself. A possible case is comparing seasonal adjustment methods, e.g. what are the revisions from X-12-ARIMA compared to those from TRAMO/SEATS.

For instance, in the course of the changeover from X-11 to X-12-ARIMA, the Deutsche Bundesbank conducted a study as to the implications for economic analysis using, among others, the automatic History procedure. (see, Deutsche Bundesbank (1999) and Kirchner (1999) for details). It creates runs from a sequence of truncated versions of the time series. The purpose is to create historical records of revisions from initial seasonal adjustments.

According to this investigation, the current domain of uncertainty of seasonal adjustment depends heavily on the time series analysed and their properties. Time series that show strong irregular effects have substantially larger revisions than those with only minor fluctuations. The new forecasting capability and the improved treatment of outlier replacement and calendar effects in X-12-ARIMA compared to its predecessor X-11 constituted a major improvement.

## 20.9 Discussion

Discussions regarding the revision of seasonally adjusted data are not restricted to statistical experts. As seasonally adjusted economic statistics form the basis for current economic analysis the world over, revisions of these data have an impact on the quantitative assessment of economic developments. Consequently, revisions to published estimates are a topic of interest for ministries, economic researchers, forecasters, capital and financial markets, commercial banks, lobbying groups, news agencies, the business press and the general public.

Debate in this area is sometimes oversimplified by the suggestion that the mathematical processes involved in seasonal adjustment are the sole cause of revisions to seasonally adjusted data. However, as outlined in Section 20.3, a variety of factors play a role in revisions. Modifications to unadjusted data, as well as changes to seasonal and calendar factors, have an impact on the seasonally adjusted results. Nonetheless, the overall revisions of seasonally adjusted data cannot simply be broken down into these building blocks, as changes to the unadjusted data generally also lead to revisions of seasonal and calendar factors. Further assumptions are therefore required in order to split revisions in this way. One approach for achieving this is outlined in Section 20.7.2. However, this is also based on simplified assumptions, as it deliberately omits some real-world reasons for revising seasonally adjusted figures (such as changes in ARIMA parameters and models, identified outliers and their replacements and, as a general point, changes to the methodological options for seasonal adjustment). In order to take all of these causes into account, the approach would need to be considerably expanded. Evaluating the significance of the individual reasons behind revisions to seasonally adjusted data is therefore no easy matter. Nonetheless, it is important to persist in this endeavour, because it can provide us with an indication of the aspects that require particular attention in order to minimise revisions for purely technical reasons. Naturally, reporting of data corrections as a result of new information should continue, as this is part of the learning process in the field of statistics.

Particularly in the case of official economic statistics, the question arises as to whether every single new piece of information should really be published as soon as it becomes available. Although we would then have an optimal base of information, the trade-off for this would be almost permanent data revisions. The provision of this information would come at a cost to both data producers and users, as the data would need to be updated continually, although with the continued development of web dissemination this would be able to automate this process. It would also create a barrier to communication within society. For example, two people discussing a particular issue and using empirical statistical data to substantiate their arguments might sooner or later want to compare their data. They might easily get bogged down in the issue of whose data is actually correct, even if the differences were only minor.

There are therefore good reasons why, in practice, a revision is not immediately carried out in response to every minor piece of new information. Following the basic principle that significant information should be incorporated as quickly as possible into published data, whereas minor changes should first be collected before being implemented, a number of revision procedures have been devised in practice, often taking the form of corrections of data from the previous month, quarter or year. Such considerations also play a role in seasonal adjustment. They are reflected, for example, in the ESS Guidelines on Seasonal Adjustment. The guidelines points out the close relationship between seasonally adjusted figures and unadjusted data:

*“Revisions to seasonally adjusted data are published in accordance with a coherent, transparent and officially published revision policy and release calendar, which is aligned with the revision policy and the release calendar for the unadjusted data.”* (item 4.1, best alternative).

This principle clearly means that official statistics should not include new seasonally adjusted data unless the raw data have changed or been supplemented, or have communicated any update in line for the published revision policy. This may seem a trivial requirement, but it is one that, in the past, large statistical institutions have not always fulfilled.

Given the importance of the revision of key seasonally adjusted macroeconomic variables, the ESS guidelines



also recommend informing the public of the average revisions of such variables that have been observed in the past. Here, the focus is on seasonally adjusted estimates, not on the revisions of seasonal and/or calendar factors. Breaking the effect down into the reasons for revision of seasonally adjusted data is mainly of interest to data producers searching for opportunities to further improve the informative value of their statistics and thus trying to reduce revisions to a technical minimum.

For seasonal adjustment itself, there are different approaches to carrying out revisions, too. This is discussed in Chapter 20.5.3. The procedure of completely re-identifying all methodological options for seasonal adjustment (regARIMA models, parameters, outliers, filters etc.) with each newly added or changed unadjusted figure is based on the idea that the estimation of seasonally adjusted data should always be founded on all available information. In practice, however, this complete openness to altering all estimation steps on a monthly or quarterly basis can lead to revisions along the length of the series, which—in the user's eyes—could undermine the credibility of the statistics. In practice, to prevent potential reputational damage and reduce the frequency of revisions due to seasonal adjustment, projected seasonal and calendar factors are sometimes used.

These two scenarios: 1) completely re-identifying all adjustment options with each additional or changed unadjusted figure (concurrent adjustment) or, 2) the use of projected seasonal and calendar factors (current adjustment) — can be considered opposite ends of a whole spectrum of possible revision approaches. As the one-sidedness of both of these strategies leads to issues with the aspects that they neglect, a middle position between the two extremes is likely to result in a more balanced procedure. This is precisely the idea expressed in item 4.2 of the ESS Guidelines on Seasonal Adjustment. Here, both extremes are rejected and the following mixed approaches are named as the best alternatives.

- Partial concurrent adjustment: The model, filters, outliers and calendar regressors are re-identified once a year and the respective parameters and factors re-estimated every time new or revised data become available.
- Controlled current adjustment: Forecasted seasonal and calendar factors derived from a current adjustment are used to seasonally adjust the new or revised unadjusted data. However, an internal check is performed against the results of the “partial concurrent adjustment”, which is preferred if a difference exists. This means that each series needs to be seasonally adjusted twice. The approach is only practicable for a limited number of important series. A full review of all seasonal adjustment parameters should be undertaken at least once a year and whenever significant revisions occur (e.g. annual benchmark).

With regard to implementation, the ESS guidelines contain the following recommendation: *“When past data are revised for less than two years and/or new observations are available, partial concurrent adjustment is preferred to take into account the new information and to minimise the size of revisions due to the seasonal adjustment process. However, if the seasonal component is stable enough, controlled current adjustment could be considered to minimise the frequency of revisions. In this case, a full review of all seasonal adjustment parameters should be undertaken at least once a year. When revisions covering two or more years occur (as observed in national accounts) model, filters, outliers and regression parameters have to be re-identified and re-estimated.”*

Revisions due to seasonal adjustment naturally extend back to the beginning of a time series. Additions or changes to raw data give rise to new models, changed parameters, differences in the identification and replacement of outliers and, as a result, to modified seasonal and calendar factors. At the same time, this raises the question of whether a newly added unadjusted figure actually improves the information content for estimating a seasonally adjusted value that is already decades old. Is a new value from 2011 really relevant for estimating data from 1960? Would it not be better to establish that, after a number of years' learning, the estimation of seasonally adjusted data should be regarded as final?

In view of this, one established practice is to refrain from changing (or “freeze”) figures in publication databases dating back many years and publish newly estimated seasonally adjusted data only for the past few years and

combine them with the frozen data from previous periods. However, several prerequisites must be fulfilled in order to prevent noticeable breaks from emerging as a result of this procedure.

First, the revision period for the seasonally adjusted results must be long enough that the adjusted data some distance away from the end of the time series can be considered at least roughly final. To specify this period more precisely, the ESS Guidelines on Seasonal Adjustment (item 4.3) refer to the properties of the seasonal filters often used in practice. As outlined in section 20.3.2, the filter weight distribution is asymmetric at series end  $t$ . However, newly added unadjusted data in  $t + 1$ ,  $t + 2$  etc gradually allow the use of increasingly symmetrical filters to estimate the adjusted figure in  $t$ . After around four years, this process can generally be regarded as virtually complete, and the resulting adjusted figure can thus also be considered more or less final. The ESS Guidelines therefore contain the following recommendation. *“A starting date for the earliest revision of the seasonally adjusted data should be set at the beginning of a year, three years before the revision period of the unadjusted data.”*

Another precondition for ensuring that there are practically no breaks during the transition from frozen data to the revised figures is to make sure that the calendar variables used to estimate the revised data do not differ from those used for the previous period, where such calendar effects occurred in the more distant past but were not modelled at the time, whether because the number of observed data was not sufficient for a sound estimate or simply because it was forgotten. To prevent such a situation from arising, attention must be paid to the uniformity of the approach. For this reason, the ESS Guidelines always regard it as acceptable to revise an entire time series, regardless of the revision period of the unadjusted data.



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# 21

## Data Presentation Issues



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## Table of Contents

21.1	Introduction	569
21.2	Data accessibility and availability	569
21.2.1	Data storage	570
21.2.2	Dissemination of metadata	571
21.3	Elements of a statistical release	572
21.3.1	Summary of the principal outputs	572
21.3.2	Tables and graphical presentation of headline information	572
21.3.3	Analysis and commentary on principal indicators	573
21.3.4	Explanatory notes	573
21.3.5	Supplementary information	573
21.3.6	Contact details	574
21.4	Choosing a headline indicator	574
21.4.1	Month on month indicators	574
21.4.2	Year on year indicators	575
21.4.3	Alternative indicators	576
21.4.4	A recommended approach	576
21.5	Graphically: Presenting time series estimates	576
21.5.1	Seasonally adjusted and/or trend-cycle estimates	577
21.5.2	Growth rates	580
21.5.3	Lollypop charts	581
21.5.4	Web based presentation	582
21.6	Statistical releases	582
21.6.1	Examples of different countries presentations	582
21.6.2	Presenting revisions to published estimates	583
21.7	Press releases and the media	586
21.7.1	Case example: Media interpreting published estimates	586
21.8	Discussion	587

## 21.1 Introduction

Appropriate presentation of data is crucial to ensuring informed decision making.

The presentation of time series data can differ greatly between countries, and even within National Statistics Institutes (NSIs) for different types of outputs. Different approaches can range from publishing just the original estimates, publishing seasonally adjusted estimates for only a few main aggregates or publishing the seasonally adjusted and trend-cycle estimates alongside the original unadjusted estimates. Different indicators can also be used as part of the presentation and outputs. For example, presentation of levels, one-period differences, and year-on-year differences are commonly used.

There is a wide range of guidelines and literature on this topic available from NSIs and also international organisations. From an international perspective, the ESS Guidelines on Seasonal Adjustment Eurostat (2015), Chapter 7, contains recommendations covering: data availability in databases, press releases, and use of metadata for seasonal adjustment. OECD (2007) handbook considers data, metadata and presentation issues based on international statistical guidelines and recommendations. This covered issues ranging from guidelines for reporting different types of data, to recommended practices for consistency of presentation and includes a comprehensive set of recommendations. The International Monetary Fund manual, Chapter 7, focuses on the “Status and Presentation of Seasonally Adjusted and Trend-Cycle QNA Estimates” (IMF (2017)). This discusses issues related to presentation of data and provides recommendations. As a national example, the United Kingdom Statistics Authority has a Code of Practice for Official Statistics (United Kingdom Statistics Authority (2009)) which includes protocols on release practices for statistics within the United Kingdom.

This Chapter in the Handbook provides a summary of relevant issues for the practitioner and publisher of official data. Topics covered include: elements of a statistical release, choosing an appropriate headline indicator, and graphically representing outputs.

## 21.2 Data accessibility and availability

This Handbook has dealt with the process from transforming a source series to a seasonally adjusted series. However, information then needs to be made available or delivered to the users. The question is, what information should be provided or made available for analysis and interpretation?

Transparency of published data is a key concept and users will not accept the published data without additional information that allows the reconstruction of the series, or at a minimum the details about the process from the raw or collected data to the adjusted data that has been published. The success of provision of data is determined by how well it can stand up to external scrutiny, such as an audit of compilation processes and methods. Most institutions publish explanatory notes on how the data are compiled and seasonally adjusted as well as providing information regarding the quality of outputs which may be circulated within its respective institution or even to the general public.

The audience of the adjusted data is very important. The requirements for colleagues within one institution or between similar institutions can be different to external users such as the media or academic researchers. It can be assumed that for one time series there are six categories of audience,

- Individual (Producer)
- Team (Collaborators)
- Department (Internal)
- Institution (General)
- Policy Makers (Government)

- General Public or Media (Public)

Each different audience may want, or need, different types and levels of data. For example, micro data will be available for access and use for the Individual, Team, Department and Institution, but typically this may not be made available to Policy Makers, or the General Public or Media. The policy makers, general public and media will most likely focus on aggregate estimates and indicators which summarise important variables, for example, GDP for a country, or the latest unemployment figures.

Each of these audiences may have access to all or a part of the information produced or published. Nevertheless, descending down the list, no audience has more information than any audience above.

When it comes to a set of time series, or data sets that are seasonally adjusted, apart from the Individual (Producer), each different audience may have access to different sets of the data but the sum of the level of the audience cannot have more information than the sum of the level above. Taking the following summary roughly illustrates the flow of the produced data from the producers to the general public.

In this example, the following simple relationships are logically true. Producer A and Producer B and Producer C, has more data available than Team A and Team B, which in turn has more data available than Department A and Department B and Department C, which has more data available than the Institute, and then Policy Makers and the General Public. In practice, between the Institute and the General Public, there may be agreements with other institutions whereby they are granted access to more information than the Public. This is happening more and more with the advent of data sharing legislation within countries and between different producers and holders of data, with one advantage being the reduction of burden associated with the cost of data collection. Within an Institution, there may be agreements whereby another team may be granted access to additional information than the Institution as a whole either due to relevancy or sensitivity of the dataset.

The simple solution would be to provide all data and allow the audiences to pick and choose the information they want. However the volume of information available could be overwhelming to the end user as well as certain information may be sensitive, and also confidentiality of respondents needs to be ensured. Therefore the minimum information to be presented to the general public should be in line with the information that influences the Policy Maker.

### 21.2.1 Data storage

Institutions have developed their own unique data repositories based on their infrastructure and production systems. These are usually developed using the state of the art technologies of the day and so the functionality of most of these storage facilities may retain the same limitations today as they had when developed. For example, historically the cost of storing data was expensive and at the same time it was not foreseen to design the data repositories looking forward to when additional data (especially metadata) could be added.

Today, most applications are not closed to future expansion although they may be victim to changes to state of the art development tools. All databases are essentially multi-dimensional data cubes, or text files with a defined format and containing controls which require interfaces to the seasonal adjustment software. Database file formats are nowadays optimised for storage and retrieval rather than space but nevertheless customised routines are still required to retrieve the output from the seasonal adjustment packages and write them into databases. This has led to only that part of the output defined by practitioners as the minimum needed to be of use to their direct end-users but not necessarily sufficient to allow another practitioner to confirm the validity of the adjustment.

For the exchange of data between institutions within the European Statistical System, their guidelines present a metadata template that describes the treatment at the dataset level (See Chapter 31). This will impact on those institutions within the system that will have to populate the template and will require an IT application to perform the task. Much of this information required may not be sourced from the same database as the data



itself and therefore the institutions will have to find ways to either adapt the data repositories to include the additional information or for their application to retrieve the information elsewhere.

The following questions must be asked in order to determine the amount of data to be made available as part of the dissemination process.

1. Relevancy, is the whole series required to reflect the economic developments today?
2. How many past observations are needed?
3. Should a moving "window" of observations be released or the whole time series?
4. Which information and output influences policy?
5. What is the revision policy used for the series and how far back in history are observations revised?
6. What additional information, such as metadata, should be included in any data release?

The answers to these questions will determine the approach for dissemination, particularly on how data and outputs are revised as part of the production process. Eurostat (2015), Section 7.1, recommends the systematic storage of unadjusted (original) data, seasonally adjusted, trend-cycle data, and also the seasonal adjustment options and prior corrections.

### 21.2.2 Dissemination of metadata

Institutions should use a data model for the storage and retrieval of data, this can be encapsulated via the use of the relevant international SDMX protocol. In practice, it may even be a simple indexing system but as more data is produced for each time period, the more detail that will need to be included in the data model. Since almost all institutions have their own data model, it is recommended that the reader become familiar with their respective data model for the series or concept of interest.

For many institutions, there is a strong link between the source (raw) data and adjusted data. For international institutions, this is simply a value change in the key reflecting the dimension "Adjustment", e.g. for the international institutions the following code values have been defined (for a full list relating to seasonal adjustment see the web link [https://sdmx.org/?page\\_id=3215](https://sdmx.org/?page_id=3215)):

- N: Raw data, or unadjusted data
- S: Seasonally Adjusted, not calendar adjustment
- W: Working day Adjusted, not seasonally adjusted
- Y: Calendar and seasonally adjusted data

When it comes to data availability, the demand from users both in academia and in policy making institutions is with respect to the metadata which describe the construction of the data and this is by design more voluminous than the indicator itself. This is even more so when it comes to providing users with adjusted data. When a user is informed that the indicator has been adjusted, they are likely to want additional information. When a media organisation provides information, there is a level of trust that what they are being provided has been vetted by the media organisation for quality and for this they rely on the metadata associated with the series even if they do not forward this to the public.

With adjusted data, the need to provide the media organisation with a description on how the indicator was compiled is important. As with all statistics, the information is based on assumptions and these assumptions and choices are made by the adjustment practitioner, e.g. model selection; treatment of outliers; calendar adjustment; or time span of series used for the adjustment.

Therefore it is necessary when producing the adjusted data from its source to catalogue each step of the process and to make the essential elements of these available to users.



The elements common to any metadata model but are not part of the software and can influence the results are:

- Method, e.g. JDemetra+, X-13-ARIMA-SEATS, TRAMO-SEATS, STAMP, BV4, etc.
- Computer Operating System, e.g. Windows; Unix; Linux
- Version of the software, e.g. release or update number
- Processor and model, e.g. internal computer specifications
- Bit Processing, e.g. 32; 64 etc.
- Calendars (if used for pre-adjustment)

The additional metadata is specific to which ever method is used but should be sufficient to allow another practitioner to reconstruct the adjusted series from the raw series, e.g. models; parameters; quality measures etc. Due to the different number of methods available to be applied it would be impossible to list everything necessary but the other chapters in the Handbook will indicate which additional metadata can be included or would be useful to the practitioner or user.

## 21.3 Elements of a statistical release

Dissemination of data is an important step in the statistical process and there are many different ways to disseminate data. Typically, specific elements relating to the data are combined together into a single statistical release. Traditionally this has been in the form of printed material, such as a booklet or statistical bulletin, which describes relevant data and also methods behind collecting the estimates.

More recently, the dissemination of relevant data is available through electronic means, often in some cases reflecting the printed format but in electronic form. Advances in electronic dissemination have meant that the traditional statistical bulletin can be reshaped to be optimal for either Internet viewing, or interactive charts or data access. However, in all cases, there are key data elements that are common.

As a general principle, any statistical release should inform users of the key outputs relating to the dataset. For example, this will include the following elements.

### 21.3.1 Summary of the principal outputs

The headline summary information should capture the most important details of the data. This should be the first piece of information available to the user. This approach assists all users of the statistics by providing a summary of the most significant contents in a readily accessible way. The preferred approach is to have a clear separation of the headline results from the greater detail which may be available elsewhere in the statistical release.

One advantage of providing a headline summary is to encourage media use of the statistics, by providing a clear and concise summary which can help ensure that the published estimates are reported accurately for other users.

### 21.3.2 Tables and graphical presentation of headline information

Tables and graphs allow the most recent published estimates to be viewed in context of the previous published, or revised, estimates. The type of table used will depend on the indicators required by all users but also the

type of statistical release. For example, a clear and concise table should only include two or three pieces of key information either at the very front, or early on in the statistical release.

More detailed tables with a larger number of indicators, different variables and time periods could be available elsewhere, such as either later in the statistical release or for electronic download. Graphical information should follow a similar principle and only present relevant information related to the statistical release or for a selected time period.

Tables and graphs of revision information are useful additions to a statistical release. All tables and graphs should have at least some basic commentary which describes the relationship between the different estimates over time.

Tables and graphs should be relevant and be specifically referred to in the statistical release.

### 21.3.3 Analysis and commentary on principal indicators

Analysis of the data is a very important component of a statistical release. This provides the opportunity to show contextual relationships around the data and provide relevant background information on why the estimates are showing the latest movements. There needs to be a careful balance to ensure that any analysis does not bring the impartiality or objectivity of the estimates or institution into question, for example, by linking recent movements to political decisions.

The time taken for the preparation of the analysis and commentary needs to be factored into the publication timetable for the statistical release. For example short indicative analysis may be appropriate to ensure that the any data release remains timely, where detailed analysis can be done at a later stage of the dissemination process and published at a later time.

### 21.3.4 Explanatory notes

Explanatory, or background, notes are important to provide context on methods used to compile the estimates, conceptual basis, and supplementary material related to producing the estimates.

For example, this could include explanations of key concepts such as source of the data (e.g. administrative or survey based), estimation methods, response rates of the survey data, seasonal adjustment parameters and revision analysis against previously published estimates.

The link: [http://www.bankofengland.co.uk/statistics/Pages/iadb/notesiadb/government\\_securities.aspx](http://www.bankofengland.co.uk/statistics/Pages/iadb/notesiadb/government_securities.aspx) provides an example of a Central Bank (Bank of England) approach to explanatory notes used for data covering monetary financial institutions. It provides information on: Overview, Availability, Sources, Definitions, Valuation and Breaks and Key resources.

### 21.3.5 Supplementary information

Where applicable, supplementary information which highlights important issues that impact on the aggregate information should be covered within the statistical release. This can include, for example, detailed information and analysis of lower level aggregates to show how this has contributed to higher level aggregates. Supplementary information can also be used to enhance users' understanding of the data, for example by providing information on relationship between different indicators or related variables.

Supplementary information can be available in different forms. This may be within the statistical release or dissemination, or as part of a separate analytical approach. Within the United Kingdom, the Office for National Statistics produces a regular Economic Review publication which summarises already published outputs,

but provides indepth additional information separate to the publication cycle. For example, the link <https://www.ons.gov.uk/economy/nationalaccounts/uksectoraccounts/articles/economicreview/october2017> includes a detailed section on “Understanding the UK economy” which not only summarises the published estimates, but links to related and wider indicators as part of the analysis.

### 21.3.6 Contact details

Contact details, such as email and phone numbers, of the individual or area within the organisation are needed to allow users to ask data-related questions. It is recommended that group-specific contact details are used to ensure continuity of user support. This information should be made available in a prominent part of the statistical release or website.

A social media presence is also strongly advisable, particularly ensuring that response times for replies to queries are monitored to ensure these are addressed promptly.

## 21.4 Choosing a headline indicator

In practice, many different estimates and indicators are available to be calculated and then presented as part of the dissemination strategy. For example, the most common are the original series  $y_t$ , and then the analytical products that can be derived from this, e.g. the seasonally adjusted  $\widehat{SA}_t$ , and the trend-cycle  $T_t$  estimates. Once these estimates are available different indicators can then be constructed.

ABS (2003) Chapter 3, extensively covers some of the most commonly used indicators presenting the advantages and disadvantages of each. Some typical indicators which are often quoted as part of the dissemination of outputs are: period to period growth, moving annual sums and averages, annual growth rates, growth in the three months and annualised growth rates. In practice, these types of indicators can delay the changes in the behavior of the series, even distorting the underlying shape of the series, and some of them can amplify the irregularity in the time series.

Monthly, quarterly and annual percentage changes can be calculated for original, seasonally adjusted and trend-cycle estimates but these measures can produce inconsistent and occasionally contradictory signals about developments in the underlying direction of a series. As a result of these inconsistent signals users of time series outputs may be confused about the direction of the series or which series they should be using for their purposes.

The follow sections discusses advantages and disadvantages of different indicators and estimates used for presenting data.

### 21.4.1 Month on month indicators

Month-on-month and quarter-on-quarter indicators can be based on: the original estimates; the seasonally adjusted estimates; or the trend-cycle estimates.

A month-to-month or quarter-to-quarter percentage change based on the original estimates can be useful in understanding the actual size of movements for a given period of time. However, if there is the presence of seasonal and irregular factors, these indicators have little value for the purpose of understanding underlying activity. This is because any seasonal influence is likely to be the dominating factor in the variation in the original estimates.

In practice, analysts often focus on the period to period movements of the seasonally adjusted series. One aspect to carefully consider with using this approach is that the seasonally adjusted estimates include both the

trend-cycle and the irregular component. The period to period movements of the seasonally adjusted series are influenced by the behaviour of the irregular fluctuations. In many instances, the irregular component may dominate the overall movement in seasonally adjusted estimates, particularly if the time series has been derived from a sample survey with either a small sample or is naturally volatile because of the concept being measured.

The period-on-period movement can be shown mathematically to actually remove the longer term cycles indicative of the trend, and amplify the irregular component, so for example, a month-on-month or quarter-on-quarter movement estimate is really just looking at the amplified irregulars.

In general, nevertheless, there are some benefits in using a month to month or quarter to quarter percentage change of the seasonally adjusted estimates as this is a reasonably good measure for the short-term variation (i.e. impact of one-off events) in the original estimates. This measure may also provide supplementary information to assess future trend-cycle estimates. Caution must be applied when interpreting the underlying direction for the most recent period(s) of any outputs as these may be subject to revisions due to revisions in the original data and/or due to additional original data becoming available.

The movement of the trend-cycle is typically the best measure of the underlying, longer-term direction of a series. This measure provides a smoothed historical perspective of the underlying pattern of behaviour without the impact of calendar related or irregular influences. Users can use this indicator to monitor the level and shape of turning points over time, which can aid timely and informed decision making.

However, caution must be applied when interpreting the trend-cycle estimates for recent periods. Revisions to the trend-cycle estimates at the current end will occur due to additional original estimates becoming available as the underlying direction will be continually reassessed as new information becomes available. For instance, in order to confirm the presence of a turning point in the underlying direction at the current end, approximately three additional observations of the trend-cycle will be required. Often it may be years before revisions reduce to zero due to either late data, annual benchmarks becoming available or being revised, or revised estimation as part of the seasonal adjustment approach.

### 21.4.2 Year on year indicators

Another common indicator used to summarise change is the year-on-year percentage change in the original estimates. This measure essentially performs a crude seasonal adjustment when applied to the original estimates as it reduces the impact of constant yearly seasonal influences. However, in practice, these effects are rarely completely eliminated due to trading day influences (changes in calendar), changing or evolving patterns in seasonality, and moving holidays which can change over the years. The year-on-year percentage change will also not cope well with seasonal effects that evolve with the level of the series by maintaining a proportional relationship.

The year-on-year percentage change measure may also be highly affected by irregular influences. This measure should only be used if seasonally adjusted or trend-cycle estimates do not exist for a given series and there is no evidence of seasonality. In such cases, caution must be applied. Also, even if the irregular and seasonal influences are minimal, the through-the-year measure will rarely detect turning points in a timely way and in practice will actually lag turning points by around six months for monthly time series.

Finally, the year-on-year percentage change provides an approximation of the average trend-cycle movement over the year. In this case, the measure of change may not reflect the most recent or current trend-cycle movement or pattern of behaviour.

**Table 21.1:** Summary of recommended approach for presenting key statistics

	Recommendation for . . .	
	One period growth	Annual growth
Original	No	No
Seasonally adjusted estimates	Yes	No
Trend-cycle estimates	Yes	Yes

### 21.4.3 Alternative indicators

There are many different indicators that can be used. Once the original data is available there is no limit on how this can be transformed, e.g. through application of specially designed filters or creation of indices to weight related outputs together. One example of a specific approach is the use of a 3-month on 3-month movement indicator. This is often used as part of the dissemination of monthly economic time series in the United Kingdom. For users, this can help relate the movement of the 3 months to corresponding or similar quarterly outputs.

ABS (2003) Chapter 3 is an excellent source of the most commonly used indicators, presenting the advantages and disadvantages of each including worked examples.

### 21.4.4 A recommended approach

Any chosen approach for a headline indicator will be driven by the institution needs, and also the user needs. Often there will need to be a provision of data in different forms to cater for all the different needs of users (see 21.2 for examples). This can be achieved by providing a range of information to meet all needs, either in a summary table as part of the main dissemination approach, or supplementary tables for users who have greater information needs.

A summary based on the discussion in this chapter is given in Table 21.1.

This approach is consistent with the IMF manual (IMF (2017), Chapter 7, point 128) which recommends to present both the seasonally adjusted and trend-cycle estimates preferably in the form of graphs incorporated into the same chart (further discussed in Section 21.5).

They do note however that there is still debate on whether to present seasonally adjusted estimates or trend-cycle estimates. Based on ease of electronic dissemination, it is advisable to provide access to at least both as part of a broad dissemination strategy. Eurostat (2015), Section 7.2, reaches the same conclusion but also recommends that the “seasonally adjusted data are the most appropriate figures to be presented in a press release... when presenting trend-cycle estimates, the most recent values should not be shown because of the end-point problem or they should be accompanied by warnings related to their end-point problem.”

## 21.5 Graphically: Presenting time series estimates

All outputs should adhere to an agreed publication policy. For example, ensuring graphs are consistent in size and format and generally confined to displaying the most recent behavior of the output, e.g. the seasonally adjusted and/or trend-cycle estimates. Line graphs of levels or bar charts of movements are the most common display of the estimates. In either case, the time span should be limited to about one year of monthly observations, or longer in the case of quarterly data.

### 21.5.1 Seasonally adjusted and/or trend-cycle estimates

It is clear that for dissemination, a graphical representation is desirable. This should at the very least consider the presentation of the seasonally adjusted estimates.

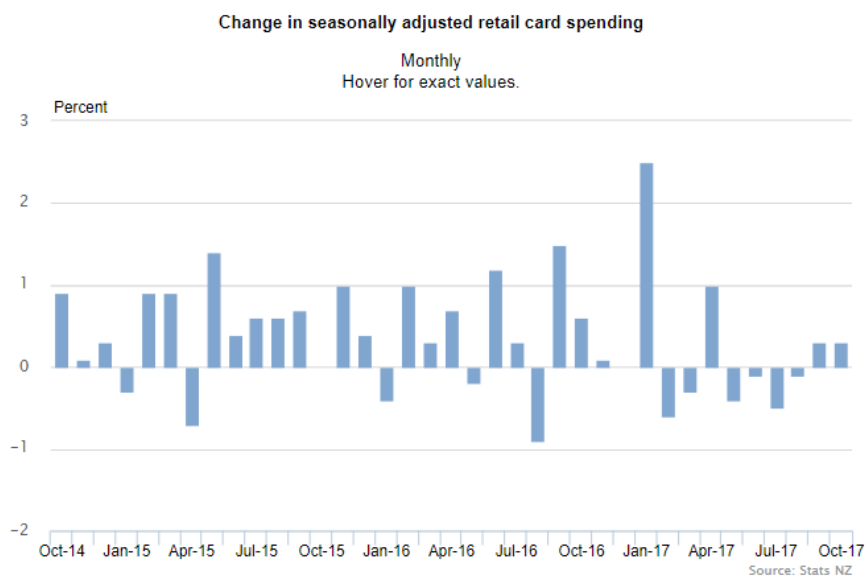
If trend-cycle estimates are published, they should be plotted together with the seasonally adjusted series from which it was derived. This means that the graph then shows the size and direction of the irregular factors present in the seasonally adjusted series. The irregular component is then clearly shown around the trend-cycle path and this assists in monitoring extreme irregular behavior that can distort trend-cycle estimates.

In practice, the trend-cycle estimates should not normally be plotted with the original (seasonal) series. This is because the difference between the two series cannot be interpreted as being either a “seasonal” or an irregular effect alone, but a mixture of both. In such a graph, this could lead to situations where the trend-cycle behavior may look quite out of step with the original series movements, and confuse the user.

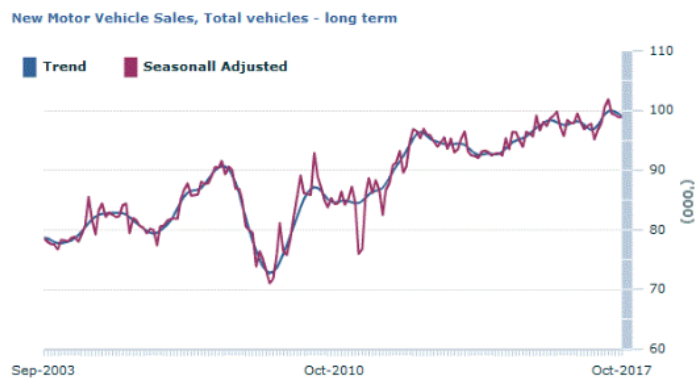
One approach to minimising concern on how trend-cycle estimates are revised at the current end is to restrict the presentation of the trend-cycle to not use the last few estimates; present them as a dashed line; or showing the trend-cycle estimates at the end with a trumpet based on estimated confidence intervals.

Examples of different graphical representations are given in Figures 21.1 to 21.4.

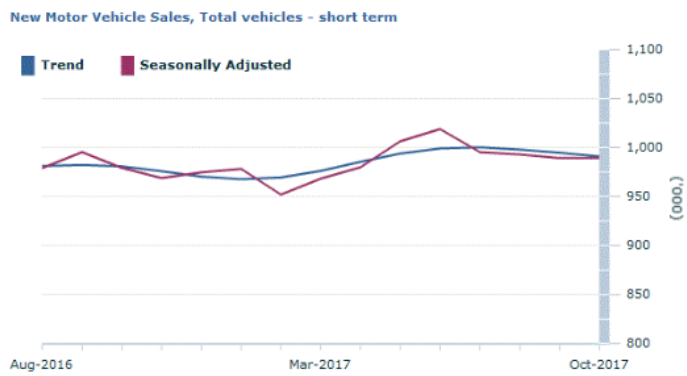
**Figure 21.1:** Example: Electronic Card Transactions, Statistics New Zealand



**Figure 21.2:** Example: New Motor Vehicle Sales, Australia

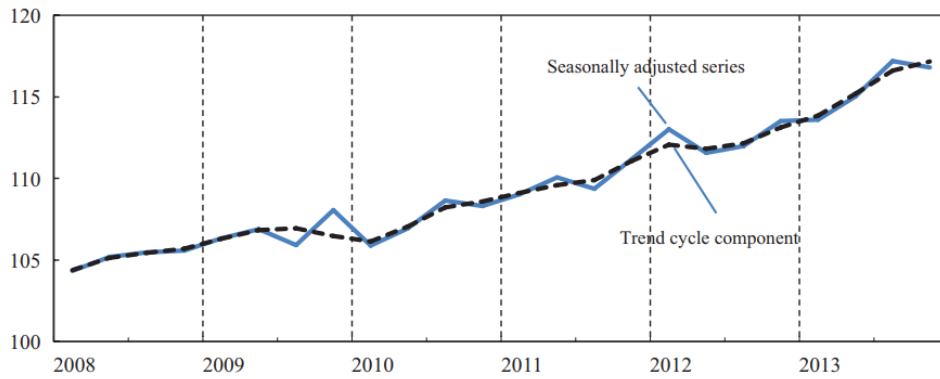


Source(s): [New Motor Vehicle Sales, Total vehicles - long term-graph\\_update](#)



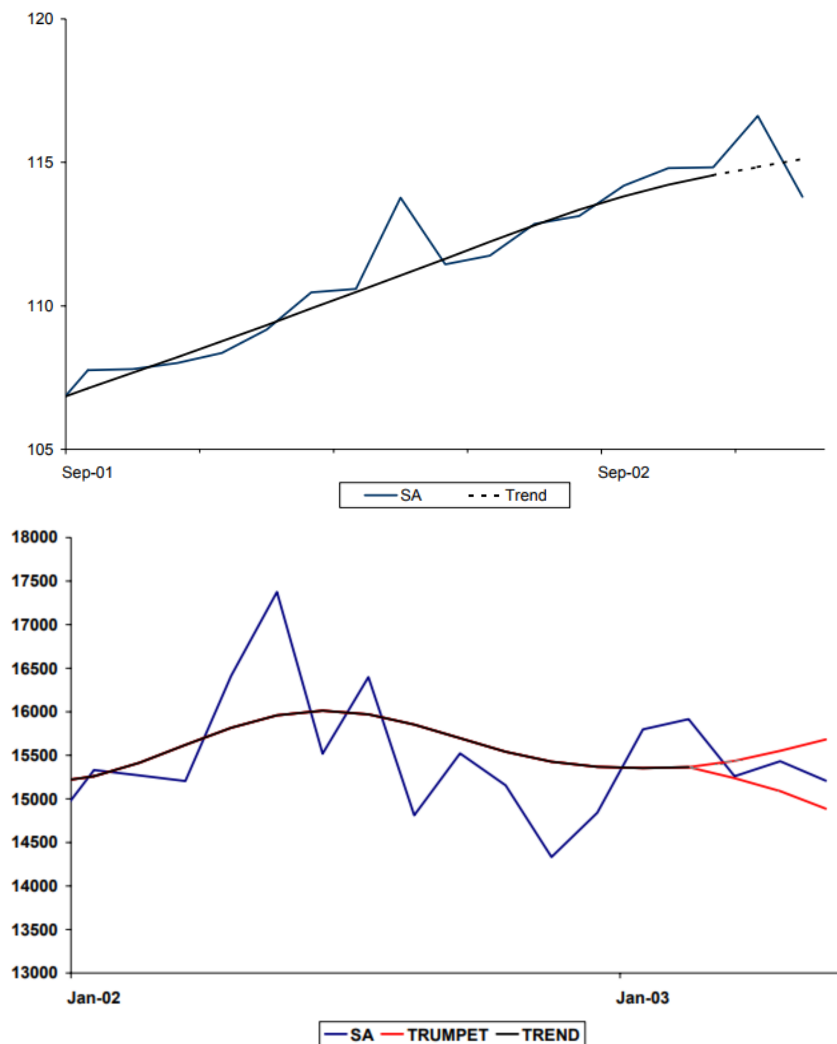
Source(s): [New Motor Vehicle Sales, Total vehicles - short term-graph\\_update](#)

**Figure 21.3:** Example: Presentation of Seasonally Adjusted Series and the Corresponding Trend-Cycle Component, Reproduced from Chapter 7 of IMF (2017) and based on simulated data.



Presenting the seasonally adjusted series and the trend-cycle component in the same chart highlights the overall development in the two series over time, including the uncertainty of the irregular component. Users should be informed that trend-cycle estimates of the latest observations are subject to high uncertainty and should be taken with caution.

**Figure 21.4:** Example: Presentation of Seasonally Adjusted Series and Corresponding Trend-Cycle Component, Office for National Statistics





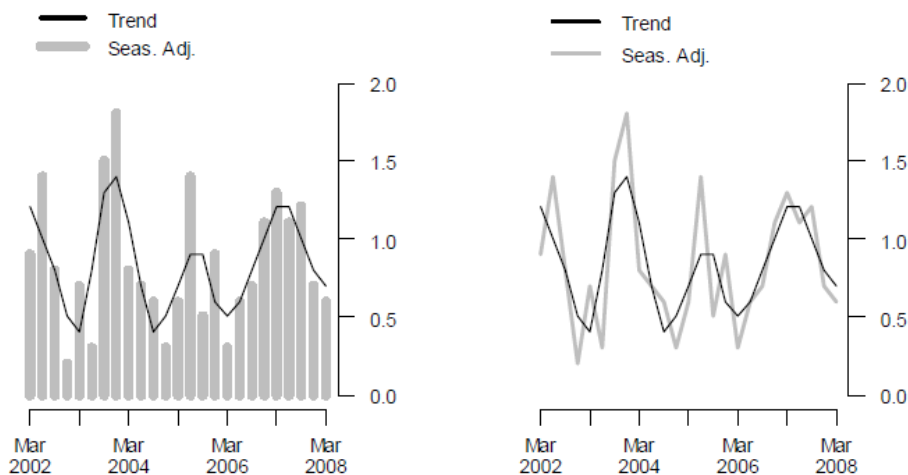
### 21.5.2 Growth rates

Percentage movements (growth rates) are useful for the interpretation of the short term direction of a time series (e.g. Figure 21.1). In practice, line graphs can be used in this capacity. A more appropriate approach is to use a bar-line graph for simultaneously presenting trend and seasonally adjusted movements. The main advantage of the bar-line movement graph is that emphasis is placed on the trend movements.

The motivation behind using bars for seasonally adjusted movements and lines for trend movements is simple: the irregularity contained in the seasonally adjusted estimates means that the seasonally adjusted movements will also contain a high degree of irregularity. Joining seasonally adjusted movements with a line can provide the inappropriate impression that the movement is in a continuous fashion rather than being a discrete short term change.

The bar plot crossing at zero provides a no-growth reference line and gives readers a perspective of growth direction against volatility. The joined line for trend movements gives a clear direction and an indication of the phase of the cyclical behaviors of the series.

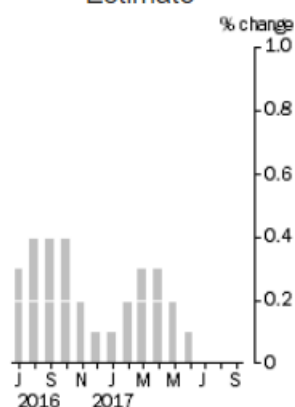
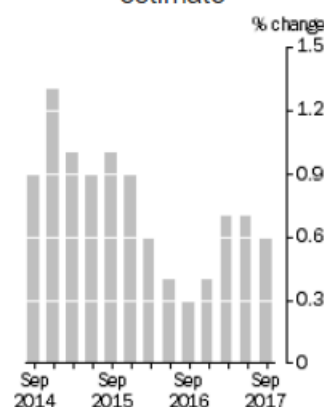
**Figure 21.5:** Example of alternative representation of movements in seasonally adjusted and trend estimates.



Traditionally, the ABS has presented time series information in statistical releases with a focus on the trend estimates. For example, presenting commentary on trend estimates first, then seasonally adjusted estimates, and then the original estimates (where applicable). An example is shown in Figure 21.6.

**Figure 21.6:** Example of trend estimate presentation, Australian Bureau of Statistics, Retail Trade estimates

	September Qtr 2017 \$m	June Qtr 2017 to September Qtr 2017 % change
<b>Turnover in volume terms</b>		
Trend	77 481.2	0.6
Seasonally Adjusted	77 387.1	0.1

**Monthly Turnover, Current Prices - Trend Estimate****Quarterly Turnover, in volume terms - Trend estimate**

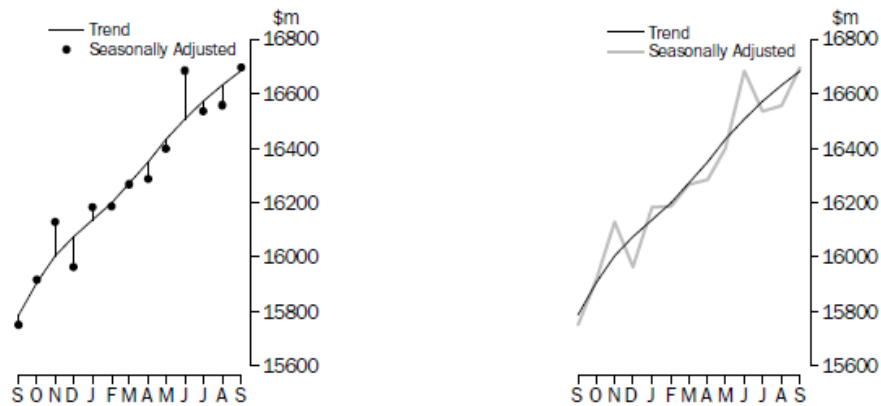
### 21.5.3 Lollipop charts

Multiple line graphs are traditionally used to present trend and seasonally adjusted estimates. However, as the seasonally adjusted estimates still contain the impact of the irregular component, this type of presentation can obscure the influence of the irregular component. The irregularity in a time series is often overlooked by analysts but can contain important information on the degree of impact from one-off events.

The typical presentation approach can inhibit and distort analysis, especially with regards to the identification of turning points in the series as often the irregular component may hide turning points. An alternative approach to presenting the seasonally adjusted and trend estimates together can be used to clearly show the impact of the irregular component by placing greater emphasis on the degree of irregularity in the series. The ABS has considered an alternative presentational approach to take this into consideration.

For example, a “lollipop chart” which presents the trend and seasonally adjusted series together but also places emphasis on the degree of irregularity in the series. Figure 21.7 shows an example of a lollipop graph (left hand side) and the traditional approach (right hand side). The emphasis of the lollipop chart is now on the trend estimates. One benefit of this type of chart is that it can help highlight turning points. For example, the typical definition of a turning point is three changes of direction in a row, so if three irregular values occur on the same side of the trend this can be considered a possible start of a turning point and hence change in the underlying direction.

**Figure 21.7:** Example of alternative representation of a seasonally and trend estimates using a lollypop graph.



### 21.5.4 Web based presentation

The use of purely web based statistical releases provides opportunities for users to interact with time series data. For example, the user could use interactive applications to plot selected variables of interest on a single graph. One widely used approach is to use the D3 visualisation approach <https://github.com/d3/d3/wiki> which can provide dynamic and interactive graphics in a web browser.

The reader is encouraged to explore the vast library of D3 examples which can be embedded into both websites or production programs. For example, within the Office for National Statistics in the United Kingdom, D3 graphical capability is embedded into the in-house production system to provide analytical graphical capability to help quality assure estimates. The line charts for the seasonally adjusted and trend estimates are dynamically available as updated by the analyst.

## 21.6 Statistical releases

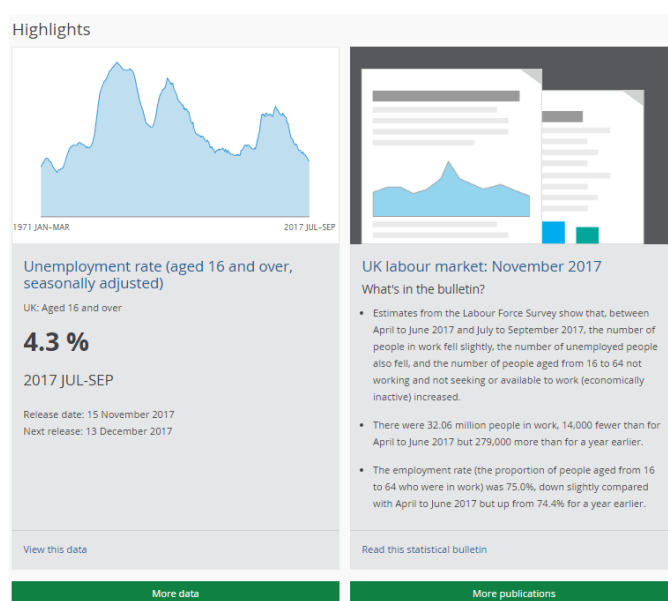
Statistical releases will differ based on the form used for dissemination. For example, paper based dissemination will be very different from the optimal presentation for the internet. Within the United Kingdom, there is a Code of Practice (United Kingdom Statistics Authority (2009), Protocol 2: Release practices) which gives specific guidance to follow. This covers advice which includes: 1. Release of statistical reports as soon as they are judged ready; 4. Issue statistical releases at a standard time on a weekday to maintain consistency; 6. Include the name and contact details of the responsible statistician in statistical reports.

### 21.6.1 Examples of different countries presentations

The reader is encouraged to explore the different approaches for country presentation by visiting the different country national statistics websites. Examples for selected countries are given in Figures 21.8 through to 21.10. This gives examples focusing on graphical presentation, to bar charts, and then textual descriptions.

In practice, different presentations for outputs will change, even within a country, based on the frequency of data (monthly, quarterly or annual); or the published headline indicators which have been chosen for dissemination.

**Figure 21.8:** Example: Extract from web presentation at [www.ons.gov.uk](http://www.ons.gov.uk) for unemployment statistics.



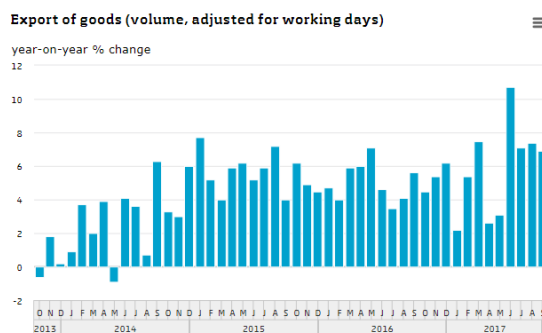
**Figure 21.9:** Example: Extract from web presentation at [www.cbs.nl](http://www.cbs.nl) for trade statistics.

## Export growth almost 7 percent

14/11/2017 15:00

Statistics Netherlands (CBS) reports that the total volume of goods exports grew by 6.9 percent in September 2017 relative to September 2016. Growth was slightly lower than in August. In September, exports of machinery, appliances and cars increased most notably. The volume of imports was 5.6 percent up on September last year.

The [CBS Exports Radar](#) indicates that circumstances for exports in November are more favourable than in September.



## 21.6.2 Presenting revisions to published estimates

Revisions are an important part of the statistical estimation process. Chapter 12 discusses revisions and presenting revisions in greater detail. The presentation of revision information is needed to ensure the transparency of statistical processes, but also so that users can understand how previously published estimates may have changed to ensure the most recent estimate is presented in context.

### 21.6.2.1 Revision triangles

Revision triangles are a useful tool to determine how the revisions of outputs change over time. There is considerable literature available, including worked examples.

Figure 21.10: Example: Extract from web presentation at www.insee.fr for price indices.

## In October 2017, the prices of frequently purchased goods were stable in hyper and supermarkets

INFORMATIONS RAPIDES  
N° 301  
Published on : 15/11/2017  
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PRINTABLE VERSION  
(pdf, 152 Ko)

### Price Index in large and predominantly-food stores - October 2017

In October 2017, the prices of frequently purchased goods in hyper and supermarkets and in large and predominantly food stores were stable over a month, after a downturn of 0.1% in September.

CONSULT DOCUMENTATION

**Table of contents**

- Year on year, prices in hyper and supermarkets grew at the same rate than in September
- Over a month, new drop in food prices in hyper and supermarkets
- Rebound in the prices of cleaning and personal care products in hyper and supermarkets

In October 2017, the prices of frequently purchased goods in hyper and supermarkets and in large and predominantly food stores were stable over a month, after a downturn of 0.1% in September.

Excluding hyper and supermarkets, prices accelerated slightly (+0.2%) after a slowdown in September (+0.1% after +0.3% in August).

Considering all types of stores, the prices of frequently purchased goods were unchanged in October. They fell back by -0.1% in September after seven months of continuous slight rise.

**Year on year, prices in hyper and supermarkets grew at the same rate than in September**

As an example, ONS (UK) regularly publish revision triangles for outputs. See <https://www.ons.gov.uk/economy/grossdomesticproductgdp/datasets/revisiontrianglesforukgdpbybhagrowthrates> for an example for quarterly growth rates for UK GDP. This is illustrated in Figure 21.11.

The OECD link <http://www.oecd.org/std/ocdeurostatguidelinesonrevisionspolicyandanalysis.htm> includes a detailed set of information related to use of revision triangles. This includes examples of pre-defined spreadsheets to enable the creation of revision triangles, e.g. <http://www.oecd.org/std/automatedprogramstoperformrevisionsanalysis.htm>.

Figure 21.11: Example: Extract from revision triangles for UK GDP

**Revisions triangle for YBHA - Gross domestic product at market prices (CP, SA)**

This table shows the percentage change, quarter on previous quarter (IHYN)

Relating to Period	2014 Q1	2014 Q2	2014 Q3	2014 Q4	2015 Q1	2015 Q2	2015 Q3	2015 Q4	2016 Q1	2016 Q2	2016 Q3	2016 Q4	2017 Q1	2017 Q2	2017 Q3
Month 2 estimate	1.2	N/A	1.6	0.4	0.9	0.7	0.6	0.0	0.7	1.6	0.8	1.4	0.7	0.8	0.7
Month 3 Estimate (first final estimate)	1.3	1.9	1.3	0.7	0.7	1.2	0.7	0.2	1.0	1.5	0.8	1.5	0.7	0.7	N/A
Value 3 years later	1.4	1.7	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Jul-16	1.4	1.8	1.1	0.4	0.7	0.8	-0.2	0.5	1.0						
Aug-16	1.4	1.8	1.1	0.4	0.7	0.8	-0.2	0.5	1.0	1.6					
Sep-16	1.4	1.8	1.1	0.4	0.4	1.2	-0.1	0.5	1.2	1.5					
Oct-16	1.4	1.8	1.1	0.4	0.4	1.2	-0.1	0.5	1.2	1.5					
Nov-16	1.4	1.8	1.1	0.4	0.4	1.2	-0.1	0.5	1.2	1.5	0.8				
Dec-16	1.4	1.8	1.1	0.4	0.3	1.5	-0.1	0.4	1.5	1.2	0.8				
Jan-17	1.4	1.8	1.1	0.4	0.3	1.5	-0.1	0.4	1.5	1.2	0.8				
Feb-17	1.4	1.8	1.1	0.4	0.3	1.5	-0.1	0.4	0.9	1.6	0.9	1.4			
Mar-17	1.4	1.8	1.1	0.4	0.3	1.5	-0.1	0.4	1.1	1.5	0.6	1.5			
Apr-17	1.4	1.8	1.1	0.4	0.3	1.5	-0.1	0.4	1.1	1.5	0.6	1.5			
May-17	1.4	1.8	1.1	0.4	0.3	1.5	-0.1	0.4	1.1	1.5	0.6	1.5	0.7		
Jun-17	1.4	1.8	1.1	0.4	0.3	1.5	-0.1	0.4	1.1	1.5	0.6	1.5	0.7		
Jul-17	1.4	1.8	1.1	0.4	0.3	1.5	-0.1	0.4	1.1	1.5	0.6	1.5	0.7		
Aug-17	1.4	1.8	1.1	0.4	0.3	1.5	-0.1	0.4	1.1	1.5	0.6	1.5	0.7	0.8	
Sep-17	1.5	1.7	0.9	0.1	0.6	1.6	0.0	0.4	1.4	1.3	0.9	1.2	0.8	0.7	
Oct-17	1.5	1.7	0.9	0.1	0.6	1.6	0.0	0.4	1.4	1.3	0.9	1.2	0.8	0.7	
Nov-17	1.5	1.7	0.9	0.1	0.6	1.6	0.0	0.4	1.4	1.3	0.9	1.2	0.8	0.7	0.7
Latest Estimate	1.5	1.7	0.9	0.1	0.6	1.6	0.0	0.4	1.4	1.3	0.9	1.2	0.8	0.7	0.7

### 21.6.2.2 Trumpet graphs (what-if charts)

The potential for data revisions can also be captured graphically. The ABS has advocated the use of a 'trumpet' graph which shows the potential size of the revision for seasonally adjusted and trend estimates.

An example of the use of what-if charts in practice is available in this link on page 25 of the pdf: [http://www.ausstats.abs.gov.au/ausstats/meisubs.nsf/0/4995198DDF96524ACA2581CB00F9F6F/\protect\T1\textdollarFile/87310\\_sep%202017.pdf](http://www.ausstats.abs.gov.au/ausstats/meisubs.nsf/0/4995198DDF96524ACA2581CB00F9F6F/\protect\T1\textdollarFile/87310_sep%202017.pdf). This shows how estimates for monthly building approvals in Australia would change at the most recent time periods, based on selected movements in the seasonally adjusted estimates. The tables show how the trend estimates are revised for the most recent periods.

**Figure 21.12:** Example: Part 1 of what-if scenarios for Monthly Building Approvals in Australia

#### WHAT IF...? REVISIONS TO TREND ESTIMATES

##### EFFECT OF NEW SEASONALLY ADJUSTED ESTIMATES ON TREND ESTIMATES

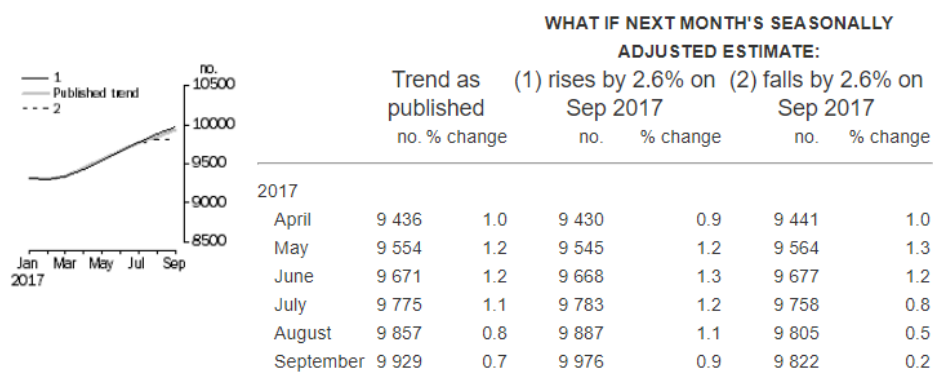
##### TREND REVISIONS

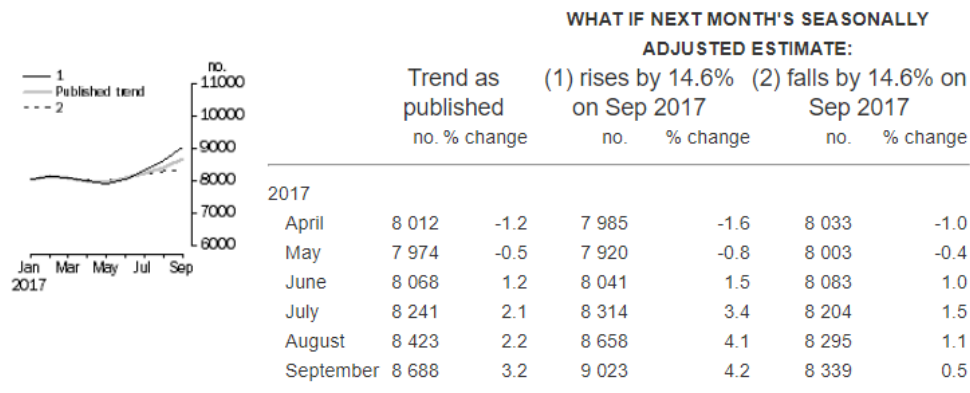
Recent seasonally adjusted and trend estimates are likely to be revised when original estimates for subsequent months become available. The approximate effect of possible scenarios on trend estimates are presented below. Generally, the greater the volatility of the original series, the larger the size of the revisions to trend estimates. Analysis of the building approval original series has shown that they can be revised substantially. As a result, some months can elapse before turning points in the trend series are reliably identified.

The graphs and tables which follow present the effect of two possible scenarios on the previous trend estimates: that the October seasonally adjusted estimate is higher than the September estimate by 2.6% for the number of private sector houses approved and 14.6% for private sector dwellings excluding houses approved; and that the October seasonally adjusted estimate is lower than the September estimate by 2.6% for the number of private sector houses approved and 14.6% for private sector dwellings excluding houses approved. These percentages represent the average absolute monthly percentage change for these series over the last ten years.

**Figure 21.13:** Example: Part 2 of what-if scenarios for Monthly Building Approvals in Australia

#### APPROVED PRIVATE SECTOR HOUSES



**Figure 21.14:** Example: Part 3 of what-if scenarios for Monthly Building Approvals in Australia**APPROVED PRIVATE SECTOR DWELLINGS EXCLUDING HOUSES**

## 21.7 Press releases and the media

The presentation of time series data may be different depending on the level of user sophistication, the need of the user, and how high profile the estimates are in terms of impact to either policy or analysis.

For example, in NSIs it is common for high profile statistical releases, such as GDP or price indices, to have dedicated briefings for the media organisations. This helps ensure that the headline estimates are communicated clearly and give a consistent message.

In practice this can either occur in lock-in briefings where a representative from the NSI responsible for deriving the estimate will brief the media slightly before the release of the estimates. The alternative is a briefing of the estimates either immediately when the data is released or shortly afterwards.

Data presentation is an important component of any media briefing and may be different from the presentation within the statistical release itself. For example, the different elements in either a media lock-in or regular media briefing should include:

- time series graphs of recent movements either as part of a presentation or hardcopy handouts for ease of access and comparison
- providing commentary on the statistical release to provide additional detail or explain the content in the release
- answering specific questions posed by the media journalists to provide additional information for articles

### 21.7.1 Case example: Media interpreting published estimates

If different outputs are published, such as the seasonally adjusted and trend-cycle estimates, then a balanced commentary on these two analytical outputs is needed because they will provide complementary information. For users of these analytical estimates there are often competing priorities. Ultimately, the public perception of official published estimates comes from the media. From a practical point of view, there is a clear interest for the media towards finding a story in the published estimates, either in a short time for publication or as part of more detailed analysis. With the advent of the 24 hour news cycle, the most recent and up-to-date information is often seized on to disseminate and interpret as rapidly as possible.

This means that the media will often focus on the one-off events and comment on the month-on-month movements in the seasonally adjusted estimates. For policy makers, there is a danger that they will also focus on one-off events. To reduce this risk, one approach is to publish all available outputs, e.g. the original, seasonally adjusted and trend-cycle estimates together to give users a choice. However, the choice of which estimate to use needs to be informed and will depend on user needs and requirements.

Continued education on the advantages and disadvantages of different indicators is important. A case example of why this is important is from a national Australian newspaper, *The Australian* (2010) which was published on April 10, 2010. The article cites a range of examples from unemployment to retail trade data, published by the Australian Bureau of Statistics, where the movement between the seasonally adjusted estimates is different to the trend-cycle estimates and hence giving a different story and interpretation and ultimately media head-line. For example,

“But ‘irregular’ influences make seasonally adjusted monthly statistics notoriously volatile and unreliable.”

In the same article it also mentions that

“Sophisticated users of statistics, such as the Reserve Bank, focus on trend numbers: that’s why it raised the cash rate on Tuesday, despite recent ‘negative’ seasonally adjusted numbers.”

This is one example of users and policy makers focusing on the trend-cycle estimates rather than single or one-off seasonally adjusted estimates.

## 21.8 Discussion

The presentation of outputs is an essential part of the dissemination process for National Statistics Institutes. The optimal approach to the presentation, and types of outputs used, will continue to evolve. This is because both graphical and tabular information is increasingly becoming more and more interactive for the users of data and the outputs that are being made available.

There is clear direction already, where the presentation of data via the internet will lead to greater control to the user to create their own indicators and analytical output to analyse for their own needs.

To balance this, there is still a role for the NSI to play to ensure important indicators are presented in a clear and concise form, but also the analytical context provided around these is made available in a clear and concise way. The availability of many different indicators, including those self-defined by users, means that NSIs will need to explain the relative advantages and disadvantages of each to ensure they remain relevant for their intended use.



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# VII

## Seasonal Adjustment in Practice





# 22

## Quality Measures and Reporting for Seasonal Adjustment



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## Table of Contents

22.1	Introduction	593
22.2	Desirable properties for a seasonal adjustment method	593
22.2.1	Some properties of “optimal” seasonal adjustment	594
22.2.2	The ESS guidelines on seasonal adjustment	595
22.3	A list of quality measures for seasonal adjustment	596
22.3.1	Common quality measures	596
22.3.2	Specific quality measures	598
22.3.3	The JDEMETRA+ quality report	599
22.3.4	Limits of a pure numerical quality approach.	601
22.4	Examples of quality assessment and quality reporting	602
22.4.1	The quality report for seasonally adjusted data used at the Deutsche Bundesbank	602
22.4.2	Quality assessment of seasonally adjusted series at the European Central Bank	606
22.4.3	Quality assessment of seasonally adjusted series at ISTAT	607
22.4.4	A quality report used at Eurostat	611
22.4.5	Quality reporting at INSEE	612
22.4.6	Statistics Canada Dashboard	614
22.5	Conclusions	617
22.6	A Detailed List of Quality Measures for Seasonal Adjustment	618



## 22.1 Introduction

Seasonally and calendar adjusted data are often considered as the key reference indicators for analysts and policy makers. They provide more understandable series for analysts revealing the “news” contained in the time series of interest; they facilitate the comparison of long-term and short-term movements among sectors and countries; and they supply users with the necessary input for business cycle analysis (i.e. output gap estimation), trend-cycle decomposition and turning points detection.

Due to their importance, their quality must be precisely checked. Unfortunately, despite the efforts of Eurostat, harmonization is far to be perfect and seasonal adjustment strategies and tools as well as revision policies may be still different from a country to another and from an indicator to another. If the need to document the seasonal adjustment process is clearly recognized, the design of a quality report faces three real difficulties:

- First, the evaluation of the “seasonal component” provided by an adjustment method is hampered by the fact that the true seasonal component remains a theoretical and imprecise concept, never liable to direct observation.
- Then, the objectives of seasonal adjustment appear multiple and implicit. Is it to obtain the best estimate of the trend-cycle component, the best estimate of the seasonal component itself or even a prediction of the next months or next year? Each objective will generate its own quality criteria.
- Finally, the expected content of a quality report usually differs according to the user. Producers, database managers, analysts, researchers, policy makers, publication units do not need and do not look for the same kind of information.

As far as the statistical output, the seasonally and calendar adjusted series, is concerned and according to the European Statistical System Quality Assurance Framework<sup>1</sup>, “*the important issues concern the extent to which the statistics are relevant, accurate and reliable, timely, coherent, comparable across regions and countries, and readily accessible by users*”. In this chapter, we will mainly focus on the accuracy and reliability aspects of quality, adopting the producer point of view who wants to check if the use of another strategy or set of parameters would not permit to improve the quality of the adjustment.

Section 22.2 reviews the literature and derives a list of possible quality measures built from a list of properties an “optimal” seasonal adjustment should have. Section 22.3 presents a first list of quality measures that are present in the main seasonal adjustment methods and software - namely X-12-ARIMA<sup>2</sup>, TRAMO-SEATS<sup>3</sup> and JDEMTRA+.<sup>4</sup> Section 22.4 presents the experience of several institutes and Section 22.5 concludes.

## 22.2 Desirable properties for a seasonal adjustment method

Seasonal adjustment is based on the notion that the observed value of an economic time series  $X_t$  may be divided into several unobserved and independent components:

- The series trend, representing the long-term evolution of the series;
- The cycle, the smooth, almost periodic movement around the trend, revealing a succession of phases of growth and recession.

<sup>1</sup> See European Statistical System (2015)

<sup>2</sup> See Findley et al. (1998)

<sup>3</sup> See Gómez and Maravall (1997)

<sup>4</sup> JDEMTRA+ is the European software for seasonal adjustment, supported by Eurostat and the European Central Bank. The software implements the 2 recommended methods X-12-ARIMA and TRAMO-SEATS; see Grudkowska (2017).

As the series studied are generally too short for both components to be easily estimated, most seasonal methods estimate the trend-cycle component, written as  $TC_t$ .

- The seasonal component, written as  $S_t$ , representing intra-year fluctuations, monthly or quarterly, that repeat more or less regularly year after year;
- A so-called “trading-days” component, written as  $TD_t$ , that measures the impact on the series of the day composition of the month or quarter;
- A component measuring the effect of moving holidays, like Easter, written as  $MH_t$ ;
- And finally, the irregular component, written as  $I_t$ , combining all the other more or less erratic fluctuations not covered by the previous components.

Several decomposition models are usually considered, including the additive model:  $X_t = TC_t + S_t + TD_t + MH_t + I_t$ . In X-12-ARIMA and TRAMO-SEATS, the calendar effects ( $TD_t$  and  $MH_t$ ) as well as outliers are detected and estimated using a regression model with ARIMA errors (regARIMA model). The decomposition in trend-cycle, seasonal and irregular is then done in a second step on the series corrected from these deterministic effects.

These definitions are qualitative and rather imprecise and they are still today the subject of controversy and various interpretations. For example, here are two quotations of eminent statisticians who apparently do not have the same objective:

- Kendall (1973): “*The essential idea of trend is that it shall be smooth.*”
- Harvey (1989): “*There is no fundamental reason, though, why a trend should be smooth.*”

Any seasonal method relies on an explicit or implicit definition of each component and, as the definitions might vary, it is therefore difficult to compare methods and to precisely judge of the quality of the adjustment.

### 22.2.1 Some properties of “optimal” seasonal adjustment

Some authors tried to solve the problem by pointing out some desirable properties that, ideally, a method of adjustment should possess. Let us note  $X_t$  the raw series and  $X_t^a$  the seasonally adjusted series.

Lovell (1963), in the context of multiple regression analysis, defines 5 properties:

1. Property 1: An adjustment procedure preserves sums if and only if  $(X_t + Y_t)^a = X_t^a + Y_t^a$  for all  $t$ .

No discrepancies in aggregation will be generated if component series rather than totals are adjusted with a procedure satisfying this requirement.

2. Property 2: An adjustment procedure preserves products if and only if  $(X_t Y_t)^a = X_t^a Y_t^a$  for all  $t$ .

With a product preserving procedure, it is equivalent to deflate a raw value series by an unadjusted price index and then adjusts the deflated value series or, alternatively, to adjust the quantity and price series before deflation.

3. Property 3: An adjustment procedure is orthogonal if and only if  $\sum_t (X_t - X_t^a) X_t^a = 0$ .

Indeed if the seasonal component ( $S_t$ ) is correlated with the adjusted series, it implies that some seasonality remains in the data.

4. Property 4: An adjustment procedure is idempotent if and only if  $(X_t^a)^a = X_t^a$  for all  $t$ .

If a seasonally adjusted time series is changed when adjusted again, we must conclude that the technique of adjustment is deficient as it either fails to filter out all seasonal movements or else distorts series already free of seasonal fluctuations.

5. Property 5: An adjustment procedure is symmetric if and only if  $\frac{\partial X_{t'}^a}{\partial X_t^a} = \frac{\partial X_t^a}{\partial X_{t'}^a}$  for all  $t$  and  $t'$ .

A new observation added to an unadjusted time series may be expected to affect a number of components of the time series obtained by seasonally adjusting the updated series. The symmetry property allows announcing in advance the seasonal correction factors and as a consequence facilitates the communication by avoiding any suggestion that the evidence is being manipulated.

Unfortunately, as demonstrated by the author, there is no non trivial procedure satisfying both properties 1 and 2<sup>5</sup>. The various properties are not independent and the author advocates for the use of multiple regression techniques: *Any sum preserving adjustment procedure that is orthogonal and idempotent (and hence symmetric) can be executed by regressing the unadjusted time series upon an appropriate set of explanatory variables; conversely, the residuals obtained through the regression of the data upon an appropriate set of explanatory variables constitutes an adjusted time series satisfying the requirements of sum preservation, idempotency, orthogonality, and symmetry.*

Other authors like Grether and Nerlove (1970) and Granger (1978), reasoning in terms of spectral analysis, point out similar properties but bring attention to some defaults of various methods that might delay the turning points (phase shifting), introduce spurious cycles and over-adjust the seasonal component:

- The coherence of the original and the seasonally adjusted series should be high at all frequencies except, possibly, seasonal and calendar ones.
- Although phase shifts are generally impossible to avoid altogether in any method of seasonal adjustment that uses past data to adjust current observations, such shifts should be minimized especially at low frequencies at which most of the power in economic time series is typically concentrated.
- Seasonal adjustment should not remove more than enough power at the seasonal frequencies thus producing "dips" at those frequencies. While this was not regarded as especially serious in and of itself, corresponding to the "dips" there must exist intermediate peaks at frequencies between the seasonal ones. Such peaks, if large enough, might induce spurious fluctuations in the adjusted series, a disturbing possibility.

Finally, Fase et al. (1973) and Kuiper (1978) mention 2 practical requirements a seasonal adjustment procedure should possess:

- The seasonal estimate should be stable;
- For any year the sum of the seasonally adjusted figures equals the sum of the original unadjusted figures.

### 22.2.2 The ESS guidelines on seasonal adjustment

All these properties are interesting but have to be confronted to the practice. For examples:

- Mainly because of the presence of outliers and/or the multiplicative decomposition scheme, seasonal adjustment is not a linear procedure: there is no reason why the outliers should be the same in a aggregated series and its sub-components. In consequence, few seasonal adjustment methods will preserve sums.
- Raw data might also be revised, quarterly national account series for example which are regularly benchmarked on annual series. In these conditions, seasonally adjusted series will also be revised and the symmetry property will not always hold.

<sup>5</sup>This result shows that the "direct versus indirect" problem has no perfect solution as you cannot assure the consistency of the rate of unemployment (product) by sector or gender (sum). See Chapter 18.



The ESS Guidelines for Seasonal Adjustment, see Eurostat (2015), translates these “optimal properties” in more practical ways and recommends checking some characteristics on any seasonally adjusted series:

*All seasonal adjustment software packages provide a wide range of measures to assess accuracy and reliability. These measures are derived to some extent from the implemented method, but many measures are common. The aim is to assess if a seasonally adjusted time series meets the following characteristics:*

- *absence of model/transformation misspecification;*
- *absence of residual seasonal/calendar effects or over-adjustment of seasonal/calendar effects;*
- *absence of under/over-treatment of outliers/seasonal breaks;*
- *absence of instability in settings of the trend-cycle/seasonal/calendar components or pattern in the irregular component;*
- *absence of irregular influences in the trend-cycle, the seasonal and calendar component;*
- *absence of residual correlation in the model residuals.*

## 22.3 A list of quality measures for seasonal adjustment

From these recommendations, and from the various software outputs, it is possible to derive a first list of quality measures. All measures mentioned hereafter are presented in Annex 22.6, with their precise definition, formula and reference.

### 22.3.1 Common quality measures

The focus is put on the detection of seasonality, user-defined regressors (including trading-day and moving holiday detection), the quality of the ARIMA model and the stability of the adjustment and the revision analysis. All these tests can be implemented in any seasonal adjustment software. A large part of them are available in the JDEMETRA+ and documented in Grudkowska (2017).

#### Testing for seasonality or residual seasonality.

Several tests can be used to check for the presence of seasonality in the raw series or of residual seasonality in the irregular and seasonally adjusted series or even in the REGARIMA model residuals.

- Ljung-Box test on seasonal autocorrelations
- F-Test for stable seasonality
- Kruskal-Wallis-Test for stable seasonality
- Canova-Hansen-Test for deterministic seasonality
- F-Test for moving seasonality
- Combined seasonality test
- Visual spectral tests
- Peaks at seasonal frequencies in the periodogram

**Testing for calendar effects or residual calendar effects.**

Various tests can be used to check for the presence of calendar effects in the raw series or of residual calendar effects in the irregular and seasonally adjusted series or even in the REGARIMA model residuals.

- Student t-tests for the significance of each trading-day and moving holiday effect
- Global F-Test for trading-day effects
- Global F-Test for the good number of trading-day regressors

Spectral tests are scarcely used in this context as they appear less powerful than the F-tests, especially for quarterly series.

**Checking for the quality of the REGARIMA model.**

Student tests can be used to check for the significance of (user-)regressors; various over tests can be used to check the REGARIMA model residuals.

- Number of outliers
- Concentration of outliers on particular month or particular year or on the end of the series
- Ljung-Box test on the autocorrelations
- Normality test
- Joint normality test
- Independence test
- Wald-Wolfowitz test (or runs test)

**Stability of the adjustment and Revision analysis.**

The stability measures have been proposed in Findley et al. (1990) and Findley et al. (1998) and have been implemented first in the “sliding pans” and “history” specifications of X-12-ARIMA. The revision measures have been proposed by McKenzie and Gamba (2008).

Revision analysis:

- Mean revision: Arithmetic mean of differences between later and preliminary estimates
- Mean squared revision
- Minimum revision
- Maximum revision
- Mean absolute revision: Arithmetic mean of absolute differences between later and preliminary estimates
- Standard deviation of revision: Root of mean of squared differences between revision and mean revision
- Root mean squared revision: Root of mean of squared differences between later and preliminary estimates
- Direction of revision: Percentage of preliminary and later estimates with the same sign
- Relative mean absolute revision: Ratio between mean absolute revision and mean absolute preliminary estimate

Stability analysis:

- X (%): Percentage of periods in which the series under review is unstable
- MM (%): Percentage of periods in which the relative month-to-month change is unstable
- QQ (%): Percentage of periods in which the relative quarter-to-quarter change is unstable
- YY (%): Percentage of periods in which the relative year-to-year change is unstable

### **Roughness measures.**

These descriptive statistics have been proposed by several authors to check the characteristics of the decomposition.

- R1: Sum of squares of the seasonally adjusted series first-differences
- R2: Sum of squares of differences between the seasonally adjusted series and a smoothed trend-cycle
- R3: Sum of squares of differences between the seasonally adjusted series and the trend-cycle
- MAR (S): Smoothness of seasonal factors
- MAR1 (TC): Sum of squares of the trend-cycle first differences
- MAR2 (TC): Sum of squares of the trend-cycle second differences

### **Other simple plausibility checks**

- Annual totals: Comparison between annual totals of unadjusted or calendar adjusted and seasonally adjusted series
- Annual average: Check if the sum (or product) of seasonal factors is equal, for each year, to 0 (or 1)
- Test for break in seasonal pattern

## **22.3.2 Specific quality measures**

These measures have been designed to check if the decomposition algorithms, SEATS for TRAMO-SEATS and X11 for X-12-ARIMA, functioned well and gave sensible results.

### **TRAMO-SEATS specific measures**

These measures are specific to ARIMA-based methods.

- Autoregressive roots: Modulus and period
- Over and under estimation test: Frequency domain test for over- and underestimation of the signal in an ARIMA based signal-plus-noise model
- Revision variance: Finite sample versions of the revision variance
- Statistical significance of seasonality: Number of significant seasonal factors in one year (central year for historical estimates, last year for preliminary estimates and one-year forecasts)
- Component variance: Comparison of variance of the component, theoretical estimator and empirical estimate

### X-12-ARIMA specific measures

These statistics have been proposed by Lothian and Morry (1978) to verify that the X11 filter was able to give a correct estimation of the seasonal component. This X11 filter is known to have problems when the series is very noisy or when the seasonality is evolving a lot; the M-statistics are a set of 11 statistics to check the irregular (statistics M1 to M6), the presence of seasonality (M7) and the stability of the seasonal component (M8 to M11). The Q-statistics are linear combinations of the M-statistics that summarize in a single number the difficulty of the decomposition.

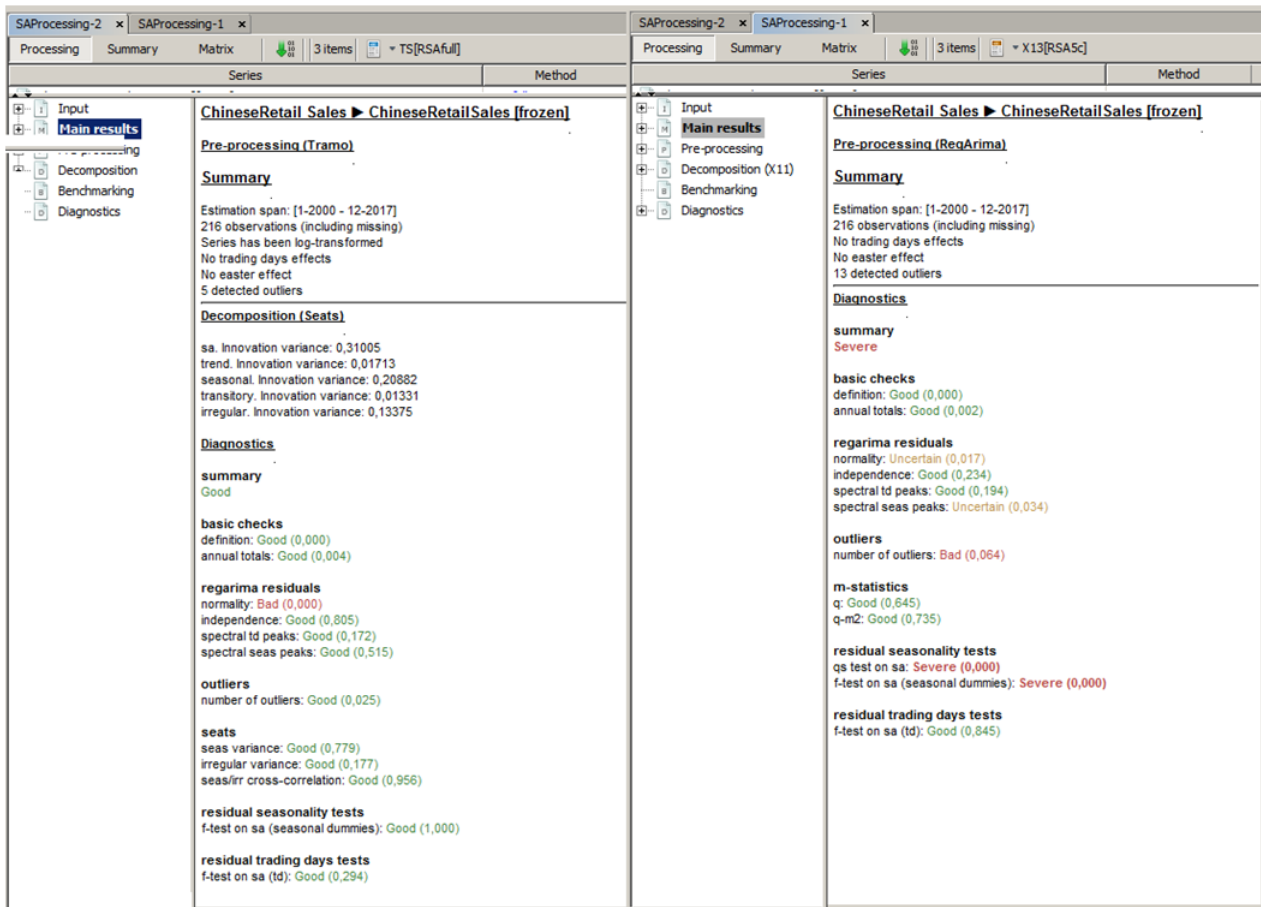
- M1: The relative contribution of the irregular over three months span
- M2: The relative contribution of the irregular component to the stationary portion of the variance
- M3: The amount of month to month change in the irregular component as compared to the amount of month to month change in the trend-cycle
- M4: The amount of autocorrelation in the irregular as described by the average duration of run
- M5: The number of months it takes the change in the trend-cycle to surpass the amount of change in the irregular
- M6: The amount of year to year change in the irregular as compared to the amount of year to year change in the seasonal
- M7: The amount of moving seasonality present relative to the amount of stable seasonality
- M8: The size of the fluctuations in the seasonal component throughout the whole series
- M9: The average linear movement in the seasonal component throughout the whole series
- M10: Same as 8, calculated for recent years only
- M11: Same as 9, calculated for recent years only
- Q and Q (without M2): linear combinations of the M-statistics

### 22.3.3 The JDEMETRA+ quality report

JDEMETRA+ proposes a generic quality report for a seasonal adjustment done with TRAMO-SEATS or X-12-ARIMA and a global assessment of the adjustment. This report, that used colors for a quick reading, is illustrated in Figure 22.1.

The quality diagnostics that can be built on the different seasonal adjustment procedures are very heterogeneous. Moreover, their interpretation might be difficult for many users. That is why the choice has been made in JD+ to give a summary of the information they provide by means of a very simple qualitative indicator, associated with a color, and defined by the following rules:

Figure 22.1: An example of JDEMETRA+ quality reporting



Value	Meaning
Undefined	The quality is undefined: unprocessed test, meaningless test, failure in the computation of the test, etc.
<b>Error</b>	There is an error in the results. The processing should be rejected; for instance, it contains aberrant values or some numerical constraints are not fulfilled.
<b>Severe</b>	There is no logical error in the results but they should not be accepted for some statistical reasons.
<b>Bad</b>	The quality of the results is bad, following a specific criterion, but there is no actual error and the results could be used.
<b>Uncertain</b>	The result of the test is uncertain. Consider it with caution.
<b>Good</b>	The result of the test is good.

Several qualitative indicators can be combined and the aggregation rule is defined by:

Result	Rule
Undefined	All diagnostics are “Undefined”.
<b>Error</b>	There is at least 1 “error” diagnostic.
<b>Severe</b>	There is at least 1 “severe” diagnostic but no error.
<b>Bad</b>	No error, no severe diagnostics; the average of the (defined) diagnostics (Bad=1, Uncertain=2, Good=3) is $< 1.5$ .
<b>Uncertain</b>	No error, no severe diagnostics; the average of the (defined) diagnostics (Bad=1, Uncertain=2, Good=3) is in $[1.5, 2.5[$ .
<b>Good</b>	No error, no severe diagnostics; the average of the (defined) diagnostics (Bad=1, Uncertain=2, Good=3) is $\geq 2.5$ .

Errors and severe diagnostics are absorbent results. The global “quality” indicator of the seasonal adjustments displayed in the multiprocessing window is the sum of all defined quality diagnostics, using the rules defined above. Finally, diagnostics can generate warnings which are indicated by exclamation marks and tooltips in the multiprocessing output panel.

As shown in figure 22.1, the different diagnostics are put in several groups focusing on:

- the coherence of the decomposition (“Basic checks” group);
- the visual spectral inspection (“Visual spectral analysis” group);
- the residuals of the regArima preprocessing model (“RegArima residuals” group)
- the number of outliers (“Outliers” group)
- the residual seasonality (“Residual seasonality tests” group)
- the residual trading-day effect (“Residual trading-day tests” group)
- the decomposition (“Seats” group for TRAMO-SEATS, “M-Statistics” group for X-12-ARIMA)

Most of the basic diagnostics use parameters, usually thresholds, that can be customized through the “Options” dialog box.

### 22.3.4 Limits of a pure numerical quality approach.

Users should consider the quality indicators as a tool to detect potential problems in a large set of adjustments quickly. For important series, a more complete investigation of the results should always be considered. This is what is suggested by the ESS guidelines on seasonal adjustment which gives the following “best” recommendation:

*“Calculate measures for all characteristics, do alternative runs of seasonal adjustment (if necessary), and take decisions based on expert judgment”.*

Maravall (2018) gives several interesting examples, in the ARIMA-based approach of TRAMO-SEATS, where the REGARIMA model looks perfect according to the various tests but gives an unacceptable seasonal component and reversely where the REGARIMA model looks bad but gives a good seasonal component.

Numerical quality measures should only be used as WARNINGS to bring the attention on potential problems that should be checked. They should not be used alone to decide if an adjustment is correct or not. A simple exercise can demonstrate this assertion easily. Let us suppose that your decision on the quality of the adjustment is based on  $n$  independent statistical tests and that each test is accepted at the 5% level<sup>6</sup>. If you accept the adjustment if the  $n$  tests are valid, then the Bayes theorem tells you that you will do it with a

<sup>6</sup>Which means that you will accept the null hypothesis with a probability of 0.95 to be right.

probability equals to  $(1 - 0.05)^n$  to be right. Which means that, if you use 6 tests with a probability of 26.5% to be wrong .... and a quality report might contain much more than 6 independent tests!

## 22.4 Examples of quality assessment and quality reporting

### 22.4.1 The quality report for seasonally adjusted data used at the Deutsche Bundesbank

#### 22.4.1.1 Preliminary remarks

The seasonal adjustment approach used at the Deutsche Bundesbank is a **controlled current adjustment** procedure using the **Census X-12-ARIMA** method. Accordingly, the quality report is embedded in a strategy that relies on re-estimation of the seasonal adjustment parameters and seasonal and calendar factors at least once per year and always together with longer revisions of the unadjusted data. In this event, the calendar and seasonal factors are typically extrapolated for one year ahead. For example, for a monthly series, factors for the following twelve months are produced. When the unadjusted figures for a new month become available, they are adjusted using the extrapolated factors, unless there are economic or statistical reasons for re-estimating the factors. The procedure takes into account the different economic and political importance of time series: Headline indicators and higher aggregates, such as 2-digit-NACE positions, undergo a more careful and resource-intensive quality control procedure than other indicators, such as 4-digit-NACE positions that are part of a system of mass production of seasonally adjusted data. Further details are given below.

#### 22.4.1.2 Quality control for headline indicators

The first time when a time series is seasonally adjusted, as well as each time when seasonal factors are re-estimated, a detailed set of graphical, descriptive, non-parametric and parametric criteria is used to validate the seasonal adjustment. The procedure is fully in accordance with the European Statistical System Guidelines on Seasonal Adjustment.

For important time series, such as industrial production and its main industrial groupings, as well as important sections such as machinery, a thorough quality control procedure is implemented each time a new data point becomes available. In accordance with the method of controlled current adjustment, extrapolated factors are compared with re-estimated factors. For this purpose, a full seasonal adjustment run is provided. This allows comparing extrapolated calendar and seasonal factors with the re-estimated factors and assessing the impact of a re-estimation on the results. Also, any statistically significant outliers in the original data can be identified on the basis of the seasonal adjustment run. In this sense, the quality control comprises also the unadjusted original data. In the case of non-plausible unadjusted original data or significant outliers, the data producers are contacted and possible errors corrected or further metadata explaining the behaviour of the time series at the current end is provided.

Particular care is taken to assess the plausibility of the outliers identified by the RegARIMA model, the statistical fit of the ARIMA model (eg by checking for absence of significant and positive autocorrelation for seasonal lags in the model residuals and inspecting the Ljung-Box statistics) or of a model with constant seasonal effects, the significance and plausibility of the coefficients of any user-specified regressors included and the appropriateness of the identified ARIMA model.

In this context, the potential occurrence of an under-adjustment as well as an over-adjustment of calendar effects is assessed. For example, comparison of the pattern of the irregular component in similar calendar

constellations might give an indication of residual calendar effects. Such effects might be explicitly modelled and adjusted for by including an adequate user-specified regressor. An over-adjustment might occur if the semi-elasticity implied by the estimated regression coefficients shows an implausibly high effect. In such cases, additional checks on the outliers detected often helps to solve problems of implausible calendar effect estimates

Also, the parametric and non-parametric tests of seasonality are checked, before a closer look is taken at the SI ratios, the adequacy of the trend and of the seasonal filters. Further attention is given to the spectra of the modified irregular series in order to identify any visually significant residual seasonal or trading day peaks. Such peaks might be an indication to take again a closer look at the seasonal adjustment procedure. Finally, the monitoring and quality statistics are checked in order to assess the quality of the seasonal adjustment procedure, the stability of the seasonal component and the influence of the irregular component on the result. The SI ratios, together with the spectral diagnostics and the quality diagnostics might give an indication of a possible seasonal over-adjustment.

In the periods in between a thorough re-estimation of the factors, the validation of the results in principle comprises all the aforementioned steps. Particular importance is given on any newly identified outliers at the current end of the time series, the appropriateness of the RegARIMA model including the calendar regressors and the comparison of the extrapolated factors with the re-estimated factors, as well as the spectral diagnostics. A significant amount of expert knowledge of the series in question and of any economic events potentially affecting the time series (such as strikes, severe weather conditions or the effects of a severe economic downturn or upswing) is taken into account in order to assess the plausibility of the time series decomposition in trend, seasonal, calendar and irregular components.

#### 22.4.1.3 Quality control in mass production

The seasonal adjustment approach used in mass production is a **limited controlled current adjustment approach**. The seasonal and calendar factors are re-estimated once per year and extrapolated for one year ahead. In the periods in between, the extrapolated factors are used for seasonal and calendar adjustment. The results are inspected visually. In the case of exceptional developments at the end of the series, a more thorough validation is performed, which resembles the procedure described above for quality control of headline indicators.

Furthermore, a log file of the Census X-12-ARIMA methods is produced. This log file produces a series of warning messages. For example, if significant seasonal peaks are visible in the spectrum of the modified irregular series or if trading day peaks are present in the spectra, the log file will produce a warning message. The log file also includes warning messages in cases of problems with the estimation of the ARIMA model parameters or with the estimation of user-defined regression coefficients. Those series with warning messages are always inspected by a statistician.

At the same time, a top-down approach is often used. For example, in case of anomalies in a higher aggregate, for example, orders received for machinery, it is checked which of the sub-series could be responsible for the anomaly.

#### 22.4.1.4 Basic plausibility checks

This category of quality measures involves three simple tests for checking that no rough mistakes occurred during the seasonal adjustment procedure. More specifically, these tests concern

- the (approximate) equality of annual totals of the calendar and the seasonally adjusted series as well as the (approximate) equality of annual totals of the unadjusted and the seasonally adjusted series (if no calendar effects are estimated)

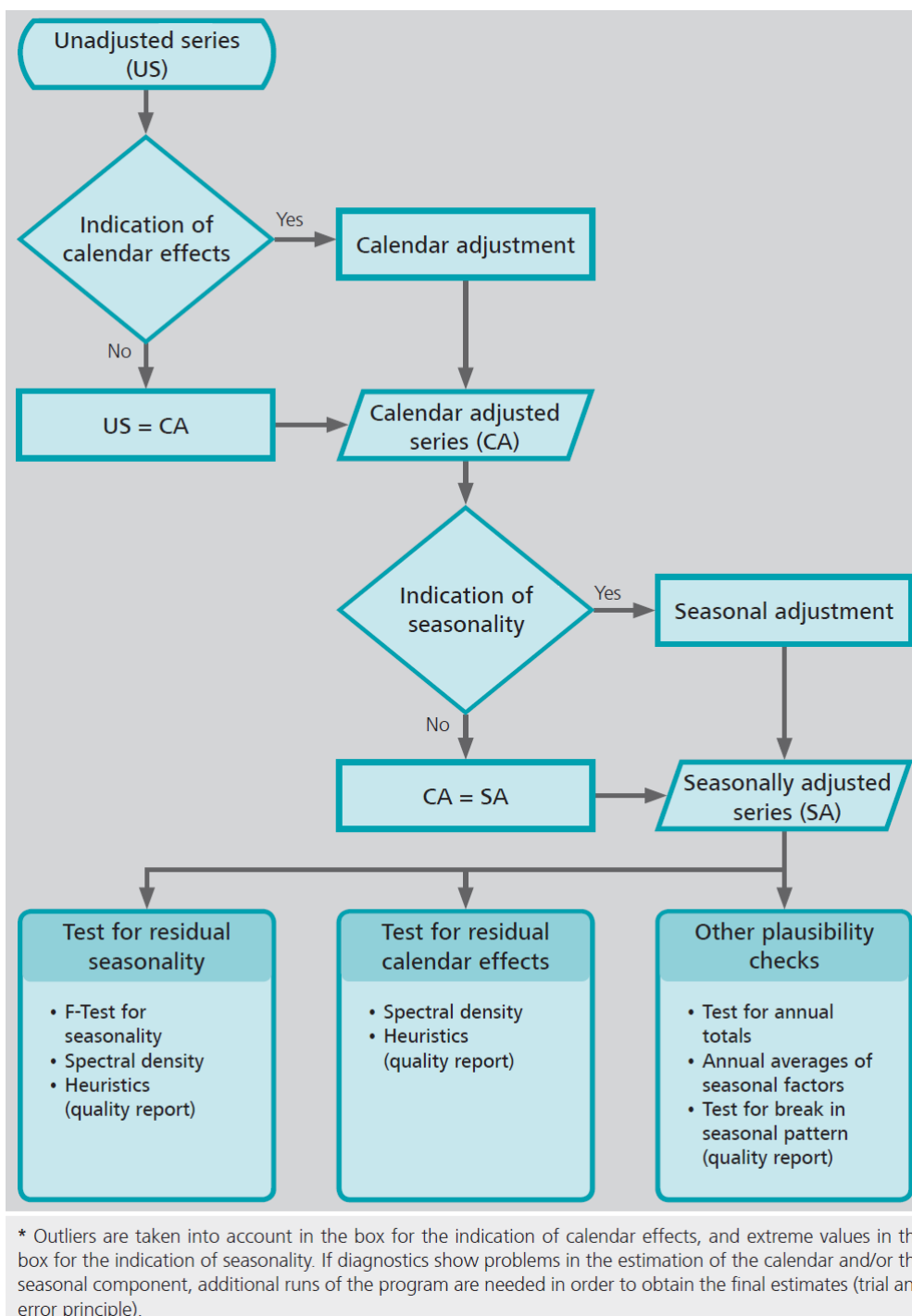


- the question if the annual average of the seasonal factors (approximately) equals 1 for a multiplicative decomposition (and 0 for an additive decomposition)
- a possible break in the seasonal pattern by inspecting the month-specific average growth rates of seasonal factors

All plausibility checks suggested could be calculated for some selected economic indicators in France and Germany, respectively. Furthermore, its critical values could be obtained with the aid of a simulation study.

The Deutsche Bundesbank Guide for judging the appropriateness of seasonal adjustment is summarized in Figure 22.2.

Figure 22.2: Deutsche Bundesbank: Guide for judging the appropriateness of seasonal adjustment



## 22.4.2 Quality assessment of seasonally adjusted series at the European Central Bank

The European Central Bank uses the JDEMETRA+ implementation of X-12-ARIMA to seasonally adjust its series. The quality control is based on a 6-page quality report, directly output from JDEMETRA+ using a specific plug-in. The report mixes charts, statistical measures and tables:

### 1. Summary Charts

- Original series, Seasonally adjusted series (concurrent) and Seasonally adjusted series (saved)
- Original series period-to-period, SA series (concurrent) period-to-period, and SA series (saved) period-to-period
- Seasonal component
- Outliers affecting Irregular component and Irregular component
- Monthly seasonal factors: Concurrent factors, Final unmodified SI ratios, Final replacement values for SI ratios and Saved factors

### 2. Main Results

- Series span
- Estimation span
- Decomposition Model
- Henderson Trend Filter
- Number of Outliers
- Presence of trading days effects
- Presence of Easter effect

### 3. RegARIMA model and regressors: Parameters, Coefficients, T-Stat and P-value.

### 4. Tests and Statistics

- Residuals from the RegARIMA model
  - Normality: Mean, Skewness, Kurtosis, Normality (P-values)
  - Independence: Ljung-Box(24), Ljung-Box on seasonality(2), Durbin-Watson statistic
  - Linearity: Ljung-Box on squared residuals(24)
- M-Statistics: M1 to M11, Q, Q-M2
- Seasonal Filters: Filters and SI Ratios
- Seasonality:
  - Tests in Original Series, Linearized Series, Regarima Residuals (complete series and last period), SA Series (complete and last period), Irregular (complete and last period):
    - \* Auto-correlation at seasonal lags
    - \* Friedman
    - \* Kruskal-Wallis
    - \* Spectral peaks

- \* Seasonal dummies
- Combined seasonality test
- Moving seasonality test
- Residual seasonality test (complete series and last 3-years)

#### 5. D Tables

- D10: Seasonal component
- D11: Seasonally adjusted series
  - Seasonally adjusted series when using the official forecast seasonal/trading day factors
  - Period to period percentage variation concurrent run
  - Period to period percentage variation when using the official forecast seasonal/trading day factors
- D16: Seasonal and calendar effects

Figures 22.3, 22.4 and 22.5 present an example of the 3 first pages of the report.

### 22.4.3 Quality assessment of seasonally adjusted series at ISTAT

#### 22.4.3.1 General considerations

Seasonally adjusted (SA) series released by Istat are processed in three principals inside two Directorates. More in particular: in the division for Seasonal adjustment method (within the Directorate for methodology and statistical process design); in other two teams, one for national accounting series the other dealing with the labor force survey (within the Directorate for statistical production).

The first is the central nucleus responsible for seasonal adjustment, while the other two conduct seasonal adjustment both independently, due to the peculiar characteristics of their data production process.

In the last few years, an inter-directorate working group on SA has guaranteed a greater harmonization of existing practices, software and policy adopted, but this process hasn't yet been fully completed.

In the current situation, for short term statistics and national accounts data, Tramo-Seats as implemented in JDemetra+ 2.2.0 is used in order to identify and review the ARIMA models and the other specifications for the annual campaign, while legacy Tramo-Seats for Linux (vs. 942) is routinely run to produce the published SA figures. As an exception to this picture, there's the labor force survey (70 monthly series) where is employed Tramo-Seats implemented in Demetra 2.0 but the transition to JD+ is expected soon.

The ESS Guidelines on SA are implemented always following alternative A with two exceptions (for which alternative B is implemented): the horizon for published revisions and data availability in databases. Since the partial concurrent approach is used to revise specifications, the measures reported in the table are checked when models and specifications are revised (once a year, generally when either the last observation of the year is available or important revisions of unadjusted data are carried out), while only a subset of those measures are routinely monitored when the model coefficients are re-estimated.

The choice between direct and indirect approach is not always based on statistical criteria because users' needs and requirements are also taken into account.

Figure 22.3: ECB Quality report: Page 1

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## 1. Summary Charts

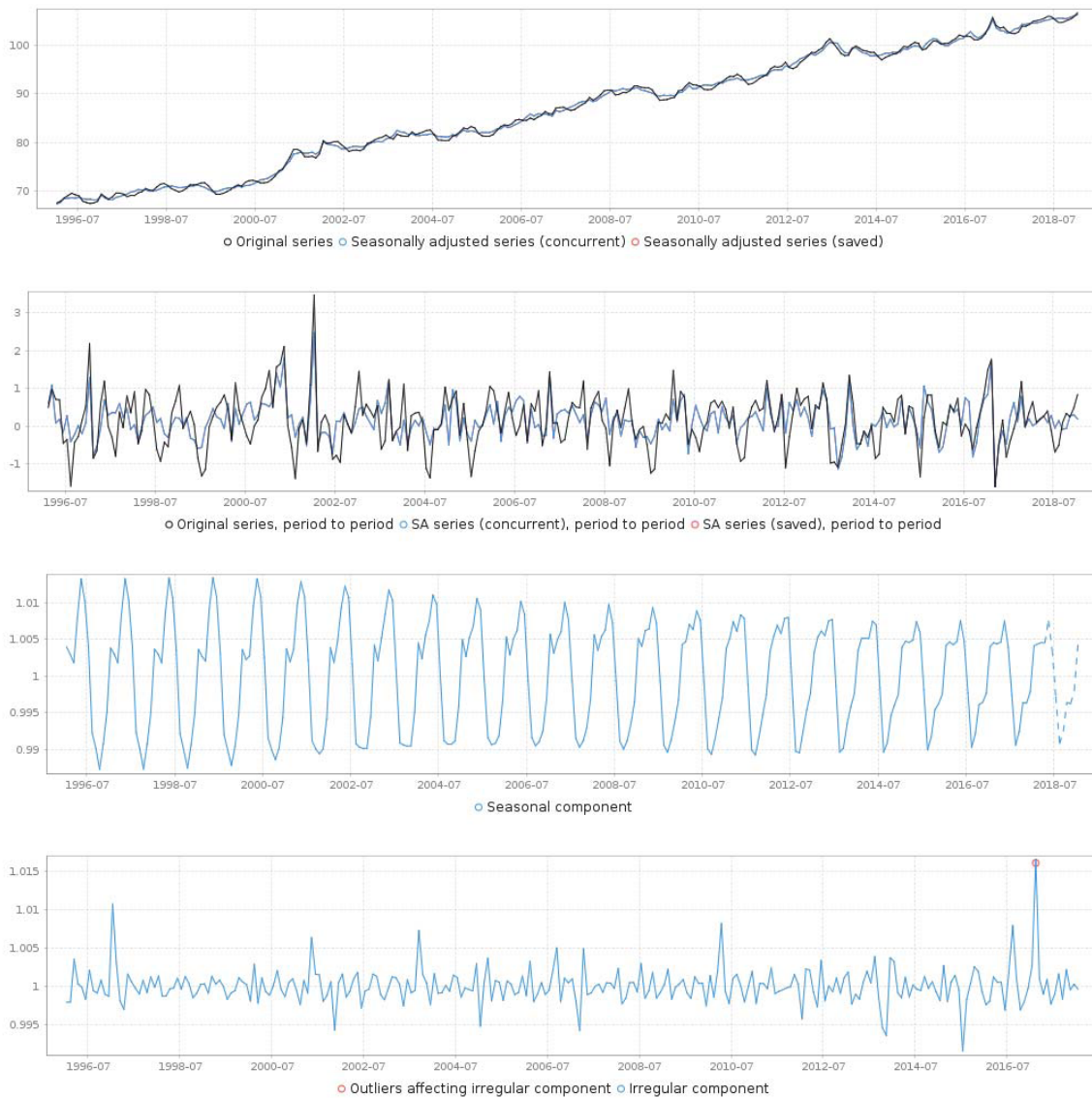


Figure 22.4: ECB Quality report: Page 2

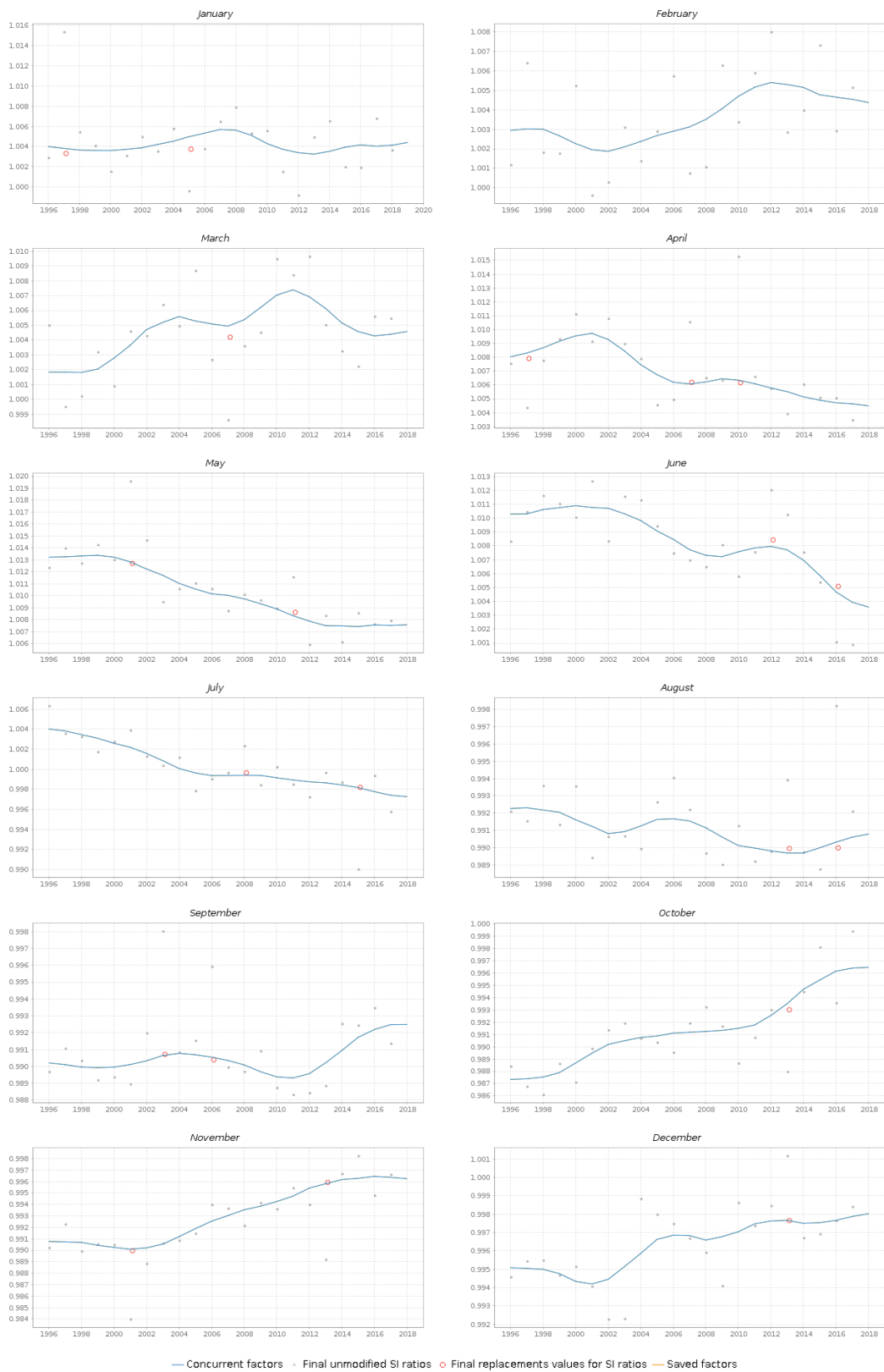


Figure 22.5: ECB Quality report: Page 3

## 2. Main Results

Series span: [1-1996 - 1-2018] (265 obs)      2 pre-specified outliers      No trading days effects  
 Estimation span: [1-1996 - 1-2018] (265 obs)      No easter effect  
 Series has been log-transformed  
 Henderson Trend Filters:13

## 3. RegARIMA model: ARIMA model: [(1,1,1)(0,1,1)] and regressors

Regression Detail	Parameters	Coefficients	T-Stat	P[ T  > t]
ARIMA model	Phi(1)	-0.310	-1.30	0.194
	Theta(1)	-0.102	-0.41	0.682
	BTheta(1)	-0.872	-24.48	0.000
Prespecified outliers	LS (1-2002)	0.024	5.08	0.000
	AO (2-2017)	0.016	5.12	0.000

## 4. Tests and Statistics

## Residuals from the RegARIMA model

Normality	P-Value	Independence	P-Value	Linearity	P-Value
Mean	0.777	Ljung-Box(24)	0.003	Ljung-Box on squared residuals(24)	0.777
Skewness	0.496	Ljung-Box on seasonality(2)	1.000		
Kurtosis	0.513	Durbin-Watson statistic:[1.968]			
Normality	0.699				

## M-Statistics

M-1	M-2	M-3	M-4	M-5	M-6	M-7	M-8	M-9	M-10	M-11	Q	Q-M2
0.263	0.075	0.000	0.586	0.056	0.285	0.222	0.460	0.262	0.555	0.541	0.250	0.272

## Seasonal Filters

Period	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Filters (prespecified)	3x5	3x5	3x5	3x5	3x5	3x5	3x5	3x5	3x5	3x5	3x5	3x5
I	0.002	0.003	0.002	0.001	0.001	0.002	0.001	0.002	0.001	0.002	0.001	0.001
S	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000
SI Ratio	6.266	5.560	6.150	3.429	4.462	3.817	4.149	6.991	3.450	3.699	3.879	5.625

## Seasonality

Test	Original Series	Linearized Series	Regarima Residuals	SA Series	Irregular	Residuals (last period)	SA Series (last period)	Irregular (last period)
Auto-corr at seasonal lags	YES	YES	NO	NO	NO	NO	NO	NO
Friedman	YES	YES	NO	NO	NO	NO	NO	NO
Kruskall-Wallis	YES	YES	NO	NO	NO	NO	NO	NO
Spectral peaks	YES	YES	NO	NO	NO	NO	NO	NO
Seasonal dummies	YES	YES	NO	NO	NO	NO	NO	NO

Combined seasonality test: Identifiable seasonality present

Moving seasonality test: Present

Residual seasonality test: No evidence in the entire series at the 10.0% level: F=0.228; No evidence in the last 3 years at the 10.0% level: F=0.256;

### 22.4.3.2 Comments on the measures

#### Important time series

- Concerning revisions of SA data, two kinds of analysis are carried out on the most important indicators through external procedure implemented neither in TS nor in JDemetra+.
  1. Given the last available unadjusted time series, the SA is recursively carried out (one observation is cut in each run) in order to study the ARIMA parameter stability and to compute the mean absolute revision and the mean absolute percentage of revision of 1,2,3,6 and 12 step revisions (for monthly series) and 1,2,3 and 4 step revisions (for quarterly series).
  2. According to the ONS-OECD approach, revisions of unadjusted, calendar adjusted and SA data are analysed, computing most of the revision measures proposed in McKenzie and Gamba (2008). For GDP, industrial production and both wages and labor costs, “triangles” and revision measures are currently updated and attached to the press releases.
- Some measures automatically available in the most recent version of TS may be useful in the preliminary analysis. They are the comparison of means and variances of first and second half of the series residuals and the comparison between forecast errors (out-of-sample) and residuals (in-sample).
- Roots of regular autoregressive polynomial in TS may affect trend, seasonality or transitory component and revision process of SA series. Both their modulus and period are considered .

#### All series

- When a log-additive decomposition is used (as in TS), the comparison between annual totals of unadjusted data and SA data is very important, since the level of trend component (and consequently of SA series) may be underestimated.
- As far as calendar adjustment is concerned, when an automatic choice of reg-ARIMA models is carried out (for more disaggregated time series) the sign of coefficient is considered and, concerning leap-year effect, the size of correction is evaluated (sometimes the correction may be very large and considered unreliable, therefore removed).
- Considering the number of significant historical, preliminary and forecast seasonal factors may help user to decide whether to adjust a series.
- Revision variance (in the miscellaneous section of the table) is used according to the infinite sample version. Together with the innovation variance is used to discriminate among competitive models on the same time series. In the current version of TS, the comparison among variance of component, estimator and estimate seldom highlights problematic decompositions.

## 22.4.4 A quality report used at Eurostat

Job vacancy statistics (JVS) provide information on the level and structure of labour demand. Eurostat publishes quarterly data on the number of job vacancies and the number of occupied posts which are collected under the JVS framework regulation and the two implementing regulations: the implementing regulation on the definition of a job vacancy, the reference dates for data collection, data transmission specifications and feasibility studies, as well as the implementing regulation on seasonal adjustment procedures and quality reports. Eurostat disseminates also the job vacancy rate which is calculated on the basis of the data provided by the countries.

The quality report has been adopted by the working group on labor market statistics in order to harmonize the seasonal adjustment of JVS series. The template includes the following information:

- the time span and number of observations adjusted;



- whether a direct or indirect method was used in case of aggregated NACE sections;
- the list of outliers detected with their type;
- the presence/absence of calendar effects with their type (trading-working day, Easter effect etc.);
- the residual seasonality test;
- the combined seasonality test;
- if TRAMO-SEATS is used: the SARIMA models identified and the overall quality of the modelling from JD+ diagnostics; if X12-related methods are used: the applied filters and the value of the combined seasonality test, as well as Q statistics;
- the maximum relative adjustment of the original series (in %), due to seasonal adjustment, calculated over all observations;

Most of this information could be created in JDEMETRA+ using the “csv matrix” output. Eurostat developed a plug-in for JDEMETRA+ which facilitates the production of the above-mentioned information required. Figure 22.6 presents an example of this quality report.

### 22.4.5 Quality reporting at INSEE

The French national statistical institute (INSEE) the JDEMETRA+ implementation of X-12-ARIMA to seasonally adjust the short-term statistics (Industrial production index and turnover statistics). An evaluation report has been developed to help the analyst during production, when there is very few time to detect and fix potential problems. The main objective of the report is to highlight the problematic series, especially those that have a large contribution to the aggregates. The expected output is a prioritized list of series the analyst will check in a very limited time.

This evaluation report is based on the same principles and quantification than the JDEMETRA+ quality report as described in section 22.3.3. Three main groups are considered: the RegArima model, the decomposition and the revisions. For each group quality measures, mostly tests, are evaluated and a global score is derived. The measures used in each case are the following:

1. For the “RegArima model” group:
  - The Ljung-Box statistic and the Box-Pierce statistic on the residuals;
  - The Ljung-Box statistic and the Box-Pierce statistic on the squared residuals;
  - The normality test on the residuals;
  - The number of outliers.
2. For the “Decomposition” group:
  - The F-test for residual seasonality on the seasonally adjusted series;
  - The F-test for residual trading-day effects on the seasonally adjusted series;
  - The F-test for residual seasonality on the irregular;
  - The F-test for residual trading-day effects on the irregular;
  - The quality of the decomposition (M and Q statistics).
3. For the “Revision” group:
  - The size of short-term revisions;
  - The size of long-term revisions.

Figure 22.6: Eurostat quality report for job vacancy statistics

Quality report for JVS seasonal adjustment (SA) - occupied posts											Country	
											UK	
NACE sections	SA time series transmitted	In case of aggregated sections (eg. B-E), was a direct/indirect <sup>(1)</sup> method used	First observation	Last observation	Number of observations	Log transformation (yes/no)	Number of outliers <sup>(2)</sup>	List of 3 main outliers <sup>(2)</sup>			Calendar effects	
								Outlier 1	Outlier 2	Outlier 3	Trading day effect	Easter effect
B-E	yes	direct	janv-12	mars-15	39	yes					No	Yes
B												
C												
D												
E												
F	yes	Not concerned	janv-12	mars-15	39	yes					NO	No
G-I	yes	direct	janv-12	mars-15	39	yes	1	ao(2014-3)			No	No
G												
H												
I												
K	yes	Not concerned	janv-12	mars-15	39	yes						No
L		Not concerned	janv-12	mars-15	39	yes	1	ls(2013-1)			No	Yes
M-N	yes	direct	janv-12	mars-15	39	yes	1	tc(2014-3)				No
M												
N												
O-Q	yes	direct	janv-12	mars-15	39	yes	1	tc(2012-4)			No	No
O												
P												
Q												
R-S	yes	indirect										
R	yes	Not concerned	janv-12	mars-15	39	yes						No
S	yes	Not concerned	janv-12	mars-15	39	yes	2	tc(2014-2)	tc(2014-3)		No	No
B-S												
B-N												
B-O												

SARIMA model						Filters used by X12-related methods <sup>(6)</sup>					Combined seasonality test	Overall quality of the modelling [from JD+ diagnostics] <sup>(8)</sup>	Max Adj. Size <sup>(7)</sup> in %
Non-seasonal part			Seasonal part			Trend filters			Seasonal filters				
P	D	Q	BP	BD	BQ	Stage 1	Stage 2	Stage 3	Stage 1	Stage 2			
0	1	0	0	1	1	2x4 <sup>(4)</sup>	H9 <sup>(5)</sup>	H13	3x3	3x5 <sup>(6)</sup>	Seasonal	Good	5%
1	1	0	0	1	1	2x4	H13	H23	3x3	3x5	Seasonal	Bad	6%
0	1	0	0	0	0	2x4	H13	H23	3x3	3x3	Not seasonal	Bad	not concerned
1	1	0	0	0	0	2x4	H13	H13	3x3	3x5	Not seasonal	Severe	not concerned
0	1	1	0	1	0	2x4	H13	H13	3x3	3x9	Seasonal	Severe	10%
0	1	0	0	1	1	2x4	H13	H23	3x3	3x3	Seasonal	Severe	25%
1	1	0	1	1	0	2x4	H9	H23	3x3	3x5	Seasonal	Severe	5%
0	1	0	0	1	1	2x4(4)	H9(5)	H13	3x3	3x5(6)	Seasonal	Good	15%
0	1	0	0	1	1	2x4(4)	H9(5)	H13	3x3	3x5(6)	Seasonal	Good	10%

Figure 22.7 presents an example of this quality report.

### 22.4.6 Statistics Canada Dashboard

Statistics Canada is currently experimenting a new quality assurance approach, see Matthews (2016), in order to monitor, maintain and report on quality of seasonal adjustment on an on-going basis.

At Statistics Canada, seasonal adjustment is performed at two different levels which have different responsibilities.

- Time Series Specialists are responsible of the development and maintenance. In particular they (1) do the analysis to initialize the system, (2) determine the options and parameters for seasonal adjustment and reconciliation, (3) conduct periodic review of seasonal adjustment options and (4) assure support for analysis, verification and explanation of results.
- Subject Matter Analysts are responsible of the ongoing production. They manage survey process to produce unadjusted estimates, run survey-specific seasonal adjustment system and analyze seasonally adjusted (and unadjusted) estimates.

In order to assure the quality of seasonally adjusted series, two kind of reviews are usually conducted:

- Scheduled Periodic Reviews (majority of updates) are done by time series specialist to update options in production system. They are conducted on a predetermined schedule, usually along with historical (annual) revisions. Due to the high volume of series for many projects, there is limited time for review and often overlapping schedules. Despite these difficulties, a regular level of reporting and approval is required.
- Ad hoc Reviews (updates made on an exceptional basis) are done in response to analyst concerns in monthly processing. As updates to options might lead to revisions and have an impact on quality, an increased level of reporting and approval is required in these cases.

A general monitoring schedule, presented in Table 22.1, is followed that can be modified based on quality requirements and operational factors.

**Table 22.1:** General monitoring schedule for the quality assurance of seasonally adjusted data at Statistics Canada.

Review type (frequency)	Mandatory Verifications	Recommended Verifications	Optional Verifications	Reporting on updates
Comprehensive (Annual)	YES	YES	Time Permitting	Subject Matter Analysts
Interim (Quarterly)	YES	Time Permitting		Senior Management
Ad hoc (On request)	YES	YES	Time Permitting	Senior Management

A simple dashboard, see Figure 22.8, has been implemented that provides a snapshot of an individual series at a point in time and points out some possible problems. It is intended for analysts but also for managers and specialists to understand the seasonal adjustment process. This dashboard is made of 3 parts:

- The first part (top panel) gives an idea of the recent history of the series (trend direction, overall volatility and obvious outliers) and summarizes the key diagnostics. These diagnostics concern both mandatory verifications (adjustability, residual seasonality and smoothness) and recommended verifications (recent and recurring outliers, moving seasonality, ARIMA model error and error autocorrelation).

Figure 22.7: INSEE STS Department quality report

Codet : XCH

Poids du groupe au niveau C5 : 8,2%

Libellé : Métallurgie et fabrication de produits métalliques à l'exception des machines et des équipements - Exports

Modèle : triplet (0 1 1)(0 1 1) Transformation : Multiplicative - Correction de la moyenne : non  
Effet jour ouvrable : 7 - Effet de pâques : NonPoints remarquables :  
2009M01 LS

Seuil outliers : Auto

Filtre saisonnier : Auto Moyenne de Henderson : Auto

Décomposition des notes des différents modèles (groupe et niveau supérieur) :

Notes Reg arima (sur 4)	XCH	C5	Notes Décompo (sur 4)	XCH	C5	Notes Révisions (sur 5)	XCH	C5
Ljung Box (bruit blanc)	4,0	3,9	Effets cjo sur cvs	4,0	3,9	Révision à court terme	0,3	1,9
Box Pierce (bruit blanc)	4,0	4,0	Effets cvs sur cvs	1,0	3,9	Révision à long terme	3,7	11,4
Ljung-Box (hétéroscédasticité)	2,0	1,5	Effets cjo sur irrég	4,0	3,9	<b>Note globale des révisions ocde</b>	<b>20,0</b>	<b>15,0</b>
Box Pierce (hétéroscédasticité)	2,0	1,5	Effets cvs sur irrég	1,0	3,8			
Normalité	4,0	3,8	Qualité	4,0	4,0			
Outliers	4,0	3,7	<b>Note globale décomposition (/20)</b>	<b>14,0</b>	<b>19,5</b>			
<b>Note globale Reg Arima (/20)</b>	<b>20,0</b>	<b>18,5</b>						

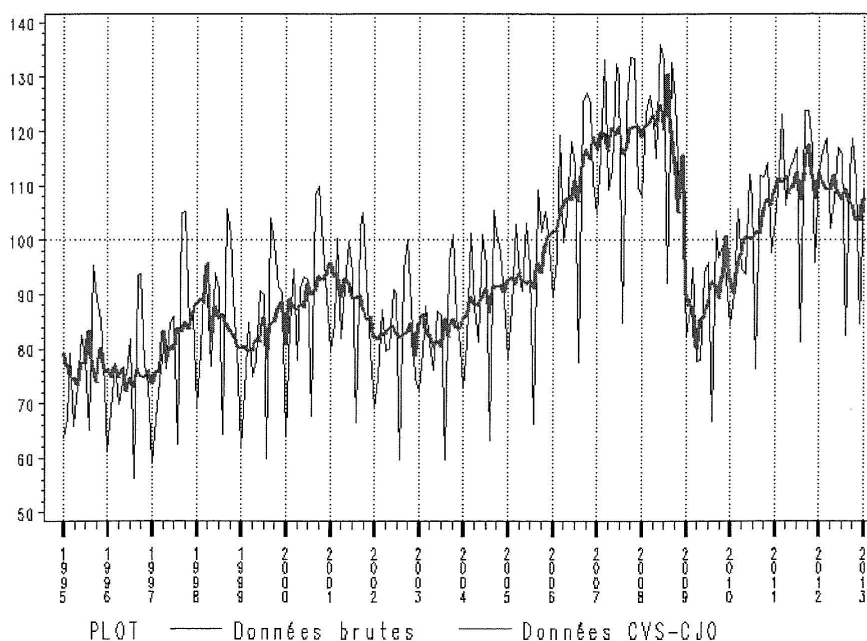
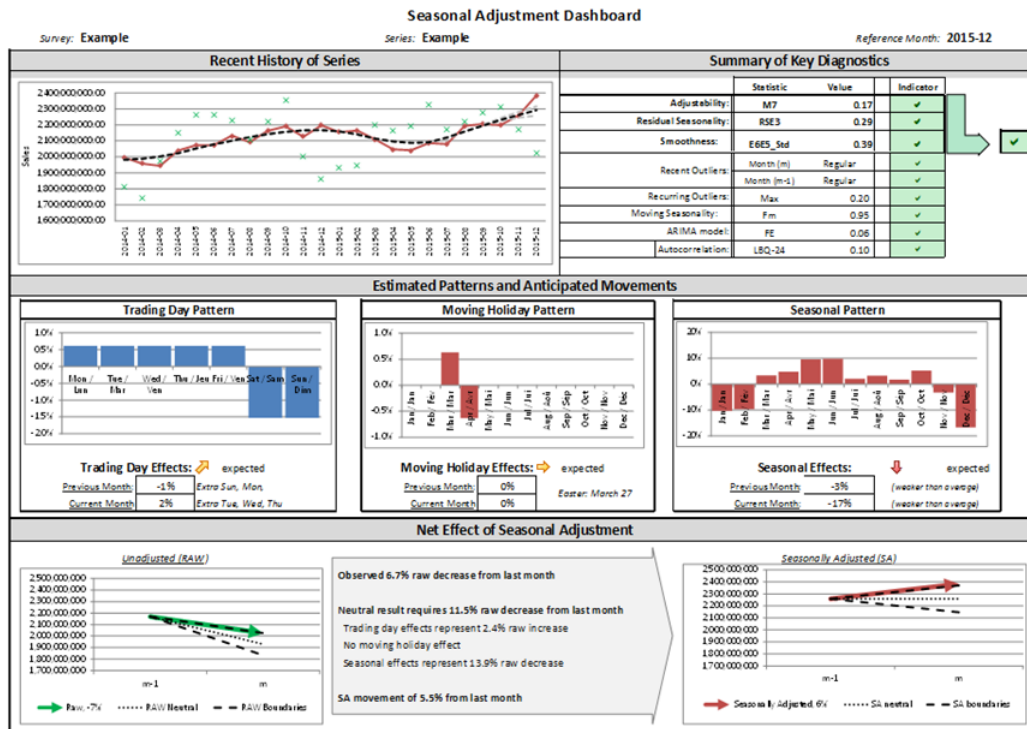
Campagne CVS de mars 2013 — Données brutes\_CVS—CJO  
Agrégat=\_XCH

Figure 22.8: Statistics Canada's Dashboard



- The second part (middle panel) concerns the estimated patterns and anticipated movements. It summarizes estimated trading day, moving holiday and seasonal pattern and presents the expected movement in unadjusted based on each pattern for the current month and the previous month.
- The third part (bottom panel) presents the net effect of seasonal adjustment in the evolution of the raw data, compared to typical ranges centered around “neutral” value, and of the seasonally adjusted series, compared to typical ranges. It also emphasizes the link between the observed and “neutral” movements in rw and adjusted data.

## 22.5 Conclusions

If the need to document the seasonal adjustment process to be able to report to user on the quality of most important indicators is clearly recognized, the design of a quality report is not easy. One important reason is the fact that the seasonal component we want to estimate and remove from the raw data, is not precisely defined and never observed; it is therefore difficult to evaluate its quality.

Several authors have proposed “ideal” properties a seasonal adjustment method should possess - sum preservation, idempotency, orthogonality, and symmetry for examples - but these properties are scarcely verified in practice. Starting from the recommendations of the ESS guidelines on seasonal adjustment, it is anyway possible to list a quite large number of tests and measures to assess the quality of the adjusted series. The variety of existing quality reports and evaluation strategies shows that we are still far away from a standard evaluation report, if such a report even exists.

It has to be emphasized that the main objective of an evaluation report, at least from the producer perspective, is to highlight potentially problematic series and the various tests and measures should always be interpreted as warnings. At the end, decisions should be based on expert judgment.

More research is still needed in order to reduce the number of tests and measures to select the most informative and relevant ones. For example, up to 7 different tests are available in JDemetra+ to check for the presence of seasonality in a time series that certainly provide a redundant information.

## 22.6 A Detailed List of Quality Measures for Seasonal Adjustment

No.	Diagnostic	Description	Regard to ESS Guideline Item	References	Availability in <sup>a</sup>
<b>Testing for seasonality</b>					
1	F-Test for stable seasonality	F-Test for equality of period-specific mean seasonal factors via	4.1, 4.2	Fisher (1921)	✓
		$F_S = \frac{\sum_{i=1}^k \frac{[(ST)_{i\cdot} - (ST)_{i\cdot}^2 / (k-1)]}{\sum_{i=1}^k \sum_{j=1}^k [(ST)_{ij} - (ST)_{i\cdot}^2 / (T-k)]}$		p. 207 ff.	✓
2	Kruskal-Wallis-Test for stable seasonality	Nonparametric test for equality of periodic-specific mean seasonal factors via	4.1, 4.2	Kruskal and Wallis (1952)	✓
		$H = \frac{12}{T(T+1)} \sum_{i=1}^k \frac{R_{i\cdot}^2}{n_{i\cdot}} - 3(T+1),$ where $R_{i\cdot} = \sum_{j=1}^T R_{ij}$ and $R_{ij}$ is the rank of $(ST)_{ij}$			✓
					p. 264

<sup>a</sup>Page information refers to the programs' manuals, i.e. the X-13ARIMA-SEATS Reference Manual, Version 1.0 (30 September 2011), and the Demetra+ User Manual, Version November 2012.

3	Regression-based F-Test for stable seasonality Canova-Hansen Test for deterministic seasonality F-Test for moving seasonality	F-Test for equality of fixed seasonal effects	4.1	Lyrtas et al. (2007)	✓ p.75 f.	
4	Hansen Test for deterministic seasonality F-Test for moving seasonality	LM-Test of the null hypothesis of no seasonal unit roots against the alternative of a unit root at either a single seasonal frequency or a set of seasonal frequencies	4.1, 4.2	Canova and Hansen (1995)		
5	F-Test for moving seasonality	F-Test for equality of yearly effects, i.e. $\alpha_1 = \alpha_2 = \dots = \alpha_T$ , in the two-way ANOVA model	4.1	Higginson, (1975)	✓ p.265	
6	Spectral test	Checks the presence of trading day and seasonal peaks based on the periodogram of the residuals	4.1, 4.2		✓ p.256	
7	F-Test for residual seasonality: F-Test for stable seasonality on irregular component and on seasonally adjusted series	F-Test for equality of period-specific mean seasonal factors via	4.1,4.2	Fisher (1921)	✓ p.207 ff	
<b>Testing for trading-day effects</b>						
8	Regression-based F-Test for trading-day effects	F-Test for equality global nullity of trading-day coefficients	1.3	Lyrtas et al. (2007)	✓ p.75 f.	
9	Regression-based F-Test for the good number of trading-day regressors	F-Test for equality of trading-day coefficients	1.3	Lyrtas et al. (2007)	✓ p.75 f.	

$$F_S = \frac{\sum_{i=1}^k [(ST)_{i, \cdot} - \overline{(ST)}]^2 / (k-1)}{\sum_{i=1}^k \sum_{j=1}^{n_i} [(ST)_{i,j} - \overline{(ST)}]^2 / (T-k)}$$

$$F = \frac{\hat{\chi}^2}{p} \times \frac{n-d-k}{n-d}$$

$$\hat{\chi}^2 = (R\hat{\beta} - c)' [RVAR(\hat{\beta}) R']^{-1} (R\hat{\beta} - c)$$

$$F = \frac{\hat{\chi}^2}{p} \times \frac{n-d-k}{n-d}$$

$$\hat{\chi}^2 = (R\hat{\beta} - c)' [RVAR(\hat{\beta}) R']^{-1} (R\hat{\beta} - c)$$



10	Test for residual trading-day effects: Regression-based F-Test for trading-day effects on irregular component and on seasonally adjusted series	$F = \frac{\hat{\chi}^2}{p} \times \frac{n-d-k}{n-d}$	F-Test for global nullity of trading-day coefficients	4.1.4.2	Lyrtas et al. (2007)	✓ p.75 f.		
		$\hat{\chi}^2 = (R\hat{\beta} - c)'[RR'AR(\hat{\beta})R']^{-1}(R\hat{\beta} - c)$						
<b>Testing for trading-day effects</b>								
11	Significance of user-defined effects	Standard-t-test for significance of single user defined regressors and F-test for significance of groups regressors	4.2	4.2		✓ p.137 ff.	✓ p.121	✓
12	Outlier detection	Automatic outlier detection for additive outliers, level shifts and temporary changes	4.2	4.2	Bell (1983), Chang and Tiao (1983), Oho and Bell (1990)	✓ p.37 ff.	✓	✓
13	Number outliers Concentration of outliers on particular month or particular year or on the end of the series	Calculates the share of outliers	4.2	4.2			✓ p.141	✓
14	Calculates the number of outliers per month, per year, and in the recent years							
15	Autocorrelation	Empirical ACF and partial ACF of regARIMA model residuals as well as Ljung-Box-Q-statistics	4.2	4.2	Ljung and Box (1978)	✓ p.74 ff	✓	✓ p.131
16	Normality test	Estimated skewness, kurtosis and Geary's $\alpha$ of regARIMA model residuals	4.2	4.2	Shedecor and Cochran (1980)	✓ p.76 f.	✓	✓ p.135

17	Joint normality test	Combined skewness and kurtosis tests of reg-ARIMA model residuals	4.2	Doornik and Hansen (2008)	✓	✓
18	Independence test	Ljung-Box and Box-Pierce test applied to reg-ARIMA model residuals	4.2	Box and Pierce (1970), Ljung and Box (1978)	✓	✓
19	Wald-Wolfowitz test	Wald-Wolfowitz test for randomness of the residuals from the reg-ARIMA model		Wallis and Moore (1941)	✓	✓
<b>M- and Q-statistics</b>						
20	M1	irregular component to the variance of the per cent change in the original series over a one-quarter span $M_1 = 10 \frac{\frac{I_3^2}{O_2^2} - \frac{I_2^2}{O_3^2}}{1 - \frac{I_3^2}{O_3^2}}$	4.3	Lothian and Morry (1978)	✓	✓
21	M2	irregular component to the variance of the stationary portion of the original series $M_2 = \frac{\text{Contribution}(I)}{\text{Contribution}(P)}$	4.3	Lothian and Morry (1978)	✓	✓
22	M3	irregular component to the variance of the stationary portion of the original series $M_3 = \frac{1}{2} \left( \frac{I}{C} - 1 \right)$	4.3	Lothian and Morry (1978)	✓	✓
23	M4	Amount of autocorrelation in the irregular component as described by the average duration of run (ADR) statistic $M_4 = \frac{\frac{T-1}{ADR} - \frac{2(T-1)}{3}}{2.57 \sqrt{\frac{16T-29}{90}}}$	4.3	Lothian and Morry (1978), Wallis and Moore (1941)	✓	✓
24	M5	Number of months it takes the average absolute change in the trend-cyclical component to dominate the average absolute change in the irregular component $M_5 = \frac{MCD' - 0.5}{5}$	4.3	Lothian and Morry (1978)	✓	✓
25	M6	Amount of year-to-year change in the irregular component as compared to the amount of year-to-year change in the seasonal component $M_6 = \frac{1}{2.5} \left  \frac{I}{S} - 4 \right $	4.3	Lothian and Morry (1978)	✓	✓
26	M7	Amount of stable seasonality present relative to the amount of moving seasonality $M_7 = \sqrt{\frac{1}{2} \left( \frac{T}{F_S} + \frac{3FM}{F_S} \right)}$	4.3	Higginson, (1975), Lothian and Morry (1978)	✓	✓

27	M8	Size of fluctuations in the seasonal component throughout the whole series $\Delta S$	4.3	Lothian and Morry (1978)	✓ p.210 f.	✓ p.135 f.	✓
		$M_8 = 10\Delta S$					
28	M9	Average linear movement in the seasonal component throughout the whole series	4.3	Lothian and Morry (1978)	✓ p.210 f.	✓ p.135 f.	✓
		$M_9 = \frac{\sum_{j=1}^k (a_{t_j} - 1)}{10} \sum_{j=1}^k  S_{t_j, j} - S_{1, j} $					
29	M10	Size of fluctuations in the seasonal component in the recent years	4.3	Lothian and Morry (1978)	✓ p.210 f.	✓ p.135 f.	✓
30	M11	Average linear movement in the seasonal component in the recent years	4.3	Lothian and Morry (1978)	✓ p.210 f.	✓ p.15 f.	✓
31	Q	Weighted sum of the eleven $M$ -statistics	4.3	Lothian and Morry (1978)	✓ p.210 f.	✓ p.15 f.	✓
32	Quality of the decomposition	Decomposition of the variance of SI component in irregular, stable seasonality and moving seasonality.	4.3				
<b>Roughness measures</b>							
33	R1	Sum of squares of differenced seasonally adjusted series	4.3	Dagum (1979)			
		$R_1 = \frac{1}{T-1} \sum_{t=2}^T (X_t - X_{t-1})^2$					
34	R2	Sum of squares of differences between seasonally adjusted series and a smoothed trend	4.3	Dagum (1979)			
		$R_2 = \frac{1}{T} \sum_{t=1}^T (X_t - H_{1.3}(X_t))^2$					
35	R3	Sum of squares of differences between seasonally adjusted series and trend	4.3	Pfefferman et al. (1984)			
		$R_3 = \frac{1}{T} \sum_{t=1}^T (SA_t - TC_t)^2$					
36	MAR(S)	Smoothness of seasonal factors	4.3	Ladray and Mazzi (2003)			
		$MAR(S) = \sum_{t=1}^T [(1 + B + B^2 + \dots + B^{11})X_t]^2$					
37	MAR1(TC)	Sum of squares of differenced trend-cyclical component	4.3	Ladray and Mazzi (2003)			
		$MAR1(TC) = \sum_{t=2}^T (X_t - X_{t-1})^2$					
38	MAR2(TC)	Sum of squares of twice differenced trend-cyclical component	4.3	Ladray and Mazzi (2003)			
		$MAR2(TC) = \sum_{t=2}^T (\Delta X_t - \Delta X_{t-1})^2$					
<b>Spectral analysis</b>							
39	Spectral density	Peaks at seasonal frequencies in the periodogram	4.3	Priestley (1981)	✓ p.49 ff	✓ p.254	✓
<b>Revision analysis</b>							
40	Mean revision	Arithmetic mean of differences between later and preliminary estimates	4.3	Mckenzie and Gamba (2008)	✓ p.53 ff	✓ p.150	
		$\bar{R} = \frac{1}{T} \sum_{t=1}^T (L_t - P_t) = \frac{1}{T} \sum_{t=1}^T R_t$					

41	Mean squared revision	$M SR = \frac{1}{T} \sum_{t=1}^T (L_t - P_t)^2 = \frac{1}{T} \sum_{t=1}^T R_t^2$	4.3		✓ p.150
42	Minimum revision	$MinR = \min_{1 \leq t \leq T} (L_t - P_t) = \min_{1 \leq t \leq T} (R_t)$	4.3		
43	Maximum revision	$MaxR = \max_{1 \leq t \leq T} (L_t - P_t) = \max_{1 \leq t \leq T} (R_t)$	4.3	McKenzie and Gamba (2008)	✓ p.53 ff
44	Mean absolute revision	Arithmetic mean of absolute differences between later and preliminary estimates $M AR = \frac{1}{T} \sum_{t=1}^T  L_t - P_t  = \frac{1}{T} \sum_{t=1}^T  R_t $	4.3	McKenzie and Gamba (2008)	✓ p.53 ff
45	Standard deviation of revision	Root of mean of squared differences between revision and mean revision $SDR = \sqrt{\frac{1}{(T-1)} \sum_{t=1}^T \sum_{t=1}^T (R_t - \bar{R})^2}$	4.3	McKenzie and Gamba (2008)	✓ p.53 ff
46	Root mean square revision	Root of mean of squared differences between later and preliminary estimates $RMSR = \sqrt{\frac{1}{T} \sum_{t=1}^T \sum_{t=1}^T R_t^2}$	4.3	McKenzie and Gamba (2008)	✓ p.53 ff
47	Direction of revision	Percentage of preliminary and later estimates with the same sign $\%S = \frac{1}{T} \sum_{t=1}^T \text{sign}(P_t) = \text{sign}(L_t)$	4.3	McKenzie and Gamba (2008)	✓ p.53 ff
48	Relative mean absolute revision	Ratio between mean absolute revision and mean absolute preliminary estimate $RMAR = \frac{\sum_{t=1}^T  R_t  / \sum_{t=1}^T  P_t }{\text{unstable}}$	4.3	McKenzie and Gamba (2008)	✓ p.259
49	X(%)	Percentage of periods in which the series under review is unstable $\frac{\max_{k \in N_t} X_t(k) - \min_{k \in N_t} X_t(k)}{\min_{k \in N_t} X_t(k)} > 0.03$	4.3	Findley et al. (1990)	✓ p.259
50	MM(%)	Percentage of periods in which the relative month-to-month change is unstable $\max_{k \in N_t} MM_t(k) - \min_{k \in N_t} MM_t(k) > 0.03$	4.3	Findley et al. (1990)	✓ p.259
51	QQ(%)	Percentage of periods in which the relative quarter-to-quarter change is unstable $\max_{k \in N_t} QQ_t(k) - \min_{k \in N_t} QQ_t(k) > 0.03$	4.3	Findley et al. (1990)	✓ p.259
52	YY(%)	Percentage of periods in which the relative year-to-year change is unstable $\max_{k \in N1(t)} YY_t(k) - \min_{k \in N1(t)} YY_t(k) > 0.03$	4.3	Findley et al. (1990)	✓ p.52 f.

		<b>Miscellaneous(signal extraction)</b>			
53	Autoregressive roots	Modulus and period			✓
54	Over and under estimation test	Frequency domain test for over- and underestimation of the signal in an ARIMA based signal-plus-noise model	Maravall (2003), Findley et al. (2004), McElroy (2008)	✓ p.161 ff.	✓ 175
55	Revision variance	Finite sample versions of the revision variance $E[(\hat{X}_{t n+h} - \hat{X}_{t n})^2]$ for $t \leq n$ and $h > 0$ in the ARIMA model based framework Number of significant seasonal factors in one year (central year for historical estimates, last year for preliminary estimates and one-year forecasts)	McElroy and Gagnon (2008)	✓ p.161 ff.	
56	Statistical significance of seasonality	Comparison of variance of the component, theoretical estimator and empirical estimate		Maravall (1995)	✓ p.175
57	Component variance				✓
		<b>Other simple plausibility checks</b>			
58	Annual totals	Comparison between annual totals of unadjusted or calendar adjusted and seasonally adjusted series by checking if (for each year) $\frac{\sum_{t=1}^{12} CA_t}{\sum_{t=1}^{12} SA_t} \approx \frac{\sum_{t=1}^{12} US_t}{\sum_{t=1}^{12} SA_t} \approx 1$		4.1	✓ p.139
59	Annual average	Check if seasonal factors satisfy (for each year) $\frac{1}{12} \sum_{t=1}^{12} SF_t \approx 1$		4.1	✓
59	Test for break in seasonal pattern	Check if month-t-specific average seasonal factor growth rate satisfies (for each month) $\sqrt{\prod_{k=1}^K \left  \frac{SF_{t-12(k-1)}}{SF_{t-12k}} - 1 \right } \approx 0$		4.1	

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# 23

## Short versus long time series: An empirical analysis



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## Table of Contents

23.1	Introduction	631
23.2	Presentation of the study	633
23.2.1	Theoretical framework	633
23.2.2	Empirical design of the study	634
23.2.3	Tools and methods	634
23.3	Quality measures and assessment criteria	635
23.4	Analysis	637
23.4.1	Results for the EA19	637
23.4.2	Results for all the series	638
23.5	Conclusions	653

## 23.1 Introduction

Seasonal adjustment procedures are usually designed for being applied on sufficiently long time series in order to obtain good quality results. This is due both to technical reasons such as the properties of the symmetric filters used and to non-technical ones such as the fact that, over a sufficiently long time period, components can be better identified and separated; consequently, the seasonal component can be more precisely estimated and eventually removed. In addition, long time series are required in order to read properly the statistics of the seasonality tests. Furthermore, on short time series the seasonality tests might be less robust.

In official statistics, available time series associated to statistical indicators are often relatively short or subject to some shortening processes. This seems to contradict one of the main quality dimensions of statistics which is the coverage, but, at the same time, official statistics need to be continuously improved to better reflect the socio-economic structure. This process unavoidably leads to regularly adapting existing official statistics to evolving socio-economic structure. Furthermore better reflecting the current socio-economic situation can also require the development and the statistical compilation of new indicators, which at least in a first phase will cover only a limited time span. These two processes imply, at least temporarily, the availability of short time series associated to the statistical indicators.

Based on the elements presented above it is possible to identify the following causes for the availability of short time series or for shortening already existing ones:

- Production of new indicators - when developing a new indicator, at least in a first phase, it is very difficult going far back in time, thus series are initially quite short;
- Changes in definitions and in the general accounting and legal framework - in this case an entire statistical domain is impacted by significant changes affecting its general structure and the definition of components indicators with obvious consequences in terms of length of series, even if a back calculation plan has been foreseen (for example the switch from ESA95 to ESA2010);
- Changing in compilation methods - this can involve the change of the statistical methods used for compiling indicators of the way in which indicators are evaluated (examples are the change from mixed base indices to previous year based one);
- Changes in the nomenclatures and consequently in the classifications adopted - this change affect (even if not necessary at the same point in time) almost all statistical domain and determines the availability of shorter time series at least for a certain period of time during which a back calculation strategy has to be defined and implemented (for example the change between the NACE 1 to the NACE 2);
- Major revisions related to multi-annual benchmarking processes - at regular intervals (such as every 5 years) benchmarking exercises can take place in various statistical domains implying change of the base year for fixed indices and other adjustment based on the availability of new and more detailed information; such process can imply some shortening of the series especially in relations to the change of the base year if any;
- Increased frequency - the development of higher frequency indicators in some cases can also determine the temporary availability of short time series (for example the implementation of quarterly accounts can, at least at the beginning, imply that quarterly data are shorter than annual ones).

In all cases mentioned above the correct identification of the unobserved components (and in particular of the seasonal one) as well as of the decomposition model can be problematic. Consequently the quality of the resulting seasonally adjusted figures might not meet the requirements of official statistics as foreseen for instance by the ESS guidelines on Seasonal Adjustment (see Eurostat , 2015, and chapter 30). This situation is determined by the fact that, when series are short, they tend to be more volatile so that often the irregular

component of the noise is dominating the signals by complicating the correct identification and estimation of components.

Seasonality tests, normally used to check the presence of seasonality before performing any seasonal adjustment, can even fail in detecting the presence of seasonality even when it is logic to assume that it has to be present in a given series. Furthermore, mainly for seasonal adjustment methods based on symmetric filters, short time series require a larger use of asymmetric filters so that the instability of seasonally adjusted data is higher. On the other hand, mainly for model based methods, when the series are short the ARIMA model can be particularly unstable by further compromising the quality of the seasonal adjusted data.

Finally when series are too short the majority of methods do not perform any seasonal adjustment. According to the ESS guidelines on Seasonal Adjustment, series with a length or 3 years or less cannot be seasonally adjusted. Such series can be labelled as too short.

By generalizing what is presented in the ESS guidelines on Seasonal Adjustment (chapter 30) and also in Mazzi and Savio (2005), one can assume the following:

- Series of 3 years or less are too short for being seasonally adjusted;
- Series between 4 and 7 years are considered short so that the results of the seasonal adjustment need to be carefully analysed;
- Series between 8 and 14 years of length are considered of average length ensuring a good degree of reliability of seasonally adjusted data;
- Series between 15 and 20 years of length are considered long so that the overall quality of seasonally adjusted data could start to be affected if the phenomenon they are measuring is evolving quite fast;
- Series having a length of 21 years or more are considered too long and, following the guidelines, it is recommended to split them in order to obtain higher quality seasonally adjusted data.

Obviously, problems related to the seasonal adjustment of long or very long time series are out of the scope of this chapter and here the attention is mainly focused on short length time series in order to compare the impact of various lengths on the overall quality of the seasonal adjustment.

Furthermore this chapter also evaluates the relative performance of alternative seasonal adjustment methods when dealing with short series. Readers interested on various aspects related to the seasonal adjustment of long or very long time series can refer to the ESS guidelines on Seasonal Adjustment.

The chapter is structured as follow: section 23.2 describes the framework of the study, in particular subsection 23.2.1 presents the theoretical framework while section 23.2.2 details the empirical one and subsection 23.2.3 is devoted to the description of the tools used in this study; section 23.3 presents the criteria and the associated measures used in the comparative study; section 23.4 provides a detailed presentation of the results of the comparative study and section 23.5 concludes.

## 23.2 Presentation of the study

This section describes in details the plan of the study, starting from the theoretical base, which provides the logical justification of the work, to the empirical part, which shows how the theoretical framework has been applied empirically. Finally, also the tool used for the empirical analysis is presented here.

### 23.2.1 Theoretical framework

Let's consider a short time series  $y_t$ , longer anyway more than 3 years so that seasonally adjusted data can be derived according one of the following standard decomposition models:

$$y_t = TC_t \cdot S_t \cdot I_t - \text{multiplicative model}$$

or

$$y_t = TC_t + S_t + I_t - \text{additive model}$$

Obviously the study could be easily extended to other more complex decomposition schemes such as the log-additive or the mixed ones. Nevertheless, since the multiplicative and additive decomposition models are largely the most used, this study limits the comparison to them without affecting the general validity of the results.

Since the series  $y_t$  is short, as already mentioned in section 1, seasonally adjusted figures could be not very reliable and subject to large revision. The stabilization of the seasonally adjusted result could be achieved only when the series will become longer. The process of transforming a short time series into a longer one can be quite long if one has to wait for the availability of new observations at the end of the series. On the other hand the adoption of a back calculations strategy can transform a short time series into a longer one from one month to the other.

In the first case, since the length of the series increases very slowly, the effects on the seasonally adjusted data will be distributed over a quite long time period so that they will be not very noticeable by the users. By contrast if a back calculation is performed the effect of the seasonally adjusted data can be very relevant determining big differences in the data from a month to another. For this reason in this chapter the attention is put on the comparison between series measuring the same phenomenon having different length, assuming that such different length is the effect of successive back calculation exercises.

The procedure used here is the following:

- First a long time series is considered:  $x_t$  ( $t=1,2,\dots,T$ ) with a length of at least 15 years;
- On the series  $x_t$  three points are identified:  $k_1, k_2$  and  $k_3$ , so that  $k_1 > k_2 > k_3$ ;
- In the original series  $x_t$  the observations respectively before  $k_1, k_2$  and  $k_3$  are taken, in order to produce three new time series respectively defined as:

$$x_{1,t} = (x_{1,k+1}, x_{1,k+2}, \dots, x_{1,T});$$

$$x_{2,t} = (x_{2,k+2}, \dots, x_{2,k+1}, \dots, x_{2,T})$$

and

$$x_{3,t} = (x_{3,k_3}, \dots, x_{3,k_2}, \dots, x_{3,k_1}, \dots, x_{3,T})$$

- The choice of  $k_i$  ( $i = 1, 2, 3$ ) should be in a way that  $x_{1,t}$  should be not longer than 6 years,  $x_{2,t}$  not longer than 10 years and  $x_{3,t}$  not longer than 15 years.



In this way three series were obtained, each of them being included in a different category of length as presented in section 1. All three series have the last part starting in  $k_1$  and ending in  $T$  in common. On this common part the comparative evaluation is conducted by using as seasonal adjustment software X-13 ARIMA-SEATS and TRAMO-SEATS as described in chapters 10, 12 and 13. The comparative analysis is conducted by using a number of quality measures as suggested in Ladiray and Mazzi (2003), Mazzi and Savio (2005) and in chapter 22.

### 23.2.2 Empirical design of the study

The aim of this study is to assess the performance of the seasonal adjustment when shortening the times series using both methods TRAMO-SEATS and X13-ARIMA-SEATS. The empirical comparison in this chapter has been carried out by using JDemetra+, the tool officially recommended by Eurostat and the ECB for seasonal adjustment, since February 2015.

The empirical study is performed on the monthly Industrial Production Index of the 28 European Union Member States, plus the euro area (EA19) and the European Union (EU28) aggregates, and the performance of the two methods is compared on three different time spans:

- Long: 15 years (180 observations);
- Medium: 10 years (120 more recent observations);
- Short: 5 years (60 more recent observations).

Another possible simulation would be to generate the short and medium series by removing the more recent data and keeping the older ones, or by keeping the observations in the centre of the series.

As mentioned in Section 1, very long series could introduce moving seasonality and therefore harm the quality of the results obtained. Therefore this study is limited to a maximum of 15 years.

Seasonal adjustment was conducted on the 30 time series for all the 3 selected time spans with both TRAMO-SEATS and X-13 ARIMA-SEATS. Firstly the EA19 series is analysed alone as an example, and then the results are generalised by considering the whole set of series.

A wide range of homogenous quality indicators for the seasonal adjustment will be described in Section 3 and presented in this empirical framework.

### 23.2.3 Tools and methods

JDemetra+ is a free open source tool for seasonal adjustment, time series analysis and much more, written in Java and developed by the Bank of Belgium with the support of Eurostat. It contains, among others, the two main seasonal adjustment methods: X-13 ARIMA-SEATS and TRAMO-SEATS, which have been reengineered, following an object-oriented approach, which allows easier handling, extensions or modifications.

Furthermore almost all criteria, tests and measures described in section 3 which will be used in the comparative analysis are also fully implemented into JDemetra+ (version 2.1 or higher). This ensures the maximum of consistency and comparability of the results, since all elaborations have been done within a unique, tested and robust IT environment. In fact JDemetra+ contains:

- A full implementation of the X-13 ARIMA-SEATS (developed by the US Census Bureau): which includes both the original moving average filter based approach (X-11) and the model based decomposition as in seats;
- A full implementation of TRAMO-SEATS (developed by Augustín Maravall for the Bank of Spain): a model based approach using the canonical decomposition;

- A complete battery of criteria, tests and measures for evaluating the goodness of the seasonal adjustment and also allowing for a comparative analysis across methods.

## 23.3 Quality measures and assessment criteria

In order to compare the different results of the seasonal adjustment performed with different techniques and different time spans, a large number of indicators is available. The choice of the criteria used is based on their sound methodology and on the possibility to apply them for the results of both X-13 ARIMA-SEATS and TRAMO-SEATS (see Ladiray and Mazzi, 2003).

The criteria here presented are considered on the seasonally adjusted series  $SA_t$ , and the comparisons are done between “long” series  $SA_t^L$ , “medium” series  $SA_t^M$  and “short” series  $SA_t^S$ , as defined in Section 2.2, but they could be easily extended to other comparisons. Formulas in this section are presented considering a multiplicative decomposition model, but could be easily adapted to other decomposition models.

The criteria and measures chosen are the following:

### 1. Distance measures

It is clear that the closer the series, more similar are the results of the seasonal adjustment. In this context, few measures of the distance between the two seasonally adjusted series are taken into account.

The distances between two results could be analysed, and several indicators could be computed for such purpose.

The series of the absolute percentage difference can be obtained:

$$APD_t = |1 - (SA_t^L / SA_t^S)|$$

Starting from this series, different descriptive statistics could be computed, like the mean, the standard deviation, the maximum and the coefficient of variation.

In order to check whether the different results are giving broadly the same message, one can focus on the growth rates of the seasonally adjusted series, which are also of great interest for the users. For this reason the concordance of growth rates can be computed, which measures, in percentage, their sign concordance:

$$C1 = \frac{\sum_{t=2}^T |SIGN(SA_t^L / SA_{t-1}^L - 1) + SIGN(SA_t^S / SA_{t-1}^S - 1)| / 2}{T - 1}$$

The absolute percentage difference of the growth rates could also be computed:

$$APD\_GR_t = \left| \left( \frac{SA_t^L - SA_{t-1}^L}{SA_{t-1}^L} \right) - \left( \frac{SA_t^S - SA_{t-1}^S}{SA_{t-1}^S} \right) \right|$$

### 2. Roughness of the components

The roughness is intended to measure the degree of smoothness or of the lack of smoothness of some unobserved components such as the seasonally adjusted, the trend-cycle and of the seasonal one.

Several indices of the roughness of all the estimated components of a time series can be computed. Such indices show how smooth the components are, according to different criteria.

A large number of indicators are available for measuring the roughness of different components.



The roughness of the seasonally adjusted series could be computed by the following index proposed by Dagum (1979), which is the  $L_2$ -norm of the differenced series:

$$R1 = \sum_{t=2}^T (SA_t - SA_{t-1})^2 = \sum_{t=2}^T (\nabla SA_t)^2$$

Pfefferman et al. (1984) proposed instead a measure of similarity between the seasonally adjusted data and the trend:

$$R3 = \sum_{t=1}^T (SA_t - TC_t)^2$$

However it is not easy to justify this indicator, as there is no real reason why the irregular component, which is inside the seasonally adjusted series, should be as smooth as the seasonally adjusted series.

Therefore one can focus on other components, like the trend-cycle and the seasonality. Gómez and Maravall (1999) propose two different indicators measuring the roughness of the trend-cycle. The first one is the  $L_2$ -norm of the first differences:

$$MAR1(TC) = \sum_{t=2}^T (\nabla TC_t)^2$$

And the second one is the  $L_2$ -norm of the second differences:

$$MAR2(TC) = \sum_{t=3}^T (\nabla^2 TC_t)^2$$

And for measuring the roughness of the seasonal component the same authors propose the following criterion:

$$MAR(S) = \sum_{t=1}^T ((1 + B + \dots + B^{11})S_t)^2$$

All these statistics are calculated not only on the entire time span of the series, but also on the last three years.

### 3. Residual seasonality

A seasonal adjustment procedure which leaves detectable residual seasonality, as well as over-adjustment, in the adjusted series is normally considered unsatisfactory. In order to check the presence of residual seasonality in the series, among all the possible alternatives, the Ljung-Box test was used. Such test is preferable as it does not only test for residual seasonality but also for over-adjustment.

**Table 23.1:** Results for the EA19

EA19 IND PROD	TS Long	TS Medium	TS Short	X13 Long	X13 Medium	X13 Short
ARIMA model	(0,1,3)(0,1,1)	(2,1,0)(0,1,1)	(3,1,0)(0,1,1)	(0,1,1)(0,1,1)	(1,1,1)(0,1,1)	(2,1,2)(0,1,1)
AO	0	1	0	0	1	0
TC	0	0	0	1	0	0
LS	1	0	0	2	0	0
Residual seasonality	NO	NO	NO	NO	NO	NO
p-Norm	0.2776	0.1232	0.1256	0.5619	0.1565	0.3511
p-Ljung-Box	0.3409	0.0108	0.6092	0.1333	0.0154	0.5378

**Table 23.2:** Distance measures for the EA19

EA19 IND PROD	TS L vs. M	TS L vs. S	TS M vs. S	X13 L vs. M	X13 L vs. S	X13 M vs. S
max APD	0.900%	1.130%	1.015%	2.283%	2.112%	1.804%
mean APD	0.141%	0.183%	0.181%	0.307%	0.295%	0.305%
CV APD	1.043	1.104	1.008	1.142	1.288	1.200
C1	93%	83%	85%	90%	80%	81%

## 23.4 Analysis

Before analysing in detail the performances of all the series, the presence of identifiable seasonality was tested by applying both the Kruskal-Wallis and the Friedman tests, and all long, medium and short series showed presence of seasonality.

Boxplots are presented in this section. In order to better visualise the differences between the different length of the series and the two methods, very extreme values have been removed (otherwise the boxes would have been too flat).

As mentioned, the analysis is carried out using both X-13 ARIMA-SEATS (X13) and TRAMO-SEATS (TS).

### 23.4.1 Results for the EA19

In this section, the results obtained for the EA19 series are presented.

The ARIMA models identified are shown in table 23.1, together with the number of different outliers detected and the analysis of the residuals. All the series presented no issues in terms of annual totals and residual seasonality. Looking at the residuals, it is possible to see that they are normally distributed, but the Ljung-Box tests give p-values very close to the critical ones for the medium-length series.

The number of identified outliers is here presented for completeness, as it is normal that short series will have in general less outliers than long ones. However some outliers could impact the results of other statistics, especially R3, as it will be seen.

The statistics on the absolute percentage differences and the concordance of growth rates are shown in table 23.2. For the EA19 the TS results are more stable than the X13, although the differences are very low in both cases for all the confrontations. The coefficients of variation are all very close each other for both TS and X13, meaning that there is no much difference between the long, medium and short series (L, M and S, respectively).

The indicators of the roughness of the components are presented in table 23.3. It is visible that R1 and R3 are always higher for X13 than with TS. For the long series, when considering the full length, Mar1(TC), Mar2(TC) and R3 are much higher for X13, but this is due to the fact that X13 has detected a transitory change outlier (see table 23.1), which is not present for TS. When looking at the seasonality and the trend-cycle, X13 gives always smoother results than TS when no transitory change is in place.

**Table 23.3:** Roughness indicators for the EA19

EA19 IND PROD	TS	X13	TS	X13	TS	X13
	Long	Long	Med.	Med.	Short	Short
R1	0.941	1.394	1.037	1.603	0.404	0.499
R1 (last 3 y.)	0.326	0.567	0.241	0.561	0.440	0.560
R3	0.084	2.858	0.168	0.432	0.089	0.183
R3 (last 3 y.)	0.061	0.189	0.042	0.178	0.103	0.202
Mar(S)	0.014	0.003	0.193	0.026	0.007	0.003
Mar(S) (last 3 y.)	0.512	2.784	0.564	0.572	0.052	0.026
Mar1(TC)	0.038	0.024	0.041	0.033	0.040	0.019
Mar1(TC) (last 3 y.)	0.038	0.024	0.041	0.033	0.040	0.019
Mar2(TC)	0.382	5.241	0.119	0.060	0.032	0.002
Mar2(TC) (last 3 y.)	0.044	0.004	0.045	0.011	0.031	0.002

### 23.4.2 Results for all the series

When looking at the models obtained on the full set of series (table 23.4), it seems that the ones identified for TS are more volatile, when moving from long to medium and to short series, than the ones identified with X13. As for TS only one series has issues with annual totals, with all the times spans, while X13 shows no issues, which means that the length of the series does not bring problems to the annual totals. The tests for the normality and the independence of the residuals have better results with short series.

The computed distance measures statistics are shown in tables 23.5 and 23.6 and charts 23.1 and 23.2. The results are clearly better for X13 than for TS. When moving from long to medium series the performances are always better than when moving from medium to short series. This is because the number of observations are not enough for short series, while the number of observations is not an issue for medium-length series. Altogether these statistics show that both methods perform relatively well when reducing the time span.

When moving from long to medium time spans, for X13, 27 series have a concordance of growth rates higher than 90%, against 22 for TS; when moving from medium to short series X13 has 23 series with C1 higher than 85%, against 19 for TS. Even when moving from long to short, the series with C1 higher than 85% are 24 for X13 against 19 for TS.

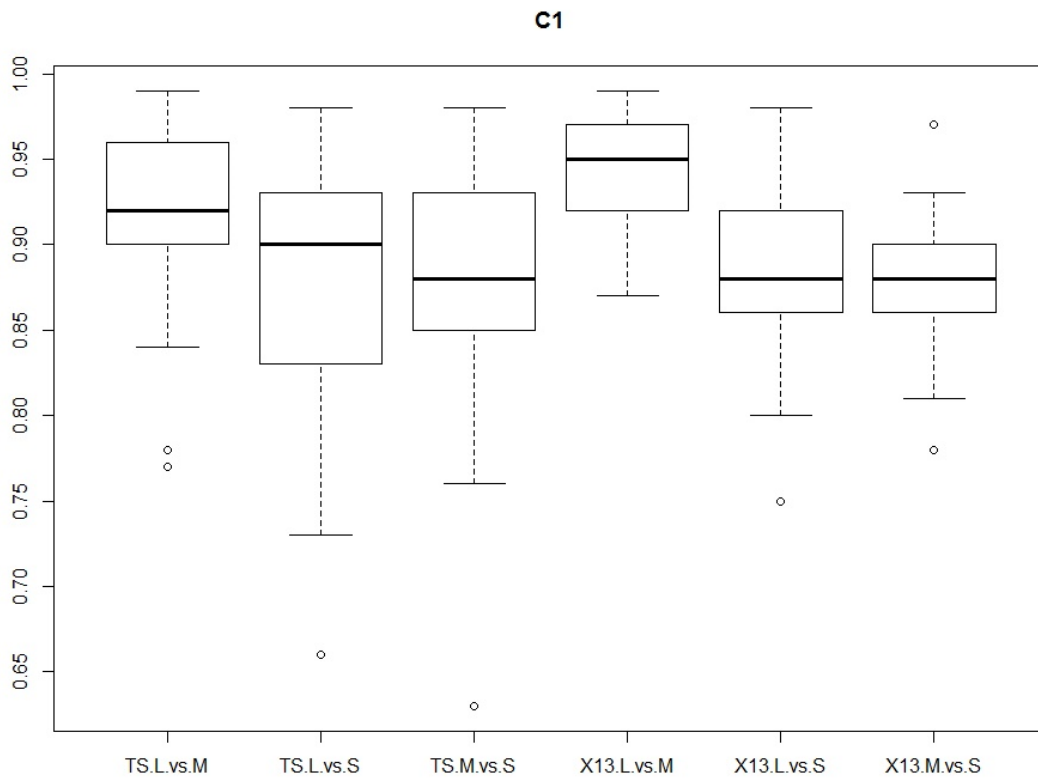
Similar but less evident results come from the mean APD statistics. Comparing long and medium series, for X13 28 series have a mean APD less than 0.5%, against 26 series for TS. While comparing medium and short series, for X13 19 series have a mean APD less than 0.5%, against 18 series for TS. Moving from long to short series gives very similar results for the two methods.

**Table 23.4:** Results for the full set of series

Statistics	TS Long	X13 Long	TS Med.	X13 Med.	TS Short	X13 Short
Series with non-zero coefficients in the non-seasonal part of the AR model	8	8	12	14	3	16
Series with non-zero coefficients in the non-seasonal part of the MA model	26	25	18	20	28	21
Series with non-zero coefficients in the seasonal part of the AR model	2	3	1	0	0	4
Series with non-zero coefficients in the seasonal part of the MA model	28	29	28	29	27	21
Residual seasonality	0	0	0	0	0	0
Residual seasonality (last 3 years)	0	1	0	1	0	0
p-norm < 5%	3	7	4	5	2	1
p-Jung-Box < 5%	3	5	8	3	2	2
Number of series with at least one AO	11	6	13	7	9	6
Number of series with at least one TC	4	8	3	3	4	3
Number of series with at least one LS	21	20	15	12	2	1

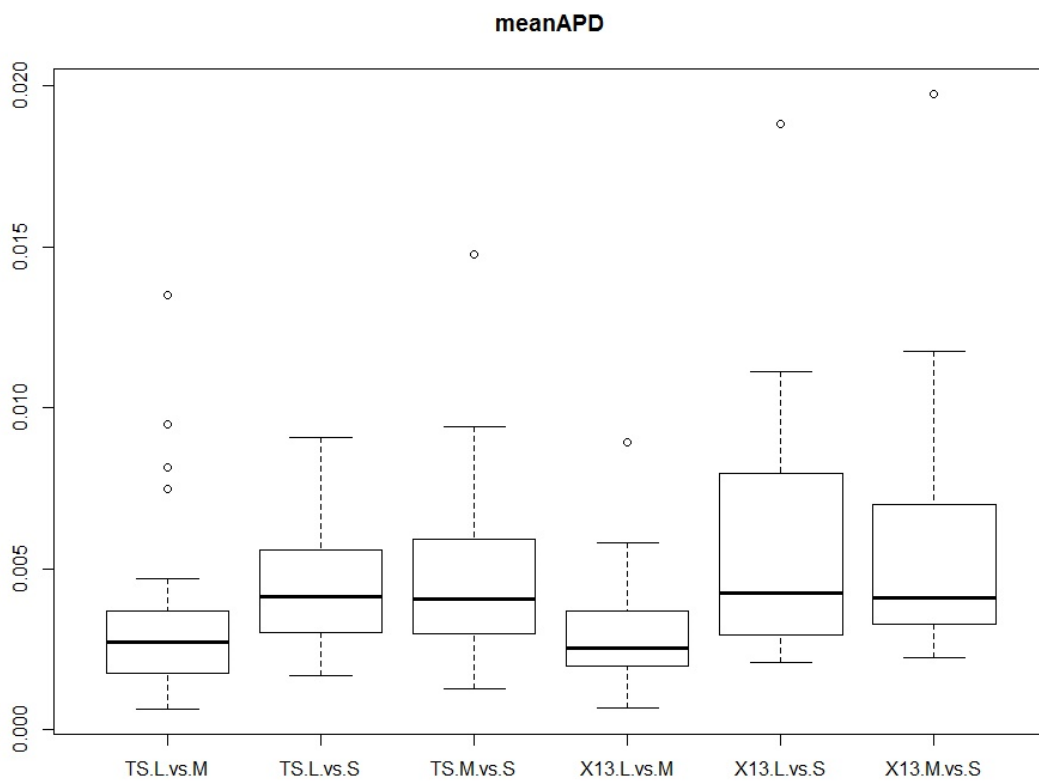
**Table 23.5:** Concordance of the growth rates for the full set of series

C1	TS	TS	TS	X13	X13	X13
	L vs. M	L vs. S	M vs. S	L vs. M	L vs. S	M vs. S
C1<80%	2	4	3	0	3	1
80%<C1<85%	1	7	8	0	3	6
85%<C1<90%	5	7	8	3	16	17
90%<C1<95%	14	11	9	17	7	5
C1>95%	8	1	2	10	1	1

**Figure 23.1:** Concordance of the growth rates for the full set of series

**Table 23.6:** Mean APD for the full set of series

mean APD	TS	TS	TS	X13	X13	X13
	L vs. M	L vs. S	M vs. S	L vs. M	L vs. S	M vs. S
APD<0.1%	2	0	0	1	0	0
0.1%<APD<0.3%	15	7	9	17	8	5
0.3%<APD<0.5%	9	13	9	10	11	14
0.5%<APD<1%	3	9	10	2	8	9
APD>1%	1	1	2	0	3	2

**Figure 23.2:** Mean APD for the full set of series

The indicators on the roughness of the components are calculated for the full time span and only for the last three years. R1 is shown in tables 23.7 and 23.8 and charts 23.3 and 23.4, R3 in tables 23.9 and 23.10 and charts 23.5 and 23.6, Mar1(TC) in tables 23.11 and 23.12 and charts 23.7 and 23.8, Mar2(TC) in tables 23.13 and 23.14 and charts 23.9 and 23.10, and finally Mar(S) is shown in tables 23.15 and 23.16 and charts 23.11 and 23.12.

As it happened for the EA19 series, the R1 tables show that TS obtains seasonally adjusted series which are smoother than X13 series for all long, medium and short series. This is true also when considering only the last 3 years of the series.

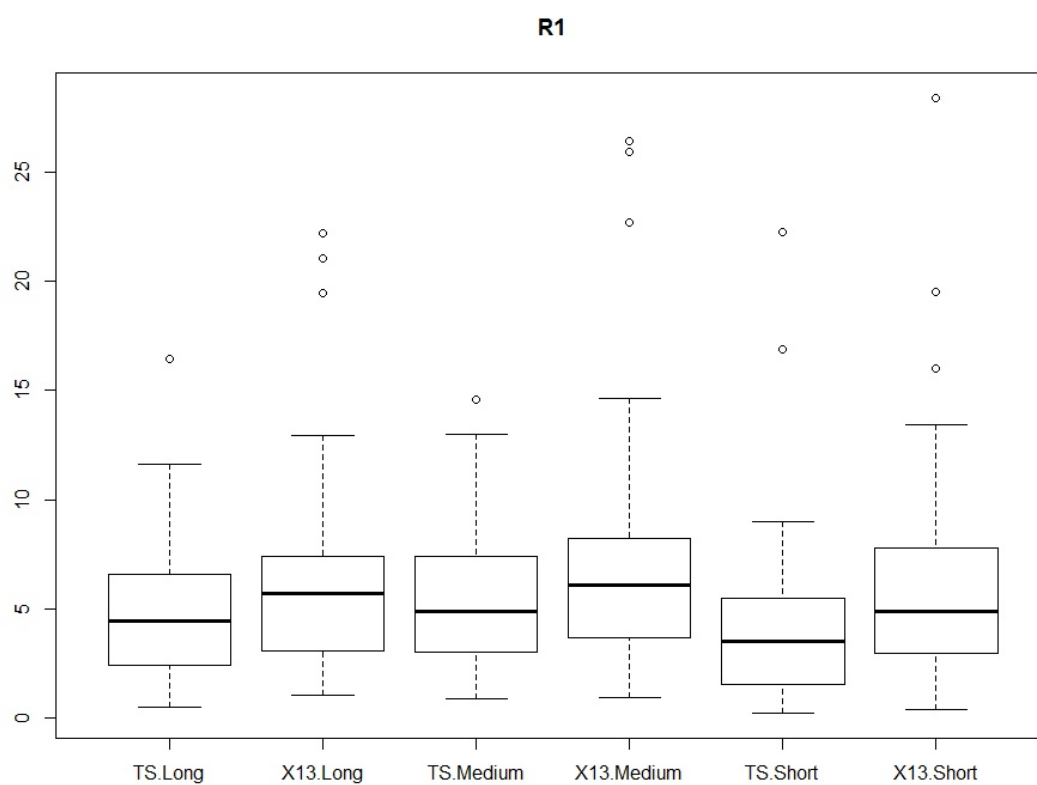
An interesting result is that short series have in general slightly smoother seasonally adjusted results than long and medium series. This is the same for R3 statistics, which means that short series identify ARIMA models with less parameters and seasonally adjusted series which are closer to the (smooth) trend series. R3 statistics are pretty much influenced by the presence of transitory changes in the series, which in case of long series is much higher for X13 than for TS.

Mar1(TC) and Mar2(TC) clearly show that short series produce much smoother trends than long and medium series. In general the trend is smoother for X13 than for TS: looking at Mar2(TC), values lower than 0.1, X13 has 8 long series against 4 for TS, while for medium X13 has 15 series against only 5 for TS, and finally for short series X13 28 against 25 for TS.

Finally, looking at the Mar(S) statistics, the seasonal component is in general much smoother for longer series, deteriorating when going to short series. In terms of comparison between the two methods, X13 obtains smoother results than TS. For long series in 28 cases X13 shows a value smaller than 0.05, against 24 for TS. For medium series, the cases where TS shows values smaller than 0.1 are 17, while there are 19 cases for X13. Finally, the number of short X13 series with values lower than 0.1 are 24, and only 17 for TS.

**Table 23.7:** R1 for the full set of series - Full time span

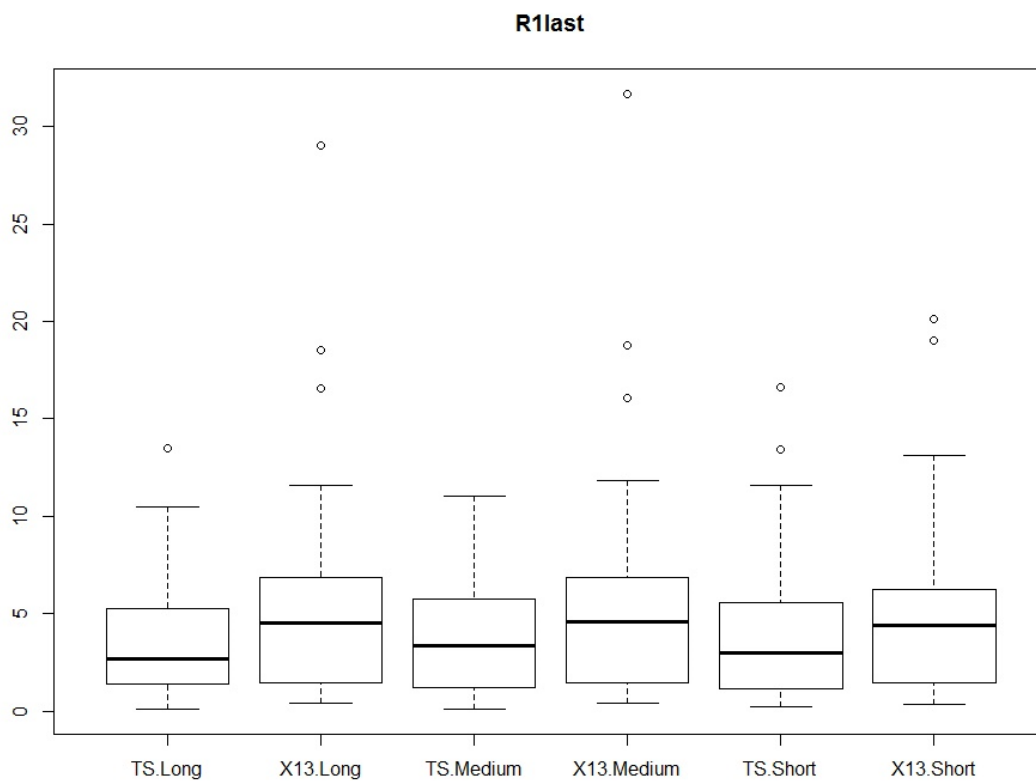
R1	TS Long	X13 Long	TS Med.	X13 Med.	TS Short	X13 Short
$R1 < 2.5$	8	4	5	4	10	7
$2.5 < R1 < 5$	8	8	11	5	10	8
$5 < R1 < 7$	7	7	4	8	2	4
$7 < R1 < 10$	3	5	5	7	5	6
$R1 > 10$	4	6	5	6	3	5

**Figure 23.3:** R1 for the full set of series - Full time span



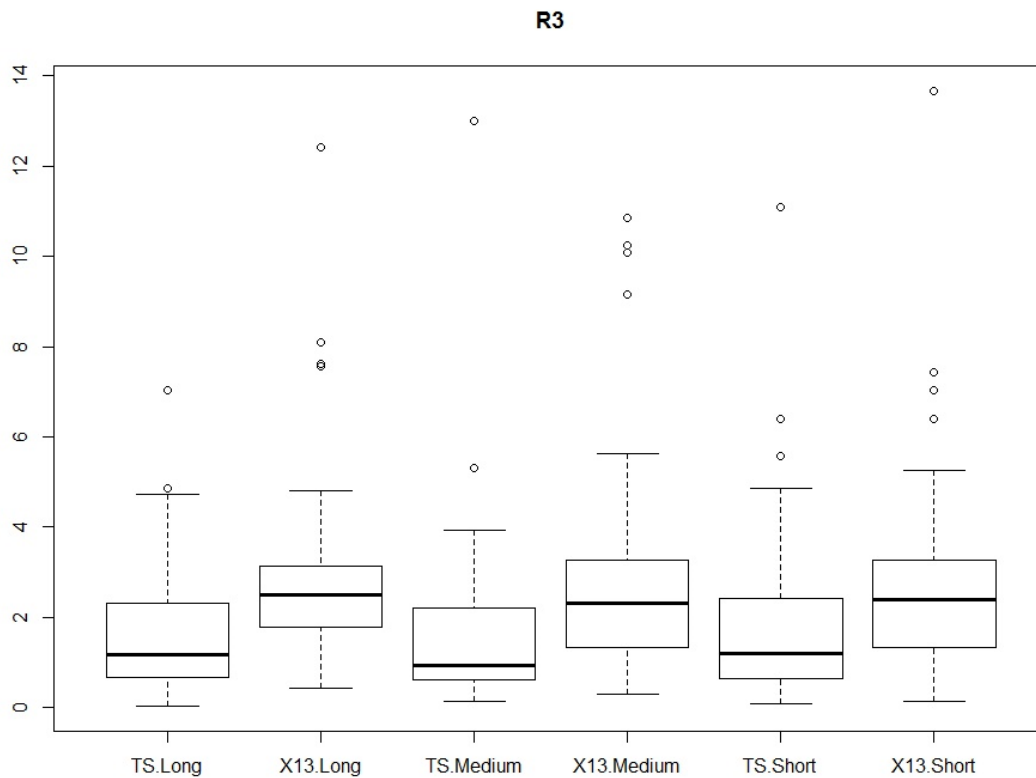
**Table 23.8:** R1 for the full set of series - Last three years

R1 last 3 years	TS Long	X13 Long	TS Med.	X13 Med.	TS Short	X13 Short
R1<2.5	14	9	12	9	13	11
2.5<R1<5	6	7	7	7	8	8
5<R1<7	4	7	4	7	2	5
7<R1<10	3	2	5	2	3	1
R1>10	3	5	2	5	4	5

**Figure 23.4:** R1 for the full set of series - Last three years

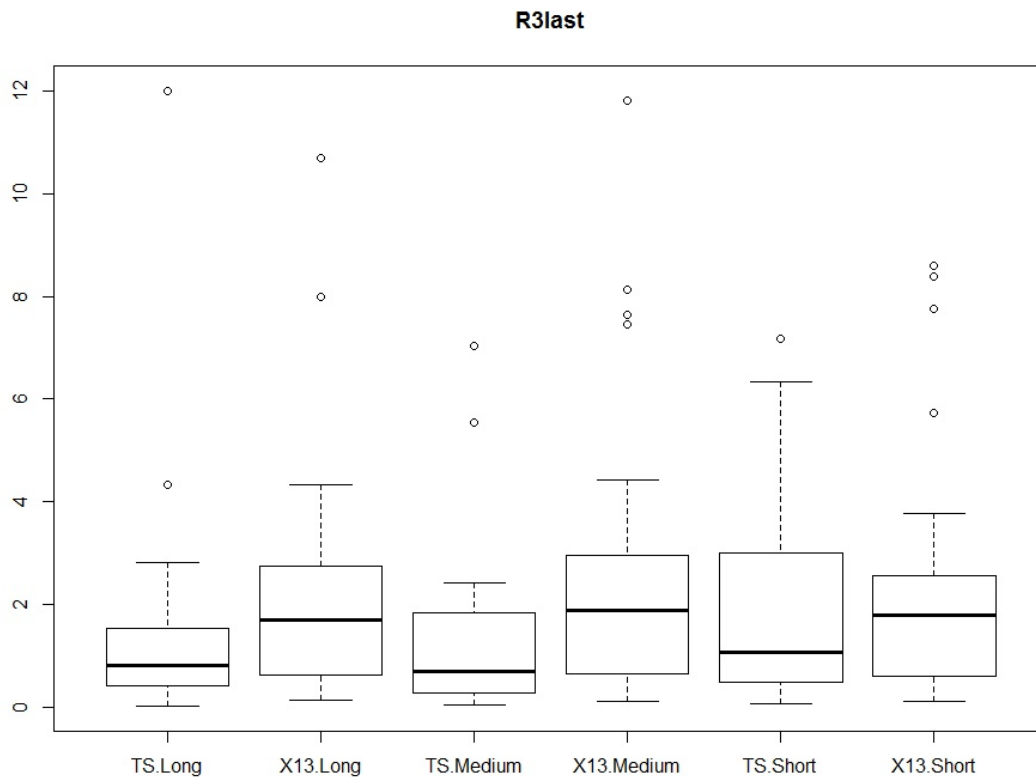
**Table 23.9:** R3 for the full set of series - Full time span

R3	TS Long	X13 Long	TS Med.	X13 Med.	TS Short	X13 Short
$R3 < 0.5$	6	1	5	3	6	3
$0.5 < R3 < 2$	15	8	16	8	14	10
$2 < R3 < 4$	4	14	6	11	4	11
$4 < R3 < 8$	4	4	1	3	4	4
$R3 > 8$	1	3	2	5	2	2

**Figure 23.5:** R3 for the full set of series - Full time span

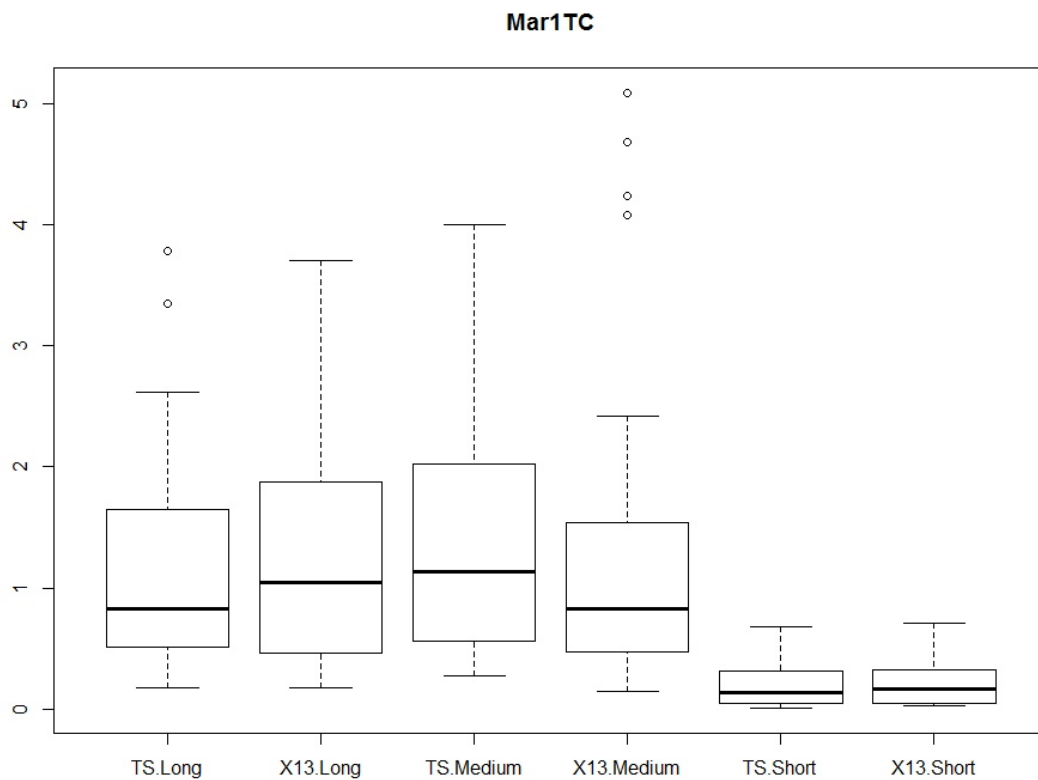
**Table 23.10:** R3 for the full set of series - Last three years

R3 last 3 years	TS Long	X13 Long	TS Med.	X13 Med.	TS Short	X13 Short
R3<0.5	12	10	6	10	8	6
0.5<R3<2	12	10	13	9	12	12
2<R3<4	3	9	4	9	5	7
4<R3<8	1	3	2	3	4	2
R3>8	2	2	1	3	1	3

**Figure 23.6:** R3 for the full set of series - Last three years

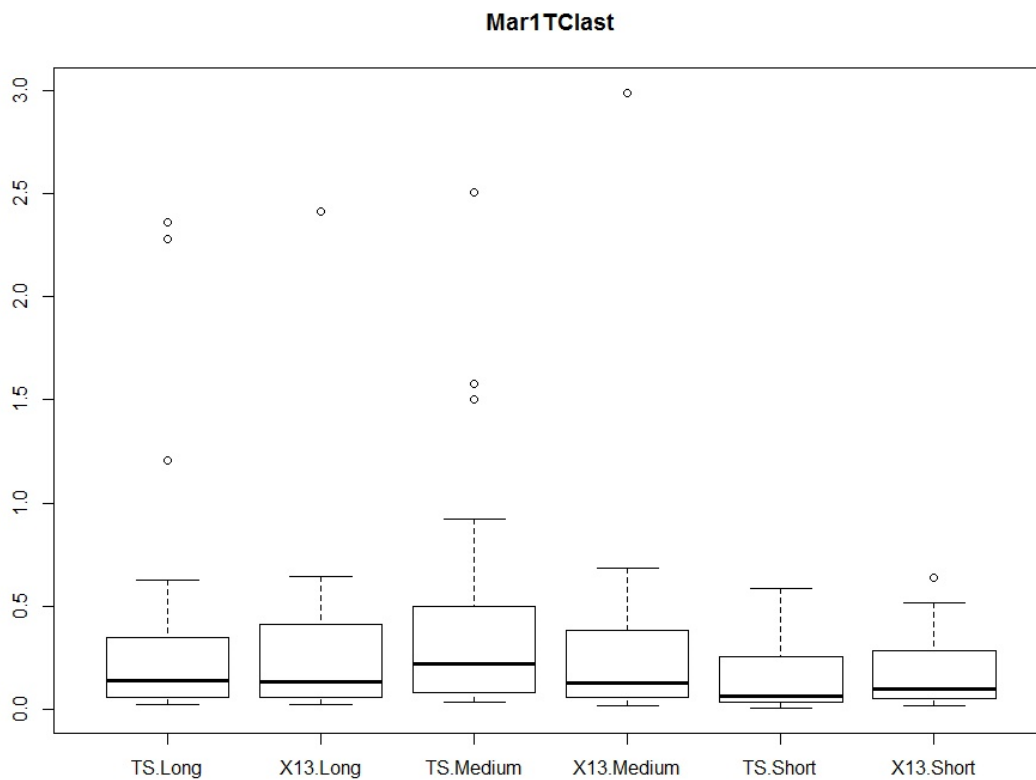
**Table 23.11:** Mar1(TC) for the full set of series - Full time span

Mar1(TC)	TS	X13	TS	X13	TS	X13
	Long	Long	Med.	Med.	Short	Short
Mar1(TC)<0.2	2	1	0	1	19	15
0.2<Mar1(TC)<0.5	4	7	5	7	8	11
0.5<Mar1(TC)<1	12	5	9	10	2	3
1<Mar1(TC)<2	7	11	8	5	0	0
Mar1(TC)>2	5	6	8	7	1	1

**Figure 23.7:** Mar1(TC) for the full set of series - Full time span

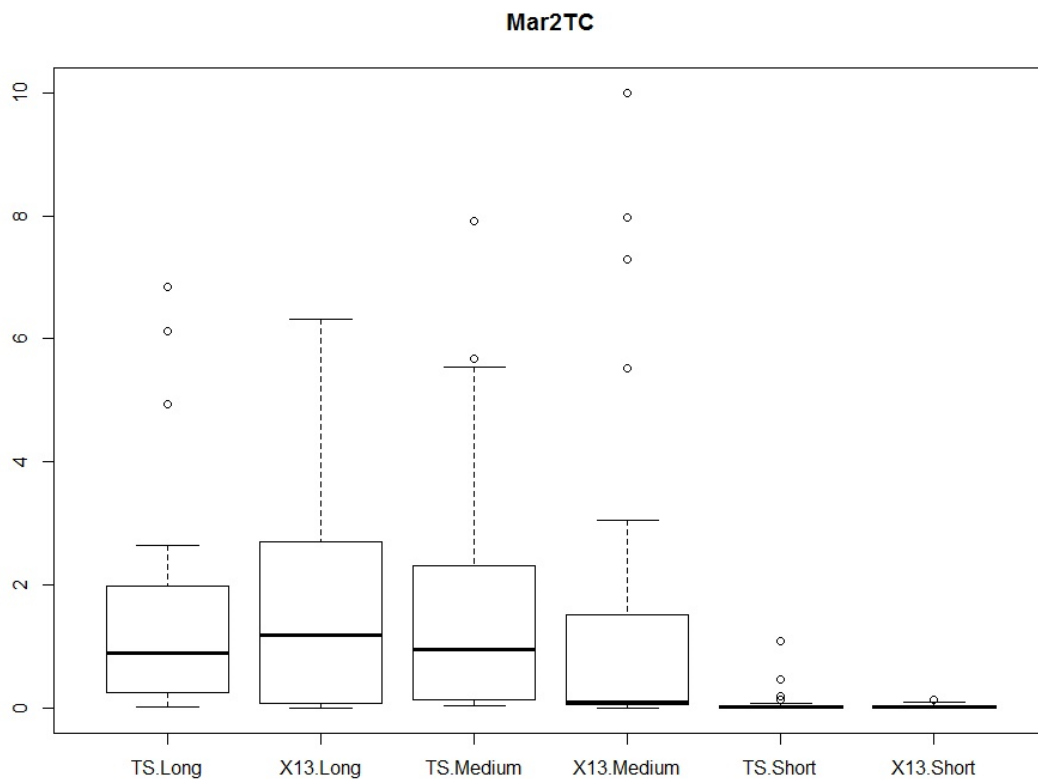
**Table 23.12:** Mar1(TC) for the full set of series - Last three years

Mar1(TC) last 3 years	TS Long	X13 Long	TS Med.	X13 Med.	TS Short	X13 Short
Mar1(TC)<0.2	16	15	14	16	19	16
0.2<Mar1(TC)<0.5	8	10	9	9	8	11
0.5<Mar1(TC)<1	2	3	4	4	2	2
1<Mar1(TC)<2	1	0	2	0	0	0
Mar1(TC)>2	3	2	1	1	1	1

**Figure 23.8:** Mar1(TC) for the full set of series - Last three years

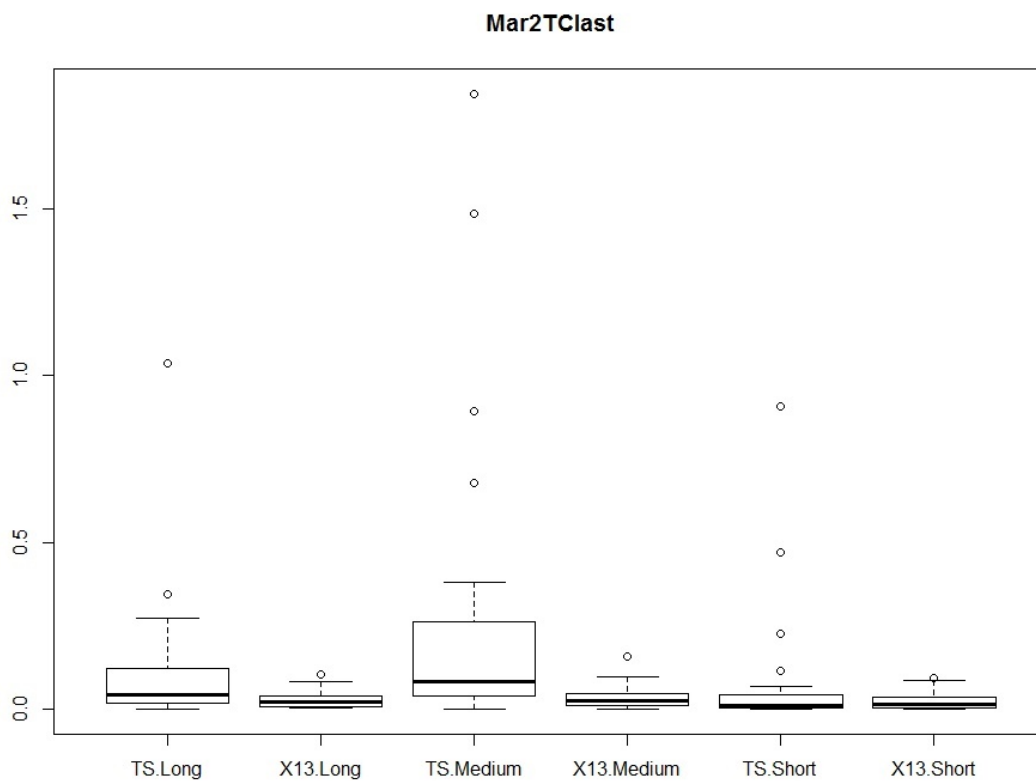
**Table 23.13:** Mar2(TC) for the full set of series - Full time span

Mar2(TC)	TS	X13	TS	X13	TS	X13
	Long	Long	Med.	Med.	Short	Short
Mar2(TC)<0.05	3	5	1	6	22	23
0.05<Mar2(TC)<0.1	1	3	4	9	3	5
0.1<Mar2(TC)<0.2	2	2	3	2	2	1
0.2<Mar2(TC)<0.5	5	1	3	1	1	0
Mar2(TC)>0.5	19	19	19	12	2	1

**Figure 23.9:** Mar2(TC) for the full set of series - Full time span

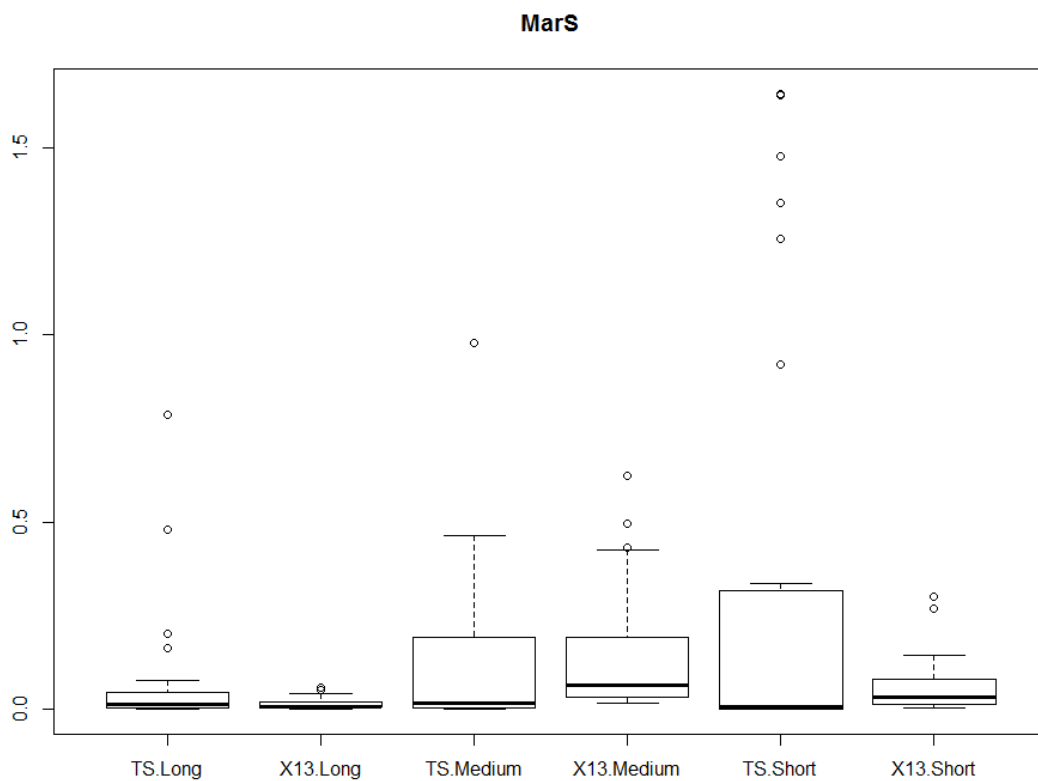
**Table 23.14:** Mar2(TC) for the full set of series - Last three years

Mar2(TC) last 3 years	TS Long	X13 Long	TS Med.	X13 Med.	TS Short	X13 Short
Mar2(TC)<0.05	3	23	9	23	22	25
0.05<Mar2(TC)<0.1	1	5	7	6	3	4
0.1<Mar2(TC)<0.2	2	1	2	1	1	0
0.2<Mar2(TC)<0.5	5	0	8	0	2	0
Mar2(TC)>0.5	19	1	4	0	2	1

**Figure 23.10:** Mar2(TC) for the full set of series - Last three years

**Table 23.15:** Mar(S) for the full set of series - Full time span

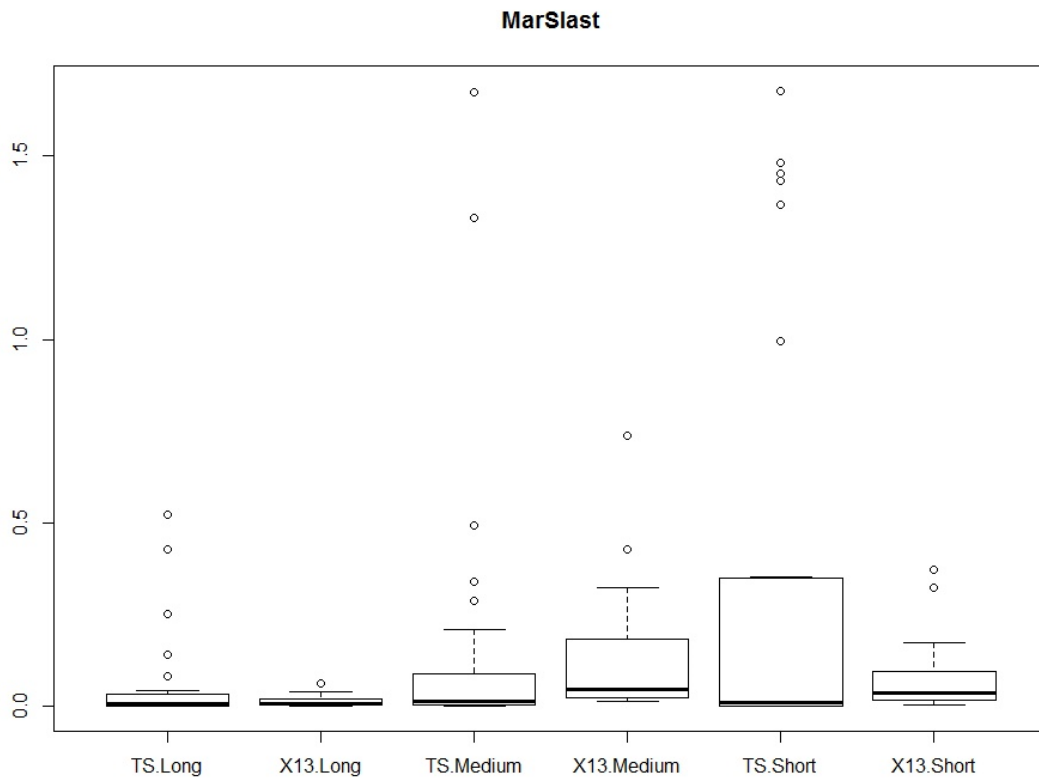
Mar(S)	TS	X13	TS	X13	TS	X13
	Long	Long	Med.	Med.	Short	Short
Mar(S)<0.01	13	18	9	0	15	7
0.01<Mar(S)<0.05	11	10	5	12	11	14
0.05<Mar(S)<0.1	2	2	3	7	1	3
0.1<Mar(S)<0.2	2	0	3	4	2	4
Mar(S)>0.2	2	0	10	7	11	2

**Figure 23.11:** Mar(S) for the full set of series - Full time span



**Table 23.16:** Mar(S) for the full set of series - Last three years

Mar(S) last 3 years	TS Long	X13 Long	TS Med.	X13 Med.	TS Short	X13 Short
Mar(S)<0.01	17	20	12	0	15	6
0.01<Mar(S)<0.05	8	9	4	16	1	10
0.05<Mar(S)<0.1	1	1	5	4	1	7
0.1<Mar(S)<0.2	1	0	0	3	2	5
Mar(S)>0.2	3	0	9	7	11	2

**Figure 23.12:** Mar(S) for the full set of series - Last three years

## 23.5 Conclusions

In this chapter the relative performance of X-13 ARIMA-SEATS and TRAMO-SEATS, when seasonally adjusting time series of different lengths (short, medium and long), is compared by using a number of widely used diagnostics and statistics.

For more details on the seasonal adjustment of very short or very long time series, see the ESS guidelines on Seasonal Adjustment (Eurostat, 2015, and chapter 30).

As a general consideration, and in accordance with the literature to date (see Matas-Mir and Osborn, 2004), the quality of the seasonal adjustment is reduced when shorter time series are considered. However few additional considerations could be done to better clarify this general statement.

The quality of seasonal adjustment is low when short time series are considered. In other words, when comparing medium and short series, the dissimilarities are much bigger than when comparing long and medium series, as visible with the C1 and APD statistics.

According to the study, the pattern of the seasonally adjusted series for the three time spans is more similar when using X-13 ARIMA-SEATS than TRAMO-SEATS. This is especially visible when comparing the medium with the short time series.

In terms of the roughness of the components, X-13 ARIMA-SEATS seems to perform slightly better than TRAMO-SEATS when looking at the smoothness of the trend-cycle and of the seasonal components, as shown by the Mar1(TC), Mar2(TC) and Mar(S) statistics. On the other hand, when looking at the smoothness of the seasonally adjusted component, TRAMO-SEATS seems to be preferred as shown by the R1 measure. However there is no theoretical support to the assumption that the quality of the seasonal adjustment will be higher when the seasonally adjusted series is smoother. Moreover when the seasonally adjusted component is too smooth there might be a risk of over-adjustment. Therefore in this article the attention was focusing more on the roughness of the trend-cycle and the seasonal components, which should be considered as good indicators of the relative performance of the seasonal adjustment procedures.

The better performance of X-13 ARIMA-SEATS, especially when looking at the short and medium series, is justified by its intrinsic non-parametric nature, so that the choice of the moving average filters is less influenced by the changes in the length of the series. On the other hand, since TRAMO-SEATS is based on the identification of a specific ARIMA model, it is likely to assume that different models can be identified on different time spans. These could be specially the case when moving from short to medium length series with the consequence of producing relatively different seasonal adjustment patterns.

Results from this empirical study cannot be generalised, but they appear to be reasonable, and are in line with the results obtained by Mazzi and Savio (2005).

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# 24

## Chain-linking of Quarterly National Accounts and Implications for Seasonal Adjustment



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## Table of Contents

24.1	Conceptual considerations	657
24.1.1	Properties of unadjusted data	657
24.1.2	Seasonal Adjustment	663
24.2	Empirical evidence	665

## 24.1 Conceptual considerations

In order to be able to understand the implications of the various chain index concepts which are available and have to be submitted to seasonal adjustment, the index methods for the unadjusted values are described first.<sup>1</sup> In the following section, they are briefly developed and presented on the basis of the critique of the fixed-price-base method.

The subsequent presentation of the index concepts used for for chain-linking or price adjustment makes no claim to being exhaustive. It concentrates on the aspects which are important for the analysis of seasonally adjusted results and play a role in the interpretation of the current economic dynamics:

- The comparison with previous periods, which is used in the description of current developments
- The decomposition of the movement of an aggregate (such as gross domestic product) into the development of its components (such as private consumption) to explain the overall dynamics
- The consistency between quarterly and annual figures (time consistency)

### 24.1.1 Properties of unadjusted data

#### 24.1.1.1 Introduction

The index concepts for capturing volume movements in a chain-linking context are most easily described in contradistinction to fixed base period methods.

There, volumes were calculated by valuing the current quantities ( $q_t$ ) at the average prices of the base year ( $\bar{p}_0$ ). The formula for the volume for the first quarter in year  $t$  was thus

$$\sum \bar{p}_0 q_t^I, \quad (24.1)$$

where the sum-total is calculated across the different goods.

A shortcoming of this procedure is that the information content of the current volume movements declines with the growing distance of the current reporting period from the base period. This is because the representativeness of the price structures from the base period can decline with the growing distance from the reporting period. For example, technical goods (such as computers) have become cheaper whereas energy prices have tended to increase. Hence, calculating current volumes on the basis of a price structure dating back many years can no longer be considered meaningful.

In order to overcome this problem, therefore, the distance between the price base year and the current period should not be too large. Consequently, in many countries the past year<sup>2</sup> is thus used for calculating the price base.

$$\sum \bar{p}_{t-1} q_t^I, \quad (24.2)$$

Because the average prices of the past year are needed to calculate current volume data, the figures for volumes normally start one year later than the corresponding nominal time series.<sup>3</sup>

If the movement within a year is being analysed, the price base does not change. Therefore, the quarter-on-quarter pure volume movement can be shown as

<sup>1</sup>An introduction to the basic concepts is also given by Bloem et al. (2001)

<sup>2</sup>Canada rebases quarterly but this can be regarded as an exception and is not treated here.

<sup>3</sup>In order to produce volume data for year  $t=0$ , average prices of the same year are used in practice, i.e.  $\sum \bar{p}_0 q_0^i (i = I, \dots, IV)$ .

$$\sum \bar{p}_{t-2} q_{t-1}^{III} \quad , \quad \sum \bar{p}_{t-2} q_{t-1}^{IV} \quad (24.3)$$

During the transition from the fourth quarter of one year to the first quarter of the following year, however, not only the quantity component but also the price component changes. Hence the pure volume movement is not shown; instead a statistically induced break occurs caused by the change in the price base.

$$\sum \bar{p}_{t-2} q_{t-1}^{IV} \quad , \quad \sum \bar{p}_{t-1} q_t^I \quad (24.4)$$

Ideally, such breaks should not appear in a pure volume series.

Actually, the change in the price base should not have any influence on volume changes. This requirement was fulfilled by the fixed-price-base concept. However, owing to the aforementioned problem of outdated price bases, this method is now no longer an option. Hence, there is a need for other approaches.

When using the total previous year as the price base and using Paasche price indices (rather than Fisher indices) for deflating quarterly figures, three different methods for deriving volume indices (so-called chain-linking methods) can be applied. Each method shows different properties and problems.

#### 24.1.1.2 Annual overlap method

In the case of the annual overlap approach, the current quantities are valued at the average prices of the past year and compared with the average nominal values of the past year. But to construct an index value which reflects the distance to the reference period, it is necessary to repeat this step several times for the annual figures. Finally, the index value is calculated by multiplying all these growth factors together. For example, the index value for the first quarter of 2003 in a series beginning in 2000 with 2000 as the reference year can be expressed as

$$100 * \frac{\sum_{i=I}^{IV} \bar{p}_{2000} q_{2001}^i}{\sum_{i=I}^{IV} \bar{p}_{2000} q_{2000}^i} * \frac{\sum_{i=I}^{IV} \bar{p}_{2001} q_{2002}^i}{\sum_{i=I}^{IV} \bar{p}_{2001} q_{2001}^i} * \frac{\sum \bar{p}_{2002} q_{2003}^I}{\frac{1}{4} \sum_{i=I}^{IV} \bar{p}_{2002} q_{2002}^i} \quad (24.5)$$

Sometimes, "chain-linked" euros are used to express a change in the volumes. If this is the case, the quarterly average of the nominal figures in the reference year ( $\frac{1}{4} \sum_{i=I}^{IV} \bar{p}_{2000} q_{2000}^i$ ) is taken instead of 100 in the formula 24.5. In the following, the index notation is used. The results for "chain-linked" euros can easily be derived.

#### 1. Time consistency

As the annual average of the thus calculated quarterly figures is the same as the autonomously calculated annual index, there is no need to adjust the quarterly results in order to reach the annual results (benchmarking). Furthermore, the percentage changes in the annual figures show no statistical breaks.

$$\frac{Index_{2002}}{Index_{2001}} = \frac{100 * \frac{\sum_{i=I}^{IV} \bar{p}_{2000} q_{2001}^i}{\sum_{i=I}^{IV} \bar{p}_{2000} q_{2000}^i} * \frac{\sum_{i=I}^{IV} \bar{p}_{2001} q_{2002}^i}{\sum_{i=I}^{IV} \bar{p}_{2001} q_{2001}^i}}{100 * \frac{\sum_{i=I}^{IV} \bar{p}_{2000} q_{2001}^i}{\sum_{i=I}^{IV} \bar{p}_{2000} q_{2000}^i}} = \frac{\sum_{i=I}^{IV} \bar{p}_{2001} q_{2002}^i}{\sum_{i=I}^{IV} \bar{p}_{2001} q_{2001}^i} \quad (24.6)$$

As the same price base is used in the current and past year, the annual overlap gives the pure volume changes when comparing the annual figures with those from the previous period.

#### 2. Quarter-on-quarter comparison

Pure volume changes are also calculated for the quarter-on-quarter movements within a year, as the following example shows.



$$\frac{Index_{2003}^{II}}{Index_{2003}^I} = \frac{100 * \frac{\sum_{i=I}^{IV} \sum \bar{p}_{2000} q_{2001}^i}{\sum_{i=I}^{IV} \sum \bar{p}_{2000} q_{2000}^i} * \frac{\sum_{i=I}^{IV} \sum \bar{p}_{2001} q_{2002}^i}{\sum_{i=I}^{IV} \sum \bar{p}_{2001} q_{2001}^i} * \frac{\sum \bar{p}_{2002} q_{2003}^{II}}{\frac{1}{4} \sum_{i=I}^{IV} \sum \bar{p}_{2002} q_{2002}^i}}{100 * \frac{\sum_{i=I}^{IV} \sum \bar{p}_{2000} q_{2001}^i}{\sum_{i=I}^{IV} \sum \bar{p}_{2000} q_{2000}^i} * \frac{\sum_{i=I}^{IV} \sum \bar{p}_{2001} q_{2002}^i}{\sum_{i=I}^{IV} \sum \bar{p}_{2001} q_{2001}^i} * \frac{\sum \bar{p}_{2002} q_{2003}^I}{\frac{1}{4} \sum_{i=I}^{IV} \sum \bar{p}_{2002} q_{2002}^i}} = \frac{\sum \bar{p}_{2002} q_{2003}^{II}}{\sum \bar{p}_{2002} q_{2003}^I} \quad (24.7)$$

By contrast, breaks occur in the transition from the fourth quarter of one year to the first quarter of the following year.

$$\begin{aligned} \frac{Index_{2003}^I}{Index_{2002}^{IV}} &= \frac{100 * \frac{\sum_{i=I}^{IV} \sum \bar{p}_{2000} q_{2001}^i}{\sum_{i=I}^{IV} \sum \bar{p}_{2000} q_{2000}^i} * \frac{\sum_{i=I}^{IV} \sum \bar{p}_{2001} q_{2002}^i}{\sum_{i=I}^{IV} \sum \bar{p}_{2001} q_{2001}^i} * \frac{\sum \bar{p}_{2002} q_{2003}^I}{\frac{1}{4} \sum_{i=I}^{IV} \sum \bar{p}_{2002} q_{2002}^i}}{100 * \frac{\sum_{i=I}^{IV} \sum \bar{p}_{2000} q_{2001}^i}{\sum_{i=I}^{IV} \sum \bar{p}_{2000} q_{2000}^i} * \frac{\sum \bar{p}_{2001} q_{2002}^{IV}}{\frac{1}{4} \sum_{i=I}^{IV} \sum \bar{p}_{2001} q_{2001}^i}} \\ &= \frac{\sum \bar{p}_{2002} q_{2003}^I}{\sum \bar{p}_{2001} q_{2002}^{IV}} * \frac{1}{\frac{\sum_{i=I}^{IV} \sum \bar{p}_{2002} q_{2002}^i}{\sum_{i=I}^{IV} \sum \bar{p}_{2001} q_{2002}^i}} \end{aligned} \quad (24.8)$$

This rate of change is thus composed of two components (in the formula after the right-hand equals sign). First, the change factor of the quantities for the first quarter of the current year valued at past-year prices is calculated vis-à-vis the quantities from the fourth quarter of the past year valued at the prices of the preceding year (t-2). As a result of the change in the price base, however, this is not a pure volume comparison. In order to at least approximate this, the price base change effect must be factored out. This is done using the reciprocal Paasche price index at the extreme right of the last formula. Yet, as the deflator relates to whole years but the volume development is to be measured for quarters, the deflator is not fully congruent with the variable that is to be deflated. As a result, a certain statistical contamination remains during the transition from the fourth quarter of one year to the first quarter of the following year. This results in a break in the volume series.

The relationship between the index change and the pure volume change can be used to express the statistical break as

$$\frac{\frac{\sum \bar{p}_{2002} q_{2003}^I}{\sum \bar{p}_{2001} q_{2002}^{IV}} * \frac{1}{\frac{\sum_{i=I}^{IV} \sum \bar{p}_{2002} q_{2002}^i}{\sum_{i=I}^{IV} \sum \bar{p}_{2001} q_{2002}^i}}}{\frac{\sum \bar{p}_{2002} q_{2003}^I}{\sum \bar{p}_{2002} q_{2002}^{IV}}} = \frac{\frac{\sum \bar{p}_{2002} q_{2002}^{IV}}{\sum \bar{p}_{2001} q_{2002}^{IV}}}{\frac{\sum_{i=I}^{IV} \sum \bar{p}_{2002} q_{2002}^i}{\sum_{i=I}^{IV} \sum \bar{p}_{2001} q_{2002}^i}} = \frac{\frac{\sum \bar{p}_{2002} q_{2002}^{IV}}{\sum \bar{p}_{2001} q_{2002}^{IV}}}{\frac{\sum \bar{p}_{2002} q_{2002}^I}{\sum \bar{p}_{2001} q_{2002}^I}} \quad (24.9)$$

The size of the statistical break can thus be derived from the relationship between two Paasche price indices, one based on the quantities from the fourth quarter of the past year and one based on the quantities over the entire past year.

Hence the more the quantity structure in the fourth quarter differs from that of the entire year, the larger the break in the volume series. Or, in other words, if the quantity of each single good over the whole year was always  $\lambda$ -times the quantity in the fourth quarter, there would be no break. This is because, assuming  $\lambda q_{2002}^{IV} = q_{2002}$ , the following holds:

$$\frac{\lambda \frac{\sum \bar{p}_{2002} q_{2002}}{\sum \bar{p}_{2001} q_{2002}}}{\frac{\sum \bar{p}_{2002} q_{2002}}{\sum \bar{p}_{2001} q_{2002}}} = 1 \quad (24.10)$$

There is likewise no statistical break if there is no change in the relative prices. If this is the case,  $\bar{p}_{2002} = \lambda \bar{p}_{2001}$  holds for all goods prices and hence.



$$\frac{\lambda \sum \bar{p}_{2001} q_{2002}^{IV}}{\sum \bar{p}_{2001} q_{2002}^{IV}} = 1 \quad (24.11)$$

The change-of-year break caused by the change in the price base is, therefore, smaller, the less the relative prices fluctuate from year to year or the less the quantity structure in the fourth quarter of the past year differs from the quantity structure of the entire past year.

### 3. Aggregation and decomposition

The aggregation of chain-linked indices and the decomposition of aggregates into subindices is not as easy as in the case of the earlier fixed-price-base concept. Chain-linked indices are not additive, i.e. their aggregates do not simply equal the sum of the components, nor do they result from the addition of their components multiplied by constant weights. Nevertheless, the aggregates are functionally dependent on their components. This can be used in the case of the annual overlap approach to calculate aggregates using components or to decompose aggregates into sub-indices.

For example, to calculate an aggregate from components, it is first necessary to de-chain the time series for the components, i.e. annual index links must be calculated. Using the structure of the nominal figures from the respective previous year, these can be summarised into annual links for the aggregate. Finally, for the aggregate, the annual components must be chain-linked again. In the annual overlap method, the only thing which is needed for such calculations are the published nominal and real figures.

The following relationship applies between values, volumes and deflators.

Index value = Annual overlap Laspeyres volume chain-linked index x annual overlap Paasche price chain-linked index / 100

#### 24.1.1.3 Quarterly overlap method

In the case of the quarterly overlap approach, the current chain link is derived by valuing the quantities of the current quarter at the average prices of the past year and then comparing them with the quantities of the fourth quarter of the past year valued at the average prices of the past year. In order to show the distance from the reference year, this step must be repeated, related each time to the quantities for the fourth quarter. The change factors which result from this process are then multiplied by one another.

The index value for the volume in the first quarter of 2003 in a time series beginning in 2000 with 2000 as the reference year is thus calculated as follows

$$100 * \frac{\sum \bar{p}_{2000} q_{2000}^{IV}}{\frac{1}{4} \sum_{i=I}^{IV} \sum \bar{p}_{2000} q_{2000}^i} * \frac{\sum \bar{p}_{2000} q_{2001}^{IV}}{\sum \bar{p}_{2000} q_{2000}^{IV}} * \frac{\sum \bar{p}_{2001} q_{2002}^{IV}}{\sum \bar{p}_{2001} q_{2001}^{IV}} * \frac{\sum \bar{p}_{2002} q_{2003}^I}{\sum \bar{p}_{2002} q_{2002}^{IV}} \quad (24.12)$$

where  $\frac{1}{4} \sum_{i=I}^{IV} \sum \bar{p}_{2000} q_{2000}^i$  acts by convention as the starting denominator for chain-linking.

#### 1. Time consistency

The year-on-year rates of change for the annual figures of an index calculated in this way for 2003 in comparison with 2002 are derived as follows

$$\frac{Index_{2003}}{Index_{2002}} = \dots = \frac{\sum_{i=I}^{IV} \sum \bar{p}_{2002} q_{2003}^i}{\sum_{i=I}^{IV} \sum \bar{p}_{2001} q_{2002}^i} \quad (24.13)$$

As a rule, they deviate from autonomously calculated annual rates (24.6). In practice, this raises the requirement that the quarterly figures calculated using the quarterly overlap approach need to be forced to equal the autonomously calculated annual figures (benchmarking) if desired. Overall, therefore, the quarterly overlap approach does not unambiguously denote a method which is used in practice. Instead, the specification of the benchmarking procedure is additionally necessary for an unambiguous description of the method if quarterly results have to be in line with annual totals.

If all quarterly figures within a calendar year are adjusted to the annual figures using the same factor (*pro rata* technique), the annual overlap is calculated. This is therefore a special case of quarterly overlap with benchmarking.

## 2. Quarter-on-quarter comparison

Without benchmarking, the quarter-on-quarter rates of change calculated using the quarterly overlap method always show the pure volume change without a break not only within a year but also between the fourth quarter and the first quarter of the following year.

$$\frac{Index_{2003}^I}{Index_{2002}^{IV}} = \dots = \frac{\sum \bar{p}_{2002} q_{2003}^I}{\sum \bar{p}_{2002} q_{2002}^{IV}} \quad (24.14)$$

This property has, however, been lost at least partially as a result of the introduction of benchmarking techniques. Depending on the extent of the differences between the average of the quarterly results and the autonomously calculated annual figures and also depending on the choice of the benchmarking method used, deviations from the pure volume comparison occur. In a *pro rata* approach, breaks can only occur during the change from the fourth quarter to the first quarter of the following year. In other methods, breaks can also occur during the quarterly movement within a particular year (albeit to a lesser extent than the *pro rata* break at the turn of the year).

## 3. Aggregation and decomposition

Figures calculated using the quarterly overlap approach without benchmarking can basically be aggregated using the same procedure as for the annual overlap method. However, it is not the structure of the past-year nominal figures that is used as a weight for the annual links but rather the structure of the quantities of the fourth quarter of the past year valued at average past-year prices. These figures are not generally released as part of the normal publishing programme of volume indices, deflators and figures at current prices. If users wish to calculate their own aggregates or disaggregates, then the quantities in the fourth quarter of the past year valued at past-year prices must be published separately and saved separately by the user.

Aggregation and decomposition are more difficult using benchmarking techniques (unless a *pro rata* approach is taken). This is because in these circumstances the user needs the figures calculated without benchmarking, the aforementioned weights as well as the benchmarking algorithm. The official statistics office would have to provide this information in addition to the benchmarked quarterly overlap results. The problems resulting from the complicated data storage and subsequent calculation steps could easily lead to a situation where third parties are not able to calculate correct aggregates or sub-components of time series.

The following relationship applies between values, volumes and deflators in the case of quarterly overlap without benchmarking.

Index value = Quarterly overlap Laspeyres volume chain-linked index x quarterly overlap Paasche price chain-linked index / 100

With benchmarking this relationship is also not necessarily satisfied. In fact, the validity of this equation depends on the approach adopted.

#### 24.1.1.4 Over-the-year technique

In the case of the over-the-year technique, the current chain link is expressed as a relationship between the current quantities valued at average past-year prices and the quantities of the same quarter of the past year, valued at the same prices. Corresponding growth factors are also calculated for earlier years. Finally, the current chain-linked index value is the product of all these factors. Therefore, the index value for the first quarter of 2003 in a series beginning in 2000 with 2000 as the reference year can be expressed as:

$$100 * \frac{\sum \bar{p}_{2000} q_{2000}^I}{\frac{1}{4} \sum_{i=I}^{IV} \sum \bar{p}_{2000} q_{2000}^i} * \frac{\sum \bar{p}_{2000} q_{2001}^I}{\sum \bar{p}_{2000} q_{2000}^I} * \frac{\sum \bar{p}_{2001} q_{2002}^I}{\sum \bar{p}_{2001} q_{2001}^I} * \frac{\sum \bar{p}_{2002} q_{2003}^I}{\sum \bar{p}_{2002} q_{2002}^I} \quad (24.15)$$

where  $\frac{1}{4} \sum_{i=I}^{IV} \sum \bar{p}_{2000} q_{2000}^i$  acts by convention as the starting denominator.

#### 1. Time consistency

The formal comparison between the annual indices calculated using the over-the-year method and the autonomously calculated annual results is made more difficult by the fact that the corresponding formulae for the over-the-year approach are very long and not easily interpreted. Applied studies do, however, show that although results calculated using this method are not fully identical with autonomously calculated annual figures, they do not differ greatly. Benchmarking is therefore necessary, although its effects on the results are likely to be limited.

#### 2. Quarter-on-quarter comparison

Even without benchmarking, the formula for quarter-on-quarter comparison, also within a particular year, is very complicated:

$$\frac{Index_{2003}^{II}}{Index_{2003}^I} = \frac{100 * \frac{\sum \bar{p}_{2000} q_{2000}^{II}}{\frac{1}{4} \sum_{i=I}^{IV} \sum \bar{p}_{2000} q_{2000}^i} * \frac{\sum \bar{p}_{2000} q_{2001}^{II}}{\sum \bar{p}_{2000} q_{2000}^{II}} * \frac{\sum \bar{p}_{2001} q_{2002}^{II}}{\sum \bar{p}_{2001} q_{2001}^{II}} * \frac{\sum \bar{p}_{2002} q_{2003}^{II}}{\sum \bar{p}_{2002} q_{2002}^{II}}}{100 * \frac{\sum \bar{p}_{2000} q_{2000}^I}{\frac{1}{4} \sum_{i=I}^{IV} \sum \bar{p}_{2000} q_{2000}^i} * \frac{\sum \bar{p}_{2000} q_{2001}^I}{\sum \bar{p}_{2000} q_{2000}^I} * \frac{\sum \bar{p}_{2001} q_{2002}^I}{\sum \bar{p}_{2001} q_{2001}^I} * \frac{\sum \bar{p}_{2002} q_{2003}^I}{\sum \bar{p}_{2002} q_{2002}^I}} \quad (24.16)$$

Consequently, the currently reported quarter-on-quarter rate depends on the entire history of these quarters. For example, if just one value from the first quarter of 2001 changes, this has an effect on the changes of the unadjusted values in the first quarter of 2003 compared with the previous quarter.

The formula, which cannot be simplified further, also proves that the same price base is not used in the comparison of the volume indices of two consecutive quarters. Instead, the historical sequence of all price bases is included in the calculation. Therefore, the results calculated using the over-the-year technique are far from the ideal of a pure volume comparison. Hence, statistically induced breaks constantly occur.

The introduction of benchmarking techniques causes further complications (use also of the history of the volumes of the other quarters). In effect, the theoretical penetration of the expanded concept for the comparison with previous periods is becoming almost impossible.

#### 3. Aggregation and decomposition

Aggregates and disaggregates can also be calculated using the over-the-year method. For this, however, the chain links for each individual quarter must first be calculated (that is, the relationship between the quantities in the quarter in question valued at average past-year prices and the quantities from the same quarter of the past year valued at the same prices). The structure of the variables in the denominator of this relationship can be used as weights. In this case, therefore, far more information is needed for the aggregation and disaggregation of chain-linked indices than in the case of quarterly overlap.

As is the case with quarterly overlap, the introduction of benchmarking techniques also leads to further complications in the over-the-year approach, with the result that users would scarcely be able to carry out their

own calculations.

The following relationship applies between values, volumes and deflators using the over-the-year technique without benchmarking.

Index value = Over-the-year Laspeyres volume chain-linked index x over-the-year Paasche price chain-linked index / 100

With benchmarking this relationship is likewise not necessarily reliable. Instead, the validity of this equation depends on the approach adopted.

### 24.1.1.5 Summary

The table 24.1 gives an overview of the results.

**Table 24.1: Overview of the results**

Method	Criterion	Consistency between autonomously calculated annual results and quarterly figures	Quarter-on-quarter rate of change	Aggregation and disaggregation (user's perspective)
Annual overlap		Given	<ul style="list-style-type: none"> <li>Unbiased within a calendar year</li> <li>Breaks occur in the transition from Q4 of one year to Q1 of the following year. These breaks are smaller, the less the relative prices fluctuate from year to year or the less the quantity structure in Q4 differs from the quantity structure of the entire year.</li> </ul>	Possible only by using published data (values and volumes).
Quarterly overlap		<ul style="list-style-type: none"> <li>Without benchmarking: not given</li> <li>With benchmarking: given</li> </ul>	<ul style="list-style-type: none"> <li>Without benchmarking: unbiased</li> <li>With benchmarking: biased, whereby the size of the bias depends on the extent and method of benchmarking.</li> </ul>	<ul style="list-style-type: none"> <li>Without benchmarking: possible with additional data concerning quantities for each fourth quarter valued at the annual average prices of the respective year.</li> <li>With benchmarking: extremely complicated.</li> </ul>
Over-the-year technique		<ul style="list-style-type: none"> <li>Without benchmarking: approximately given</li> <li>With benchmarking: given</li> </ul>	Not meaningful	<ul style="list-style-type: none"> <li>Without benchmarking: possible with additional data concerning quantities for all four quarters of each year valued at the annual average prices of the respective year.</li> <li>With benchmarking: extremely complicated</li> </ul>

## 24.1.2 Seasonal Adjustment

Direct seasonal adjustment for chain-linked data can be carried out in the usual way described in this handbook. The only technical requirement is the existence of a sufficiently long time series, which is necessary for estimating the typical seasonal fluctuations and, therefore, for seasonal adjustment. However, it is not so easy to interpret the seasonal component and the seasonally adjusted results for chain-linked unadjusted data.

The breaks in the series due to the chain-linking process, in particular, create problems. In the following, only those aspects of seasonal adjustment that are new in the context of chain-linked data are described<sup>4</sup>.

#### 24.1.2.1 Annual overlap method

As presented above, the application of the annual overlap technique in chain-linking normally causes breaks at the beginning of each year. From a theoretical point of view, it is an open question whether these breaks should be treated as level shifts or partly as irregular or seasonal effects. If such breaks were to recur to a similar extent year after year, the seasonal adjustment programs would assign them to the seasonal component and would spread them across the year. If the breaks were sizeable, this would make the task of interpreting the quarter-on-quarter rates of change more difficult. This is because the rates of change would have to be decomposed into two parts, one representing the actual volume movement and one reflecting the distribution of the statistically induced breaks. This is, however, not possible without additional information.

If the breaks do not always occur in the same direction, but often change their sign, they would not be included in the seasonal component and, therefore, would be visible in the seasonally adjusted series. The actual volume movement would then be contaminated by statistical effects at the turn of the year but would not be contaminated within a year.

In practice, a mixture of these extreme cases can be expected. Part of the statistical breaks is modelled in the seasonal component and another part is not. It goes without saying that it is extremely difficult to give an economical interpretation of the seasonally adjusted movement in such a situation. However, the practical problems should not be overemphasised. If the relative price change from year to year is limited or if the volume structure in the fourth quarter is similar to that of the entire year, the problematic breaks at the beginning of a year are limited in size.

The aggregation of the seasonally adjusted time series (i.e. the indirect approach) generally follows the rules for aggregating the unadjusted data. However, there is a difference in the procedure if the calendar components of the time series under review differ. In such a case, it is not the structure of the unadjusted nominal figures from the respective previous year which should be used for weighting the annual index links, but rather the structure of the calendar adjusted nominal figures that matters. Otherwise there would be a residual calendar effect in the aggregated adjusted series coming from the aggregation procedure.

#### 24.1.2.2 Over-the-year-technique

The data constructed by the over-the-year chain-linking technique normally include breaks from quarter to quarter. Therefore, the problem of splitting the actual economic movement from statistical effects in the seasonally adjusted figures is more pronounced here than in the annual overlap method. Of course, how much this conceptual problem matters in practice depends on the pattern and on the size of such distortions.

The calculation of indirectly seasonally adjusted data is also more complex than in the annual overlap method. Because there is a different quarterly weighting structure in the over-the-year-method for the chain links, it is possible that the weights have a seasonal and/or a calendar component. In order to avoid residual seasonality in an indirectly adjusted time series, the weights have to be seasonally and calendar adjusted too.

#### 24.1.2.3 Quarterly overlap method

This is the only method which does not contain any break in the quarter-on-quarter movement. Hence, the problems of statistically induced jumps mentioned for the other methods are not relevant in this case.

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<sup>4</sup>Chain-linking and seasonal adjustment is also intensively discussed in the Final Report of the Task Force on Seasonal Adjustment of Quarterly National Accounts (2008) Task Force (2008).

However, the problem of benchmarking the seasonally adjusted quarterly figures with the autonomously calculated annual results is more pronounced. Firstly, it has to be decided whether the seasonal adjustment should be based on the unadjusted data before or after benchmarking. Both approaches have pros and cons. Using the unadjusted data before benchmarking has the advantage that the estimated seasonal component is perhaps somewhat easier to interpret. However, unadjusted data before benchmarking have to be published in order to understand and replicate the official seasonal adjustment. The availability of these data might cause confusion, if benchmarked unadjusted data are published too. All in all, there is no overall consensus regarding which kind of unadjusted data (benchmarking or not benchmarking) should be the basis for the seasonal adjustment.

Secondly, it has to be decided whether the annual total of the seasonally and calendar adjusted data should be equal to the annual results that have only been calendar adjusted, to the unadjusted yearly totals or should not be benchmarked at all. This question refers to chapter 10 and is not discussed here. **Drafting note: this sentence has to be confirmed at the final stage of the handbook**

With regard to the calculation of indirectly adjusted aggregates, the structure of the quantities of the fourth quarter of the past year valued at average past-year prices has to be used for weighting the annual links. Because the structure refers to the same quarter in each year, there is no need to seasonally adjust it. However, the calendar pattern can be different from year to year. Hence, the weights should be calendar adjusted in order to avoid a residual calendar effect in the indirectly adjusted data.

Of course, using the mathematically correct algorithm for aggregating and disaggregating seasonally adjusted chain-linked indices is complex. In practice, sometimes methods are used that are not completely correct. These are not described here.

## 24.2 Empirical evidence

As mentioned above, the size of the breaks in the time series caused by the different chainlinking methods depends on the extent of the changes in both the price structure and the quantities. In the literature, only a few empirical studies concerning the extent of such breaks exist as yet. Scheiblecker (2010) calculated these distortions for the Austrian GDP after chain-linking it according to all three methods. He found only minor differences between the annual and the quarterly overlap method but larger differences when compared with the over-the-year method. In general, automated time series modelling routines can react very sensitively to the various chain-linked series because of differences in detected outliers. In times of strong economic changes, the possibility of larger differences between the outcome of all three methods should be higher than during normal times.

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# VIII

## Seasonal Adjustment and Business Cycles







# 25

## The Effect of Alternative Seasonal Adjustment Methods on Business Cycle Analysis



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## Table of Contents

25.1	Introduction	671
25.2	The data	671
25.3	The different seasonal adjustment methods	671
25.4	The graphical inspection and the cluster analysis of the different seasonal adjustment methods	672
25.5	Turning points dating analysis	676
25.6	Conclusions	679

## 25.1 Introduction

Most of the literature on business cycle analysis relies, as input, on the seasonally adjusted (SA) data of the main economic indicators. The rationale is that the seasonal frequencies are different from the frequencies of the cycles, then seasonal movements do not carry useful information, moreover they can hide the information on the frequencies of interest in business cycle analysis. This idea is coherent with the literature on SA that rests on the hypothesis of orthogonality among the seasonal and the others components.

For several reasons this hypothesis can fail and an interaction between seasonal and business cycles can arise. This work address the plausibility of this hypothesis and a first study on the effect that different seasonal adjustment algorithms can have on the business cycle analysis due to their different ability in separating the seasonal from the other frequencies. Empirically the evidence or the presence of interactions can be hardly detectable. There are several reasons: components are unobservable as well as their connections, consequently. Moreover, the series are often characterized by instability, and/or evolutionary behaviour of the components.

Aim of this work can be summarized in: empirical investigation of the effects of a variety of SA methods on two aspects: the cyclical shape of the series and the turning point dating. The focus is on the growth cycle. The approach will be historical, and then no real time exercises is run.

## 25.2 The data

The comparison exercise has been run over 23 long monthly economic time series of some European countries plus a European aggregate. Only monthly series, and no quarterly, have been considered because the high frequency makes more evident differences in cyclical shape and, mainly, in turning point dating. The most relevant two economic / social indicators available at monthly frequency are the Industrial Production Index (IPI) and the Unemployment rate. Discharged the series not long enough and or presenting various problems the final list is:

- For the IPI:
  - Austria (AUT\_IPI), Belgium (BEL\_IPI), Germany (DEU\_IPI), Denmark (DNK\_IPI), Euro Area-17 countries (EMU\_IPI), Spain (ESP\_IPI), Finland (FIN\_IPI), France (FRA\_IPI), United Kingdom (GBR\_IPI), Greece (GRC\_IPI), Ireland (IRL\_IPI), Italy (ITA\_IPI), Luxembourg (LUX\_IPI), Netherlands (NLD\_IPI), Portugal (PRT\_IPI) and Sweden (SWE\_IPI).
- For the Unemployment rate:
  - Belgium (BEL\_UNEMPL), France (FRA\_UNEMPL), United Kingdom (GBR\_UNEMPL), Ireland (IRL\_UNEMPL), Luxembourg (LUX\_UNEMPL), Netherlands (NLD\_UNEMPL) and Sweden (SWE\_UNEMPL)

The selected series have different lengths; their source is the Eurostat on line database.

Since the target of the work is the seasonal adjustment algorithm, the pre-treatment has been standardized for all the series and for all the methods. A RegARIMA model based pre-treatment has been applied in all cases, in order to reach a better comparability of the result, even for the methods for which no pre-treatment module is available.

## 25.3 The different seasonal adjustment methods

The effects of eight different methods on the analysis of the business cycle have been compared:

- BAYSEA (Akaike and Ishiguro (1980)), BAYEsian SEasonal Adjustment program, is based on a model based Bayesian algorithm. It is used at the Bank of Japan.
- DAINTRIES (Fischer (1995); Hylleberg (1986)), a deterministic method, developed in the late 70's at Eurostat. Belongs to the seasonal adjustment procedures based on moving averages derived from local regressions. DAINTRIES treat trend and seasonality as deterministic components: the trend can be modelled as a third-degree polynomial and the seasonality is constant.
- DECOMP (Kitagawa and Gersch (1984)). Decom is a seasonal adjustment method using State-Space-Modeling.
- STAMP (Koopman et al. (1995)), which uses the Kalman filter and related algorithms to fit unobserved component time series, it is then a structural time series model based method.
- TRAMO-SEATS (Gómez and Maravall (1997)). TRAMO (Time Series Regression with ARIMA Noise, Missing Observations and Outliers) and SEATS (Signal Extraction in ARIMA Time Series). It is a program for decomposing a time series into its unobserved (trend-cycle, seasonal, transitory and irregular) components (i.e. for extracting the different signals from a time series), following an ARIMA-model-based method.
- X12v3 (U. S. Census Bureau. (1997)), developed by US Census Bureau as an extended and improved version of the X11-ARIMA method of Statistics Canada (Dagum (1980)). As derived from X11 belong to the family of non-parametric smoothing based on iterative application of moving averages filters.
- X13, a development of the X12, belonging to the same family.
- X13 SEATS, a further development of the X12 that give the possibility to apply, in some cases, an ARIMA model based methods.

After having seasonally adjusted the series a cyclical extraction, for each series and for each methods, has been carried out by means of a Baxter-King filter. Again, the cycle extraction methods has been the same for all the occurrences in order not to introduce other sources of differences and to keep comparable all the results. In total 184 runs (23 times 8) of the procedure (pre-treatment, seasonal adjustment and cyclical extraction) have been realised and analysed.

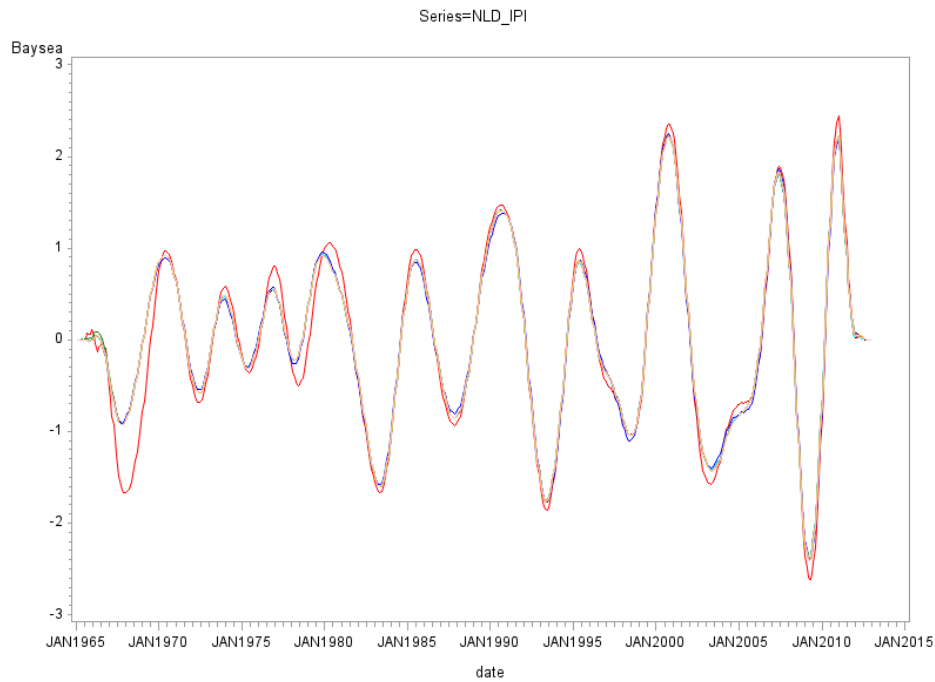
## 25.4 The graphical inspection and the cluster analysis of the different seasonal adjustment methods

The first part of the comparison consists in a graphical inspection of the cyclical estimates and a cluster analysis of the eight methods based on the reciprocal distances of the resulting cyclical components. The purpose is to detect significant differences among the methods; this would proof the interferences among the seasonal and the cyclical frequencies. As an example in figure 25.1 the industrial production of Netherlands is shown. Differences in shape and in dating of the peaks and the troughs can be noticed. In the Italian industrial production (figure 25.2) some methods generate a cycle, not detected by the other methods, at the beginning of the series, the resulting profiles are completely different.

Also in the Swedish unemployment rate (figure 25.3) there are non-negligible differences in the cyclical detection.

Graphical inspection results can be summarised by means of a cluster analysis applied to the different seasonal adjustment methods using as distance the semi-partial R-squared between each couple of cyclical estimates. A simple hierarchic method has been applied for every indicator and finally to all the indicators at

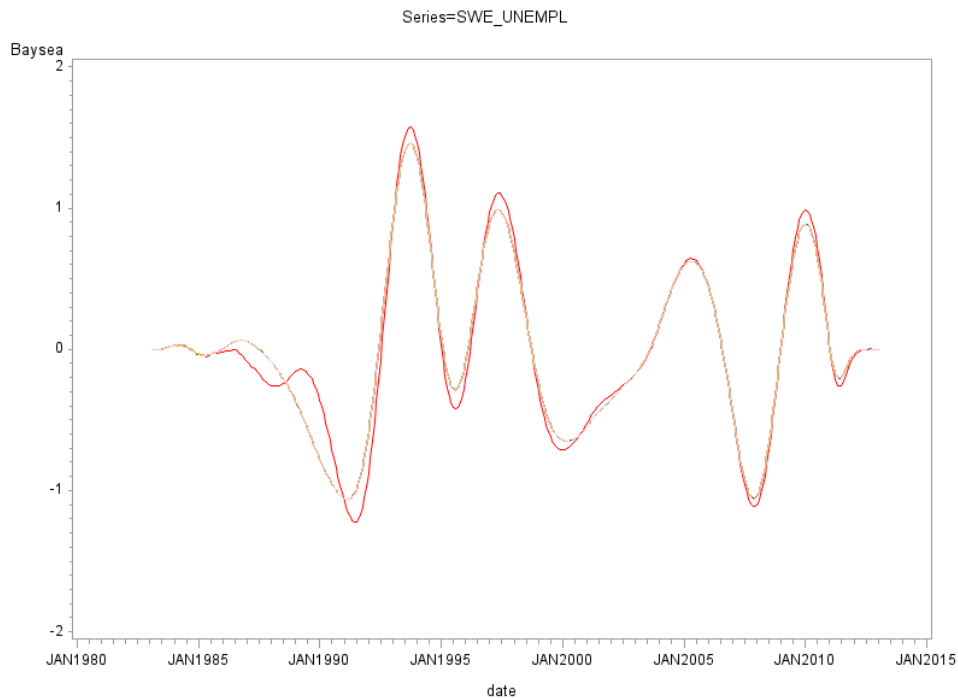
**Figure 25.1:** Graphical inspection of the cyclical estimates of the industrial production series of Netherlands derived by different seasonal adjustment of the input series



**Figure 25.2:** Graphical inspection of the cyclical estimates of the industrial production series of Italy derived by different seasonal adjustment of the input series



**Figure 25.3:** Graphical inspection of the cyclical estimates of the unemployment rate series of Sweden derived by different seasonal adjustment of the input series



the same moment for an overall evaluation. Some clusters is shown as example: The dendrogram built on the industrial production of Austria (figure 25.4) shows that Dainties is a cluster apart, far from the other methods and this is a feature appearing in all the analysis. In this particular case also Tramo-Seats is rather far from the other methods. The X family behave coherently forming one sigle cluster from the beginning while the Bayesian/structural time series methods are close together.

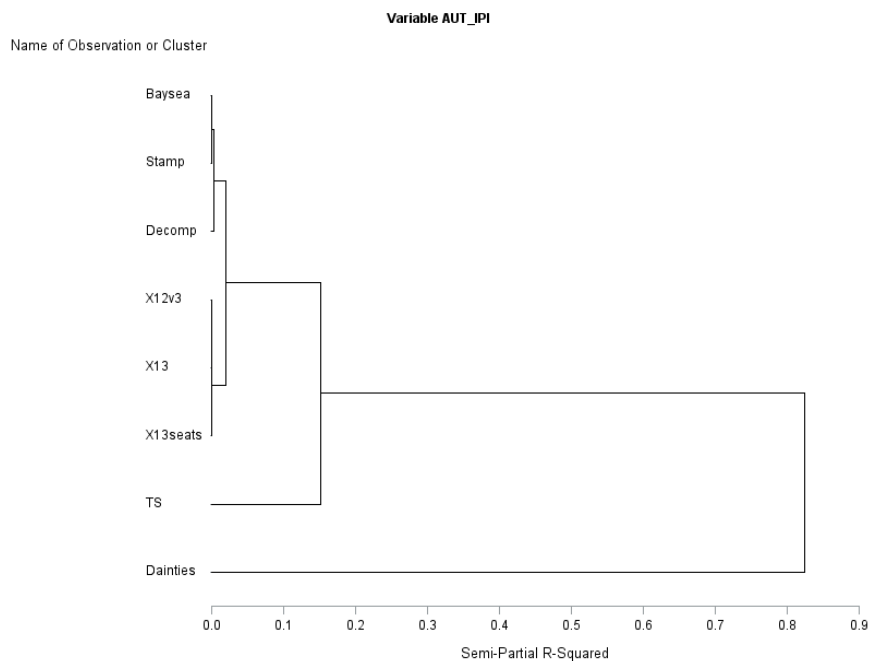
In the dendrogram for the industrial production of Germany (Figure 25.5) Tramo-Seats and the Bureau of Census methods soon join each other in the same cluster while Decomp appears to be further from the others, apart of course Dainties that is the last cluster to merge with the others.

Results are not univocal, it can happen that the Bureau of Census methods belong to different clusters and that Tramo-Seats join X13 Seats before the other members of the X family. The overall dendrogram is shown in figure 25.6. In this graph, in order to enhance the readability, the horizontal axis represents the number of clusters instead of the distances. When merging all the results evidences become consolidated. Dividing the set in four groups the subsets are the Bureau of Census family, even if X13 seats is a bit more far away, showing some similarity with Tramo-Seats; the Bayesian/Structural time series based methods, where Decomp is unexpectedly distant from Stamp more than Baysea; Tramo-Seats and, last and farthest, Dainties, the deterministic method.

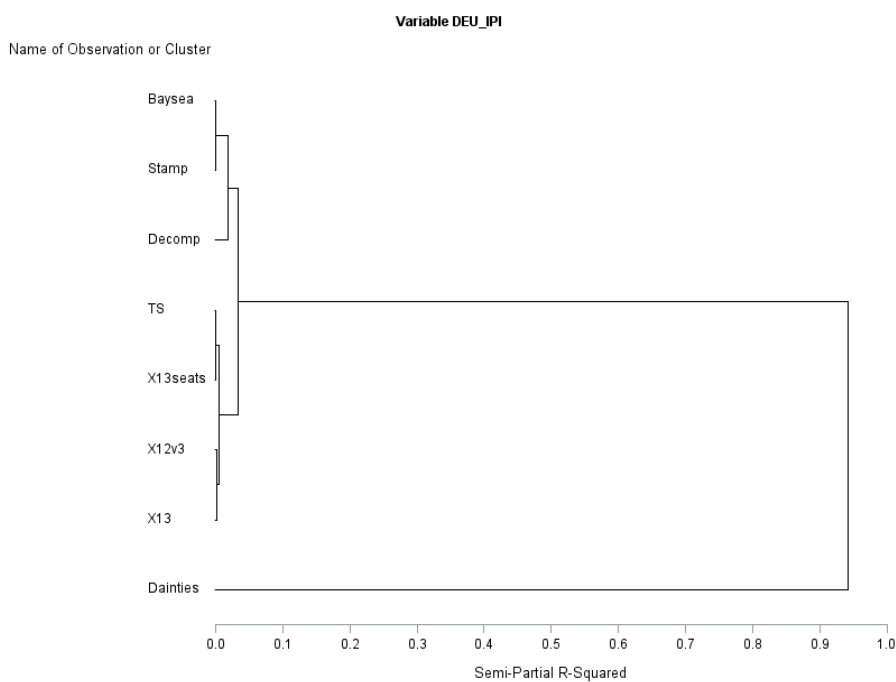
Main evidence from the graphical inspection and the cluster analysis are:

- Dainties is clearly the most idiosyncratic method
- Often TS and Bureau of Census methods perform in a very similar way
- The State Space based methods perform quite similar each other and, time to time, they appear close to the cluster of TS and BoC methods.

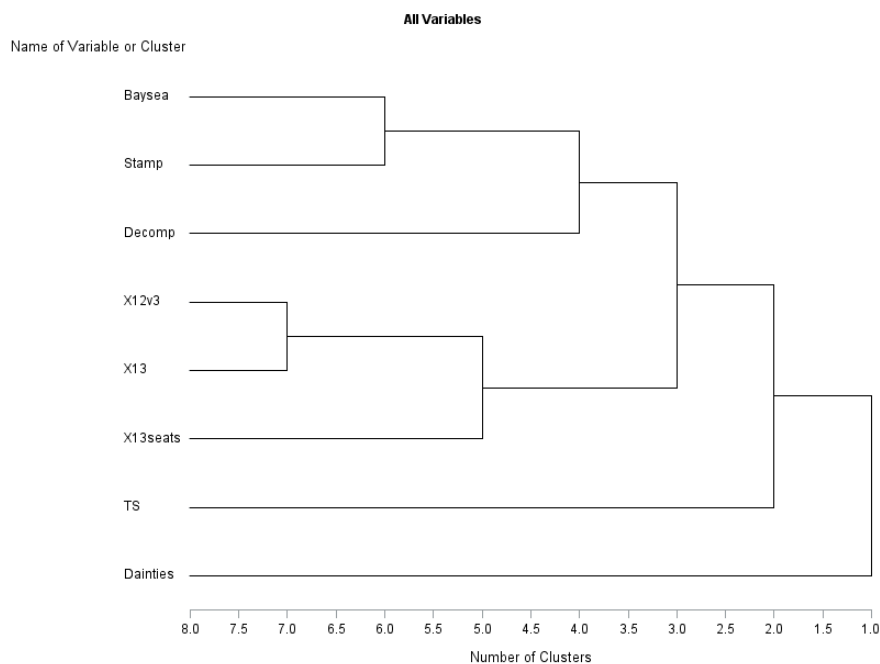
**Figure 25.4:** Cluster analysis dendrogram of the SA methods, Industrial production Austria



**Figure 25.5:** Cluster analysis dendrogram of the SA methods, Industrial production Germany





**Figure 25.6:** Cluster analysis dendrogram of the SA methods, all series together

## 25.5 Turning points dating analysis

Turning point dating has been run on all the series and methods by means of a Bry-Boschan algorithm. The analysis studied leading/lagging effects and missed/spurious cycles. In order to simplify this part the X12v3 results have been used as benchmark and all the other methods have been compared with them. In table 25.1 the results from X12v3 are detailed.

**Table 25.1:** Detailed analysis of the resulting historical dating for the benchmark method: X12v3

benchmark method: X12v3	Abs. Values
Number of series	23
Number of SA methods	8
Total number of observations	11304
Average number of obs. per series	491.5
Number of peaks in X12	227
Number of trough in X12	226
Number of turning points in X12	453
Average number of turning points per series in X12	19.7

In table 25.2 all the comparisons of the other seasonal adjustment methods with X12:

**Table 25.2:** Detailed analysis of the resulting historical dating, other methods vs. X12

	Abs. Values	%
<b>Baysea vs. X12v3</b>		
Concordance in no turning point observation	10794	95.49
Concordance in peaks	201	88.55
Concordance in trough	195	86.28
Concordance in TP	396	87.42
Average delay for non synchronous peaks	-0.57	
Average delay for non synchronous trough	0.62	
False peaks	1	
Missed peaks	3	
False trough	1	
Missed trough	2	
<b>Dainties vs. X12v3</b>		
Concordance in no turning point observation	10624	93.98
Concordance in peaks	117	51.54
Concordance in trough	118	52.21
Concordance in TP	235	51.88
Average delay for non synchronous peaks	-0.20	
Average delay for non synchronous trough	-0.19	
False peaks	9	
Missed peaks	6	
False trough	11	
Missed trough	3	
<b>Decomp vs. X12v3</b>		
Concordance in no turning point observation	10766	95.24
Concordance in peaks	189	83.26
Concordance in trough	176	77.88
Concordance in TP	365	80.57
Average delay for non synchronous peaks	-0.31	
Average delay for non synchronous trough	-0.10	
False peaks	1	
Missed peaks	2	
False trough	1	
Missed trough	3	
<b>Stamp vs. X12v3</b>		
Concordance in no turning point observation	10794	95.49
Concordance in peaks	201	88.55
Concordance in trough	195	86.28
Concordance in TP	396	87.42
Average delay for non synchronous peaks	-0.52	
Average delay for non synchronous trough	0.34	
False peaks	3	
Missed peaks	1	
False trough	0	
Missed trough	2	
<b>TS vs. X12v3</b>		
Concordance in no turning point observation	10758	95.17
Concordance in peaks	184	81.06
Concordance in trough	173	76.55

Concordance in TP	357	78.81
Average delay for non synchronous peaks	-0.17	
Average delay for non synchronous trough	0.00	
False peaks	1	
Missed peaks	1	
False trough	0	
Missed trough	3	
<hr/>		
X13 vs. X12v3		
Concordance in no turning point observation	10844	95.93
Concordance in peaks	224	98.68
Concordance in trough	222	98.23
Concordance in TP	446	98.45
Average delay for non synchronous peaks	-0.33	
Average delay for non synchronous trough	-1.00	
False peaks	0	
Missed peaks	0	
False trough	0	
Missed trough	0	
<hr/>		
X13Seats vs. X12v3		
Concordance in no turning point observation	10784	95.40
Concordance in peaks	195	85.90
Concordance in trough	192	84.96
Concordance in TP	387	85.43
Average delay for non synchronous peaks	0.06	
Average delay for non synchronous trough	-0.06	
False peaks	3	
Missed peaks	0	
False trough	0	
Missed trough	2	

Main evidence from the analysis of the resulting historical dating are:

- Dainties is again the most idiosyncratic method.
- Often TS and X13 Seats perform in very similar way.
- State Space methods are closer to X12 and X13 than TS and X13 Seats.
- All the other methods anticipate the peaks with respect to X12.

## 25.6 Conclusions

A comparison among eight seasonal adjustment methods has been conducted in order to detect a significant effect of the choice of the SA methods over the results of the business cycle analysis due to a non-complete orthogonality between the seasonal and the cyclical frequencies of a series or the inability of the SA methods to filter just the exact seasonal frequencies. Empirical evidences seem to confirm the hypothesis that the choice of the SA method can influence the outcome of the business cycle analysis. In particular, there is not fully concordance of findings when looking at cyclical shapes and at turning points location.

Some relevant findings can be addressed:

- Census Bureau SA methods always perform in similar way.
- Often results from Tramo-Seats and Census Bureau SA methods are close to each other.
- Danties has always an idiosyncratic behaviour, being the SA method more distant from all the others.

Further investigation should be devoted to the possibility of running the business cycle analysis over the raw data instead on the seasonal adjusted ones in order to get rid of eventual interferences. At least in order to have a benchmark to compare the results of the traditional, seasonal adjusted input based business cycle analysis.

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# 26

## The Effect of Seasonal Adjustment on Turning-Point Detection



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## Table of Contents

26.1	Introduction	683
26.2	Growth Cycle Coincident Indicator	683
26.2.1	Growth Cycle Coincident Indicator components	683
26.2.2	Revision policy	685
26.3	Coincident indicators from alternatively seasonal	685
26.3.1	Description of the time-series	685
26.4	Application	687
26.4.1	Research strategy	687
26.4.2	Evaluation criteria	687
26.4.3	Reference dating chronology of the growth cycle	688
26.4.4	GCCI: X-12-ARIMA, version 0.3, with the "automdl" specification	689
26.4.5	GCCI: X-12-ARIMA, version 0.3, with the "pickmdl" specification	692
26.4.6	GCCI: X-13-AS using X11 and the "pickmdl" specification	692
26.4.7	GCCI: Tramo-Seats	695
26.4.8	GCCI: Dainties	695
26.4.9	GCCI: Baysea	699
26.4.10	GCCI: Stamp	701
26.4.11	GCCI: Decomp	702
26.4.12	GCCI: Results discussion	704
26.5	Conclusions	706

## 26.1 Introduction

This chapter aims at assessing whether and to what extent coincident indicators (in particular the Growth Cycle Coincident Indicator<sup>1</sup>) hinge upon the seasonal adjustment procedures applied to the time-series that are used to construct them.

To accomplish this task, a comparative analysis is carried out between the coincident indicators that are released on a monthly basis among the Principal European Economic Indicators and a set of alternative coincident indicators that are obtained by replacing the usually employed component variables with the same variables but treated according to distinct seasonal adjustment methods. In particular, we consider eight different seasonal adjustment procedures, which are applied to the component variables of the Growth Cycle Coincident Indicator before these variables are modelled as Markov-switching models.

A dynamic rather than a static perspective is adopted in this analysis; in fact, the exercise is repeated for a set of pseudo vintage releases that are backcasted for each of the component variables of the Growth Cycle Coincident Indicator.

In Section 26.2, we briefly recall the component variables and the procedure followed to derive the Growth Cycle Coincident Indicator. Section 26.3 is concerned with a description of the alternative seasonal adjustment procedures that are used in this chapter to replicate the Growth Cycle Coincident Indicator. In Section 26.4 we construct as many alternative coincident indicators of the growth cycle as the number of seasonal adjustment procedures. Finally some conclusions are drawn.

## 26.2 Growth Cycle Coincident Indicator

First of all, we remind the variables that are involved in the construction of the Growth Cycle Coincident Indicator (hereafter referred to as the GCCI) and summarize the procedure followed to aggregate these component variables to form the coincident indicator.

On one hand, a statistical description of the variables on which is based the coincident indicator is necessary as the main goal pursued in this chapter is to grasp the role played by seasonal adjustment methods on the coincident indicators themselves; therefore, we need to be aware of the procedures actually used.

On the other hand, the presentation of the procedure used to combine these variables to get the GCCI is propaedeutic since the same approach will be applied to derive alternative coincident indicators of the growth cycle.

### 26.2.1 Growth Cycle Coincident Indicator components

For what regards the GCCI, the five variables used in its construction are the following:

- Industrial Production Index (IPI)
- Importations of intermediate Goods from outside the Euro area (ImportInter)
- Employment Expectations for the Months ahead in the Industry Survey (IndEExp)
- Construction Confidence Indicator in the Building Survey (BuildCI)
- Financial Situation over last 12 Months in the Consumer Survey (ConsFS)

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<sup>1</sup>See Anas et al. (2007) and Anas et al. (2017).



**Table 26.1:** Variables employed in the construction of the GCCI

	IPI	ImportInter	IndEEExp	BuildCI	ConsFS
Unit	Volume Index (2005 = 100)	Volume Index (2000 = 100)	Balance of Opinions	Balance of Opinions	Balance of Opinions
Statistical Treatment	SWDA	SWDA	SA	SA	SA
Starting Date	1990:01	2000:01 (1989:01)	1985:01	1985:01	1985:01
Timeliness Data Transformation	2-month delay Difference over 6 months of the growth rate over 12 months	3-month delay Difference over 6 months of the growth rate over 12 months	1-month delay Difference over 6 months	1-month delay Difference over 6 months	1-month delay Difference over 6 months
Markov switching Model	MSI(2)-AR(0)	MSI(2)-AR(0)	MSI(2)-AR(0)	MSI(2)-AR(0)	MSI(2)-AR(0)
Weight	0.20	0.20	0.20	0.20	0.20

26.1 reports in more detail a description of each of variable. We stress the fact that the ImportInter time-series drawn from the EuroInd database starts in January 2000 and, following the approach proposed by Caporin and Sartore (2006), it is historically reconstructed back to January 1989.

As far as the two macroeconomic variables related to the real economy, namely, the IPI and ImportInter, they are drawn already seasonally and working days adjusted from the EuroInd database. For what regards the three surveys, they are drawn already seasonally adjusted from the DG-EcFin database.

For what regards the timeliness entry in Table 26.1, it denotes the delay between the issuance month and the month of the last available observation; for instance, a delay of two months means that a time-series released in month  $t$  contains observations up to month  $(t - 2)$ .

Among the five variables that makes up the GCCI, the most delayed one is ImportInter, namely, it is 1- and 2-month lagged with respect to the IPI and the three surveys, respectively. In order not to further worsen the timeliness of the GCCI, an estimate for month  $(t - 2)$  is provided for the ImportInter; it is obtained by resorting to the statistical properties of the Markov-switching model that is presented below.

In the lower panel of Table 26.1, we detail the transformation that is taken for each time-series before a Markov-switching model is fitted; as for the model specification, for all the five variables we specify a homoskedastic Markov-switching model whose state-variable is a Markov chain of the first order that assumes values only in the discrete finite set 0, 1. As a by-product of the estimation process, which is performed by means of the Expectation Maximization (EM) algorithm, we get filtered and smoothed probabilities. To be consistent with the coincident timeliness of the indicator, filtered rather than smoothed probabilities are used to infer the probabilities of being in a contraction phase of the growth cycle. Since the latent state-variable has only two regimes, the regime that returns the lowest value for the state-dependent intercept coefficient is assumed to identify contraction phases of the growth cycle. Finally, the GCCI is obtained as an equally weighted average of the contraction probabilities stemming from each of its five component variables, as it is shown in formula 26.1 below:

$$GCCI_t = \frac{1}{5} \sum_{k=1}^5 PR(\text{Contraction})_t^k, \quad (26.1)$$

where  $PR(\text{Contraction})_t^k$  is the probability that the  $k$ -th component of the GCCI is in a contraction phase of the growth cycle at time  $t$ , with  $k \in \{1, 2, 3, 4, 5\}$ .

## 26.2.2 Revision policy

Formula 26.1 is used to compute the GCCI. Nonetheless, it must be said that the coincident indicator released in a given month is actually made up of two parts. More in detail, consider the coincident indicator issued in a given month  $t$ ; due to the time-delay of its component variables, it contains contraction probabilities until month  $(t - 2)$ . This coincident indicator is the result of the merge of two sets of contraction probabilities. The first one ranges from the first available month for which the coincident indicator is estimated (July 1991) to month  $(t - 8)$ ; the second set covers the remaining period, that is, the six observations from month  $(t - 7)$  to month  $(t - 2)$ ; to this second set belong the contraction probabilities computed using the information available in month  $t$ , whereas the first set contains the contraction probabilities estimated in the previous monthly assessments, that is, resorting to the information available in month  $(t - 1)$  and before. This composition is valid for all the sequence of coincident indicators that are released on a monthly basis, which means that the contraction probabilities estimated for the last six months of the sample covered by the coincident indicator issued on a given month are replaced when the next coincident indicators are released, whereas the remaining probabilities remain unchanged.

## 26.3 Coincident indicators from alternatively seasonal adjusted time-series

### 26.3.1 Description of the time-series

The procedures described in the previous section are also followed to construct a set of alternative coincident indicators of the growth cycle. We refer to this coincident indicators as alternative because, even though, they are derived by aggregating the same component variables used in the GCCI, these time-series are treated with different seasonal adjustment methods.

In this section we briefly present time-series and the seasonal adjustment methods applied to them. For what regards the time-series, they are the same variables previously listed when describing the GCCI. However, they differ in terms of either the time frame covered, that is, in the number of observations available or the seasonal adjustment procedure according to which they are treated. Table 26.3 below resembles Table 26.1 and presents the time-series that will be used to derive a set of alternative coincident indicators of the growth cycle.

The (Euro area 16) ImportInter has been backcasted up to January 1995, using the longer Euro area 12 aggregate.

When analysing in comparative terms data reported in Table 26.1 and Table 26.3, some differences arise. In particular, the ImportInter time-series employed in the construction of the (usual) GCCI starts in January 1989, whereas the one backcasted starts only in January 1995. This exerts a major effect on the respective coincident indicators to which they will be included; in fact, besides the effect on the estimated contraction probabilities due to the different information set, the shorter estimation sample constrains the first date for which the coincident indicators can be computed. In fact, while the GCCI includes probabilities of being in a contraction phase of the growth cycle since July 1991, the coincident indicators that includes the shorter ImportInter time-series can start only in July 1996. This time difference will be taken into account in the remaining.

We conclude this section by presenting the different seasonal adjustment methods that have been considered to treat the time-series. As most of the seasonally adjusted methods do not handle very well outliers and calendar effects, time-series are first corrected from these effects. The data are linearized using X12-ARIMA. The outlier component is kept and will be re-introduced later in the time-series once this has been seasonally

**Table 26.2:** Variables employed in the construction of the alternative coincident indicators of the growth cycle.

	IPI	ImportInter	IndEExp	BuildCI	ConsFS
Data Source	EuroInd	EuroInd	DG-EcFin	DG-EcFin	DG-EcFin
Unit	Volume Index (2005 = 100)	Volume Index (2000 = 100)	Balance of Opinions	Balance of Opinions	Balance of Opinions
Statistical Treatment	NSA	NSA	NSA	NSA	NSA
Starting Date	1990:01	1999:01 (1995:01)	1985:01	1985:01	1985:01
Timeliness	2-month delay	3-month delay	1-month delay	1-month delay	1-month delay
Data Transformation	Difference over 6 months of the growth rate over 12 months	Difference over 6 months of the growth rate over 12 months	Difference over 6 months	Difference over 6 months	Difference over 6 months
Markov switching Model	MSI(2)-AR(0)	MSI(2)-AR(0)	MSI(2)-AR(0)	MSI(2)-AR(0)	MSI(2)-AR(0)
Weight	0.20	0.20	0.20	0.20	0.20

adjusted. This step is performed on the available span of data. Each linearized series is seasonally adjusted using eight different seasonal adjustment methods or procedures. These are listed as follows:

- X-12-Arima version 0.3, according to the "*automdl*" specification
- X-12-Arima version 0.3, following the "*pickmdl*" specification
- X-13-AS using X11 and the "*pickmdl*" specification <sup>2</sup>
- Tramo-Seats
- Dainties
- Baysea
- Stamp
- Decomp

For the first step, the longest available sample is considered for each variable; the starting date of the variables are reported in the tables above, whereas the last observation available for all of them is in July 2009. Once the seasonally adjusted series are computed, the outlier component removed in the first step of the procedure, be either additive or multiplicative, is re-introduced in the seasonally adjusted series.

As vintage time-series are not available for all the variables considered, the "X12 history specification" is mimicked by coming back 3 years in the past and then by adding a new observation at each month as we move ahead in time. As a result, we get 36 pseudo vintage releases for each of the component variables of the coincident indicator. As we will make clear in the next section, this allows us to perform a pseudo real-time estimation to compare to the GCCI actually released in real-time from April 2007 to September 2009.

<sup>2</sup>It is worth noting that the "*automdl*" specification is discarded due to the many problems it brought.

## 26.4 Application

In this section, we derive as many alternative coincident indicators of the growth cycle, as the number of seasonal adjustment methods that are applied to their component time-series.

Before presenting results, we stress two points: the first one merely concerns the terminology, that is, with the term "*alternative*" we mean that these coincident indicators are built on the same variables appearing in the GCCI, but these time-series are treated with different seasonal adjustment methods than the procedure actually employed for the variables that made up the GCCI. The second point makes clear the biunique relation between the alternative coincident indicators and the seasonal adjustment methods; in fact, a coincident indicator is built on time-series that are all seasonally adjusted with the same method, that is, as far as possible, we do not construct a coincident indicator whose component variables are treated with different seasonal adjustment procedures. This choice is motivated by the need to reduce the number of possible alternative coincident indicators; in fact, if we allow coincident indicator to include time-series treated with different seasonal adjustment procedure, there is no reason for disregarding any given combination of all the possible ones, which are  $8^5 = 32,768$  for the coincident indicator of the growth cycle. It immediately follows that these great numbers cannot be easily handled.

### 26.4.1 Research strategy

In accordance with our purpose of assessing whether and what role the seasonal adjustment procedure plays on the coincident indicators of the growth cycle, our research strategy is based on comparing the GCCI that was actually released with a set of alternative coincident indicators constructed following eight different procedures to remove the seasonal component. In doing so, as stated in the introduction, we adopt a dynamic approach in the sense that the comparison above is extended to the 30 releases of the GCCI released from April 2007 to September 2009. These releases are compared with the alternative coincident indicators of the growth cycle we compute resorting to the pseudo real-time data base described in the previous section. This data base provides us the possibility to estimate a sequence of 36 coincident indicators of the growth cycle, which corresponds to the coincident indicators that would have been computed between April 2007 and September 2009 if this exercise were actually performed in real-time.

### 26.4.2 Evaluation criteria

We present the criteria that are used to appraise the effect of the seasonal adjustment method on the coincident indicators. The first of them is the classical Quadratic Probabilistic Score of Brier (1950), hereafter shortened as QPS, which is defined as follows:

$$QPS = \frac{1}{T} \sum_{t=1}^T (P_t - RC_t)^2, \quad (26.2)$$

where, for  $t \in 1, \dots, T$ ,  $P_t$  is the filtered probability of being in contraction in month  $t$  and  $RC_t$  is a random variable that is equal to 1 during contractionary phases of the growth cycle and 0 otherwise, according to the reference chronology.

The Concordance Index is the second measure considered; following Harding and Pagan (2002), it is defined as follows:

$$CI = \frac{1}{T} \left[ \sum_{t=1}^T I_t \times RC_t + \sum_{t=1}^T (1 - I_t) \times (1 - RC_t) \right], \quad (26.3)$$

where  $RC_t$  is the same variable already employed in the QPS, which represents the turning points of the reference chronology, while  $I_t$  is a binary random variable that assumes value 1 if the coincident indicator is in the contractionary phase of the growth cycle and 0 otherwise.

Moreover, we explicitly take into account two more statistics that, in our opinion, are helpful in overcoming the drawbacks inherent to the QPS and Concordance Index and that allow us to investigate at a deeper level the causes of the values provided by the QPS and Concordance Index. The first one (hereafter referred to as the Lag) counts the number of months by which the turning point dating stemming from a given coincident indicator lags behind the reference dating chronology. According to its definition, this statistic measures by how many months the peaks are delayed with respect to the ones pinpointed by the reference dating chronology. However, it must be stated that this statistic is unable to distinguish between a delay in the detection of a peak and a missed contraction, as it also increases with the months of missed contractions.

If we take into account only this statistic we could incur in the error of judging positively a coincident indicator that is not lagged with respect to the reference dating but that returns a high number of false contraction signals. Consider for instance the paradoxical situation of a coincident indicator that conveys a contraction signal throughout the whole sample: in this case, the count of the lags is zero, but the coincident indicator is clearly unable to deal with economic cycles. In order to avoid being misled, a second statistic is introduced (hereafter referred to as the Excess): it counts the number of months for which a given coincident indicator does emit a contraction signal, while the reference chronology does not. According to its definition this statistics takes into account by how many months the troughs obtained from a coincident indicator lag behind the trough dates of the reference dating chronology. However, it should be stressed that this statistics increases as a false contraction signal is emitted and therefore it is unable to separate out this feature from the lags of the troughs.

Due to the dynamic perspective adopted in our analysis, the statistics are computed for each of the 30 releases of the GCCI and also for the sequence of 36 alternative coincident indicators obtained as a result of the pseudo real-time exercise described above.

Moreover, the mean and the standard deviation of these statistics are computed. By taking into account a measure of the volatility we are able to individuate whether a coincident indicator produces estimates of the contraction probabilities that vary considerably while moving through the sequence of monthly releases. The same statistics are provided also for each component of the coincident indicators, in such a way we can get an insight into the causes of the pattern shown by the coincident indicators and also to gauge, at a more detailed level, the effect of the different seasonal adjustment procedures on each component.

However, we must advise that the statistics are not provided with a confidence interval, which could allow us to judge the significance of the statistics themselves and the differences between them.

### 26.4.3 Reference dating chronology of the growth cycle

Before moving to the application part of this report, it is worth noting that we adopt as reference dating chronology, that is, as benchmark to compute the statistics described above, a dating chronology that is built by merging a final dating chronology of the growth cycle and a provisional one. The final dating chronology is the one proposed in Anas et al. (2007) and we denominate it final because it is not subject to be revised. However, this dating chronology is limited to the time horizon between 1971 and 2002, therefore it is not useful to assess the ability of coincident indicators in picking out turning points of the last period. As stated above, this problem is overcome by merging the final dating chronology with a provisional one. The latter dating chronology was issued in October 2009<sup>3</sup>. Contrary to the final dating chronology, this one is only provisional, which means it is subject to be revised as new releases will be available. The provisional dating chronology of the growth cycle, in the period not covered by the (final) dating chronology, locates a peak in February 2008. Each of the eight seasonal adjustment procedures previously described is used to remove the seasonal component from all the five variables that are involved in the construction of the GCCI. As previously

<sup>3</sup>"Current cyclical assessment of the Euro zone economy: Autumn assessment 2009".

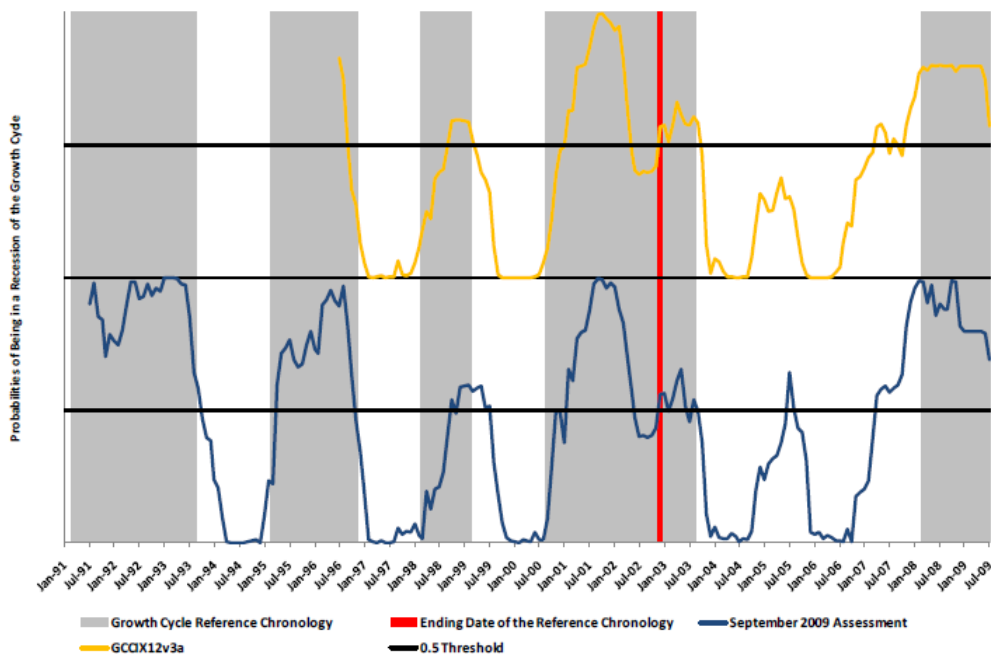
illustrated, a 2-regime Markov-switching model is fitted to each of these seasonally adjusted time-series and the resulting contraction probabilities are combined following formula 26.1 so as to obtain as many coincident indicators of the growth cycle as the number of seasonal adjustment procedures considered.

#### 26.4.4 GCCI: X-12-ARIMA, version 0.3, with the "automdl" specification

The first method we consider to remove the seasonal component is the X-12-ARIMA, in its version 0.3, with the "automdl" specification. For ease of exposition, the coincident indicator that is obtained using this method is shortly labelled after its specification as GCCI(X12v3a).

In order to get a first insight of the GCCI(X12v3a) we graphically represent it in the upper panel of Figure 26.1 and compare it with the GCCI that was actually released in September 2009 (hereafter referred to as the September 2009 GCCI), which is traced in the lower panel. In both cases, the coincident indicator that is shown in Figure 26.1 is the last one of its respective sequence; as such, according to the revision policy put in place, its last six observations are the contraction probabilities computed using the information available in September 2009, whereas the remaining contraction probabilities are the ones estimated with the information available in previous months.

**Figure 26.1:** Probabilities of being in contraction of the growth cycle either from July 1991 to July 2009 as they are produced by the September 2009 GCCI (lower panel) and from July 1996 to July 2009 as they are returned by the GCCI(X12v3a) (upper panel).



Due to the time constraint imposed by the ImportInter, which is the component with the shortest available sample, the GCCIX12v3a starts only in July 1996, whereas the GCCI(X12v3a) released in September 2009 starts in July 1991.

Figure 26.2 replicates the same comparison, but contraction signals implied by the coincident indicators are shown instead of the contraction probabilities. Contraction signals are obtained by filtering the contraction probabilities following to the 0.5 "natural rule", according to which a contraction signal (1) is emitted at time  $t$  if the coincident indicator at time  $t$  exceeds the 0.5 threshold, otherwise a no contraction signal (0) is conveyed.

A graphical inspection of Figure 26.1 and Figure 26.2 suggests that, throughout the overlapping period from July 1996 to July 2009, the GCCI(X12v3a) resembles quite well the September 2009 GCCI. In particular,

when compared to the reference dating chronology, on one hand, both of them<sup>4</sup> anticipate the occurrence of the peak that precedes the last contraction of the growth cycle to March 2007, whereas February 2008 is the peak date provided by the reference dating chronology; on the other hand, both of them are delayed with respect to the reference dating chronology in detecting the peak of the contraction triggered by the Asian crisis and the peak of the long-lasting slowdown between 2000 and 2003.

However, some differences also arise; more in detail, the September 2009 GCCI detects a short-lived contraction signal between July and August 2005, which is not located in the reference dating chronology. Moreover, on one hand, the September 2009 GCCI postpones more than what the GCCI(X12v3a) does the trough date that, according to the reference dating chronology, concludes the contraction period between February 1998 and February 1999. However, on the other hand, the GCCI(X12v3a) locates in September 1996 the trough of the growth cycle that the reference dating chronology identifies in November 1996 and the September 2009 GCCI in October 1996.

**Figure 26.2:** Contraction signals of the growth cycle either from July 1991 to July 2009 as they are implied by the September 2009 GCCI (lower panel) and from July 1996 to July 2009 as they are implied by the GCCI(X12v3a) (upper panel).

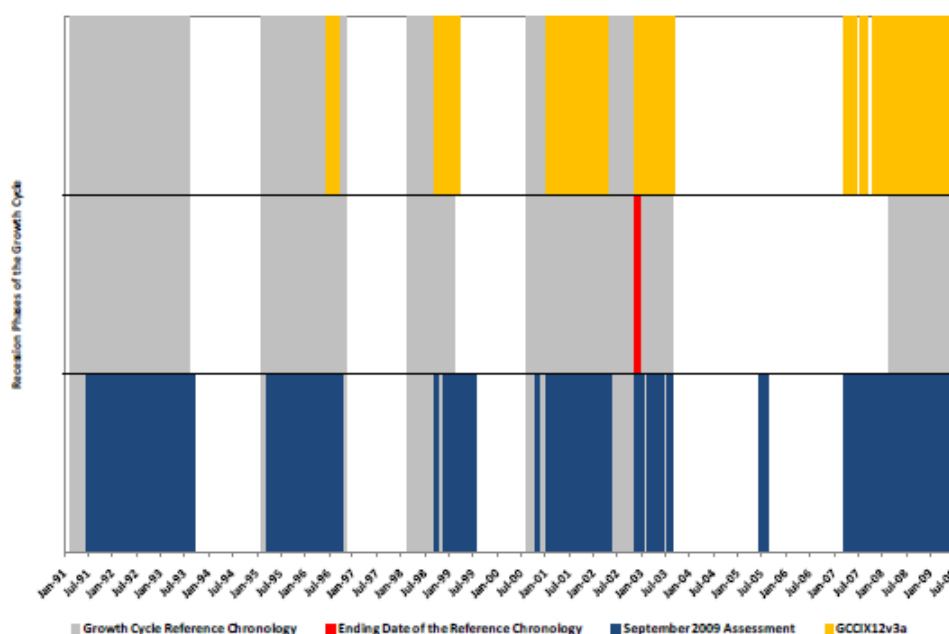


Table 26.5 reports the mean and standard error of the four statistics previously described, namely, the QPS, Concordance Index (CI), Lag and Excess, when they are computed for the (usual) GCCI. More in detail, the upper panel shows these statistics for the GCCI. It is worth noting that we provide these statistics for the sequence of coincident indicators actually released from April 2007 to September 2009, that is, the coincident indicators that are updated according to the revision policy previously described; furthermore, the statistics above are provided also for the sequence of coincident indicators as they are computed (not released) in each month, that is, by aggregating the contraction probabilities estimated in this same month. The lower panel is devoted to the five component variables of the coincident indicator. In this case, we propose the mean and standard deviation of the four selected criteria without applying the above revision mechanism.

The first column lists the coincident indicators and the component variables. The time frame over which the statistics are computed is shown in the second column; to facilitate the comparison with the counterpart statistics of the GCCI(X12v3a) in Table 26.3, we consider two time frames: the first time frame is the longest available one over which contraction probabilities can be computed and it varies from coincident indicator and

<sup>4</sup>Nevertheless, the signal conveyed by the GCCI(X12v3a) in the first months of the slowdown is not as constant as the one emitted by the September 2009 GCCI, indeed, the former coincident indicator fluctuates above and below the 0.5 threshold.



**Table 26.3:** Growth Cycle Coincident Indicator actually released/computed from April 2007 to September 2009 : Mean and standard deviation of the QPS, Concordance Index, Lag and Excess statistics.

GCCl - Components	Time Frame	QPS		CI		Lag		Excess	
		Mean	StD	Mean	StD	Mean	StD	Mean	StD
GCCl (as computed)	1991:7 - 2009:7	0.123	0.010	0.815	0.018	21.9	6.8	15.7	5.7
	1996:7 - 2009:7	0.150	0.010	0.754	0.021	20.5	6.1	14.7	5.7
GCCl (6-month revised)	1991:7 - 2009:7	0.122	0.010	0.811	0.014	22.0	0.0	16.3	4.0
	1996:7 - 2009:7	0.151	0.010	0.746	0.019	21.0	0.0	15.3	4.0
IPI	1991:7 - 2009:7	0.211	0.018	0.749	0.021	25.5	31.4	25.4	15.0
ImportInter	1990:7 - 2009:7	0.276	0.014	0.656	0.018	40.5	7.1	33.5	8.9
	1996:7 - 2009:7	0.304	0.023	0.619	0.030	28.0	5.9	33.5	8.8
IndEExp	1985:7 - 2009:7	0.187	0.017	0.781	0.019	36.0	20.0	24.2	4.0
BuildCI	1985:7 - 2009:7	0.195	0.009	0.785	0.010	35.6	0.9	23.5	0.7
ConsFS	1985:7 - 2009:7	0.21	0.010	0.754	0.011	51.1	0.6	16.5	0.5

**Table 26.4:** Coincident Indicator if the growth cycle whose component variables are seasonally adjusted by applying the X-12-Arima version 0.3, according to the "automdl" specification: Mean and standard deviation of the QPS, Concordance Index, Lag and Excess statistics.

GCCl - Components	Number of Releases	QPS		CI		Lag		Excess	
		Mean	StD	Mean	StD	Mean	StD	Mean	StD
GCCl(X12v3a) (as computed)	From 1 to 36	0.131	0.012	0.789	0.022	22.8	6.6	6.8	4.7
	From 7 to 36	0.134	0.012	0.783	0.022	23.4	7.1	7.7	4.7
IPI	From 1 to 36	0.217	0.020	0.749	0.026	25.8	14.4	24.5	10.6
	From 7 to 36	0.223	0.020	0.749	0.026	26.6	15.7	25.8	11.2
ImportInter	From 1 to 36	0.260	0.030	0.690	0.033	27.3	8.2	16.1	8.0
	From 7 to 36	0.270	0.030	0.679	0.033	28.1	8.7	17.6	7.8
IndEExp	From 1 to 36	0.179	0.004	0.794	0.006	37.0	3.6	18.8	3.4
	From 7 to 36	0.178	0.004	0.796	0.006	37.3	3.9	18.6	3.7
BuildCI	From 1 to 36	0.194	0.010	0.778	0.012	41.3	1.6	19.2	5.1
	From 7 to 36	0.197	0.010	0.774	0.012	41.3	1.8	20.8	3.9
ConsFS	From 1 to 36	0.253	0.011	0.723	0.014	62.1	4.8	13.2	2.0
	From 7 to 36	0.254	0.011	0.721	0.014	63.1	4.7	13.6	1.8

component variables, the period between July 1996 and July 2009 is the second time frame and it is chosen to be of the same length as the longest sample over which the GCCl(X12v3a) is available, this choice is made in order to allow a fair comparison between the (usual) GCCl and the GCCl(X12v3a). The remaining eight columns reports the mean and standard error of the QPS, Concordance Index, Lag and Excess statistics.

To foster the comparison between the (usual) GCCl and the GCCl(X12v3a), statistics that are directly comparable, having been computed over the same time frame, are highlighted in bold text .

We do not directly comment figures reported in Table 26.3 but, coherently to the comparative perspective adopted, we discuss them in comparative terms with the values shown in Table 26.4.

Statistics for the GCCl(X12v3a) are reported in Table 26.4. This table has a structure that is almost identical to the one of Table 26.3, the only difference is in the second column, where we report the number of releases over which the statistics are computed. Also in this case, we highlight in bold text the statistics that can be fairly compared with the ones related to the (usual) GCCl, that is, the statistics that are computed over the last 30 pseudo vintage releases, therefore discarding the first six backcasted releases which range from August 2006 to March 2007.

Figures contained in Table 26.3 and Table 26.4 confirms the insight gained by graphical inspection. On one hand, as far as the timely detection of peaks and/or missing cycles are concerned, both the (usual) GCCl and the GCCl(X12v3a) produce quite close values of the Lag statistic. Actually, as far as the policy revision is



concerned, the Lag statistic assumes value equal to 21.0 for both coincident indicators. On the other hand, when the timely detection of troughs and/or false signals are concerned, the (usual) GCCI returns a value of the Excess statistic that almost doubles the one provided by the GCCI(X12v3a), namely, 15.3 versus 8.9. The graphical evidence suggests that this can be explained with the fact that the former coincident indicator postpones the occurrence of the trough of the Asian crisis recession and it detects a false contraction signal between July and August 2009. Overall, the GCCI(X12v3a) is characterized by both a lower (mean) QPS and higher (mean) Concordance Index than the (usual) GCCI, while returning close values for the standard deviation of these statistics.

Moving to a more disaggregated level, four out of the five component variables treated with the X-12-ARIMA, version 0.3, with the "*automdl*" specification, that is, all the component variables except the IPI, produce values of the Excess statistic that are on average lower than the ones returned by the same variables comprised in the (usual) GCCI.

What we cannot infer by graphical inspection is the stability of the signal over time. In order to get a measure of the persistence of the signal conveyed either by the coincident indicators and by their components, we adopt an indirect approach and take into exam the coherence over successive releases of the four statistics as measured by their standard deviation. As far as the coincident indicators are concerned, three out of the four statistics computed for the (usual) GCCI have a standard deviation that is not lower than the respective value associated to the GCCI(X12v3a). This higher volatility is mainly due to the IndEExp variable, in fact, with the exception of the Excess statistic, the remaining three statistics computed for the (usual) IndEExp have a standard deviation that is at least three times higher than the value returned by the same statistic for the IndEExp(X12v3a), where we use the same notation introduced for the alternative coincident indicator, therefore IndEExp(X12v3a) denotes the IndEExp time-series treated with the X-12-ARIMA, version 0.3, with the "*automdl*" specification.

#### 26.4.5 GCCI: X-12-ARIMA, version 0.3, with the "*pickmdl*" specification

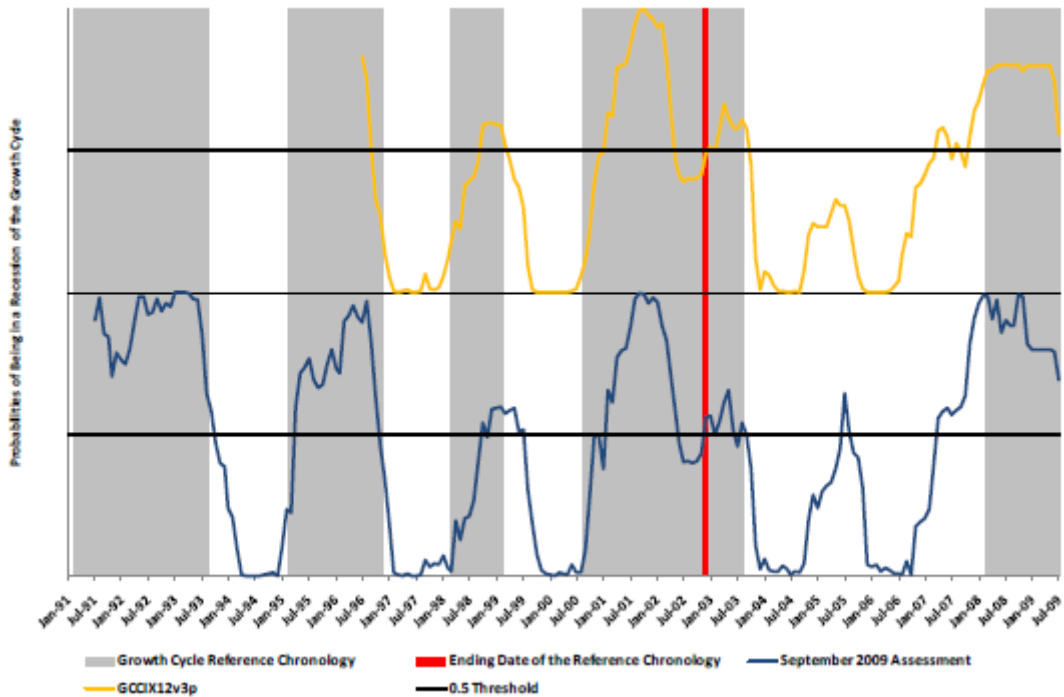
The X-12-ARIMA in its version 0.3 with the "*pickmdl*" specification is the second method of seasonal adjustment that is applied to remove the seasonal component from the five component variables of the alternative coincident indicator of the growth cycle. The alternative coincident indicator of the growth cycle whose component variables are seasonally adjusted following this method is labelled GCCI(X12v3p). The pattern of the GCCI(X12v3p) between July 1996 and July 2009 is traced in the upper panel of Figure 26.3. In the lower panel we show the September 2009 GCCI, which ranges from July 1991 to July 2009. The pattern of the GCCI(X12v3p) almost matches the trajectory of the GCCI(X12v3a), therefore we refer the reader to the discussion on the former coincident indicator. The 0.5 "*natural rule*" is applied to the coincident indicators above in order to transform continuous contraction probabilities into binary signals of being (1) or not (0) in a contraction phase of the growth cycle. These contraction signals are depicted in Figure 26.4. The close similarity between the GCCI(X12v3a) and GCCI(X12v3p) obviously recurs also when the contraction signals stemming from them are considered. Therefore, also in this case, we refer the reader to the previous discussion on the GCCI(X12v3a).

Table 26.5 reports the QPS, Concordance Index, Lag and Excess statistics computed for the GCCI(X12v3p) and its five components. Although not completely coincident with the figures previously shown in Table 26.4, values in Table 26.5 are very close to the ones discussed for the GCCI(X12v3a), to which the reader is referred to for a detailed explanation.

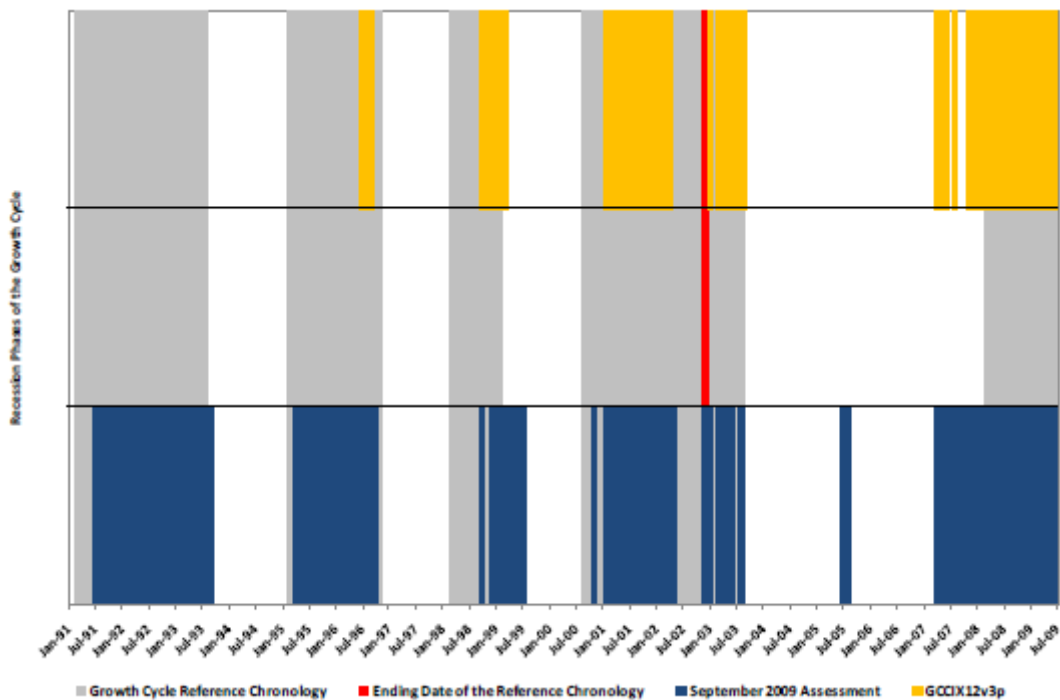
#### 26.4.6 GCCI: X-13-AS using X11 and the "*pickmdl*" specification

The third coincident indicator is the one obtained by seasonally adjusting its five component time-series by resorting to the X-13-AS method, using X11, with the "*pickmdl*" specification. This coincident indicator is

**Figure 26.3:** Probabilities of being in a contraction phase of the growth cycle either from July 1991 to July 2009 as they are produced by the September 2009 GCCI (lower panel) and from July 1996 to July 2009 as they are returned by the GCCI(X12v3p) (upper panel).



**Figure 26.4:** Contraction signals of the growth cycle either from July 1991 to July 2009 as they are implied by the September 2009 GCCI (lower panel) and from July 1996 to July 2009 as they are implied by the GCCI(X12v3p) (upper panel).



shortened as GCCI(X13asp). The pattern of the GCCI(X13asp) between July 1996 and July 2009 is traced in the upper panel of Figure 26.5. In the lower panel we show the September 2009 GCCI, which ranges from July

**Table 26.5:** Coincident Indicator if the growth cycle whose component variables are seasonally adjusted by applying the X-12-Arima version 0.3, according to the "pickmdl" specification: Mean and standard deviation of the QPS. Concordance Index, Lag and Excess statistics.

GCCl - Components		Number of Releases	QPS		CI		Lag		Excess	
			Mean	StD	Mean	StD	Mean	StD	Mean	StD
GCCl(X12v3p)	(as computed)	From 1 to 36	0.131	0.012	0.787	0.021	23.5	6.2	6.4	4.3
		From 7 to 36	0.133	0.012	0.783	0.021	23.9	6.8	7.2	4.3
	(6-month revised)	From 1 to 36	0.128	0.006	0.784	0.013	23.0	0.0	7.3	3.4
		From 7 to 36	0.130	0.006	0.780	0.013	23.0	0.0	8.3	2.7
IPI	1991:7 - 2009:7	From 1 to 36	0.217	0.020	0.749	0.025	25.7	14.2	24.7	10.6
		From 7 to 36	0.223	0.020	0.742	0.025	26.5	15.5	26.0	11.2
ImportInter	1990:7 - 2009:7	From 1 to 36	0.260	0.030	0.690	0.033	27.3	8.2	16.1	8.0
		From 7 to 36	0.270	0.030	0.679	0.033	28.1	8.7	17.6	7.8
IndEEExp	1985:7 - 2009:7	From 1 to 36	0.178	0.004	0.798	0.006	38.3	3.2	16.6	2.5
		From 7 to 36	0.177	0.004	0.800	0.006	38.5	3.4	16.5	2.7
BuildCI	1985:7 - 2009:7	From 1 to 36	0.194	0.010	0.778	0.012	41.3	1.6	19.2	5.1
		From 7 to 36	0.197	0.010	0.774	0.012	41.3	1.8	20.8	3.9
ConsFS	1985:7 - 2009:7	From 1 to 36	0.253	0.011	0.721	0.012	62.6	4.3	13.2	1.9
		From 7 to 36	0.255	0.011	0.720	0.012	63.4	4.3	13.6	1.8

1991 to July 2009. By comparing the upper panel of Figure 26.1, Figure 26.3 and Figure 26.5, it turns out that the GCCl(X12v3a), the GCCl(X13asp) and the GCCl(X13asp) almost match one another. It follows that, for ease of exposition, we refer the reader to the discussion carried out for the GCCl(X12v3a).

**Figure 26.5:** Probabilities of being in a contraction phase of the growth cycle either from July 1991 to July 2009 as they are produced by the September 2009 GCCl (lower panel) and from July 1996 to July 2009 as they are returned by the GCCl(X13asp) (upper panel).

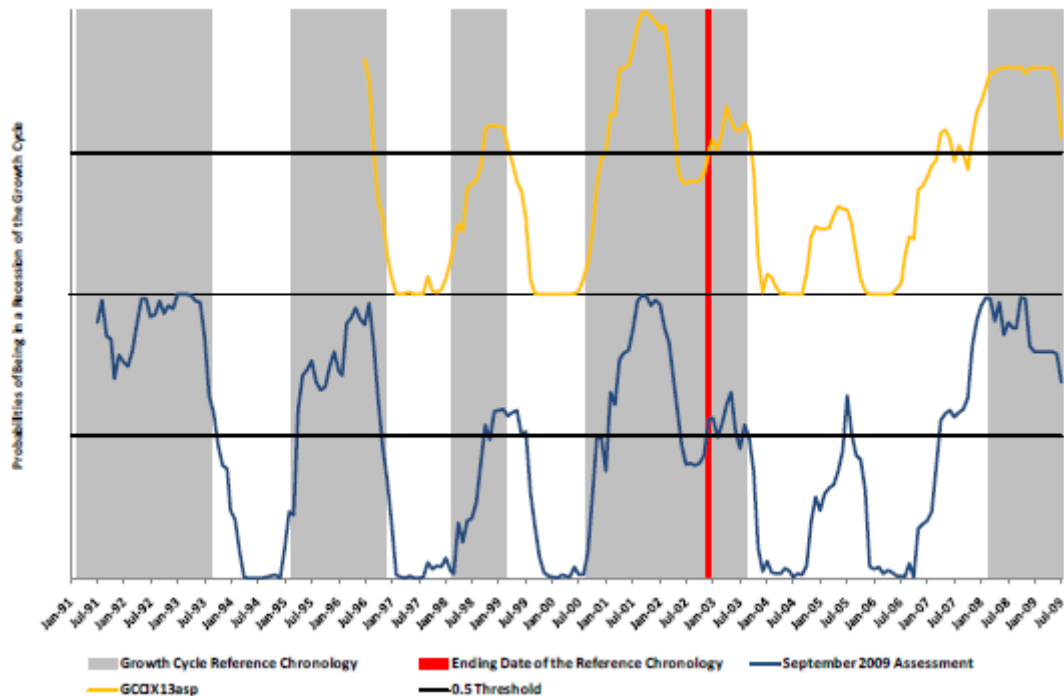
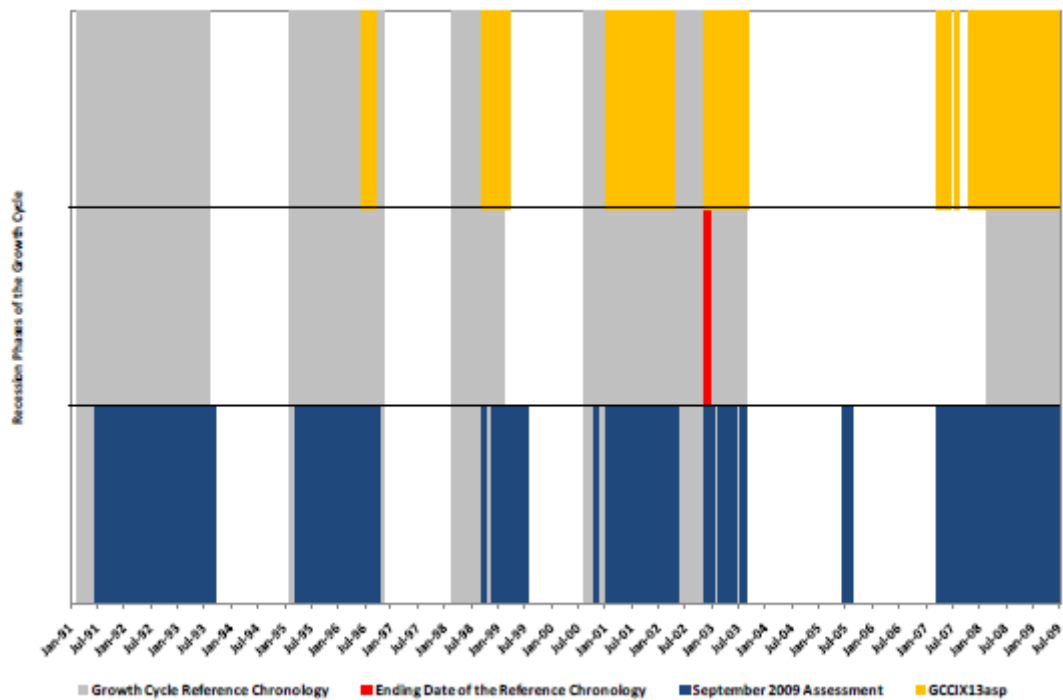


Figure 26.6 below shows the contraction phases obtained by filtering the GCCl(X13asp) and (usual) GCCl with the 0.5 "natural rule".

Table 26.6 below, where the QPS, Concordance Index, Lag and Excess statistic are reported, confirms the close similarity among the GCCl(X12v3a), GCCl(X12v3p) and GCCl(X13asp).

**Figure 26.6:** Contraction signals of the growth cycle either from July 1991 to July 2009 as they are implied by the September 2009 GCCI (lower panel) and from July 1996 to July 2009 as they are implied by the GCCI(X13asp) (upper panel).



### 26.4.7 GCCI: Tramo-Seats

Tramo-Seats is the fourth method to adjust the seasonal pattern of time-series considered. The resulting alternative coincident indicator is labelled as GCCI(TS).

In the upper panel of Figure 26.7 below we chart the pattern of the GCCI(TS), which spans from July 1996 to July 2009, and compare it to the September 2009 GCCI, which is shown in the lower panel. Also in this case, the pattern of the GCCI(TS) only slightly differs from the trajectory of the three alternative coincident indicators previously presented.

Figure 26.8 shows the contraction phases of the growth cycle that are derived either from the GCCI(TS) (upper panel) and from the September 2009 GCCI (lower panel) by filtering the contraction probabilities according to the 0.5 "natural rule".

As far as the overall evaluation of the coincident indicator is concerned, figures reported in Table 26.7 are very close to the statistics' values previously shown for the GCCI(X12v3a), GCCI(X12v3p) and GCCI(X13asp). This confirms the judge based on graphical inspection only.

### 26.4.8 GCCI: Dainties

In this section we deal with the Dainties procedure and we refer to the corresponding coincident indicator as GCCI(Dainties). Its pattern from July 1996 to July 2009 is depicted in the upper panel of Figure 26.9 below. In the lower panel the September 2009 GCCI is shown.

Figure 26.10 below shows the contraction phases of the growth cycle derived by filtering either the GCCI(Dainties) (upper panel) and the September 2009 GCCI (lower panel) with the 0.5 "natural rule".

When compared to the four alternative coincident indicators presented in the previous four sections, the

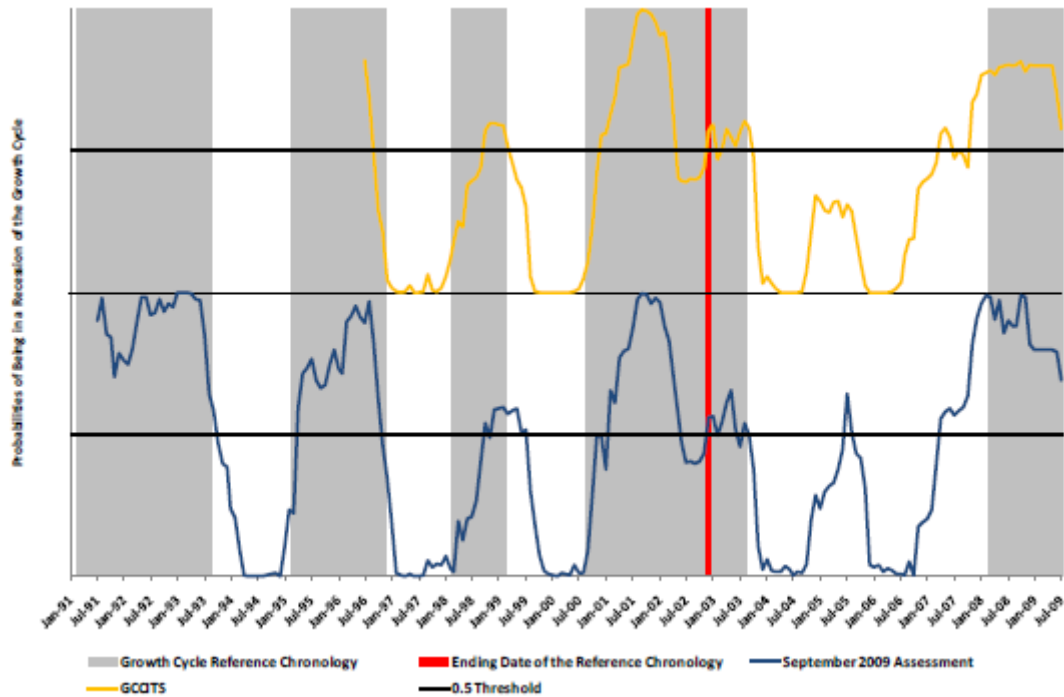
**Table 26.6:** Coincident Indicator if the growth cycle whose component variables are seasonally adjusted by applying the X-13-AS using X11, according to the "pickmdl" specification: Mean and standard deviation of the QPS. Concordance Index, Lag and Excess statistics.

GCCl - Components		Number of Releases	QPS		CI		Lag		Excess	
			Mean	StD	Mean	StD	Mean	StD	Mean	StD
GCCl(X13asp)	(as computed)	From 1 to 36	0.131	0.012	0.794	0.025	22.8	6.7	6.2	4.1
		From 7 to 36	0.133	0.012	0.788	0.025	23.4	7.1	6.9	4.0
	(6-month revised)	From 1 to 36	0.127	0.006	0.798	0.013	21.0	0.0	7.3	3.4
		From 7 to 36	0.129	0.006	0.795	0.013	21.0	0.0	8.3	2.7
IPI	1991:7 - 2009:7	From 1 to 36	0.217	0.020	0.749	0.025	25.7	14.2	24.7	10.6
		From 7 to 36	0.223	0.020	0.742	0.025	26.5	15.5	26.0	11.2
ImportInter	1990:7 - 2009:7	From 1 to 36	0.260	0.030	0.690	0.033	27.3	8.2	16.1	8.0
		From 7 to 36	0.270	0.030	0.679	0.033	28.1	8.7	17.6	7.8
IndEEExp	1985:7 - 2009:7	From 1 to 36	0.177	0.004	0.800	0.004	37.9	2.9	16.3	2.1
		From 7 to 36	0.176	0.004	0.801	0.004	38.2	3.1	16.4	2.3
BuildCI	1985:7 - 2009:7	From 1 to 36	0.194	0.010	0.778	0.012	41.3	1.6	19.2	5.1
		From 7 to 36	0.197	0.010	0.774	0.012	41.3	1.8	20.8	3.9
ConsFS	1985:7 - 2009:7	From 1 to 36	0.256	0.010	0.717	0.010	63.6	3.7	13.2	1.9
		From 7 to 36	0.257	0.010	0.716	0.010	64.3	3.6	13.6	1.8

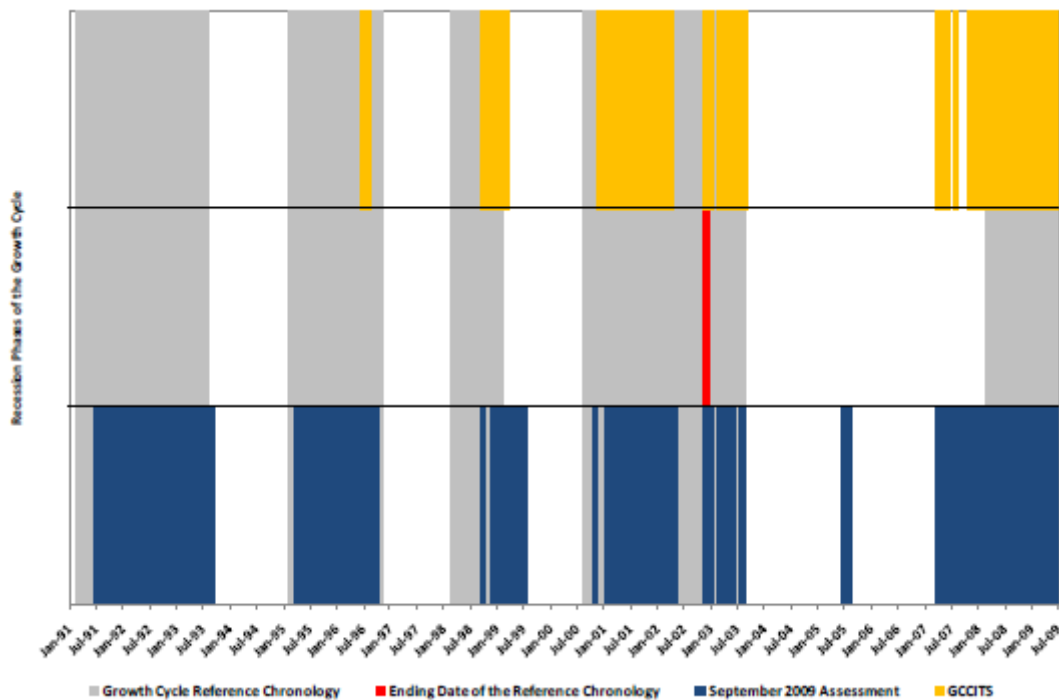
**Table 26.7:** Coincident Indicator of the growth cycle whose component variables are seasonally adjusted by applying the TRAMO-SEATS procedure: Mean and standard deviation of the QPS. Concordance Index, Lag and Excess statistics.

GCCl - Components		Number of Releases	QPS		CI		Lag		Excess	
			Mean	StD	Mean	StD	Mean	StD	Mean	StD
GCCl(TS)	(as computed)	From 1 to 36	0.133	0.011	0.788	0.020	23.2	6.6	6.5	4.3
		From 7 to 36	0.135	0.011	0.783	0.020	23.6	7.2	7.4	4.2
	(6-month revised)	From 1 to 36	0.132	0.006	0.798	0.013	21.0	0.0	7.3	3.4
		From 7 to 36	0.134	0.006	0.795	0.013	21.0	0.0	8.3	2.7
IPI	1991:7 - 2009:7	From 1 to 36	0.216	0.021	0.749	0.024	25.4	14.0	24.8	10.5
		From 7 to 36	0.221	0.021	0.742	0.024	26.4	15.2	26.0	11.2
ImportInter	1990:7 - 2009:7	From 1 to 36	0.253	0.029	0.704	0.035	27.3	7.1	14.2	6.8
		From 7 to 36	0.262	0.029	0.694	0.035	28.1	7.5	15.6	6.6
IndEEExp	1985:7 - 2009:7	From 1 to 36	0.178	0.009	0.798	0.011	37.7	4.1	17.3	3.4
		From 7 to 36	0.176	0.009	0.800	0.011	37.9	4.5	17.1	3.7
BuildCI	1985:7 - 2009:7	From 1 to 36	0.200	0.009	0.777	0.010	43.5	0.7	17.2	4.6
		From 7 to 36	0.203	0.009	0.774	0.010	43.4	0.7	18.6	3.6
ConsFS	1985:7 - 2009:7	From 1 to 36	0.249	0.006	0.728	0.008	59.9	3.0	14.0	2.0
		From 7 to 36	0.249	0.006	0.728	0.008	60.3	3.1	14.4	1.9

**Figure 26.7:** Probabilities of being in a contraction phase of the growth cycle either from July 1991 to July 2009 as they are produced by the September 2009 GCCI (lower panel) and from July 1996 to July 2009 as they are returned by the GCCI(TS) (upper panel).

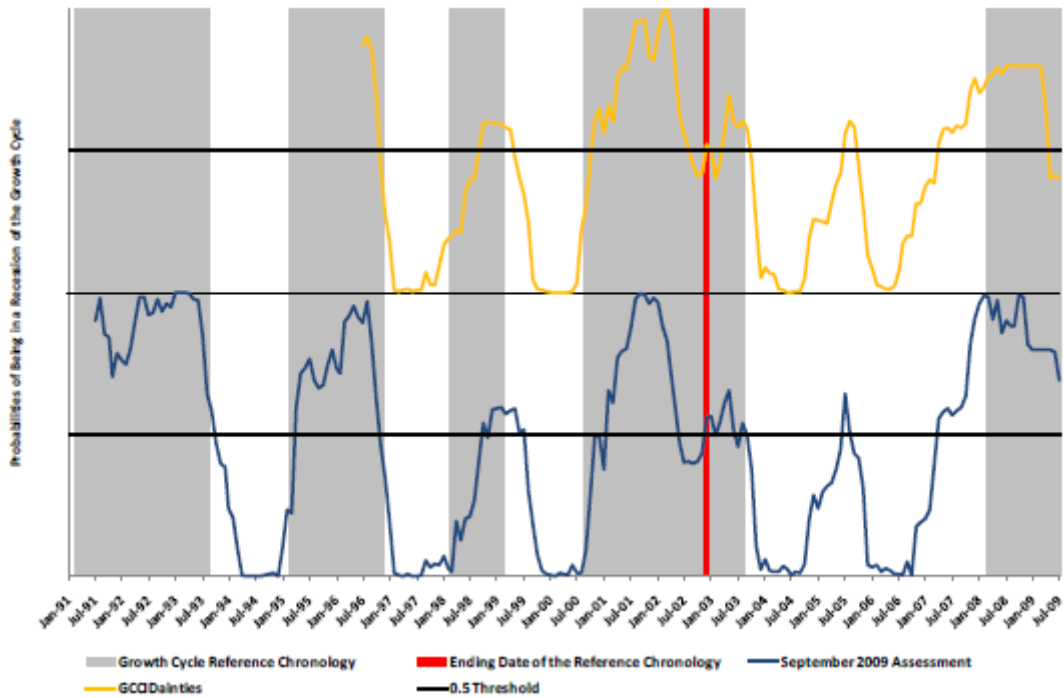


**Figure 26.8:** Contraction signals of the growth cycle either from July 1991 to July 2009 as they are implied by the September 2009 GCCI (lower panel) and from July 1996 to July 2009 as they are implied by the GCCI(TS) (upper panel).

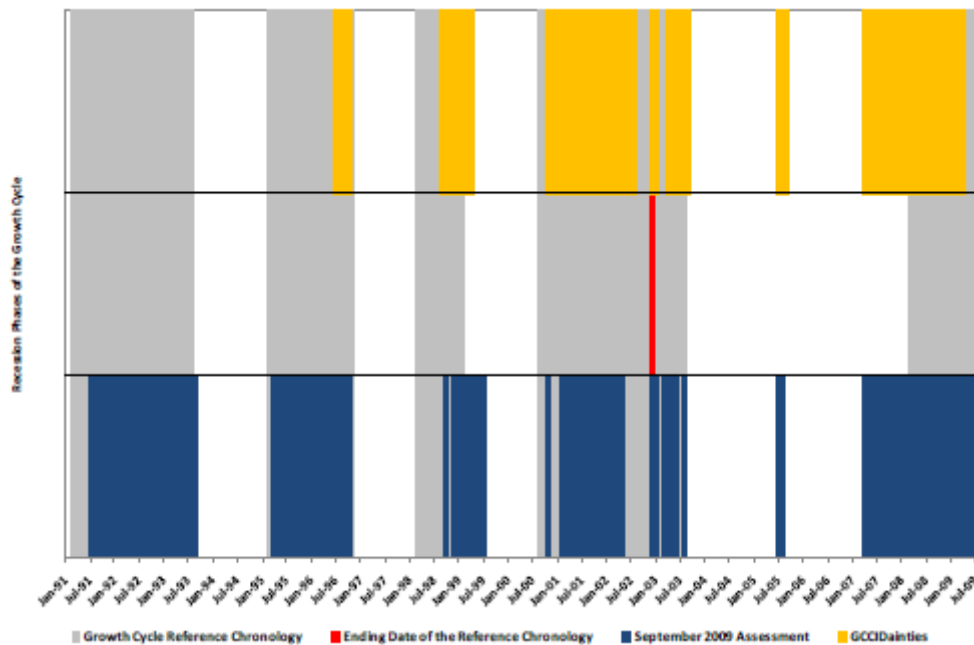


GCCI(Dainties) is somewhat different; in fact, contrary to the former coincident indicators, but like the September 2009 GCCI, the GCCI(Dainties) detects a short-lived contraction signal in mid-2005. Furthermore, the

**Figure 26.9:** Probabilities of being in a contraction phase of the growth cycle either from July 1991 to July 2009 as they are produced by the September 2009 GCCI (lower panel) and from July 1996 to July 2009 as they are returned by the GCCI(Dainties) (upper panel).



**Figure 26.10:** Contraction signals of the growth cycle either from July 1991 to July 2009 as they are implied by the September 2009 GCCI (lower panel) and from July 1996 to July 2009 as they are implied by the GCCI(Dainties) (upper panel).



GCCI(Dainties) locates with more delay than the previous four alternative coincident indicators the trough of the so-called Asian crisis recession; in this respect, the GCCI(Dainties) is similar to the September 2009 GCCI. These two features, which made the GCCI(Dainties) to resemble the September 2009 GCCI, explain why the (mean) Excess statistic associated to the former coincident indicator is close to the value of the same



**Table 26.8:** Coincident Indicator if the growth cycle whose component variables are seasonally adjusted by applying the Dainties procedure: Mean and standard deviation of the QPS, Concordance Index, Lag and Excess statistics.

GCCl - Components		Number of Releases	QPS		CI		Lag		Excess	
			Mean	StD	Mean	StD	Mean	StD	Mean	StD
GCCl(Dainties)	(as computed)	From 1 to 36	0.128	0.011	0.791	0.029	18.6	6.8	10.8	5.1
		From 7 to 36	0.131	0.011	0.784	0.029	19.3	7.3	11.8	5.0
	(6-month revised)	From 1 to 36	0.126	0.008	0.807	0.023	14.2	0.6	13.0	4.8
		From 7 to 36	0.128	0.008	0.800	0.023	14.2	0.7	14.4	3.9
IPI	1991:7 - 2009:7	From 1 to 36	0.222	0.019	0.739	0.028	19.6	0.8	32.7	8.1
		From 7 to 36	0.228	0.019	0.730	0.028	19.5	0.9	35.3	6.1
ImportInter	1990:7 - 2009:7	From 1 to 36	0.263	0.035	0.690	0.035	18.1	13.5	25.4	11.4
		From 7 to 36	0.275	0.035	0.679	0.035	19.6	14.3	26.3	12.2
IndEEExp	1985:7 - 2009:7	From 1 to 36	0.236	0.012	0.736	0.018	50.4	9.1	21.3	2.5
		From 7 to 36	0.235	0.012	0.735	0.018	51.5	9.6	21.3	2.7
BuildCI	1985:7 - 2009:7	From 1 to 36	0.214	0.008	0.762	0.008	43.0	1.0	21.5	4.8
		From 7 to 36	0.216	0.008	0.760	0.008	42.8	0.9	23.6	3.7
ConsFS	1985:7 - 2009:7	From 1 to 36	0.225	0.008	0.754	0.007	50.6	4.5	16.2	1.2
		From 7 to 36	0.226	0.008	0.754	0.007	51.3	4.6	16.2	1.3

statistic for the (usual) GCCl, namely, 14.4 and 15.3, respectively. The GCCl(Dainties) differs both from the previous four alternative coincident indicators and from the (usual) GCCl for a lower value of the Lag statistic, see Table 10. This is indeed mainly due to the fact that the GCCl(Dainties) detects with greater timeliness than the other coincident indicators the peak of the slowdown between August 2000 and August 2003; furthermore, when compared to the previous four alternative coincident indicators, the GCCl(Dainties) conveys a more stable contraction signal throughout this period. It is worth noting that the GCCl(Dainties) produces the lowest (mean) value of the Lag statistic among the previous four alternative coincident indicators despite the fact that it detects in April 2009 the trough of the last contraction of the growth cycle, whereas the latter coincident indicators as well as the September 2009 GCCl do not locate it yet.

Table 26.8 reports the QPS, Concordance Index, Lag and Excess statistics for the GCCl(Dainties) and its five component variables.

### 26.4.9 GCCl: Baysea

The case in which the Baysea procedure is followed to seasonally adjust the five component variables of the coincident indicator of the growth cycle is addressed in this section. The resulting coincident indicator is labelled GCCl(Baysea).

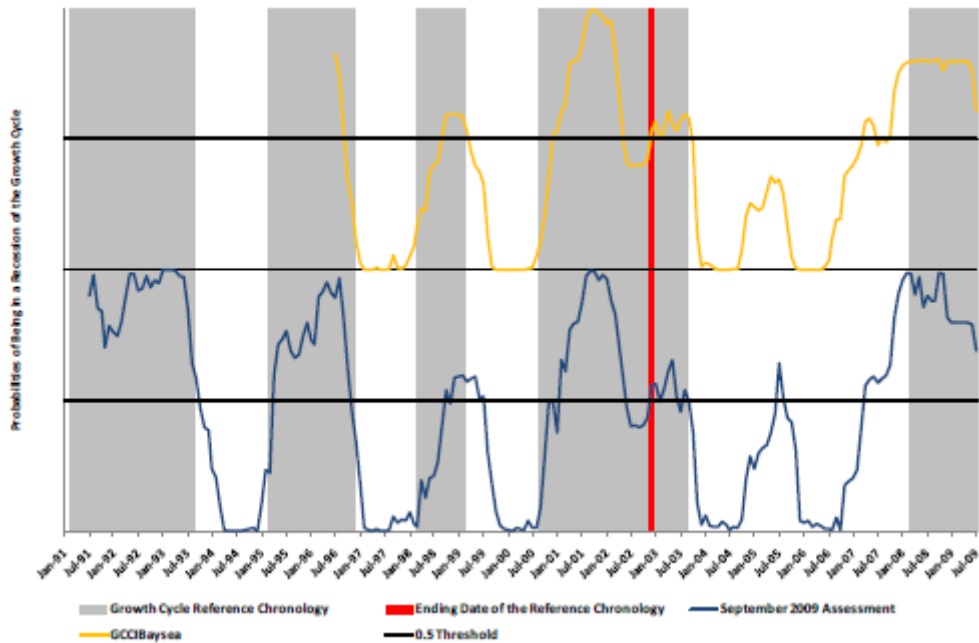
Figure 26.11 shows the GCCl(Baysea) (upper panel), which returns an estimate of the probability for the Euro area economy to be in a contraction phase of the growth cycle from July 1996 to July 2009. In the lower panel we deal with the case when the same inference is drawn from September 2009 GCCl over the period July 1991 – July 2009. Graphical evidence suggests that the GCCl(Baysea) almost matches the first four alternative coincident indicators, namely, the GCCl(X12v3a), GCCl(X12v3p), GCCl(X13asp) and GCCl(TS), we previously presented.

Figure 26.12, where contraction phases implied by the GCCl(Dainties) (upper panel) are shown, confirms the relation between the GCCl(Dainties) and the first four alternative coincident indicators also in terms of contraction signals.

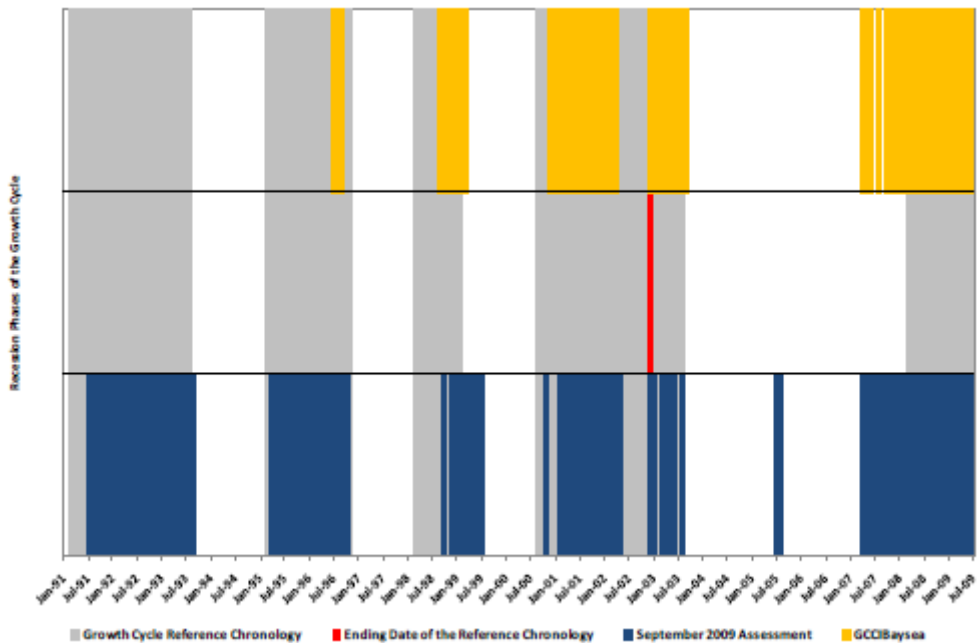
The insight gained by graphical inspection of Figure 26.11 and Figure 26.12 is grounded on quantitative terms by providing a set of statistics, which are contained in Table 26.11. It turns out indeed that the (mean) QPS, Concordance Index, Lag and Excess statistics associated with the GCCl(Dainties) are close to the ones



**Figure 26.11:** Probabilities of being in a contraction phase of the growth cycle either from July 1991 to July 2009 as they are produced by the September 2009 GCCI (lower panel) and from July 1996 to July 2009 as they are returned by the GCCI(Baysea) (upper panel).



**Figure 26.12:** Contraction signals of the growth cycle either from July 1991 to July 2009 as they are implied by the September 2009 GCCI (lower panel) and from July 1996 to July 2009 as they are implied by the GCCI(Baysea) (upper panel).



reported for the GCCI(X12v3a), GCCI(X12v3p), GCCI(X13asp) and GCCI(TS). Nevertheless, the (mean) Lag statistic provided by the GCCI(Baysea) is 18.0, that is, although higher than the one pertaining to the GCCI(Dainties), it is lower than the values associated to the remaining alternative coincident indicators seen above.

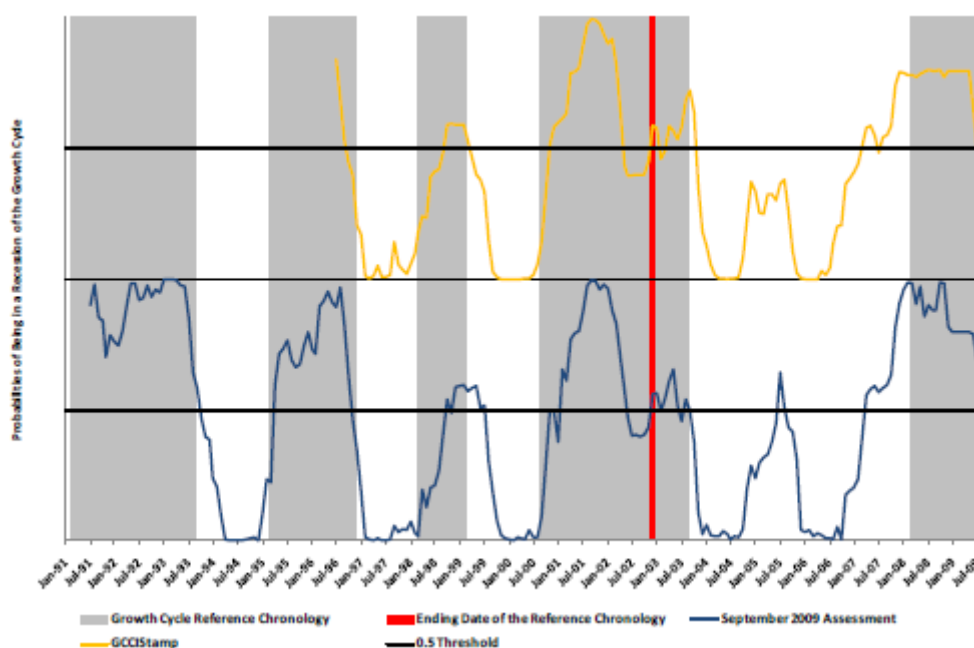
**Table 26.9:** Coincident Indicator if the growth cycle whose component variables are seasonally adjusted by applying the Baysea procedure: Mean and standard deviation of the QPS, Concordance Index, Lag and Excess statistics.

GCCl - Components		Number of Releases	QPS		CI		Lag		Excess	
			Mean	StD	Mean	StD	Mean	StD	Mean	StD
GCCl(Baysea)	(as computed)	From 1 to 36	0.133	0.012	0.798	0.028	21.3	7.5	7.1	4.6
		From 7 to 36	0.135	0.012	0.791	0.028	21.9	8.1	8.0	4.6
	(6-month revised)	From 1 to 36	0.130	0.007	0.816	0.016	18.0	0.0	7.8	3.8
		From 7 to 36	0.133	0.007	0.812	0.016	18.0	0.0	8.9	3.0
IPI	1991:7 - 2009:7	From 1 to 36	0.218	0.024	0.753	0.027	25.0	15.1	24.6	10.5
		From 7 to 36	0.224	0.024	0.746	0.027	25.8	16.4	25.8	11.1
ImportInter	1990:7 - 2009:7	From 1 to 36	0.258	0.031	0.693	0.038	27.4	7.9	15.6	7.9
		From 7 to 36	0.269	0.031	0.680	0.038	28.3	8.4	17.3	7.5
IndEEExp	1985:7 - 2009:7	From 1 to 36	0.178	0.004	0.797	0.005	35.2	4.6	19.8	3.9
		From 7 to 36	0.177	0.004	0.798	0.005	35.5	5.0	20.0	4.2
BuildCI	1985:7 - 2009:7	From 1 to 36	0.202	0.008	0.775	0.010	42.1	1.7	19.0	5.2
		From 7 to 36	0.204	0.008	0.772	0.010	41.9	1.9	20.6	4.0
ConsFS	1985:7 - 2009:7	From 1 to 36	0.248	0.009	0.732	0.013	59.5	4.6	13.3	2.1
		From 7 to 36	0.248	0.009	0.732	0.013	60.0	4.9	13.8	1.9

### 26.4.10 GCCl: Stamp

The seventh method to remove the seasonal component from the time-series on which the GCCl is built is the Stamp procedure. We indicate with GCCl(Stamp) this coincident indicator of the growth cycle. As it appears from Figure 26.13, where the pattern of the GCCl(Stamp) is depicted in the upper panel, this alternative coincident indicator covers the period from July 1996 to July 2009.

**Figure 26.13:** Probabilities of being in a contraction phase of the growth cycle either from July 1991 to July 2009 as they are produced by the September 2009 GCCl (lower panel) and from July 1996 to July 2009 as they are returned by the GCCl(Stamp) (upper panel).



Contraction probabilities provided by the coincident indicator are transformed into contraction signals by applying the 0.5 "natural rule". The signals implied by the GCCl(Stamp) are represented in the upper panel of Figure 26.14, whereas the lower panel shows the signals implied by the September 2009 GCCl.

This graphical tool makes apparent that the GCCI(Stamp) detects with some delay the trough of the recession between August 2000 and August 2003. This adverse feature is in common with the GCCI(Dainties) and contributes to increase the (mean) Excess statistic, which is higher than the ones related to the GCCI(X12v3a), GCCI(X12v3p), GCCI(X13asp) and GCCI(TS) and GCCI(Baysea).

**Figure 26.14:** Contraction signals of the growth cycle either from July 1991 to July 2009 as they are implied by the September 2009 GCCI (lower panel) and from July 1996 to July 2009 as they are implied by the GCCI(Stamp) (upper panel).

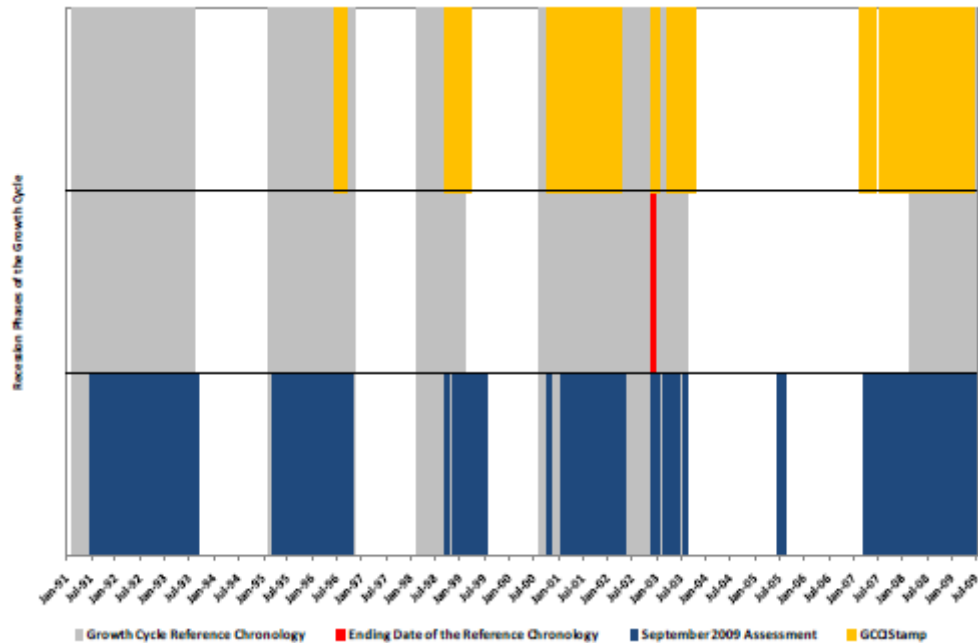


Table 26.10 reports the mean and standard deviation of the QPS, Concordance Index, Lag and Excess statistics produced by the GCCI(Stamp). Figures contained in Table 26.10 allow us to conduct a more detailed analysis into the causes of the increase of the (mean) Excess statistic. It turns out that the INDEA(Stamp) and more the BuildCI(Stamp) components are to be held accountable for the rise of the (mean) Excess statistic.

### 26.4.11 GCCI: Decomp

The last method that we consider to seasonally adjust the five component variables of the GCCI is the Decomp procedure. We indicate with GCCI(Decomp) the resulting coincident indicator of the growth cycle. In the upper panel of Figure 26.15 we represent the pattern of the GCCI(Decomp), which spans from July 1996 to July 2009. It is compared with the September 2009 GCCI, which is shown in the lower panel. On one hand, the pattern of the GCCI(Decomp) slightly differs from the trajectories of the GCCI(X12v3a), GCCI(X12v3p), GCCI(X13asp), GCCI(TS), GCCI(Baysea) and also from the GCCI(Stamp); however, for what regards the comparison with this last coincident indicator, these differences are muted when the contraction probabilities are transformed into contraction signals. These signals are shown in Figure 26.16.

The similarity between the GCCI(Decomp) and GCCI(Stamp) is confirmed by the statistics reported in Table 26.11, where the mean and standard deviation of the QPS, Concordance Index, Lag and Excess statistics computed for the GCCI(Decomp) and its five component variables are shown. As far as the Lag statistic is concerned, the mean value associated to the GCCI(Decomp) is 21.0, while it is 20.0 for the GCCI(Stamp). The values of the mean Excess statistic are even more close, namely, 11.6 and 11.5, respectively.

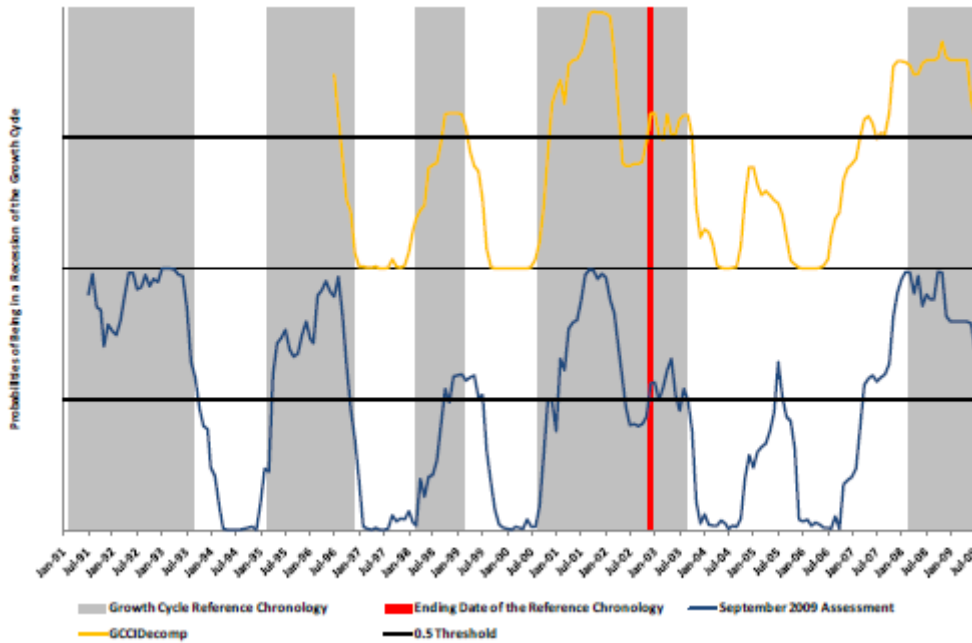
**Table 26.10:** Coincident Indicator if the growth cycle whose component variables are seasonally adjusted by applying the Stamp procedure: Mean and standard deviation of the QPS, Concordance Index, Lag and Excess statistics.

GCCl - Components		Number of Releases	QPS		CI		Lag		Excess	
			Mean	StD	Mean	StD	Mean	StD	Mean	StD
GCCl(Stamp)	(as computed)	From 1 to 36	0.137	0.014	0.778	0.027	22.4	8.3	8.8	5.6
		From 7 to 36	0.141	0.014	0.771	0.027	22.9	9.1	9.8	5.6
	(6-month revised)	From 1 to 36	0.133	0.009	0.785	0.021	20.0	0.2	10.1	4.7
		From 7 to 36	0.136	0.009	0.779	0.021	20.0	0.2	11.5	3.8
IPI	1991:7 - 2009:7	From 1 to 36	0.218	0.024	0.751	0.030	24.9	15.4	25.0	10.8
		From 7 to 36	0.224	0.024	0.743	0.030	25.9	16.7	26.4	11.3
ImportInter	1990:7 - 2009:7	From 1 to 36	0.258	0.028	0.688	0.034	27.0	7.3	16.5	7.8
		From 7 to 36	0.267	0.028	0.679	0.034	27.8	7.8	17.8	7.7
IndEEExp	1985:7 - 2009:7	From 1 to 36	0.191	0.014	0.774	0.018	40.4	10.3	21.1	5.5
		From 7 to 36	0.191	0.014	0.773	0.018	41.1	11.1	21.3	6.0
BuildCI	1985:7 - 2009:7	From 1 to 36	0.214	0.017	0.750	0.023	35.1	2.2	32.6	7.7
		From 7 to 36	0.217	0.017	0.747	0.023	35.2	2.3	34.2	7.2
ConsFS	1985:7 - 2009:7	From 1 to 36	0.245	0.008	0.735	0.009	58.3	3.4	13.6	2.2
		From 7 to 36	0.245	0.008	0.734	0.009	58.8	3.6	14.2	2.1

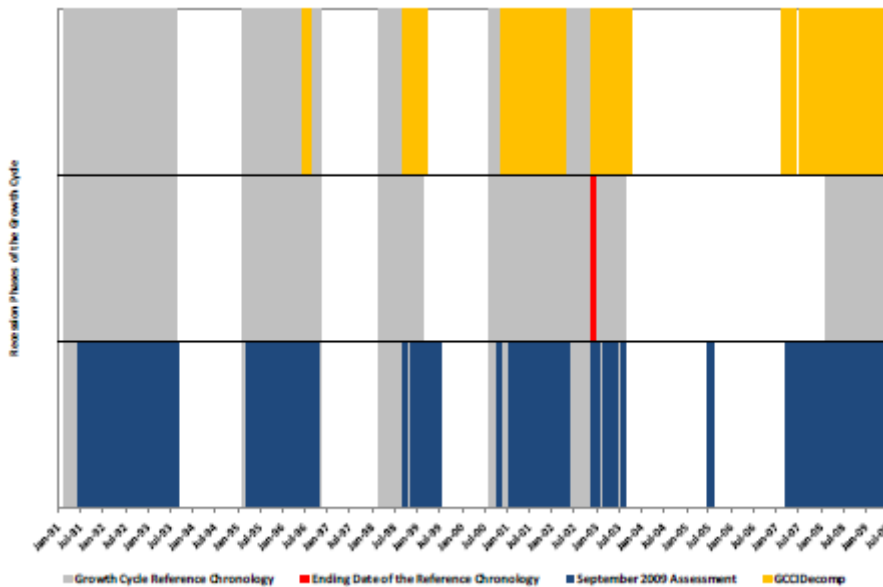
**Table 26.11:** Coincident Indicator if the growth cycle whose component variables are seasonally adjusted by applying the Decomp procedure: Mean and standard deviation of the QPS, Concordance Index, Lag and Excess statistics.

GCCl - Components		Number of Releases	QPS		CI		Lag		Excess	
			Mean	StD	Mean	StD	Mean	StD	Mean	StD
GCCl(Decomp)	(as computed)	From 1 to 36	0.135	0.009	0.785	0.022	22.5	6.6	7.7	4.6
		From 7 to 36	0.138	0.009	0.780	0.022	22.8	7.8	8.7	4.4
	(6-month revised)	From 1 to 36	0.135	0.009	0.778	0.021	21.0	0.0	10.1	4.7
		From 7 to 36	0.137	0.009	0.772	0.021	21.0	0.0	11.6	3.8
IPI	1991:7 - 2009:7	From 1 to 36	0.226	0.021	0.741	0.023	27.4	13.7	24.4	10.4
		From 7 to 36	0.232	0.021	0.735	0.023	28.1	14.9	25.7	11.0
ImportInter	1990:7 - 2009:7	From 1 to 36	0.259	0.025	0.691	0.037	26.4	4.9	16.6	6.9
		From 7 to 36	0.268	0.025	0.682	0.037	27.1	5.1	18.0	6.5
IndEEExp	1985:7 - 2009:7	From 1 to 36	0.193	0.004	0.780	0.008	38.9	4.9	20.9	3.9
		From 7 to 36	0.192	0.004	0.782	0.008	39.0	5.3	20.9	4.2
BuildCI	1985:7 - 2009:7	From 1 to 36	0.197	0.009	0.783	0.010	41.9	1.1	17.1	4.6
		From 7 to 36	0.200	0.009	0.780	0.010	41.8	1.2	18.5	3.5
ConsFS	1985:7 - 2009:7	From 1 to 36	0.242	0.008	0.735	0.010	56.8	3.7	15.2	1.9
		From 7 to 36	0.242	0.008	0.734	0.010	57.4	3.8	15.7	1.8

**Figure 26.15:** Probabilities of being in a contraction phase of the growth cycle either from July 1991 to July 2009 as they are produced by the September 2009 GCCI (lower panel) and from July 1996 to July 2009 as they are returned by the GCCI(Decomp) (upper panel).



**Figure 26.16:** Contraction signals of the growth cycle either from July 1991 to July 2009 as they are implied by the September 2009 GCCI (lower panel) and from July 1996 to July 2009 as they are implied by the GCCI(Decomp) (upper panel).



### 26.4.12 GCCI: Results discussion

We finally conclude our discussion on the impact of seasonal adjustment procedures to the coincident indicator of the growth cycle by summarizing the results above.

As previously stated, we did not perform a formal test to judge the statistical significance of the statistics constructed to measure the ability of the alternative coincident indicators to timely locate turning points. Nev-

ertheless, these statistics are helpful in providing us suggestions on the effect of seasonal adjustment on the coincident indicator.

Seven out of the eight alternative coincident indicators we consider and analyse closely resemble one another. Only the GCCI(Dainties) differs in some respect from the remaining seven and also from the (usual) GCCI. More in detail, when comparing to these latter coincident indicators, the coincident indicator whose component variables are seasonally adjusted with the Dainties procedure produces the lowest mean value of the Lag statistic, namely, 14.2. In the light of the definition of the Lag statistic, this means that the GCCI(Dainties) is either able to locate with a greater timeliness the peaks of the growth cycle reference dating chronology and to reduce the number of missed (or partly missed) cycles. However, this is partly offset by the fact that, at the same time, the GCCI(Dainties) returns the highest value of the Excess statistic among the alternative coincident indicators, although this value is slight lower than the one associated to the usual GCCI, namely 14.4 versus 15.3. Given the definition of the Excess statistic we infer that the GCCI(Dainties) either detects with a lower timeliness the trough of the growth cycle or locates some false contraction signals.

When the sum of Lag and Excess statistics is considered, the GCCI(Baysea) is the alternative coincident indicator that returns the best performance. This sum is also lower than the value provided by the usual GCCI, namely, 26.9 versus 36.3. As a result the GCCI(Baysea) also produces the highest value of the Concordance Index.

At the more disaggregated level of the component variables, the Dainties procedure is the one that delivers an outcome that considerably differs for three out of five component variables from the remaining seasonal adjustment methods and also from the usual approach. More in detail, both the IPI and ImportInter when treated with the Dainties procedure return the lowest (mean) Lag and the highest (mean) Excess statistics. This behaviour is coherent to the ones previously shown for the overall coincident indicator, namely the GCCI(Dainties). In the opposite direction drives the IndEExp component, in fact, in this case the time-series seasonally adjusted with the Dainties procedure produces the highest value of the (mean) Lag statistic among the eight seasonal adjustment methods and also with respect to the usual time-series seasonally adjusted by Eurostat.

Until now the dynamic analysis has not been extensively developed, this is because the standard deviation of the four statistics considered does not markedly differ among the eight alternative coincident indicators. This is true not only for the coincident indicators whose last six monthly estimates are the only ones to be revised, but also for the coincident indicators that are completely revised moving from one monthly assessment to the next. Nonetheless, in this latter case the standard deviation is obviously increased with respect to the former case.

When the single component variables are taken into exam, a mixed picture emerges. Consider the IPI variable, when this time-series is seasonally adjusted with Dainties procedure, it returns by far lowest standard deviation of the Lag statistic. On the contrary, as far as the ImportInter component is concerned, both the Lag and Excess statistics associated to this time-series treated with the Dainties method are characterized by the highest standard deviation.

Finally, with the exception of the IndEExp, the remaining two surveys, namely the BuildCI and ConsFS, show quite close values of the standard deviation of the four statistics considered.

## 26.5 Conclusions

In this chapter we assess the role played by seasonal adjustment on the coincident indicator of the growth cycle. This task has been accomplished by comparing the coincident indicators regularly released among the Principal European Economic Indicators with a set of alternative coincident indicators. The latter ones have been constructed resorting to several different seasonal adjustment procedures to treat the components.

The eight seasonal adjustment procedures can be divided into two sets, depending on their comparative relation with the usual GCCI. To the first set belongs only the Dainties procedure; in fact, among the eight seasonal adjustment methods considered, this is the one that produces a coincident indicator that detects with a greater timeliness than the usual GCCI the peaks of the growth cycle. All the seven remaining seasonal adjustment methods belong to the second set as they share a common feature, namely, when compared to the usual GCCI they either detect with a greater timeliness the troughs of the growth cycles or reduce the false contraction signals. All in all, the seasonal adjustment procedure adopted seems to not markedly affect the resulting coincident indicators.

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# 27

## Seasonal Adjustment and Business Cycles



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## Table of Contents

27.1	Introduction	711
27.2	Seasonality and Business Cycles Interactions	712
27.2.1	Nonparametric methods for trend-seasonal interaction	713
27.2.2	Parametric Models of Interactions	714
27.2.3	Formal decompositions	715
27.3	The effects of Seasonal Adjustment on Business Cycle Measurement	716
27.3.1	The effect on trend-cycle interactions	717
27.3.2	Seasonal Adjustment and the Reliability of the Cycle Estimates	718
27.3.3	Empirical Evaluation of the Variance Inflation Factor	722
27.3.4	Empirical illustration	722
27.3.5	Discussion	725
27.4	Conclusions	727

## 27.1 Introduction

Seasonality is a prominent feature of economic time series that are surveyed at the monthly or quarterly frequency, such as production, sales and employment. Its adjustment serves, however, a variety of useful purposes and satisfies well established information requirements from the users. Indeed, most of the literature on the business cycle (BC) relies on seasonally adjusted data. The latter simplify the specification and estimation effort of the analyst, who can concentrate directly on BC features of interest. Moreover, seasonality is often considered as a nuisance feature and as the most predictable component of a time series.

It is nowadays standard practice among statistical agencies to carry out seasonal adjustment (SA) either using X-12-ARIMA (see Findley et al. (1998)) and Tramo-Seats (see Gomez and Maravall (1996), and Caporello and Maravall (2004)). The first method has a very long tradition and its roots are essentially nonparametric, in that there is no formal underlying model to commit oneself<sup>1</sup>. On the contrary, Tramo-Seats is an ARIMA model-based method, since an ARIMA model is decomposed into mutually uncorrelated unobserved components, namely trend-cycle, seasonality and irregular, by performing what is known as the canonical decomposition; see Hillmer and Tiao (1982). The components are extracted by the Wiener-Kolmogorov filter.

The dichotomy between these two approaches disguises a much larger variety of methods and approaches. Among these we mention SABL and STL (Cleveland et al. (1978), Cleveland et al. (1990), robust nonparametric seasonal adjustment methods based on loess, a nonparametric locally weighted polynomial regression filter, and the structural approach popularized by Harvey (1989) and West and Harrison (1997), which is also a model based method, but with the important difference that the decomposition of the time series into unobserved components is specified and estimated at the outset, rather than derived ex-post from an estimated reduced form. The structural approach is implemented in the software package STAMP (Structural Time series Analyser, Modeller and Predictor) from Koopman et al. (2008).

A collection of relevant papers on seasonality and its adjustment is the volume edited by Hylleberg (1992); for the nonparametric approach, and in particular for a thorough exposition of the X-11 method, a very useful reference is Ladiray and Quenneville (2001). A general reference for the problem of seasonal adjustment is den Butter and Fase (1991); for the role of seasonality in econometric modelling see Osborn Ghysels and Osborn (2001). Older, but never out of date, references are Nerlove et al. (1979) and the volumes edited by Zellner (Zellner (1978), Zellner (1983)). Findley (2005) points out several directions for future research.

The standard practice of basing business cycle measurement and analysis on seasonally adjusted series is not uncontroversial. In fact, a fundamental issue is whether seasonal adjustment affects the main stylized facts concerning the business cycle. This is also the main issue that permeates this report. In order to understand its relevance and the plurality of facets it has, we need to reflect on the nature of the operation known as seasonal adjustment.

As it has been clearly remarked by Bell and Hillmer (1984) (sec. 4.2), seasonal adjustment rests upon two basic assumptions:

1. Additivity, perhaps after a transformation.
2. Orthogonality with the nonseasonal component.

They make this point so forcefully that they conclude: “*Someone who does not want to make these assumptions is working on a different problem*”. On the other hand, a strand of the econometric literature has questioned the realism of the two assumptions, especially the latter, both from the empirical and the theoretical ground. We mention in passing that the two aspects are related, in that the failure of the orthogonality may arise as a consequence of non additivity on a particular scale: for instance, if the series is log-additive

<sup>1</sup>There is however a model based feature, in that a regression model with time series errors, belonging to the Box and Jenkins seasonal ARIMA class is entertained for forecast extension, calendar modelling, outlier detection and removal, and so forth.

but a purely additive decomposition is used, then we may find evidence for a positive association between the seasonal amplitude and the level of the series.

The objective of this chapter is to review this literature and present new statistical evidence on two related issues: the interactions between seasonal and business cycle fluctuations; the role of seasonal adjustment filters on the measurement and analysis of the characteristics of the business cycle (turning point estimation, assessment of cyclical stance, nonlinearity of the business cycle, asymmetry, and so forth).

In this investigation, the role of the following three elements has to be identified and characterized: 1. The nature of seasonality. It may be the case that additivity and orthogonality do not hold, so that the adjustment can take place only at the cost of distorting the cyclical information. 2. The seasonal model and seasonal adjustment method: the seasonal model may be misspecified, or the SA method inadequate. Additivity and orthogonality can be achieved by using a more flexible seasonal model or on a suitably transformed scale. 3. The definition of the business cycle. It should be clarified at the onset of the analysis which definition of the business cycle (classical, growth, growth rate cycle) is considered and what measure is adopted.

We will try to address basic questions such as:

- Does the presence of interactions prevents seasonal adjustment?
- What is the role of misspecified purely seasonal features such as seasonal heteroscedasticity or the need for a data transformation (such as the Box-Cox transformation) on the evidence for interactions?
- Is seasonal adjustment an important component of the reliability of the business cycle measurement?

The chapter is divided in two essential parts, the first devoted to the theoretical and empirical work concerning the interactions between the seasonal and the business cycle (section 27.2). The second deals with the effects of seasonal adjustment on business cycle measurements.

## 27.2 Seasonality and Business Cycles Interactions

There is a vast literature on the interactions between seasonality and cycles. We distinguish between two different strands of literature. The first is grounded in economic theory, or starts from economic arguments. The second has a more statistical flavour and aims at detecting interactions, without assuming a particular economic mechanism for them.

Economic models accounting for seasonality postulate that either preferences, technological parameters or endowments, or a combination thereof have periodic features. Examples are Osborn (1988), Ghysels (1988), Miron and Zeldes (1988), Todd (1990), Hansen and Sargent (1991), Cecchetti et al. (1997). The latter ascribe the interactions to the shape of the marginal cost curves faced by an industry: for instance, if marginal costs are increasing, then in cyclical highs firms (facing a capacity constraint) will find convenient to reduce the seasonal amplitude by appropriate manipulation of the stock of inventories. Christiano and Todd (2002) investigate the conditions under which the seasonal adjustment by regression methods has no distortionary effects on business cycle stylized facts: for this purpose they simulate data from a time to build model driven by technological shocks, taste shocks and government consumption shocks that have both a perfectly predictable and an indeterministic seasonal component, and evaluate the consequences of seasonal adjustment.

Among the empirical studies concerning the cycle-seasonal interactions, we mention Barsky and Miron (1989), who argue that the seasonal cycle displays the same characteristics as the business cycle, in that certain stylized facts, such as comovements, hold at the seasonal frequencies as well. The seasonal cycle is deterministic and modeled by linear regression methods. The comovements in the seasonal cycle are measured by the correlation of the estimated seasonal effects.

Canova and Ghysels (1994) detect significant changes in the seasonal patterns of US macroeconomic variables and also document that the variation of amplitude is related to the business cycle phases. Similar results

are reported in Osborn and Matas-Mir (2004). These references explore the possibility that seasonality is affected by the business cycle, intended in a classical sense, as the alternation of expansions and recessions in the level of economic activity. On the contrary, van Dijk et al. (2003) find support for the fact that most changes in seasonal patterns are due to institutional and technological factors, rather than to the state of the business cycle.

Ghysels (1994), using a Markov chain model with periodically varying transition probabilities for the classical NBER binary indicator of the classical cycle, argues that the duration of the business cycle phases has a seasonal feature, being more likely to make a transition from recession to expansion during spring and December. Hence, turning points are more likely to occur in certain seasons of the year. The same author (Ghysels (1997)) draws the same conclusions using duration data, originating from the NBER data (and an additional source).

The plausibility of the two assumptions has also been challenged from another standpoint by the characterisation of seasonal time series using periodic autoregressive models, featuring periodic integration, that have applied to economic time series (see Franses and Paap (2004)) for a comprehensive review. In this framework, the trending and seasonal behaviour of the series is due to the presence of unit roots nested in the periodic process, and periodicity is proposed as an alternative to a seasonal model, rather than a further feature that needs to be brought into the model. No adjustment is possible, since the seasonality cannot be disentangled from the periodic autoregressive mechanism.

Krane and Wascher (1999) investigate the cyclical sensitivity of seasonality in US employment, using the payroll data published by the Bureau of Labor Statistics. They adopt a multivariate dynamic factor model, which is a variant of the model by Stock and Watson (1991), for the logarithmic changes in industry specific employment, featuring a single common cycle with periodic mean. The measurement equation has a deterministic idiosyncratic seasonal component for each of the series and an interaction term due to the product of seasonal dummies and the latent cycle. Their conclusion is that the seasonal idiosyncratic component absorbs a sizable amount of variation; however, the interaction term is significant for a number of industries.

The papers that we have just reviewed appeared in economic and applied econometrics journals. There is also some more statistically oriented literature that has considered the possibility of interactions between the cycle and the seasonal component. The next three paragraphs will review it. A distinguishing common feature of this literature is that it essentially aims at extending seasonal adjustment to the case when the orthogonality assumption is relaxed. In other words, the presence of interactions does not hinder the possibility of adjusting the series. In the statistical approach, moreover, more emphasis has been placed on the trend-seasonal interaction, rather than the cycle-seasonal one.

### 27.2.1 Nonparametric methods for trend-seasonal interaction

One of the earliest attempts to design suitable seasonal adjustment filters in the presence of a trend-seasonal interaction is due to Durbin and Murphy (1975), who refer to the mixed additive- multiplicative decomposition of the series:

$$y_t = \mu_t + \gamma_t + \gamma_t^* \mu_t + \epsilon_t$$

where  $\mu_t$  denotes the trend,  $\gamma_t$  is the seasonal component, and  $\epsilon_t$  is the irregular. The interaction is due to  $\gamma_t^* \mu_t$ , where  $\gamma_t^*$  is an additional zero mean seasonal component that interacts with the trend.

Let  $m_t$  denote a preliminary estimate of the trend (the authors use an ad hoc moving average filter that passes a cubic polynomial and suppresses a deterministic seasonal component); Durbin and Murphy propose running the following regression model

$$y_t - m_t = \sum_{j=1}^{s-1} \delta_j \tilde{D}_{jt} + \sum_{j=1}^{s-1} \delta_j^* \tilde{D}_{jt} m_t + u_t.$$



Let

$$g_t = \hat{\delta}_j D_{jt}, \hat{\delta}_s = - \sum_{j=1}^{s-1} \hat{\delta}_j$$

and

$$g_t^* = \hat{\delta}_j^* D_{jt}, \hat{\delta}_s^* = - \sum_{j=1}^{s-1} \hat{\delta}_j^*$$

where the coefficients are estimated by least squares or by frequency domain regression (the authors also propose a variable selection procedure to increase the estimation precision).

The seasonally adjusted series is obtained by the nonlinear filter:

$$\frac{y_t - g_t}{1 + g_t^*}.$$

The denominator is the a correction term that removes the effect of the interaction. If  $g_t^*$  were equal to one, the traditional additive adjustment would take place.

A pseudo-additive decomposition has also been recently incorporated, more or less experimentally, in the X-12 seasonal adjustment program. The decomposition deals with situations when some seasons have extremely small values (due to holidays or climate, for example), whereas the remaining months appear to have multiplicative seasonality. The traditional multiplicative seasonal adjustment becomes highly unstable in those situations. The pseudo-additive decomposition is based on the following representation:

$$y_t = \mu_t(\gamma_t + \varepsilon_t - 1),$$

where  $\gamma_t$  is a seasonal factor with geometric mean equal to one. The estimate of the SA series is  $y_t - m_t(g_t - 1)$ , where  $m_t$  and  $g_t$  are estimates of the trend and the seasonal factors, respectively. See Findley et al. (1998) for more details.

## 27.2.2 Parametric Models of Interactions

We now consider three parametric unobserved components models that have been proposed in the statistical literature.

Shephard (1994) proposed for the analysis of a monetary UK series a mixed multiplicative-additive model taking the following form:

$$y_t = \mu_t(1 + \gamma_t^* + \varepsilon_t^*) + \varepsilon_t, \quad (27.1)$$

where  $\mu_t$  is a local linear trend, with IMA(2,1) representation (see e.g. Harvey (1989)),  $\gamma_t^*$  is a stochastic dummy seasonal component,  $S(L)\gamma_t^* = \omega_t \sim \text{NID}(0, \sigma^2\omega)$ ,  $\varepsilon_t \sim \text{NID}(0, \sigma^2\varepsilon)$ , and  $\varepsilon_t^* \sim \text{NID}(0, \sigma^{2*}\varepsilon)$ . Seasonality enters multiplicatively and, according to Shephard, the total seasonal effect is  $\mu_t\gamma_t^*$ , whereas the irregular is  $\mu_t\varepsilon_t^* + \varepsilon_t$ , so that the nonseasonal part of the series is  $\mu_t(1 + \varepsilon_t^*) + \varepsilon_t$ . The model has a conditionally Gaussian state space representation and can be estimated by the simulated EM algorithm.

Ozaki and Thomson (2002) specify a non linear model with multiplicative interactions that aims at obtaining estimates of the seasonally adjusted series free from bias. The bias of the log-additive decomposition arises from the fact that it is a property of this decomposition that the yearly average of the minimum mean square estimate of the seasonally adjusted series is different from the yearly average of the unadjusted series. It is questionable whether this should be considered as a problem (see the discussion in Proietti and Riani (2008)). Let  $y_t$  denote the series in its original scale of measurement, e.g. the index of industrial production. Ozaki and Thomson consider the following decomposition:

$$y_t = \mu_t^*(1 + \gamma_t^*)(1 + \varepsilon_t^*) = \mu_t^* + \mu_t^*\gamma_t^* + \mu_t^*(1 + \gamma_t^*)\varepsilon_t^*,$$

such that  $\mu_t^*$  is generated by  $\Delta^p \mu_t = \eta_t$ , where  $\mu_t = \ln \mu_t^*$ ,  $\gamma_t^*$  is a stochastic seasonal component, e.g.  $S(L)\gamma_t^* \sim \text{WN}(0, \sigma^2\omega)$ ,

$$\ln(1 + \epsilon_t^*) = \varepsilon_t - \frac{\sigma\varepsilon^2}{2}, \varepsilon_t \sim \text{NID}(0, \sigma^2\varepsilon)$$

i.e.  $(1 + \epsilon_t^*)$  is lognormal with mean 1 ( $\epsilon_t^*$  is a zero mean r.v.).

Replacing onto the previous expression, yields a nonlinear state space model with nonlinear measurement equation:

$$y_t = \mu_t^*(1 + \gamma_t^*) \exp\left(\varepsilon_t - \frac{\sigma\varepsilon^2}{2}\right).$$

Taking logarithms

$$\ln y_t = \mu_t - \sigma\varepsilon^2/2 + \ln(1 + \gamma_t) + \varepsilon_t;$$

hence, the difference with the log-additive decomposition arises from the term  $\ln(1 + \gamma_t)$ . The model is estimated by approximate maximum likelihood, using the extended Kalman filter.

Koopman and Lee (2009) have recently proposed a model that accounts for an interaction between the seasonal component and the trend and cyclical components. Let  $y_t$  denote the logarithm of the series. The model is specified as follows:

$$y_t = \mu_t + \exp(b\mu_t)\gamma_t + \varepsilon_t, \quad (27.2)$$

where  $\mu_t$  is a local linear trend,  $\gamma_t$  is a trigonometric seasonal component, and  $\varepsilon_t \sim \text{NID}(0, \sigma^2)$ .

When a cycle is present, the specification is extended so as to allow for interaction also between seasonality and cycle. This models the amplification or dampening of the seasonal fluctuations according to the state of the business cycle.

$$y_t = \mu_t + \psi_t + \exp(b\mu_t + c\psi_t)\gamma_t + \varepsilon_t, \quad (27.3)$$

where  $\psi_t$  is the cycle.

The model is a nonlinear state space model, with nonlinear measurement equation and linear transition. Inferences are carried out using the Extended Kalman filter, i.e. via a linearization around the filtered estimates of the components, e.g.

Koopman and Lee (2009) present applications concerning the logarithm of the monthly visits abroad by UK residents (BSM with cycle and trend-seasonal interaction) and the logarithm of US unemployment (BSM plus cycle and trend-seasonal and cycle-seasonal interactions) US industrial production (interaction is not present) index and dwellings production (both trend and cycle interactions are significant). Notice that normality is always rejected, thus it may be the case that a Box-Cox transform is needed.

### 27.2.3 Formal decompositions

There has been renewed interest in what is termed the Beveridge-Nelson decomposition of a seasonal process. This is a particular case of what is known in the time series literature as a formal (as opposed to statistical) decomposition; see Brewer et al. (1975), Brewer (1979) and Piccolo (1982). Its distinguishing features are twofold: the components are correlated (Piccolo (1982), provides an explicit formula for their correlation) and the filters for the extraction of the components are one-sided, since they take into account only the current and past observations. This may induce a phase shift which must be carefully considered especially for business cycle dating and real time analysis.

For the quarterly case, Proietti (1995) showed that the process

$$\begin{aligned} \Delta\Delta_4 y_t &= w_t \\ &= C(L)\xi_t \end{aligned}$$



can be decomposed as:

$$y_t = \mu_t + \gamma_t + c_t,$$

where  $\mu_t$  is an IMA(2,1) stochastic trend,  $\gamma_t$  is the stochastic seasonal component with defining property  $S_4(L)\gamma_t \sim MA(2)$  and  $c_t$  is a stationary process.

In particular, the trend is defined by the recursions

$$\begin{aligned} \mu_t &= \mu_{t-1} + \beta_t + \eta_t \\ \beta_t &= \beta_{t-1} + \zeta_t \end{aligned}$$

$\beta_t$  is the stochastic drift of the trend and is generated by a random walk with innovations  $\zeta_t = C(1)\xi_t/4$  and  $\eta_t = [\frac{1}{4}C^*(1) + \frac{3}{8}C(1)]\xi_t$ .

The seasonal component is the sum of two integrated cycles:  $\gamma_t = \gamma_{1t} + \gamma_{2t}$ , where

$$\gamma_{1t} = \frac{1}{4} \frac{(\Gamma_1 - \Gamma_2) + (\Gamma_1 + \Gamma_2)L}{1 + L^2} \xi_t, \quad \gamma_{2t} = \frac{1}{8} \frac{C(-1)}{1 + L} \xi_t.$$

The proof is given in Proietti (1995). Gregoir (2001) extended the derivation to the monthly case and provided more general results. Applications of these algorithms can be found in Proietti (1995) and Gregoir (2001).

We remark that these methods since they postulate that all the components are driven by the same disturbance, and thus are cross-correlated. These methods have not received much attention. For instance Piccolo (1982), p. 581, concludes that the aim of the formal decomposition "is to reach a quick assessment of cycle, trend and seasonal components; on the other hand, the phase shift induced and the correlation among the components may cause serious problems". Seasonal adjustment of time series using one-sided filters obtained from the canonical decomposition of a seasonal ARIMA model is presented in Cupingood and Wei (1986). This is different from the Beveridge-Nelson decomposition, as it is based on the principle of maximising the variance of the irregular component.

## 27.3 The effects of Seasonal Adjustment on Business Cycle Measurement

As we have stated in the introduction, the availability of official seasonally adjusted data is their widespread use for business cycle analysis have generated a very rich and relevant literature assessing the effects of seasonal adjustment on subsequent econometric modeling and analysis.

One of the most influential contribution is Wallis (1974), who dealt with the effects of SA on the relationship between variables. Two main important conclusions came out of his analysis: the two-sided nature of the adjustment is a potential source of distortion of the lead-lag relationships between economic variables, which could be remedied upon by using one sided filters (see the previous discussion on the Beveridge-Nelson decomposition). Secondly, the least distortionary effects arise when the same filter is applied to all the series.

Several other contributions have looked at the possible distortions that may arise as a consequence of the adjustment. Ghysels and Perron (1993) consider the effect on unit root tests. Ghysels et al. (1996) propose the interpretation of seasonal adjustment by X11 as a nonlinear data filtering process that may induce spurious nonlinearity. That seasonal adjustment can influence the evidence for nonlinear BC features is documented also in Proietti (2003). The Monte Carlo evidence presented in Proietti (2004) provides support to the finding that seasonal adjustment biases the evidence for correlated trend and cycle disturbances; Jaeger and Kunst (1990) show that SA biases upwards the measures of persistence. Ooms and Hassler (1997) consider the effects on the estimation of the long memory parameter by log-periodogram regression. Franses and de Bruin

(2000) argue instead that SA has little or no effect on the business cycle chronology estimated from STAR and seasonal STAR models of the unemployment rate.

### 27.3.1 The effect on trend-cycle interactions

The analysis of macroeconomic fluctuations usually relies on quarterly seasonally adjusted series. This raises the obvious issue as to whether seasonal adjustment can be considered as a neutral operation, in the sense that it does not alter the main stylised facts. The presence of a correlation between trend and cycle disturbances is one of those facts, given the relevance that the literature attaches to it.

Morley et al. (2003) contributed to this by considering a class of UC decompositions of U.S. real gross domestic product (GDP) into a random walk trend and a purely AR(2) cycle, that depends on the identifiable correlation between the trend and cycle disturbances and that produces an ARIMA(2,1,2) reduced form. Within this class, MNZ compare the fit and the components arising from the UC model assuming orthogonal disturbances and the BN decomposition of the unrestricted ARIMA model, which features perfectly and negatively correlated disturbances. The resulting decompositions produce different stylised facts, and in particular the BN cycle is characterised by a much smaller amplitude and a shorter periodicity.

Since a degree of freedom is allowed from the fact that the UC model has one parameter less than the ARIMA reduced form, they estimate the correlation between the trend and cycle disturbances and find out that the estimated value is negative, about -0.92, and significantly different from zero. The resulting *real time*, or concurrent, estimates of the trend and cycle in U.S. GDP closely resemble the BN components, which allows us to reconcile the UC with the unrestricted reduced form.

They interpret this empirical evidence as an expression of the dominant role of real shocks, which shift the long run path of output, whereas short term fluctuations reflect only the adjustment to the new path.

To investigate this issue we perform a Monte Carlo experiment, by which 1000 series of length  $T = 140$  are generated according to the unobserved components model  $y_t = \mu_t + \psi_t + \gamma_t$ , with orthogonal trends and cycles. In particular,  $\mu_t$  is a random walk trend and  $\psi_t$  is an autoregressive second order cycle with stationary roots:

$$\begin{aligned}\mu_t &= \mu_{t-1} + \beta + \eta_t, \\ \psi_t &= \phi_1 \psi_{t-1} + \phi_2 \psi_{t-2} + \kappa_t,\end{aligned}\tag{27.4}$$

$$\begin{pmatrix} \eta_t \\ \kappa_t \end{pmatrix} \sim \text{NID} \left[ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\eta^2 & \sigma_{\eta\kappa} \\ \sigma_{\eta\kappa} & \sigma_\kappa^2 \end{pmatrix} \right], \quad \sigma_{\eta\kappa} = r\sigma_\eta\sigma_\kappa.$$

$r$  denotes the correlation between the trend and cycle disturbances, which is equal to zero in the simulation.  $\gamma_t$  is a quarterly seasonal component, with trigonometric representation:

$$\gamma_t = \gamma_{1t} + \gamma_{2t},$$

resulting from the sum of an annual non stationary cycle

$$(1 + L^2)\gamma_{1t} = \varpi_{1t}, \quad \varpi_{1t} \sim \text{NID}(0, \sigma\omega^2)$$

and a biannual one,

$$(1 + L)\gamma_{2t} = \varpi_{2t}, \quad \varpi_{2t} \sim \text{NID}(0, 0.5\sigma\omega^2)$$

with  $\varpi_{1t}$  and  $\varpi_{2s} = 0$  independent and independent of  $\eta_t$  and  $\kappa_t$ .

The cycle autoregressive parameters are written as  $\phi_1 = -2 \cos \lambda_c$ ,  $\phi_2 = \rho^2$ , where  $\rho = 0.9$  and  $\lambda_c$  can take the two values  $2\pi/12$  and  $2\pi/32$  corresponding to a period of 3 (12 quarters) and 8 years (32 quarters), respectively. The trend-seasonal signal ratio is always kept at  $\sigma_\eta^2/\sigma_\omega^2 = 20$ , whereas for  $\sigma_\eta^2/\sigma_\kappa^2$  we consider three values, Low:  $\sigma_\eta^2/\sigma_\kappa^2 = 1/3$ ; Medium:  $\sigma_\eta^2/\sigma_\kappa^2 = 3$ ; High:  $\sigma_\eta^2/\sigma_\kappa^2 = 30$ . The combination of these values

with the two cycle periods gives 6 data generating processes in total.

For each simulation we fit the true model model by maximum likelihood and construct a seasonally adjusted (SA) series,  $y_t^{(SA)}$  by removing from the simulated series the smoothed estimates of the seasonal component, obtained by the Kalman filter and smoother using the estimated parameters. The non seasonal model

$$y_t^{(SA)} = \mu_t + \psi_t,$$

with components as in expressions (27.4) is fitted to the series. In the presentation of the results we label this experiment as SA-UC( $r,0$ ), since we estimate the trend cycle correlation on seasonally adjusted data. Moreover, to characterise the small sample distribution of the correlation coefficient when the true value is  $r = 0$ , we estimate model a trend plus cycle plus seasonal model with correlated trend and cycle disturbances, that is (27.4) plus an orthogonal seasonal component. We shall refer to this experiment with TCS( $r, 0$ ).

Figure 27.1 plots the distribution of the estimated  $r$  for SA-UC( $r,0$ ) and TCS( $r,0$ ) in the six cases. The histograms clearly point out that seasonal adjustment biases the estimates of the correlation coefficient, increasing the the evidence for a negative correlation. In general, the problem is lessened as we move away from the fundamental seasonal frequency (a yearly cycle), as the histograms for the 32 quarters cycle suggest.

Also, the panels in the second and the fourth columns highlight that the small sample distribution of  $r$  estimated on the unadjusted data is highly nonstandard, suffering from a pile-up problem at  $\pm 1$ . Experimentation suggests that we need a much larger sample size to have  $r$  distributed symmetrically around its true zero value.

### 27.3.2 Seasonal Adjustment and the Reliability of the Cycle Estimates

The business cycle is often measured by applying an ad hoc filter to seasonally adjusted time series. The aim of this section is to assess the contribution of seasonal adjustment to the uncertainty of the measurement. An well-known filter has been popularized by Baxter and King (1999): this is a band-pass filter that aims at selecting the fluctuations with a specified range of periodicities, namely those ranging from one and a half to eight years. Thus, if  $s$  is the number of observations in a year, the fluctuations with periodicity between  $1.5s$  and  $8s$  are included. Given the two business cycle frequencies,  $\omega_{c1} = 2\pi/(8s)$  and  $\omega_{c2} = 2\pi/(1.5s)$ , the BK filter cycle filter is

$$w_{bp}(L) = \frac{\omega_{c2} - \omega_{c1}}{\pi} + \sum_{j=1}^{3s} \frac{\sin(\omega_{c2}j) - \sin(\omega_{c1}j)}{\pi j} (L^j + L^{-j}). \quad (27.5)$$

up to a proportionality factor, which is  $[w_{bp}(1)]^{-1}$ .

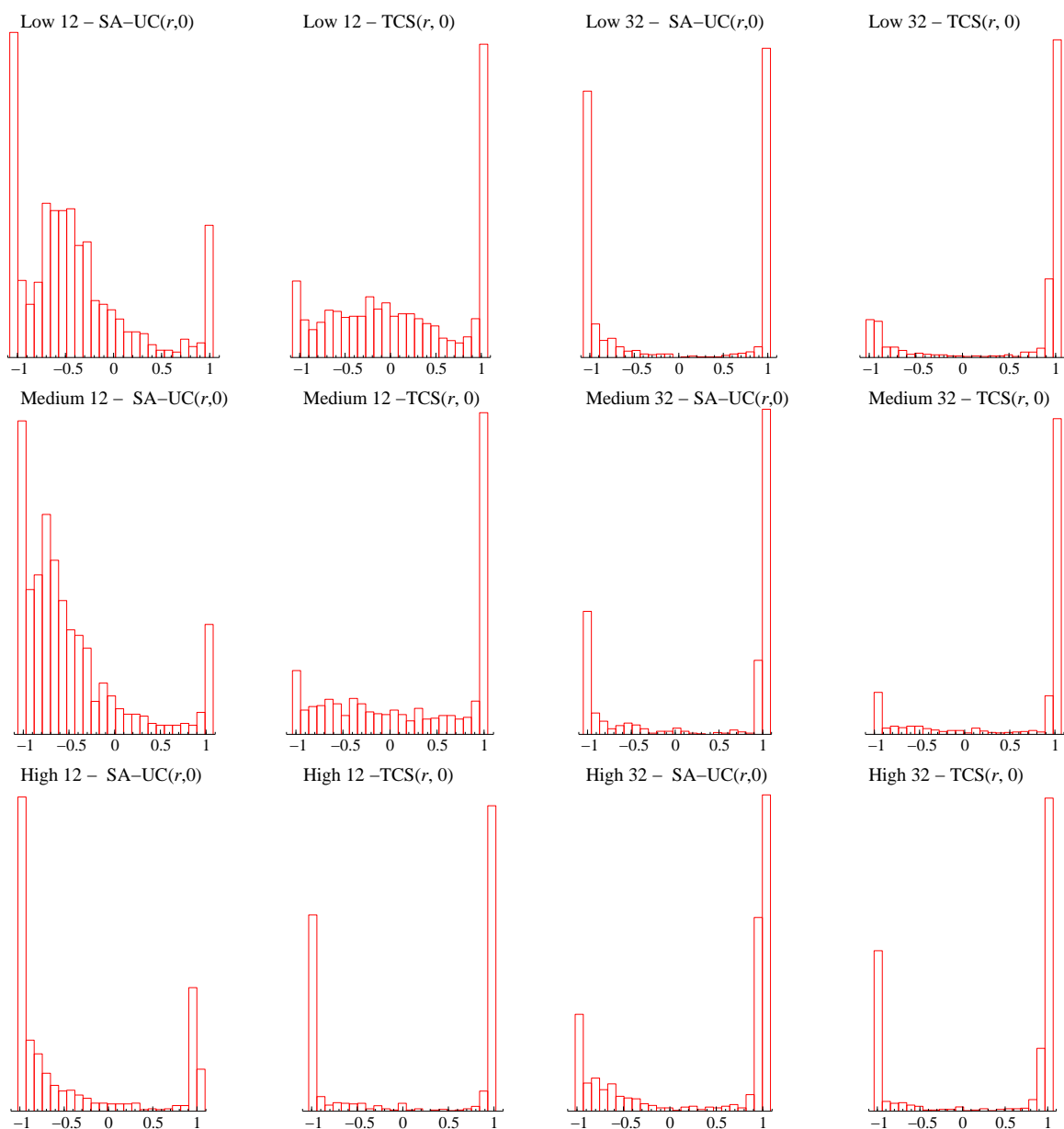
Another important filter in macroeconomics is the Hodrick and Prescott (HP) filter. The HP trend minimises the penalised least squares criterion:

$$\begin{aligned} PLS &= \sum_{t=1}^n (y_t - \mu_t)^2 + \lambda \sum_{t=3}^n (\Delta^2 \mu_t)^2 \\ &= (y - \mu)'(y - \mu) + \lambda \mu' D^2 D^2 \mu \end{aligned}$$

$D^2$  is the  $n - 2 \times n$  matrix corresponding to the 2nd differences filter, with  $d_{ii} = 1$ ,  $d_{i,i-1} = -2$ ,  $d_{i,i-2} = 1$  and zero otherwise. Differentiating with respect to  $\mu$ , the first order conditions yield:  $\tilde{\mu} = (I_n + \lambda D^2 D^2)^{-1} y$ . The *smoothness* or *roughness penalty* parameter,  $\lambda$ , governs the trade-off between fidelity and smoothness. HP purposively select the value  $\lambda = 1600$  for quarterly time series. Ravn and Uhlig (2002) discuss the choice of  $\lambda$  for any frequency  $s$  of observations.

It is well known that, assuming the availability of a doubly infinite sample,  $y_{t+j}$ ,  $j = -\infty, \dots, \infty$ , the above HP filter tends to the Wiener-Kolmogorov optimal signal extraction filter for the trend component  $\mu_t$  of the

**Figure 27.1:** Distribution of the estimated correlation coefficient,  $r$ , between the cycle and the trend disturbance. 1000 quarterly series of length  $T = 140$  are generated according to orthogonal trend plus cycle plus seasonal models with low, medium and high signal ratios, and cycle periods equal to 12 and 32 quarters.



following local linear trend model:

$$\begin{aligned} y_t &= \mu_t + \epsilon_t, & t = 1, 2, \dots, n, \\ \Delta^2 \mu_t &= \zeta_t, & \zeta_t \sim \text{NID}(0, \sigma_\zeta^2), \\ \epsilon_t &\sim \text{NID}(0, \lambda \sigma_\zeta^2), & \text{E}(\zeta_t, \epsilon_{t-j}) = 0, \forall j, \end{aligned} \quad (27.6)$$

The HP cycle filter can thus be represented (see Whittle (1983)) as :

$$\tilde{\epsilon}_t = w_{HP}(L)y_t, \quad w_{HP}(L) = \frac{\lambda|1-L|^4}{1+\lambda|1-L|^4} \quad (27.7)$$

and we have written  $|1-L|^2 = (1-L)(1-L^{-1})$ .

In the sequel we shall generically denote by  $w_c(L)$  the cycle extraction filter under investigation. The important point is that we are not able to observe  $y_t$ , but rather a contaminated version of it,

$$z_t = y_t + \gamma_t,$$

where  $\gamma_t$  is the seasonal component. Let us also assume that the components are orthogonal, so that the pseudo-autocovariance generating function (ACGF) of  $z_t$  decomposes as  $g_z(L) = g_y(L) + g_\gamma(L)$ , where  $g_y(L)$  and  $g_\gamma(L)$  are ACGFs of the nonseasonal and the seasonal components, respectively, which we assume to have a known parametric form. The minimum mean square linear estimator of the nonseasonal component based on a doubly infinite sample is

$$\tilde{y}_t = \frac{g_y(L)}{g_z(L)} z_t.$$

We mention in passing that with little effort we could cover the case in which there are interactions between the seasonal and the nonseasonal component, for which  $g_z(L) = g_y(L) + g_\gamma(L) + g_{y\gamma}(L) + g_{\gamma y}(L)$ , where  $g_{y\gamma}(L) = g_{\gamma y}(L^{-1})$  is the cross-covariance generating function of  $(y_t, \gamma_t)$ .

Business cycle analysis is customarily carried out by applying the filter  $w_c(L)$  to the seasonally adjusted series,  $\tilde{y}_t$ , rather than  $y_t$ , which is unobserved. Let

$$c_t = w_c(L)y_t$$

denote the true cycle, which arises from applying the cycle filter to  $y_t$ . The estimator  $w_c(L)z_t$  would have very poor properties due to leakage from the seasonal frequencies; in particular, the fundamental frequency (corresponding to a period of one year) lies very close to the business cycle frequency range that characterizes the BK filter (1.5 years to 8 years), and the HP cycle is a high-pass filter that will retain to great extent the spectral power at the seasonal frequencies.

We thus focus on the estimator of the cycle

$$\tilde{c}_t = \text{E}(c_t|z) = w_c(L)\tilde{y}_t, \quad (27.8)$$

where  $z = (\dots, z_{t-1}, z_t, z_{t+1}, \dots)$ , and similarly  $y = (\dots, y_{t-1}, y_t, y_{t+1}, \dots)$ ,  $\tilde{y} = (\dots, \tilde{y}_{t-1}, \tilde{y}_t, \tilde{y}_{t+1}, \dots)$ . This is not without consequences for business cycle analysis. In particular, the seasonal component is estimated with nonzero estimation error variance, and this constitutes an additional source of variation for the above estimator.

It is important to illustrate how seasonal adjustment affects the reliability of the cycle measurement. Orphanides and van Norden (2002) have stressed the relevance of the uncertainty assessment for the estimation of the output gap, which is often measured by applying an ad hoc filter  $w_c(L)$ , such as BK or HP to seasonally adjusted data. Hereto we are not aware of studies that aimed at assessing the role of seasonal adjustment and this section aims at bridging the gap.

We shall be exclusively concerned with the variability due to estimation of the seasonally adjusted series due to smoothing. We will consider the additional source arising from the estimation of seasonal model parameters at a later stage.

Let  $e_t = c_t - \tilde{c}_t$  denote the estimation error. Its ACGF is

$$g_e(L) = |w_c(L)|^2 \frac{g_y(L)g_\gamma(L)}{g_z(L)}.$$

The  $\tilde{c}_t$  estimator is (un)conditionally unbiased, since  $E(e_t|z) = 0$ , but the variance of  $\tilde{c}_t$  is a downward biased estimator of  $\text{Var}(c_t)$ . Denoting by  $g_y(\omega)$ ,  $\omega \in [0, \pi]$ , the spectral generating function (SGF) of  $y_t$ , and by  $w_c(\omega) = |w_c(e^{-i\omega})|^2$  the squared gain of the cycle filter, i.e. the squared modulus of the frequency response function of the filter, with  $e^{-i\omega} = \cos(\omega) - i \sin(\omega)$ , the unconditional variance of the true cycle is

$$\text{Var}(c_t) = \int_0^\pi \pi w_c(\omega) g_y(\omega) d\omega.$$

Also, the SGF of  $\tilde{y}_t$  is  $g_{\tilde{y}}(\omega) = [g_y(\omega)]^2 / g_z(\omega)$ , as the estimator of the SA series is  $\tilde{y}_t = g_y(\omega) / g_z(\omega) z_t$ . The variance of the cycle estimator is thus:

$$\begin{aligned} \text{Var}(\tilde{c}_t) &= \int_0^\pi \pi w_c(\omega) g_{\tilde{y}}(\omega) d\omega \\ &= \int_0^\pi \pi w_c(\omega) \frac{g_y(\omega)}{g_z(\omega)} g_y(\omega) d\omega \\ &\leq \text{Var}(c_t) \end{aligned}$$

The last inequality follows from the following basic identity:

$$\begin{aligned} \text{Var}(c_t) &= \text{Var}[E(c_t|z)] + E[\text{Var}(c_t|z)] \\ &= \text{Var}(\tilde{c}_t) + \text{Var}(e_t|z) \\ \text{Var}(e_t|z) &= \text{Var}(c_t|z) = \int_0^\pi \pi w_c(\omega) \frac{g_y(\omega)g_\gamma(\omega)}{g_z(\omega)} d\omega \end{aligned} \tag{27.9}$$

where the expectations are taken with respect to the distribution of  $y$  given  $z$ . A first conclusion is that seasonal adjustment implies an underestimation of the cycle volatility, i.e. the amplitude of the cycle estimate is lower than the true amplitude.

The conditional variance of the cycle estimator, given the data  $z$ , is thus smaller than the variance of the true cycle. The components  $\text{Var}(c_t)$  and  $\text{Var}(e_t|z)$  can be evaluated as a by-product of the SA modeling effort, as it will be illustrated shortly. Hence, in the sequel, we will assume that a parametric UC model is available postulating a decomposition  $z_t = y_t + \gamma_t$ .

If  $c_t$  had an embedded model based representation,  $\text{Var}(e_t|z)$  would be computed by the Kalman filter and smoother adapted to the state space representation of the model, or by the expressions provided by Bell and Martin (2004).

The variance inflation factor that is attributable to seasonal adjustment (i.e. to the fact that  $y_t$  is replaced by  $\tilde{y}_t$ ) is

$$VIF = \frac{\text{Var}(c_t)}{\text{Var}(\tilde{c}_t)} = 1 + \frac{\text{Var}(e_t|z)}{\text{Var}(\tilde{c}_t)}.$$

The increase in the variance depends on the filter, and the proportion of the total variation that is attributable to seasonality. If  $y_t$  were observed, then  $VIF = 1$ . Otherwise  $VIF$  is greater than 1.

### 27.3.3 Empirical Evaluation of the Variance Inflation Factor

The VIF can be computed analytically; however, the analytic expressions are valid for the cycle estimates in the middle of a long time series; analogous formulae could be derived using optimal prediction theory (see e.g. Whittle (1983)), but are extremely cumbersome and analytically intractable.

However, in finite sample they can be evaluated by Monte Carlo methods, by setting up the following simulation scheme.

1. Formulate the model  $z_t = y_t + \gamma_t$  and estimate it by maximum likelihood, under the assumption of Gaussianity (Bayesian MCMC estimation is also possible). Obtain  $\tilde{y} = E(y|z)$  using the Kalman filter and smoothing algorithm.
2. For  $i = 1, \dots, M$ , run the following simulation smoother (Durbin and Koopman (2002)):
  - a) Draw  $y^{(i)}, \gamma^{(i)}$ , from  $g_y(L)$  and  $g_\gamma(L)$ , respectively. Obtain  $z^{(i)} = y^{(i)} + \gamma^{(i)} \sim g_z(L)$ .
  - b) Obtain the estimate of the nonseasonal component  $\tilde{y}^{(i)}, i = 1, \dots, M$ , (using the Kalman filter and smoother as in step 1).
  - c) Compute  $\dot{y}^{(i)} = \tilde{y} + (y^{(i)} - \tilde{y}^{(i)}) \sim y|z$ .  $\dot{y}^{(i)}$  is a draw from the conditional distribution of  $y$  given  $z$ . The latter is normal with mean  $\tilde{y}$ .
  - d) Compute

$$\begin{aligned}\dot{c}_t^{(i)} &= w_c(L)\dot{y}_t^{(i)} \sim c_t|z \\ c_t^{(i)} &= w_c(L)y_t^{(i)} \sim c_t\end{aligned}$$

3. Estimate  $\text{Var}(c_t|z)$  using

$$\widehat{\text{Var}}(c_t|z) = \frac{1}{M} \sum_i \left( \dot{c}_t^{(i)} - \tilde{c}_t \right)^2$$

4. Estimate  $\text{Var}(c_t)$  using

$$\widehat{\text{Var}}(c_t) = \frac{1}{M} \sum_i \left( c_t^{(i)} - \bar{c}_t \right)^2$$

The estimate of the conditional variance,  $\widehat{\text{Var}}(c_t|z)$ , provides the assessment of the reliability required.

The actual implementation of this scheme entails a parametric model for the decomposition  $z_t = \mu_t + \gamma_t$ . This is the case of ARIMA model based seasonal adjustment (see Hillmer and Tiao (1982)) and structural time series models (Harvey (1989)).

### 27.3.4 Empirical illustration

We present an empirical illustration which deals with the extraction of the cycle by the BK filter and the HP filter for three monthly industrial production series referring to Germany, France and Italy. The series are taken from the Eurostat database Europa and are available for the sample 1990.1-2009.4 (France and Italy) and 1991.1-2009.4 (Germany). We restrict our analysis to the sample period up to 2007.12.

We assume that  $z_t$  follows a basic structural model:

$$\begin{aligned}z_t &= y_t + \gamma_t, \\ y_t &= \mu_t + \epsilon_t, \\ \gamma_t &= s_t + x'_{it} \delta_i,\end{aligned}$$

where the nonseasonal component,  $y_t$  is a local linear trend plus irregular,

$$\begin{aligned}\mu_t &= \mu_{t-1} + \beta_t + \eta_t, & \eta_t &\sim \text{NID}(0, \sigma_\eta^2) \\ \beta_t &= \beta_{t-1} + \zeta_t, & \zeta_t &\sim \text{NID}(0, \sigma_\zeta^2)\end{aligned}\quad (27.10)$$

and  $\epsilon_t \sim \text{NID}(0, \sigma_\epsilon^2)$ . The seasonal component is made up of a purely seasonal trigonometric cycle plus the calendar component, which is obtained from the regression on suitable explanatory variables.

The component  $s_t$  arises from the combination of six stochastic cycles defined at the seasonal frequencies  $\lambda_j = 2\pi j/12$ ,  $j = 1, \dots, 6$ ,  $\lambda_1$  representing the fundamental frequency (corresponding to a period of 12 monthly observations) and the remaining being the five harmonics (corresponding to periods of 6 months, i.e. two cycles in a year, 4 months, i.e. three cycles in a year, 3 months, i.e. four cycles in a year, 2.4, i.e. five cycles in a year, and 2 months):

$$s_t = \sum_{j=1}^6 s_{jt}, \quad \begin{bmatrix} s_{j,t} \\ s_{j,t}^* \end{bmatrix} = \begin{bmatrix} \cos \lambda_j & \sin \lambda_j \\ -\sin \lambda_j & \cos \lambda_j \end{bmatrix} \begin{bmatrix} s_{j,t-1} \\ s_{j,t-1}^* \end{bmatrix} + \begin{bmatrix} \omega_{j,t} \\ \omega_{j,t}^* \end{bmatrix}, \quad j = 1, \dots, 5, \quad (27.11)$$

and  $s_{6,t} = -s_{6t} + \omega_{6t}$ . The disturbances  $\omega_{jt}$  and  $\omega_{jt}^*$  are normally and independently distributed with common variance  $\sigma_\omega^2$  for  $j = 1, \dots, 5$ , whereas  $\text{Var}(\omega_{6t}) = 0.5\sigma_\omega^2$  (see Proietti (2000), for further details). Calendar effects are accounted for by six trading day regressors, measuring the number of days of type  $j$ ,  $j = 1, \dots, 6$ , occurring in month  $t$  in excess of the number of Sundays. One regressor picks up the Easter effect, and an additional one captures the length of month (LOM) effect.

All the disturbances in the model are mutually and serially uncorrelated.

The model is fitted to each of the time series by maximum likelihood. Conditional on the maximum likelihood estimates, we compute the estimate of the cycle (27.8). To assess its reliability, we perform the simulation scheme proposed in the previous section to get the estimated conditional variance of  $\tilde{c}_t$ .

Figure 27.2 displays the 95% interval estimates of the seasonally adjusted series, the HP cycle with  $\lambda = 129600$ , the BK cycle, and the HP bandpass cycle. It should be noticed that the SA series displays a lot of variation at the high frequency. This is responsible for the high variability of the HP cycle estimates (see the top right panel). The latter is obtained by adopting the value  $\lambda = 129600$  for the smoothness parameter, which, according to Ravn and Uhlig (2002), is the exact monthly analogue of the traditional HP with  $\lambda = 1600$  for quarterly time series. As a matter of fact the HP cycle filter is a high-pass filter that leaves unchanged the amplitude of the high frequency fluctuations. As a result a lot of high frequency variation leaks from the SA series to the cycle estimates.

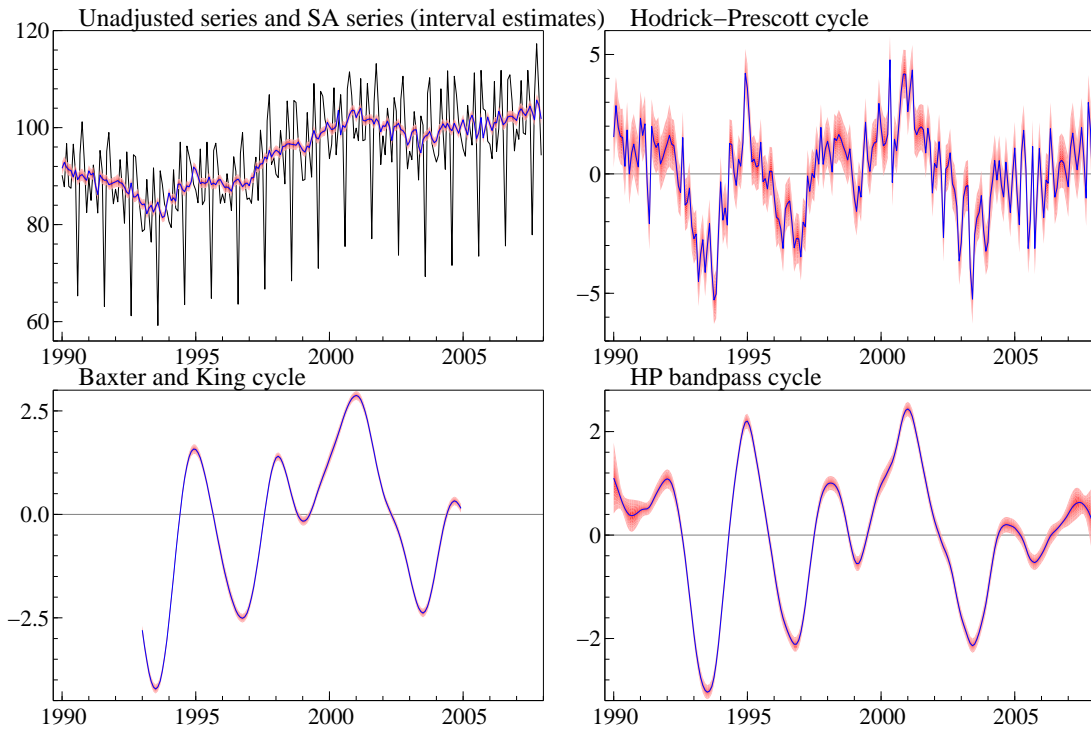
The situation improves a lot when we consider cycle measures that suppress the high frequency variation. The bottom left plot displays the point estimates of the BK cycle, along with their 95% confidence bands. These estimates are little affected by the adjustment, being very stable. It should be noticed that the BK filter is two-sided and fails to produce the estimates at the beginning and at the end of the sample. These limitations are overcome by the bandpass version of the HP filter, which originates from the difference of two low-pass HP filters with cut-off periods equal to 18 months (1.5 years) and 92 months (8 years), corresponding respectively to  $\lambda = 68.7$  and 54535. The bottom right plot confirms that in the middle of the sample seasonal adjustment does not contribute much to the variability of the estimates. However, as it might be anticipated, the variance of the estimates is much higher at the extremes of the sample period.

Figure 27.3 compares the 95% interval estimates of the HP bandpass cycle for Germany, Italy and France. The German series is available starting from 1991. The plot reveals that the uncertainty is about the same, the series behaving very similarly, and that the business cycles are very synchronized.

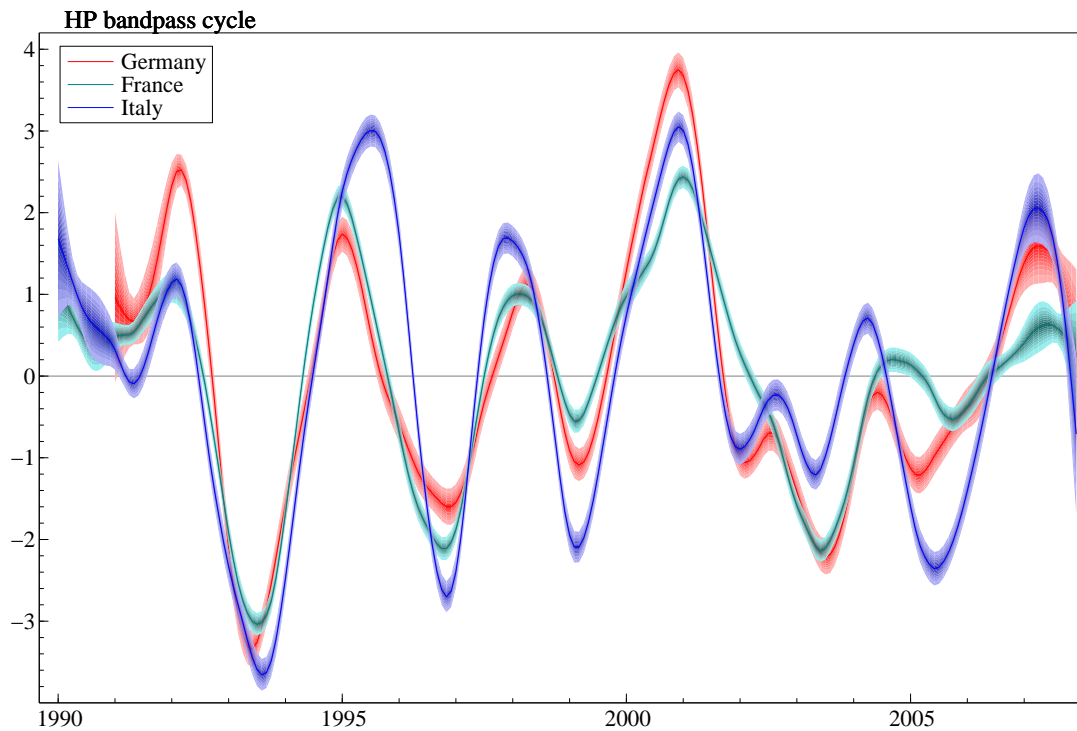
Figure 27.4 reproduces the pattern for the variance inflation factors. These are higher for the HP highpass filter, and close to one for the BK filter. The plot confirms that seasonal adjustment does not contribute much to the precision of the cycle estimates if a bandpass component is defined which is not affected by the high



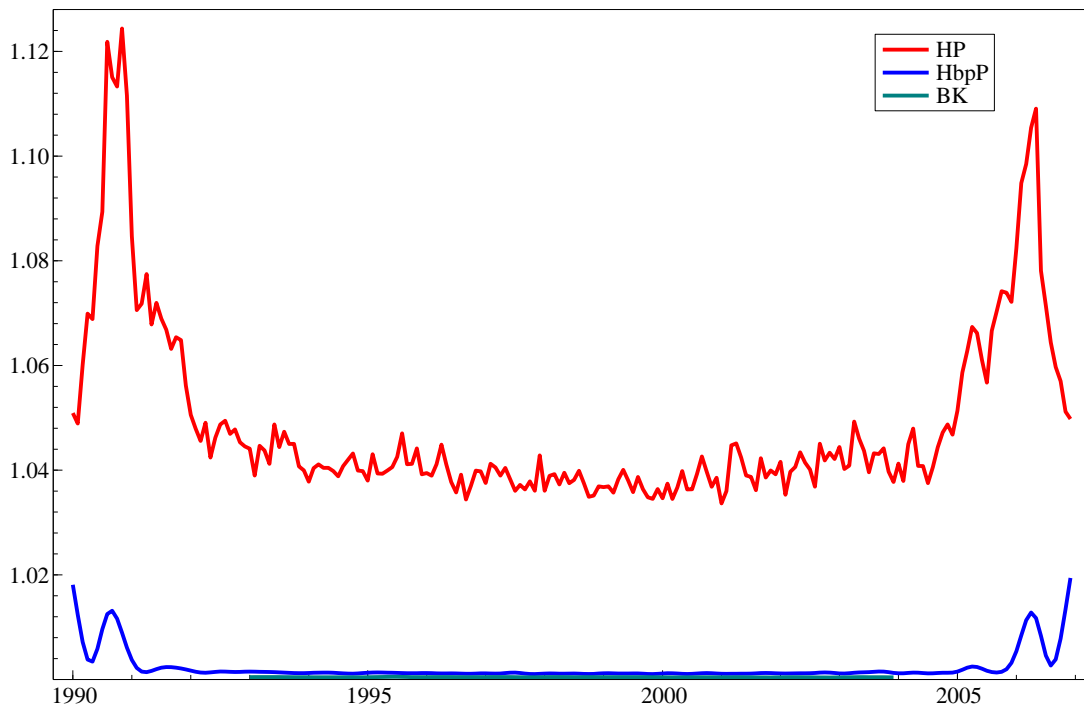
**Figure 27.2:** France, Index of Industrial Production (Manuf. total, Source Eurostat). Interval estimates of the seasonally adjusted series, the HP cycle with  $\lambda = 129600$ , the BK cycle, and the HP bandpass cycle.



**Figure 27.3:** Germany, France and Italy, Index of Industrial Production (Manuf. total, Source Eurostat). Comparison of the estimates of the HP bandpass cycle.



**Figure 27.4:** Germany, Index of Industrial Production (Manuf. total, Source Eurostat). Variance inflation factors for three cycle estimates: HP, HP bandpass and BK.



frequency components of the original series.

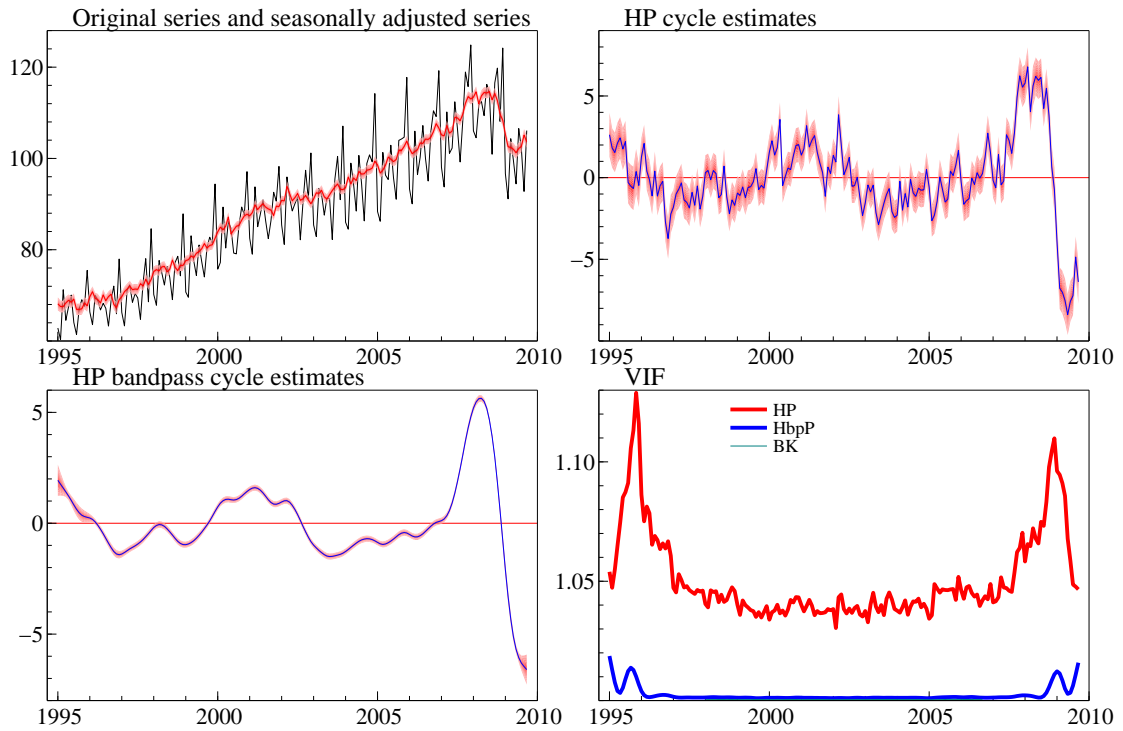
Our final illustration deals with the unadjusted turnover of the French retail sector. The series is available from Jan-1995 to Sept-2009. In this case we include the recent recession in the sample period. The results, displayed in figure 27.5 confirm by and large the previous conclusions. In particular, the HP cycle estimates are less reliable.

### 27.3.5 Discussion

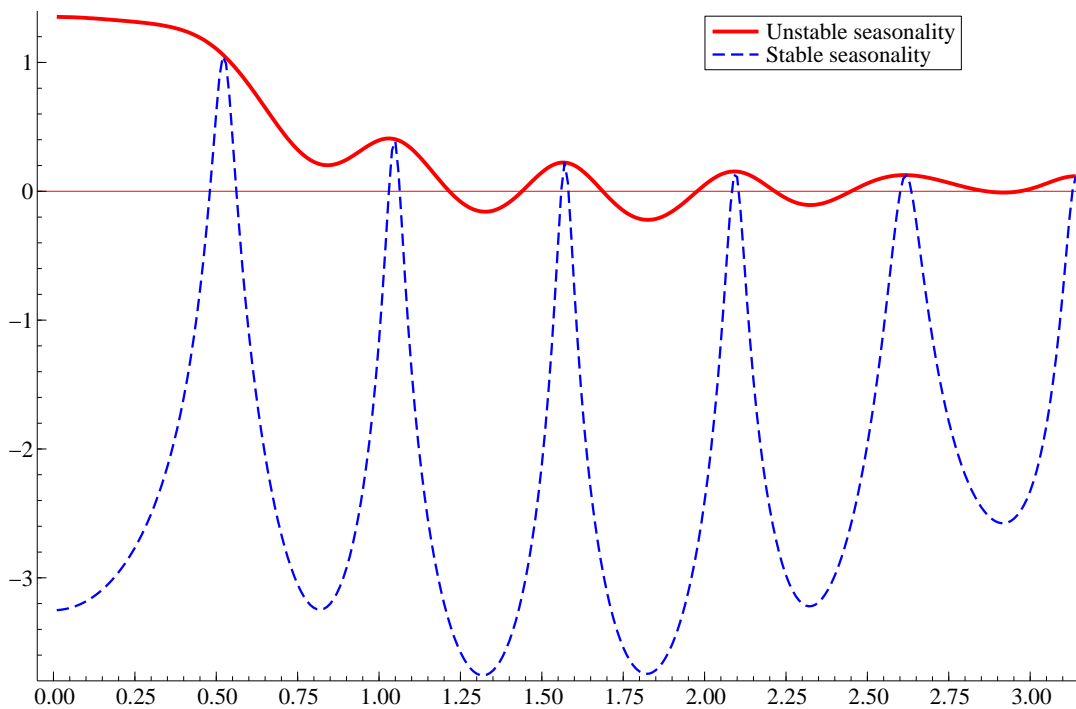
In general, the contribution of seasonal adjustment to the uncertainty of the cycle estimates will depend on the extent to which  $\gamma_t$  contaminates  $y_t$ . A more variable seasonal component will imply that the estimation error will have a Figure 27.6 compares the logarithm of the spectral generating function of the estimation error  $y_t - \tilde{y}_t$  when the size of the seasonal component differs. In particular, the process  $z_t$  is generated with  $\sigma_\zeta^2 = 0$  and by two different values of the signal to noise ratio  $\sigma_\omega / (2\sigma_\epsilon^2 + \sigma_\eta^2)$ : 0.001 (stable seasonality) and 0.1 (unstable seasonality).

When the seasonal component is more stable, then the spectrum of the estimation error will display low power at the low frequencies. Viceversa, if the seasonality is subject to a lot of variation, the estimation uncertainty will be higher at the low frequencies. In particular, the HP cycle estimates are less reliable.

**Figure 27.5:** France, Index of Turnover, Retail sector (Source Eurostat). Interval estimates of the seasonally adjusted series (first panel), the HP cycle with  $\lambda = 129600$ , (top right panel), the HP bandpass cycle (bottom left panel), and Variance Inflation Factors (bottom right panel).



**Figure 27.6:** Spectral generating function (decibels) of the estimation error of the seasonally adjusted series for seasonal components having different evolution.



## 27.4 Conclusions

This report has concentrated mainly on a systematic review of the literature on the interaction between cycle and seasonality and on assessing the contribution of seasonal adjustment as a (often neglected) source of the reliability of the cycle estimates. The main conclusion is that this contribution is sizable for highpass filters, like the Hodrick-Prescott filter which are strongly affected by the high frequency fluctuations in a time series. These are the one that are more affected by seasonal adjustment. If a band-pass component is considered, the effect is much less sizable, although it is stronger at the extremes of the sample period.

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# IX

## Seasonal Adjustment of High Frequency Data





# 28

## Weekly Seasonal Adjustment: A Locally-weighted Regression Approach



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## Table of Contents

28.1	Introduction	737
28.2	The Method	737
28.3	Holidays and Outliers	740
28.4	Diagnostics	741
28.5	Example: Initial Claims, U.S. Unemployment Insurance Program	743
28.6	Software	749
28.6.1	Description	749
28.6.2	Input and output files	751
28.6.3	Run instructions with the SAS interface program	753



## 28.1 Introduction

The *Economist* (Feb 25, 2010) and others have spoken of a "data deluge" in these times. Statistical agencies extend their coverage to smaller and smaller geographical areas. Financial data are available at very high frequencies. In like fashion, weekly time series offer timely updates for government and private observers of the economy while they wait for the release of monthly or quarterly economic indicators. The unemployment insurance (UI) claims series produced by the U.S. Department of Labor and money supply series from the U.S. Federal Reserve are prominent examples.

Weekly series have characteristics which make seasonal adjustment inaccessible or unsatisfactory with most available software. These series are typically compiled for weeks ending on a given day of the week, Saturday in the application presented here. The number of Saturday's within a year can be either 52 or 53 and their position varies from year to year. These features violate the basic periodic time series structure assumed by X-13ARIMA-SEATS, TRAMO/SEATS, and STAMP. The SABL method of Cleveland, W.S., Dunn, D.M., and Terpenning, I.J. (1978) transforms weekly data to create a period 52 series and applies robust versions of the seasonal and trend smoothers of X-11. The Kalman filter methods of Gersch, W. and Kitagawa, G. (1983) also assume a fixed number of periods, but could be extended to add multiple regressions and their corresponding hyperparameters. Additionally, holidays require more attention; in the U.S., most switch weeks across years, which must be accounted for.

Periodic splines are used in work by Poirier, D.J. (1973), Harvey, A. and Koopman, S.J. (1993), and Harvey, A., Koopman, S.J., and Riana, M. (1997). The last paper develops structural models with stochastic trend, seasonal, and holiday effects, allowing for estimation using Kalman filter techniques. The seasonal component is modeled as a periodic spline with time-varying parameters. The spacing of the spline knots and values of the variance ratios have significant implications for the results, adding complexity to the method.

A locally-weighted least squares procedure is suggested here, which can be used with a weekly design matrix having 52 or 53 observations in a year. The procedure starts from the regression method in Pierce, D.A., Grupe, M.R. and Cleveland, W.P. (1984), which assumes a deterministic seasonal component. The method is currently being applied at the Bureau of Labor Statistics, the Federal Reserve, and the Bank of Canada. Much of this material has appeared in a JOS article, Cleveland and Scott (2007).

The method is developed in the next section, followed by sections on holidays/outliers and diagnostics. Section 28.5 contains a detailed application. Section 28.6 describes a software implementation MoveReg based on a FORTRAN executable and a SAS interface. The interface makes execution more convenient and provides valuable displays, but the FORTRAN program can be run independently.

## 28.2 The Method

The basic model is a regression on sine and cosine terms evaluated over the days of a year. Using a sufficient number of terms of increasing frequency in cycles per year allows representation of any regular seasonal pattern. Since a given day of the month has a corresponding day of the year (leap years will be dealt with later), within month patterns can also be represented in this manner. The design matrix with columns of sine and cosine terms is augmented by columns specifying holiday and outlier effects.

While the basic seasonal effects are tied to the day of the year, weeks ending on a particular day of the week are not. For a series with weeks ending on Saturdays, the program determines on which day of the year the first Saturday falls. With this information, weekly data values can be associated with the correct days of the year with corresponding sine and cosine values. If Saturday fell on the first day of the year, there would be 53 weeks that year. Weeks ending on Saturday will be used as illustration, but the algorithm and computer program implementing it are general and can be used for weeks ending on any day of the week.

When approaching a new series, a fixed weight regression with only 12 or 24 sine and cosine terms is run. This can be done by running the software described below with simple settings or even with an external program. The fixed weight regression captures enough of the seasonality to reveal much about what outlier or holiday specifications will be needed. After adding appropriate additional terms, the locally weighted regression is carried out to estimate regression coefficients so that seasonal effects can evolve over time. In this context one can determine better how many trigonometric coefficients will ultimately be needed and proceed with further specification of outliers and holidays.

The analysis begins with a regression model for a series  $y$ , which is the observed series after suitable transformation and detrending. Series used in this paper were logged and differenced. Thus, the model for  $y$  consists of a seasonal component and error.

$$y = X\beta + e \tag{28.1}$$

For the seasonal component of year  $s$ , we employ trigonometric variables with fundamental frequency  $1/365$ ,

$$X_s(t, 2j - 1) = \sin(2\pi ij/365), X_s(t, 2j) = \cos(2\pi ij/365) \tag{28.2}$$

where  $i = i(s, t)$  is the day of the year  $s$  on which week  $t$  ends and  $j = 1, 2, \dots, p/2$ . We choose  $p$  sufficiently large to capture the dynamics of the seasonal pattern. The index  $t$  runs from 1 to  $n$ , which is 52 or 53. We stack the yearly matrices into an overall design matrix in  $X$  in levels. For leap years, the index  $i$  is set to  $i - 1$  for days after February 29 so that these days have the same day-of-the-year index in computing the trigonometric terms for leap years as for other years. Now let  $X$  represent the first difference of this matrix. Assuming  $X$  is defined for  $K$  complete years, it has dimension  $n \times p$ , where  $n = \sum_{s=1}^K n_s$ . To achieve a weighted regression, we employ an  $n \times n$  diagonal weight matrix  $W$  and apply the standard solutions.

$$\hat{\beta} = (X'WX)^{-1}X'Wy, \hat{y} = X\hat{\beta} \tag{28.3}$$

The regression parameter estimates  $\hat{\beta}$  minimize  $(y - X\beta)'W(y - X\beta)$ . The term of (28.3) requiring an inverse corresponding to a three-year series may be expanded as

$$\begin{bmatrix} X'_1 & X'_2 & X'_3 \end{bmatrix} \begin{bmatrix} w_1 I_{n_1} & & \\ & w_2 I_{n_2} & \\ & & w_2 I_{n_2} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \sum_s w_s X'_s X_s \tag{28.4}$$

For  $\sum_s w_s = 1$  and identical  $X$  matrices (e.g., for monthly data),  $X'WX$  simplifies to  $X'_s X_s$ . Even though the  $X_s$  vary from year to year for weekly series, the  $X'_s X_s$  matrices are identical for years with 52 weeks. Years with 53 weeks are infrequent enough so we can evaluate (28.4) simply as  $X'X$ . Also, we have

$$X'Wy = w_1 X'_1 y_1 + w_2 X'_2 y_2 + w_3 X'_3 y_3$$

Looking back at (28.3), we see  $\hat{\beta}$  is a weighted sum of regression coefficients for individual years. Our estimated seasonal component for year  $s$  becomes

$$\hat{y}_s = X_s (X'X)^{-1} \sum_i w_i X'_i y_i$$

Use of identical weights  $w_i$  would correspond to the results in Pierce, D.A., Grupe, M.R. and Cleveland, W.P. (1984). To allow for moving seasonality, we can apply the above method once for each year, choosing a weight matrix  $W$  geared to that particular year and using the results only for that year's seasonal component. Let  $I_p$  represent a  $p \times p$  identity matrix, and let  $w_{ij}$  be the weight applied to year  $j$  to estimate year  $i$  factors. With three years, we may write

$$\begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \end{bmatrix} = \begin{bmatrix} X_1 (X'X)^{-1} & & \\ & X_2 (X'X)^{-1} & \\ & & X_3 (X'X)^{-1} \end{bmatrix} \begin{bmatrix} w_{11} I_p & w_{12} I_p & w_{13} I_p \\ w_{21} I_p & w_{22} I_p & w_{23} I_p \\ w_{31} I_p & w_{32} I_p & w_{33} I_p \end{bmatrix} \begin{bmatrix} X'_1 y_1 \\ X'_2 y_2 \\ X'_3 y_3 \end{bmatrix} \tag{28.5}$$

The choice of the  $w_{ij}$  for each year is to be determined. Years close to the year being estimated should get the most weight. The X-11 procedure uses variations of the 3x5 seasonal filter for this purpose, applying asymmetric versions near the ends of series. We use a formula from signal extraction theory, see Cleveland (W.P. and Tiao) and references. The seasonal factors for a given week of the year (or month for monthly data) are assumed to follow an autocorrelated random walk. The detrended data are this seasonal part plus white noise, which means no autocorrelations in the

detrended, seasonally adjusted series of lag one year. Given the model

$$y_t = u_t + e_t$$

$$(1 - B)(1 - \phi B)u_t = a_t$$

with white noise terms  $e_t$  and  $a_t$ , the weights to estimate  $u_t$  given  $y$  form the desired  $W^*$  matrix. These are obtained from

$$E(u|y) = (I + \nu \Sigma_u^{-1})^{-1}y = W^*y$$

where  $\nu = \sigma_e^2 / \sigma_a^2$  and  $\Sigma_u$  is the autocorrelation matrix of  $u$ . The values  $w_{ij}^*$  are the weights  $w_{ij}$  in (28.5). The weights are more concentrated (distant years have less impact) for smaller values of  $\phi$  and  $\nu$ , but the pattern is much more sensitive to changes in  $\nu$ . Two examples of  $W^*$  for a series of length 9 years are given in Table 28.1. The rows are labeled for the year being estimated and contain the weights for that year (bold) and adjacent years. Note that the last rows show symmetric weight patterns. For seasonal series with more noise, it makes sense to use more data to extract an estimate of the seasonal signal. With  $\nu = 10$ , the first three years provide more than 80 percent of the weight for year 1; with  $\nu = 24$ , they provide about two-thirds of the weight.

Table 28.2 shows the weights for X-11 seasonal adjustment with 3x5 and 3x9 seasonal filters; cf. Shiskin, J., Young, Alan H., and Musgrave, J.C. (1967). While there is a rough correspondence with the filters from the signal extraction formula, the signal extraction weights exhibit exponential decay and tend to concentrate more on the year being estimated.

The results of these operations might be termed a seasonal kernel regression, with the shape of the kernel and smoothing parameter determined by  $\nu$  (or  $\phi$  and  $\nu$  if  $\phi$  is also allowed to vary). Use of the signal extraction formula automatically supplies correct kernel shapes at the ends of the series. It is up to the analyst to choose the number of trigonometric terms and the value of  $\nu$ . It is best to set some reasonable values for  $\phi$  and  $\nu$  (e.g., 0.5 and 15) and adjust the number of frequencies first, as likelihood ratio tests for adding new frequencies based on the increased degrees of freedom used are fairly straightforward and the results are fairly sensitive to the number of frequencies used. It is reasonable to start with 6 frequencies (12 terms) and add new frequencies 6 at a time rather than one at a time.

**Table 28.1: Year Weights for Two Choices of  $\phi$  and  $\nu$**

a. $\phi = 0.5, \nu = 10$									
Year									
1	<b>0.384</b>	0.271	0.169	0.096	0.049	0.023	0.008	0.002	-0.002
2	0.271	<b>0.264</b>	0.197	0.127	0.074	0.039	0.019	0.007	0.002
3	0.169	0.197	<b>0.218</b>	0.170	0.114	0.068	0.037	0.019	0.008
4	0.096	0.127	0.170	<b>0.203</b>	0.163	0.111	0.068	0.039	0.023
5	0.049	0.074	0.114	0.163	<b>0.200</b>	0.163	0.111	0.074	0.049
Phase Shift	1.254	0.706	0.358	0.146	0.000	-0.146	-0.358	-0.706	-1.254
b. $\phi = 0.5, \nu = 24$									
Year									
1	<b>0.285</b>	0.226	0.165	0.116	0.078	0.052	0.035	0.024	0.018
2	0.226	<b>0.215</b>	0.174	0.130	0.092	0.064	0.044	0.031	0.024
3	0.165	0.174	<b>0.177</b>	0.148	0.113	0.083	0.060	0.044	0.035
4	0.116	0.130	0.148	<b>0.160</b>	0.138	0.109	0.083	0.064	0.052
5	0.078	0.092	0.113	0.138	<b>0.155</b>	0.138	0.113	0.092	0.078
Phase Shift	2.001	1.319	0.788	0.366	0.000	-0.366	-0.788	-1.319	-2.001



**Table 28.2:** Year Weights for X-11 Seasonal Filters

a. $3 \times 5$									
Year									
1	<b>0.283</b>	0.283	0.283	0.150	0.000	0.000	0.000	0.000	0.000
2	0.250	<b>0.250</b>	0.250	0.183	0.067	0.000	0.000	0.000	0.000
3	0.150	0.217	<b>0.217</b>	0.217	0.133	0.067	0.000	0.000	0.000
4	0.067	0.133	0.200	<b>0.200</b>	0.200	0.133	0.067	0.000	0.000
5	0.000	0.067	0.133	0.200	<b>0.200</b>	0.200	0.133	0.067	0.000
b. $3 \times 9$									
Year									
1	<b>0.246</b>	0.221	0.197	0.173	0.112	0.051	0.000	0.000	0.000
2	0.208	<b>0.192</b>	0.176	0.160	0.144	0.092	0.028	0.000	0.000
3	0.173	0.163	<b>0.154</b>	0.143	0.133	0.123	0.079	0.032	0.000
4	0.141	0.137	0.132	<b>0.128</b>	0.123	0.117	0.113	0.075	0.034
5	0.111	0.111	0.111	0.111	<b>0.111</b>	0.111	0.111	0.111	0.111

The problem of choosing  $\nu$  remains. In kernel or smoothing spline regressions, the smoothing parameter is optimized by minimizing some sort of penalized residual sum-of-squares, or a cross-validation technique. As suggested in Härdle et al. (1988), convergence is slow and the surface rather flat. Given that there is no simple objective criterion for a seasonally adjusted series, it may be helpful to create seasonal factors for a set of  $\nu$  values. One can compute the smoothness of the resulting seasonally adjusted series using concurrent and projected factors and the size of revisions when new data are added. This is similar to the approach in Grillenzoni, C. (1994) of minimizing one-step-ahead projection errors.

In a standard regression, the number of degrees of freedom used in the regression is given by

$$tr(X(X'X)^{-1}X').$$

In the case of weighted regression this generalizes to  $tr(X(X'\Omega X)^{-1}X'\Omega)$ , where  $\Omega$  is a diagonal matrix containing the weights. For our application we used the trace of the matrix in (28.5), which equals the trace of the annual weight matrix partially illustrated for particular choices of  $\phi$  and  $\nu$  in Table 28.2. This is in agreement with calculations described in Zhang (2003).

The program begins by running an unweighted or global regression on the specified model. The observed series is adjusted for all variables in the model other than the trigonometric variables, so that what remains is, at least in principle, weekly seasonality plus noise. Then, the weighted regressions described above are carried out to obtain estimates of trigonometric coefficients for successive years, one year at a time. As mentioned, holiday coefficients are estimated with a fixed-coefficient regression. Estimating an "average" holiday effect across calendar variations helps stabilize holiday adjustment. Also, some holiday events do not occur every year, so a long span provides more occurrences for estimation. Section 28.5 has some examples. The trigonometric and built-in holiday sections of the design matrix are automatically extended two years beyond the input data span to compute projected seasonal factors. These projected factors are needed when seasonal adjustment is not being done contemporaneously.

Last, the weighted regressions are carried out. With this procedure, all but the seasonal variables are treated as deterministic, with all years contributing equally to estimation of their effects. Typically, for holidays or other special variables, it is helpful to base estimation on as much data as possible. The program also removes the holiday effects from the final seasonally adjusted series.

## 28.3 Holidays and Outliers

The program contains calendar features to assist in handling holidays and outliers. A flexible treatment of U.S. holidays is built into the program, and holidays of other countries or other adjustments can be entered as user-supplied variables. A list of the precoded holidays is in Table 28.3. The application in Section 28.5 will illustrate these features. It is important

to estimate holiday effects, because many change weeks of the year from one year to the next. The program asks for a holiday code (which of the precoded holidays is at issue or 0 for a user-supplied variable), and for the parameters of a daily weight pattern around the holiday. The first parameter is the number of weights to be used (often 1), then the index in the pattern of the day of the holiday, and finally the weights. If the effect of the holiday is only to cancel activity on that day, a single weight is fine. The value of the weight is not important, as its effect is scaled by the regression coefficient estimated. A weight value of 1.0 or 0.1 usually gives a regression coefficient with a good printable size. In the case of Christmas (or Easter) sales in previous days might be boosted. If so, one could specify several positive weights with Christmas falling on the last one or on a final zero weight. The program determines in which week of the year and where in the week the holiday falls, and assigns the weights to the correct week or weeks. While statistical tests could be tried comparing various weight patterns, the patterns are often dictated by external sources of information.

In addition to the holidays which are coded into the program, the user may read in a column of the design matrix as a user-specified holiday effect. This would be a column of data with non-zero entries in rows corresponding to dates when the effect was present. This column should include dates two years beyond the latest date being used in estimation to allow for projected seasonal factors.

**Table 28.3: Holiday Codes**

Code	Holiday	Code	Holiday
0	User-supplied effect	6	Christmas
1	Easter	7	July 4 <sup>th</sup>
2	Labor Day	8	President's Day
3	New Year's Day	9	Thanksgiving
4	Memorial Day	10	M.L. King Day
5	April 15th Tax Day	11	Veterans Day
		12	Columbus Day

The program also includes a way to compensate for additive outliers and level shifts; it does not do automatic identification. Identification of outliers may come from known events or analysis of graphs. The program takes as input parameters the number of outliers to be treated and a corresponding sequence of dates on which they occur in terms of year and week of the year. The week of the year can be easily identified from the data listings generated by the program (see Section 28.6 for more details).

## 28.4 Diagnostics

As described in Section 28.2, the first step in the method is to fit a global regression model. A standard ANOVA table exhibits the overall contribution from seasonal, holiday, outlier, and linear trend variables. T-statistics and p-values are available for individual trig coefficients, holidays, additive outliers, intercept and slope variables, and, when included, user-supplied variables, while In addition,  $R^2$  and Ljung-Box statistics help assess overall suitability of the model. The diagnostics allow the user to assess the overall model and each individual variable in the model.

The SAS interface part of the MoveReg program produces 10 graphs by default and outputs them to a pdf. Some of the graphs are displayed for the Unemployment Insurance application in Section 28.5. Table 28.4 has an annotated list of the graphs in the order that they are produced.

**Table 28.4: MoveReg Graphs**

Plot	Description
Time series plot	Shows the unadjusted and seasonally adjusted series for the full time span. Additive outliers are marked by a circle.
Time series plot	Same as the first, but only for the last 5 years.
Time series plot	Same as the first, but only for the last 2 years.
Plot of initial and revised seasonally adjusted series, along with the unadjusted series, for the last year	The initial or preliminary adjusted values use trig coefficients based on data through the next-to-last year (so that values for the last year come from projected factors); the revised values use trig coefficients from data through the last year.
Average seasonal factors by week; plot of seasonal factors	The plot of averages highlights the overall seasonal pattern across the year. The time series plot (not shown) can be helpful in assessing whether strength of seasonality is increasing or not.
Projected seasonal factors	Time series plot of projected seasonal factors (typically 2 years), with 3 previous years of factors.
Seasonal sub-plots by week	Assesses movement of factors for individual weeks. The straight lines are the weekly means for the factors.
Trig coefficient sub-plots by frequency	Indicates which frequency terms are significant and how much volatility they exhibit. The straight lines are the means by frequency.
Autoregressive spectrum	The SAS interface utilizes X13ARIMA-SEATS to calculate the spectrum for the differenced seasonally adjusted and unadjusted data. The horizontal lines are the medians.

## 28.5 Example: Initial Claims, U.S. Unemployment Insurance Program

Among important U.S. economic time series is Initial Claims from the Unemployment Insurance program. Individuals apply for unemployment benefits at government offices across the country or through the internet. Claims data from these local offices are forwarded first to the state office, and then to the Department of Labor (DOL), Washington, D.C. Every Thursday ([http://www.doleta.gov/ETA\\_News\\_Releases/](http://www.doleta.gov/ETA_News_Releases/)), Initial Claims (IC) are reported for the previous week ending on Saturday and Continuing Claims for the week before that. With less than a week's time lag, the Initial Claims series is clearly one of the timeliest indicators of the state of the economy. Press releases and many media accounts feature weekly figures and 4-week moving averages, both seasonally adjusted. During the economic recession beginning in 2007 and continuing through the recovery period, these numbers have been scrutinized closely.

Figure 28.1 shows the observed and seasonally adjusted series from January 2010 to January 2015, plus any additive outliers. Seasonality and holiday effects are the main features that are apparent. In previous years, large recession effects are also present. Figure 28.2 graphs average weekly seasonal factors across the span January 1988 to January 2015. An extreme peak occurs at the beginning of each year, while seasonality is low during the second quarter. In the third quarter, a short peak occurs near the beginning, followed by a trough around the end. Figure 28.2 also contains the time series of the seasonal component. Evident in both the unadjusted and adjusted series, volatility is due to differences in both reporting practices and economic behavior across the individual states. Naturally enough, the number of claims tends to increase in difficult economic times, and fall during upturns. Adopted in 2002 at BLS, the locally-weighted regression approach accommodating moving seasonality has significantly improved the seasonal adjustment, in particular in terms of (1) smoothness of the seasonally adjusted series and (2) less evidence of residual seasonality, cf. Cleveland and Scott (2007). Figure 28.3 has sub-plots of the seasonal factors by week, showing how the factors move across years. Recall that even with deterministic trig coefficients these plots would show variability, due to changing positions of the weeks. Estimating a stochastic seasonal component continues to be pertinent, since the recent recession and changes in economic activity appear to be affecting seasonality.

As described in Section 28.2, after differencing, the data are modeled as a seasonal time series, along with calendar effects and outliers. The model can produce a seasonal factor for each day of the year. For week  $i$ , we use the factor corresponding to  $d(i)$ , where  $d(i)$  is the day of the year on which the Saturday of week  $i$  falls. For example, in 2011, weeks 1, 2, and 6 have their Saturdays on January 1, January 8, and February 5, so the seasonal factors for those weeks are computed for days 1, 8, and 36. We first address selection of the seasonal variables. A conservative approach is to add trig terms in sets of 6 frequencies, beginning with the fundamental frequency  $\lambda_i = 1/365$ . A more selective approach would be more parsimonious, but with over a thousand observations, parsimony is not a serious issue. Figure 28.4 graphs spectra for differenced unadjusted and seasonally adjusted series. The gray lines denote the seasonal frequencies for weekly data, computed as  $7k/365$ ,  $k = 1, \dots, 26$ . The spectrum for the unadjusted series contains peaks at or near several of the seasonal frequencies, even some corresponding to periods nearly as short as "every two weeks". The adjustment yielding this spectrum is based on setting  $nfs=60$ , that is, including the first 30 seasonal frequencies. There is little or no evidence of residual seasonality in the seasonally adjusted spectrum.

Figure 28.5 shows the coefficients for each sine and cosine variable. Most of the visually significant values come from the first 12 frequencies, but others from each set of six seem different from 0. For almost all of the 30 frequencies, at least one of the trig terms has a significant t-statistic. These statistics come from the global regression model (1) with uniform weights.

Table 28.5 provides further confirmation for inclusion of all these seasonal variables. It contains an example of a set of residual F-tests from adding successive sets of 6 frequencies to the regression model. The change in residual sum of squares is attributable to the 12 added variables and compared to the mean residual sum of squares for the expanded model. We see that each F-test supports the inclusion of additional variables.

Figure 28.1: Initial Claims

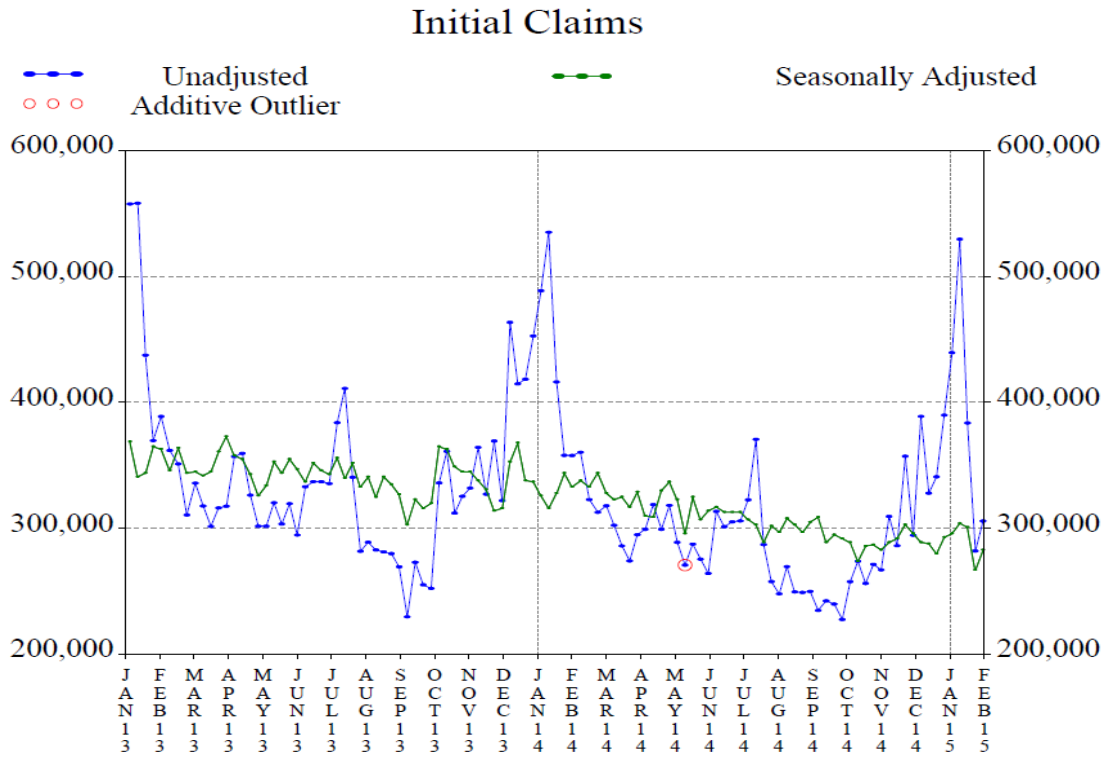


Figure 28.2: Average Seasonal Factors by Week of Year

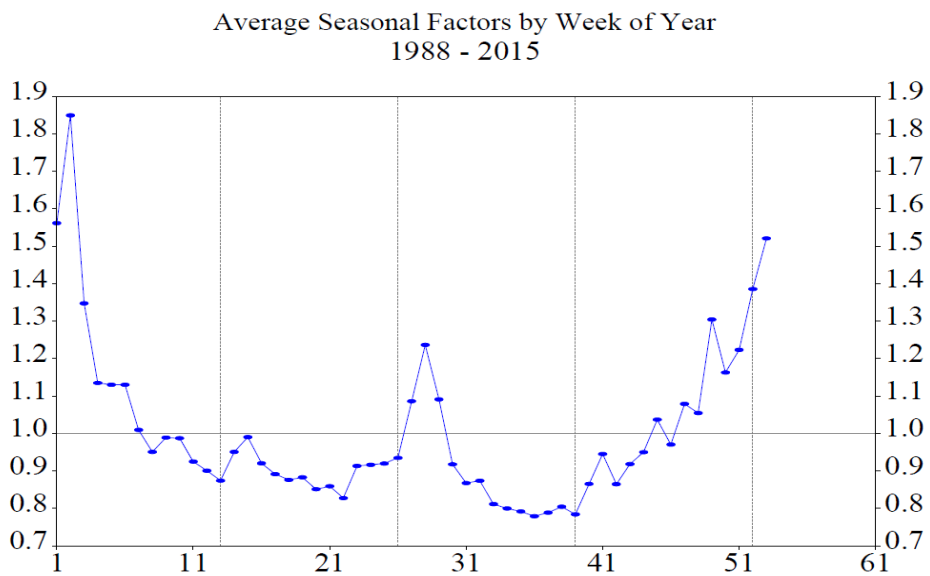


Figure 28.3: Seasonal Factor Sub-Plots

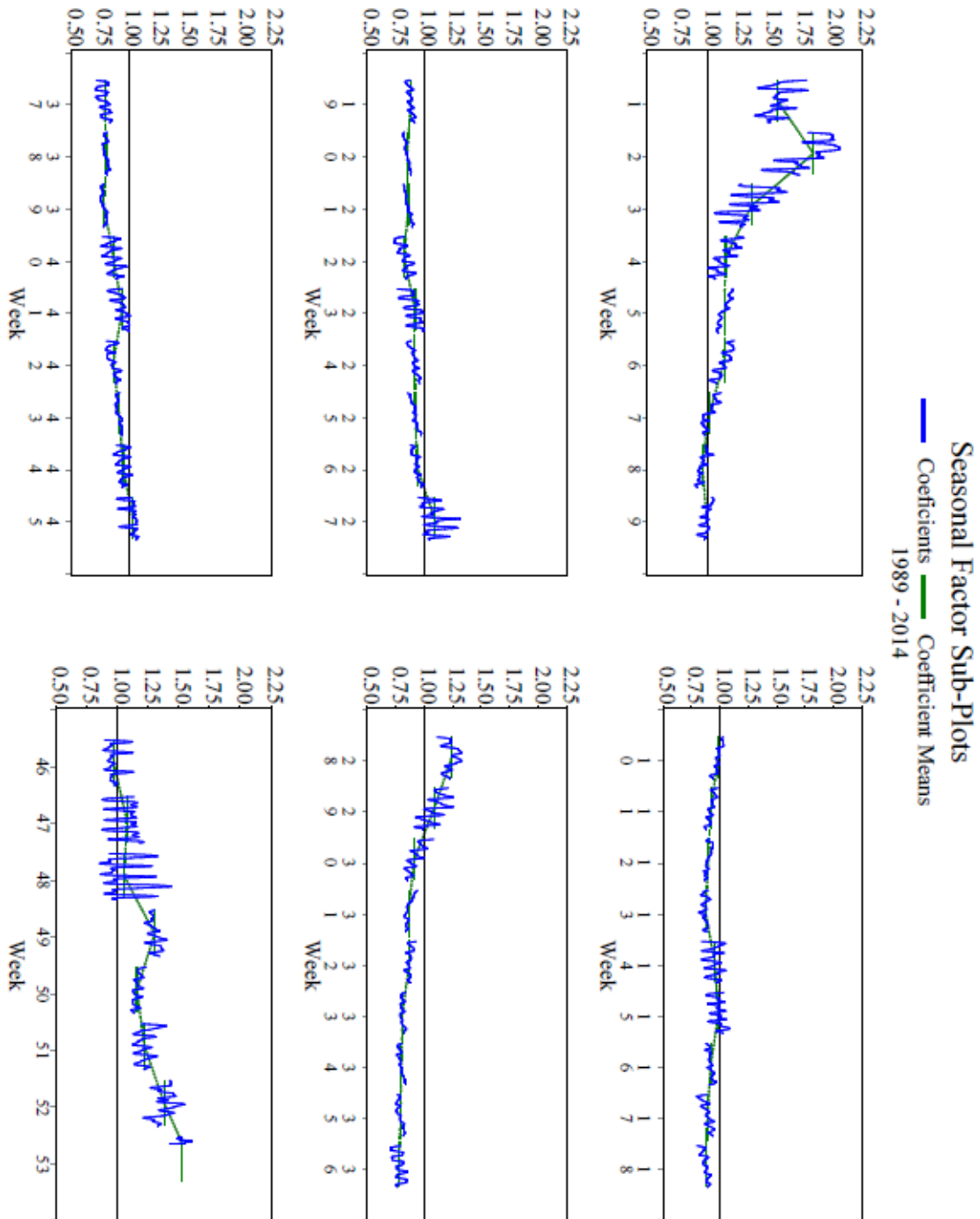


Figure 28.4: Autoregressive spectrum

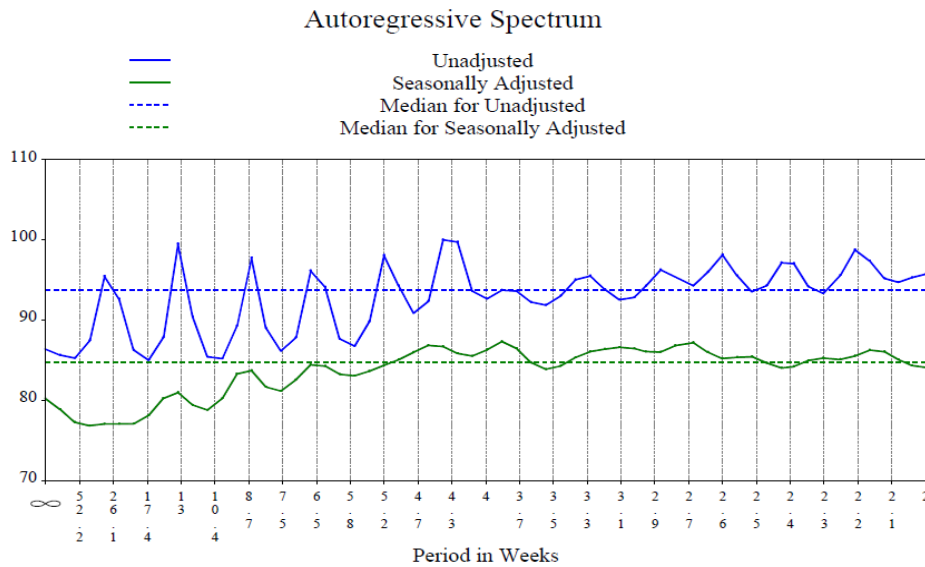


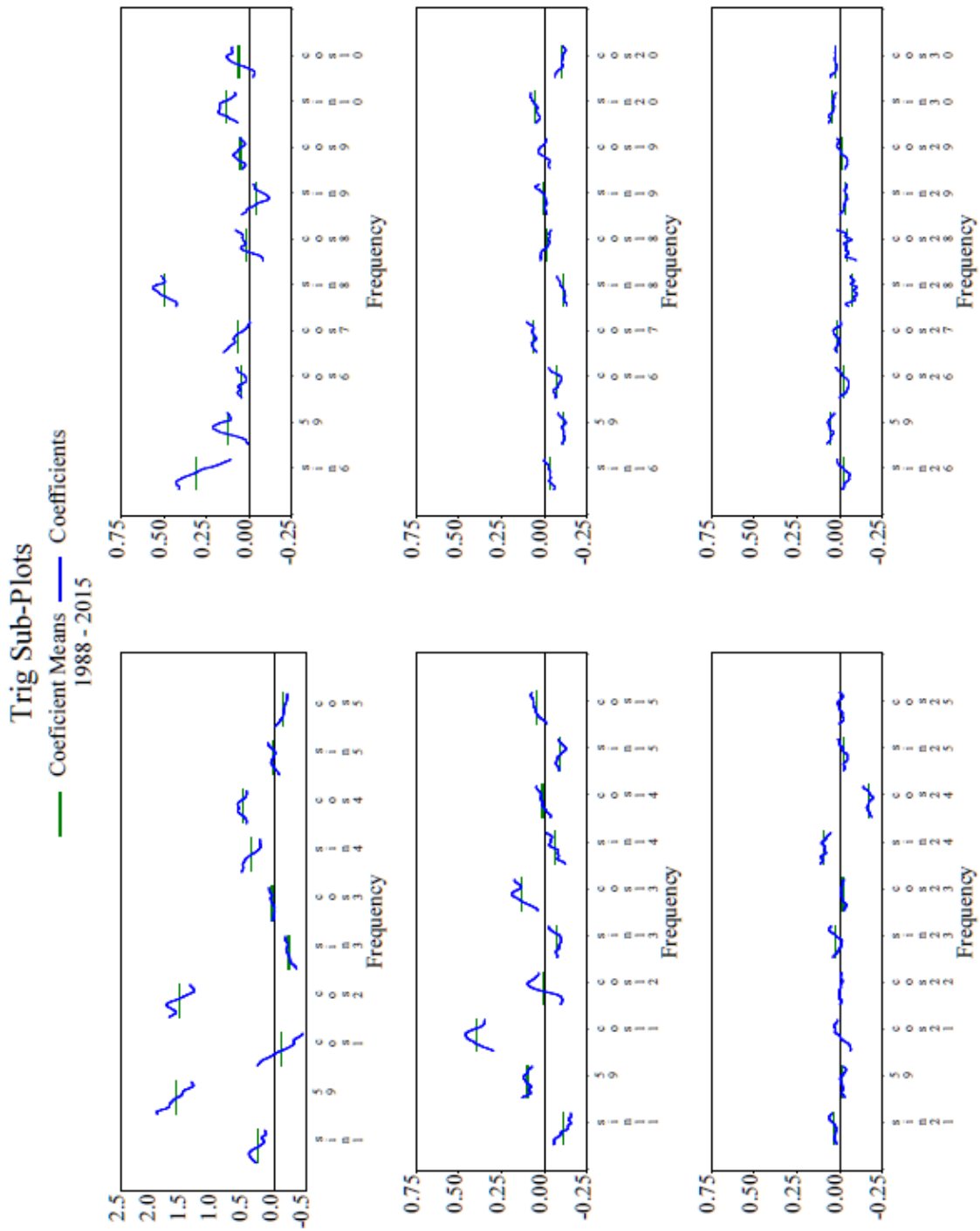
Table 28.5: Sequential F-tests for Inclusion of Seasonal Frequencies  $\lambda$

# $\lambda$ 's	Model(df)	Sum of Squares	Error(df)	Added $\lambda$ 's	Mean Squares	Error	Seq'l F	Added $\lambda$ 's
6	10.73 (48)	7.02 (996)	-	-	.0071	-	-	-
12	13.78 (60)	3.98 (984)	3.05	3.05	.0040	.2540	63.5	
18	14.73 (72)	3.03 (972)	0.95	0.95	.0031	.0789	25.4	
24	15.40 (84)	2.36 (960)	0.67	0.67	.0025	.0561	22.4	
30	15.70 (96)	2.06 (948)	0.29	0.29	.0022	.0248	11.3	

Every government holiday is potentially significant for this series. In the past, holidays generally represented a loss of a day for filing claims. Although all claims are now filed electronically, holidays continue to have a similar (if not stronger) effect on the initial claims series. Table 28.6 shows that holiday variables have t-statistics ranging from 2.1 to 16.9 in absolute magnitude. All require special treatment, since they affect different weeks in different years. July 4<sup>th</sup> can occur in either week 27 or 28. While not a Federal holiday, Easter affects claims activity and moves between weeks 12 and 18. The strongest effect comes from Thanksgiving, since in addition to the Thursday holiday, many state offices are closed on Friday. Even if an office is open, applicants may be less likely to appear that Friday. Special user variables are introduced for occurrence of July 4<sup>th</sup> on Wednesday and for particular patterns related to Christmas.

A surge in claims activity occurs when the Christmas season comes to a close. This is modeled as a New Year's Day effect. From our definition of weeks, New Year's Day is always in week 1. The New Year's effect is modeled as an increase generally assigned to week 2. When January 1<sup>st</sup> falls on Saturday, the rest of week 1 is in December, so clearly the surge comes in week 2. The surge seems to fall in week 2 for other positions of January 1<sup>st</sup> as well, except when it is on Sunday. In this case, the effect is split between weeks 1 and 2. The holiday specification which gives this result is 8 1 0 0 0 0 0 1 1, where 8 is the number of weights, 1 is the position of January 1<sup>st</sup> in the weight sequence, and the following 0's and 1's are the weights. When January 1<sup>st</sup> is Sunday, one nonzero weight falls in week 1 and the other in week 2. Otherwise, both nonzero weights correspond to days in week 2. From this example, we see that the program can handle a variety of holiday effects. A limitation is that holiday effects are treated as time-invariant.

Figure 28.5: Trig-Sub Plots





**Table 28.6:** Holiday Effects and Associated t-statistics

Holiday	Factor	t-statistic	p-value
New Year's Day	1.09	9.5	0.0000
Martin Luther King Day	0.84	-12.1	0.0000
President's Day	0.95	-3.9	0.0000
Easter	0.96	-6.7	0.0000
Memorial Day	0.89	-8.2	0.0000
July 4 <sup>th</sup>	0.96	-3.1	0.0010
July 4 <sup>th</sup> on Wednesday (user)	1.10	4.9	0.0000
Labor Day	0.90	-7.8	0.0000
Columbus Day	0.97	-2.1	0.0163
Veterans Day	0.88	-9.5	0.0000
Thanksgiving	0.79	-16.9	0.0000
Christmas in Week 53 (user)	1.04	2.1	0.0175
Christmas on Friday (user)	0.92	-3.9	0.0000

**Table 28.7:** Outlier Effects and Associated t-statistics

Year	Week	Factor	t-statistic	p-value
1989	40	1.19	5.1	0.0000
1992	30	1.41	10.0	0.0000
1993	30	1.36	8.8	0.0000
1993	52	0.89	-3.3	0.0001
1994	5	1.11	3.2	0.0002
1996	3	1.16	4.1	0.0000
2001	38	1.13	2.8	0.0030
2001	39	1.28	4.3	0.0000
2001	40	1.16	2.5	0.0063
2001	41	1.14	2.4	0.0093
2001	42	1.12	2.1	0.0188
2001	43	1.10	2.3	0.0108
2001	47	1.17	3.8	0.0001
2001	48	1.15	3.6	0.0002
2005	37	1.29	5.8	0.0000
2005	38	1.25	4.0	0.0000
2005	39	1.08	1.2	0.1117
2005	40	1.14	2.2	0.0141
2005	41	1.12	2.4	0.0090
2006	1	1.09	2.8	0.0022
2007	2	0.88	-4.1	0.0000
2012	45	1.17	4.4	0.0000
2014	19	0.92	-2.5	0.0002

As usual in seasonal adjustment, outlier and intervention specification is an iterative process. We can take advantage of the automatic outlier detection in a modified version of X-13ARIMA-SEATS (<http://www.census.gov/srd/www/x13as/>) by submitting residuals from the overall regression model. Candidate outliers are then tested within an expanded overall regression model. The process is repeated until a final selection is made. Dates and t-statistics for the 21 selected outliers are given in Table 28.7. All are simple additive outliers.

Hurricane Katrina struck the U.S. Gulf Coast on August 29, 2005 with devastating effects in Louisiana, Mississippi, and Alabama. This resulted in a spike in Initial Claims, modeled as five outliers. Two runs have been carried out with data

through 2006, week 4, one with and one without outlier treatment. We do not want the hurricane, an extraordinary event, to influence estimation of the seasonal component. Figure 28.6 compares seasonal factors from both runs for the last half of 2005. Differences from the run ending on August 27 appear minor, except for three weeks in September and two weeks in October, where clearly the “Treatment with 5 Outliers” factors agree better.

For the weights matrix, we follow the suggestion in Section 28.2 of setting the AR parameter  $\phi = .5$ ; Box, G.E.P., and Jenkins, G.M. (1970), sign convention. For  $\nu$ , a comparison of smoothness with  $\nu = 10, 16$  and  $24$  leads us to select  $\nu = 16$ . Based on the discussion in Section 28.2, this yields a weighting scheme intermediate between X-11's 3x5 and 3x9 seasonal filters.

To analyze stability, we compute revisions to seasonal adjustment from two successive runs. Figure 28.7 overlays observed and seasonally adjusted data from 2014, week 5 through 2015, week 5. The series in green is the initial or preliminary adjusted series, based on projected factors derived from data through 2014, week 4. The series in red comes from the adjustment we have been examining, with data through 2015, week 5. The largest revisions occur in February and April 2014 and January 2015. For all three months the revised series is smoother.

Table 28.8 shows revision statistics from two runs for 2007 and earlier years to demonstrate the effects of the weights. The median absolute revision is 4,118 for the year 2007, referring to the period 2007, week 5 to 2008, week 4. The maximum absolute revision is 21,873 (6.46%), occurring in 2007, week 6. We see the revisions decline going back in time as we approach use of the central filter for weighting. The drop in these statistics is especially strong moving back to 2004. For 2002, the median value is below 1000. These numbers are large, reflecting considerable volatility in the data. Still, the seasonally adjusted values themselves are quite large, often above 300,000.

**Table 28.8: Absolute Revisions by Year in Seasonally Adjusted Series, Based on 2007 Run**

Year	Median	Root Mean Square	Maximum
2002	654	1345	4304
2003	1007	1976	4391
2004	1279	2726	6797
2005	2080	3697	11085
2006	2931	4973	15790
2007	4118	6951	21873

Note. The statistics are based on "offset" years. For instance, year 2007 refers to the 52 week period of 2007, week 5 to 2008, week 4. Input series for official annual adjustments end with January weeks, since data tend to stabilize somewhat following the Christmas and New Year holiday periods.

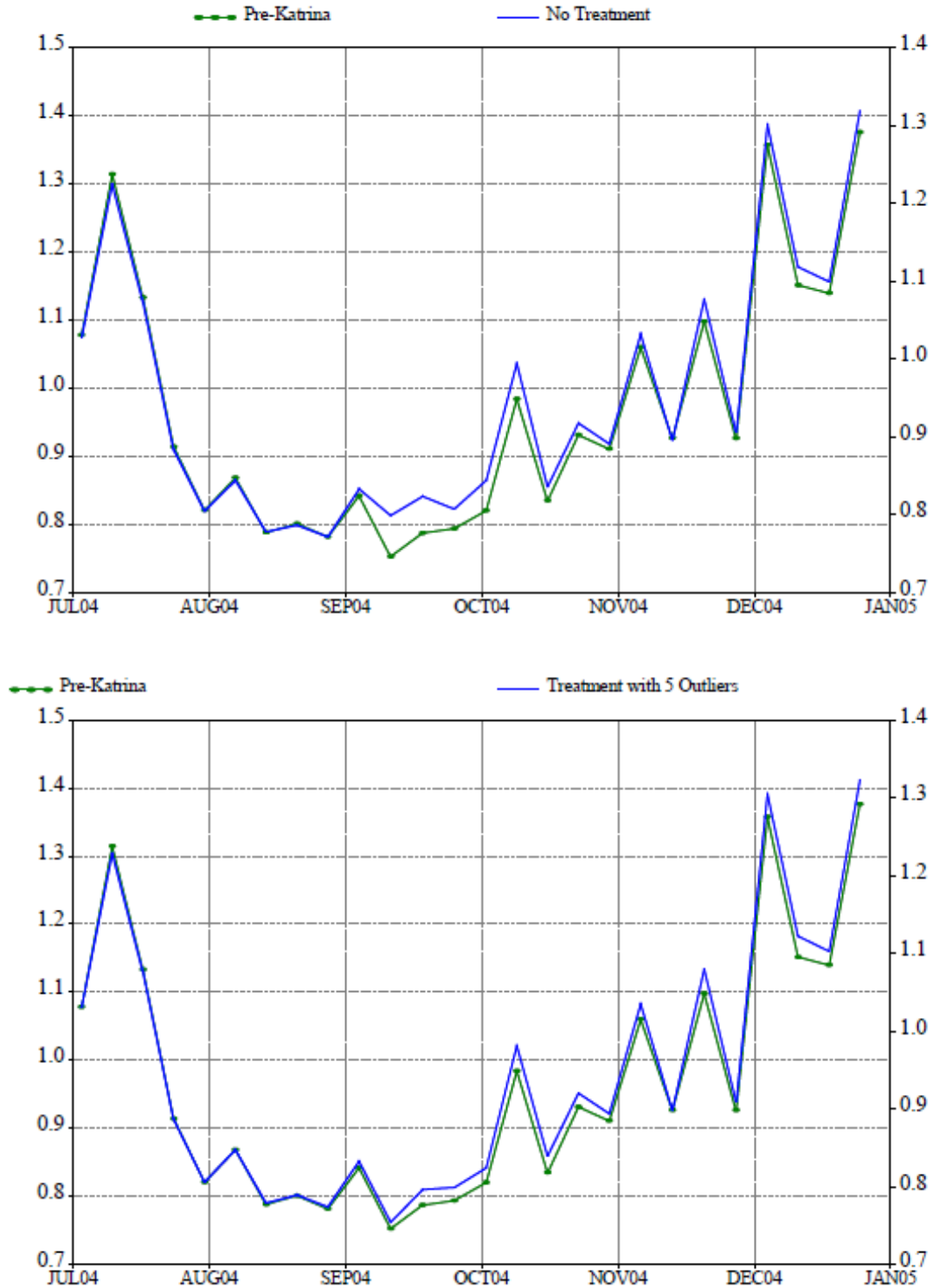
## 28.6 Software

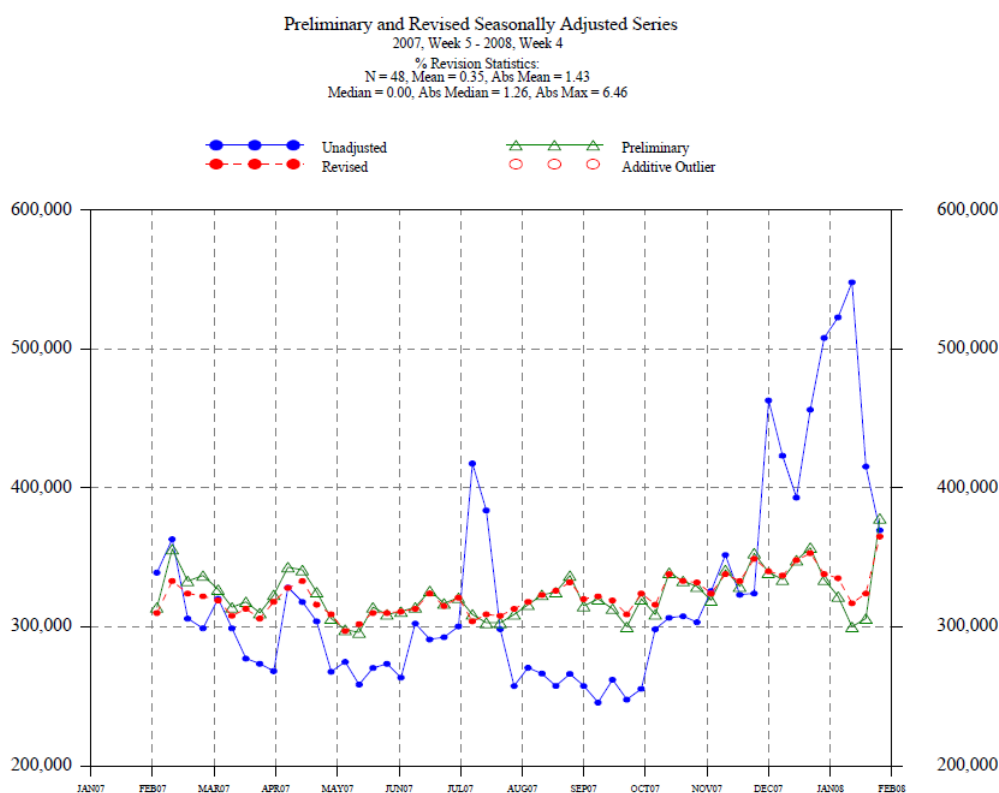
### 28.6.1 Description

The MoveReg software for carrying out weekly seasonal adjustment consists of a FORTRAN executable and a SAS interface. The original code for weekly seasonal adjustment with moving seasonality was written by William P. Cleveland in the FORTRAN programming language. This code is available for both Linux and Windows, but can be compiled for other platforms. Since BLS began using this program, modifications have been made to the FORTRAN code to add diagnostics and improve the machine readability of the output files. The FORTRAN program requires two input files and prints seven output files in text format. The SAS interface simplifies execution and aids in the preparation of the input files.

The rest of this section presents in some detail features and instructions for the software. More complete documentation is available from Evans. This package contains (1) a MoveReg software manual, (2) FORTRAN executables, (3) sample SAS programs, and (4) a full example with data, input and output files, and analysis of the results.

Figure 28.6: Effects on Seasonal Component from Outlier Treatment for Hurricane Katrina



**Figure 28.7: Initial Claims: Preliminary and Revised Seasonally Adjusted Series**

## 28.6.2 Input and output files

The two input files are called CONTROL.TXT and DATA.TXT. Eight output files report the results.

1. CONTROL.TXT specifies program options and parameters. A complete example for the Initial Claims series of Section 28.5 is shown in Example 1.
2. DATA.TXT contains the input data plus any user-defined events. A shortened version of this file is presented in Example 2.
3. User-defined variables (if any) follow the input data series. An example of the coding is given in Example 3. The variable value is zero except when Christmas falls in week 53. This series must be extended to cover the forecast period. Here, the length is 1149, 1045 for the input data, plus 104 for the forecast period.
4. ANOVA.OUT is a text file with the test statistics discussed in Section 28.4. There are seven other output files, some in table form and some more amenable to further calculations.

**Example 1: Sample CONTROL.TXT file.**

```

52 0
21 0 13 2 2 nout hol nfilt mxtype
0.4 16 phi sigratio
60
1988 01 16 2008 01 19 7
40 1989 30 1992 30 1993 52 1993 5 1994 3 1996 38 2001 39 2001
40 2001 41 2001 42 2001 43 2001 47 2001 48 2001 37 2005 38 2005
39 2005 40 2005 41 2005 1 2006 2 2007
0 0 0 3 10 8 1 4 7 2 12 11 9
8 1 0 0 0 0 0 0 1. 1. New Year
1 1 1. M. L. King Day
1 1 1. President's Day
8 8 1. 0 0 0 0 0 0 0 0 Easter
1 1 1. Memorial Day
1 1 1. July 4
2 2 0. 1. Labor Day
1 1 1. Columbus Day
1 1 1. Veterans Day
1 1 1. Thanksgiving

```

Key:

Line		Description
1	NPER	52 for weekly, 12 for monthly
	LOC	1 for fame database <sup>1</sup> , 0 for ascii file (data.txt)
2	NOUTAO	no. of AO's
	NOUTLS	no. of LS's (100 total outliers maximum)
	HOL	no. of built-in holidays
	NFILT	width of detrending filter (2 for the factor 1-B)
	MXTYPE	1 for trapezoidal weighting (not documented), 2 for ARIMA weighting
3	PHI	AR parameter (only when MXTYPE=2)
	SIGR	variance ratio (only when MXTYPE=2)
4	NFS	no. of terms in trig seasonal = 2 x no. of frequencies
5	KBYR,KBMO,KBDAY	starting & ending dates for input data (4-digit year, month,
	LYR,LMO,LDAY,IWKD	day of month), days in week
6		dates of outliers (week & year for each)
7		holiday codes preceded by '0' for each user variable
8		information on built-in holidays, 1 line per holiday: length of holiday effect, position of holiday in weight pattern, weight pattern (the most common weight pattern is 1 1 1, which simply generates a 1 in the weeks where the holiday occurs and zeroes elsewhere).

**Example 2: Sample DATA.TXT file (extract).**

```

ic
      1149  52  1988  01  30  2010  01  30  7  0
(46x,f8.0)
30JAN1988      ic          1 30  1988  5      1  395000
06FEB1988      ic          2 6  1988  6      2  381000
13FEB1988      ic          2 13 1988  7      3  335000
20FEB1988      ic          2 20 1988  8      4  316000
27FEB1988      ic          2 27 1988  9      5  324000
05MAR1988      ic          3 5  1988  10     6  312000
12MAR1988      ic          3 12 1988  11     7  294000
19MAR1988      ic          3 19 1988  12     8  276000
26MAR1988      ic          3 26 1988  13     9  269000

```

Key:

Line	Description
1	title, identifying info
2	NOBS, NPER, KKBYR, KKBMO, etc., IWKD, KBB number of observations, number of periods, start year, start month, start day, end year, end month, end day, number of days in a week, '0'
3	FORTRAN input format (change as needed); note that the data column is read following the format line
4	date, variable name, month, day, year, week of year, cumulative week, data value(46x, f8.0)

**Example 3: Sample specification of a user variable.**

```

ic USER FACTORS FOR CHRISTMAS IN WEEK 53
      1149  52  1988  01  16          2010          1          16 7 0
(46x,f6.0)
XMAS IN WEEK 53  January  16  1988  3      1      0
XMAS IN WEEK 53  January  23  1988  4      2      0
XMAS IN WEEK 53  January  30  1988  5      3      0
XMAS IN WEEK 53  February 6  1988  6      4      0
XMAS IN WEEK 53  February 13  1988  7      5      0
.....
XMAS IN WEEK 53  December 17  1988  51     49     0
XMAS IN WEEK 53  December 24  1988  52     50     0
XMAS IN WEEK 53  December 31  1988  53     51     1
XMAS IN WEEK 53  January   7  1989  1      52     0
XMAS IN WEEK 53  January  14  1989  2      53     0

```

Key:

Line	Description
1	title of user variable
2	number of observations, number of periods, start year, start month, start day, end year (include forecasts), end month, end day, '7', '0'
3	FORTRAN input format
4	variable title, month in letters, day, year, week in year, cumulative week, data value (only the last column is actually read)

### 28.6.3 Run instructions with the SAS interface program

This section describes how to carry out seasonal adjustment with the interface. The input control and data files are automatically created by the SAS code which can significantly reduce the possibility of errors. Note that the interface is available for both UNIX and Windows. If one chooses to execute the program from the command line without the interface, control and data files need to be created with a programming language like SAS or with a text editor. Requirements and examples for setting up these files are in the detailed MoveReg documentation.

Required input files are created with SAS, and the time series to be adjusted can be supplied as a SAS data set or a text file. Example 2 exhibits the format for the latter option. If "Num of Additive Outliers" is positive, these are to be specified in a file OUTLIERS.TXT. The format should be that of lines 6-8 in Example 1. If treatment for U.S. holidays is desired, a text file HOLIDAYS.TXT needs to be supplied. This file should have one line for each holiday, as in Example 1, with the same logic (but not necessarily the same weight structure). User-defined events can be added to the data input file by providing SAS code to create dummy variables; a supplied program USER\_EVENTS.SAS can be modified for this purpose.

To begin MoveReg, execute the supplied SAS program. Next, as shown, select "Start" from the Program menu. This will bring up a window to specify program options seen in Figure 28.8. This "Start" window allows the user to specify required program parameters. All available settings can be specified with this window except for holiday weights and user-defined events as explained above.

**Figure 28.8: MoveReg Interface Start Window**

Start

Enter the date for the beginning week to use for seasonal adjustment using the following numerical format: MM/DD/YYYY (2 digits each for month and day and 4 digits for the year)

Enter the date for the last week to use for seasonal adjustment using the following numerical format: MM/DD/YYYY

NOTE: When entering dates, use the date for Saturday.

Check box if output desired:  Graphs

Type in a title for graphs: \_\_\_\_\_

Enter the name of the input variable (max 8 chars): \_\_\_\_\_

If you want to change any of the parameter values, fill in the blanks below. If left blank, the default values shown will be used.

	Default	User-Specified
Num of Seasonal Periods (52 or 12):	52	<input type="text"/>
Num of Additive Outliers (70 max):	24	<input type="text"/>
Total Num of Holidays (including user events):	13	<input type="text"/>
Width of Detrending Filter (2 for (1-B)):	2	<input type="text"/>
AR Parameter:	0.4	<input type="text"/>
Variance Ratio:	16	<input type="text"/>
Num of Seasonal Frequencies:	60	<input type="text"/>
Num of Forecasts for Seasonal Factors:	104	<input type="text"/>

Is your input data file a SAS file or text?  SAS  Text

Enter input data file name: \_\_\_\_\_

Did you specify any user-defined effects?  Yes  No

Select factor for scaling in graphs:  1000  100  10  None

OK Cancel

The beginning and ending dates are entered at the top of the window. Next, one can indicate if graphs are desired, type in a title, and specify a name for the time series. The eight small boxes that follow are for settings in the control file. The "Width of Detrending Filter" is to specify differencing; the "AR parameter"  $\phi$  controls how fast the seasonal factors move; the "Variance Ratio" is a noise-to-signal ratio. It works in a similar way to the seasonal moving averages in X-11; the lower the ratio, the more concentrated the weights. The "Num of Seasonal Frequencies" indicates how many sine and cosine pairs are needed for the trig seasonal component. The input data can be in either a SAS database or a text file (check the appropriate radio box), and the name of the data file is entered in the next box. Then, if user-defined events are needed for the input data file, one selects the "Yes" radio button in the next line. Finally, scaling choices are provided through radio buttons in the last line for the graphs that are output to a pdf.

A SAS macro is provided to change the default options for the "Start" window. This macro also allows the user to select graphs for the pdf. The choices include time series plots for 1) the unadjusted series, the seasonally adjusted series and outliers; 2) revisions for the last year; 3) average seasonal factors by week; 4) seasonal factors over time; 5) projected seasonal factors; 6) seasonal sub-plots by week, 7) trig coefficient sub-plots; and 8) a spectral plot.

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# 29

## Seasonal Adjustment of Daily and Weekly Data



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## Table of Contents

29.1	Introduction	759
29.2	The Main Characteristics of High Frequency Data	760
29.2.1	Why to use High Frequency Data?	760
29.2.2	Characteristics of Daily and Weekly Data	760
29.2.3	Checking for the Various Periodicities in the Data	762
29.3	Pretreatment of Daily and Weekly Data	765
29.3.1	Decomposition Model	767
29.3.2	Calendar Effects	767
29.3.3	Dealing with outliers	769
29.4	Seasonal Adjustment Based on Non-parametric Methods	770
29.4.1	Adapting X11 to multiple and non integer periodicities	770
29.4.2	Improving STL for multiple periodicities	773
29.4.3	Pros and cons of the X11 and STL approaches	774
29.5	Seasonal adjustment based on parametric models	775
29.5.1	Seasonal adjustment based on ARIMA Models	775
29.5.2	Canonical decomposition	775
29.5.3	Estimation of the components	775
29.5.4	Pros and Cons of the current implementation	776
29.5.5	Seasonal adjustment based on unobserved components models	776
29.6	Application: Seasonal adjustment of the Electricity series	778
29.6.1	The seasonal patterns	778
29.6.2	Validation of the results	780
29.7	Conclusions	782

## 29.1 Introduction

High frequency data, i.e. data observed at infra-monthly intervals, have been used for decades by statisticians and econometricians in the financial and industrial worlds. Weekly data were already used in the 20's by official statisticians to assess the short-term evolution of the Economy. For example, Fisher (1923) proposed a weekly index number of wholesale prices that was the first general weekly index number to appear. The index was based on the price quotations of 200 commodities and published in the press each Monday, covering the week ending the previous Friday noon. Crum (1927) studied the series of weekly bank debits outside New York city from 1919 to 1926 and proposed a method to seasonally adjust these data based on the median-link-relative method developed by Persons (1919).

Nowadays, these data are ubiquitous and concern almost all sectors of the Economy. Numerous variables are collected weekly, daily or even hourly, that could bring valuable information to official statisticians in their evaluation of the state and short-term evolution of the Economy.

But these data also bring challenges with them: they are very volatiles and show more outliers and breaks; they present multiple and non integer periodicities and their correct modeling implies numerous regressors: calendar effects, outliers, harmonics.

The current statistician's traditional toolbox, methods and algorithms, has been developed mainly for monthly and quarterly series<sup>1</sup>; how should these tools be adapted to handle time series of thousands observations with specific characteristics and dynamics efficiently?

This chapter presents some ideas to adapt the main seasonal adjustment methods, namely "the X11 family" i.e. methods based on moving averages like X11, X11-ARIMA, X12-ARIMA and X-13ARIMA-SEATS<sup>2</sup> and methods based on Arima models like TRAMO-SEATS<sup>3</sup>. STL, a non-parametric method proposed by Cleveland et al. (1990) and very similar to the "X11 family" in philosophy, is also considered. Unobserved component models are used to complete the set of seasonal adjustment methods. The chapter also makes some recommendations about the most appropriate methods for pretreatment and filtering of daily and weekly data.

Section 29.2 explains why high frequency data are important for the short term evolution of the Economy and highlights the main characteristics of these data. Section 29.3 focuses on the usually necessary pretreatment of the data and more specifically on the detection and estimation of outliers and calendar effects. Sections 29.4 and 29.5 present adapted methods that can be used to seasonally adjust these data. These methods are illustrated and compared on an example in Section 29.6 and section 29.7 concludes.

Not all problems have been resolved at this stage and further research is needed, mainly on the "tuning" of the methods, to assure a good seasonal adjustment of daily and weekly data. For examples, an automatic determination of the length of the filters in non-parametric methods like STL or X11 has still to be found and the use of unobserved component models still requires a difficult selection of relevant harmonics or regressors to obtain a parsimonious model.

<sup>1</sup>With some notable exceptions like the method used by the Bureau of Labor Statistics to seasonally adjust weekly time series, see Cleveland et al. (2018); and of course unobserved component models that can handle any kind of periodicity by design, see Harvey (1989) and Harvey (2018).

<sup>2</sup>See Ladiray and Quenneville (2001), Ladiray (2018) and Findley et al. (1998).

<sup>3</sup>See Gómez and Maravall (1997) and Maravall (2018)

## 29.2 The Main Characteristics of High Frequency Data

### 29.2.1 Why to use High Frequency Data?

Even if daily data have been available for a long time, for examples in the financial and industrial domains, they have not been really incorporated in the indicators constructed to analyze the short-term evolution of the economy. Nowadays, the “Big Data” phenomena makes numerous high frequency data available at a small costs and official statisticians seriously consider them.

Some of these data might for example be used:

- To improve the timeliness of usual indicators: could we use some high frequency data to get an estimate of the quarterly GDP 15 days after the end of the quarter, or even before?
- To get higher frequency estimates of important variables: a weekly indicator of job vacancies, of consumer price index, a monthly GDP etc.
- To construct new indicators such as climate or sentiment indexes.

It has to be noted that if you want to use these data to build a rapid estimate, you have to remove the components which have also been removed in the target monthly and quarterly series, and only these components. In particular, if the objective is to get a rapid estimate of the seasonally adjusted quarterly GDP, the high frequency data used should be corrected from any calendar and periodic effects.

### 29.2.2 Characteristics of Daily and Weekly Data

Figure 29.1 presents the consumption of electricity in France recorded from January 1<sup>st</sup>, 1996 to April 30<sup>th</sup>, 2016: the top graph shows the daily figures and the bottom graph the monthly values. The series shows a clear periodic effect linked to the seasons: the electricity consumption is higher in Winter than in Summer. But there is also a “hidden” weekly effect due to the fact many factories and businesses are closed during the week-end which explains a lower consumption on Sundays and Saturdays.

#### 29.2.2.1 Multiple and Non-integer Periodicities

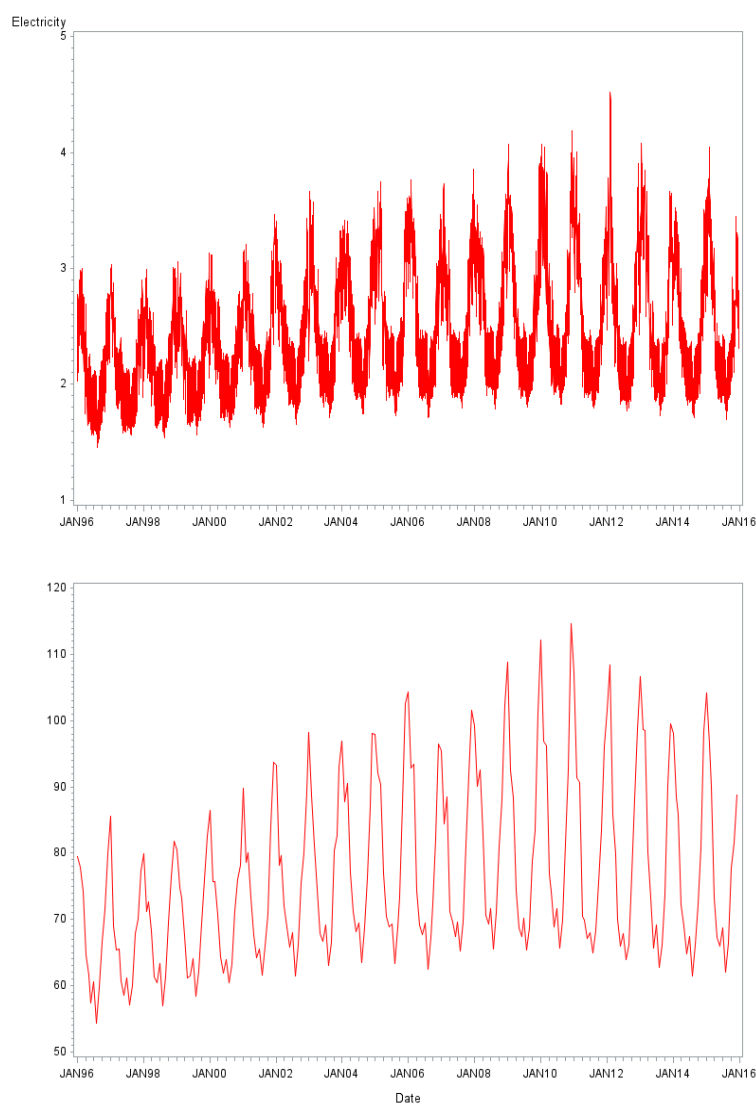
Daily data usually present several “regularities”. In many economic series, the 7 days of the week have different behaviors. In Christian countries, many businesses and retail shops are closed on Sunday which directly impacts the turnover of the concerned economic sectors. Apart this “week cycle” of period 7 days, daily series can show an intra-monthly effect of average period 30.436875 days: for example many wages and revenues are paid at the end of the month and this will have an effect on the sales, the money in circulation and other economic indicators. We also have the “solar cycle” mentioned before that is likely to be present through a “year cycle” of average period 365.2425 days.

Table 29.1 shows the potential periods that could be present in the data according to the data collection rhythm. For example, a hourly series might have up to 5 periodic components: a “Day cycle” (24 hours), a “Week cycle” (168 hours), a “Month cycle” (average length of 730.485 hours), a “Quarter cycle” (average length of 2191.455 hours), and a “Year cycle” (average length of 8765.82 hours).

#### 29.2.2.2 Important Remarks

1. Hopefully, for daily data at least, the various possible periodicities even if they can be non-integer, are co-primes and it should be possible to separate the various periodic components. The problem might be more complex for hourly and other higher frequency data.

**Figure 29.1:** Consumption of electricity in France since 1996: the daily values are in the upper panel, the monthly values in the lower panel.



**Table 29.1:** Periodicity of the series according to the data collection rhythm.

Data	Period (number of observations per cycle)						
	Minute	Hour	Day	Week	Month	Quarter	Year
Annual							1
Quarterly							4
Monthly						3	12
Weekly					4.348125	13.044375	52.1775
Daily				7	30.436875	91.310625	365.2425
Hourly			24	168	730.485	2191.455	8765.82
Half-hourly			48	336	1460.97	4382.91	17531.64
Minutes		60	1440	10080	43829.1	131487.3	525949.2
Seconds	60	3600	86400	604800	2629746	7889238	31556952



2. More generally, an important problem of the seasonal adjustment of high frequency data will be the proper identification of the various effects and the distinction between the component frequencies: between the various periodic components but also between the trend-cycle frequencies and the annual frequencies.
3. The high number of observations usually available with high frequency data is somehow misleading. For example, if you have 2 years of daily data, you have 730 observations but the estimation of the annual effect might be tricky even if this effect is evident as in the Electricity series. To illustrate the problem, remember that in non-parametric methods like STL or X11, a periodic effect is estimated by smoothing the values corresponding to a modality of the period. Thus, the weekly effect will be obtained smoothing 7 different series: one for each day of the week (Monday values, Tuesday values etc.). This should not be a problem in our 2-year series as we have about 100 observations for each day. But, when it comes to the annual effect, things are more complicated as you just have 2 observations for each day of the year.

It turns out that the span of the series, the number of annual cycles, is very important and in fact guides the modelling of the series and the quality of the results.

### 29.2.3 Checking for the Various Periodicities in the Data

The detection of the various periodicities must be done before any modelling of the time series. Among the statistical tools that can be used in this respect, the most efficient are certainly: the spectrum of the series, the Ljung-Box test and the Canova-Hansen test. Of course, these tools must be adapted to the characteristics of high frequency data.

#### 29.2.3.1 Spectral Analysis

Spectral analysis is commonly used to check for the presence of seasonality in monthly and quarterly series. But in this context, they do not seem to be very efficient as they might miss a periodicity and have a quite high “false alarm rate” as shown in Lytras et al. (2007).

But, in the context of high-frequency data, the large number of observations improves the quality of the spectrum estimate and might make this tool more efficient.

Figure 29.2 represents the Tukey spectrum of the Electricity series (upper panel); the periodic behaviour shown in Figure 29.1 is partly explained by a weekly periodic component: the peaks that can be observed are located at frequency  $2\pi/7$  and its harmonics.

The annual periodicity we are expecting in these data is in fact hidden by the 7-day periodic component. The 365-day frequency is associated with fundamental frequency  $2\pi/365$  ( $360/365 = 0.9863$  degrees) and its harmonics. If we focus on the low frequencies, as shown in the lower panel of Figure 29.2, we can see the expected but small peaks at the annual frequencies.

#### 29.2.3.2 Statistical Tests

##### Ljung-Box Seasonality Test

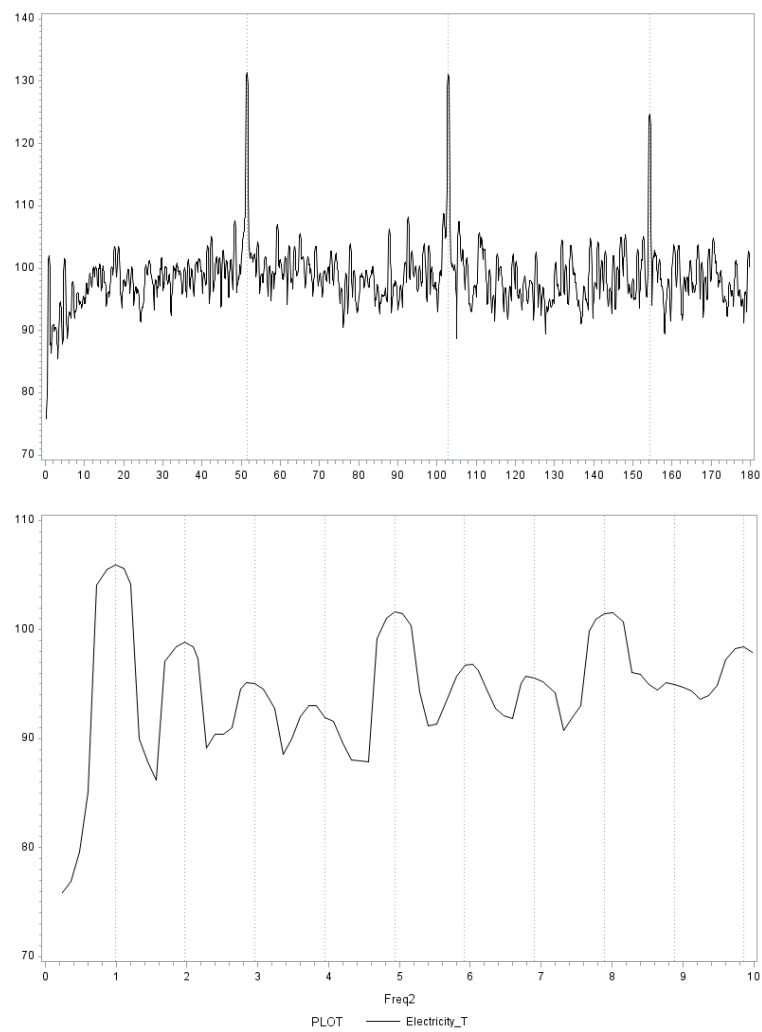
The Ljung-Box seasonality test checks for the presence or absence of auto-correlation at the seasonal lags. For a time series potentially presenting a seasonality of order  $k$ , the test statistic is:

$$LB = n(n+2) \sum_{j=1}^h \frac{\rho_{j*k}^2}{(n-j*k)}$$

where  $n$  is the number of observations,  $\rho_{j*k}$  is the autocorrelation at lag  $j*k$ , and  $h$  is the number of lags being tested. Under  $H_0$  the statistic  $LB$  follows a chi-squared distribution with  $h$  degrees of freedom. Usual implementations of the Ljung-Box seasonality test focus on the 2 first seasonal lags and  $QS$  follows a  $\chi^2(2)$ . Thus,  $QS > 5.99$  and  $QS > 9.71$  would suggest rejecting the null hypothesis at 95% and 99% significance levels, respectively.

Maravall (2012) suggests that a significant  $LB$  statistic can be used to conclude that a seasonality is present only when the sign of the autocorrelation coefficients is consistent with such a hypothesis. He introduces the  $QS$  statistics, derived from the Ljung-Bix statistics where, for monthly data, negative values of  $\rho_1$  and  $\rho_{24}$  are replaced by zero, and  $QS = 0$  for

**Figure 29.2:** Tukey's Spectrum of the Electricity series (upper panel) and focus on the low frequencies (lower panel).





**Table 29.2: Autocorrelations and Ljung-Box statistics**

Period P	Ljung-Box	LB Pvalue	QS stat	QS Pvalue	Autocorrelation at lag			
					P	2*P	3*P	4*P
7	7314.9749	0.000	7314.9749	0.000	0.63502	0.47239	0.43973	0.46221
365	1128.6499	0.000	1128.6499	0.000	0.23905	0.19045	0.15697	0.13547

$\rho_{12} \leq 0$ . This refinement implies that the presence of seasonality will be detected if there is a statistically significant positive autocorrelation at lag 12 or if there are a non negative sample autocorrelation at lag 12 and a statistically significant positive autocorrelation at lag 24.

Ljung-Box seasonality tests can be adapted to daily and weekly data, after proper differencing of the data:

- You can test all the possible relevant periods. For example, for weekly data, you compute the test using lags (52, 53, 104, 106, ...). For daily data, it would be on lags (7, 14, 21, ...) for the weekly periodicity and on lags (365, 366, 730, 732, ...). for the yearly periodicity.
- You can restrict the test to the closest integer. For weekly data, the average yearly periodicity is 52.18, so you use lags (52, 104, ...). For daily data, the weekly periodicity is 7 so you still use lags (7, 14, 21, ...) and as the average length of the year is 365.2475 days, you use for the yearly periodicity lags (365, 730, ...).

Table 29.2 presents the autocorrelations and associated Ljung-Box statistics for the Electricity series. The results confirm the presence of a significant effect at the weekly and annual periodicities.

### Canova-Hansen Test

The stochastic nature as well as the presence of seasonality can be subjected to formal statistical tests. Canova and Hansen (2003) and Buseti and Harvey (2003) derive the locally best invariant test of the null that there is no seasonality against a permanent seasonal component, that can be either deterministic or stochastic, or both.

Consider a seasonal cycle with period  $p$  and angular frequency  $\lambda = 2\pi/p$ . The seasonal component is decomposed into a deterministic term, arising as a linear combination with fixed coefficients of sines and cosines defined at the frequency  $\lambda$ , plus a nonstationary stochastic term, which is a linear combination of the same elements with random coefficients:

$$\gamma_t = \gamma_t^D + \gamma_t^S.$$

Defining  $z_t = [\cos \lambda t, \sin \lambda t]'$ ,  $\gamma_t^D = z_t' \gamma_0 + \alpha_0$ , where  $\gamma_0$  and  $\alpha_0$  are fixed coefficients. The stochastic component is  $\gamma_t^S = z_t' \sum_{i=1}^t k_i$  where  $k_t$  is a bivariate vector of serially independent disturbances with zero mean and covariance matrix  $\sigma^2 W$ .

The null hypothesis is then formulated as  $H_0 : \gamma_0 = 0, \sigma^2 = 0$ ; a permanent seasonal component is present under the two alternatives:  $H_a : \gamma_0 \neq 0, \sigma^2 = 0$  (deterministic seasonality),  $H_b : \gamma_0 = 0, \sigma^2 > 0$  (stochastic seasonality).

The test statistic proposed by Buseti and Harvey (2003) is consistent against both alternative hypotheses, and it is computed as follows:

$$\varpi = \frac{1}{n^2 \sigma^2} \sum_{t=1}^T \left[ \sum_{i=1}^t (e_i \cos \lambda i)^2 + \sum_{i=1}^t (e_i \sin \lambda i)^2 \right], \quad (29.1)$$

where  $e_i$  are the OLS residuals obtained from the regression of  $y_t$  on a set of explanatory variables accounting for a constant or a linear deterministic trend. Under the null  $\varpi$  is asymptotically distributed according to a Cramer von Mises (CvM) distribution with 2 degrees of freedom.

For the frequencies  $\lambda = 0$  and  $\lambda = \pi$  the test takes the form

$$\varpi = \frac{1}{n^2 \sigma^2} \sum_{t=1}^T \sum_{i=1}^t (e_i \cos \lambda i)^2, \quad (29.2)$$

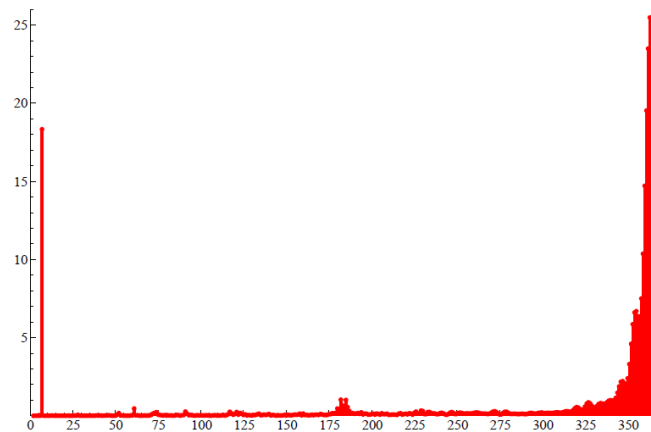
and the null distribution is Cramer von Mises (CvM) with 1 degree of freedom. When  $\lambda = 0$ , the test statistic is the usual KPSS test of stationarity at the long-run frequency.

The test of the null that the seasonal component is deterministic ( $H_a$ ) against the alternative that it evolves according to a nonstationary seasonal process ( $H_b$ ), i.e. characterised by the presence of unit roots at the seasonal frequencies  $\omega_j$ , is based on the Canova-Hansen (CH) test statistic which is (29.1), with  $e_i$  replaced by the OLS residuals that are obtained by including the trigonometric functions in  $z_t$  as additional explanatory variables along with  $x_t$ . Under the null of deterministic seasonality, the test has again a Cramér von Mises distribution.

The test statistic in (29.1) requires an estimate of  $\sigma^2$ . A nonparametric estimate is obtained by rescaling by  $2\pi$  the estimate of the spectrum of the sequence  $e_t$  at the frequency  $\lambda$ , using a Bartlett window.

Figure 29.3 displays the values of the CH test for the daily Electricity series, versus the period  $p$  of the periodic component. Significant values are detected at the weekly and annual frequencies only, for which the null is rejected.

**Figure 29.3:** Consumption of electricity in France. Canova-Hansen tests for components with period  $p = 2, 3, \dots, 366$ .



## 29.3 Pretreatment of Daily and Weekly Data

The seasonal adjustment of high frequency time series poses several challenges. One is the detection of outliers that, if they are not exhibited and imputed, could hamper the proper estimation of the seasonal component. The robustness can be enforced either by an outlier detection procedure or by robust filtering methods. Another is the detection and correction of trading-day effects and moving-holiday effects that are directly linked to the calendar and make periods (quarter, month, week, day etc.) not directly comparable.

Usual seasonal adjustment packages propose an automatic detection facility to detect and correct for outliers and calendar effects. These statistical algorithms have usually been designed specifically for monthly and quarterly time series and are applied before the decomposition of the time series in trend-cycle, seasonal component and irregular. The basic process adopted by both TRAMO-SEATS and X-13ARIMA-SEATS is summarized in Figure 29.4.

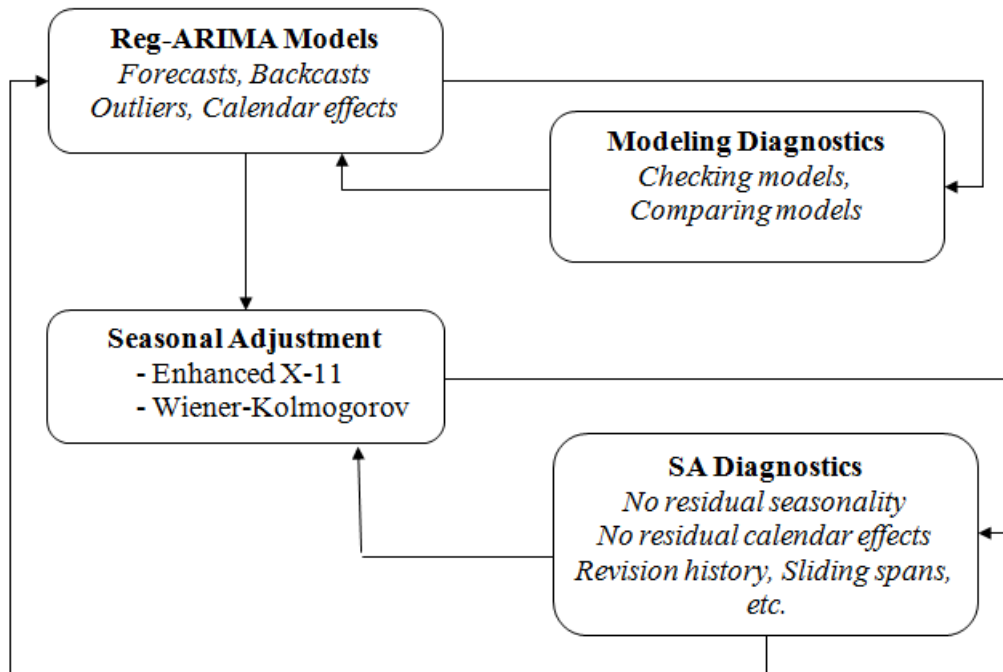
A common approach is based on Reg-Arima models where each effect (outliers, calendar effects) is associated to a specific regression and where the residuals of the model are supposed to follow a general Arima model. The algorithm proposed by Gómez and Maravall (1998), which is implemented in the software TRAMO-SEATS, is probably the most popular and the most efficient. This algorithm performs an automatic detection of the decomposition model (additive, multiplicative), an automatic detection and correction of outliers (additive outlier, level shifts, transitory changes, ramps, seasonal outliers), an automatic detection and correction of usual trading-day effects, an automatic adjustment of the Arima model and produces forecasts and backcasts of the series.

TRAMO fits a seasonal Arima model  $(p, d, q)(P, D, Q)_s$  to a monthly or quarterly series  $y_t$ . This model can be written:  $\Phi(B)\delta(B)y(t) = \theta(B)\epsilon(t)$ , where:

$$\begin{aligned}\delta(B) &= \Delta^d \Delta_s^D = (1 - B)^d (1 - B^s)^D \\ \Phi(B) &= (1 + \phi_1 B + \dots + \phi_p B^p)(1 + \Phi_1 B^s + \dots + \Phi_P B^{Ps}) \\ \theta(B) &= (1 + \theta_1 B + \dots + \theta_q B^q)(1 + \Theta_1 B^s + \dots + \Theta_Q B^{Qs})\end{aligned}$$

Figure 29.4: TRAMO-SEATS and X-13ARIMA-SEATS 2-step process.

### X-13ARIMA-SEATS and TRAMO-SEATS Seasonal Adjustment Process



In these equations,  $s$  is the integer periodicity of the series. The seasonal difference operator is noted  $\Delta_s y_t = y_t - y_{t-s}$  and  $\Delta_1$  is simply denoted  $\Delta$ .

For daily and weekly data,  $s$  is not any more a single and constant integer. To interpret the peculiar trait of the seasonality, the class of seasonal fractionally integrated ARMA processes, or ARFIMA in short, has been advocated in various contribution, among which Koopman et al. (2007). Following this approach, it is possible to extend the model:

1. Focusing on the “average” periodicities: for a daily series, the possible periodicities will therefore be  $s_1 = 7$ ,  $s_2 = 30.436875$  and  $s_3 = 365.2425$ ; for a weekly series, the average yearly periodicity is  $s = 52.1775$ .
2. Focusing in a first step on a generalization of the airline model with fractional periodicities. The basic models are here:

- For a daily series (D1):

$$(1 - B)(1 - B^7)(1 - B^{30.436875})(1 - B^{365.2425})y_t = (1 + \theta_1 B)(1 + \theta_2 B^7)(1 + \theta_3 B^{30.436875})(1 + \theta_4 B^{365.2425})\epsilon(t)$$

- For a weekly series (W1):

$$(1 - B)(1 - B^{52.1775})y_t = (1 + \theta_1 B)(1 + \theta_2 B^{52.1775})\epsilon(t)$$

Let us note  $s + \alpha$  the periodicity (for example 365.2475 for daily data or 52.1775 for weekly data) where  $s$  is an integer (365 or 52) and  $\alpha$  a real number belonging to the interval  $]0, 1[$  (0.2475 or 0.1175).

Using the Taylor expansion of  $x^\alpha$ , we have:

$$x^\alpha = 1 + \alpha(x - 1) + \frac{\alpha(\alpha + 1)}{2!}(x - 1)^2 + \frac{\alpha(\alpha + 1)(\alpha + 2)}{3!}(x - 1)^3 + \dots$$

And, if we limit to the first two terms of the expansion, we have  $x^\alpha \cong (1 - \alpha) + \alpha x$ . So, we can define the “tilde difference operator”:

$$\tilde{\Delta}_{s+\alpha} y_t = y_t - B^{s+\alpha} y_t = y_t - B^s B^\alpha y_t \cong y_t - (1 - \alpha)B^s y_t + \alpha B^{s+1} y_t$$

The previous high-frequency models can therefore be rewritten:

1. For weekly series:

$$W2: \Delta \tilde{\Delta}_{52.1775} y_t = (1 - \theta_1 B)(1 - 0.8225\theta_2 B^{52} - 0.1775\theta_2 B^{53}) \epsilon_t$$

2. For daily series:

$$D2: \Delta \Delta_7 \tilde{\Delta}_{30.436875} \tilde{\Delta}_{365.2475} y_t = (1 - \theta_1 B)(1 - \theta_2 B^7)(1 - 0.563125\theta_3 B^{30} - 0.436875\theta_3 B^{31})(1 - 0.7525\theta_4 B^{365} - 0.2475\theta_4 B^{366}) \epsilon_t$$

3. And for infra-daily series, similar extensions could be considered with additional periodicities.

### 29.3.1 Decomposition Model

Following the genuine TRAMO algorithm it is possible to choose between an additive and multiplicative decomposition model by adjusting the fractional airline model to the raw data and to the log-transformed data and comparing the quality of the adjustments.

But high frequency data are usually very volatile and the presence of outliers might biased the choice towards a multiplicative decomposition. Moreover, because of the presence of several periodicities, the data could present more complex decomposition models: additive for one periodicity, multiplicative for another etc.

To handle these issues, more robust tests like the “spread versus level” test proposed by Hoaglin et al. (1983), Box-Cox transformations might be more appropriate. In this case, robust tests like the “ladder of powers” proposed by Tukey (1977) could also be implemented.

### 29.3.2 Calendar Effects

The trading-day effect was originally defined for monthly, quarterly and even annual series. The fact is that months (or quarters) are not directly comparable. They do not have the same number of days (mainly a seasonal effect) and the day composition of months varies from one month to another and from one year to another. For example, May 2015 had 5 Saturdays, one more than May 2014, April 2015 and June 2015. In the retail trade sector, this extra Saturday can make more difficult the year-to-year and month-to-month turnover comparisons. This effect directly linked to the day composition of the month (or quarter) defines the trading-day effect.

Therefore, weekly and daily data cannot theoretically present any pure trading-day effect: weeks always have one day of each kind and the differences between days that could be observed in daily data is a periodic/seasonal component. But it is without taking into account the specificity of the National calendar.

National holidays are often linked to a date, not to a specific day. For example, in catholic countries, Christmas is always the 25<sup>th</sup> of December, but not always a Sunday. As these National days are usually off, they might impact the activity of some sectors of the economy in different ways. For a weekly (monthly, quarterly) time series, a National holiday contributes to the trading-day effect as the weeks (months, quarters) do not have any more the same number of working days. And this contribution might be significant for a given week.

The way National holidays should be considered might change according to the periodicity of the series. For a daily series, the “Christmas effect” on December 25<sup>th</sup> and around is a periodic/seasonal effect and should be directly taken into account in the seasonal component. But for a weekly series, this is different as Christmas might impact different weeks and even be in different weeks (the 51<sup>st</sup> or the 52<sup>nd</sup>) according to the year. This effect must be very carefully modeled and eliminated. In a similar way, the impact of moving holidays like Easter and other related events, Ramadan, Chinese new year etc. should be specifically modeled.

It is quite easy to derive general impact models<sup>4</sup>. For a given event, the parameters of a model are:

- The date of the event in the Gregorian calendar;

<sup>4</sup>For a detailed presentation of these models, see Ladiray (2018).

Figure 29.5: A general constant impact model

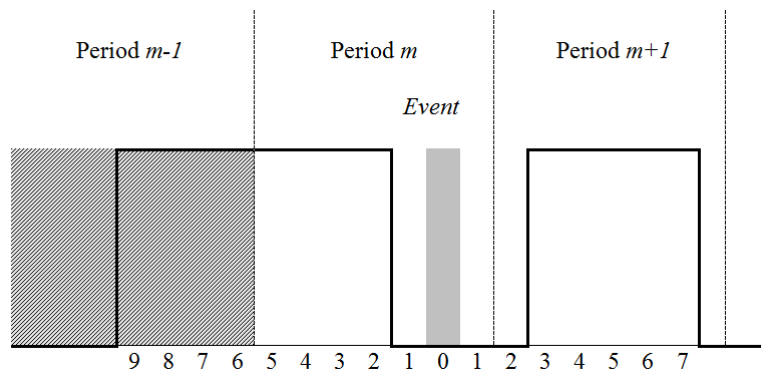
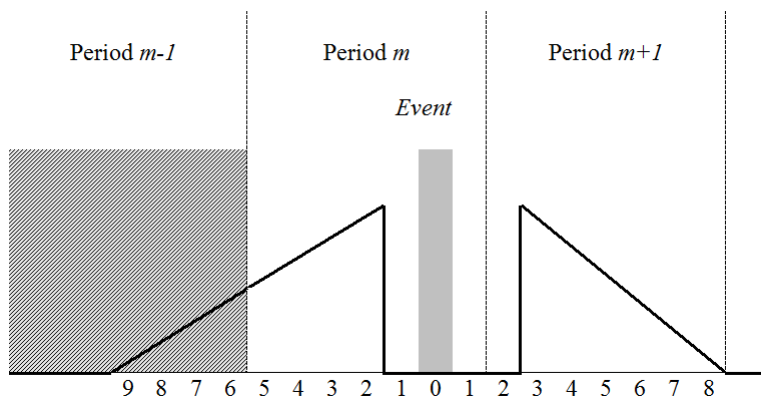


Figure 29.6: a general “increase-decrease” linear impact model



- The span of time on which the event is supposed to have an impact. It can be determined by 4 parameters: the starting date of the impact before the event, the ending date of the impact before the event, the starting date of the impact after the event and the ending date of the impact after the event;
- The nature of the impact: constant, linear, quadratic, increasing, decreasing etc.

Figure 29.5 and Figure 29.6 illustrate a general constant impact model and a general “increase-decrease” linear model. The constant impact model supposes the event occurs in period (week)  $m$  and has an impact on 8 days in the past, starting 2 days before the event, and on 5 days in the future, starting 3 days after the event. The regressor  $MH$  which will be used in the model is, for a given period, proportional to the number of days of impact in this period. Therefore,  $MH(m-1) = 4/13$ ,  $MH(m) = 4/13$ ,  $MH(m+1) = 5/13$  and  $MH(w) = 0$  for all other periods.

The modeling should also take into account the fact that moving holidays often present a periodical behavior in the Gregorian calendar. For example, the catholic Easter always falls in March or April and more often in April than in March, the orthodox Easter always falls in April or May and more often in April than in May etc. This annual effect should be removed from the regressor that must take into account non-periodical effects.

### Calendar Effects in the Electricity series

France celebrates 11 National holidays per year: January 1<sup>st</sup>, May 1<sup>st</sup>, May 8<sup>th</sup>, July 14<sup>th</sup>, August 15<sup>th</sup>, November 1<sup>st</sup>, November 11<sup>th</sup>, December 25<sup>th</sup>, Easter Monday, Pentecost Monday and Ascension Thursday.

Just to explore the possible calendar effects, a regressor was created for each of the 11 holidays and for each of the 7 days before and the 7 days after the holiday. We therefore have  $11 * 15 = 165$  such regressors. The significance (t-value) of each regressor is reported in Table 29.3 and we can draw important conclusions for the modelling of these calendar effects.

- As expected, all holidays have a negative effect on the consumption of electricity. But the effects are different which advocates for specific regressors.

- The effect of Pentecost and July 14<sup>th</sup> is concentrated on the day itself (Lag 0);
- Other holidays show have an impact on the day itself and on the following day;
- The week between Christmas and New Year has a very specific behaviour.

This simple example shows that a proper modelling of a daily series requires to take into account the specificities of the National calendar. Of course, in this case, only the effect of moving holidays (Easter Monday, Ascension and Pentecost) should be taken into account as the effect of the other holidays will be captured by the annual periodic component.

When we restrict the Calendar effect to these 3 regressors, the t-values are:

- -20.128 for the Easter Monday
- -20.963 for Pentecost
- -18.200 for Ascension.

**Table 29.3:** National holiday effects in the daily consumption of electricity in France

Lag	Jan1	May1	May8	Jul14	Aug15	Nov1	Nov11	Dec25	EasterMonday	Ascension	Pentecost
-7	3.581	-1.553	-0.279	0.163	-1.183	-0.013	-0.92	-0.711	0.197	1.401	0
-6	-4.933	-1.313	2.422	0.565	-0.698	-0.382	0.317	-0.822	-0.089	1.228	0
-5	1.013	-1.194	-0.177	1.141	-1.07	-0.385	-0.542	-1.294	1.594	0.8	0
-4	-1.506	0.194	-0.135	1.649	-1.336	-0.364	2.055	-1.581	1.255	0	0
-3	-0.196	0.611	0.728	1.462	-1.769	-0.145	0.081	-1.759	0.006	1.757	3.86
-2	-0.977	1.953	1.809	1.669	-2.346	-0.306	0.183	-5.742	1.038	2.25	0.782
-1	-1.095	-0.168	0.419	-0.595	-3.372	-2.943	-1.81	-13.247	0	2.729	0
0	-5.732	-6.881	-6.644	-17.457	-16.706	-16.265	-15.455	-12.639	-20.188	-17.218	-18.976
1	-7.371	-6.412	-2.562	-1.092	-4.908	-4.18	-2.712	-1.309	-2.151	-7.614	1.262
2	-1.429	1.198	0.626	2.692	-1.682	-0.2	-0.832	-4.853	-0.438	-1.53	5.262
3	0.356	0.839	0.98	2.121	-1.406	0.497	-0.097	-1.569	0.598	3.732	3.45
4	0.178	0.03	0.615	2.129	-1.069	-0.275	0.139	-2.352	1.308	0	3.809
5	-0.437	-1.276	0.419	1.88	-1.217	0.968	0.14	-1.966	0.481	4.091	0.329
6	-0.65	-0.412	0.632	1.452	-0.903	2.069	0.111	-3.858	0.949	3.978	1.726
7	-1.001	1.116	1.519	1.795	-0.361	-0.25	-0.634	-1.181	0	4.354	0

### 29.3.3 Dealing with outliers

Both parametric methods (Reg-Arima models or unobserved component models) and non parametric models (X11 and STL) use automatic procedures to detect and correct series for outliers.

These procedures are quite efficient and can be used as they are for daily and weekly data.

- Reg-Arima methods, like TRAMO-SEATS, and unobserved component models use intervention analysis and step-wise iterations;
- X11 has a robust built-in procedure based on trimmed means to detect and correct for additive outliers that is usually preceded by a Reg-Arima correction procedure;
- STL has a robust built-in procedure based on LOESS, a local weighted regression procedure that can be also complemented if needed by a Reg-Arima correction procedure.

#### Outliers in the Electricity series

As shown by Table 29.4, only 8 outliers are detected, a very small number considering the length and the volatility of the series. This is a sign that the modeling strategy fits quite well the data.

Moreover, most of the detected outliers (6 out of 8) concern the Christmas/New Year week or National holidays for specific years. Once again these results advocate for a precise modeling of the National holidays.

**Table 29.4:** Outliers in the daily consumption of electricity in France

Type	Date	Estimate	Pvalue
AO	29/12/1996	0.152	4.399
AO	08/05/1997	0.184	5.066
AO	01/05/2008	0.202	5.548
AO	20/12/2009	0.155	4.5
AO	30/12/2009	-0.192	-5.5
AO	31/12/2009	-0.205	-5.858
AO	25/12/2010	0.182	5.138
AO	12/02/2012	0.155	4.499

## 29.4 Seasonal Adjustment Based on Non-parametric Methods

X11 and STL share a common non parametric philosophy. The 2 methods are based on an iterative estimation of the series components. The difference lies in the choice of the filters: a set of moving averages for X11, and the use of local weighted regressions (LOESS) for STL. Both software can be extended to daily and weekly data.

### 29.4.1 Adapting X11 to multiple and non integer periodicities

The iterative process of X11 is precisely described in Ladiray and Quenneville (2001) and seems to be easily extended to multiple periodicities.

#### 29.4.1.1 An X11-like algorithm for multiple periodicities

Let us suppose our time series  $X_t$  presents 2 different seasonalities of order  $p$  and  $q$  and can be represented by the following additive model:

$$X_t = TC_t + S_{p,t} + S_{q,t} + I_t.$$

A basic algorithm to derive an estimation of the various components, and therefore of the seasonally adjusted series, can be summarized in 10 steps.

##### 1. Estimation of the trend-cycle by a composite moving average

$$TC_t^{(1)} = M_{p \times q}(X_t)$$

The symmetric moving average used here is a so-called  $p \times q$  moving average, of order  $(p + q) - 1$ , which preserves linear trends, eliminates order- $p$  and order- $q$  constant seasonalities and minimizes the variance of the irregular component. Its coefficients are the coefficients of the product of the 2 polynomials  $\frac{1}{p}(1 + t + t^2 + \dots + t^p)$  and  $\frac{1}{q}(1 + t + t^2 + \dots + t^q)$ <sup>5</sup>.

##### 2. Estimation of the global seasonal-irregular component

A first estimation of the global seasonal-irregular component can be easily obtained removing the trend-cycle estimation from the raw data.

$$(S_{p,t} + S_{q,t} + I_t)^{(1)} = X_t - TC_t^{(1)}$$

##### 3. Estimation of each basic seasonal-irregular component

<sup>5</sup>If the order  $(p+q)-1$  is even, we use a  $2 \times p \times q$  moving average.

Smoothing the global seasonal-irregular estimate with a simple moving-average of order  $p$ , which removes order- $p$  constant seasonalities, gives an estimate of a seasonal-irregular component corresponding to periodicity  $q$ .

$$(S_{q,t} + I_t^\bullet)^{(1)} = M_p \left[ (S_{p,t} + S_{q,t} + I_t)^{(1)} \right]$$

And, by difference, we obtain an estimate of the seasonal-irregular component corresponding to periodicity  $p$ .

$$(S_{p,t} + I_t^\bullet)^{(1)} = (S_{p,t} + S_{q,t} + I_t)^{(1)} - (S_{q,t} + I_t^\bullet)^{(1)}$$

Note that it is better to begin with the smallest periodicity in order to minimize the number of lost points at the ends of the series.

#### 4. Estimation of each basic seasonal component, by a 9-term Henderson moving average over each period

For seasonality  $p$ , we smooth each of the  $p$  series corresponding to each period with a 9-term Henderson moving average which preserves local quadratic trends. The seasonal factors are then normalized so that their sum over each  $p$ -period span is approximately zero.

$$(S_{p,t})^{(1)} = H_9 \left[ (S_{p,t} + I_t^\bullet)^{(1)} \right]$$

and

$$\left( \tilde{S}_{p,t} \right)^{(1)} = (S_{p,t})^{(1)} - M_p (S_{p,t})^{(1)}$$

We do the same for seasonality  $q$ .

$$(S_{q,t})^{(1)} = H_9 \left[ (S_{q,t} + I_t^\bullet)^{(1)} \right]$$

and

$$\left( \tilde{S}_{q,t} \right)^{(1)} = (S_{q,t})^{(1)} - M_q (S_{q,t})^{(1)}$$

#### 5. Estimation of the irregular component, detection and correction of the extreme values

A first estimate of the irregular component is derived, removing the seasonal factors from the estimate of the global seasonal-irregular component.

$$I_t^{(1)} = (S_{p,t} + S_{q,t} + I_t)^{(1)} - \left( \tilde{S}_{p,t} \right)^{(1)} - \left( \tilde{S}_{q,t} \right)^{(1)}$$

A robust moving standard error  $\sigma_t$ , the F-pseudosigma<sup>6</sup>, is computed for each data  $t$ . Each value of the irregular component  $I_t$  is assigned a weight  $w_t$ , function of the standard deviation associated with that value, calculated as follows:

- Values which are more than  $2.5\sigma_t$  away in absolute value from the average 0 are assigned zero weight.
- Values which are less than  $1.5\sigma_t$  away in absolute value from the average 0 are assigned a weight equal to 1.
- Values which lie between  $1.5\sigma_t$  and  $2.5\sigma_t$  in absolute value from the average 0 are assigned a weight that varies linearly between 0 and 1, depending on their position.

and the irregular is therefore corrected:

$$\tilde{I}_t^{(1)} = w_t \times I_t^{(1)}$$

#### 6. Estimation of the global seasonal-irregular component corrected from extreme values

<sup>6</sup>The F-pseudosigma is defined as  $\sigma_t = \frac{IQR}{1.349}$  where  $IQR$  is the interquartile range.



A new estimation of the global seasonal-irregular component, corrected from extreme values is then derived:

$$(S_{p,t} + S_{q,t} + I_t)^{(\bullet)} = \left(\tilde{S}_{p,t}\right)^{(1)} + \left(\tilde{S}_{q,t}\right)^{(1)} + \tilde{I}_t^{(1)}$$

### 7. Estimation of each basic seasonal-irregular component

Smoothing the global seasonal-irregular component corrected from extreme values with a simple moving-average of order  $p$  gives a new estimate of a seasonal-irregular component corresponding to periodicity  $q$ .

$$(S_{q,t} + I_t^{\bullet})^{(2)} = M_p \left[ (S_{p,t} + S_{q,t} + I_t)^{(\bullet)} \right]$$

And, by difference, we obtain an estimate of the seasonal-irregular component corresponding to periodicity  $p$ .

$$(S_{p,t} + I_t^{\bullet})^{(2)} = (S_{p,t} + S_{q,t} + I_t)^{(\bullet)} - (S_{q,t} + I_t^{\bullet})^{(2)}$$

### 8. Estimation of each basic seasonal component, by a 9-term Henderson moving average over each period

For seasonality  $p$ , we smooth each of the  $p$  series corresponding to each period with a 9-term Henderson moving average. The seasonal factors are then normalized so that their sum over each  $p$ -period span is approximately zero.

$$(S_{p,t})^{(2)} = H_9 \left[ (S_{p,t} + I_t^{\bullet})^{(2)} \right]$$

and

$$\left(\tilde{S}_{p,t}\right)^{(2)} = (S_{p,t})^{(2)} - M_p (S_{p,t})^{(2)}$$

We do the same for seasonality  $q$ .

$$(S_{q,t})^{(2)} = H_9 \left[ (S_{q,t} + I_t^{\bullet})^{(2)} \right]$$

and

$$\left(\tilde{S}_{q,t}\right)^{(2)} = (S_{q,t})^{(2)} - M_q (S_{q,t})^{(2)}$$

### 9. Preliminary estimation of the seasonally adjusted series

A first estimate of the seasonally adjusted series can be easily computed by removing the seasonal factors from the raw data:

$$A_t^{(1)} = X_t - (S_{p,t})^{(2)} - (S_{q,t})^{(2)}$$

#### Remark

Another simple idea to extend the genuine X11 algorithm to multiple periodicity is to use it successively to remove each seasonality. For a daily series, you use the genuine algorithm to remove first the periodicity 7 after choosing the relevant filters. The intra-monthly and annual periodicities will then be part of the trend-cycle component. Then, you use the adjusted series to remove the intra-monthly periodicity etc.

#### 29.4.1.2 Dealing with non integer periodicities

To illustrate the problem, let us suppose that our daily series presents an intra-monthly seasonality which means that the days of the month (first, second, third etc.) are not similar. For example, the pay-day (every 2 weeks, end of the month etc.) might have an impact on the retail sales.

To estimate the effect of each day of the month, X11 will smooth a “ragged matrix” similar to the one presented in Table 29.5 which presents 2 problems:

- The first one, due to a length-of-month effect, is the presence of missing values in the last columns of the matrix which makes the smoothing by moving averages impossible.

- The second problem is that, if you assume a strong end-of month effect, the values for the 27<sup>th</sup> day of the month are not completely coherent as they might be affected by the end-of-month effect for February and not for the other months.

Therefore, a correct imputation of missing values depends of the nature of the effect you want to estimate. You can either interpolate the missing values per column or use some kind of “time wrapping” for each line putting the last values of the month in the last columns and interpolating the missing values.

**Table 29.5:** An example of “ragged matrix” encountered in the estimation of an intra-monthly periodicity

Date	D01	D02	D03	D04	.....	D26	D27	D28	D29	D30	D31
12/2011	0.28	0.40	0.34	0.38	.....	0.42	0.34	0.29	0.32	0.30	0.35
01/2012	0.29	0.20	0.09	0.02	.....	-0.51	-0.45	-0.48	-0.45	-0.41	-1.21
02/2012	-1.17	-0.29	-0.28	-0.26	.....	-0.58	-0.58	-0.57	-0.57	.	.
03/2012	-0.47	-0.40	-0.44	-0.37	.....	-0.78	-0.77	-0.73	-0.62	-0.41	-0.45
04/2012	-0.47	-0.41	-0.41	-0.24	.....	-0.50	-0.30	-0.30	-0.27	-0.22	.
05/2012	-0.23	-0.21	-0.15	-0.16	.....	-0.04	-0.10	-0.07	-0.14	-0.05	0.03
06/2012	0.11	0.10	0.06	0.02	.....	-0.26	-0.22	-0.05	0.11	0.15	.
07/2012	0.08	0.08	0.05	0.04	.....	0.18	0.27	0.26	0.20	0.26	0.28
08/2012	0.35	0.45	0.48	0.48	.....	0.27	0.15	0.13	0.17	0.39	0.49
09/2012	0.47	0.47	0.48	0.38	.....	0.32	0.48	0.61	0.63	0.62	.
10/2012	0.73	0.77	0.77	0.78	.....	0.34	0.35	0.37	0.39	0.43	0.49
11/2012	0.51	0.59	0.61	0.56	.....	-0.16	-0.16	-0.09	0.11	0.23	.
12/2012	0.23	0.24	0.20	0.20	.....	0.97	1.01	1.12	1.12	1.09	1.06
01/2013	1.09	0.94	0.77	0.68	.....	-0.49	-0.51	-0.53	-0.52	-0.45	-0.27
02/2013	-0.22	-0.24	-0.25	-0.29	.....	-0.74	-0.68	-0.54	.	.	.
03/2013	-0.43	-0.49	-0.50	-0.54	.....	-0.64	-0.40	-0.07	-0.07	-0.09	-0.09

The annual periodicity presents the same problem linked to the Leap Year as the matrix to smooth will have missing values in its 366<sup>th</sup> columns. But, in this case, it is suggested to ignore the problem by smoothing only the 365 columns and interpolating the missing values in the annual seasonal component.

## 29.4.2 Improving STL for multiple periodicities

STL<sup>7</sup> and X11 share the same iterative and nonparametric philosophy . . . and face the same problems. The idea of iteratively adjusting several seasonal movements in STL has already been put forward by Cleveland et al. (1990); and a generalization of STL to multiple periodicities has been proposed by Ollech (2016).

In the case of daily time series, the main obstacle is the constraint of STL to have a constant period length, i.e. the number of observations per period has to be constant. This is not unique to STL though, as we already saw X11 has the same requirements. While this is unproblematic in the case of intra-weekly seasonality, the number of days per month and the number of days per year are not identical for all periods. In any case, the period length has to be standardized, either by omitting a subset of the data or by artificial prolongation.

Before decomposing the series and estimating the various seasonal components with STL, the series may have to be pre-adjusted for calendar effects, outliers and missing values. The proposed generalized Reg-ARIMA solution could be use for this purpose. If the time series has not been outlier adjusted, the robust version of STL should be used, i.e. the number of iterations of the outer loop needs to be higher than 0.

The iterative procedure proposed by Ollech is the following:

- **Step 1: Removing Intra-Weekly Seasonality** For most of the LOESS regressions in STL there exist sensible default values for the degree of smoothing (see Cleveland et al. (1990)), i.e. the number  $q$  of neighbourhood points. In the case of intra-weekly seasonality,  $q$  has to be large enough so that the week-day effects do not get confounded by other effects such as moving holidays.
- **Step 2: Removing Intra-Monthly Seasonality** While the length of any year is always the same except for one day, the number of days in a given month is quite irregular ranging from 28 to 31 days with an average of 30.4 days

<sup>7</sup>See Cleveland et al. (1990) for a detailed presentation of the method.

per month. STL is flexible enough to handle a time series with a frequency of 30.4, but this estimation strategy will blend together the effects of different dates.

As in X11, the problem of confused effects can be circumvented by extending each month to cover 31 days. For the extension of the time series, several different methods could be used. While regression based techniques have the advantage of possibly incorporating much or all of the information that is contained in the actual series, in the context of high frequency time series they can become computationally burdensome. Therefore, Ollech suggests to use a simpler approach, namely cubic splines. Their benefit is an extremely fast algorithmic implementation and a high degree of smoothness, which seems preferable to a simple linear interpolation. A Forsythe-Malcolm-Moler algorithm is for example applied to obtain values for the additional data points needed to have 31 days in each month. The only parameter that needs to be specified is the value for  $q$ , which determines the length of the seasonal filter.

- **Step 3: Removing Intra-Annual Seasonality**

As the time series has been extended in the pre-treatment step, the excess days including every 29<sup>th</sup> of February have to be removed so that each year contains 365 days. Then  $q$  has to be chosen. As outliers and the influence of moving holidays have already been removed from the series, the main objective in choosing  $q$  is the variability of the intra-monthly seasonality.

Yet, the robust version of STL may be chosen to ensure that intra-annual seasonal peaks do not affect the estimation of the intra-monthly seasonal factors.

- **Step 4: Producing the Seasonally Adjusted Series**

The 29<sup>th</sup> of February is added back to the seasonally adjusted series via spline interpolation. Finally, the effects of the outliers are added to the original and seasonally adjusted time series.

### 29.4.3 Pros and cons of the X11 and STL approaches

The 2 approaches are computationally very efficient and they can easily handle long and complex high frequency time series.

Taking into account non integer periodicities requires the imputation of missing values but solutions have been proposed that seem efficient. The real problems are mainly in the choice of the filters and in the tuning of their length according to the component to estimate.

- From this point of view, STL is slightly simpler as it essentially uses the LOESS robust regression. The number of points  $q$  to take into account in the regression must anyway be precised.
- X11 is certainly more complex to tune as the moving averages might be different according to the component you want to estimate. In the genuine X11 algorithm, and for monthly and quarterly series, the order of the moving averages is selected according to a "signal to noise" ratio (the  $I/C$  ratio for the order of the trend cycle moving averages and the  $I/S$  ratio for the seasonal moving averages). Large scale simulations have still to be done to understand the behavior of these ratios and to elaborate a decision rule for high frequency data.

**Table 29.6:** Auto-Regressive Polynomials used by the new implementation of SEATS

Component	Auto-regressive polynomial
Trend	$(1 - B)^3$
Weekly	$1 + B + B^2 + \dots + B^6$
Intra-monthly	$(1 + B + \dots + B^{30})$ or $(1 + B + \dots + B^{29} + 0.436875B^{30})$
Annual	$(1 + B + \dots + B^{364})$ or $(1 + B + \dots + B^{364} + 0.2475B^{365})$
Noise	1

## 29.5 Seasonal adjustment based on parametric models

### 29.5.1 Seasonal adjustment based on ARIMA Models

The TRAMO algorithm has been adapted to account for multiple periodicities and adapted to daily and weekly data (see Section 15.3). The challenge is more difficult for SEATS as the current algorithm cannot handle high degree AR and MA polynomials.

### 29.5.2 Canonical decomposition

For weekly models, the usual canonical decomposition of SEATS applies but for daily models, we have to introduce a additional models corresponding to the intra-monthly and weekly periodicities. The different components are derived using the auto-regressive polynomials precised in Table 29.6.

It has to be noted that 365, 30 and 7 are relatively prime numbers, so that the decomposition is well defined.

### 29.5.3 Estimation of the components

Taking into account the size of the problem, the estimation of the components corresponding to the canonical decomposition is tricky. The Burman's algorithm can be used for the weekly models provided that the moving average coefficients are not too close to 1 (we can of course force them to be lower than a given value, for instance 0.95, as it is done in the current SEATS). The main advantages of that algorithm is that it is very fast, that it uses few resources and that it does not imply the computation of the actual models of the canonical decomposition (the pseudo-spectra are sufficient). However, the Burman's algorithm can become rapidly unstable especially in the case of daily models.

For such models, we recommend to use the Koopman's disturbance smoother. Unfortunately, this algorithm yields other numerical challenges:

- The factorization of very large polynomials (to get the actual models of the components);
- The diffuse initialization of complex state space forms;
- The stabilization of the disturbance smoother itself.

These challenges have been solved by using some non-standard solutions:

- Polynomial reduction by least squares techniques;
- Diffuse squared root initialization;
- Small correction mechanism in the disturbance smoother.

### 29.5.4 Pros and Cons of the current implementation

1. The “TRAMO-SEATS” algorithm applied to daily and weekly data works quite well, is fast and reliable. It is based on a clear methodology.
2. It gives in particular a fast and sound way to clean the raw data from outliers and trading-day effects.
3. At the moment, the adapted program is based on a generalization of the airline model to multiple and non-integer periodicities only. It is likely that this model will not suit all series. In particular, the effect of the differencing on the annual frequencies should be investigated more precisely.
4. The stability of the model should also be checked very carefully.

### 29.5.5 Seasonal adjustment based on unobserved components models

The unobserved components (UC) approach provides a flexible and easily customizable approach to modelling and adjusting daily and weekly time series. For the series analysed in the course of the project, a “basic” specification, encompassing most of the stylized features of the series, is the following:

$$y_t = \mu_t + \sum_{j=1}^m \gamma_{jt} + \beta' x_t + \epsilon_t. \quad (29.3)$$

- The trend component,  $\mu_t$ , is usually specified as a local linear trend, as in the monthly case. The trend model is already flexible enough, featuring two sources of disturbances, one for the level and one for the rate of change (velocity, or slope) of the trend.
- The seasonal component,  $\sum_{j=1}^m \gamma_{jt}$ , for daily data, consists of the annual cycle and its harmonics, the intra-monthly cycle and the weekly cycle. Two representations for this component can be considered, the trigonometric representation and the periodic spline representation. These are interchangeable in regular cases, but offer comparative different advantages in nonstandard cases.
- The regression component accounts for the effect of moving festivals, such as Easter, and other calendar components. It can be used also for modelling special seasonal effects, such as the Christmas effect.
- The irregular component,  $\epsilon_t$ , is assumed to be uncorrelated.

#### 29.5.5.1 Pros of the UC approach

As stated above, the main comparative advantage of the UC approach, over the reduced form ARIMA approach and the nonparametric one, is the flexibility by which special features can be incorporated. Nothing prevents, in principle, to model the trend using a cubic spline with fixed knots, or a segmented trend, subject to slope changes, rather than having a model where the level and the velocity are subject to small and frequent shocks. Also, the definition of the components is well delineated and the identifiability of the various effects is imposed a priori. Furthermore, the variability of the components is learnt from the data, as the model is estimated by maximum likelihood. As a result the filter used for seasonal adjustment is not fixed, but it is adapted to the specific nature of the series under investigation.

The seasonal component can be represented by the combination of trigonometric cycles, which are defined at the seasonal frequencies. As the cosine and sine functions at the harmonic and the fundamental frequencies are orthogonal, model selection is simplified, and the decision on the number of trigonometric terms to be included can be carried out by the usual information criteria. Hence, the trigonometric specification makes variable selection relatively easy, due to the orthogonality of the seasonal cycles.

An alternative effective representation for the seasonal component is the periodic spline model. Here the model selection problem deals with locating the knots. In many application, a quite natural choice is locating the knots at the end of each month. The advantage of the periodic spline model, over the trigonometric one, is the possibility of entertaining special periods of the year, e.g. Christmas, by allocating a specific knot. This however comes with a disadvantage, as the effect associated to a new knot is not orthogonal to the other effects already present. As a result, model selection is more complicated.

Finally, it has to be stated that the UC approach can handle missing values and can easily incorporate a complete pretreatment of the series, with the purpose of adjusting for outliers and structural breaks.

### 29.5.5.2 Limitations of the UC approach

In the actual implementation of the UC approach we can envisage two classes of limitations: those related to the estimation of the model and those related to the specification of the model.

The model depends on a set of hyperparameters, typically a transformation of the variance parameters of the disturbances driving the evolution of the latent components, which have to be estimated by maximum likelihood. Usually, the optimization takes place by a quasi-Newton algorithm, such as BFGS, using numerical first and second derivatives, which is iterated until convergence. The following problems may arise:

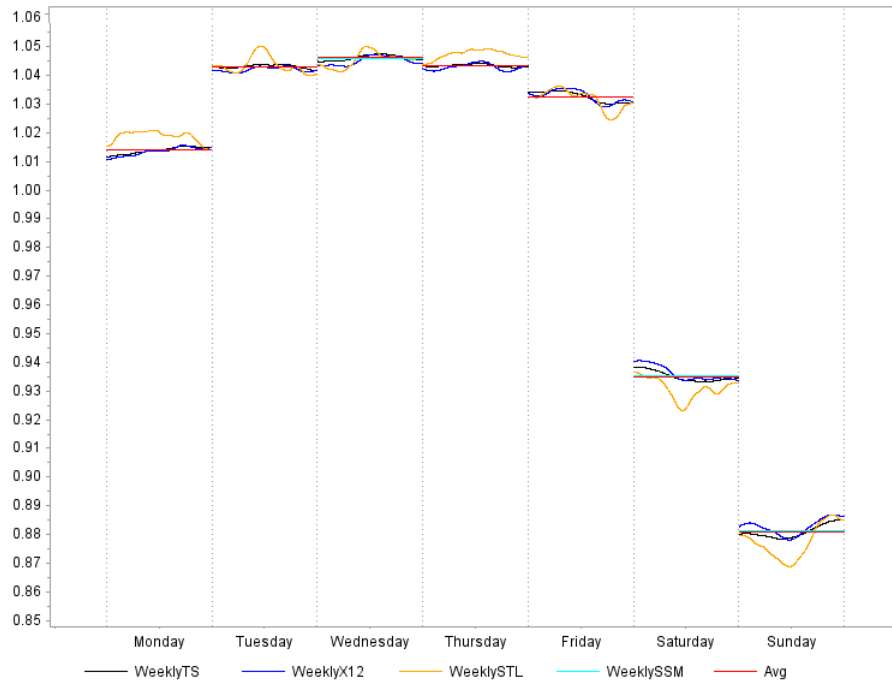
1. The maximization algorithm requires initial parameter values. It is not trivial, in general, to provide good starting values of the hyperparameters.
2. When the number of observations is very large, and or the state vector is high-dimensional, each iteration of the algorithm may take a substantial computational time.
3. The evaluation of the likelihood for certain hyperparameter values may fail.
4. There is always the possibility that several local maxima are present and that the algorithm converges to a local maximum.

One problem with the specification of the unobserved components is that the models tend to be very “series specific”, i.e. rather ad hoc. For instance, knots selection can be rather arbitrary; we may or may not have a knot corresponding to the 25<sup>th</sup> of December; or we may add an intervention variable for the Christmas effect, which spreads the effects over neighbouring days and repeats itself constantly over the years.

As a consequence, it appears that it is difficult to implement a general user friendly software that can be applied to a vast number of situations. The user could get the impression that modelling daily and weekly time series is an art, rather than a routine operation.

A specification issue we often face is deciding a priori whether a particular component is deterministic or stochastic. This actually is a problem when the frequency of the observations is high, then the permanent of trend shocks (to the component  $\mu_t$ ) will tend to be small, so that a time evolutive trend may be actually very difficult to estimate. As a matter of fact, it is indeed very difficult to distill what is permanent from a daily observation: a needle in a haystack. Hence, the estimates of the size of the disturbances to  $\mu_t$  tend to pile up at zero.

When the number of years of available data for a daily series is not large, it is a problem to identify the trend from the annual cycle: as a matter of fact, the annual cycle corresponds to the frequency  $2\pi/365.25 = 0.0055\pi$ , which is very close to the zero frequency. As a result, it will be difficult to isolate the trend from the fundamental annual cycle. Specifying a model with both time evolving trend and annual cycle is asking too much from the data.

**Figure 29.7:** Various estimates of the weekly pattern in the Electricity series

The patterns obtained with X11, STL, TRAMO-SEATS and UCM are represented respectively in black, blue, orange and cyan. The red line represents the average of the TRAMO-SEATS weekly pattern.

## 29.6 Application: Seasonal adjustment of the Electricity series

The Electricity series has been decomposed using the 4 methods: X11, STL, TRAMO-SEATS and UCM. As already noticed, the series has 2 different periodicities: a weekly component (Sunday and Tuesday do not have the same behavior) and an annual component (Winter and Summer have different behaviors).

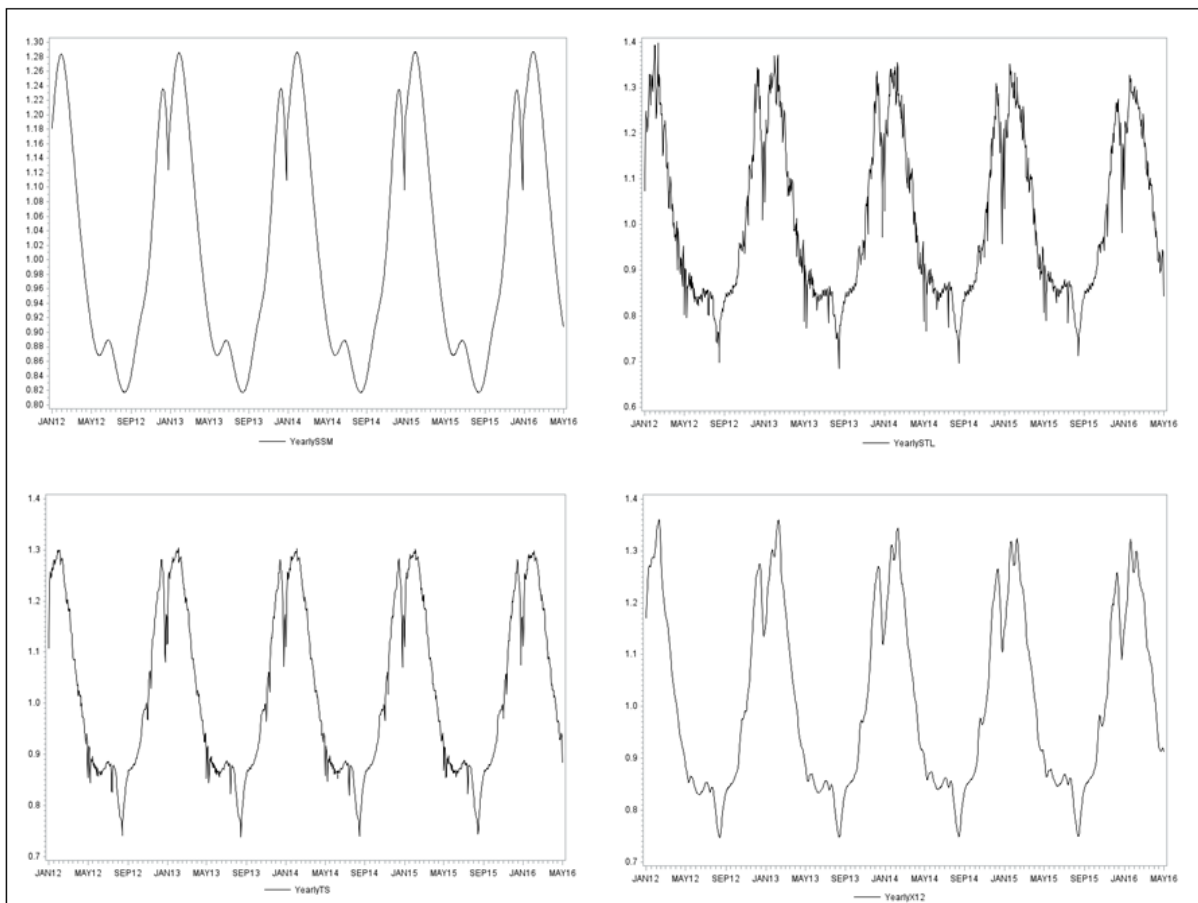
### 29.6.1 The seasonal patterns

The weekly patterns estimated by the 4 methods are presented in Figure 29.7. It clearly appears that the 4 methods deliver a similar message; the weekly pattern is very stable. We observe a decreasing of the consumption of electricity during the weekend (-5% on Saturday and -12% on Sunday) a slight augmentation on Monday (+1%) and a stable consumption the 4 other days (about +4%). This common weekly pattern is clearly linked to the fact that businesses and factories are closed during the weekend when retail sale shops are mainly closed on Sunday and Monday.

On the opposite the yearly patterns shown in Figure 29.8, if they deliver roughly the same message, are quite different:

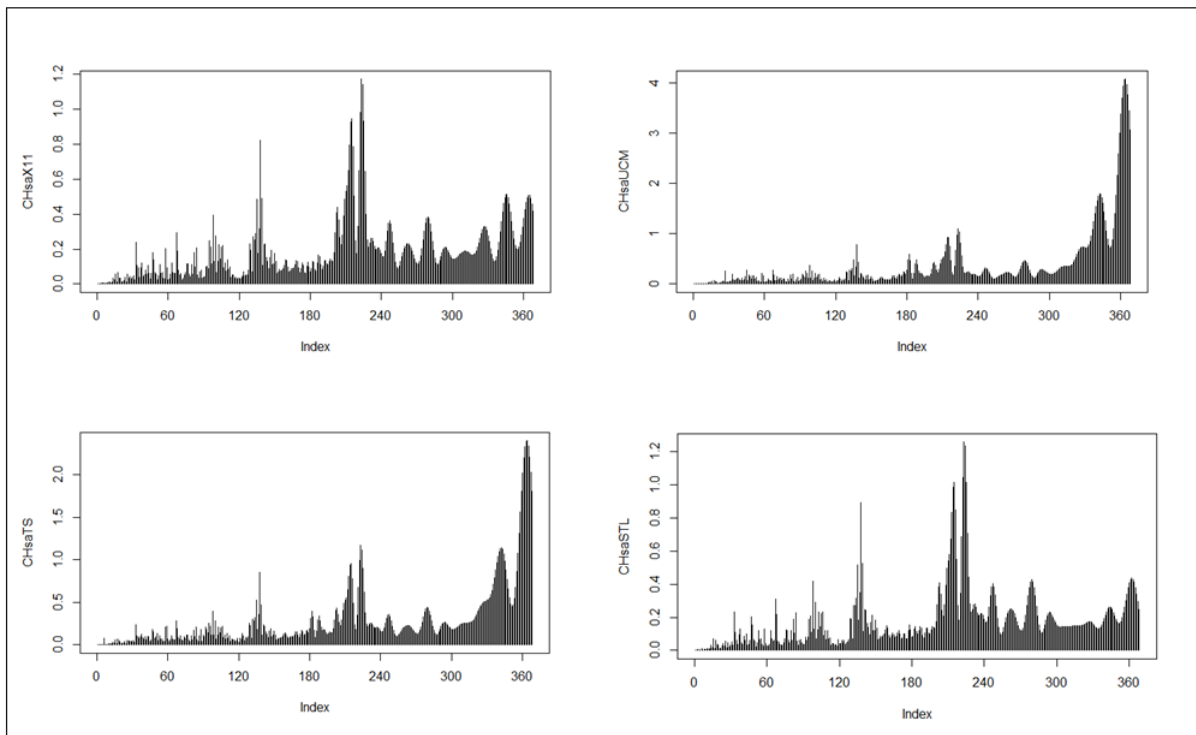
- All patterns present a maximum in the Winter period (January-February) with a consumption of electricity about 30% above the average and a minimum in Summer about 15% under the average. The patterns also show 2 specific periods where the consumption of electricity is very low: the week between Christmas and New year period (only 10% above the average) and the big period of holidays in August (more than 25% under the average).
- Yearly patterns are supposed to reflect the different behavior of each day of the year. In particular, National holidays should appear as specific as most of the businesses, shops and factories are closed. From this perspective, the UCM yearly pattern appears too smooth.

Figure 29.8: Various estimates of the yearly pattern in the Electricity series



These patterns were obtained with UCM (upper left panel), STL (upper right panel), TRAMO-SEATS (lower left panel) and X11 (upper right panel)



**Figure 29.9:** Canova-Hansen test on the 4 seasonally adjusted series of the Electricity series.

These tests concern the estimates obtained with X11 (upper left panel), UCM (upper right panel), TRAMO-SEATS (lower left panel) and STL (upper right panel)

### 29.6.2 Validation of the results

The validation of the results, the quality of the decomposition, must first respect some basic ideas. In particular, the “seasonally adjusted” series, as well as the irregular series, should present no residual seasonality and no residual calendar effect. This can be assessed in particular using the statistical tests or graphs previously presented: Ljung-Box seasonality tests, Canova-Hansen test, spectra etc.

The results of the Canova-Hansen tests performed on the 4 estimates of the seasonally adjusted series are presented in Figure 29.9.

- No problem of residual weekly periodicity is detected in the 4 seasonally adjusted series;
- On the opposite, a residual yearly periodicity appears in the UCM and in the TRAMO-SEATS seasonally adjusted series

Fisher tests on dummy seasonals and QS tests have also been performed to check for residual seasonality in the 4 seasonally adjusted series. Results are presented in Table 29.7.

- As far as the weekly periodicity is concerned, the QS test detects a residual seasonality in all seasonally adjusted series. And the Fisher test validates the absence of residual seasonality in the UCM estimate.
- For the annual periodicity, the Fisher test validates all seasonally adjusted series except the UCM estimate when the QS test finds significant residual seasonality in all series except the STL one.

It turns out that the adjustments are not really satisfactory.

**Table 29.7:** Tests on the presence of residual seasonality in the 4 seasonally adjusted series

Test	Period	X11		UCM		TS		STL	
		Value	Pvalue	Value	Pvalue	Value	Pvalue	Value	Pvalue
SFtest	7	17.64	0.00	0.07	0.99	0.35	0.91	62.49	0.00
QStest	7	409.47	0.00	275.28	0.00	375.48	0.00	386.85	0.00
SFtest	365	0.90	0.89	1.19	0.01	0.45	1.00	0.69	1.00
QStest	365	389.18	0.00	473.40	0.00	28.51	0.00	0	1

## 29.7 Conclusions

This chapter demonstrated that the main seasonal adjustment methods used for monthly and quarterly series, namely TRAMO-SEATS and X12-ARIMA, as well as STL, can be adapted to high frequency data which present multiple and non integer periodicities. TRAMO-SEATS can be modified using fractional Arima models and more efficient numerical algorithms; and the non parametric and iterative processes of X11 and STL can also be easily adapted after imputation of the missing values induced by the different lengths of months and year.

But the tuning of the multiple parameters of the methods might be cumbersome.

- Unobserved component models can by design handle any kind of high frequency data but the models tend to be very “series specific”, i.e. rather ad hoc. In particular, the selection of harmonics or knots can be rather arbitrary. As a consequence, it appears that it is difficult to implement a general user friendly software that can be applied to a vast number of situations.
- The tuning of X11 and STL parameters, namely the length of the filters used in the decomposition, needs to be improved. In the genuine X11 algorithm, and for monthly and quarterly series, the order of the moving averages is selected according to a “signal to noise” ratio (the  $I/C$  ratio for the order of the trend cycle moving averages and the  $I/S$  ratio for the seasonal moving averages). Thresholds have been defined by simulations and practice. Large scale simulations have still to be done to understand the behavior of these ratios and to elaborate a decision rule for high frequency data.
- At the moment, the adapted TRAMO-SEATS program is based on a generalization of the airline model to multiple and non-integer periodicities only. It is likely that this model will not suit all series. The stability of the model should also be checked very carefully.

Despite these shortcomings, it seems clear that methods for the treatment of high frequency time series will improve and be more relevant in the near future, due to the advances in automated data collection and the progress in statistical and econometric analysis.

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# X

## ESS Guidelines on Seasonal Adjustment







# 30

## The ESS Guidelines on Seasonal Adjustment





## Table of Contents

30.1	Introduction	791
30.1.1	Motivation for guidelines	791
30.1.2	Scope of guidelines	791
30.1.3	Costs and risks	791
30.1.4	Background to guidelines and basic definitions	792
30.2	Annex: Principles for seasonal adjustment	793
30.3	A policy for seasonal adjustment	794
30.3.1	A general seasonal adjustment policy	794
30.3.2	The need for domain specific seasonal adjustment policies	794
30.3.3	Consistency of general and domain specific policies	795
30.3.4	Stability of seasonal adjustment policies	796
30.3.5	Dissemination of general and domain specific seasonal adjustment policies	797
30.3.6	Quality framework for seasonal adjustment	797
30.4	Pre-treatment	799
30.4.1	General aspects for choosing between detailed and automatic pre-treatment	799
30.4.2	Graphical analysis of the series	800
30.4.3	Calendar adjustment	800
30.4.4	National and EU/euro area calendars	801
30.4.5	Choosing the frequency of time series for calendar adjustment	802
30.4.6	Other calendar related and weather effects	803
30.4.7	General principles of outlier detection and correction	805
30.4.8	Treatment of outliers at the end of the series and at the beginning of a major economic change	806
30.4.9	Model selection	807
30.4.10	Decomposition scheme	807
30.5	Seasonal adjustment	809
30.5.1	Choice of seasonal adjustment method	809
30.5.2	Choice of the software	809
30.5.3	Temporal consistency between unadjusted and seasonally adjusted data	810
30.5.4	Direct and indirect approaches	811
30.5.5	Direct versus indirect approach: dealing with data from different agencies	812
30.5.6	Different seasonal filters for different months/quarters	813
30.6	Revision policies	814
30.6.1	General revision policy and release calendar	814
30.6.2	Concurrent versus current adjustment	815
30.6.3	Length for routine revisions	816
30.6.4	Length for major revisions	816
30.7	Accuracy of seasonal adjustment	818
30.7.1	Validation policy for seasonal adjustment	818
30.7.2	Measurement for individual series	818
30.7.3	Comparison of alternative approaches/strategies	819
30.8	Specific issues on seasonal adjustment	821
30.8.1	Seasonal adjustment of short and very short time series	821
30.8.2	Seasonal adjustment of long time series	821
30.8.3	Treatment of problematic series	822
30.8.4	Seasonal heteroskedasticity	823

30.8.5	Seasonal adjustment of annually chain-linked series (Laspeyres-type)	824
30.9	Data presentation issues	826
30.9.1	Data availability in databases	826
30.9.2	Press releases	826
30.9.3	Documenting metadata for seasonal adjustment	827



## 30.1 Introduction

### 30.1.1 Motivation for guidelines

The European Statistical System (ESS) developed these guidelines to promote best practice in seasonal adjustment to:

- achieve harmonisation across national processes;
- enhance comparability between results;
- increase robustness of European aggregates.

The guidelines are aimed at all infra-annual statistics produced by the ESS which are politically and/or economically important.

The guidelines provide a consistent framework for seasonal adjustment, taking advantage of similarities in the process to define a common vocabulary to facilitate communication and comparison between practitioners.

A standard metadata template would improve user understanding of seasonal adjustment by providing transparency in revisions policies, modelling options chosen, and reliability of outputs. Such metadata template will be developed after the endorsement of the guidelines.

### 30.1.2 Scope of guidelines

The guidelines are aimed at anyone whose work involves seasonal adjustment. Both for experts and others, the framework for seasonal adjustment remains the same: only the level of detail in the analysis varies.

The guidelines cover issues related to seasonal adjustment of monthly or quarterly time series, from pretreatment, through seasonal adjustment, to revisions, quality measurement, documentation and publication.

Each stage of the seasonal adjustment process is detailed and options described. Out of these options three alternative courses of action are highlighted: (A) Best alternative (B) Acceptable (C) To be avoided.

- A) The best alternative should always be the feasible target for producers. It should always be achievable with a reasonable effort, unless some production or institutional constraints prevent it.
- B) The acceptable alternative should be used as an intermediate step towards the achievement of alternative A. It could also be seen as the target for a limited number of cases when specific data issues, user requests, time or resource constraints, prevent the achievement alternative (A).
- C) The alternative to be avoided is not recommended.

The objective of the guidelines is help producers move to alternative (A). Careful considerations and, possibly, prompt measures should be taken whenever alternative C is in use.

### 30.1.3 Costs and risks

The costs of applying the guidelines' recommendations are considerable as seasonal adjustment is time consuming in terms of human resources and requires a common and well defined IT structure.

The risks of not applying the guidelines' recommendations are that inappropriate or low-quality seasonal adjustment can generate misleading results, for example over-smoothing or residual seasonality, increasing the probability of false signals leading to misinterpretation of seasonally adjusted data. This will reduce credibility and hence, ultimately, trust in statistics.

### 30.1.4 Background to guidelines and basic definitions

Seasonal adjustment is a fundamental process in the interpretation of time series to inform policy making.

Seasonal fluctuations and calendar effects can mask short and long-term movements in a time series and impede a clear understanding of underlying phenomena. Seasonal adjustment filters out usual seasonal fluctuations and typical calendar effects from a time series.

- Usual seasonal fluctuations mean those movements which recur with similar intensity in the same season each year and which, on the basis of the past movements of the time series in question and under normal circumstances, can be expected to recur.
- Calendar effects arise from annual differences in the number of working or trading days in a month or a quarter, or the dates or days of public holidays.

Movements due to exceptionally strong or weak seasonal influences, for example extreme weather conditions or atypical holiday patterns, will continue to be visible in the seasonally adjusted series. Other random disruptions and unusual movements with real-world interpretations, for example strikes or large orders, will also continue to be visible.

Hence, the seasonally adjusted results do not show "normal" and repeating events, but do show the "news" in the time series, for example turning points in the trend, the business cycle or the irregular component.

The downside of seasonal adjustment is that seasonality cannot be precisely defined and different approaches — such as the signal extraction approach (Burman, 1980; Gomez and Maravall, 1996) and the semi-parametric approach based on a set of predefined moving averages (Shiskin et al, 1967; Findley et al, 1998) — may result in different outcomes. The expertise of an analyst will also impact on the quality of seasonal adjustment, although the primary drivers are the quality of the unadjusted time series and the production timetable: in a mass production environment, when thousands of time series are seasonally adjusted in a short period of time, often only alternative (B) can be achieved, although alternative (A) may still be possible for some. These guidelines are designed to guide producers through this process, to achieve a more comparable end result.

## 30.2 Annex: Principles for seasonal adjustment

1. The objectives of seasonal adjustment are to identify and remove seasonal fluctuations and calendar effects which can mask short and long-term movements in a time series and impede a clear understanding of underlying phenomena. Seasonal adjustment is therefore a fundamental process in the interpretation of time series to inform policy making.
2. As seasonal adjustment is performed both at European and Member States levels in several domains, it is important to ensure consistency between different seasonal adjustment policies. A general ESS set of principles must be defined and published.
3. Seasonal adjustment policies compliant with the principles must be defined at Member States and domain levels, taking care of inter-domain constraints, and published. These policies must be as stable over time as possible.
4. To avoid misleading results, seasonal adjustment should be applied only when seasonal and/or calendar effects can be properly explained, identified and estimated. Where none of these effects can be identified and estimated, unadjusted and calendar/seasonally adjusted series are identical.
5. The use of regARIMA models is recommended to estimate and remove outliers before estimating the seasonal effect.
6. It is also recommended to use regARIMA modelling to calculate calendar adjustment factors. These calendar adjustment factors should take into account the different characteristics of national calendars.
7. Seasonally adjusted series should have neither residual seasonality nor residual calendar effects and should show both the full trend-cycle and irregular component.
8. The quality of seasonally adjusted data must be regularly checked. The results of this monitoring should be made available to the public.
9. A stable and publicly available revision policy for seasonally adjusted data must be defined and followed.
10. Seasonally adjusted data should be published with unadjusted data according to an announced release calendar.
11. The recommended seasonal adjustment methods are parametric methods based on signal extraction like Seats (Gomez and Maravall (1996)) and semi-parametric methods based on a set of predefined moving averages like Census II X 11 family (Findley et al. (1998)) and X-13ARIMA-SEATS.

## 30.3 A policy for seasonal adjustment

### 30.3.1 A general seasonal adjustment policy

#### Description

Seasonal adjustment is performed both at European and Member State level in several domains. It is important to define a general seasonal adjustment policy based on a set of principles.

A general policy for seasonal adjustment describes which issues (not depending on data characteristics) should be decided in a consistent way when performing seasonal adjustment in different domains and/or institutions. It should include at least the need for consistency among different seasonal adjustment policies, when and on the basis of which methods to perform seasonal adjustment, the need for assessment of the seasonally adjusted data quality, the existence of a stable and publicly available revision policy for seasonally adjusted data, the need for dissemination of metadata on the seasonal adjustment process in a ESS standardised format.

#### Options

- A general seasonal adjustment policy is adopted in line with the principles for seasonal adjustment, including at least when and on the base of which methods to perform seasonal adjustment, the need for assessment of the seasonally adjusted data quality, the existence of a stable and publicly available revision policy for seasonally adjusted data, and the need for dissemination of metadata on the seasonal adjustment process.
- A general seasonal adjustment policy in line with the principles for seasonal adjustment is adopted; however exceptions are performed based on justified needs. Exceptions are documented and efforts are undertaken in order to reduce their impact and their occurrence. The trade-off between harmonisation and particular needs is carefully considered, guaranteeing the maximum possible degree of harmonisation.
- Lack of a general seasonal adjustment policy.

#### Alternatives (\*)

- Adopt a general seasonal adjustment policy fully compliant with the principles for seasonal adjustment, specifying at least when and on the base of which methods to perform seasonal adjustment, the need for assessment of the seasonally adjusted data quality, the existence of a stable and publicly available revision policy for seasonally adjusted data, the need for dissemination of metadata on the seasonal adjustment process.
- Adopt a seasonally adjustment policy only partially compliant with the principles for seasonal adjustment; exceptions to the general principles are limited to when they are considered unavoidable; those exceptions are documented and efforts are undertaken in order to reduce their impact and their occurrence.
- No seasonal adjustment policy is adopted, or adoption of a seasonal adjustment policy not compliant with the seasonal adjustment principles.

(\*) A) Best alternative; B) Acceptable; C) To be avoided

### 30.3.2 The need for domain specific seasonal adjustment policies

#### Description

Statistical producers and users are confronted with very varied production processes and data sources; even in the presence of related concepts, the production processes can differ to the degree of requiring some effort to get a comparable and consistent national or European system of statistics. Each statistical domain can be characterised by specific data/survey characteristics as well as by constraints derived from existing legal acts.

Statistical institutions should adopt harmonised domain specific seasonal adjustment policies. A domain specific seasonal adjustment policy will address the issues related to data characteristics or to the specific data production process of the domain that need to be harmonised in order to guarantee data comparability at ESS level. For example, in National Accounts a domain specific seasonal adjustment policy should define the strategy for dealing with the seasonal adjustment of volumes data expressed in chain-linked form.

Domain specific seasonal adjustment policies must be compliant with the general seasonal adjustment policy and harmonised at ESS level.

### Options

- Adopt domain specific seasonal adjustment policies compliant with the general seasonal adjustment policy and harmonised at ESS level.
- Adopt domain specific seasonal adjustment policies taking into account the general seasonal adjustment policy, but with some well justified exceptions; constant effort to reduce discrepancies and to move towards harmonisation of policies is performed and monitored.
- Adopt domain specific seasonal adjustment policies not harmonised at ESS level.
- Do not adopt any domain specific seasonal adjustment policy.

### Alternatives (\*)

- A) Adopt domain specific seasonal adjustment policies fully compliant with the general seasonal adjustment one and harmonised at ESS level.
- B) Adopt domain specific seasonal adjustment policies only partially compliant with the general seasonal adjustment policy or not harmonised at ESS level. Reasons for the lack of compliance and/or harmonisation should be clearly justified.
- C) Lack of domain specific seasonal adjustment policies, or domain specific seasonal adjustment policies not compliant with the general seasonal adjustment policy, or not harmonised at ESS level.

(\*) A) Best alternative; B) Acceptable; C) To be avoided

## 30.3.3 Consistency of general and domain specific policies

### Description

When looking at consistency several aspects have to be considered; consistency of domain specific seasonal adjustment policies across institutions and with the general policy has been considered in the previous item; however, we have also to consider the cross domain consistency. When looking at domain level, it could be relevant for the user to compare different statistics stemming from related domains, for example for base statistics and derived ones, in order to assess the degree of comparability between them; this aspect is related to the quality assessment of a system of statistics. It is then necessary that statistical institutions adopt domain specific seasonal adjustment policies consistent across domains, taking into account the implications that domain specific seasonal adjustment policies have on other domains at national and European level.

### Options

- Adopt domain specific seasonal adjustment policies consistent across domains in order to ensure the comparability of final results with data stemming from related domains and the harmonisation of practices at national and ESS level.



- Adopt domain specific seasonal adjustment taking account of consistency across domains, but with some well justified exceptions; constant effort to reduce discrepancies and to move towards harmonisation of practices with other domains is performed and monitored.
- Adopt domain specific seasonal adjustment policies not coordinated with other domains.
- Adopt domain specific seasonal adjustment only partially consistent across domains.

#### Alternatives (\*)

- A) Adopt domain specific seasonal adjustment policies consistent across domains at national and ESS level.
- B) Adopt domain specific seasonal adjustment policies, only partially consistent across domains. Reasons for the lack of full consistency should be clearly justified; constant effort to reduce discrepancies and to move towards harmonisation of practices with other domains is performed and monitored.
- C) Adoption of domain specific seasonal adjustment policies not consistent across domains.

(\*) A) Best alternative; B) Acceptable; C) To be avoided

### 30.3.4 Stability of seasonal adjustment policies

#### Description

Maintaining the stability of the general and domain specific seasonal adjustment policies over time is important to foster user confidence and ensure transparency of the seasonal adjustment process. The general policies for seasonal adjustment should rarely be revised; when this happens, domain specific seasonal adjustment policies should be reviewed accordingly.

At domain level and when looking at the data production side, the stability of a domain specific seasonal adjustment policy is an essential element of a well-established production process which will allow for better planning of activities and resources. When looking at the user side, the stability of a domain specific seasonal adjustment policy ensures that users generally know in advance when, which and why seasonally adjusted data will be revised. However, it could be necessary to change domain specific seasonal adjustment policies in order to keep them in line with relevant improvements in the production process. This could be the case when changes could enhance accuracy and/or reduce the statistical burden or be necessary to fulfil national laws.

When a seasonal adjustment policy needs to be changed, it is better to adopt the new policy in correspondence with major seasonal adjustment revision. Changes in seasonal adjustment policies should be communicated in advance, well documented and justified and should be, as far as possible, coordinated at ESS level.

#### Options

- The general seasonal adjustment policy and the domain specific seasonal adjustment ones are stable over time. If changes occur, they are announced in advance and coordinated as far as possible at ESS level.
- Domain specific seasonal adjustment policies are validated from year to year and eventually revised.
- Changes in the general seasonal adjustment policy or in the domain specific seasonal adjustment ones happen often and/or irregularly in time.

#### Alternatives (\*)

- A) The general seasonal adjustment policy and the domain specific ones are stable over time; when changes are required (new legal acts, new definitions, new methods of estimation, etc.), they should be coordinated as far as possible at ESS level and announced in advance. Important changes of domain specific seasonal adjustment policies at the member state level that are necessary to foster accuracy, to reduce the reporting burden or to fulfil national laws should be preannounced too. Those cases should be combined as far as possible.

- B) The general seasonal adjustment policy is stable over time at ESS level. Domain specific seasonal adjustment policies are validated annually, eventually revised and coordinated as far as possible at ESS level.
- C) Lack of coordination/stability of general seasonal adjustment policy and/or domain specific seasonal adjustment ones.

(\* A) Best alternative; B) Acceptable; C) To be avoided

### 30.3.5 Dissemination of general and domain specific seasonal adjustment policies

#### Description

Seasonal adjustment is a very relevant process for the user; even if some aspects of seasonal adjustment may appear quite technical, it is important to inform the user about the adopted approach at least for the seasonal adjustment of main indicators. What can appear as a small change, e.g. the introduction of a new public holiday in the pre-treatment process, may have significant impact on results and on data treatment at user level. It is important to publish and keep updated the general and domain specific seasonal adjustment policies; they should be standardised at ESS level and made easily available, e.g. publishing them on the statistical agencies' websites.

#### Options

- The general seasonal adjustment policy and the domain specific seasonal adjustment ones are publicly available at least for main indicators. Users are promptly informed about any relevant change in the policies (e.g. new quality assessment criteria, new model revision criteria, etc.); documentation on the seasonal adjustment process is published in an ESS standard format, publicly available and kept up to date, focusing on the transparency of the process.
- The general seasonal adjustment policy and the domain specific seasonal adjustment ones are available in a non-standard format and/or only on request, even for main indicators; some information on the seasonal adjustment process is published but only on very general aspects.
- Information on the seasonal adjustment of main indicators is not disseminated, even on request.

#### Alternatives (\*)

- A) The general seasonal adjustment policy and the domain specific ones are publicly available in a ESS standardised format, promptly informing the user of any change.
- B) The general seasonal adjustment policy and the domain specific ones are publicly available; efforts are performed to keep the information up to date in a reasonable delay.
- C) The general principles for seasonal adjustment and the domain specific seasonal adjustment policies are available only on request or not available at all; changes are eventually communicated with long delay.

(\* A) Best alternative; B) Acceptable; C) To be avoided

### 30.3.6 Quality framework for seasonal adjustment

#### Description

Quality measurement of seasonal adjustment needs to consider all five ESS dimensions of statistical output quality, as listed in the European Statistics Code of Practice:

- relevance

- accuracy and reliability
- timeliness and punctuality
- coherence and compatibility
- accessibility and clarity

Measures can be qualitative or quantitative — qualitative measures will normally be "Yes" or "No", and quantitative measures will normally be test statistics with the direct interpretation of "Pass" or "Fail".

Relevance is measurable qualitatively through consultation with users, for example the perception of quality depends on users' satisfaction that the outputs meet their needs. This quality dimension is considered in section 30.7.

Accuracy and reliability are measurable quantitatively through statistical tests to assess whether the seasonally adjusted time series display suitable characteristics; the measures should not be limited by software choice. For example, if they are not embedded in the seasonal adjustment software, they should be defined elsewhere. These quality dimensions are considered in section 30.7. Timeliness and punctuality are measurable quantitatively relative to publication timetables. For example, in the case of mass production of seasonally adjusted time series, run-times of processes may limit the number of quality measures that can be validated due to the sheer number of series involved. These quality dimensions are considered in section 30.9.2.

Coherence and comparability are measurable quantitatively through statistical tests: using measures within software packages to assess coherence over time/domain/across European member states; and using common measures to assess comparability over methods. These quality dimensions are considered in section 30.7.

Accessibility and clarity are measurable both quantitatively through enumeration of outputs and qualitatively through consultation with users and producers: accessibility in terms of what statistics are available; clarity in terms of user satisfaction with the interpretability of the final seasonally adjusted series, and producer satisfaction with the quality of the seasonal adjustment settings in production systems, for example in the case of concurrent adjustment where settings are reviewed annually by independent experts. These quality dimensions are considered in section 30.9.1.

### Options

- Measure quality comprehensively for the qualitative/quantitative ESS dimensions of statistical output quality.
- Measure quality partially for the qualitative/quantitative ESS dimensions of statistical output quality.
- Measure quality for none of the qualitative/quantitative ESS dimensions of statistical output quality.

### Alternatives (\*)

- Measure quality comprehensively for all the ESS dimensions of statistical output quality.
- Measure quality comprehensively for the quantitative ESS dimensions of statistical output quality, and partially for the qualitative dimensions.
- Measure quality partially (or not at all) for the quantitative ESS dimensions of statistical output quality, and/or not at all for the qualitative dimensions.

(\*) A) Best alternative; B) Acceptable; C) To be avoided

## 30.4 Pre-treatment

### 30.4.1 General aspects for choosing between detailed and automatic pre-treatment

#### Description

Most seasonal adjustment filters: ARIMA models, moving averages, regression analysis, state-space models, etc. These linear tools are optimal under certain assumptions but also have some weaknesses:

- They are not robust, i.e. they are sensitive to the presence of atypical values (outliers).
- They are sensitive to any misspecification of the underlying model.

The main objective of pre-treatment of the series is to ensure a reliable estimation of the seasonal and calendar component. This is done in particular by detecting and correcting the series for data and/or components, sometimes called "non-linearities", which could hamper the estimation of the seasonality and the calendar effects.

Outliers are a clear example of data that could greatly affect the quality of the seasonal estimate.

Various kinds of outliers (i.e. additive outliers, transitory changes, level shifts etc.) should be detected and corrected for. RegARIMA models have proved a successful method of doing this.

Economic time series are usually recorded each month (or each quarter) but months (or quarters) are not equivalent. In particular, they have neither the same length nor the same composition in number of days. These details, strictly linked to the calendar, may affect the unadjusted data. For example, one more Saturday in a month may explain an increase in the retail trade turnover. RegARIMA models can also be used to detect and correct the series for these calendar effects (different number and structure of working or trading days in different periods, moving national or religious holidays). It should be noted that a part of these calendar effect is seasonal (the length of most months repeats itself every year, the non-Orthodox Easter falls more often in April than in March, etc.) and that the calendar component should only concern the non-seasonal part of the effect, whereas the seasonal part of the calendar influences should be assigned to the seasonal component. It is also important to note that the analyst has very few doubts about the future of the calendar which is periodical with a period of 400 years. (This does not apply to the date of Easter, but even this can be calculated with certainty in advance.) Therefore, an official calendar adjustment should include good estimates of the calendar effects, as these also will improve forecasts of the unadjusted data and lead to more stable estimates of the seasonal component.

Most of the statistical tools used in seasonal adjustment procedures rely, at least in one step of the adjustment, on the stationarity of the series. The stationarity in mean can usually be achieved by appropriate differencing. The stationarity in variance may require a further transformation of the series, specifically testing for log-transformation to guide the choice of the decomposition scheme (see item 30.4.10).

#### Options

- Running a detailed pre-treatment based on RegARIMA models using statistical criteria complemented by the use of economic and calendar information.
- Running an automatic pre-treatment based on selected statistical strategies only (unit root testing, information criteria, tests on statistical significance as mentioned in subsequent items, tests on the white-noise-property of the model residuals, using calendar regressors and outlier variables, etc.).
- Ignoring economic and statistical information (i.e. no pre-treatment of the series).

#### Alternatives (\*)

- A) A detailed non automatic pre-treatment at least once a year for the most important macroeconomic indicators based on RegARIMA models and the best alternatives mentioned in the items of this section.
- B) Automatic pre-treatment.

C) No pre-treatment.

(\* ) A) Best alternative; B) Acceptable; C) To be avoided

## 30.4.2 Graphical analysis of the series

### Description

A first graphical analysis of the series provides the analyst with some useful information on how to perform the seasonal adjustment and choose the parameters, and reveals possible problems in the data.

This analysis should be carried out on unadjusted data and the initial run of the seasonal adjustment software.

The analyst should also consider information on:

- The length of the series and model span;
- The presence of zeros or outliers or problems in the data;
- The structure of the series: presence of a trend-cycle, of a seasonal component, volatility etc.;
- The presence of possible breaks in the seasonal behaviour;
- The decomposition scheme (additive, multiplicative).

More sophisticated graphs, such as the spectrum or the autocorrelograms, could provide information on the presence of a seasonal component and/or a calendar effect. Based on RegARIMA residuals, these graphs are also tools for checking that the seasonal and calendar effects are taken into account in the model, i.e. they usually disappear after modelling. Additionally, histograms of the residuals can be checked.

### Options

- Not considering graphical evidence.
- Use of basic graphs in the time domain (i.e. unadjusted time series, log-transformed time series, outlier adjusted time series, year-on-year rates of change for the unadjusted and calendar adjusted data).
- Use of additional graphs (including the spectrum, the autocorrelograms and histograms) before and after a suitable transformation of the series and for the RegARIMA residuals.

### Alternatives (\*)

A) A detailed graphical analysis for the unadjusted data and the RegARIMA residuals, based on basic graphs, autocorrelograms, spectra and histograms, is performed for the most important series to be adjusted at least once a year and the related outcomes should be documented.

B) A first graphical analysis in the time domain, performed on most important series and, whenever possible, on all of them and the related outcomes should be documented.

C) No graphical analysis.

(\* ) A) Best alternative; B) Acceptable; C) To be avoided

## 30.4.3 Calendar adjustment

## Description

The aim of calendar adjustment is to obtain a seasonally adjusted series whose values are independent of the length and the composition in days (number of Mondays, Tuesdays, etc. or number of working days and weekend days) of the month/quarter.

It should be noted that the length and day-of-week composition of the month/quarter is partly seasonal: March has always 31 days and has, on average, more Mondays than February. Since the seasonal part is already captured by the seasonal adjustment filters, it should not be removed during calendar adjustment.

Working- or trading-day effects - in the narrow sense - should therefore be associated with the nonseasonal part of the effect. This partial effect can be estimated by centring the calendar regressor, i.e. removing its long-term monthly/quarterly average.

Additionally, the number of working days and national holidays are complementary. Given the length of month, a higher number of working days always implies a lower number of non-working days (Sundays, national holidays which are not Sundays, and in most fields of economic activity, Saturdays). Therefore, a normal working day adjustment implicitly adjusts for moving national holiday effects. However, non- Orthodox and Orthodox Easter, for example, may have differing effects on neighbouring months or quarters. This can cause problems for the interpretation of data in the respective periods. Hence, such effects are part of a separate calendar adjustment.

The calendar adjustment should not result in frequent large revisions when additional data become available, if it does, it is an indication that the method's estimates are not reliable.

## Options

- Proportional working day adjustment - in this case, the effects of working days are estimated by counting the proportion of them in the month/quarter.
- Regression-based adjustment - in this case, the effect of the calendar is estimated in a regression framework. Within the regression approach, the effect can be estimated by using a correction for the length of the month or leap year, regressing the series on the number of working days, etc.
- RegARIMA-based adjustment, same as before but with an ARIMA structure for the residuals.
- No adjustment.

## Alternatives (\*)

- A) RegARIMA approach, with all pre-tests for number of regressors, length and composition of month, national and religious holiday effects, check of plausibility of effects (sign and size of estimated coefficients), etc. The calendar adjustment should be done for those time series for which there is an economic rationale for the existence of calendar effects and statistical evidence.
- B) Regression approach for all calendar effects based on the (provisional) irregular component (e.g. X11Regression included in X-12-ARIMA). The calendar adjustment should be done for those time series for which there is statistical evidence and an economic explanation for the existence of calendar effects.
- C) Proportional adjustment, other adjustment or no adjustment at all (when this leaves evidence of calendars effects in the adjusted series).

(\*) A) Best alternative; B) Acceptable; C) To be avoided

## 30.4.4 National and EU/euro area calendars

### Description

In order to take into account the national and EU/euro area idiosyncrasies, different calendars are needed. They are used to calculate calendar regressors for calendar adjustment.

An EU/euro area calendar, built from national calendars, i.e. by averaging the national numbers of working or trading days using appropriate weights, can be considered an alternative in cases of direct seasonal adjustment of unadjusted EU/euro area aggregates. It is not easy to create and maintain national and European calendars and their effectiveness is strongly dependent on their regular and accurate maintenance.

Member States should compile, maintain and update their national calendars or, as a minimal alternative, supply a historical list of public holidays including, whenever possible, information on compensation holidays. Moreover they should provide, in advance, the calendar for the year  $t+1$  or the corresponding holidays list.

### Options

- Use of default calendars.
- Use of national calendars or the EU/euro area calendar as appropriate.
- Identification of series not requiring calendar adjustment.

### Alternatives (\*)

- The use of national calendars is recommended at the Member State level or for European aggregates when an indirect approach is chosen. The use of EU/euro area calendars is recommended when a direct approach is chosen for the seasonal adjustment of European aggregates in particular if national calendar adjusted series are not available, incomplete or of insufficient statistical quality. The calendar information used should be available to the public (at least upon request).
- Use of default calendars (defined within the tool chosen for seasonal adjustment) complemented by an historical list of national public holidays to be corrected for (through the use of appropriate regressors).
- Use of default calendars, without any reference to national and European public holidays, as well as no calendar adjustment irrespective of diagnostic evidence of calendar effects.

(\*) A) Best alternative; B) Acceptable; C) To be avoided

## 30.4.5 Choosing the frequency of time series for calendar adjustment

### Description

The calendar effects of a quarterly time series that is calculated from monthly data (sum or average) can be estimated in two different ways: either directly by estimating the calendar effects using the quarterly figures, or indirectly by estimating the monthly calendar adjusted results and transforming them to the quarterly frequency (sum or average).

Of course, the effects of individual days can be quantified exactly if and only if all statistics are reported daily. In monthly or quarterly time series calendar day influences can only be estimated. In a monthly time series, for example, the effect of 29 February is mixed with all influences from 1 to 28 February. In a quarterly time series, the effect of 29 February is also combined with the influences of January and March. Additionally, the effects of Easter in a monthly time series can be estimated using the figures for March and April only. Estimates based on quarterly data, however, are also influenced from the figures of January, February, May and June which are included in the first and second quarter, respectively. Therefore, the precision of a calendar effect estimate is normally lower at quarterly frequency and increases with the number of observations within a year.

Calendar effects do not exactly balance out each other in each calendar year, because of the leap year and of different numbers of working days. Therefore, calendar adjusted annual data can be consistently calculated from the corresponding monthly or quarterly results. A direct estimate of calendar adjusted annual results, however, is not possible in practice because estimation techniques cannot separate the calendar effects from business cycle influences in this case.

## Options

- Calculate calendar factors separately for each frequency.
- Use the highest frequency available (monthly, quarterly) in order to estimate calendar effects and derive lower frequency calendar adjusted results (quarterly, annual).
- No calendar adjustment for data at a quarterly or annual frequency.

## Alternatives (\*)

- A) Use the highest frequency available for estimating calendar effects and derive lower frequency calendar adjusted results.
- B) Estimate quarterly calendar adjusted figures directly and derive annual calendar adjusted figures indirectly.
- C) Do not calendar adjust quarterly data, irrespective of whether such effects exist. Adjust annual figures directly.

(\*) A) Best alternative; B) Acceptable; C) To be avoided

## 30.4.6 Other calendar related and weather effects

### Description

The effects of bridging day, school holiday and weather effects that are not part of the seasonal or the calendar component can also be estimated with RegARIMA models and appropriate regressors.

Bridging days are days lying between a public holiday and a weekend. They are counted in purely calendar terms as full working days, but the fact that they lie between public holiday and a weekend means that such days can be used to "work off" overtime that has been accumulated or for taking a long weekend. In this sense, they would be expected to have an influence on the time series, e.g. a negative impact on the industrial production. Empirical investigations show clear evidence of these effects. However, the same investigations indicate an over-adjustment in months which have two single bridging days. Additionally, the use made of single bridging days might depend on the prevailing economic situation. In times when the economy is weak, bridging days could be used, in particular, to stop production temporarily, whilst, in times of considerable growth, there would be a tendency to continue working. If the effect of the concentration of leave on single bridging days is eliminated, the countermovement of less leave should actually also be adjusted over the rest of the year so that no distortion of the business cycle occurs. However, the estimation of this countermovement is often not possible in practice.

The basic idea of the vacation adjustment is that the economic activity in a month/quarter is likely to depend on the timing of the school holidays. Workers with school-age children take leave mainly during the school holidays, and hence interrupt their work. Empirical investigations show clear evidence of these effects. However, monthly-specific estimates of the influences of the school holidays are based in each case on only a very limited number of observations. A small number of values can hence exert a relatively major influence on the result. What is more, the addition of new values may lead to significant changes, and hence revisions. Since more holidays in a month always correspond to fewer holidays in other months, one might accept that the estimated positive and negative holiday effects roughly balance each other out throughout the year. In empirical examples, however, this is not the case.

Similar to school holidays, weather-induced effects do not occur repeatedly with exactly the same intensity in the same month each year. Rather, the impairment of construction activity in the cold season depends on the intensity and, above all, on the length of the extreme weather periods. In this sense, one may attempt to model the weather dependency of, for instance, construction output using suitable regressors in order to make it easier to draw conclusions as to economic developments. However, exceptionally severe weather-related production impairments in the cold season frequently lead to positive catch-up effects in the spring. If the winter shortfall was adjusted, the indirect knock-on effect would also have to be removed from the spring calculation in order not to unilaterally distort the business cycle picture.



### Options

- Not adjusting for bridging day, school holiday and weather-induced effects in order to avoid the problems described and the high costs of constructing the variables.
- Adjusting for as many effects as possible in order to smooth the time series.
- Estimating as many effects as possible and deleting them from the unadjusted data in order to ensure seasonal and calendar estimates are not influenced by these effects. Then, results are produced which are only adjusted with these normal seasonal and calendar factors. These seasonally and calendar adjusted figures show the full effects estimated in the first step.

### Alternatives (\*)

- A) Only calendar and seasonal effects are adjusted (no additional bridging day, school holiday and weather-induced effects). Studies on the latter effects, however, are done in order to inform data users.
- B) Estimate these effects and delete them from the unadjusted data in order to better estimate seasonal and calendar factors which are not influenced by these effects (Then, results are produced which are adjusted only with these normal seasonal and calendar factors.).
- C) Adjust for as many effects as possible.

(\*) A) Best alternative; B) Acceptable; C) To be avoided

### 30.4.7 General principles of outlier detection and correction

#### Description

Outliers are abnormal values of the series. They can be modelled in a number of ways, the most important being:

- additive outliers (abnormal values in isolated points of the series);
- temporary changes (series of outliers with temporarily decreasing effects on the level of the series);
- level shifts (series of innovation outliers with a constant long-term effect on the level of the series, where for innovation outlier is meant anomalous values in the innovation series);
- ramps (which describe a smooth, linear or quadratic transition between two time points unlike the abrupt change associated with level shifts);
- temporary level shifts (where the level shift has short-term rather than a long-term effect).

Seasonal adjustment methods are likely to be severely affected by the presence of such outliers. Therefore they should be detected and replaced simultaneously or before estimating the seasonal and calendar components in order to avoid a distorted or biased estimation of them. However, outliers should remain visible in the seasonally adjusted data (unless they can be associated with data errors) because they give information about some specific events (e.g. strikes). Therefore, the outliers should be reintroduced in the time series after having estimated the calendar and/or seasonal component (which is the normal procedure in commonly used methods): additive outliers and temporary changes are assigned to the irregular component, level shifts, ramps, and temporary level shifts are part of the trend-cycle. This means that outliers due to data errors in the unadjusted data have to be corrected before starting the seasonal adjustment procedure.

Seasonal breaks (sometimes called seasonal outliers or change of regimes) are a special case. They describe an abrupt increase or decrease of the seasonal component for a specific month or quarter and are of permanent nature. Therefore, seasonal breaks belong to the seasonal component and are removed from the unadjusted data in the normal process of seasonal adjustment. RegARIMA models provide a possibility for modelling outliers identified by the user and an automatic procedure to detect outliers and to correct for their effects. A relatively large number of identified outliers as compared to the time series' length might indicate inappropriateness of the ARIMA model chosen. If problems occur, it might help to shorten the time span for outlier detection or to change the critical value of the statistical tests which are used for identifying outliers.

#### Options

- Selecting the types of outliers to be considered for pre-testing.
- Removal of outliers before seasonal adjustment is carried out.
- Including the most important outliers in the regression model as intervention variables.
- No outlier identification.

#### Alternatives (\*)

- A) The series should be checked for outliers of different types (see description). Once identified, outliers caused by data errors should be corrected in the unadjusted (raw) data before pre-treatment. Remaining outliers should be explained/modelled using all available information. Outliers for which a clear interpretation exists (e.g. strikes, consequences of changes in government policy, territory changes affecting countries or economic areas, etc.) are included as regressors in the model, even if their effects are somewhat below the general significance threshold.
- B) As A), but with a completely automatic procedure for detecting and correcting outliers.
- C) No preliminary treatment of outliers.

(\*) A) Best alternative; B) Acceptable; C) To be avoided

### 30.4.8 Treatment of outliers at the end of the series and at the beginning of a major economic change

#### Description

Outliers at the ends of the series present unique problems:

- 
- a level shift at the first data point cannot be estimated since the level of the series prior to the given data is unknown;
- a level shift at the last data point cannot be distinguished from an additive outlier there;
- a level shift at the second data point cannot be distinguished from an additive outlier at the first data point;
- a temporary change at the last data point cannot be distinguished from an additive outlier there etc.

These conceptual limitations create problems concerning the estimation of the trend-cycle and/or the irregular component because a level shift at the end of a series can be wrongly treated as an additive outlier and, hence, wrongly assigned to the irregular component and not the trend-cycle. However, these problems do not affect the seasonally adjusted results because the latter contain both the trend-cycle and the irregular component. Which outlier at the end of a time series is assigned to the trend-cycle and which outlier to the irregular component is not relevant for estimating the seasonal and calendar component and therefore not relevant for calculating seasonally adjusted data. All that matters is that atypical values are treated as outliers.

Major economic changes, errors or problems in the reported data or the statistical compilation process firstly appear as an additive outlier at the end of the series. Additional observations are needed before changing the outlier type from an additive outlier to a transitory change or a level shift. However, changing the outlier type can have an impact on the series revisions and the choice of the type of outlier can influence turning point identification. Caution is necessary in these cases.

A seasonal outlier is an exception. This kind of outlier is assigned to the seasonal component and cannot be distinguished from an additive outlier at the end of a time series using statistical tests. Fortunately, seasonal outliers are extremely rare because most of the reasons for seasonality (length of a month, Christmas etc.) are stable over time. Therefore, there should be clear indications of the reasons for seasonality change at the end of a series before modelling seasonal outliers.

Based on the assumption that the abrupt extraordinary effects of financial and/or economic crises do not happen year after year with roughly the same intensity they should not influence the seasonal estimate. Hence, using appropriate outlier variables is important. This approach ensures that the full effects of the crises are visible in the seasonally adjusted results.

#### Options

- Never modelling outliers at the end/beginning of a series because the type of outlier cannot be identified automatically.
- Fully trusting automatic outlier detection and replacement procedures.
- Modelling outliers at the end/beginning of the time series in accordance with statistical criteria (ttest) and economic information, especially strong economic changes are modelled.

#### Alternatives (\*)

- A) Outliers are modelled at the end of a time series based on statistical criteria and economic information, especially in times of strong economic changes.
- B) Using fully automatic outlier detection procedures.
- C) Never model outliers at the beginning/end of a series.

(\* ) A) Best alternative; B) Acceptable; C) To be avoided

### 30.4.9 Model selection

#### Description

Model selection pertains to: criteria to select the appropriate model for pre-adjustment and seasonal adjustment or forecast extension for seasonal adjustment; log versus non-log specification of the model; order of differencing for the seasonal and non-seasonal part; use of additive or multiplicative components (see item 30.4.10); statistical checking of the adequacy of the estimated model; analysis of decomposition on the basis of the chosen model; etc.

There are various ways of selecting an appropriate model. Unit root tests and information criteria can be used, forecast properties can help to select a model from a list of models, and non-automatic procedures are useful (e.g. ACF, PACF of model residuals for different orders of integration). All these different possibilities aim at finding a parsimonious model which describes the relevant features of the data generating process that is assumed to underlie the time series in question.

This item is much more important for model-based methods than for non-parametric ones.

#### Options

- Automatic model selection.
- Model selection based on a model-set.
- Manual model selection.

#### Alternatives (\*)

- A) Selection of a model from a large number of models, after checking for model adequacy using standard statistical tests (e.g. normality, heteroskedasticity, serial correlation, etc.) and spectrum diagnostics for the model residuals. Using non automatic model selection for important or problematic series.
- B) As before, but with a completely automatic procedure.
- C) Selection based on restricted number of pre-defined models that have not been tested for adequacy with the set of series being adjusted.

(\* ) A) Best alternative; B) Acceptable; C) To be avoided

### 30.4.10 Decomposition scheme

#### Description

The decomposition scheme specifies how the various components - basically trend-cycle, seasonal, calendar component and irregular — combine to form the original series. Usually, the decomposition scheme is multiplicative (either pure multiplicative or log-additive), because in most economic time series, the magnitudes of the seasonal component appear to vary proportionally to the level of the series. If these two components are independent from each other, the additive scheme is used. The pseudoadditive approach is preferable for time series that generally show a multiplicative behaviour, where, however, at least one period always goes down close to zero.

The algorithms underlying model based and moving averages based methods provide the user with an automatic test for log-transformation. The result of this test will also suggest the choice of the decomposition scheme.

For series with zero or negative values the additive decomposition is automatically selected by seasonal adjustment procedures, regardless of the real decomposition scheme.

The choice of the decomposition scheme and the choice of the differencing orders aim to achieve weak stationarity. These two decisions have the greatest impact on forecasts and on model-based seasonal adjustments and trend-cycle estimates at the end of series.

### Options

- Automatic decomposition scheme selection.
- Manual decomposition scheme selection after graphical inspection of the series.
- For series with zero or negative values, adding a constant to make the series positive and select the appropriate decomposition scheme.
- For stationary series (with no trend in mean and in variance) the additive decomposition is used.

### Alternatives (\*)

- A) Automatic decomposition scheme selection using appropriate criteria (e.g. information criteria) after graphical inspection of the series. Special investigations for series with zeros or negative values (i.e. adding a constant before testing for the decomposition scheme and checking the impact on the seasonally adjusted series). Use of non-automatic selection for more problematic series.
- B) Fully automatic decomposition scheme selection using information criteria.
- C) Use of fixed decomposition scheme (e.g. multiplicative for positive series, additive for series with zeros or negative values).

(\*) A) Best alternative; B) Acceptable; C) To be avoided

## 30.5 Seasonal adjustment

### 30.5.1 Choice of seasonal adjustment method

#### Description

Currently the most commonly used seasonal adjustment methods are the signal extraction approach (Burman, 1980; Gomez and Maravall, 1996) that starts from an ARIMA modelling of the complete series, and the semi-parametric approach based on a set of predefined moving averages (Shiskin et al, 1967; Findley et al, 1998).

Unobserved component methods (Harvey 1980) based on state space models represent a reasonable alternative, provided they allow for a complete calendar and outlier treatment and include an adequate set of diagnostics.

#### Options

- The semi-parametric method based on a predefined set of symmetric moving averages.
- The signal extraction method based on an ARIMA modelling of the series.
- Unobserved component methods based on state space models.
- Regression methods.
- Spectral methods.

#### Alternatives (\*)

- A) The signal extraction method based on an ARIMA modelling of the series and/or the semi-parametric method based on a predefined set of symmetric moving averages should be used for seasonal adjustment. The choice between the methods should take into account statistical investigations and past practices.
- B) Use of unobserved component methods based on state space models, provided they allow for a complete calendar and outlier treatment and include an adequate set of diagnostics.
- C) Use of other methods.

(\*) A) Best alternative; B) Acceptable; C) To be avoided

### 30.5.2 Choice of the software

#### Description

There are many software packages implementing the recommended seasonal adjustment methods. To choose from them, the user should take into account several aspects: versioning, maintenance and support, compatibility with these guidelines, documentation, costs, open-source architecture, completeness, user-friendliness, suitability for mass production, computational efficiency, etc. Software should be updated according to a well-defined release strategy. Using the same software release across domains and countries is beneficial for coherence and transparency. Methods and tool versions currently used in data production should be clearly communicated to users.

Software packages officially released by statistical institutions and designed to fully implement the recommended methods being in line with these guidelines should be favoured.

When migrating software, the impact on data should be assessed in the specific IT environment where it will be used.

### Options

- Using the official software implementing the recommended methods.
- Using software packages officially approved at ESS level and implementing recommended methods.
- Using old releases of these software.
- Using commercial software implementing the recommended methods.

### Alternatives (\*)

- A) Using freely available up-to-date software officially released by statistical institutions, preferably open-source, which fully contains the various recommended methods, follows a clear release strategy and has been thoroughly tested.
- B) Using complete and well tested implementations of recommended methods included in statistical commercial or free packages.
- C) Using incomplete or obsolete versions of official software or the use of commercial packages based on incomplete, obsolete or old versions of official software, or any other software implementing a non-recommended method.

(\*) A) Best alternative; B) Acceptable; C) To be avoided

## 30.5.3 Temporal consistency between unadjusted and seasonally adjusted data

### Description

It is unrealistic to assume that seasonal adjustment is neutral over the whole year (either calendar or financial), especially in presence of evolving seasonality, calendar effects and outliers. It is possible to force the sum (or average) of seasonally adjusted data over each year to equal the sum (or average) of the unadjusted data, but, from a theoretical point of view, there is no justification for this.

The disadvantages in forcing equality over the year between the seasonally adjusted data and the unadjusted data (sum or average) are:

- Bias in the seasonally adjusted data, especially where calendar and other non-linear effects are relevant;
- The final seasonally adjusted data are not optimal;
- Additional post-processing calculations are required.

The only benefit of this approach is that there is consistency over the year between adjusted and unadjusted data. This can be of particular interest when low-frequency (e.g. annual) benchmarking figures officially exist (e.g. National Accounts, Labour market) where users' needs for temporal consistency are stronger.

### Options

- Do not apply any constraint.
- Apply constraining techniques.
- Constrain equality over the year of seasonally adjusted data to unadjusted data (sum or average).
- Constrain equality over the year of seasonally adjusted data to calendar (only) adjusted data (e.g. sum or average).

### Alternatives (\*)

- A) In principle do not constrain the seasonally adjusted data to the unadjusted data or the calendar adjusted data over the year, unless strong users' requirements justify the benchmarking. In this case, in the presence of calendar effects, constrain the seasonally and calendar adjusted data to the calendar adjusted data over the year. Otherwise, constrain the seasonally adjusted data to the unadjusted data over the year. Recognised benchmarking methods preserving short-term movements should be used.
- B) Do not constrain the seasonally adjusted data to the unadjusted data or the calendar adjusted data over the year.
- C) Constrain data even in absence of users' requirements; use a benchmarking technique that generates seasonality or a benchmarking technique that do not preserve short-term movements.

(\*) A) Best alternative; B) Acceptable; C) To be avoided

## 30.5.4 Direct and indirect approaches

### Description

Economic indicators are often computed and reported according to a certain classification or breakdown: most short-term statistics are computed according to the NACE classification. Unemployment data are published by sex and age etc. In this case, the seasonally adjusted aggregates can be computed either by aggregating the seasonally adjusted components (indirect adjustment) or adjusting the aggregate and the components independently (direct adjustment). These two strategies result in different seasonally adjusted aggregates. The issue of direct versus indirect adjustment has a great relevance for users who consider the consistency between disaggregated and aggregated data to be important. Which of these approaches is preferred is still an open question since neither theoretical nor empirical evidence uniformly favours one approach over the other. In practice a mix of the two approaches may be used. As the quality of the adjustments cannot always be guaranteed at the lower level of the classification, a direct approach is used up to a certain level and the upper aggregated series are then computed indirectly. The choice of the cutoff level is usually linked more to user needs than to statistical considerations. For an informed choice between the direct and the indirect approach producers should consider:

- Descriptive statistics on the quality of the indirect and direct seasonally adjusted estimates, e.g. the smoothness of aggregates, presence of residual seasonality, stability of the model and measures of revisions;
- Characteristics of the seasonal pattern in the component time series and similarities/differences between them;
- User demand for consistent and coherent outputs, especially where they are additively related;
- The cut-off level.

### Options

- Direct approach where all series at the various aggregation levels are directly seasonally adjusted using the same method and software.
- Direct approach, as described above, with the distribution of discrepancies by means of multivariate benchmarking techniques (if discrepancies are small enough).
- Indirect approach where the seasonal adjustment of components occurs using the same approach and software, and then totals are derived by aggregation of the seasonally adjusted components.
- Direct approach applied to the disaggregated data until a certain level and the indirect approach applied to upper aggregated series.

### Alternatives (\*)

- A) Producers should carefully consider the application of either direct or indirect adjustment and make an informed choice based both on all mentioned statistical criteria to assess the quality of the adjustment and user demand.



The direct approach should be preferred for clarity, especially when component series show similar seasonal patterns. The production of consistent seasonally adjusted data and the use of coherent seasonal adjustment parameters should be monitored, especially if the direct approach is used. The indirect approach should be preferred where component series show significantly different seasonal patterns. The presence of residual seasonality and calendar effects should be monitored, especially in the indirectly adjusted series. If the quality of the adjustment cannot be guaranteed at the lower level of disaggregation and there is a need of ensuring the consistency between aggregates and components at macro-level, the direct adjustment can be used at lower disaggregation level and the indirect one at upper disaggregation level.

- B) The choice follows only user requirements for consistency between lower and higher level aggregates (e.g. additivity). The use of either the direct approach, associated with benchmarking techniques to remove discrepancies, or the indirect approach is acceptable. The presence of residual seasonality and calendar effects should be monitored, especially in the indirectly adjusted series.
- C) Choosing either direct or indirect approach without any justification.

(\* ) A) Best alternative; B) Acceptable; C) To be avoided

### 30.5.5 Direct versus indirect approach: dealing with data from different agencies

#### Description

Seasonal adjustment can be performed at different geographical aggregation levels (horizontal aggregation). This case is relevant for European aggregates, which are usually derived as an aggregation of corresponding national ones. The question of direct or indirect seasonal adjustment is even more relevant in the case of geographical aggregation for those users who consider the consistency between disaggregated and aggregated geographical data to be priority especially for their forecasting exercise of the geographical aggregate.

#### Options

- Seasonal adjustment can be performed either by national or European statistical institutions (e.g. NSIs and Eurostat) on geographical component series by using the same method and software, and then totals derived by their aggregation (decentralised or centralised indirect approach).
- All time series, including geographical aggregates, are seasonally adjusted on an individual basis.
- Same as before, but aggregation constraints imposed ex-post by means of multivariate benchmarking techniques.
- Each geographical component is seasonally adjusted, even by using disparate methods and software, and the seasonally adjusted geographical aggregates are derived from the seasonally adjusted components (mixed indirect approach).

#### Alternatives (\*)

- A) The direct approach is recommended when geographical component series show similar seasonal patterns or where there is a lack of harmonisation in the use of national practices. The production of consistent seasonally adjusted data and the use of coherent seasonal adjustment parameters should be monitored, especially if the direct approach is used. The centralised indirect approach is recommended for specific cases where it has been agreed that seasonal adjustment should be delegated to the centralised agency. The decentralised indirect approach can also be used in the presence of a satisfactory degree of harmonisation of national seasonal adjustment practices and where component series do not show similar seasonal patterns. In both centralised and decentralised indirect approaches, aggregates should be checked for the presence of residual seasonality.
- B) Under strong user's requirements for consistency between aggregates and geographical components, and in the presence of a satisfactory degree of harmonisation of national seasonal adjustment practices, the decentralised indirect approach can be also used even when national series show similar seasonal patterns. However, indirectly adjusted aggregates should be checked for the presence of residual seasonality.

C) The use of the mixed indirect approach.

(\* ) A) Best alternative; B) Acceptable; C) To be avoided

### 30.5.6 Different seasonal filters for different months/quarters

#### Description

In the standard approach of seasonal adjustment, one seasonal filter is applied to all individual months/quarters. The estimation of seasonal movements is therefore based on sample windows of equal length for each individual month/quarter (i.e. for each month/quarter the seasonal filter length or the number of years representing the major part of the seasonal filter weights is identical). This approach relies on the assumption that the number of past periods, in which conditions causing seasonal behaviour are sufficiently homogenous, is equal for all months/quarters.

However, this assumption does not always hold. Seasonal causes may change in one month, while staying the same in others. In German retail sales, for example, the peak in December had been steadily decreasing over years, especially because Christmas bonuses were being paid by fewer companies or were simply being lowered compared to the regular monthly salary. The Easter business in March and April, respectively, and the traditionally moderate shopping behaviour in a summer vacation month such as July are barely affected by this. In such cases it makes sense to use a shorter seasonal filter for December and longer (normal) ones for the remaining months of the year.

Additionally, different filters for different months/quarters can be used in the context of seasonal heteroskedasticity (see item 30.8.4). In correspondence to the overall moving seasonality ratio used in the X-11 algorithm for automatic selection of seasonal filters, monthly/quarterly specific moving seasonality ratios can be calculated in order to select monthly/quarterly specific seasonal filters. Care should be taken because these monthly/quarterly specific ratios may be highly dependent on individual observations.

Seasonal outliers can be interpreted as an extreme form of period-specific seasonal filters. Their usage can change estimated seasonal movements abruptly in one month/quarter, while the others are hardly affected.

#### Options

- Graphical analysis to determine the necessity of different filter lengths (period-specific SI-graphs).
- Calculation of monthly/quarterly moving seasonality ratios.
- Acquiring information about state and development of period-specific causes of the seasonal figure.
- Persistent use of standard analysis tools.

#### Alternatives (\*)

- A) Information about state and development of period-specific causes of the seasonal figure is actively acquired. Together with monthly/quarterly moving seasonality ratios and graphical analysis it forms the basis for the decision on the use of period-specific seasonal filters, at least for the adjustment of important macroeconomic aggregates.
- B) Available information about state and development of period-specific causes of the seasonal figure as well as monthly/quarterly moving seasonality ratios and graphical analysis form the basis for the decision on the use of period-specific seasonal filters, at least for the adjustment of major macroeconomic aggregates.
- C) Available information about special developments of the seasonal figure is not considered for seasonal adjustment.

(\* ) A) Best alternative; B) Acceptable; C) To be avoided

## 30.6 Revision policies

### 30.6.1 General revision policy and release calendar

#### Description

Revisions of seasonally adjusted data take place for two main reasons. First, seasonally adjusted data may be revised due to a revision of the unadjusted data, which may occur due to the availability of an improved information set (in terms of coverage and/or reliability). Second, revisions of seasonally adjusted data can take place because of a better estimate/identification of the seasonal pattern due to new information provided by new unadjusted data and/or due to the characteristics of the filters and procedures removing seasonal and calendar components. As far as revisions are solely based on new information, they are mostly welcome. However, in seasonal adjustment it may be the case that just one additional observation results in revisions of the seasonally adjusted data for several years, which sometimes confuses users.

The challenge is to find a balance between the need for the best possible seasonally adjusted data, especially at the end of the series, and the need to avoid insignificant revisions that may later be reversed (the trade-off between the accuracy of seasonally adjusted data and their stability over time).

Before developing a revision policy, consideration needs to be given to the needs of users and resources available to implement the policy. The policy should refer to and possibly define at least the following points: the frequency and relative size of revisions due to seasonal adjustment; the accuracy of the seasonally adjusted data, the time period over which the unadjusted data have been revised and the relationship between the timing of publication of revisions to the seasonally adjusted data and publication of the revisions to the unadjusted data.

It is important that the revision policy is as coherent and transparent as possible and that it does not mislead the interpretation of the economic picture.

#### Options

- Revise seasonally adjusted data in accordance with a well-defined and publicly available revision policy and release calendar.
- Revise both unadjusted and seasonally adjusted data between two consecutive scheduled releases of the release calendar.
- Perform revisions on an irregular basis and/or do not revise at all.

#### Alternatives (\*)

- A) Revisions to seasonally adjusted data are published in accordance with a coherent, transparent and officially published revision policy and release calendar, which is aligned with the revision policy and the release calendar for the unadjusted data. Revised seasonally adjusted data should not be released more often than unadjusted data. The public is informed about the size, direction and volatility of past revisions of important seasonally adjusted macroeconomic variables.
- B) Revisions to seasonally adjusted data are published in accordance with a coherent, transparent and officially published revision policy and release calendar.
- C) No revision of seasonally adjusted data, absence of a clear and public revision policy, absence of a public release calendar, or policies leading to the publication of misleading information especially for the current period.

(\*) A) Best alternative; B) Acceptable; C) To be avoided

## 30.6.2 Concurrent versus current adjustment

### Description

The way in which seasonal adjustment is carried out has implications for the revisions of seasonally adjusted data. The possible strategies range between the following extremes:

- Current adjustment. The model, filters, outliers and regression parameters are re-identified and the respective parameters and factors re-estimated at appropriately set review periods. The seasonal and calendar factors to be used to adjust for seasonal and calendar effects of new unadjusted data in-between the review periods are those estimated in the previous review period and forecasted up to the next review period.
- Concurrent adjustment. The model, filters, outliers, regression parameters are re-identified and the respective parameters and factors re-estimated every time new or revised data become available.

The current adjustment strategy minimises the frequency of revisions and concentrates the revisions coming from seasonal adjustment on the review period. The concurrent adjustment strategy generates the most accurate seasonally adjusted data at any given time point but will lead to more revisions, many of which will be small and perhaps in opposing directions.

Both of these strategies have drawbacks: for example, the current adjustment strategy can lead to a lack of precision in the estimation of the latest adjusted figures and the concurrent adjustment strategy can lead to a high instability of the seasonal pattern. Therefore, in practice, balanced alternatives between these two are followed in order to cope with data peculiarities and aiming to provide good quality adjustment:

- Partial concurrent adjustment. The model, filters, outliers and calendar regressors are re-identified once a year and the respective parameters and factors re-estimated every time new or revised data become available.
- Controlled current adjustment. Forecasted seasonal and calendar factors derived from a current adjustment are used to seasonally adjust the new or revised unadjusted data. However, an internal check is performed against the results of the "partial concurrent adjustment", which is preferred if a significant difference exists. This means that each series needs to be seasonally adjusted twice. The approach is only practicable for a limited number of important series.

A full review of all seasonal adjustment parameters should be undertaken at least once a year and whenever significant revisions occur (e.g. annual benchmark).

### Options

- Current adjustment with review on annual basis.
- Current adjustment with review less frequent than one per year.
- Concurrent adjustment.
- Partial concurrent adjustment.
- Controlled current adjustment.

### Alternatives (\*)

- When past data are revised for less than two years and/or new observations are available, partial concurrent adjustment is preferred to take into account the new information and to minimise the size of revisions due to the seasonal adjustment process. However, if the seasonal component is stable enough, controlled current adjustment could be considered to minimise the frequency of revisions. In this case, a full review of all seasonal adjustment parameters should be undertaken at least once a year. When revisions covering two or more years occur (as observed in national accounts) model, filters, outliers and regression parameters have to be re-identified and re-estimated.
- Current adjustment with a full review every year.

C) Current adjustment without annual review as well as concurrent adjustment.

(\* ) A) Best alternative; B) Acceptable; C) To be avoided

### 30.6.3 Length for routine revisions

#### Description

As a rule, when seasonal factors are re-estimated the seasonally adjusted results from the beginning of the time series change. Two factors speak in favour of always carrying out a revision for the whole series: the methodically identical treatment of all values and the fact that the calculation of the seasonally adjusted results is easy to understand and to replicate. It is, however, questionable whether a current newly added figure really contains relevant information for significant revisions of the estimation of the usual seasonal fluctuations in previous decades. As a way of balancing the information gain and the revision length, the revision period for the seasonally adjusted data is often, in practice, limited.

The date for the earliest revision of the seasonally adjusted data should be set at the beginning of a calendar year, not less than three years before the revision period of the unadjusted data. This date should be kept fixed for up to five years for transparency reasons. Statistical agencies should periodically investigate the existence of breaks in the revised series. For the earlier periods, seasonal factors could be frozen. This choice takes into account the extent of revisions of unadjusted data as well as the normal convergence properties of seasonal adjustment filters and the period for the filter to become symmetric.

#### Options

- Limit the revision period of the seasonally adjusted data to three years before the revision period of the unadjusted data and freeze the older data.
- Fix a starting date for the earliest revision.
- Revise the whole time series.
- Revise the seasonally adjusted data for a period no longer than the revision period of the unadjusted data.
- Do not revise seasonally adjusted data when unadjusted data are revised.

#### Alternatives (\*)

A) A starting date for the earliest revision of the seasonally adjusted data should be set at the beginning of a year, three years before the revision period of the unadjusted data. This date should be kept fixed for up to five years. Statistical agencies should periodically investigate for the existence of breaks in the revised series. When breaks are detected, statistical agencies can decide to reset the starting date.

B) Revise the whole time series.

C) Do not revise seasonally adjusted data when unadjusted data are revised, or revise for a shorter period than the revision period of the unadjusted data plus three years.

(\* ) A) Best alternative B) Acceptable C) To be avoided

### 30.6.4 Length for major revisions

#### Description

Major revisions of seasonally adjusted data are exceptional and substantial changes in published results occurring for one or more of following reasons:

- Major revisions of unadjusted data due to changes/updates of definitions/ concepts/nomenclatures/sampling scheme/legal acts, etc.;
- Change of seasonal adjustment method;
- Change of seasonal adjustment approach, such as moving from direct to indirect;
- Inclusion of a user-defined variable in pre-treatment to account for country/ domain-specific holidays.

Data producers usually take advantage of a major revision to introduce methodological improvements. This is good practice as it prevents revisions from occurring more often than necessary. Producers should identify the impact of each single change on the total revisions of the time series and inform the users. Major revisions affect a large part of the time series (unadjusted and seasonally adjusted) and sometimes even the complete time series.

When major methodological breaks in the unadjusted data occur, the seasonal adjustment could be separated into two different parts: one before and one after the break (provided that these periods are long enough for seasonal adjustment).

Major revisions are expected and planned well in advance. Users should be informed in advance and warned that considerable changes are expected in the time series. A policy for major revisions of seasonally adjusted data should specify at least the following: the pre-announcement strategy, how to communicate information on causes and impacts, expected length and depth.

### Options

- Revise the whole time series.
- Revise the seasonally adjusted data for a period as long as the one for the unadjusted data.
- Do not perform any revision.

### Alternatives (\*)

- A) In situations where the unadjusted data are substantially revised, the seasonally adjusted series should be revised accordingly. If major methodological breaks in the unadjusted data occur, the seasonal adjustment should account appropriately for the methodological break; when there is a change in the seasonal adjustment methodology or software, the need to revise the whole time series has to be carefully considered; users are informed in advance when a major revision will take place.
- B) Revise the whole time series in case of major revisions or when there is a change in the seasonal adjustment methodology; users are informed in advance when a major revision will take place.
- C) The impact of a major revision is not checked, or the seasonally adjusted data are revised for a period shorter than the one of the unadjusted data, or no revision of seasonally adjusted data is performed in the case of a major revision of unadjusted data; users are not informed in advance that a major revision will take place.

(\*) A) Best alternative B) Acceptable C) To be avoided

## 30.7 Accuracy of seasonal adjustment

### 30.7.1 Validation policy for seasonal adjustment

#### Description

Seasonal adjustment is a complex statistical process. Given the reliance of users on seasonally adjusted data, it is essential to validate seasonal adjustment before results are published.

The quality of seasonal adjustment can be evaluated only by using a wide range of measures. The graphical, descriptive, non-parametric and parametric criteria included in the output of the seasonal adjustment software can be complemented with additional graphical diagnostics and statistical tests.

The specific measures, such as absence of residual seasonality and stability of seasonality are discussed in item 30.7.2.

Furthermore, seasonally adjusted data must have a meaningful interpretation. As a consequence implausible data should not be validated even when statistical tests are successful.

#### Options

- Use a detailed set of graphical, descriptive, non-parametric and parametric criteria, across statistical packages, to validate the characteristics of seasonal adjusted data.
- Restrict validation to the measures included in the software used for seasonal adjustment.
- Use simple graphical inspection and descriptive statistics to validate seasonal adjustment.
- Do not validate seasonal adjustment.

#### Alternatives (\*)

- A) Use a detailed set of graphical, descriptive, non-parametric and parametric criteria, across statistical packages if necessary, to validate the seasonal adjustment. If validation fails, repeat the seasonal adjustment process in order to solve the problem (if possible).
- B) Use only default criteria included within the software used for seasonal adjustment. If validation fails, repeat the seasonal adjustment process in order to solve the problem (if possible).
- C) No validation of seasonal adjustment or use of only restricted graphical and descriptive statistics to validate the seasonal adjustment OR Not repeating the seasonal adjustment process if validation fails in cases A) or B) above OR validation of implausible data.

(\*) A) Best alternative; B) Acceptable; C) To be avoided

### 30.7.2 Measurement for individual series

#### Description

All seasonal adjustment software packages provide a wide range of measures to assess accuracy and reliability. These measures are derived, to some extent, from the implemented method — but many measures are common. The aim is to assess if a seasonally adjusted time series meets the following characteristics:

- absence of model/transformation misspecification;
- absence of residual seasonal/calendar effects or over-adjustment of seasonal/ calendar effects;
- absence of under/over-treatment of outliers/seasonal breaks;

- absence of instability in settings of the trend-cycle/seasonal/calendar components or pattern in the irregular component;
- absence of irregular influences in the trend-cycle, the seasonal and calendar component;
- absence of residual correlation in the model residuals.

Each of these characteristics should be tested for.

### Options

- Calculate measures for all characteristics.
- Calculate measures for some characteristics.
- Do not calculate measures.
- Make decisions based on expert judgement.
- Rely on automated decisions.

### Alternatives (\*)

- Calculate measures for all characteristics, do alternative runs of seasonal adjustment (if necessary), and take decisions based on expert judgement.
- Calculate measures for all characteristics relying on automated decisions rules or calculate measures for some characteristics taking decisions based on expert judgement.
- Do not calculate measures or calculate measures for some characteristics only relying on automated decisions.

(\*) A) Best alternative; B) Acceptable; C) To be avoided

## 30.7.3 Comparison of alternative approaches/strategies

### Description

To compare the accuracy and reliability of seasonal adjustment alternatives, a set of common quality measures as wide as possible should be used. The set of common quality measures should contain at least the following:

- M-statistics;
- roughness measures (smoothness of trend-cycle and of seasonal components, and in the context of indirect adjustment: R1 and R2);
- spectral diagnostics;
- pattern stability (history of revisions, sliding spans);
- presence of seasonality (for example Kendall and Friedman, Harvey Canova Hansen, Kruskal and Wallis);
- graphical inspection.

### Options

- Calculate all common measures to compare approaches/strategies.
- Calculate some common measures to compare approaches/strategies.
- Calculate no common measures, or only alternative/strategy-specific measures, to compare alternatives/strategies.
- Make decisions based on expert judgement.



- Make automated decisions.

#### Alternatives (\*)

- A) Calculate all common measures making decisions on approaches/ strategies based on expert judgement.
- B) Calculate all common measures making automated decisions on approaches/strategies or calculate some common measures making decisions based on expert judgement.
- C) Calculate no measures or calculate some common measures relying on automated decisions of approaches/strategies.

(\*) A) Best alternative; B) Acceptable; C) To be avoided

## 30.8 Specific issues on seasonal adjustment

### 30.8.1 Seasonal adjustment of short and very short time series

#### Description

Very short series (less than 3 years) cannot be seasonally adjusted using either moving average or model based methods. Nevertheless, they could be adjusted using alternative, less standard, procedures. Short series (3-7 years) are long enough to run moving average or model based methods but remain quite short and some instability problems can occur. Several empirical comparisons have investigated the relative performance of moving average and model based methods on short time series. Moreover, backcalculated time series (even non-official) could be used to extend the sample of short or very short time series and stabilise seasonal adjustment, when they are reliable enough.

As a general rule, when the series are shorter than seven years, the specification of the parameters used for the pre-treatment and the seasonal adjustment have to be checked more often (e.g. twice a year) than in normal situations.

#### Options

- Use back-calculation in order to extend the time series to be seasonally adjusted.
- Do not adjust time series when they are shorter than the minimum requirement for moving average or model based methods.
- Use of alternative procedures to seasonally adjust very short time series.
- Re-specify all parameters involved in the pre-treatment and seasonal adjustment of short time series more often than in the standard case.
- Conduct comparative studies on the performances of moving average and model based methods on short time series.
- Inform users about instability problems when series are shorter than 7 years.

#### Alternatives (\*)

- A) Perform seasonal adjustment of very short series by using standard tools conditional to the availability of reliable back-calculated series. Short time series must be seasonally adjusted by using standard tools with a more frequent parameter review. Enhanced stability of short seasonally adjusted series can often be achieved by means of back-calculation. Users should be informed about problems related to the seasonal adjustment of short and very short time series.
- B) Do not perform any seasonal adjustment of very short time-series; seasonally adjust short time series by means of standard tools with a more frequent parameter review.
- C) Use of non-standard methods for very short time series or merely automatic use of standard methods for short ones.

(\*) A) Best alternative; B) Acceptable; C) To be avoided

### 30.8.2 Seasonal adjustment of long time series

#### Description

The availability of long time series is of great importance for users but the maintenance of long time series is not an easy task for statistical agencies for several reasons, such as changes in definitions, methodology and classifications.

Nevertheless, long time series are provided by statistical agencies either because relevant changes do not happen or because they are overcome by means of back-calculation.

In the context of seasonal adjustment it is possible to assume heuristically that long time series are those exceeding twenty years of length. Performing seasonal adjustment of long time series can be difficult. Over such a long period the underlying data generating process may change, determining changes also in the components and in the components structure. In this case, to perform the adjustment over the whole series may produce sub-optimal results, mainly in the most recent and the initial parts of the series.

### Options

- Perform the seasonal adjustment on the whole time series using a unique set of settings and parameters for the pre-treatment and the seasonal adjustment.
- Perform the seasonal adjustment by partially overlapping sub-periods, each possibly longer than seven years, identified by means of an accurate investigation using statistical tests and graphical inspection.
- Perform the seasonal adjustment by sub-periods identified by either a simple, equal-length, cut rule or any subjective evaluation.
- Perform the seasonal adjustment only over the most recent period of the series.

### Alternatives (\*)

- A) Perform the seasonal adjustment on partially overlapping sub-periods, each possibly longer than seven years, selected by means of tests and graphical inspection. Link the seasonally adjusted data of each sub-period by using the information from overlapping parts to avoid breaks. Freeze the seasonally adjusted data of former sub-periods and regularly update the seasonally adjusted data of the current sub-period.
- B) Perform the seasonal adjustment by sub-periods identified by either a simple, equal-length, cut rule or any subjective evaluation. Freeze the seasonally adjusted data of former sub-periods and regularly update the seasonally adjusted data of the current sub-period.
- C) Perform the seasonal adjustment on the whole time series, using a unique set of settings and parameters, or only over the most recent period of the series.

(\*) A) Best alternative; B) Acceptable; C) To be avoided

## 30.8.3 Treatment of problematic series

### Description

Some series can be characterised by very specific features such as:

1. High non-linearity, which does not allow the identification of a model with acceptable modelling diagnostics, even by shortening the series;
2. Significantly large irregular component which makes it difficult to split normal seasonal from irregular effects;
3. Unstable seasonality (e.g. visible in graphs or in inconsistent adjustments from overlapping spans of data);
4. Large number of outliers compared with the length of the series (i.e. more than 10% of data points).

These series cannot be submitted to standard seasonal adjustment: individual treatment should be carried out, both in terms of method and set of options. The quality of the seasonally adjusted data depends on the suitability of the adopted strategy.

## Options

- To seasonally adjust only recent years of the series, if deleting earlier data makes it possible to find a model/adjustment of reasonable quality.
- To perform individual seasonal adjustment for all the problematic series.
- To perform individual seasonal adjustment only when the problematic series are relevant.
- No individual seasonal adjustment is performed.
- No seasonal adjustment is performed at all.

## Alternatives (\*)

- A) Seasonal adjustment is performed for problematic series. A case-by-case approach to seasonal adjustment is preferred to an automatic one. The literature, the manuals and experts should be consulted in order to develop a solution. Users should be informed of the adopted strategy. If a sufficient level of quality of the output series is not achieved, even with an individual treatment, no seasonally adjusted series are published.
- B) Seasonal adjustment is performed only on relevant problematic series, where failure to adjust these series leads to residual seasonality in important higher level aggregates. Other problematic series are treated in a standard way. If a sufficient level of quality of the output series is not achieved, even with an individual treatment, no seasonally adjusted series are published.
- C) Seasonal adjustment is performed automatically for all series or seasonal adjustment is not performed at all on problematic series.

(\*) A) Best alternative; B) Acceptable; C) To be avoided

## 30.8.4 Seasonal heteroskedasticity

### Description

Seasonal heteroskedasticity is present when the variance of a time series is dependent on the time of year (month or quarter). This phenomenon is observed when values of specific months or quarters are determined by volatile conditions, which do not occur in other months/quarters. The irregularly fluctuating duration and intensity of the frost and snow period, for example, affect the production in the construction industry in the northern European countries, while there are no such volatile factors in the warm season. Therefore, in this case, the variance in winter is higher than in summer.

This issue influences, among other things, the detection of outliers. If a time-independent constant variance is assumed for their identification (as for example in the framework of RegARIMA-models), outliers are usually automatically detected only in those months/quarters that exhibit a higher variance than the others. In periods of low volatility (such as the warm season with production in the construction industry), special influences are rarely automatically identified.

Seasonal heteroskedasticity can be identified using hypothesis testing, graphs and information on the causes of seasonal behaviour and their variability. In seasonal adjustment, it can be taken into account within the X-11 algorithm for automatic detection of extreme values.

### Options

- Examining time series for seasonal heteroskedasticity using:
  - statistical hypothesis testing;
  - graphs;
  - information on the causes of seasonal behaviour and their variability.
- Taking seasonal heteroskedasticity into account in seasonal adjustment, i.e. using different variances for filters for different month/quarters in order to detect extreme values.
- Ignoring the issue.

### Alternatives (\*)

- A) Examination for seasonal heteroskedasticity using hypothesis testing, graphs and information on the causes of seasonal behaviour and their variability, at least for important macroeconomic aggregates. Taking identified seasonal heteroskedasticity into account in seasonal adjustment and detecting extreme values.
- B) Automatic modelling of seasonal heteroskedasticity dependent on the outcome of a standard test.
- C) Ignoring the issue.

(\*) A) Best alternative; B) Acceptable; C) To be avoided

## 30.8.5 Seasonal adjustment of annually chain-linked series (Laspeyres-type)

### Description

Annually chain-linked Laspeyres(-type) price or volume indices are used in several macroeconomic statistical areas, inter alia, in consumer price statistics, national accounts, short-term statistics and labour cost statistics (detailed explanations of the chain-linking techniques are provided, for example, in Chapter 6 of the Eurostat Handbook on QNA; in Chapter 2 of Understanding National Accounts by the OECD; in Chapter 9 of the Consumer Price Index Manual; in Chapter 5 of the Eurostat Handbook on Price and Volume Measures in National Accounts and in Chapter 9 of the IMF's QNA Manual).

Chain-linking has implications for seasonal adjustment. For example, chain-linking of Laspeyres(-type) indices may matter for seasonal adjustment if seasonal adjustment of a chain-linked aggregate series is derived indirectly from its chain-linked component series. As a result, the indirectly seasonally adjusted aggregate series may exhibit a different trend level than the respective unadjusted series. Such deviations in the trend levels should be monitored on a regular basis. Furthermore, chain-linking of Laspeyres(-type) series may matter also for temporal consistency between infra-annual and annual data, depending on the chain-linking technique applied.

### Options

- Perform the seasonal adjustment directly on each individual chain-linked Laspeyres(-type) series.
- Perform the seasonal adjustment directly on each individual chain-linked Laspeyres(-type) series imposing temporal consistency only if unadjusted data are temporally consistent.
- Derive indirectly seasonally adjusted chain-linked Laspeyres (-type) aggregate series by aggregating the seasonally adjusted chain-linked Laspeyres (-type) component series following a sequential approach, allowing for preservation of aggregation and temporal consistency, whenever it is considered necessary.
- Perform the seasonal adjustment on unchained series.

### Alternatives (\*)

- A) Perform seasonal adjustment on annually chain-linked Laspeyres (-type) series and decide between direct and indirect adjustment following the criteria of item 30.5.4 and 30.5.5. When deriving indirectly seasonally adjusted chain-linked Laspeyres(-type) aggregate series by aggregating seasonally adjusted chain-linked Laspeyres(-type) component series, the following steps need to be performed:
  1. seasonally adjust the chain-linked Laspeyres(-type) component series;
  2. 'un-chain' the seasonally adjusted Laspeyres(-type) component series (with respect to the chain-linking technique);
  3. aggregate the unchained seasonally adjusted Laspeyres(-type) component series;
  4. chain-link the resulting seasonally adjusted Laspeyres(-type) aggregate series (again, with respect to the chain-linking technique);

5. re-reference the seasonally adjusted chain-linked Laspeyres(-type) aggregate series to the index reference year. In the presence of calendar effects, normalise the seasonally and calendar adjusted data to the calendar adjusted aggregate of the reference year, otherwise normalise the seasonally adjusted data to the unadjusted data of the reference year (e.g. 2005=100).

When strong users' requirements justify it and unadjusted (or calendar adjusted) data are additive, force the annual total of the seasonally adjusted (or seasonally and calendar adjusted) data to be equal to the annual total of the unadjusted (or calendar adjusted) data. Finally, check for overall quality of the adjustment.

- B) Perform seasonal adjustment directly on each individual annually chain-linked Laspeyres(-type) series with no specific consideration to temporal and sectoral/ geographical consistency.
- C) Seasonal adjustment is performed on unchained series (non-meaningful approach — strictly speaking, unchained series are not time series). Impose temporal, accounting or geographical constraints on chain-linked seasonal adjusted data.

(\* A) Best alternative; B) Acceptable; C) To be avoided

## 30.9 Data presentation issues

### 30.9.1 Data availability in databases

#### Description

Outputs from the seasonal adjustment process should be stored within a secure and usable database environment. The minimal output that should be stored is unadjusted and seasonally adjusted data using a time series nomenclature allowing users to associate the respective data, either by including a relevant dimension in or as a suffix to the raw data identifier. Additional output that could be stored include: calendar adjusted, trend-cycle data, seasonal factors, calendar factors, parameters/options for rerunning the process; prior corrections and prior versions (vintages). The database should be accessible for the purposes of re-producing; updating; and revising. The stored information should be consistent with any dissemination strategy and be accessible to users on request, respecting any confidentiality issues.

For a single time series, accessibility is measured as the number of available outputs, in the case that user requirements are not met the relevance dimension will fail. Clarity is measured qualitatively through producer satisfaction with seasonal adjustment settings, and quantitatively through the measures to support interpretation of outputs, for example Months (or Quarters) for Cyclical Dominance (MCD and QCD respectively) for trend movements in the seasonally adjusted series, standard errors for any single or combined component or real time data revisions.

The comparative quality of the accessibility of alternative approaches/strategies is measured as the difference in the number of relevant outputs available. Measures to compare clarity are the relative sizes of MCD (or QCD), standard errors (or even the existence of these) and revisions as these will directly impact the quality of user interpretation.

#### Options

- Storage and availability of additional time series output. For example, prior corrections, calendar adjusted data.
- Storage of all associated metadata information relating to an individual time series.
- Storage of data vintages to allow revision analysis.

#### Alternatives (\*)

- A) Systematic storage of unadjusted data, seasonally adjusted, seasonal adjustment options, prior corrections and trend-cycle data in a database with related nomenclatures. Ideally data vintages should be included. Metadata standards should be followed to ensure that all data can be exchanged easily and comply with the Metadata Template (see item 30.9.3). The database information should be secure but be accessible as required. The principles of ensuring transparency and enabling all users to understand and replicate the seasonal adjustment process should be followed.
- B) Systematic storage of unadjusted and seasonally adjusted data with associated metadata identifier. Additional data and metadata required to replicate the process can be stored or documented. The information should be made available on request and should allow for replicating the seasonally adjusted figures.
- C) No systematic storage of unadjusted and seasonally adjusted time series.

(\*) A) Best alternative; B) Acceptable; C) To be avoided

### 30.9.2 Press releases

## Description

Press releases aim to provide news and the figures on which policy is based. Data can typically be presented either as unadjusted or seasonally-adjusted. The unadjusted data contain all characteristics of the time series. The adjusted data contain the "news" of the series, i.e. the trend-cycle and the irregular component.

Much of the discussion on trend-cycle analysis focuses on the so-called end-point problem. Since the trend-cycle values at the end of the series are usually estimated by extrapolation, the estimated trend-cycle for the most recent data is very uncertain and can suffer from phase-shift problems. Particular care is required at turning points, where it is often months before the new correct direction of development appears. In all cases, the information contained within the press release should adhere to the principles of ensuring transparency and assisting users in making informed decisions.

Timeliness and punctuality are an issue for publication of a single time series if the time needed for seasonal adjustment delays data release.

Timeliness and punctuality will be an issue for alternative seasonal adjustment approaches/strategies only if either alternative impacts directly on the publication timetable. For example, validation of indirect adjustment might take longer than validation of direct adjustment. Further details on data presentation recommendations for press releases are available in the OECD Data and Metadata Reporting and Presentation Handbook, Chapter 5.

## Options

- Include only unadjusted data in press releases.
- Extend the informative content of press releases with one or more of the following transformations: seasonally adjusted series; calendar adjusted series; trend-cycle series.
- Present only levels or different forms of growth rates.
- Include empirical revision errors for the seasonally adjusted and/or trend-cycle series.

## Alternatives (\*)

- A) Seasonally adjusted data are the most appropriate figures to be presented in press releases. In addition, users should be provided with directions to the full historical unadjusted, calendar adjusted and trend-cycle time series, by reference and/or by internet download. When presenting trend-cycle estimates, the most recent values should not be shown because of the end-point problem or they should be accompanied by warnings related to their end-point problem. Analysis of real time revision errors of at least the seasonally adjusted estimates should be included. Period-on-period growth rates and changes in level should be computed on seasonally adjusted data and used with caution if the time series has high volatility. Year-on-year comparisons should be computed on calendar adjusted or, in the case of absence of calendar effects, on unadjusted data.
- B) Presentation of seasonally adjusted data and presentation of the trend-cycle in a graphical way which includes estimates for the current end of the series. In this case the end-point problem of the trend-cycle estimate should be made very clear. Year-on-year comparisons could be computed on seasonally adjusted data, in case of strong user demand. Annualised growth rates can also be used, especially for well justified reasons (e.g. for monetary aggregates). Particular attention has to be paid in cases of highly volatile series. Users should be informed of the specific characteristics of annualised growth rates.
- C) Presentation of the unadjusted data only, for series with seasonal components, or trend-cycle data only, as well as the computation of early period to period growth rates on either the raw or trend-cycle data.

(\*) A) Best alternative; B) Acceptable; C) To be avoided

### 30.9.3 Documenting metadata for seasonal adjustment



### Description

It is important that seasonally adjusted data are appropriately documented using the SDMX structure. Seasonal adjustment metadata are essential for communication with users. Additionally they are very useful for the exchange of information between institutions, but also for monitoring the implementation of the ESS Guidelines on Seasonal Adjustment.

In SDMX the adjustment concept (Concept 3 of Annex 1 of the SDMX guidelines — "Cross-Domain Concepts") is defined as:

"The set of procedures employed to modify statistical data to enable it to conform to national or international standards or to address data quality differences when compiling country specific data sets".

A seasonal adjustment metadata template should be designed to fulfil the requirement of the adjustment concept. It should record, in a standard form, the metadata on how seasonal adjustment is performed for different groups of series. Both the SDMX structure and the seasonal adjustment metadata template should be reviewed regularly.

### Options

- Use the SDMX structure.
- Use the seasonal adjustment metadata template.
- Use another metadata structure.
- Do not document metadata.

### Alternatives (\*)

- A) Use the SDMX structure supplemented by the standard metadata template for seasonal adjustment for all groups of series. Update the information using both the SDMX and the seasonal adjustment metadata template regularly to reflect changes in the seasonal adjustment process.
- B) Use only the SDMX structure, reviewed regularly.
- C) Do not compile any standard metadata.

(\*) A) Best alternative; B) Acceptable; C) To be avoided

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# Handbook on Seasonal Adjustment

Seasonal variation is one of the key factors, if not the key factor, that can impact the analysis of times series. In order to derive a meaningful analysis of the data under consideration, this seasonality has to be taken into account and adjusted for. The purpose of this handbook is to take stock of the existing techniques for seasonal adjustment and to propose amendments to the conventionally used methods. In addition, the handbook provides recommendations on the choice of the appropriate methods for seasonal adjustment and offers guidance on how these systems can be implemented in practice.

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