Secundum quid and *contingentia:* Scholastic Reminiscences in Early Modern Mechanics.

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According to medieval theocentric worldviews, the concept of Nature as God's Creation implied the contingency of its very existence. However, Scholastic thinkers did not limit their discussion on contingency to the onto-theological dimension, that is, the foundation of reality upon God's Will. Rather, contingency also implied a certain mental model for physical causality, regarded as a not-necessary but determined concatenation of natural events. Heated debates raised in the framework of medieval philosophy concerning divine prescience and human freedom, God's omnipotence and natural order, the distinction between logical and ontological necessity as well as determinism and indetermination in natural chains of events. All these issues gravitated around the problematic of contingency.[1] The investigation of the scholastic model of contingent causality is a premise to the understanding of long-living explanations of natural phenomena produced within the conceptual framework of Scholasticism and of later natural philosophies stemming from it more or less overtly. My present undertaking is to briefly assess in what form a 'principle of contingency' entered the science of weights and mechanics between the Middle Ages and the Early Modern Period.

[1] Among other sources, see Anneliese Maier, "Notwendigkeit, Kontingenz, Zufall," in *Die Vorläufer Galileis im 14. Jahrhundert: Studien zur Naturphilosophie der Spätscholastik* (Roma: Edizioni di Storia e Letteratura, 1949), pp. 219-250, Margaret J. Osler, "Divine Power and Divine Will in the Middle Ages: Historical and Conceptual Background," in *Divine Will and the Mechanical Philosophy: Gassendi and Descartes on Contingency and Necessity in Created World* (Cambridge: UP, 1994), pp. 15-35, and Magali Roques, "Contingence et déterminisme dans le commentaire d'Ockham au *Perihermeneias.* Essai de reconstruction," *Medioevo* 40 (2015).

Contingentia: A Principle of Causality in the Medieval Conceptions of Nature

In order to understand why the problem of contingency is crucial for the medieval debates on nature and the natural science of the time, it is expedient to begin by looking at early codifications of the concept and the philosophical problems surrounding it. The Latin word '*contingentia*' is a translation of the Aristotelian concept of 'possibility,' both as modal logical ἐνδεχόμενον as well as physical-metaphysical δύναμις within a hylemorphic framework. In the Christian context of creationist theology, this terminus was transformed and received an onto-theological connotation, which went far beyond the original meaning. In late Scholasticism, *contingentia* signifies the worldly reality, or nature as Creation. Nature is contingent; it exists *de facto* but could also not, because its existence depends on God's Will. As John Duns Scotus (1266-1308) put it,

> So then, the first issue has become clear: how there is contingency in things - because it comes from God - and what is in God which is the cause of this contingency - because it is his will.[2]

In Aristotle, there was a tension between two meanings of 'possibility.' According to *Prior Analytics* (13: 32 a 18-20) possible is that which is "neither necessary nor impossible," whereas according to *De interpretatione* (On Interpretation) (13: 22 a 14-13 a 26) possibility is exclusively that which is opposed to "impossibility" and therefore includes also that which is necessary. As a reminiscence of this original blurriness, one can find in Scholastic philosophy two different definitions of contingency either as "*quod est nec impossibile nec necessarium*" (that which is neither impossible nor necessary) or "*quod non est impossibile*" (that which is not impossible).[3] Both meanings were kept in the Latin rendering of the Aristotelian possibility as *contingentia* by Marius Victorinus

[2] John Duns Scotus, *Contingency and Freedom: Lectura I 39* (Dordrecht: Springer, 1994), p. 140: "Sic igitur apparet primum, quomodo est contingentia in rebus, quia a Deo, - et quid est in Deo quod est causa huius contingentiae, quia voluntas eius."

[3] Cf. Peter Vogt, *Kontingenz und Zufall. Eine Ideen- und Begriffgeschichte* (Berlin: Akademie Verlag, 2011), p. 52. The entire Chapter One is relevant for a historical overview of the reception and transformation of the Aristotelian concept of "possibility" as "contingency" in the Latin tradition.

(III-IV cent. CE) and Boethius (IV-V cent. CE), but the Latin expression also suggested affinity between that which is contingent (*contingit*) and that which occurs (*evenit* or *accidit*).[4] This third connotation would eventually prevail through the late-Scholastic differentiation between *contingentia* and *possibilitas* and its reception in the philosophical systems of the 17th century (and most notably by Leibniz).[5] Unlike abstract (purely logical) possibility, contingency referred only to that which is real but not so by necessity: "*id, quod (est sed) potest non esse*" (that which is but could not be). In the Christian perspective of the Almighty's Creation, contingency happened to include all that is not God himself in His absoluteness, that is to say, nature, or the universe.

This background is fundamental to understand not only theological disputes but also natural philosophical and scientific developments during the Middle Ages and the Early Modern Period. The connotation of nature as contingent—as that "which could not be"—is theological and metaphysical in its essence, since it points to the dependency of the world on God. However, from the point of view of natural conceptualizations, not only the 'vertical' dimension of metaphysics is relevant but also the 'horizontal' dimension of causality within nature. On the *horizontal* plane of the interrelation among finite beings, contingency refers to a degree of indetermination, and certain unpredictability in the connection between causes and effects. Moreover, whereas a theological perspective focuses on the radical contingency of that which exists as created being, natural reflections addressed the *relationship* between contingency and necessity within nature, that is, between divine order and phenomenal imperfection. This relationship between that which is not necessary and that which is necessary *had to* be conceptualized and indeed was conceptualized as the relationship between the *absolutum* and the *conditionale* or *secundum quid*. In the following I would like to stress that this perspective affected natural theorizations and explanations, such as those of the *scientia de ponderibus* (science of weights). I will soon deal with. But before addressing this discipline, in order to pinpoint the historical endurance of basic mental models, I should consider the views on natural contingency by two influential theologizing philosophers, one from the Middle Ages and one from the Renaissance: Thomas Aquinas in the *Summa contra gentiles* (1270 ca.) and Philipp Melanchthon in the *Initia doctrinae physicae* (1549[1]).

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[4] Ibid., p. 50.
[5] Heirich Schepers, "Zum Problem der Kontingenz bei Leibniz: Die beste der möglichen Welten,"

Theologizing Approaches to Natural Contingency

In the first book of the *Summa contra gentiles*, Thomas defined contingency through its distinction from necessity. In the case of the contingent beings, as one reads in *Summa contra gentiles* I 67, a cause can produce its effect or not, whereas in the case of necessary beings, their cause cannot but produce them:

> The contingent differs from the necessary according to the way each of them is found in its cause. The contingent is in its cause in such a way that it can both not-be and be from it; but the necessary can only be from its cause. [...] Just as from a necessary cause an effect follows with certitude, so it follows from a complete contingent cause if it be not impeded.[6]

A contingent cause, as one reads, will fulfill its tendency to produce a certain effect "*si non impediatur*," that is, if no impediment hinders its realization.

In the second book of the *Summa contra gentiles*, Thomas dealt extensively with the contingent being (II 15: "*omne quod est possibile esse et non esse*" and *"[id quod] ad utrumlibet se habet"*).[7] According to him, the world is contingent insofar as it is created. In this general sense, "*Deus est omnibus causa essendi*" (*Summa contra gentiles* II 15). In particular, God's free will is the origin of this world. Nonetheless, Thomas does not exclude that natural reality is populated by both necessary and contingent beings. Absolute necessity (*necessitas absoluta*), he writes in *Summa contra gentiles* II 29, does not pertain to God, since His decision and action is independent from any constriction (*debitum*). Rather, absolute necessity pertains to the immaterial, or 'separated' beings as well as to those bodies in which the *form* fulfills all potentialities of their *matter*, as is the case with the heavenly bodies transported in circles. As for terrestrial (sublunary) bodies, their forms are imperfectly realized. Matter, as the potentiality to take *different* forms, is at the origin of their contingency, that is, it is the source of the possibility to realize or not to realize certain effects (II 30): "But in things whose form does not fulfill the total potentiality of the matter, there still remains in the matter potentiality to

[6] Thomas Aquinas, *Summa contra gentiles* (Notre Dame-London: University of Notre Dame Press, 1975), 221 f: "Contingens a necessario differt secundum quod unumquodque in sua causa est: contingens enim sic in sua causa est ut non esse ex ea possit et esse; necessarium vero non potest ex sua causa nisi esse. [...] Ex causa necessaria certitudinaliter sequitur effectus, ita ex causa contingenti completa si non impediatur."

[7] Ibid., 48: "everything that can be and not-be" and "it is indifferent to either."

another form."[8] For the low realm of birth, corruption and change, Thomas speaks of conditional necessity (*necessitas conditionalis*). In the sublunary sphere, contingency cohabits with absolute necessity (e.g. the inevitability of the death for all animals and the hylemorphic composition of all bodies). Whereas necessity pertains to the formal determinations of natural phenomena, contingency is the partial fulfillment of necessary tendencies (II 23):

> For the power of every agent which acts by natural necessity is determined to one effect; that is why all natural things invariably happen in the same way, unless there be an obstacle; while voluntary things do not. God's power, however, is not ordered to one effect only […]. Therefore, God acts, not out of natural necessity, but by His will.[9]

Thomas points to natural regularities as a sign of the causal determination of the effects. Nonetheless, he adds, *impedimenta* can hinder the production of these effects. Furthermore, the will, in particular *divine* will, is not determined (*non ordinatur*) to produce one specific effect. Note that the freedom of will (the *liberum arbitrium*) has a positive connotation that conditional necessity cannot receive.

The natural reflections on contingency by Philipp Melanchthon are an instance of the lasting influence of Scholastic conceptions on nature even in a post-Scholastic environment, such as reformed Wittenberg. In fact, Melanchthon's intention as a Lutheran reformer of university curricula was to renounce Scholasticism, especially in theology, whereas he did not renounce the Aristotelian framework in philosophy. Nonetheless, his reflection on contingency documents a striking continuity between his philosophy and that of his medieval predecessors.[10]

An entire chapter of his *Initia doctrinae physicae* is dedicated to the issue "*De contingentia*" (On Contingency). It begins with the distinction between celestial necessity (*Non est animadversum, aliqua coeli partem mutatam esse, et motus coelestes servant perpetuas leges*) and sublunary corruption. In order to overcome this dissymmetry, according to Melanchthon, the ancient philosophers tried to reduce the

[8] Ibid., 87: "In quibus [rebus] vero forma non complet totam potentiam materiae, remanet adhuc in materia potentia ad aliam formam."
[9] Ibid., p. 68: "Omnis enim agentis per necessitatem naturae virtus determinatur ad unum effectum. Et inde est quod omnia naturalia semper eveniunt in eodem modo, nisi per impedimentum: non autem voluntaria. Divina autem virtus non ordinatur ad unum effectum tantum […]. Deus non agit per necessitate naturae, sed per voluntatem."
[10] Cf. sources on German Aristotelianism

conditions of one of these two realms to the other's, incurring in opposite mistakes. The Epicureans, on the one hand, attributed an earth-like condition to the heavens by claiming that chance (*casus*) is the principle of both celestial and terrestrial phenomena. By contrast, the Stoics did not limit necessity to the motions of the heavenly bodies and judged all of nature to be ruled by the inescapable laws of the *fatum* (fate).[11] Embracing an Aristotelian perspective, Melanchthon opposed to these ancient schools the view that nature is both necessitated and contingent in its different realms.

'Necessity,' as one reads, has three distinct meanings. As *necessitas absoluta* (absolute necessity), it simply refers to that whose opposite is impossible (*cuius contradictorium simpliciter est impossibile*). Among the 'absolute necessities,' Melanchthon includes the truth that God is free (*Deus est libere agens*), defining freedom as the faculty to act, to suspend an action, or to act differently (*Libertas est facultas agendi, aut suspendendi actionem, aut aliter agendi*).[12] Second, natural necessity (*necessitas physica*) refers to ordinary regularities, in particular in astronomy. Melanchthon calls these regularities "laws of nature."

> Physical necessity is an acting manner [*modus agendi*] ordered according to natural causes in such a manner that, if it is not interrupted by God, causes cannot act in a different way. E.g., the solar path is said necessary, because it cannot be changed otherwise than by divine intervention.[13]

Thus, certain phenomena can be said necessary relative to their ordered occurrence, although they are not necessary as to their being. God's action, which is not limited by natural constraints, is indeed the source of natural contingency. From His viewpoint as *architectus et servator universi opificii mundi* (architect and maintainer of the universal worldly contruction), nature is intrinsically contingent as a whole.

> It is uttermost certain that God is a very free agent. He is not bound to second causes, as the Stoics wrongly believed. This freedom of God's will is the first origin of contingency. In fact, contingent is that which does not exist by necessity but, once it has been posited, it does not imply anything impossible.

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[11] Philipp Melanchthon, *Initia doctrinae physicae* (Wittenberg, 1550), f. 31*r*-*v*.
[12] Ibid., f. 32*r*.
[13] Ibid.: "Necessitas physica est modus agendi ordinatus in causis naturalibus, quo modo non interrupto a Deo, non possunt illae causae aliter agere, ut Solis cursus dicitur necessarius, quia mutari non potest, nisi divinitus."

> Moreover, although the heavens move freely and contingently from the viewpoint of the doctrine of the Church, nonetheless we say in philosophy that the heavens move by necessity. By that we refer to physical necessity. In fact we speak about the order that has already been established.[14]

The third meaning of necessity is that of causal concatenation, for which if certain causes are given, determined effects will follow (*certo sequitur aliquid, quod tamen sua natura contingens est*).

According to Melanchthon, there are two sources of contingency in nature. One is material, and depends on the various motions of the elements and their compounds (*motus in elementis et mixtis*). The other one is human freedom. The difference between the two is that, whereas natural beings are directed toward one determined effect that they can realize or not (e.g. the vertical fall of a stone striving toward the center of the elements), human will can act in various directions and can also refrain from an action thanks to an inner refrainment or impulse to the contrary.[15] This freedom implies responsibility, since it includes the possibility to commit a sin, that is, the possibility to deviate from the laws of God. The distinction between voluntary freedom and natural necessity implies that, in Melanchthon's termini, the 'first' causes (e.g., the will) can act independently from the 'second' causes (natural order). "The artisan acts differently than the matter it uses; the physician freely acts whereas his remedy [acts] physically."[16] In a similar but perfect manner, the highest *artifex*, that is, God, freely imposes onto nature the laws of physics.

Secundum quid and Circularity as Contingented Straightforwardness in the Scholastic *scientia de ponderibus*

[14] Ibid., f 32*v*: "Certissimum est igitur, Deum esse agens liberrimum, non alligatum causis secudis, ut Stoici fingebant. Haec libertas voluntatis divinae, primus fons est Contingentiae. Est autem Contingens, quod non necessario existit, cum vero ponitur, nihil accidit impossibile. Quanquam igitur iuxta Ecclesiae doctrinam, Coelum libere et contingenter movetur, tamen cum nos in philosophia dicimus necessario moveri coelum, intelligimus, hanc necessitatem physicam, loquimur enim de ordine iam instituto."

[15] Ibid., f. 33*r*-*v*.

[16] Ibid., f. 35*r*: "Aliter agit artifex, aliter materiam quam adhibet. Medicus libere agit, remedium vero naturaliter."

The medieval *scientia de ponderibus* (science of weights) drew heavily on the idea of the conditional limitation of natural necessity depending on circumstances (*secundum situationem*, also literally meant as "depending on the position"). In particular, the concept of *gravitas secundum quid*, or positional heaviness, had a powerful explanatory function, most notably in the Aristotelian treatment of weights by Jordanus Nemorarius (XIII cent.), and continued to be essential, during the Renaissance, in the reflections on mechanics by scholars such as Niccolò Tartaglia, Girolamo Cardano and Giovanni Battista Benedetti.[17]

The meaning and function of contingency in mechanics was parallel to that in other disciplines such as ethics and logic. In ethics it was assumed that, whereas there can be no obstacle impeding the realization of God's will, which is 'absolute' (*voluntas absoluta*), human will, or *voluntas secundum quid*, is conditioned by circumstances.[18] In other words, the realization of its higher aims, revealed by God and reason, is intrinsically contingent, as Dante expressed in verses in the *Divina commedia*:

Vero è che, come forma non s'accorda
molte fiate a l'intenzion de l'arte,
perch'a risponder la materia è sorda,
così da questo corso si diparte
talor la creatura, c'ha podere
di piegar, così pinta, in altra parte;
e sì come veder si può cadere
foco di nube, sì l'impeto primo
l'atterra torto da falso piacere.[19]

Apart from ethical contingency, Scholasticism also used *secundum quid* in logic. For instance, Petrus Hispanus (XIII cent.), explained the meaning of the so-called fallacy *secundum quid* in his *Tractatus sive summule logicales*, commenting on Aristotle's *On Sophistical Refutations* V (166b36-167a14)[20] In logic, *secundum quid* meant either a 'diminution' of a concept through restriction of its definition (*secundum quid et*

[17] See Jürgen Renn and Peter Damerow, *The Equilibrium Controversy*: *Guidobaldo del Monte's Critical Notes on the Mechanics of Jordanus and Benedetti and their Historical and Conceptual Background* (Berlin: epubli, 2012), especially the sections from 3.6 to 3.8.
[18] Cf. ibid., *Paradiso* IV 87, IV 109 and IV 113 and *Purgatorio* VII 57.
[19] Dante Alighieri, Paradiso I 126-132.
[20] Petrus Hispanus, *Tractatus sive summule logicales*, ed. by Ch. Rapp and K. Corcilius (Assen: van Gorcum, 1972), pp. 157-158.

simpliciter), or the designation of a subject through one of its parts or characteristics (*denominatio totius per partem*).[21]

In mechanics the 'limitation' or 'determination' *secundum quid* implied that the dynamic tendency of a body was reduced or enhanced depending on intervening constraints or circumstances, in particular mechanical ones. The rotations of a lever around a pivot or of a balance around its fulcrum were conceptualized as constrained motions. In such displacements, the necessary vertical tendency of a weight resulted into a circular motion due to external constraints. At the same time, the heaviness (*gravitas*) of the bodies suspended at the extremities of a simple machine varied in relation to their changing positions within the system. In such cases, a *necessary* straightforward motion in accordance with natural order resulted *contingently* into a circular one. The implicit mental model for this kind of displacement was that circular motion is constrained rectilinear motion. This means that, in the sublunary sphere of contingency, straightforwardness and rectilinear tendency had a higher onto-epistemological status than circularity since straightforwardness was necessarily rooted in natural order. By contrast, circularity, as the deviation from such order, had to be explained. As a consequence, circularity (in the elementary sphere) was allotted a derived and subordinated onto-epistemological status. In other words, circularity was an instance of *necessitas secundum quid*. From this viewpoint, it was seen as a deviant realization of determined potentialities similar to moral deviation from the necessary laws of uprightness, or to the external regulation of physiological processes through medical intervention. In order to stress the embedding of the mechanical treatment of the *scientia de ponderibus* in the scholastic framework of contingency, one could also formulate the principle as follows: "circular motion is *contingented* rectilinear motion."

At the beginning of his treatise "On weights," Nemorarius pointed to his Aristotelian background. As one reads, his approach is based on the opposition between the *natural* vertical motion of the elements and the *violent* hindrances producing circular deviation. At the same time, he introduced the key concept of *gravitas secundum quid* (in same cases, also of *levitas secundum quid*), which could be referred to as 'positional heaviness.'

[21] A fallacy *secundum quid* occurs if an identity is established between something considered in a particular respect and the same thing considered absolutely (or *simpliciter*). For instance, the existence of a depicted animal does not imply the existence of the animal *simpliciter*. Thus, the argument "est animal pictum, ergo est animal" is not correct.

> [...] if equal arcs are taken on a greater circle, and on a smaller one, the chord of the arc of the greater circle is longer. From this I can then show that a weight on the arm of a balance becomes lighter, to the extent that it descends along the semicircle. For let it descend from the upper end of the semicircle, descending continuously. I then say that since the longer arc of the circle is more contrary to a straight line, than is the shorter arc, the fall of the heavy body along the greater arc is more contrary to the fall which the heavy body would have along the straight line, than is a fall through a shorter arc. It is therefore clear that there is more violence in the movement over the longer arc, than over the shorter one; otherwise the motion would become heavier. Since something moves with more violence in the ascent [along the arc], it is apparent that there is more positional heaviness [*gravitas secundum situm*] and, as it is like that depending on position [*secundum situationem*], one can aptly call it 'positional heaviness' [*gravitas secundum situm*].[22]

In its circular descent along a circular path, a weight deviates from its natural tendency, or *intentio*. The more the arm of the balance departs from the horizontal position, the more it departs from the natural tendency. It is assumed therefore that the 'violence' is greater the longer the arc of displacement is, while the weight progressively looses its weight if the vertical component in its motion is reduced (Fig. 1).

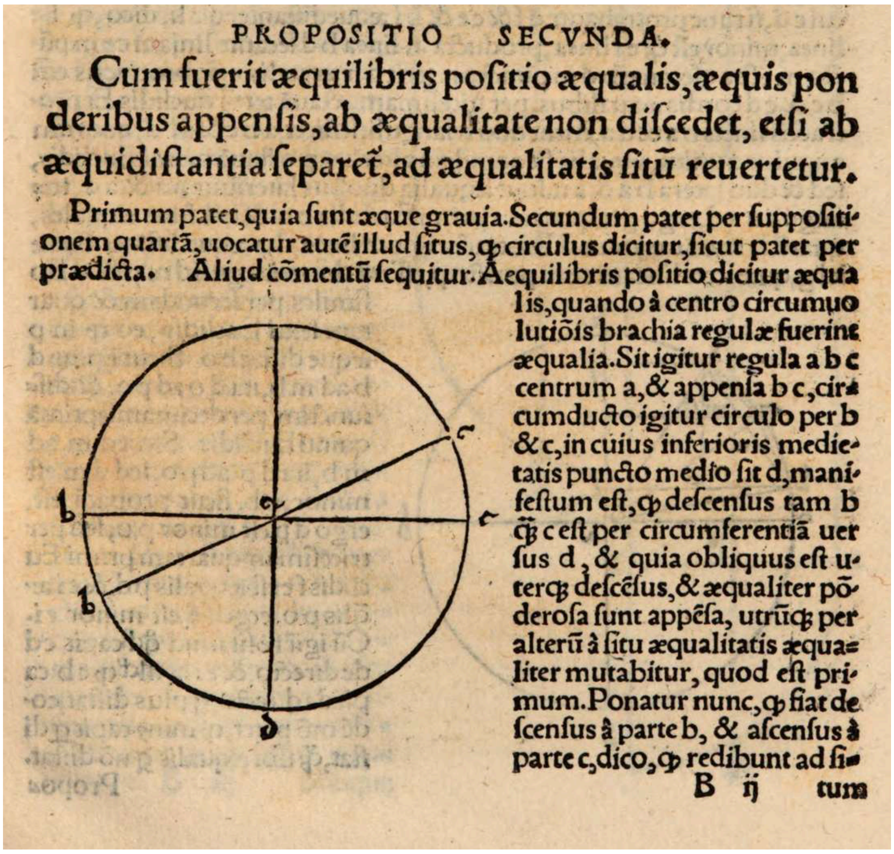
Nemorarius argued that a weight that reaches the bottom of the circular arc described by the arm in its displacement is not "at rest" but only "lighter." In fact, a natural being is at rest only if it fully realizes the aim (or act) toward which its power is directed teleologically. By contrast, a body is always in motion, or strives to move, if it has not reached its end: "All motion strives toward its aim—indeed the whole nature strives towards actuality and is realized [in it]—hence the opposition occurs against [a displacement] contrary [to the natural tendency]."[23] If a body on one arm of the balance becomes lighter during its downward motion than an equal one located on the other

[22] Johannes Nemorarius, *Liber de ponderibus*, ed. Petrus Apianus (Nuremberg: Petreius, 1533), f. A3*v* (emphasis added): "[...] si sumantur de circulo maiori et minori arcus aequales, corda arcum maioris circuli longior est. Propeterea posset ex hoc ostendi, quod pondus in libra tanto sit levius, quanto plus descendit in semicirculo. Incipiat igitur mobile descendere a summo semicirculi, et descendat continue. Dico tunc quod maior arcus circuli plus contrariatur rectae lineae quam minor, et casus gravis per arcum maiorem, plus contrariatur casui gravis, qui per rectam fieri debet, quam casus per arcum minorem. Patet ergo maior est violentiam in motus secundum arcum maiorem, quam secundum minorem. Aliter enim fieret motus magis gravis. Cum ergo plus in ascensu aliquod movetur violentie [violentia?], patet, quam maiore est gravitas secundum situm, et quia secundum situationem talium sic sit, dicatur gravitas secundum situm." Here and in the follow, Latin grammar, capitalization and punctuation have been modernized and standardized.

[23] Ibid., ff. A3*v*-A3*r*: "In termino enim cuiscunque motus intenditur, intenditur et viget tota natura in actu, qui in motu sit quasi in potentis, secundum quem fiebat contrarietatis suae oppositio."

extremity, as Nemorarius takes pains to demonstrate, then a balance removed from its state of equilibrium will tend to restore the original state. As one reads in the *propositio secunda* (second proposition, which is referred to the diagram in Fig. a), which is the second of a series of propositions developing the details of Nemorarius's doctrine of weights,

> Suppose now that the descent occurs on the side B and the ascent on the side C. I say that both will go back to the [horizontal] position of equality. In fact, B will not further descent, because its descent towards D is more oblique than the ascent of C towards the [horizontal position of] equality; in fact, B and C are equidistant from the place of equality.[24]


PROPOSITIO SECVNDA.

Cum fuerit æquilibris poſitio æqualis, æquis ponderibus appenſis, ab æqualitate non diſcedet, etſi ab æquidiſtantia ſeparetᷣ, ad æqualitatis ſitū reuertetur.

Primum patet, quia ſunt æque grauia. Secundum patet per ſuppoſitionem quartā, uocatur autē illud ſitus, ꝙ circulus dicitur, ſicut patet per prædicta. Aliud cōmentū ſequitur. Aequilibris poſitio dicitur æqualis, quando à centro circumuolutiōis brachia regulæ fuerint æqualia. Sit igitur regula a b c centrum a, & appenſa b c, circumducto igitur circulo per b & c, in cuius inferioris medietatis puncto medio ſit d, manifeſtum eſt, ꝙ deſcenſus tam b q̄ꝫ c eſt per circumferentiā uerſus d, & quia obliquus eſt uterqꝫ deſcēſus, & æqualiter pōderoſa ſunt appēſa, utrūqꝫ per alterū à ſitu æqualitatis æqualiter mutabitur, quod eſt primum. Ponatur nunc, ꝙ fiat deſcenſus à parte b, & aſcenſus à parte c, dico, ꝙ redibunt ad ſi-

B ij tum

Figure 1. Diagram accompanying preposition two in Apianus's 1533 edition of Nemorarius's *Liber de ponderibus* (1533, f. B2*r*).

Nemorarius's reasoning becomes clearer in the light of propositions four and five:

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[24] Ibid., ff. B2*r*-*v*: "Ponatur nunc, quod fiat descensus a parte B, et ascensus a parte C, dico quod redibunt ad situm aequalitatis. Non enim ulterius descendet B, eo quod descensus eius versus D magis obliquus est, quam ascensus C ad aequalitatem; B enim et C iam aequaliter distant a situ aequalitatis."

> The fourth [proposition]: It is positionally heavier, insofar as its descent, in the same position, is less oblique.
> The fifth [proposition]: But a more oblique descent partakes less of the straight [descent], for the same quantity [of the path]."[25]

In proposition five, it is suggested that the vertical components of the potential descents of the two beams could be identified and compared. This was the source of the idea that also the variation of heaviness could be determined. In this respect, one can refer to Niccolò Tartaglia's reworking, in the *Questiti et inventioni diverse* (Various Questions and Inventions) (1546), about the manner to ascertain the positional heaviness of two weights on the basis of the so-called angles of contact. These are the 'curvilinear' or 'mixed' angles between the circular path of the arms of a balance and the vertical lines connecting the weights to the cosmological center of gravity (see Fig. 3). LITERATURE? Tartaglia compared the angles of contact of two equal weights located on the extremes of a balance, and argued that the lifted one is always smaller than the lowered one. Thus, the lifted weight would face a descent path that is more oblique. It would acquire a greater positional heaviness than its lowered counterweight and, as a further consequence, the inclined system would reestablish its horizontal balance, if not hindered to do so.

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[25] Ibid., f. A3*r*: "Quarta [propositio]: Secundum situm gravius esse, quanto in eodem situ minus obliquus est descensus. Quinta [propositio]: Obliquiorem autem descensum minus capere de directo, in eadem quantitate." Translation from Renn-Damerow, *Equilibrium Controversy*, p. 63. For proposition four, cf. Nemorarius, *Liber de ponderibus*, *cit.*, f. B3*v*-B4*r* and, for proposition five, f. B4*r*-C2*v*.

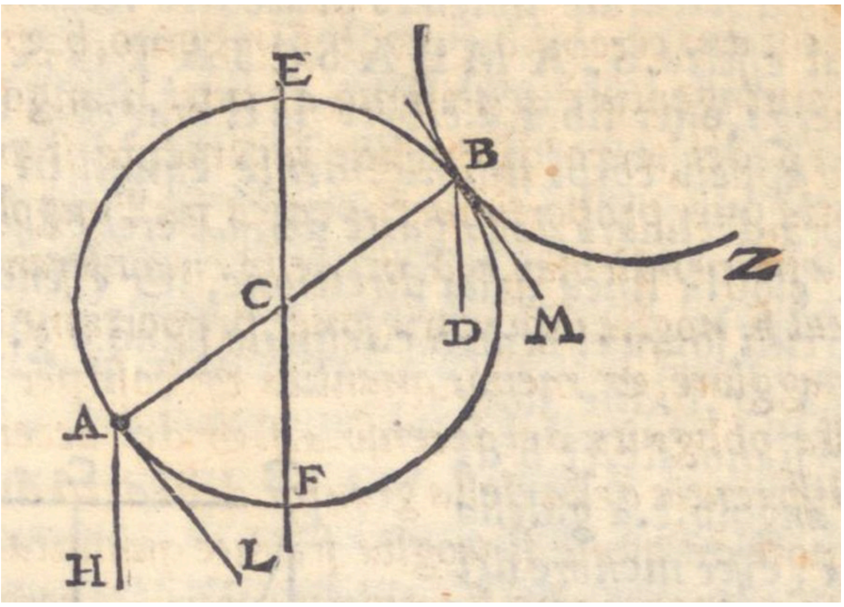

Figure 3. In the *Quesiti et inventioni diverse*, Tartaglia argued that the relative positional heaviness of the weights A and B on a balance could be determined on the basis of the 'mixed' angles of contact HAF and DBF. Since it is argued that DBF < HAF, the weight B will be heavier than A. Thus, the inclined system will strive toward the restoration of a horizontal equilibrium.

In spite of his attempt to quantify the *quid* accounting for the alleged restorative motion of the inclined balance, Tartaglia's geometrical quantification maintained a margin of indetermination. As he stated, the ratio between the two mixed angles is less than that between any determined quantities. Therefore, it is impossible to stabilize the system in its inclined position by adding a small (no matter how small) weight on the lowered side of the balance. In other words, it is impossible to counterweight the positional heaviness of the lifted weight. Quite on the contrary, any additional weight added to the lowered side would make the balance rotate and reach the vertical position.[26]

[26] Tartaglia's approach was controversial, already in his time. Cf. Renn and Omodeo, Guidobaldo Del Monte's Controversy with Giovan Battista Benedetti on Positional Heaviness," in *Guidobaldo del Monte (1545–1607). Theory and Practice of the Mathematical Disciplines from Urbino to Europe*, ed. Antonio Becchi, Domenico Bertoloni Meli and Ezio Gamba (Berlin: Edition Open Access, 2013), pp. 53-94, section 3.6.

Inclinatio recte eundi: Benedetti's generalization from weights to forces

As I have argued so far, in the medieval *scientia de ponderibus* circular motion is conceived of as constrained linear motion. Yet, this mental model is restricted to the sublunary sphere, where motions cannot fulfill their nature. This is indeed the sphere of contingency, where a gap is to be witnessed between the necessary order of things (or 'nature' as actuality) and the effective phenomena (subjected to 'violence' or to external constraints). In fact, the four elements naturally tend toward their places through a straightforward descent or ascent. Heavy bodies, for instance, strive toward the center of gravity, which is, at the same time, the center of the cosmos. If their motion is hindered, as is the case with mechanical constraints, a *quid* (a determination) has to be taken into account, which explains the deviation from the rule. In this theoretical context, contingency is the concept expressing the relationship between the natural law (to use Melanchthon's expression) and phenomenal reality, which follows a norm while deviating from it. The *secundum quid* explains this deviation. Possibly, it has to be expressed through geometrical means. However, it might result in unintelligibles or infinitesimals, as was the case with Tartaglia's ratio between mixed angles accounting for the *gravitas secundum quid* of the weights of a balance. In the treatment of weights, in particular of those on a balance, Nemorarius and his followers made a limited use of the mental model of curvilinear motion as constrained linear motion. In fact they employed it to account for phenomena linked to gravity (i.e., the vertical fall of bodies explained in Aristotelian terms). It is with Giovanni Battista Benedetti that a decisive step was made toward the generalization of this model in the direction of inertial dynamics. I will consider first his application of the model to balances and then to centripetal forces.

In his book *De mechanicis* (On Mechanics), published as a section of the *Diversarum speculationum mathematicarum et physicarum liber* (Various Mathematical and Physical Speculations) (1585), Benedetti renewed the concept of *gravitas secundum quid*. Guidobaldo Del Monte had already criticized Nemorarius's and his followers' conclusion that an inclined balance hinged on its fulcrum (as its center of gravity) would return to the horizontal position, but his criticism went so far as to renounce the concept of positional heaviness altogether.[27] Relying on the Archimedean concept of center of gravity of a body, Del Monte came to the conclusion that an equal-arms balance hinged on its fulcrum would remain stable in any position (a correct conclusion only if it is assumed, in modern terms, that the gravitational field is homogeneous): "Fourth Proposition: [Consider] a balance that is equidistant to the horizon, bearing equal weights at its extremities and at the same distance from the pole, which is in the center of the balance. Whether it is displaced from its position or not, it will remain in whatever position."[28] Benedetti shared the criticism of Nemorarius and Tartaglia on their specific argumentation about the tendency of such an inclined balance to reach the horizontal position but based it on a varied treatment of positional heaviness.

The first chapter of Benedetti's *De mechanicis* begins with the statement: "Every weight placed at the end of an arm of a balance has a greater or a lesser heaviness depending on differences in the position of the arm itself."[29] The commitment in favor of a mechanical theory of equilibrium based on positional heaviness is evident. Although his terminology is not always consequent, note that in this case (as in most cases in the text), *pondus* has the essentialist meaning of a substance (a substratum or ὑποκείμενον). It is the body or weight on the balance, whose special property of being heavy, namely the *gravitas*, varies according to a *quid*. This *quid* is the position, or *situm*. Benedetti makes the effort to quantify it using a method of his invention. He considers the line, which he calls *linea inclinationis* or *linea itineris*, connecting a weight on an inclined beam of the balance to the cosmological center of gravity. Note that Benedetti calls the elementary downward tendency an *iter* from a merely cinematic viewpoint, but also an *inclinatio* from a physical and more proper one. According to Benedetti, the degree of heaviness of the weight can be assessed through the projection of the *linea inclinationis* on the horizontal line passing through the fulcrum (Fig. 4). The more distant it is from the fulcrum, the heavier the positional heaviness becomes. Thus, the weight reaches a maximum of heaviness when the balance is horizontal, and its minimum when it is vertically resting (*nititur*) on the fulcrum or hanging (*pendet*) from it. Notably, this approach anticipates that based on the determination of the torque in classical physics,

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[27] See Renn and Damerow, *Equilibrium Controversy*, *cit.*, especially pp. 86-92.

[28] Own translation. Cf. Jürgen Renn and Peter Damerow, *Guidobaldo del Monte's Mechanicorum Liber* (Berlin: epubli, 2010), p. 65: "Propositio IV: Libra horizonti aequidistans aequalia in extremitatibus, aequaliterque a centro in ipsa libra collocato, distantia habens pondera; sive inde moveatur, sive minus, ubicunque relicta manebit."

[29] Giovanni Battista Benedetti, *Diversarum speculationum mathematicarum et physicarum liber* (Taurini: Apud Haeredem Nicolai Bevilaquae, 1585), p. 141: "Omne pondus positum in extremitate alicuius brachii librae maiorem, aut minorem gravitatem habet."

and comes to the same conclusions.[30] In particular, Benedetti objected to Nemorarius and Tartaglia that no difference in the effectiveness of the weights could result from the inclination of a balance (hinged on the fulcrum) since the vertical projections from the extremities would be equidistant from the fulcrum. According to Benedetti, however, this is true if one assumes for convenience that the vertical tendencies of the weights are parallel, neglecting that in reality they converge and eventually meet in the center of the Earth.[31]

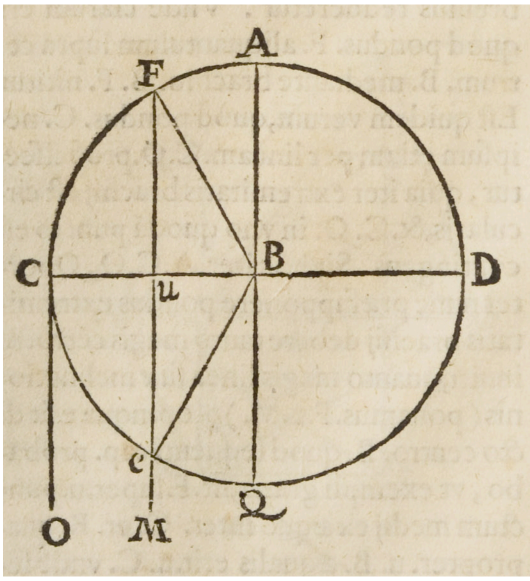

Figure 4. Benedetti's diagram showing a balance CBD or FBD. The lines CO and FUEM are the so-called lines of inclination connecting the weights C and F with the center of the elements. The length of the projection on the horizontal is proportional to the positional heaviness.

Additionally, Benedetti equates the heaviness to a *virtus*, *vis* or *vigor*, i.e. a force, which might act also in different directions (in *De mechanicis*, Ch. 3) and is applied to the extremity of a constrained mechanical system, like a lever or a balance. This is already a significant generalization from weights to forces, but for my present discussion the most important generalization relates to rectilinear tangential tendencies in systems set in circular motion.

The relevant treatment is a famous epistle that Benedetti addressed to the Savoy courtier Giovanni Paolo Capra. It deals with the rotation of a millstone and the question of whether its motion could be perpetual. Benedetti denies it arguing that the rotation is

[30] Cf. Renn and Damerow, *Equilibrium Controversy*, *cit.*, p. 138.
[31] Benedetti discusses this case in *De mechanicis* 7-8 coming to the conclusion that, if one takes into consideration the cosmological context, the inclined balance will be tilted to the vertical.

impeded first by the friction of the air and, second and more important, by the resistance of the millstone's parts. The latter have a straightforward tendency, an *inclination recte eundi*, along the tangential lines of their rotation. As one reads, this rectilinear inclination or impulse (*impetus*) can be bent only by violence. Moreover the centripetal tendency grows proportionally with the augmentation of the speed, as witnessed by other cases, among them the rotation of a catapult or a sling (*machina missilis*). A centripetal tendency is therefore a rectilinear natural inclination (*naturalis inclinatio recte eundi*).

> You ask me this question in your letter. Suppose a millstone rested on a virtually mathematical point an was set in circular motion; could that circular motion continue without end, it being assumed that the millstone is perfectly round and smooth.
>
> I answer that this kind of motion will certainly not be perpetual and will not even last long. For apart from the fact that the wheel is constrained by the air which surrounds it and offers resistance to it, there is also resistance from the parts oft he moving body itself. When these parts are in motion, they have by nature a tendency [impetus] to move along a straight path. Hence, since all the parts are joined, and any one of them is continuous with another, they suffer constraint in moving circularly and they remain joined together in such motion only under compulsion. For the more they move, the more there grows in them the natural tendency to move in a straight line, and therefore the more contrary to their nature is their circular motion. And so they come to rest naturally: for, since it is natural to them, when they are in motion, to move in straight lines, it follows that, the more they rotate under compulsion, the more does one part resist the next one and, so to speak, hold back the one in front of it.[32]

[32] Stillman Drake and Israel Edward Drabkin, *Mechanics in Sixteenth-Century Italy, Selection from Tartaglia, Benedetti, Guido Ubaldo and Galileo* (Madison: University of Wisconsin Press, 1969), p. 229. Cf. Benedetti, *Diversarum speculationum... liber*, *cit.*, p. 285: Quaeris a me literis tuis, an motus circularis alicuius molae molendinarie, si super aliquod punctum, quasi mathematicum, quiesceret, posset esse perpetuus, cum aliquando esset mota, supponendo etiam eandem esse perfecte rotundam, et laevigatam. Respondeo huiusmodi motum nullo modo futurum perpetuum, nec etiam multum duraturum, quia praeterquam quem ab aere qui ei circumcirca aliquam resistentiam facit stringitur, est etiam resistentia partium illius corporis moti, quae cum motae sunt, natura, impetum habent efficiendi iter directum, unde cum simul iunctae sint, et earum una continuata cum alia. Dum circulariter moventur patiuntur violentiam, et in huiusmodi motu per vim unitae manent, quia quanto magis moventur, tanto magis in iis crescit naturalis inclinatio recta eundi, unde tanto magis contra suammet naturam volvuntur, ita ut secundum naturam quiescant, quia cum eis proprium sit, quando sunt motae, eundi recta, quanto violentius volvuntur, tanto magis una resistit alteri, et quasi retro revocat eam, quae antea reperitur habere."

The mental model of dynamic circularity as constrained straightforwardness receives in Benedetti's treatment a higher degree of generalization. In this case he argues that, since it contrasts a natural inclination, it cannot be eternal. Note that this is an Aristotelian assumption (violent motion cannot be eternal) emerging in a context in which this legacy is explicitly rejected.[33] Another Aristotelian echo underlies Benedetti's statement that the linear tendency makes a body 'lighter' since, if it were freed from the constraint hindering its projection, it would not fall vertically but rather travel through a more or less rectilinear trajectory tangent to the circular motion of the constrained rotation. In the conclusion of his reflection on the natural rectilinear strive of the parts of a body set in circular motion, Benedetti stressed the originality of his treatment "without precedents" and its opposition to Aristotelian dynamics (according to which, the projection of a body through a medium presupposes the support of the medium itself).

> But if you wish to see this truth more clearly, imagine that while the body, i.e., the top, is spinning around very rapidly, it is cut up or divided into many parts. You will observe not that those parts immediately fall toward the center of the universe, but that they move in a straight line, and, so to speak, horizontally. No one, so far as I know, has previously made this observation on the subject of the top.
>
> From such motion of the top or of a body of this kind it may be clearly seen how mistaken are the Peripatetics on the subject of the forced motion of a body. They hold that the body is driven forward by the air which enters [behind it] to occupy the space left by the body. But actually the opposite effect [that is to say, resistance] is produced by the air.[34]

I have so far observed two instances in Benedetti's work on mechanics in which a tension between mathematical laws of nature and their empirical realization emerges: his treatment of the rotation of beam about its pole and that of a turning wheel. In both

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[33] Benedetti's anti-Aristotelianism is well known. Cf. Carlo Maccagni, "Contra Aristotelem et omnes philosophos," in *Aristotelismo veneto e scienza moderna*, vol. 2, ed. by Luigi Olivieri (Padova: Antenore, 1983), pp. 717–727.

[34] Drake and Drabkin, *Mechanics in Sixteenth-Century Italy*, *cit.*, 229-230. Benedetti, *Diversarum speculationum... liber*, *cit.*, p. 285: "Sed si clarius, hanc veritatem videre cupis, cogita illud corpus, trochum scilicet, dum velocissime circunducitur secari, seu dividi in multas partes, unde videbis illas omnesque, non illico versus mundi centrum descendere, sed recta orizontaliter, ut ita dicam, moveri. Id quem a nemine adhuc (quem sciam) in trocho est obseruatum. Ab huiusmodi motu trochi, aut huius generis corporis, clare perspicitur, quam errent peripatetici circa motum violentum alicuius corporis, qui existimant aerem qui subintrat ab occupandum locum a corpore relictum, ipsum corpus impellere, cum ab hoc, magis effectus contrarius nascatur."

cases, natural straightforward tendencies are constrained and deviated into violent circular ones. The epistemological meaning of these concepts lies in the possibility of a geometrical treatment of natural contingency seen as the connection between the *necessity* of the rules and of the principles and their *necessitation* in the empirical reality of curvilinear motions. However, the element of indeterminacy characteristic of Tartaglia's infinitesimal geometry tends to disappear in Benedetti's exact determination of the outcome of constrained motion. The contingent determination of the *secundum quid* shifts toward its necessary determination.

Galileo's Cosmologization of Mechanics

Galileo Galilei's physical work is a crucial passage towards the cosmologization of explanatory models descending from Renaissance mechanics, like those of Benedetti, on whose work he relied in many ways. REFERENCES

In *Le Mecaniche* (The Mechanics) (1593, printed 1634), positional heaviness was transformed into the concept of 'momento,' defined as "*propensione di andare al basso cagionata non tanto dalla gravità del mobile, quanto dalla disposizione che abbino tra di loro diversi corpi*."[35] Galileo used this conceptual tool in the explanation of simple machines by determining the *momento* of a weight in a manner that is akin to Benedetti's determination of the *gravitas secundum quid*. Just as Benedetti's positional heaviness is determined by the projection of the line of inclination on the horizontal, Galileo's momentum depends on the distance between the vertical projection of the downward tendency of a body and the center of gravity of the mechanical system (e.g., of a balance or a sphere on an inclined plane). In his concept of 'momento,' however, the Scholastic flavor disappeared, disguised by a technical term encapsulating the *secundum quid* without making its Aristotelian origin explicit.

Not only positional heaviness was transformed by Galileo's physics, but also the status of circular motion and, as a consequence, the treatment of the relation between circularity and straightforwardness in dynamics. In the *Giornata seconda* (Second Day) of the *Dialogo sopra i massimi sistemi del mondo* (Dialogue Concerning the Two Chief World Systems) (1632), Galileo embraced Nicholas Copernicus's (1473-1543)

[35] Galileo Galilei, *Le Mecaniche*, in *Opere* (Torino: Utet, 2005), vol. 1, p. 147.

suggestion that circular motion is natural not only for heavenly bodies but also for the elements, in particular the element earth and, as a consequence, for the terrestrial globe.[36] In *De revolutionibus orbium coelestium* (1543), this assumption served as a means to bypass the Aristotelian criticism that, since bodies can have only one natural tendency and the elements have natural downward and upward drives ('heaviness' and 'lightness'), they cannot have an additional circular 'natural' tendency. Along this line of reasoning, Aristotelians argued for the alleged impossibility of terrestrial motion. As to Galileo, he responded to Aristotle's geostatic objection that the daily rotation of the Earth would be a 'violent' displacement and therefore could not last forever. Galileo responded that,

> But if that which is forced cannot be eternal, then by the converse that which cannot be eternal cannot be natural; but there is no way for the Earth's downward motion to be eternal, and so much the less can it be natural, nor can any motion be natural to it which cannot be eternal to it. But if we make the Earth circularly movable, this can be eternal to it and to its parts, and therefore natural.[37]

Furthermore, in order to solve the 'tower argument' (how can a stone fall vertically from a tower, if the Earth moves westwards during the descent of the stone?), Galileo developed a sort of 'circular inertia.' It is actually a principle of indifference according to which heavy bodies preserve either their state of rest or their uniform motion around the center of gravity, if they are not hindered by external causes, because on a plane that is not inclined (*né acclive né declive*) they are not forced by the gravitational tendency to move downward with an accelerated motion.

Actually, Galileo treated the circular motion of the bodies on the Earth also in a different manner. In another passage of the Dialogue, he faced the Ptolemaic argument in the first book of the *Almagest* that terrestrial motion would be disruptive. He interpreted this reasoning as based on the analogy between the spinning Earth and turning objects by which centripetal tendencies are observable.

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[36] REF.

[37] Galileo Galilei, *Dialogue Concerning the Two Chief World Systems*, ed. by Stillman Drake (New York: The Modern Library, 2001), p. 156. Cf. idem, *Dialogo sopra i due massimi sistemi del mondo*, in *Le Opere* (Firenze: Barbera, 1968), vol. 7, p. 160: "Ma se quello che è violento non può essere eterno, pel converso quello che non può essere eterno non potrà essere naturale: ma il moto della Terra all'ingiù non può essere altramente eterno: adunque meno può essere naturale, né gli potrà essere naturale moto alcuno che non sia anco eterno. Ma se noi faremo la Terra mobile di moto circolare, questo potrà esser eterno ad essa e alle parti, e però naturale."

Here is another very ingenious argument taken from certain experiences. Circular motion has the property of casting off, scattering, and driving away from its center the parts of the moving body, whenever the motion is not sufficiently slow or the parts not too solidly attached together. If, for example, we should very rapidly spin one of those great treadmills with which massive weights are moved by one or more men walking within them [...], then if the parts of the rapidly turned wheel were not very solidly joined, it would all come apart. Or, if many rocks or other heavy materials were strongly attached to its external surface, they would not be able to resist the impetus, and it would scatter them with great force to various places far from the wheel, and accordingly from tis center. If, then, the Earth were to be moved with so much greater a velocity, what weight, what tenacity of lie or mortal would hold rocks, buildings, and whole cities so that they would not be hurled into the sky by such precipitous whirling? And neb and beasts, none of which are attached to the Earth; how would they resist such an impetus?[38]

Note that Galileo treated this issue in the same terms as Benedetti. He also clarified that the centripetal motion is a tangential rectilinear outward tendency. "The projectile acquires an impetus to move along the tangent of the arc described by the motion of the projectile at the point of its separation from the thing projecting it."[39]

This is Galileo's generalization from mechanics to cosmology, which he ascribed to Ptolemy in order to reinforce it, before he discarded it. Observations and theories derived from objects like wheels, slings or millstones could serve to treat the terrestrial globe in its entirety. The question he raised here is: Given the extreme rapidity of the daily rotation, why do terrestrial bodies not receive a centripetal trust sufficient to project them from the ground along the tangential line of the circular motion as is the case with a stone thrown from a sling? Galileo's solution is based on a distinction between the rotation of a wheel *on* the Earth and the rotation of the 'Big Wheel, Earth.'

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[38] Galilei, *Dialogue Concerning the Two Chief World Systems*, pp. 153-154. Cf. idem, *Dialogo sopra i due massimi sistemi del mondo*, p. 158: "Ècci un'altra molto ingegnosa ragione, presa da certa esperienza, ed è tale. Il moto circolare ha facoltà di estrudere, dissipare e scacciar dal suo centro le parti del corpo che si muove, qualunque volta o 'l moto non sia assai tardo o esse parti non sian molto saldamente attaccate insieme [...]. Quando dunque la Terra si movesse con tanto e tanto maggior velocitá, qual gravità, qual tenacità di calcine o di smalti, riterrebbe i sassi, le fabriche e le città intere, che da sì precipitosa vertigine non fusser lanciate verso il cielo? E gli uomini e le fiere, che niente sono attaccati alla Terra, come resisterebero ad un tanto impeto?"

[39] Ibid., p. 224. Cf. idem, *Dialogo sopra i due massimi sistemi del mondo*, p. 219: "Il proietto acquista impeto di muoversi per la tangente dell'arco descritto dal moto del proiciente nel punto della separazione di esso proietto dal proiciente."

The crucial difference rests on the fact that, in the cosmological generalization, the center of the circular motion coincides with the center of gravity. At this point, could Galileo not simply remark as earlier that no centripetal tendency would result due to the already developed proto-inertial idea that heavy bodies circulating around the gravitational center do not need to change their state? And therefore no centrifugal tendency should be taken into account *a priori*, assuming the naturalness of the circular motion around the center? Galileo develops his reasoning in a very different way.

He assumes that every body on the rotating Big Wheel is subjected to two tendencies: One is the centrifugal 'impeto' along the tangent, and the other is the centripetal 'inclinazione' along the perpendicular. The gravitational inclination, so Galileo, will always prevail over the centrifugal impetus. This can be appreciated by considering the circular angle of contact—called *angolo del contatto* or *angolo di contingenza*—between the tangent and the circle. No matter how rapid the circular motion (and thus the potential motion along the tangent) is, no body will ever have a tangential outward motion bigger than the simultaneous vertical fall. The reason has to be found in the infinitesimal littleness of the angle of contact between the rectilinear motion and the circular one, for which gravitation, no matter how small, will always be sufficient to keep the body on the ground (Figs 6 and 7).

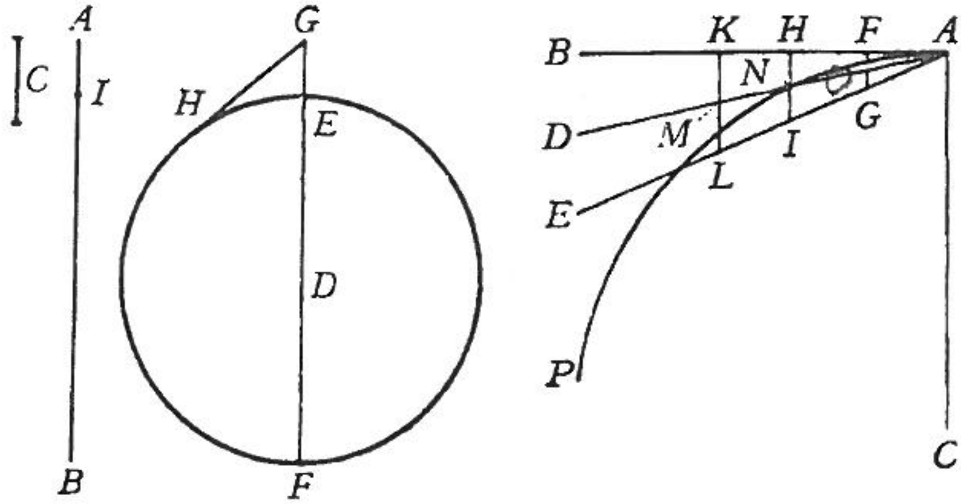
> So that if the rock thrown from a rapidly moving wheel had any such natural tendency to move toward the center of the wheel as it has to go toward the center of the Earth, it might very well return to the wheel, or rather never leave it. For the distance traveled being so extremely small at the beginning of its separation (because of the infinite acuteness of the angle of contact), any tendency that would draw it back toward the center of the wheel, however small, would suffice to hold it on the circumference. [40]

And:

[40] Ibid., p. 226. Cf. idem, *Dialogo sopra i due massimi sistemi del mondo*, p. 221: "Talché quel sasso che scagliato da quella ruota mossa in giro con velocità grande, avesse così propension naturale di muoversi verso il centro dell'istessa ruota, sì come e' l'ha di muoversi verso il centro della Terra, sarebbe facil cosa che e' ritornasse alla ruota, o più tosto che e' non se ne partisse; perché essendo, su 'l principio della separazione, l'allontanamento tanto minimissimo, mediante l'infinita acutezza dell'angolo del contatto, ogni poco poco d'inclinazione che lo ritirasse verso il centro della ruota, basterebbe a ritenerlo sopra la circonferenza."

If even assuming that the tangent lie removed from the Earth except at one point, it has been proven that the projectile would not be separated, because of the extreme acuteness of the angle of contact (if it can indeed be called angle) [...].[41]

One could expect that a quicker revolution of the Earth would give the body a centrifugal speed sufficient to eject it but, as Galileo argues, the bigger the ratio between the line representing the tangential speed and the vertical line representing the vertical acceleration is, the smaller the curvilinear angle of contact becomes, so that an infinitesimally small gravitational tendency is sufficient to keep a body on the ground.



Figures 5 and 6. Galilean diagrams from the *Dialogo* representing the tangential and vertical tendencies of a body on the rotating Earth.

In this place, I would like to point to the terminological (and conceptual) difference between Galileo and Benedetti. Whereas for the latter Benedetti both gravitational and centrifugal tendencies are *inclinationes naturales*, Galileo rigorously distinguishes between a downward "inclinazione" and a centrifugal "impeto." The *inclinazione* is an inner tendency of heavy bodies directed to the terrestrial center with an accelerated motion, while the impetus is a communicated tendency. Both tendencies are linear but none is 'natural' if one assumes the definition of natural motion as a never-ending one. Only the circular motion around the center can be eternal, therefore

[41] Ibid., p. 236. Cf. idem, *Dialogo sopra i due massimi sistemi del mondo*, p. 230: "Posto che la tangente, da un sol punto in fuori, fusse separata dalla superficie della Terra, si è [...] dimostrato che per la grande strettezza dell'angolo della contingenza (se però si deve chiamare angolo) il proietto non si separerebbe [...]."

‘natural’ in this strict sense. The conceptual framework has shifted away from the Aristotelian conceptualizations of ‘natural’ and ‘violent’ expressing the relation between straightforwardness and circularity in authors such as Nemorarius, Tartaglia and Benedetti. For them, circular motion is the *explanandum*. Apparently, Galileo’s statement that circular motion is natural makes any explanation of it superfluous, at least at a cosmological scale, since natural motion only needs to be postulated. But evidently he did not rest with this principle of circular inertia. In fact, he also offered an explanation of why the bodies and the parts of the Big Wheel Earth continue in their circular motion despite centrifugal tendencies acting upon them. In his account, the circular motion of the terrestrial globe and of its parts results from the composition of centrifugal impetus and gravitational inclination. Note that this is not the ‘constrained straightforwardness’ of his Aristotelian forerunners. Circular motion is not just a deviation but it is the resultant of different tendencies acting on one body. This is a passage from contingency (i.e., from motion as violently constrained) to the composition of motions.

From *inclinatio* to *inertia*: Cartesian Perspectives

In René Descartes’s *Le Monde* (1632-1633, printed posthumously, 1664), circular motion is, once again, ‘contingented rectilinear motion.’ His treatment is at the same time a step behind and a step forward with respect to Galileo. In Descartes’ case, the application is cosmological in a much wider sense than in the instances I have derived from Galileo’s *Dialogo*, as he develops a general theory of the world. In this view, circular motions are those of the particles of matter as well as of planets revolving about the centers of their orbits.[42]

> […] when a body is moving, even if its motion most often takes place along a curved line and, as we said above, it can never make any movement that is not in some way circular, nevertheless each of its parts individually tends always to continue moving along a

[42] On the Cartesian cosmos, see E.J. Aiton, *The Vortex Theory of Planetary Motions* (London-New York: MacDonald-American Elsevier Inc, 1972), pp. 30-64, and Stephen Gaukroger, *The Emergence of a Scientific Culture. Science and the Shaping of Modernity 1210-1685* (Oxford: Clardendon, 2006), pp. 304-317.

straight line. And so the action of these parts, that is the inclination they have to move, is different from their motion.[43]

This is the third of Descartes's three laws of nature (*loix* or *règles de la Nature*) as exposed in Chapter Seven ("Des loix de la nature de ce nouveau Monde"). It comes after the inertial law of the bodies' conservation of their state and that of the conservation of the quantity of motion.

The third law is particularly relevant from the viewpoint of our epistemological inquiry since it clearly expresses the gap between the law and the effective reality, between the straightforward tendency of all bodies and their real circular motions. Note that Descartes, unlike Galileo, calls the rectilinear tendency "*inclination*" just as Benedetti called it "*inclinatio recte eundi*." This terminological choice is apt to express its character as a natural inner tendency. The examples that Descartes choses to illustrate his claim are familiar to readers of Renaissance sources on mechanics: the wheel (*une roue*) and the sling (*fronde*) (Fig. 7). Furthermore, Descartes explains that the inclination of a body (or its tendency, action, and effort) does not refer to a voluntary action or a vitalistic drive. Rather, it has to be conceived as a constrained mechanical tendency accorded to a general and impersonal law of nature.[44] Nature itself is no imaginary and personified power ("*par la Nature je n'entens point icy quelque Déesse, ou quelque autre sorte de puissance imaginaire*"). It is just matter, and its properties ("*la Matiere mesme, entant que je la considere avec toutes les qualitez que je luy ay attribuée, comprises toutes ensamble*").[45]

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[43] René Descartes, *The World and Other Writings* (Cambridge: Cambridge University Press, 1998), p. 29. Cf. idem, *Le Monde ou Traité de la Lumière*, in *Œuvres de Descartes*, vol. 11 (Paris: Vrin, 1986), ed. by Charles Adam and Paul Tannery, pp. 1-118, Ch. VII, pp. 43–44: "Lors qu'un corps se meut, encore que son mouvement se fasse plus souvent en ligne courbe, et qu'il ne s'en puisse jamais faire aucun, qui ne soit en quelque façon circulaire [...], toutesfois chacune de ses parties en particulier tend toujours à continuer la sien en ligne droite. Et ainsi leur action, c'est à dire l'inclination qu'elles ont à se mouvoir, est differente de leur mouvement."

[44] Ibid., Ch. XIII.

[45] Ibid., Ch. VII.

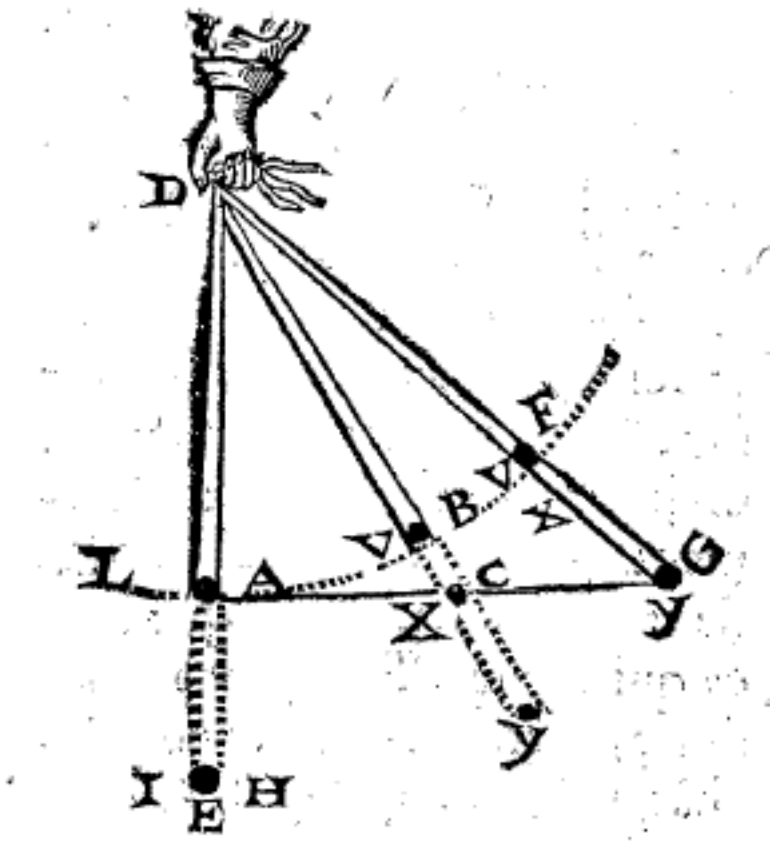

Figure 7. Descartes visualization of the centripetal tendency of the bodies through a sling, in *Le Monde*, Ch.7.

Descartes indicates the hindrances resulting from motion through the plenum as the reason for the circularity of all real natural motions.

> It is true that this equality could not be completely perfect. For, first, because there is no void at all in this new world, it was not possible for all the parts of matter to move in a straight line. Rather, since they were all just about equal and as easily divisible, they all had to form together into various circular motions.[46]

The underlying view of natural processes is intrinsically contingent, in the medieval sense that they result from concrete interactions bending the necessary laws of nature. Just as in the Scholastic tradition the difference between necessary order and contingent phenomena is expressed in theological terms:

[46] René Descartes, *The World*, pp. 32-33. Cf. idem, *Le Monde*, Ch. VIII, p. 49: "Il est vray que cette égalité n'a pû totalement estre parfaite. Car, premierement, à cause qu'il n'y a point du tout de vuide en ce [...] Monde, ila esté impossible que toutes les parties de la Matiere se soient muës en ligne droite; mais estant égales à peu prés, et pouvant presque aussi facilement estre détournées les unes que les autres, elles ont dû s'accorder toutes ensemble à quelque mouvements circulaires."

> If God always acts in the same way and consequently always produces substantially the same effect, many differences in this effect occur, as if by accident. And it is easy to accept that God, who is, as everyone must know, immutable, always acts in the same way.[47]

The Scholastic reminiscences in Descartes *système* include also the ethical dimension of contingency. Just as God is the source of natural order, he is also the source of the moral laws and, just as natural phenomena deviate from straightforwardness in their circular displacements, so the human will deviates from the uprightness of morality:

> We must say that God alone is the author of all the motions in the world in so far as they exist and in so far as they are straight, but that it is the various dispositions of matter that render the motions irregular and curved. Likewise, the theologians teach us that God is also the author of all our actions, in so far as they exist and in so far as they have some goodness, but that it is the various dispositions of our wills that can render tem evil.[48]

Conclusive Remarks: Contingency, and beyond

In the *Ètudes geliléennes*, Alexandre Koyré affirmed the complete independence of the law of inertia, which is only *in nuce* in Galileo's physics, from experience since rectilinear motion is never observed in nature. "Contrary to what we so often say, the law of inertia does not have its origin in the experience of common sense and is neither a generalization of this experience, nor even its idealization. What we find in our experience is circular motion, or more generally curvilinear movement. We are never—except in the exceptional case of free fall, which is not an inertial movement—in the presence of rectilinear movement." "Contrairement à ce qu'on affirme bien souvent, la

[47] Ibid., p. 25; cf. *Le Monde*, pp. 37-38: "Dieu agissant toujours de mesme, et par consequent produisant toujours le mesme effet en substance, il se trouve, comme par accident, plusieurs diversitez en cét effet. Et il est facile à croire que Dieu, qui, comme chacun doit sçavoir, est immuable, agit toujours de mesme façon."

[48] Ibid., p. 30; cf. *Le Monde*, pp. 46-47: "Il faut dire que Dieu seul est l'Autheur de tous les mouvemens qui sont au monde, entant qu'ils sont, et entant qu'ils sont droits ; mais que ce sont les diverses dispositions de la matiere, qui les rendent irreguliers et courbez. Ainsi que le Theologiens nous apprennent, que Dieu est aussi l'Autheur de toutes nos actions, entant qu'elles sont, et entant qu'elles ont quelque bonté : mais que ce sont les diverses dispositions de nos volontez, qui les peuvent rendre vicieuses."

loi d'inertie n'a pas son origine dans l'expérience du sens commun et n'est ni une généralisation de cette expérience, ni même son idéalisation. Ce que l'on trouve dans l'expérience, c'est le mouvement circulaire ou, plus généralement, le mouvement curviligne. On n'est jamais—sauf le cas exceptionnel de la chute, qui n'est justement pas un mouvement inertiel—en présence d'un mouvement rectiligne."[49] This is a strange statement, since, as we have seen, the vertical fall of a heavy body is not the only observable straight motion: also the beginning of the trajectory of a projectile thrown with great speed looks rectilinear. Slings and catapults are in fact the instruments with which turning wheels and rotating millstones were compared and wherefrom Benedetti, Galileo and Descartes derived the centrifugal tendencies of the parts of rotating objects. Is this not a generalization from experience? Such generalization went so far as to include the explanation of the behavior of bodies on a rotating Earth, in the case of Galileo, and the conceptualization of corpuscular and planetary motions, as was the case for Descartes. Moreover, before an inertial principle could be defined, what counted more was the observation of rectilinear motions—either the vertical fall or centrifugal tendencies—and of their circular deviations. A major physical problem faced by Scholastic and post-Scholastic mechanics was precisely that to reflect the relationship between curves and straight lines. In particular, against the Aristotelian backdrop, curvilinear motion appeared as constrained. It was a derived displacement, resulting from a *violent* external intervention bending the straightforward *natural* tendency of a moving body. As I said, in this Aristotelian and post-Aristotelian context, circular motion was seen as contingent. That is to say, it is the deviation from natural order depending from an obstacle, indicated as *secundum quid*.

As I have argued, the concept of 'secundum quid' is embedded in the Scholastic reflection on natural necessity, order and contingency. In the medieval philosophy, 'contingency' expressed first of all an ontological condition. It is the condition of all created beings the existence of which depends from the free decision of God. Second, and more important for natural philosophy, contingency referred to a model of causality in which the observed phenomena represent a partial fulfillment of an underlying order, or of natural laws. Accordingly, elementary bodies express their necessary laws in a limited manner, that is, they have to be explained through the so-called *necessitas conditionata* or *necessitas secundum quid*. Contingency is the relation between necessary

[49] Alexandre Koyré, *Études galiléennes* (Paris: Hermann, 1986), p. 206.

order and phenomenal reality. The gap has to be explained, and was explained with a *quid*, a factor or a determination. Accordingly, a *quid* was introduced into mechanics to account for circular motions in terms of mechanical constraints.

According to the medieval *scientia de ponderibus*, for the treatment of the balance, two determinations were considered: first, the circle resulting from the inclusion of the vertical motions of the weights in a mechanical system, and second, the *situm* (location) of the weights in a mechanical system determining a variation in heaviness. The reflection on *gravitas secundum situm* (positional heaviness), from Nemorarius through Benedetti, presupposes this twofold *quidditas*, and focuses on the latter aspect (the variation of the heaviness).

The conviction that circular motion, as a violent motion, requires an explanation, is based on the mental model that "circular motion is constrained (or *contingented*) straightforwardness." However embedded in the medieval discourse on contingency, the several attempts to quantify the *quid* accounting for the deviation are witnesses to the common effort to overcome the qualitative and undeterminable characterization of contingency as a form of causality. What was maintained, for instance in Descartes, was the idea of a gap between law and phenomenon. Yet, if the deviation from the law can be perfectly quantified, then also the separation between order of nature and its realization is virtually eliminated, that is, the fracture between absolute necessity and conditional necessity is recomposed. To be sure, this step toward the necessitation of nature, resulting from the abandonment of ontological contingency, was accomplished only after Descartes. As for the original theological element of contingency, the dependency of the world from God's free decision, was also to disappear with Baruch Spinoza, representative of one of the most radical positions in seventeenth-century philosophy. But these considerations bring us far beyond the discussion about the embedding of Renaissance mechanics in a natural conception marked by the principle of contingency.

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