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Abstract

Credible implementation of climate change policy, consistent with the 2°C limit, requires a large proportion of current fossil fuel reserves to remain unused. This issue, named the Carbon Bubble, is usually presented as a required asset write-off, with implications for investors. For the first time, we discuss its implications for macroeconomic policy. We embed the Carbon Bubble in a macroeconomic model exhibiting a financial accelerator: if investors are leveraged, the Carbon Bubble may precipitate a fire-sale of assets across the economy, and generate a large and persistent fall in output and investment. We find a role for policy in mitigating the Carbon Bubble.

Keywords

Carbon Bubble, fire-sale, deleveraging, resource substitution, 2°C target.

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1 Introduction

In 1996, EU national governments set a global temperature target of two degrees Celsius (°C) above pre-industrial level, which was made international policy at the 2009 United Nations Climate Change Conference in Copenhagen. If we want to reduce the probability of exceeding 2°C warming to 20%, then only one-fifth of the earth's proven fossil fuel reserves can be burned unabated (Carbon Tracker Initiative, 2011). Since this translates broadly into a near term cessation of all coal use and only a partial exploitation of proven oil and gas reserves (McGlade and Ekins, 2015), the imposition of a climate policy consistent with these calculations would mean that the fundamental value of many fossil fuel assets must be written-off. With an inappropriate terminology for an economist familiar with the theory of bubbles (e.g. Tirole, 1985), this issue has come to be known by the wider public as the Carbon Bubble (Carbon Tracker Initiative, 2011, Rolling Stone, 2012): the reasoning is that the current positive market value of these "stranded assets" is a "bubble", because their fundamental value is zero under the 2°C target. Whereas the majority of the discussions of the Carbon Bubble that have appeared to date focus upon the risks to investors off climate policy (e.g. Carbon Tracker Initiative, 2011, 2013), we consider, for the first time, the impact of climate policy on financial markets and the macroeconomy, and consequently the appropriate macro-financial policies that should accompany serious climate policy.

In this paper, we model the consequences of the implementation of a climate policy in the spirit of the 2°C target in a modified version of Kiyotaki and Moore's (1997) model, where entrepreneurs borrow from savers using their current asset holdings as collateral. We take as given that climate science mandates a severe climate policy response, such that society has a limited "carbon budget" relative to its ability to emit carbon pollution. We show that naïvely imposing this carbon budget damages the balance sheets of entrepreneurs, and in the presence of financial frictions, this has major macroeconomic implications. Indeed, these write-downs decrease the debt-capacity of the non-financial sector, which must then reduce the level of leveraged investment. As economic activity worsens, the asset price drop fuels further debt capacity reductions in a downward spiral.

Forward looking markets could turn an announced carbon budget into a "sudden stop" akin to, or worse than, the 2008 Financial Crisis.

We underline that a recessionary response is particularly damaging with respect to the implementation of the climate policy itself, as one of its aims is to provide the incentives for investments in alternative energy capital, in order to replace the current fossil fuel based energy infrastructure. A substantial stock of zero carbon productive capacity will need to be in place at the point at which the carbon budget is exhausted, but the bursting of the Carbon Bubble could throw the economy into a deep recession, thus depriving green technology of investment funds when they are most needed. Even if the fossil fuels assets really should be written-off to avoid disastrous global warming, the implementation of such a policy must pay cognisance to the impact that it will have upon investment.

In this respect, we then consider the social planner's problem in facilitating the transition from a high carbon economy to the carbon-free era, by choosing policies that maximise social welfare. Alongside the credible implementation of climate policy consistent with the 2°C limit, we consider policies that transfer investors' debts to the government, subsidise investment, and provide government guarantees on investors' borrowings. We show that these macroeconomic policies mitigate the impact of the Carbon Bubble upon the balance sheets of investors: their improved net asset position allows them to invest more in replacement zero carbon energy capital than in the no-policy case, driving the economy faster out of the recession. These policies are welfare enhancing, even if such policies are welfare destroying under normal circumstances.

Our main contribution is to begin to answer the questions posed when considering the Carbon Bubble in a macroeconomic context: is there a role for policy in mitigating the impact of the Carbon Bubble? We find that policy protecting investors' balance sheets mitigates the macroeconomic downturn, and leads to higher investment in the replacement zero carbon productive capacity over the period in which we still use carbon emitting productive capacity.

The remainder of this article is organised as follows. Section 2 reviews the literature. Section 3 presents the theoretical model we employ. Section 4 describes the calibration.

Section 5 sets out the Carbon Bubble scenario, and also examines policies the planner can implement additionally. Finally, Section 6 concludes.

2 Previous Literature

This article belongs to the literature studying the potential effects of environmental regulations on the economy, combining insights from environmental economics and macroeconomics. Among these, the standard framework is the integrated-assessment model (IAM). Indeed, since the pioneering work of Nordhaus, a plethora of IAMs have been constructed to evaluate optimal paths for carbon dioxide emissions and associated prices.[1] However, IAMs do not consider irreversible catastrophic events by construction (Pindyck, 2013, Stern, 2013, Weitzman, 2013), and thus are not generally consistent with the concept of a binding carbon budget.[2] We take the proposed policy seriously, and impose it exogenously rather than deriving it from an IAM. This is based upon evidence that emphasises the impact of cumulative emissions (Allen et al., 2009, Matthews et al., 2009, Zickfeld et al., 2009), and is predicated upon the risk of catastrophic damages associated with highly non-linear threshold effects or runaway feedback effects (e.g. IPCC, 2018).

This article is also connected to a literature using dynamic stochastic general equilibrium models with pollution. For example, Angelopoulos et al. (2010, 2013), Fischer and Springborn (2011), Heutel (2012), Annicchiarico and Di Dio (2015), and Dissou and Karnizova (2016) have compared the macroeconomic performances of different emission policies in an economy with exogenous productivity shocks (see Fischer and Heutel, 2013 for a literature review). Annicchiarico and Di Dio (2017) have extended these analyses by studying the optimal environmental and monetary policy mix. Our novelty is to study

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[1] See Nordhaus and Boyer (2000) for a description of the original modelling, Golosov et al. (2014) and Hassler et al. (2016) for the new generation of "analytical" IAMs, and Nikas et al. (2019) for a literature review.

[2] For this to arise, one needs both a dependence upon cumulative emissions, and marginal damages tending to infinity at the carbon budget. Whereas IAMs can easily incorporate a dependence on cumulative emissions (e.g. Golosov et al., 2014), adding a catastrophic threshold renders their conclusions trivial. With low marginal climate damages below the threshold, these models would balance marginal damages against the marginal costs of eliminating emissions, at the singularity in the marginal damages induced by the damage function threshold.

the optimal policy in response to endogenous macroeconomic fluctuations arising from the environmental policy itself. Central Banks and financial regulators are also investigating this issue (see Campiglio et al., 2018).

Finally, this article is related to the recent literature investigating the impact of climate change on the financial system and the macroeconomy. Campiglio (2016) discusses the role of monetary and macro-prudential financial regulation in expanding credit towards low-carbon firms. Dietz et al. (2016) and Mercure et al. (2018) provide estimates about the climate-induced or climate policy induced loss of assets or asset value at the economy-wide level. van der Ploeg and Rezai (forthcoming) propose a model of exploration investment, discoveries, and depletion to analyse the effects of different climate policy instruments on asset stranding. Dafermos et al. (2018) use an ecological stock-flow consistent model to gauge the consequences of severe global warming on the liquidity of firms, financial stability, and credit, and argue that a global green quantitative easing programme could ameliorate the financial distress caused by climate change.[3]

3 The Model

We extend Kiyotaki and Moore's (1997) full model by considering two types of investment good (high carbon and zero carbon) and by introducing a simple policymaker. The model has two types of agent, which we label entrepreneurs and savers, and two types of capital, which we label fixed capital and energy capital. Credit-constrained entrepreneurs borrow from unconstrained savers in the fixed capital market, and combine the fixed capital and the energy capital to produce a final good. The final good can either be consumed or invested in energy capital.

The model follows Kiyotaki and Moore (1997) closely, and we argue that it is a highly appropriate first-order model in this context. First, it does not include an endogenous climate sector, and we impose climate policy exogenously. We take a carbon budget

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[3] Despite a very different focus and modelling framework, Dafermos et al. (2018) argue for similar macroeconomic policy interventions, with their green quantitative easing programme being akin to a combination of our "transfer of investors' debt" and "subsidy" policies in Section 5.

scenario seriously, and deliberately exclude the degenerate solution of "not implementing a carbon budget" from the analysis of solutions to the problems thrown up by the Carbon Bubble. If a particular carbon budget is always the optimal policy, then the simplest, most tractable way of implementing this is to impose it exogenously.

Second, the model has a stock of "fixed capital" that cannot be invested in, nor does it depreciate. Our focus is on understanding the impact of a restriction in the availability of some "energy capital", via the financial accelerator mechanism at the macroeconomic level, on new investment in "alternative energy capital": we study the impact of the Carbon Bubble on investment in the transition of the energy system to zero carbon, under conditions of a fixed stock (though not a fixed valuation) of "all other capital". Again, this is a parsimonious treatment of this issue.[4]

3.1 Primitives of the Model

Time is discrete and indexed by t. There are two types of infinitely lived agents: a continuum of entrepreneurs of mass one, and a continuum of savers of mass m. Entrepreneurs and savers maximize the expected discounted utilities from consumption,

$$\max_{\{x_s\}} E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} x_s \right] \qquad \text{and} \qquad \max_{\{x'_s\}} E_t \left[\sum_{s=t}^{\infty} (\beta')^{s-t} x'_s \right] \tag{1}$$

where x_t and x'_t represent consumption at date t of the entrepreneur and the saver respectively; $\beta, \beta' \in (0,1)$ indicate the discount factors; and E_t indicates expectations formed at t. Both types of agent are risk neutral but they differ in their rates of time preference: entrepreneurs are more impatient,[5]

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[4] The model is appropriate for (at least) two other reasons. First, most models of the financial accelerator share qualitatively similar features (Gerke et al., 2013), and, among these, Kiyotaki and Moore (1997) is a tractable model with fire-sale dynamics. Second, the Leontief specification for the entrepreneurs' technologies, outlined in the next section, captures the very low elasticity of substitution between energy and other inputs to production, at least in the short-run, measured by Hassler et al. (2015). In a nested CES aggregate production function model, they estimate an elasticity of substitution between energy and a labour-capital composite of 0.0044, not significantly different from zero.

[5] Exogenous ex-ante heterogeneity on the subjective discount factors allows us to keep the model tractable and ensures the model simultaneously has borrowers and lenders. This is in line with many dynamic general equilibrium models of financial friction e.g. Iacoviello (2005), Iacoviello and Neri (2010), Devereux and Yetman (2009), Pariès et al. (2011), and Liu et al. (2013).

Assumption A *(Impatient entrepreneurs)* $\beta < \beta'$.

There are three types of goods: fixed capital (K), energy capital (Z), and non-durable commodity. The non-durable commodity cannot be stored but can be consumed or invested in energy capital. The energy capital has two flavours: high carbon and zero carbon, indexed by H and L respectively. The fixed capital does not depreciate and is available in a fixed aggregate amount, given by $\bar{K}$, while both types of energy capital depreciate at rate $1-\lambda$ per period. For example, the energy capital could include coal mines, power stations, and a balanced electricity grid, while the fixed capital includes the factory using the electricity, but does not directly care how this electricity has been produced.[6]

The government can levy a tax on the output of an entrepreneur who uses high carbon energy capital, i.e. a carbon tax, and can provide a green subsidy to entrepreneurs using zero carbon energy capital. The net position of the government is either financed through a lump-sum tax or distributed through a lump-sum transfer on a per capita basis, i.e. the government runs a balanced budget.

At the end of each time period $t-1$, there is a competitive asset market and a competitive one-period credit market. In the former, one unit of the fixed capital is exchanged for q_{t-1} units of the commodity; in the second, one unit of the commodity at date $t-1$ is exchanged for R_{t-1} units of the commodity at date t. The commodity is assumed to be the numeraire, so that its price is normalised to unity. Then q_t represents the price per unit of fixed capital, and R_t is the gross interest rate. At the start of a new period t, markets are closed: stocks of fixed capital, energy capital, and debt holdings are state variables. Production then takes place over period t.

3.2 Entrepreneurs

An entrepreneur produces a quantity of the commodity, y, with a one-period Leontief production function: fixed capital, k, is combined with energy capital, z, in $1:1$ proportion.

This period's decisions affect next period's production. The entrepreneur can choose

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[6] Energy capital depreciation can be understood as the aggregate of what we normally view as capital depreciation, plus exhaustible resource depletion (of e.g. coal and oil).

between two technologies. Choosing the first, k_{t-1} units of fixed capital are combined with z^H_{t-1} units of the high carbon energy capital, producing y_t units of the commodity. However, this choice implies that the after tax output available to the entrepreneur will be reduced by any proportional carbon tax implemented, $\tilde{\tau}_t$:

$$y_t = \left(a^H + c\right) \times \min\left(k_{t-1}, z^H_{t-1}\right) \tag{2a}$$

$$\left(1 - \tilde{\tau}_t\right) y_t = \left(a^H - \tau_t + c\right) \times \min\left(k_{t-1}, z^H_{t-1}\right). \tag{2b}$$

The productivity of this technology is $a^H + c$, but a portion $c/(a^H + c)$ of the y_t units of output are not tradable and must be consumed by the entrepreneurs (who must therefore pay any carbon tax levied out of tradable output).[7]

Choosing the second technology, the entrepreneur combines k_{t-1} units of fixed capital with z^L_{t-1} units of the zero carbon energy capital and benefits from a proportional subsidy, $\tilde{\varsigma}_t$.[8] The output available to the entrepreneur, however, will be increased by only a fraction $\delta \in [0, 1]$ of the subsidy implemented, where δ is a structural parameter representing the effectiveness of the subsidy:

$$y_t = \left(a^L - (1 - \delta)\,\varsigma_t + c\right) \times \min\left(k_{t-1}, z^L_{t-1}\right) \tag{3a}$$

$$\left(1 + \tilde{\varsigma}_t\right) y_t = \left(a^L + \delta\varsigma_t + c\right) \times \min\left(k_{t-1}, z^L_{t-1}\right). \tag{3b}$$

As before, $c \times \min(k_{t_1}, z^L_{t-1})$ units of the y_t units produced are not tradable and must be consumed by the entrepreneurs.

In line with Acemoglu et al. (2012), we assume that zero carbon energy capital is intrinsically less productive than high carbon energy capital,

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[7] The ratio $a^H/(a^H + c)$ represents an upper bound on the entrepreneur's savings rate. This is introduced in Kiyotaki and Moore (1997) to avoid the possibility that the entrepreneur keeps postponing consumption. While this assumption and the presence of linear preferences, but different discount factors, can be considered as unorthodox modelling choices, Kiyotaki and Moore (1997, Appendix) show that the same qualitative results can be obtained using an overlapping generations model with standard concave preferences and conventional saving/consumption decisions.

[8] In the remainder of the article, we discuss only $\tau_t = \tilde{\tau}_t(a^H + c)$ and $\varsigma_t = \tilde{\varsigma}_t\left(a^L - (1 - \delta)\,\varsigma_t + c\right)$, positive bijective transformations of the proportional tax rate and subsidy rate into units that can be compared to the productivities of the two alternative technologies.

Assumption B *(Productivity advantage of the carbon sector)* $a^H > a^L$.

The commodity can be consumed or invested. For that portion of their invested output, the entrepreneur converts ϕ units of the commodity into one unit of energy capital: ϕ is the output cost of investing. Note that, instead of writing the model in terms of differing productivities of high and zero carbon technologies and a single cost of investment, $\{a^H, a^L, \phi\}$, we would reach qualitatively the same results by writing the model in terms of a single productivity and differing output costs of investing in energy capital, $\{a, \phi^H, \phi^L\}$, with $\phi^H < \phi^L$ as in van der Zwaan et al. (2002).[9]

We impose an upper limit on the obligations of the entrepreneurs. Suppose that, if the entrepreneur repudiates their contract, the lender can only repossess the fixed asset. Entrepreneurs are therefore subject to the following borrowing constraint:

$$b_t \leq \frac{q_{t+1}k_t}{R_{t+1}}. \tag{4}$$

Consider an entrepreneur who holds k_{t-1} units of fixed capital, $z_{t-1} = k_{t-1}$ units of energy capital (since the Leontief production function means that any other quantity of energy capital is suboptimal), and has gross debt b_{t-1} at the end of period $t-1$. At date t they receive net income of $a_t^i k_{t-1}$ (depending on the technology used), they incur a new loan b_t and acquire more fixed capital, $k_t - k_{t-1}$. Having experienced depreciation and having increased their fixed capital holdings, the entrepreneur will have to convert part of the tradable output to energy capital. In general, entrepreneurs will have to invest $\phi(k_t - \lambda k_{t-1})$ in order to have enough energy capital to cover depreciation and new fixed capital acquisition; they then repay the accumulated debt, $R_t b_{t-1}$, and choose how much to consume in excess of the amount of non-tradable output, $(x_t - ck_{t-1})$. In addition, they receive a per capita transfer from the government or pay the per capita tax, g_t, depending on the net position of the government. Thus, the entrepreneur's flow-of-funds constraint,

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[9] That $a^H > a^L$ (or $\phi^H < \phi^L$) should be an uncontroversial assumption. While the costs associated with adding a unit of electricity generation using renewables can be competitive with the equivalent fossil fuel costs, once we factor in the cost of storage requirements or reworked electricity grid architecture, renewables are still more expensive. And lower costs per unit output here are equivalent to a higher productivity parameter.

as at the end of period t, is given by either

$$q_t(k_t - k_{t-1}) + \phi(k_t - \lambda k_{t-1}) + R_t b_{t-1} + (x_t - c k_{t-1}) + \tau_t k_{t-1} = a^H k_{t-1} + b_t + g_t \tag{5a}$$

$$q_t(k_t - k_{t-1}) + \phi(k_t - \lambda k_{t-1}) + R_t b_{t-1} + (x_t - c k_{t-1}) = a^L k_{t-1} + \delta \varsigma_t k_{t-1} + b_t + g_t. \tag{5b}$$

The first equation refers to an entrepreneur who uses the high carbon energy capital, while the second relates to the use of the zero carbon energy capital.

Finally, in each period only a fraction $\pi \in (0,1)$ of entrepreneurs have an investment opportunity.[10] Thus, with probability $1-\pi$, the entrepreneur cannot invest and must downsize the scale of operation, since the depreciation of their energy capital implies $z_t^i = \lambda z_{t-1}^i$. This probabilistic investment assumption, combined with Leontief production, means that with probability $1-\pi$ the entrepreneur also faces the constraint

$$k_t \leq \lambda k_{t-1}. \tag{6}$$

3.3 Representative Saver

Savers are willing to lend commodities to entrepreneurs in return for debt contracts, and they also produce commodities by means of a decreasing return to scale technology, which uses only the fixed capital as an input, k'_{t-1}, and takes one period, according to

$$y'_t = -\frac{m}{2\beta'}\left(k'_{t-1}\right)^2 + \frac{\bar{K} - \nu}{\beta'} k'_{t-1} - \frac{const}{m}, \tag{7}$$

where ν and *const* are positive parameters.[11] Savers are never credit constrained because they can trade all their output and no particular skill is required in their production process. Savers solve the relevant maximization problem in (1), subject to their budget

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[10] This assumption is introduced by Kiyotaki and Moore (1997, page 229 - 230) to capture the idea that "investment in fixed assets is typically occasional and lumpy".

[11] Equation (7) is implicit in Kiyotaki and Moore (1997), who assume a marginal productivity that is linearly decreasing in the fixed capital stock owned by the savers, $dy'_t/dk'_{t-1} \equiv (\beta')^{-1}(K_{t-1} - \nu) = (\beta')^{-1}[(\bar{K} - \nu) - m k'_{t-1}]$, since $k'_{t-1} = (\bar{K} - K_{t-1})/m$. Equation (7) is obtained integrating this up.

constraint,

$$q_t\left(k'_t - k'_{t-1}\right) + R_t b'_{t-1} + x'_t = y'_t + b'_t + g_t. \tag{8}$$

A saver who produces y'_t units of the commodity, incurs (issues) new debt, b'_t, and receives (pays) the per capita government expenditure (tax), g_t, can cover the cost of buying fixed capital, $q_t\left(k'_t - k'_{t-1}\right)$, repaying (collecting on) the previous debt (including interest), $R_t b'_{t-1}$, and consuming, x'_t.

3.4 Competitive Equilibrium

An equilibrium consists of a sequence of prices $\{q_t, R_t, \tau_t, \varsigma_t\}$, allocations for the entrepreneur $\{x_t, k_t, z_t, b_t\}$ and the saver $\{x'_t, k'_t, b'_t\}$ such that, taking the prices as given, each entrepreneur solves the relevant maximization problem in (1) subject to the appropriate constraints in (2) - (5); each saver maximizes the relevant part of (1) subject to (7) and (8); the government always runs a balanced budget; and the goods, asset, and credit markets clear.

Using $\gamma_t \in [0,1]$ to indicate the share of aggregate entrepreneurs' fixed capital holdings, which are combined with high carbon energy capital at t, let I_t^H, I_t^L, B_t, $mb'_t \equiv B'_t$, K_t, $mk'_t \equiv K'_t$, X_t, $mx'_t \equiv X'_t$, Y_t, $my'_t \equiv Y'_t$, $\tau_t\gamma_t K_{t-1} \equiv T_t$, $\varsigma_t(1-\gamma_t)K_{t-1} \equiv P_t$, $(1+m)g_t \equiv G_t$ be aggregate investment flows, borrowing, fixed capital holdings, consumption, output, carbon tax, green subsidy, and aggregate lump-sum transfer (positive) or tax (negative). The government budget constraint and the market clearing conditions for assets, credit, and goods are then, respectively,

$$T_t - P_t = G_t \tag{9a}$$

$$K_t + K'_t = \bar{K} \tag{9b}$$

$$B_t + B'_t = 0 \tag{9c}$$

$$I_t^H + I_t^L + X_t + X'_t + G_t - T_t + P_t = Y_t + Y'_t. \tag{9d}$$

Note that, given assumption A, the impatient entrepreneurs borrow from the patient savers in equilibrium. Moreover, given that savers are risk neutral and there is no uncertainty, the rate of interest, R_t, is constant and determined by the patient saver's rate of time preference i.e. $R_t = 1/\beta' \equiv R$.

To characterize equilibrium, we start with the savers. Since they are not credit constrained, their fixed capital holdings are such that they are indifferent between buying and selling this capital. This is the case if the rate of return from buying fixed capital is equal to the rate of return of selling,

$$\frac{dy'_{t+1}}{dk'_t} = Rq_t - q_{t+1}. \tag{10}$$

This implies the following forward-looking equation of motion for the capital price,

$$q_t = K_t - \nu + \frac{1}{R} q_{t+1}. \tag{11}$$

Entrepreneurs who can invest at date t will prefer borrowing up to the limit and investing, rather than saving or consuming, hence limiting their consumption to the current non-tradable output ($x_t = ck_{t-1}$): for them, the credit constraint in (4) is binding. Conversely, an entrepreneur who cannot invest at t, given that they will not want to waste their remaining stock of energy capital, will adjust their levels of debt and fixed capital such that Equation (6) will hold with equality.[12] At the aggregate level, these imply that entrepreneurs' aggregate fixed capital holdings and borrowing evolve according to

$$\begin{aligned} K_t = \frac{\pi}{q_t + \phi - \frac{q_{t+1}}{R}} \left[(q_t + \phi\lambda + a_t) K_{t-1} - RB_{t-1} + \frac{\gamma_t \tau_t - (1-\gamma_t)\varsigma_t}{1+m} K_{t-1} \right] + \\ + (1-\pi)\lambda K_{t-1} \end{aligned} \tag{12}$$

$$B_t = q_t (K_t - K_{t-1}) + \phi (K_t - \lambda K_{t-1}) + RB_{t-1} - a_t K_{t-1} - \frac{\gamma_t \tau_t - (1-\gamma_t)\varsigma_t}{1+m} K_{t-1}, \tag{13}$$

where a_t represents the net productivity of the entrepreneurial technology.[13]

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[12] We refer the interested reader to Kiyotaki and Moore (1997, footnote 22) for the full proof of the claims on the behaviour of investing and non-investing entrepreneurs.

[13] Formally, $\gamma_t = 1 \Rightarrow a_t = a^H - \tau_t \geq a^L + \delta\varsigma_t$, $\gamma_t = 0 \Rightarrow a_t = a^L + \delta\varsigma_t \geq a^H - \tau_t$, and $\gamma_t \in (0,1) \Rightarrow a_t = a^L + \delta\varsigma_t = a^H - \tau_t$. For convenience, we just use $a_t = a_t(a^H, a^L, \tau_t, \varsigma_t)$ i.e. the entrepreneur net or private productivity is a function of intrinsic technological productivities plus government tax and subsidy policy.

Equations (11), (12), and (13) give us a three dimensional system of dynamic equations, which characterises the model.

3.5 Steady State

Given constant a, τ, and ς there exists a steady state equilibrium, $(q^\star, K^\star, B^\star)$, the value of which is a function of $\gamma^\star \in [0,1]$ (which we take parametrically), where[14]

$$q^\star = \frac{R}{R-1}\left\{\frac{\pi\left[a + \frac{\gamma^\star\tau-(1-\gamma^\star)\varsigma}{1+m}\right] - \phi(1-\lambda)(1-R+R\pi)}{\pi\lambda + (1-\lambda)(1-R+R\pi)}\right\} \tag{14a}$$

$$K^\star = \frac{R-1}{R}q^\star + \nu \tag{14b}$$

$$B^\star = K^\star\left\{\frac{\phi\lambda - \phi + a + \frac{\gamma^\star\tau-(1-\gamma^\star)\varsigma}{1+m}}{R-1}\right\}. \tag{14c}$$

Once the government has set its tax and subsidy policies, τ and ς, which determine the net private productivity for the entrepreneurial sector, a, Equation (14c) reports that in steady state the entrepreneurs use the amount of tradable output, $aK^\star$, together with (net of) the transfer (tax) from the government, $K^\star[\gamma^\star\tau - (1-\gamma^\star)\varsigma]/(1+m)$, to repay the interest on the debt, $(R-1)B^\star$, and to replace the amount of energy capital that has depreciated in the period, $\phi(1-\lambda)K^\star$.

Figure 1 provides a visual representation. The horizontal axis shows demand for fixed capital from the entrepreneurs, from left to right, and from the savers, from right to left. Since the market for fixed capital clears, the sum of the two demands is equal to $\bar{K}$. The vertical axis consists of the marginal product of fixed capital, which is constant at $a^i + c$, $i = \{H, L\}$, for entrepreneurs but decreasing with fixed capital usage for savers.

Were the debt enforcement problem absent, and absent any government policy, the economy would be able to reach the first best allocation, E_{FB}, in which the entirety of the aggregate entrepreneurs' fixed capital holdings are used with high carbon energy capital. In this scenario, entrepreneurs are not constrained in the amount they can borrow. Thus,

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[14] See Appendix A.1 for further assumptions used in obtaining the model's equilibrium and A.7 for the stability of the system. Note that, for any $(\gamma^\star, a)$, this interior steady state is unique.

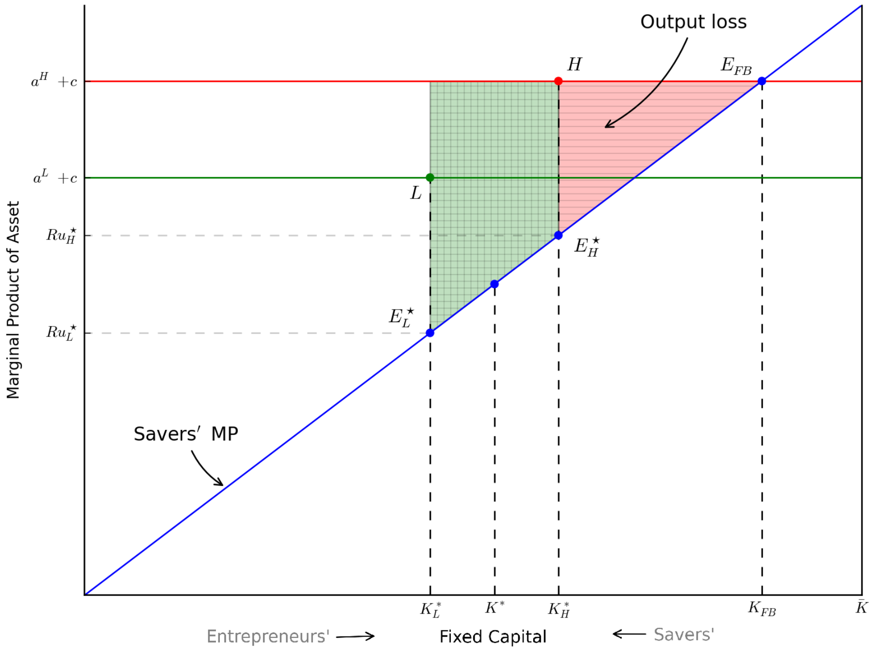

Figure 1: Comparison of steady states

the marginal products of the two sectors are identical. In contrast, in the constrained economy too much of fixed capital is left in the hands of the savers and entrepreneurs have a higher marginal product than savers.

Consider two particular equilibria. In a world that uses only the high carbon energy capital and no carbon tax, the equilibrium is given by $E_H^\star$, where the aggregate entrepreneurs' fixed capital holding is $K_H^\star$. Conversely, the fully decarbonised equilibrium with no green subsidy is $E_L^\star$, with corresponding $K_L^\star$. It is easy to show that the former equilibrium provides a larger share of fixed capital to the entrepreneurs compared to the latter, $K_H^\star > K_L^\star$. As a consequence, output, investment, borrowing and consumption are higher. Intuitively, having a lower net private productivity in the entrepreneurial sector, means entrepreneurs not only earn less revenue with respect to a higher net private productivity, but they also have lower net worth. Thus, in general, entrepreneurs can borrow, invest, and produce less. To clear the market, the demand for fixed capital by the savers must be higher in the decarbonised world, which requires a lower user cost. But a lower user cost is associated with a lower fixed capital price and thus with a lower net worth of the constrained sector, which translates into less collateral. Less collateral means lower investment and production, and so on in a vicious circle.[15]

4 Calibration Strategy

In this section, we present briefly our calibration strategy, but an interested reader can find more details in Appendix A.5. We match energy data, and broadly match the experience of the 2008-09 Financial Crisis.[16]

4.1 Definition of Welfare and Time

We calibrate *const* in the savers' production function under the assumption that, arbitrarily, the (decarbonised) steady state consumption flow is the same for both individual savers and entrepreneurs.

The utilitarian social welfare function maximised by the policymaker at $t = 0$ is assumed to be the discounted present value of future consumption. We assume that savers are infinitesimally more patient than the entrepreneurs and thus, for any practical calculation, their discount factors are the same. Therefore, policy is chosen by the policymaker to maximise the present discounted value of all future "Net National Income" flows in the model,

$$W_t = \sum_{s=t+1}^{\infty} \left[\beta^{s-t} E\left[x_s\right] + \left(\beta'\right)^{s-t} mx'_s \right] \approx \sum_{s=t+1}^{\infty} R^{t-s} \left[E[x_s] + mx'_s\right].$$

We follow Kiyotaki and Moore (1997), and set the depreciation rate of energy capital, $\lambda = 0.975$, and the interest rate, $R = 1.01$, so that time periods can be interpreted as quarter years. These correspond to a depreciation rate of 10% per annum for energy capital, and to an annual interest rate on debt of 4%. Finally, we normalise productivity, $a^L = 1$.

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[15] See Appendix A.4 for some comparative statics on the value of γ.

[16] Qualitatively, there is very little dependence upon the calibration: our general conclusions about policy effectiveness are robust to the particulars of this calibration.

4.2 The Energy Sector

According to Newell et al. (2016) and EIA (2016, Table 1.2), fossil fuels represent around 80% of energy generation, and thus $\gamma^\star = 0.8$. The EIA (2015, Table 1) provides figures on the "total system levelized costs of electricity", which we apply to their energy mix figures to estimate that fossil fuel generation costs around 10% less per unit of energy supplied. Thus, $a^H = 1.10$.

Both fossil fuels and alternative energy generating capacity exist in the data, and we can replicate this in the model only if their net private productivities, after taxes and subsidies, have been equalised. This means that net private productivity is $a = a^L$, and carbon tax $\tau = a^H - a^L$.

EIA (2016, Table 1.2) suggests that energy expenditure as a percentage of total GDP is around 7.5%. In order to interpret this within the model, we imagine that in steady state the expenditure by the entrepreneurial final goods production sector on energy intermediate goods is equal to the flow value of the energy capital value i.e. $(R-1)\phi K^\star$ and that this represents total expenditure on energy. Therefore, we use the following object as a calibration target:

$$\frac{(R-1)\phi K^\star}{GDP} = \frac{(R-1)\phi K^\star}{(a+c+\gamma^\star\tau)K^\star + R(\bar{K}-\nu)(\bar{K}-K^\star) - 0.5R(\bar{K}-K^\star)^2 - const} = 7.5\%.$$

4.3 The Financial Crisis

As explained by Kiyotaki and Moore (1995, 1997), the contemporaneous response to a shock and the persistence of its effects are influenced by both π and ϕ (but also by ν). We calibrate these parameters to the impact of the financial crisis on output and upon asset values.

In particular, we hit our steady state with a one-time initial unexpected shock to the wealth of the entrepreneurs, loosely based on the experience of the 2008-09 Financial Crisis. The financial crisis began with the realisation that the fundamental value of subprime mortgages (and the CDOs into which they were bundled) was much lower than had previously been recognised. Hellwig (2009) estimated that the total value of subprime mortgages outstanding was $1.1tn in the second quarter of 2008. Dietz et al. (2016) use data from the Financial Stability Board to claim that the total value of global non-bank financial assets in 2013 was $143.3tn. Since both figures are an approximation, we use 1/150 as a rounded figure for the loss in entrepreneurial wealth. The initial shock is $\Delta K^\star$, where Δ is given by the solution to

$$\frac{\Delta K^\star}{\phi K^\star + q^\star \bar{K}} = \frac{1}{150}.$$

Data from FRED[17] suggests that annual percentage changes in "Constant GDP per capita for the World" were consistently just below 3% prior to the financial crisis, but fell to less than -3% when the crisis struck. Thus, our financial crisis scenario involves a fall in output of around 6%, since there is no growth in per capita incomes in our steady state model. Moreover, a back of the envelope calculation suggests that a well diversified investor experienced a fall in asset values of around 20-25%,[18] which we also match.

4.4 Stranded Fossil Fuel Assets

The Carbon Tracker Initiative (2013) estimates that 65-80% of listed companies' current reserves cannot be burnt unmitigated; IEA (2012) argues that no more than one-third of proven reserves of fossil fuels can be consumed. According to Robins et al. (2012), carbon constraints could impact the valuations of coal assets by as much as 44%, with average impact to the stock market valuation of mining companies between 3% and 7%. Moreover, "given that the mining sector comprises around 12% of the FSTE100 index,

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[17] Accessible at `https://research.stlouisfed.org/fred2/series/NYGDPPCAPKDWLD`.

[18] The loss of 1/150 should represent a relatively mild adverse event to a well diversified investor. Instead we saw the S&P500 decline by 40% (`https://research.stlouisfed.org/fred2/series/SP500`), and corporate bond spreads rise. For example, "Moody's Seasoned Baa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity" rose from just over 1.5% prior to the financial crisis, to more than 5.5% at the height of the crisis (`https://research.stlouisfed.org/fred2/series/BAA10YM`). On the other hand, the effective value of public assets, inferred from government bond prices, rose as interest rates fell (at least in non-Eurozone periphery countries). Assume public assets are 20% of total assets, and private assets are funded 50:50 debt and equity. Public assets grow in value by around 10% (calculated as 4% income, plus an interest rate fall of 2% on a bond portfolio with a discounted mean term of 5 years). Equity falls by 40% (with no income, since the S&P500 is a total return index), and corporate debt falls by 10% (calculated as income of 4%, plus an interest rate rise of 2% on a bond portfolio with a discounted mean term of around 5 years).

the risk is potentially also relevant to the broader market" (Robins et al., 2012, page 6).[19]

However, the financial importance of some of these write-offs can be questioned. Most of the fossil reserves that will have to stay in the ground are coal, which is quite costly to extract (relative to the price obtained for the energy that can be so generated). Further, unconventional oil and gas fracking is currently a key marginal oil and gas resource: its extraction seems to start as soon as the price is only marginally over extraction costs; which suggests that the value of the unexploited reserves is very low.[20] Conversely, the fossil fuel assets with significant value (e.g. conventional oil and gas with low extraction costs) are the very assets that will still be allowed to be used within a global carbon budget. Furthermore, not all the non-fossil fuel assets that are strongly complementary with fossil fuels will be exposed to the Carbon Bubble. Assets that depreciate fairly quickly, e.g. cars, will be used almost fully even under a Carbon Bubble scenario. Some assets that depreciate slowly, e.g. the road network, may be valuable in the post-fossil fuel era, though some proportion of its value is likely at risk (e.g. it may be in a relatively sub-optimal location, or it may be utilised at a very low capacity). A Carbon Bubble scenario, compared to laissez-faire, will strongly constrain the future working life of many slowly depreciating assets, such as newly built coal power stations, whose financial value is therefore at great risk.

Estimating the financial value of the stranded fossil fuel assets is beyond the scope of this paper. Fortunately, recent estimates are provided by Mercure et al. (2018). Relying on empirical data on socio-economic and technology diffusion trajectories, they suggest that a Carbon Bubble "could lead to a discounted global wealth loss of \$1-4tn, a loss comparable to the 2008 financial crisis". In the rest of the paper, we thus assume that our Carbon Bubble scenario is precipitated by the need to write-off a fraction of carbon

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[19] See Battiston et al. (2017) for a calculation of the exposures of financial actors to fossil fuel and climate policy relevant sectors.

[20] Unexploited reserves are valuable if owners expect to be able to exploit them for sale at a price which exceeds the costs of extraction. However, currently we see that as prices rise and fall, so do quantities extracted. This suggests that whenever prices rise above the extraction cost of a particular resource, it is exploited, rather than held "in reserve" in the expectation of future price rises. If all exploitation is done when price is equal to extraction cost plus an arbitrary small margin, then the value of the unexploited reserves is approximately zero.

emitting energy capital equal to 1/150 of total asset value (see Section 4.3).[21,22]

4.5 Calibration Steps

Before moving to the issue of the Carbon Bubble, we quickly recap the calibration strategy we have followed. Table 1 presents parameter values.

First, we defined the interest rate R, the depreciation rate λ, and the zero carbon technology productivity a_L, based on normalization and to allow the interpretation of periods as quarter years. Second, we chose the constant in the savers' linear marginal productivity ν, the cost of investing ϕ, and the fraction of entrepreneurs with an investment opportunity each period π, such that a loss in wealth of 1/150 of total asset value causes a decline of approximately 22.5% in value of total capital. Third, we defined the high carbon technology productivity a_H and the baseline share of fossil fuels in the energy mix γ using energy data. Finally, we contemporaneously chose the size of the stock of fixed capital $\bar{K}$, the constant in the savers' production function $const$, the relative size of the population of savers m, and the non-tradable entrepreneurial productivity c, such that the maximum output impact in the Financial Crisis run is around 6%, energy expenditure over GDP is 7.5%, and the consumption of individual savers and entrepreneurs is equal in the decarbonised steady state.

5 Dynamic Simulations

Now that we have developed and calibrated the analytic framework, we turn to the issue of the Carbon Bubble.

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[21] Simulations with write-offs ranging from 10% to 80% of the initial steady state amount of energy capital, reflecting different sources, are available on request. Obviously, the quantitative results depend on the size of the shock, but the qualitative results are unchanged: the policies that we consider in the next subsections are always associated with a welfare increase (the bigger the shock, the bigger the potential welfare increase).

[22] When considering this asset write-off, note that we are referring to new policy or to a crystallisation of policy expectations. We do not claim that asset values in this sector have not already been impaired by the market attaching some probability to a Carbon Bubble scenario (e.g. Atanasova and Schwartz, 2019, Carattini and Sen, 2019), but that the realisation that the probability of such a scenario is 100%, rather than substantially lower, causes further asset loss.

Table 1: Parameter Values

Description	Parameter	Value	Source/Target
Interest Rate	R	1.01	Kiyotaki and Moore (1997)
Depreciation Rate	$1-\lambda$	0.025	Kiyotaki and Moore (1997)
Zero Carbon Productivity	a^L	1.0	Normalization
Savers' Marginal Productivity	ν	1.5	Fall in asset values in the financial crisis run
Investment Cost	ϕ	33	
Investment Opportunity	π	0.02	
High Carbon Productivity	a^H	1.1	EIA (2015)
Share of High Carbon	γ	0.8	EIA (2016), Newell et al. (2016)
Stock of Fixed Capital	$\bar{K}$	15	Equal consumption, energy expenditure over GDP (EIA, 2016), and fall in output in the financial crisis run (FRED)
Non-Tradable Output	c	1.14	
Relative Number of Savers	m	2.08	
Savers' Production Function	$const$	87	

Here we imagine a scenario loosely modelled upon the current state of the global economy's capital stock: efforts have been made to provide incentives to develop and deploy zero carbon energy capital, but at the global level, the stock of high carbon energy capital is not falling; global reserves of fossil fuels are more than sufficient to exceed some carbon budget; and energy capital investments that lock the economy into high carbon patterns of use are still being made. Therefore, as described in Section 4.2, in the periods prior to the start of our dynamic simulations, we consider the global economy to be in a steady state in which the private returns from investment in both high and zero carbon energy capital are equalised via the imposition of a carbon tax, but that we are in a steady state characterised by $\gamma_F^\star \equiv 0.8$. We use the subscript F (mnemonic for fossil fuel) to indicate that we are in a steady state where high carbon energy capital stock is used. The values for the fixed capital used by, and the debt holdings of, entrepreneurs in this steady state are $K_F^\star$ and $B_F^\star$ respectively. Therefore, the steady state high carbon energy capital stock is $Z^H = \gamma_F^\star K_F^\star$.

The carbon production in period t is linear in the amount of high carbon investment energy capital, Z_t^H, used in production at t. Since we can choose units, let this amount of carbon production also equal Z_t^H for simplicity, so that the cumulative carbon production implied by the steady state amount of high carbon energy capital stock is $S = \sum_{t=0}^{\infty} \lambda^t Z^H$.

At the start of our simulation, the planner announces a cumulative emissions constraint: future investment in high carbon energy capital is banned, and the total future use of high carbon energy capital is limited. Effectively, the policymaker announces a carbon budget, $\bar{S}$, which satisfies

$$\bar{S} = (1-\kappa) \times \sum_{t=0}^{\infty} \lambda^t Z^H = (1-\kappa) \times \frac{\gamma_F^\star K_F^\star}{1-\lambda},$$

where κ is the share of high carbon good that must be left in the ground, whereas the share $1-\kappa$ can be used until it depreciates completely.[23]

As explained in Section 4.3, this means that our Carbon Bubble scenario is precipitated by writing-off a fraction κ of the current stock of carbon emitting energy capital, such that

$$\kappa \left(\frac{\gamma_F^\star \phi K_F^\star}{\phi K_F^\star + q_F^\star \bar{K}} \right) = \frac{1}{150}.$$

This gives $\kappa \approx 12\%$.[24]

We let entrepreneurs choose the optimal way to satisfy this carbon budget. In particular, entrepreneurs must decide the share of the current high carbon energy capital they will use in this period, the rate at which they will retire these goods in the subsequent periods, and the share of the remaining high carbon energy capital they will retire in each subsequent period. These choices, which are clearly not independent, are a function of prices, q_t, and determine the values of the state variables for the next periods, K_t and B_t for $t \geq 1$. As explained in Appendix A.6, entrepreneurs always choose to produce at

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[23] A cumulative emissions constraint may be subject to the so-called "Green Paradox" issue (Sinn, 2012). The fundamental problem raised by the Green Paradox, in this context, is the ability of a policy-maker to implement a cumulative emissions constraint and a fossil fuel investment ban. In this first-order exercise, we assume a certain omnipotence for the policy-maker, and leave complications such as the ability to implement and enforce this policy for future research. As well as this being a first-order treatment, there are also contentions in the literature (van der Ploeg, 2013, Di Maria et al., 2014, Tietenberg and Lewis, 2014) that the Green Paradox does not seem to be a serious obstacle to climate policy.

[24] Note that this is a conservative interpretation of the impact of the Carbon Bubble scenario in that we are reducing the stock of high carbon energy capital that can be used, but this capital is not collateralisable. Entrepreneurial net worth is affected, but their stock of collateralisable assets is not affected. The impact of the Carbon Bubble in this scenario comes because the reduction in entrepreneurial net worth reduces entrepreneurial demand for credit which feeds through into reduced demand for fixed capital, lower prices for fixed capital, and hence fire-sale dynamics.

the maximum rate that they are able to, and delay the retirement of the remaining high carbon asset to the last period available.

Figure 2 gives an overview of the responses of the economy to announcing $\bar{S}$ at $t = 0$.[25] It shows movement in $K/K^\star$, $Y/Y^\star$, $B/B^\star$, $q/q^\star$, and $I/I^\star$ i.e. the ratios of entrepreneurs' fixed capital, total output, investors' debt, price of fixed capital, and aggregate investment flow, to their respective *decarbonised* steady state values.

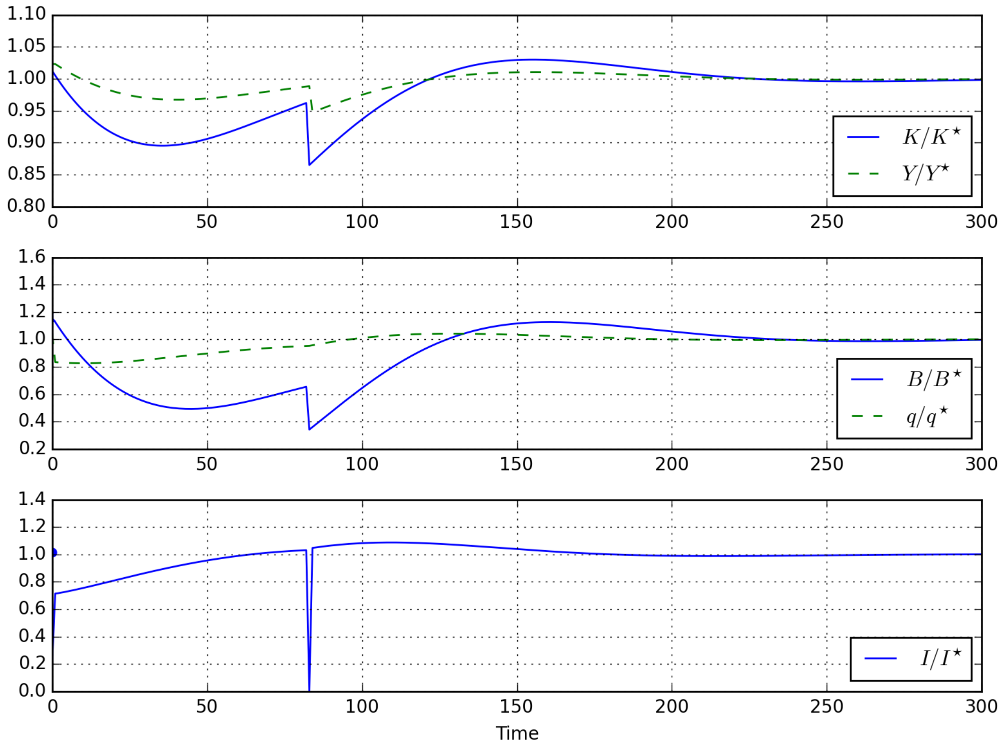

Figure 2: The burst of the bubble

Notes. In the lower panel, the line represents investment in zero carbon energy capital weighted over its decarbonised steady state level. However, the dot represents the ratio of the steady state level of investment at $t = 0$, which consists of both low and high carbon energy capital, to the decarbonised steady state level, which by definition is composed of investment in zero carbon energy capital alone.

As soon as the carbon budget is announced, the price of fixed capital collapses by approximately 20%. The price fall has damaged the entrepreneurs' balance sheets such that they choose to sell fixed capital back to the savers and repay debt. The fall in asset values precipitates forced sales to ensure borrowing and collateral requirements are aligned, but this forced sale causes prices to fall again, which causes further forced sales;

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[25] The dynamics of the model are solved for using numerical simulations of the forward shooting method (see Appendix A.8).

and further price falls, and so on, i.e. we see fire-sale dynamics. The process stops when fixed capital becomes so unproductive in the hands of the savers that the entrepreneurs can once again afford the lowered price, and the economy recovers towards the new steady state.

In the dynamics associated with the announcement of the Carbon Bubble, the entrepreneurial sector deleverages, reducing both assets and debt, until around period 40 (10 years after the announcement). At this point debt levels and the holdings of fixed capital in the hands of the entrepreneurs are around 49% and 88% of steady state levels. Since a large share of fixed capital is employed in the low productivity sector, output is low. Output bottoms out at almost 6% below the previous steady state value, and at more than 3% below the new steady state value. Investment levels fall markedly, by around 30%, even though the economy is in short supply of energy capital.

After approximately 80 periods, the economy reaches its carbon budget: the remaining high carbon assets must be retired, and thus entrepreneurs return fixed capital and debt to the savers, but prices do not fall because this time the shock is “expected”. Given the substantial deleveraging of the entrepreneurs, investment stops for this period, before returning to the previous level. The new decarbonised economy then takes approximately 40 periods to fully recover and start stabilising around the new decarbonised steady state.

In the next subsections, we consider four possible additional actions for the planner that mitigate some of the welfare loss associated with writing-off the high carbon energy capital.

5.1 Tax Funded Transfer of Investors’ Debt

The entrepreneurial sector is credit constrained, and following the imposition of climate policy, it is burdened with excessive debt relative to its assets. Perhaps the planner can achieve a better outcome if the burden of this debt is shifted to an economic actor who is not credit constrained.

We suppose that the planner first announces the carbon budget $\bar{S}$, and then takes over some share $\omega \in [0, 1]$ of the entrepreneurs’ debt, funding the debt repayments by raising a constant per capita tax, τ^G, over $T = 100$ periods.[26] The social planner chooses the value of ω to maximise our measure of social welfare. Figure 3 gives an overview of the responses of the economy to implementing the optimal policy, $\omega = 35\%$, at $t = 0$.

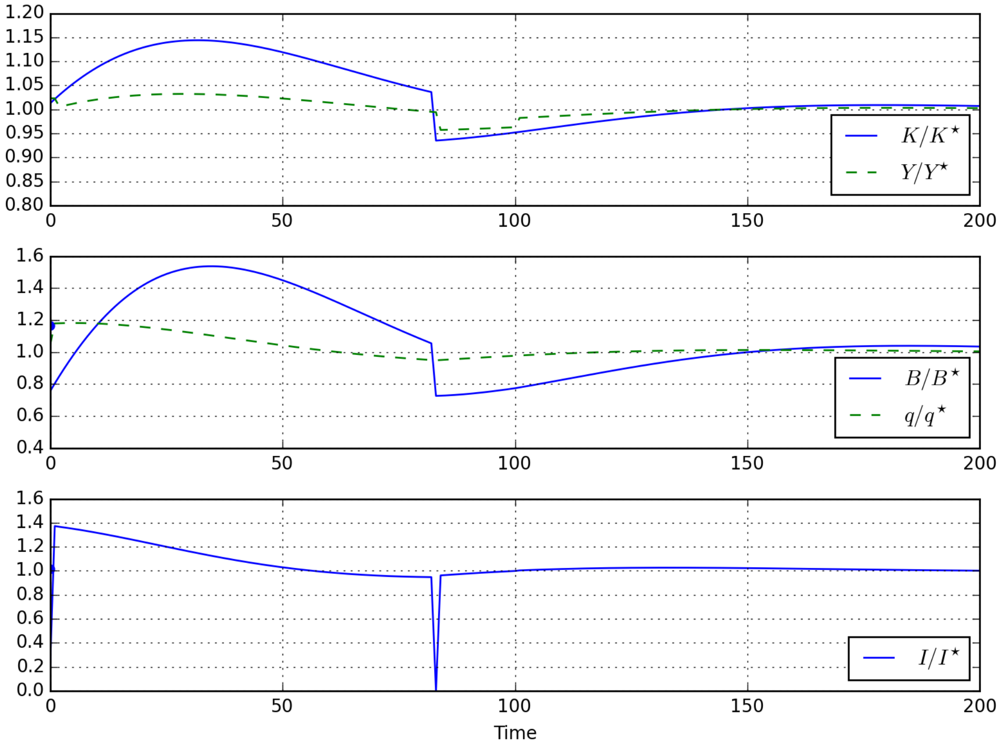

Figure 3: Transferring entrepreneurs' debt

Notes. In the middle panel, the dot represents the ratio of the steady state value of debt before the write-off of high carbon energy capital to the decarbonised steady state value, while the line starts at the post-transfer value.

Following the policymaker's actions, the price of fixed capital increases by around 12%. As a consequence of the debt transfer, entrepreneurs' borrowing starts at 65% of the steady state value, and slightly increases over the first period as the entrepreneurs use their cash flow to balance their fixed capital holdings with their energy capital stocks. This involves slightly increasing their fixed capital holdings and increasing investment levels to 1.35 times the steady state level. The entrepreneurs start to take on debt, to operate with more of the fixed asset than in steady state, and to build up stocks of zero carbon

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[26] The choice of $T = 100$ is relatively arbitrary (but 25 years is a common term for new issues of government debt). A less arbitrary choice would have been the issue of perpetuities, but this would have changed the steady state, which is problematic since we are running a numerical rather than analytic analysis.

energy capital to a level above their steady state value. As before, around period 80, entrepreneurs retire the remaining high carbon asset. After that, the economy is heading towards a "steady state" with taxes, which is characterised by lower output, entrepreneur fixed asset holdings, debt, asset values, and investment levels, than in the true steady state. Once the taxes and government intervention in the debt market cease at $t = 100$, the economy converges to the model's true steady state. Over the course of 200 periods, the cumulative investment in zero carbon energy capital is approximately 7% higher than in the no-policy scenario.

The welfare gain over 200 periods induced by implementing this optimal $\omega = 35\%$ policy is +2.5%.[27] Implementing this debt transfer policy does not however represent a Pareto improvement over the Carbon Bubble with a no-policy scenario: the welfare improvement is composed of +12.3% for entrepreneurs, and −2.2% for savers. Savers have limited upside from this policy, but they still pay taxes to fund it.

5.2 Subsidy

After banning new high carbon investment and announcing the carbon budget that constrains the use of existing high carbon energy capital, in this policy scenario the social planner also announces an increased level of subsidy paid to entrepreneurs to boost the net private productivity of their production.[28] This subsidy, for simplicity, will linearly decrease back to its optimal level over 100 periods, as in the debt transfer policy.

We choose the subsidy induced distortion parameter, δ, such that the optimal subsidy

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[27] We want to underline that any benefits from this policy is a consequence of the Carbon Bubble issue, but debt redistribution would be also welfare increasing if applied in the steady state. Thus we have introduced a deadweight loss associated with τ^G in the production function of the entrepreneurs, $y_t = (a + c - \mu(\tau^G)^2)k_{t-1}$. The deadweight loss parameter μ is calibrated in such way that it is optimal for the social planner to use zero debt redistribution in steady state: any benefits from this policy must be a result of the Carbon Bubble issue.

[28] Alternatively the extra subsidy could be targeted to output produced with only zero carbon energy capital. Since new investment in high carbon energy capital is banned, there is no incentive problem with simply paying a general production subsidy. The only difference between these policies is that the targeted subsidy provides a lower boost to entrepreneurs' incomes.

rate from the planner's perspective in the initial steady state is $\varsigma = 0$.[29] We make this choice so that when we look at this subsidy policy, which mitigates the problem of the Carbon Bubble, we ensure that if there are any benefits in applying a subsidy, these must be due to the Carbon Bubble issue.

We find that, in the case of the Carbon Bubble, there is a clear optimal subsidy boosting net private productivity by, initially, around 40%. Figure 4 shows the dynamics following the carbon budget announcement, when the planner implements this optimal subsidy program.

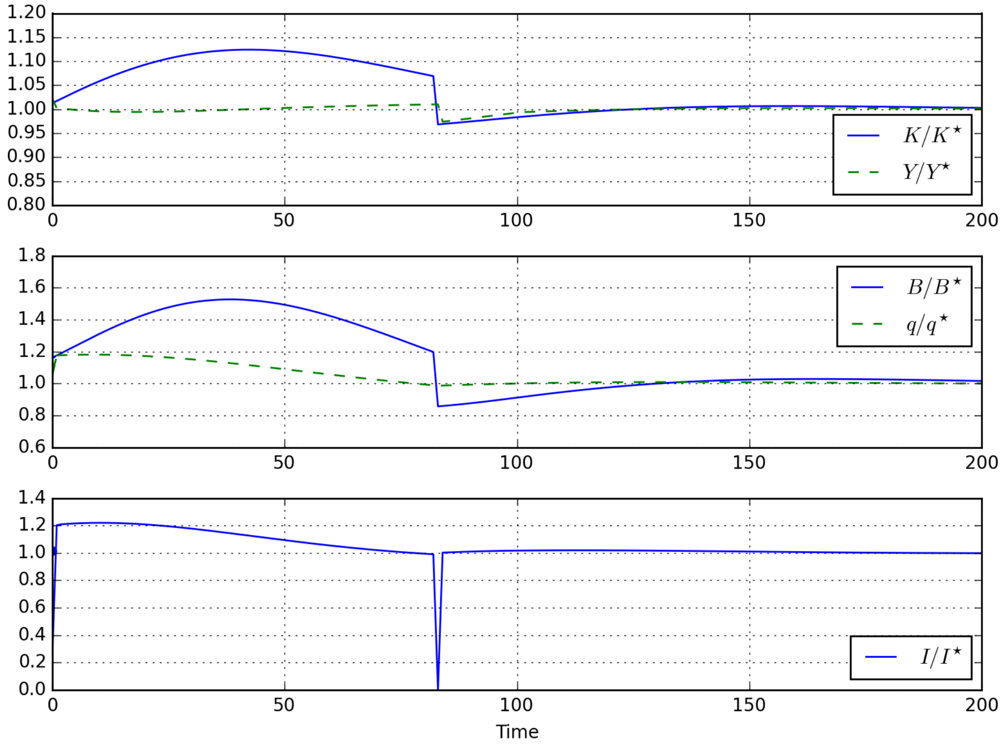

Figure 4: Subsidising entrepreneurs

The dynamics are similar to the debt reallocation scenario. Over 200 periods, the cumulative investment in zero carbon energy capital is approximately 7% higher, and welfare is almost 2% higher, than in the no-policy scenario. Again, this is not a Pareto

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[29] This means that the optimal subsidy from the planner's perspective in the decarbonised steady state is actually negative: this optimal $\varsigma = 0$ calibration target requires a distortion large enough to offset the benefits of a higher net private productivity applied to the 80% of output that is produced using the undistorted high carbon energy capital. However, we do not allow negative distortions, so the optimal subsidy from the planner's perspective in the decarbonised steady state is again zero. See Appendix A.2 for more information on the subsidy.

improvement: savers are worse off (-2.4%) because of the increased tax they have to pay to fund the subsidy, whereas the entrepreneurs benefit from the subsidy ($+10.8\%$).

5.3 Government Guarantee

In this policy scenario, we model a government guarantee that reassures lenders and relaxes credit constraints. Specifically, we imagine a guarantee that effectively multiplies an entrepreneur's collateral: for a given quantity of collateral, the entrepreneurs can borrow more. Analytically, Equation (4) is modified to $b_t \leq R^{-1} q_{t+1} k_t \left(1 + gtee_t\right)$, where $gtee_t$ is the government guarantee in t.

Immediately after the same carbon budget announcement as in the no-policy scenario, the planner announces a linearly reducing guarantee, which reaches zero after 100 periods.[30] The steady state to which the economy is converging is therefore unchanged. Figure 5 shows the dynamics following the carbon budget announcement, when the planner implements a guarantee starting at $gtee_0 = 5\%$, which is approximately optimal given our parameters.

The price jump is slightly smaller than in the no-policy scenario, at approximately -19%. However, the guarantee allows entrepreneurs to have access to more debt, and the entrepreneurs use the proceeds of this borrowing to maintain higher fixed capital holdings and higher investment levels than in the no-policy scenario. Over the course of 200 periods, the cumulative investment in zero carbon energy capital is approximately 3% higher, and welfare is almost 3% higher, than in the no-policy scenario.

This policy does produce a Pareto improvement with respect to the no-policy scenario: both savers ($+0.4\%$) and entrepreneurs ($+8.2\%$) are better off. This is because this policy relaxes credit constraints at no cost, which may of course be unrealistic.

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[30] Again, 100 periods is chosen for consistency with the previous two policies. This implies $gtee_t = \max\{0, (100 - t)/100\} \times gtee_0$, and it ensures that the final steady state does not have a government guarantee. Indeed, the model with a guarantee behaves counter-intuitively: a higher guarantee makes entrepreneurs rely more on external borrowings, leading to a reduction in their long-run net worth (Luangaram, 2003).

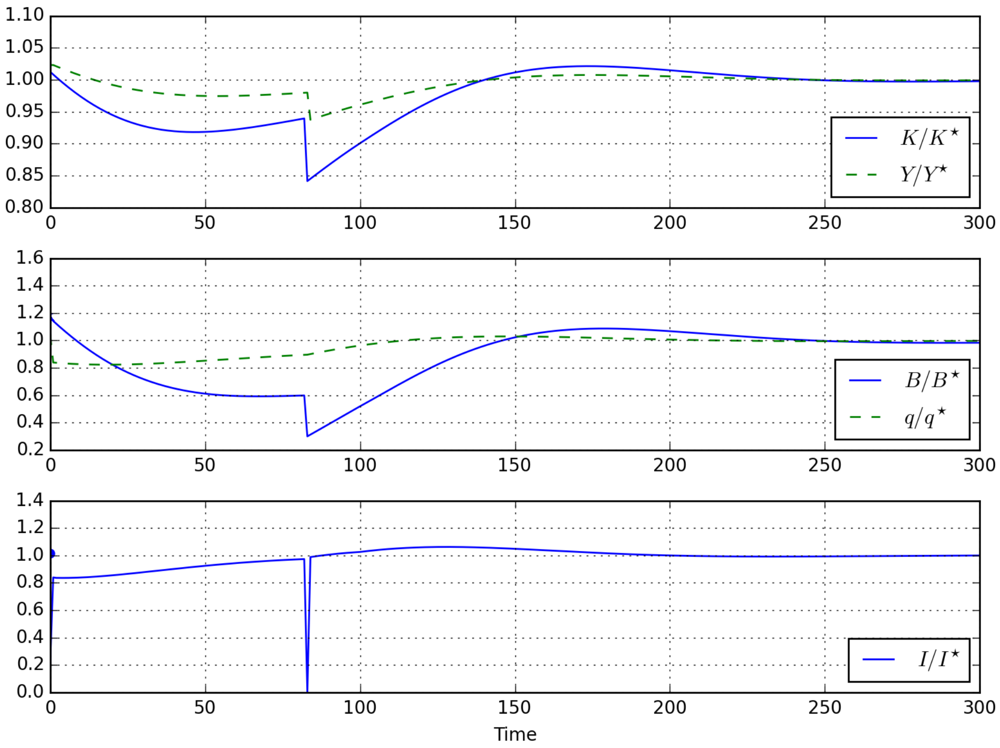

Figure 5: Providing a government guarantee

5.4 Deception

Here we consider a different possible action for the planner: she can vary the amount of current high carbon energy capital that she tells the market is allowed to be used, $\hat{S} \geq \bar{S}$. For $\hat{S} > \bar{S}$, the economy's actual carbon budget, $\bar{S}$, is used at some time T. When this happens, $\bar{S}$ is revealed to all agents and the entrepreneurs are compelled to leave unused their remaining high carbon energy capital: high carbon production is abruptly banned in a desperate attempt to avoid catastrophic climate change.

In a canonical model, the social planner does not have any incentive to lie: stating an $\hat{S} > \bar{S}$ would cause a welfare destroying discontinuity in consumption across the period in which $\bar{S}$ is revealed. In this model, conversely, overstating the actual carbon budget limits the fall in the price of fixed capital and thus the decrease in the value of the collateral.[31] This allows higher investment in zero carbon technology, and potentially generates enough productive capacity between period 0 and T, when $\bar{S}$ is revealed, to mean that the present

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[31] In addition to risk neutrality.

value of consumption flows is higher under deception.

Figure 6 presents the simulation for the welfare-maximizing value of $\hat{S}$, consistent with approximately 105% of the size of the actual carbon budget being announced at $t = 0$ (equivalently, the announced write-off is approximately 60% of the truthful write-off).

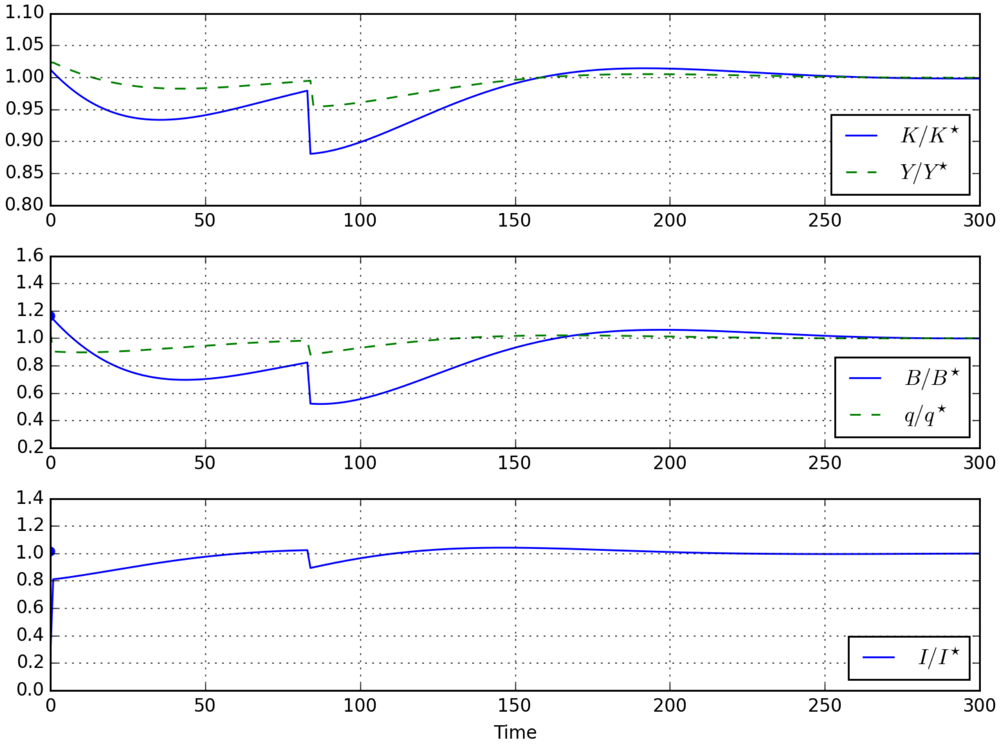

Figure 6: Dishonest social planner

The dynamics are similar to the ones following the truthful announcement, but the initial shock is smaller. The price of fixed capital falls immediately by approximately 13%. Investment does not fall as much as in the no-policy scenario, and when the actual carbon budget runs out, despite a large negative shock inducing further deleveraging, investment can soon continue. At T, around 90% of aggregate entrepreneurs' asset holdings are already dedicated to zero carbon energy capital so that, when the remaining high carbon resources must be left unused, an alternative productive capacity already exists. This limits the magnitude of the recession that results.

Over 200 periods, the cumulative investment flow in zero carbon energy capital is almost 3% higher than in the no-policy scenario; and welfare is around 3% higher (+8.6% for entrepreneurs and +0.6% for savers). This policy is a Pareto improvement on the no-policy scenario.

6 Conclusions

We analyse the effects of the credible implementation of climate change targets, which imply that a substantial proportion of fossil fuel assets become "stranded" in an economy characterized by collateral constraints. To do this, we extend Kiyotaki and Moore's (1997) model to allow for two investment goods representing high carbon and zero carbon energy capital. This framework allows us to model, for the first time in the economics of climate change, the so-called "Carbon Bubble". This was introduced as a warning to investors: climate change mandates a policy response, and you, as an investor, should protect your portfolio from this policy response. By incorporating the Carbon Bubble issue within a macro-financial model, we start the conversation around appropriate macroeconomic policies that should accompany the Carbon Bubble.

We show that policies that mitigate the impact of the Carbon Bubble upon investors' balance sheets can be beneficial. The "global balance sheet" will be used to fund the zero carbon infrastructure, which must be built to replace our fossil fuel based economy, and the bursting of the Carbon Bubble could throw the economy into a deep recession, depriving green technology of investment funds when they are most needed. Thus, even if the fossil fuels assets really should be written-off to avoid disastrous global warming, it is likely to be sub-optimal to do this naïvely.

We acknowledge that there are many areas of the model that could be made more sophisticated. However, we reiterate that embedding the financial accelerator mechanism within a standard IAM is not the most important next step for this research agenda. Indeed, incorporating a dependence upon cumulative emissions and the risk of catastrophic damages within an IAM would output the carbon budget as an optimal policy: this seems like an unnecessary complication relative to simply imposing this policy exogenously. More importantly, the supply side of the model could be made more realistic, with the depreciation of, and investment in, the non-energy capital, and a time-varying degree of substitutability between clean and dirty inputs. Endogenous growth is likely an important aspect: with learning-by-doing, any under-utilisation of capital induced by the Carbon Bubble could be more damaging than in the model presented here (e.g. Ghisetti et al., 2017). The "black box" distortion associated with the subsidy could also be micro-founded in an endogenous growth framework.

On the macro-financial side, perhaps the next step should be the addition of a banking sector (e.g. Gertler and Kiyotaki, 2015).[32] One could also properly model the bankruptcy process, incorporating a fuller description of the capital structure of investors' balance sheets with different priority creditors, and costs of financial distress. Furthermore, one could assume that green-tech projects are perceived as riskier by creditors (and thus more credit constrained) than fossil fuel projects. Heterogeneous agents may be an important element to add to the model, as in Punzi and Rabitsch (2015). The costless government guarantee suggests that aggregate uncertainty or stochastic noise should be added, so that such a guarantee had an expected cost along the equilibrium path.

On the political economy side, the Pareto sub-optimality of some of the policies and the fact that we have ignored Green Paradox effects (see Footnote 23), suggest that a political process is important. Yet, perhaps the Pareto sub-optimality problem would be reduced if savers also supplied labour, and there was the possibility of unemployment in recessions. International spillovers (e.g. Ganelli and Tervala, 2011, Annicchiarico and Diluiso, 2019), and some measure of the costs of acting through the planner,[33] might also be important.

There remains much to do in fully specifying a model that will allow a macroeconomic forecast of the impact of the Carbon Bubble, and will allow the design of an optimal policy response. This article has started this modelling, and shown that there is a role for policy in mitigating its impact. Policy that protects investors' balance sheets mitigates the macroeconomic downturn, and leads to a higher investment in the replacement zero carbon

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[32] The key mechanism in this article is the fall in the collateral value of the entrepreneurs following a tighter climate policy. But, in reality, it is not only entrepreneurs who own fossil capital or fossil reserves. In a model with a financial intermediation sector, we would observe costlier bank credit and bank runs if the tighter climate policy influences, directly or indirectly, the balance sheets of the intermediaries. See, for example, Gertler and Kiyotaki (2015).

[33] In our model, an obvious optimal policy would be to nationalise investment in energy so that the unconstrained government maintains investment in the face of the Carbon Bubble.

productive capacity over the period in which we still use carbon emitting productive capacity.

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Online Appendix to “The Carbon Bubble: Climate Policy in a Fire-Sale Model of Deleveraging”

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A Online Appendix

A.1 Model assumptions not given in main text

We specify below some further model assumptions that are omitted from the main text and that are relevant restrictions in the derivation of the steady state.

We avoid a corner solution i.e. we ensure that, in the neighbourhood of the steady state, both types of agent produce,

Assumption A. $\left.\frac{dy'}{dk'}\right|_{k'=\frac{\bar{K}}{m}} < \frac{\pi\left[a+\frac{\gamma\tau-(1-\gamma)\varsigma}{1+m}\right]-\phi(1-\lambda)(1+R\pi-R)}{\pi\lambda+(1-\lambda)(1-R+R\pi)} < \left.\frac{dy'}{dk'}\right|_{k'=0};$

we assume that the tradable output is at least enough to substitute the depreciated energy capital,

Assumption B. $a^L > (1-\lambda)\phi,$

and that the probability of investment is not too small,

Assumption C. $\pi > \frac{R-1}{R}.$

Assumptions B and C are also used to ensure that the steady state values ($q^\star$, $K^\star$, $B^\star$) are positive.

To guarantee that the entrepreneur will not want to consume more than the non-tradable output, we assume

Assumption D. $c > \frac{1-\beta R\lambda(1-\pi)}{\beta R[\pi\lambda+(1-\lambda)(1-R+R\pi)]}\left(\frac{1}{\beta}-1\right)(a^L+\lambda\phi).$

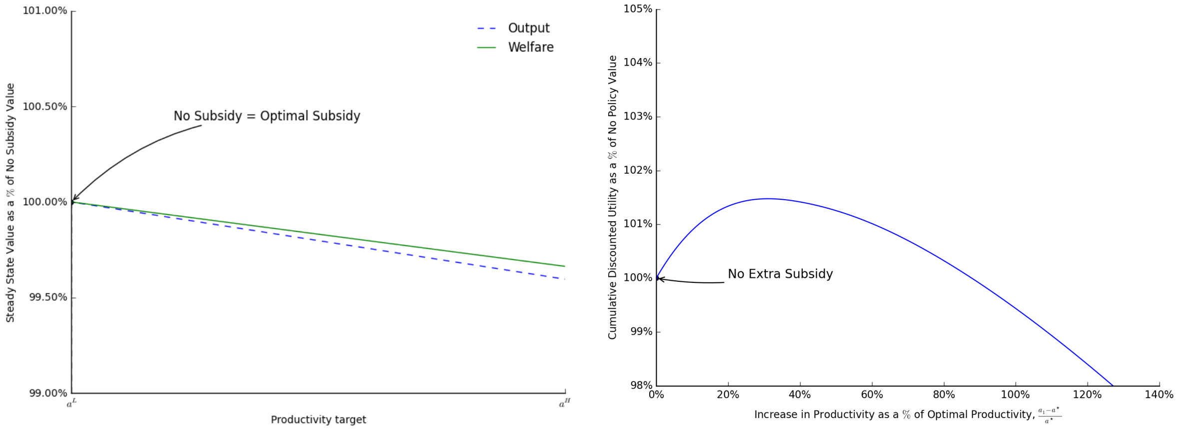
Note that, since β and R are close to one, both Assumptions C and D are weak. Finally, we avoid the explosion in asset prices with the following transversality condition,

Assumption E. $\lim_{s\to\infty} E_t(R^{-s}q_{t+s}) = 0.$

A.2 Subsidy induced distortion

In a decarbonised world ($\gamma = 0$), since $K^\star$ is a monotonically increasing function of the productivity target set by the government, the amount of fixed capital used by the entrepreneurs increases with the subsidy. Indeed, an entrepreneur benefits fully from the presence of the subsidy while contributing only partly to the per capita tax (since this is paid by savers too). However, when there is a cost associated with the subsidy, an increase in the target productivity set by the government has an ambiguous effect on social welfare. Indeed, while entrepreneurs' utility always increases in a (as it is a constant multiple, c, of $K^\star$), the increase in savers' income from increased debt interest as lending increases may not compensate the decrease due to increasing taxes. We choose the subsidy induced distortion parameter, δ, such that the optimal subsidy rate from the social planner's perspective is $\varsigma = 0$.

Panel (a) of Figure A1 shows that any positive subsidy is welfare destroying in the neighbourhood of the decarbonised steady state. Conversely, Panel (b) shows there is a clear optimal subsidy following the announcement of the carbon budget, because a positive subsidy ameliorates the balance sheet position of the entrepreneurs, and thus allows more investment in alternative productive capacity.



(a) Decarbonized Steady State

(b) Carbon Bubble Scenario

Figure A1: Welfare effects of the subsidy

A.3 Tax Induced Deadweight Loss

Similarly to the above, we add a deadweight loss parameter associated with the transfer of entrepreneurs' debt, such that the optimal transfer in the decarbonised steady state is zero.

Consider a social planner taking over a share $\omega \in [0, 1]$ of the entrepreneurs' debt, $B_G^\star$, in the decarbonised steady state. The debt repayments are funded through lump-sum taxes, τ^G, over T years. This implies

$$\tau^G = \frac{(1-\beta)\,\omega B_G^\star}{(1+m)\,\beta\,(1-\beta^T)}.$$

The deadweight loss associated with the tax, $\mu\left(\tau^G\right)^2$, modifies the production function of the entrepreneurs to

$$y_t = \left[a^L - (1-\delta)\,\varsigma_t + c - \mu\left(\tau^G\right)^2\right] \times \min\left(k_{t-1}, z_{t-1}^L\right)$$

and thus modifies for T periods the equations of motion of K_t and B_t. We numerically find the value of μ that makes $\omega = 0$ (subject to $\omega \geq 0$) optimal in the decarbonised steady state.

A.4 Steady state zero carbon energy investment

The amount of fixed capital used by the entrepreneurs, $K^\star(a, \gamma^\star)$, for any $a \in \left[a^L, a^H\right]$, $\gamma^\star \in [0,1]$, is within the interval $[K_L^\star, K_H^\star]$ and is a monotonically increasing function of the share of fixed capital used in conjunction with high carbon energy capital, $\gamma^\star \in [0,1]$.[1] Increasing $\gamma^\star$ results in a higher net worth of the entrepreneurial sector, through an increase in the carbon tax revenues and the per capita transfer made by the government. As a consequence, the representative entrepreneur can afford higher fixed capital holdings with positive repercussions on investment and output.

One interesting result in the initial steady state is that, in an economy with credit frictions, unlike its frictionless equivalent, the high carbon proportion, γ, which maximises steady state zero carbon investment, may be greater than zero. Under certain conditions, the relationship between the proportion of high carbon production, γ, and the absolute value of zero carbon investment is not monotonic: indeed, the higher the share, γ, of entrepreneurs using high carbon production and investing in high carbon energy capital, the higher is the net productivity of the fixed capital, and the higher are tax revenues and so the per capita transfer. This means that entrepreneurs have higher net worth and so can hold more of the fixed capital. Since the fixed capital is more productive in the hands of the entrepreneurs, its value increases. This potentially allows the entrepreneurs who are using zero carbon production and investing in zero carbon energy capital to borrow more, invest more and produce more. Crucially, we show that this non-monotonic relationship is due to the presence of credit frictions.

The steady state value of aggregate entrepreneurs' fixed capital holdings is

$$K^\star = \frac{\pi\left[a + \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m}\right] - \phi(1-\lambda)(1-R+R\pi)}{\pi\lambda + (1-\lambda)(1-R+R\pi)} + \nu.$$

Since zero carbon investment is given by $I_t^L = (1-\gamma)\phi(K_t - \lambda K_{t-1})$, in steady state this value is $I^{L\star} = (1-\gamma)\phi(1-\lambda)K^\star$. Therefore, investment in zero carbon energy capital can be expressed as

$$I^{L\star} = (1-\gamma)\phi(1-\lambda)\left\{\frac{\pi\left[a + \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m}\right] - \phi(1-\lambda)(1-R+R\pi)}{\pi\lambda + (1-\lambda)(1-R+R\pi)} + \nu\right\}.$$

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[1] The same argument does not need to hold for the net private productivity of the entrepreneurs' technology, $a \in \left[a^L, a^H\right]$, because of the distortionary impact of the subsidy.

Differentiating it with respect to γ gives

$$\frac{\partial I^{L\star}}{\partial \gamma} = \phi(1-\lambda)\left\{\frac{\frac{\pi(1-\gamma)(\tau+\varsigma)}{1+m} - \pi a + \phi(1-\lambda)(1-R+R\pi)}{\pi\lambda + (1-\lambda)(1-R+R\pi)} - \nu\right\}.$$

It is then easy to see that under certain conditions (depending on e.g. the difference between the productivities of the two technologies, the fraction of entrepreneurs with respect to savers, the net private productivity), $I^{L\star}$ increases for low levels of γ before starting to decrease, as shown by the solid line in Figure A2.

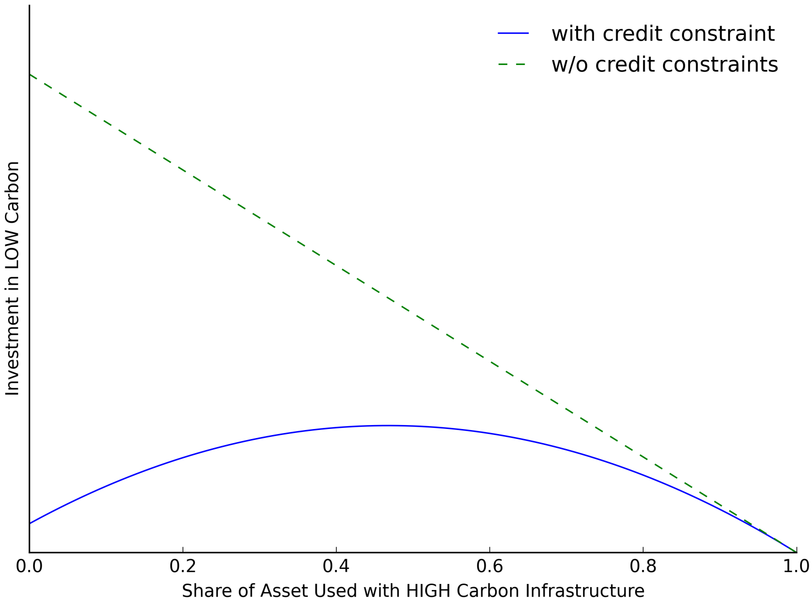


Figure A2: Absolute investment in zero carbon energy capital as a function of γ

We now want to show that this result is a consequence of the presence of the credit constraint. Consider an economy in which there are no debt enforcement problems so that capital can be optimally allocated. In such an allocation the marginal products of the two technologies would be equalised and the fixed capital price would be given by the discounted gross return from using the entrepreneurs' technology, $q^0 = (a+c)/(R-1)$. It follows that $K^0 = (a+c)/R + v$. Therefore, without the inefficiency caused by the presence of borrowing constraint, investment in zero carbon energy capital would be given by the following relationship

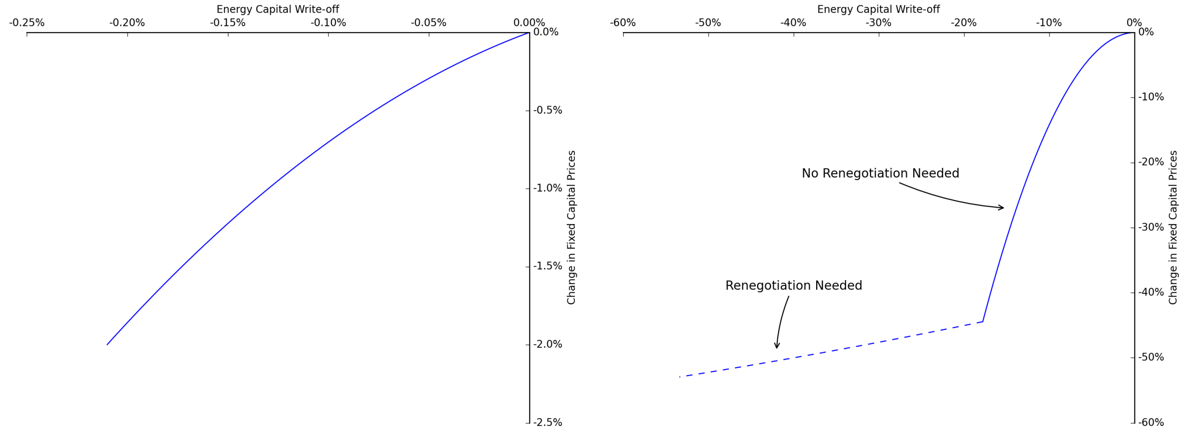
$$I^{L0} = (1-\gamma)\phi(1-\lambda)\left\{\frac{a+c}{R} + v\right\}$$

which is strictly increasing in $1-\gamma$, the proportion of fixed capital used by entrepreneurs in conjunction with zero carbon energy capital, as shown by the dashed line in Figure A2. Since the policymaker equalises the private return from using fixed capital with either high or zero carbon energy capital, which is optimally set equal to the returns from the savers' use of fixed capital, it is clear that the proportion γ of high carbon energy capital use cannot affect the amount of fixed capital used overall by the entrepreneurs. Therefore, in steady state, the flow of zero carbon energy capital investment is monotonically decreasing in the high carbon share, γ.

To the extent that the policy target is to maximise investment in zero carbon energy capital, this result shows that the optimal policy may be counter-intuitive: we may get more zero carbon investment if we allow high carbon investment to continue.

A.5 Calibration Strategy

The "calibrations" presented in Kiyotaki and Moore (1997, Chapter III) are not suitable for our exercise because the economy can only return to steady state for extremely small negative shocks: as shown in Panel (a) of Figure A3, the maximum write-off of energy capital that the model can sustain is only approximately 0.2%. Any write-off exceeding this amount requires the introduction of a debt renegotiation mechanism to ensure that the economy can converge to the interior steady state.



(a) Original Parametrization

(b) Our Parametrization

Figure A3: The effect of a energy capital write-off

Not only do we introduce a debt renegotiation mechanism based on Cordoba and Ripoll (2004) (see Section A.8), we also define an alternative calibration strategy. This is explained in Section 4, but it broadly consists of the following steps. First, we define the parameters R, λ, and a_L based on definitional convenience, normalization, and to maintain the interpretation of periods as quarter years.

Second, we set δ and μ such that the optimal subsidy and the optimal debt transfer are zero in the decarbonised steady state, and a_H and γ using energy data. Third, we try to match broadly the experience of the 2008-09 Financial Crisis. In order to do so, we modify the model slightly and set $\gamma = 0$. This means that m (and the constant in the savers' production function) does not influence this stage of the calibration since $a = a^L = 1.0$ and $g(m) = [\gamma\tau - (1-\gamma)\varsigma]/(1+m) = 0$. We choose ν, ϕ, and π such that a loss in wealth comparable to the total value of the subprime mortgages causes a decline of approximately 20-25% in the value of total capital. In doing so, we try to find a combination of these parameters such that the model can withstand a large write-off of energy capital before needing debt renegotiation. The reason being that relying too heavily upon debt renegotiation ruins the story that models of this sort tell, as when debt renegotiation is introduced, this absorbs most of the impact of the shock, while prices change only marginally: see Panel (b) of Figure A3. Additionally, we choose parameters such that we observe that debt holdings are positive in the maximum write-off run. Most calibrations satisfying these conditions produce extremely long economic cycles (when interpreting a period as a quarter). We therefore also try to minimise cycle length subject to satisfy these other conditions.

Fourth, contemporaneously we choose $\bar{K}$, *const*, m, and c such that the consumption of individual saver and entrepreneur, i.e.[2]

$$E[x_t] = cK_{t-1}$$

$$x_t' = \frac{R\left[(\bar{K}-\nu)(\bar{K}-K_{t-1}) - \frac{(\bar{K}-K_{t-1})^2}{2}\right]}{m} - \frac{const}{m} + g_t + \frac{RB_{t-1} - B_t}{m} + \frac{q_t\,(K_t - K_{t-1})}{m},$$

are equal in the new decarbonised steady state, the maximum output impact in the Financial Crisis run is around 6%, and energy expenditure over GDP is 7.5%. Figure A4 shows the dynamics on the variable of interest following a shock comparable to the 2008-09 Financial Crisis. The calibrated parameters are presented in Table 1. Targeted and model moments are given in Table A1.

Table A1: Calibrated values

	Target	Model
Energy Expenditure	7.50%	7.50%
Asset Impact of FinCrisis	-20/25%	-22.47%
Output Impact of FinCrisis	-6.00%	-6.00%
Difference in Individual Consumption	0.00%	0.00%

Figure A5 shows that the effect of moving from the initial steady state, where more productive high energy capital is used, to the decarbonised steady state, without any

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[2] Entrepreneurs are not all identical as their personal holdings of energy and fixed capital will depend upon their idiosyncratic histories, but in expectation (given a unit mass of entrepreneurs) their individual fixed capital holdings are represented by the aggregate entrepreneurial fixed capital holding. Conversely, if we imagine that all savers save through some financial intermediary institution, then they are all homogeneous.

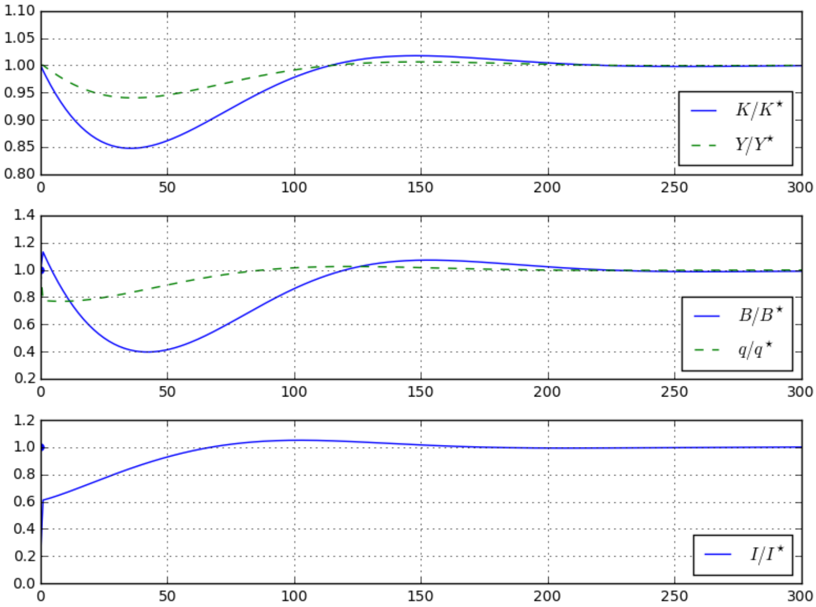

Figure A4: Dynamics induced by the financial crisis shock

write-off, is important but significantly smaller that the effect caused by the write-off, as we can see comparing Figure A5 with Figure 2.

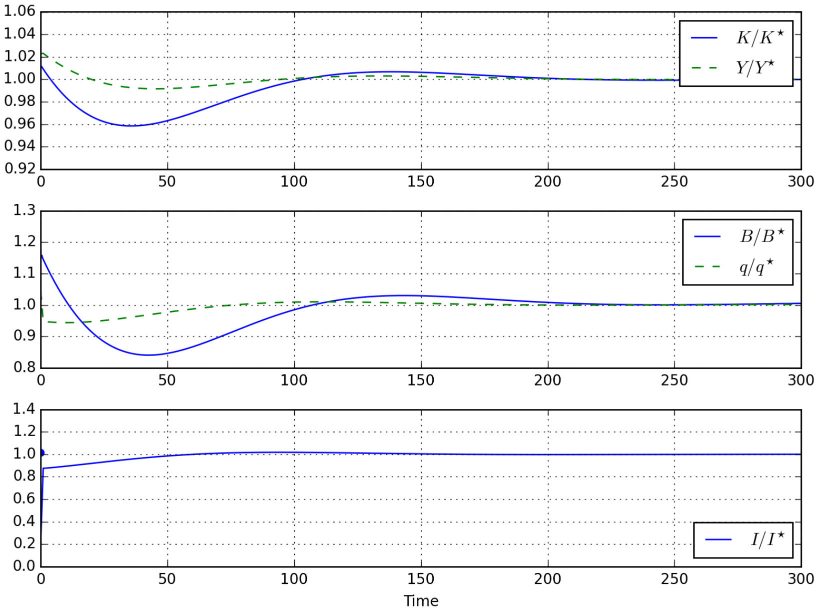

Figure A5: Dynamics induced by the productivity differential

A.6 Dynamics and Timing

In the first period, $t = 0$, timing is as described in Figure A6. At the time of the announcement (the very start of the $t = 0$ period), asset and credit markets have just closed, so that $K_0 = K_F^\star$ and $B_0 = B_F^\star$ are state variables. The announcement affects prices, which hugely impair entrepreneurs' balance sheets. However, after the announcement there is a debt renegotiation opportunity that changes B_0 to $B_{0+} \leq B_F^\star$, but cannot alter K_0: savers and entrepreneurs adjust their credit positions given the entrepreneurs' net worth implied by their fixed real holdings of the asset. Renegotiation does not take place if the economy can converge back to the steady state given $B_{0+} = B_F^\star$, because savers have no incentive to renegotiate. However, if the economy cannot converge back to its steady state then both parties have the incentive to renegotiate the outstanding value of the debt:[3] the debt level retained by the entrepreneurs is then reduced to $B_{0+} < B_F^\star$, where this B_{0+} value is the maximum value for debt levels consistent with the economy being able to reach steady state. Production then takes place with entrepreneurs using $Z_0^H = \gamma_0 K_F^\star$ high carbon energy capital.[4]

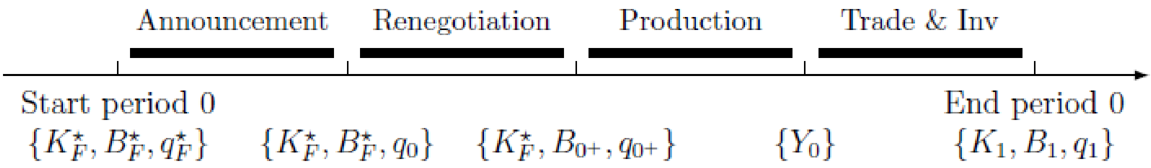

Figure A6: Timing

At the end of the period, savers and entrepreneurs receive their output, the asset and credit markets open, and agents make consumption and investment decisions. Entrepreneurs must also decide the share $\rho \in [0, 1]$ of the remaining $\lambda\gamma_0 K_F^\star$ high carbon energy capital they will use, and the rate, $1 - \lambda_H \in [1 - \lambda, 1]$, at which they will retire these goods. These choices are a function of prices, q_1, and determine the values of the state variables for the next period: K_1 and B_1. These are choices that are not independent and must satisfy the carbon budget set by the planner and are optimally taken

---

[3] Entrepreneurs would always like to reduce their debt. If the economy cannot converge back to the interior steady state, then the outside option for the representative saver is to accept the economy converging to the steady state with no fixed capital in the hands of the entrepreneurs. Savers prefer to reduce their debt rather than accept this, and so engage in renegotiation. They write-off the minimum quantity of debt such that the interior steady state can be reached. We refer an interested reader to Appendix A.5 for how renegotiation influences the dynamics following a shock, and to Appendix A.8 for more information about the renegotiation process.

[4] Note that, alternatively, entrepreneurs could leave some of the fixed asset unused in this first period, since markets are closed and it cannot be traded back to the savers. We checked and verified that this option is not optimal for the entrepreneurs, as the reduced consumption in the first period (and the lower net worth) is enough to offset the positive effect of an increased remaining carbon budget.

by the entrepreneurs: it turns out that it is optimal for the entrepreneurs to delay the retirement of the remaining carbon goods to the last period, i.e. $\rho = 1$ and $\lambda_H = \lambda$.

Given the choice of the entrepreneurs, there will be a time period t (where $t \approx 80$ in our simulations) at which the cumulative emissions constraint is reached. At $t - 1$, entrepreneurs must retire (enough of) the remaining high carbon energy goods, in order to stay within the carbon budget. After this date, production will use only low carbon energy goods.

A.7 Stability

In this section, we follow Kiyotaki and Moore (1995, Appendix) to linearise the model around the steady state in order to examine the dynamics. The procedure requires using the equations of motion for capital price in (11), for aggregate entrepreneurs' asset holdings in (12), and for entrepreneurs' borrowings in (13), to find (K_t, B_t, q_{t+1}) as function of (K_{t-1}, B_{t-1}, q_t):

$$\begin{aligned}
q_{t+s} =& R q_{t+s-1} - R(K_{t+s-1} - \nu) \\
B_{t+s} =& q_{t+s}(K_{t+s} - K_{t+s-1}) + \phi(K_{t+s} - \lambda K_{t+s-1}) + RB_{t+s-1} - aK_{t+s-1} + \\
& - \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m} K_{t+s-1} \\
K_{t+s} =& (1-\pi)\lambda K_{t+s-1} + \\
& + \frac{\pi}{\phi + (K_{t+s} - \nu)} \left[(q_{t+s} + \phi\lambda + a) K_{t+s-1} - RB_{t+s-1} + \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m} K_{t+s-1} \right].
\end{aligned}$$

Consider taking a first-order Taylor series expansion to this system around the steady state,

$$\begin{aligned}
\frac{q_{t+s} - q^\star}{q^\star} \approx & \frac{\partial q_{t+s}}{\partial q_{t+s-1}}\bigg|_{SS} \frac{q^\star}{q^\star} \frac{q_{t+s-1} - q^\star}{q^\star} + \frac{\partial q_{t+s}}{\partial K_{t+s-1}}\bigg|_{SS} \frac{K^\star}{q^\star} \frac{K_{t+s-1} - K^\star}{K^\star} = \\
=& R\frac{q_{t+s-1} - q^\star}{q^\star} - R\frac{K^\star}{q^\star} \frac{K_{t+s-1} - K^\star}{K^\star} = \\
=& R\frac{q_{t+s-1} - q^\star}{q^\star} - (R-1)\frac{K^\star}{(K^\star - \nu)} \frac{K_{t+s-1} - K^\star}{K^\star}
\end{aligned}$$

$$
\begin{aligned}
\frac{B_{t+s}-B^\star}{B^\star} \approx & \left.\frac{\partial B_{t+s}}{\partial B_{t+s-1}}\right|_{SS}\frac{B^\star}{B^\star}\frac{B_{t+s-1}-B^\star}{B^\star} + \left.\frac{\partial B_{t+s}}{\partial q_{t+s-1}}\right|_{SS}\frac{q^\star}{B^\star}\frac{q_{t+s-1}-q^\star}{q^\star}+ \\
& + \left.\frac{\partial B_{t+s}}{\partial K_{t+s-1}}\right|_{SS}\frac{K^\star}{B^\star}\frac{K_{t+s-1}-K^\star}{K^\star} = \\
& = \left[R + (q^\star+\phi)\frac{\partial K_{t+s}}{\partial B_{t+s-1}}\right]\frac{B_{t+s-1}-B^\star}{B^\star} + (q^\star+\phi)\frac{\partial K_{t+s}}{\partial q_{t+s-1}}\frac{q^\star}{B^\star}\frac{q_{t+s-1}-q^\star}{q^\star}+ \\
& + \left[-\left(q^\star+\lambda\phi+a+\frac{\gamma\tau-(1-\gamma)\varsigma}{1+m}\right)+(q^\star+\phi)\frac{\partial K_{t+s}}{\partial K_{t+s-1}}\right]\frac{K^\star}{B^\star}\frac{K_{t+s-1}-K^\star}{K^\star}
\end{aligned}
$$

$$
\begin{aligned}
\frac{K_{t+s}-K^\star}{K^\star} \approx & \left.\frac{\partial K_{t+s}}{\partial q_{t+s-1}}\right|_{SS}\frac{q^\star}{K^\star}\frac{q_{t+s-1}-q^\star}{q^\star} + \left.\frac{\partial K_{t+s}}{\partial B_{t+s-1}}\right|_{SS}\frac{B^\star}{K^\star}\frac{B_{t+s-1}-B^\star}{B^\star}+ \\
& + \left.\frac{\partial K_{t+s}}{\partial K_{t+s-1}}\right|_{SS}\frac{K^\star}{K^\star}\frac{K_{t+s-1}-K^\star}{K^\star} = \\
& = \left[\frac{R\pi K^\star}{\phi+(K^\star-\nu)} - \frac{K^\star(1-\lambda+\pi\lambda)}{\phi+(K^\star-\nu)}\frac{\partial K_{t+s}}{\partial q_{t+s-1}}\right]\frac{q^\star}{K^\star}\frac{q_{t+s-1}-q^\star}{q^\star}+ \\
& - \left[\frac{\pi R}{\phi+(K^\star-\nu)} + \frac{K^\star(1-\lambda+\pi\lambda)}{\phi+(K^\star-\nu)}\frac{\partial K_{t+s}}{\partial B_{t+s-1}}\right]\frac{B^\star}{K^\star}\frac{B_{t+s-1}-B^\star}{B^\star}+ \\
& + \left\{(1-\pi)\lambda + \frac{\pi\left(q^\star+\phi\lambda+a+\frac{\gamma\tau-(1-\gamma)\varsigma}{1+m}\right)}{\phi+(K^\star-\nu)} + \frac{\pi K^\star}{\phi+(K^\star-\nu)}\frac{\partial q_{t+s}}{\partial K_{t+s-1}}\right. \\
& \left. - \frac{K^\star(1-\lambda+\pi\lambda)}{\phi+(K^\star-\nu)}\frac{\partial K_{t+s}}{\partial K_{t+s-1}}\right\}\frac{K_{t+s-1}-K^\star}{K^\star}.
\end{aligned}
$$

From the last approximation, it follows that

$$
\begin{aligned}
\frac{\partial K_{t+s}}{\partial q_{t+s-1}} &= \frac{R\pi K^\star}{\phi+(K^\star-\nu)} - \frac{K^\star(1-\lambda+\pi\lambda)}{\phi+(K^\star-\nu)}\frac{\partial K_{t+s}}{\partial q_{t+s-1}} = \\
&= \frac{R\pi K^\star}{\phi+(K^\star-\nu)}\left[1+\frac{K^\star(1-\lambda+\pi\lambda)}{\phi+(K^\star-\nu)}\right]^{-1}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial K_{t+s}}{\partial B_{t+s-1}} &= -\frac{\pi R}{\phi+(K^\star-\nu)} - \frac{K^\star(1-\lambda+\pi\lambda)}{\phi+(K^\star-\nu)}\frac{\partial K_{t+s}}{\partial B_{t+s-1}} = \\
&= -\frac{R\pi}{\phi+(K^\star-\nu)}\left[1+\frac{K^\star(1-\lambda+\pi\lambda)}{\phi+(K^\star-\nu)}\right]^{-1}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial K_{t+s}}{\partial K_{t+s-1}} =& (1-\pi)\lambda + \frac{\pi(q^\star+\phi\lambda+a+\frac{\gamma\tau-(1-\gamma)\varsigma}{1+m})}{\phi+(K^\star-\nu)} + \frac{\pi K^\star}{\phi+(K^\star-\nu)}\frac{\partial q_{t+s}}{\partial K_{t+s-1}}+ \\
& - \frac{K^\star(1-\lambda+\pi\lambda)}{\phi+(K^\star-\nu)}\frac{\partial K_{t+s}}{\partial K_{t+s-1}} = \\
=& \left[1+\frac{K^\star(1-\lambda+\pi\lambda)}{\phi+(K^\star-\nu)}\right]^{-1} \\
& \left[(1-\pi)\lambda + \frac{\pi(q^\star+\phi\lambda+a+\frac{\gamma\tau-(1-\gamma)\varsigma}{1+m})}{\phi+(K^\star-\nu)} + \frac{\pi K^\star}{\phi+(K^\star-\nu)}\frac{\partial q_{t+s}}{\partial K_{t+s-1}}\right].
\end{aligned}
$$

The system can be expressed more compactly as

$$\begin{pmatrix} \hat{q}_{t+s} \\ \hat{B}_{t+s} \\ \hat{K}_{t+s} \end{pmatrix} = J \begin{pmatrix} \hat{q}_{t+s-1} \\ \hat{B}_{t+s-1} \\ \hat{K}_{t+s-1} \end{pmatrix}$$

where an hatted variable indicates proportional deviation from the steady state and J is the Jacobian in elasticity form. An element of the Jacobian is indicated with J_{mn}, where $m, n = (q, B, K)$, so that $J_{mn} = (\partial m_{t+s})/(\partial n_{t+s-1}) \times (n^\star)/(m^\star)$. More specifically,

$$J_{qq} = R \qquad J_{qB} = 0 \qquad J_{qK} = -(R-1)\frac{K^\star}{(K^\star - \nu)}$$

$$J_{Bq} = (q^\star + \phi)\frac{\partial K_{t+s}}{\partial q_{t+s-1}}\frac{q^\star}{B^\star} = (q^\star + \phi)J_{Kq}\frac{K^\star}{q^\star}\frac{q^\star}{B^\star} = (q^\star + \phi)J_{Kq}\frac{K^\star}{B^\star}$$

$$J_{BK} = \left[-\left(q^\star + \lambda\phi + a + \frac{\gamma\tau - (1-\gamma)\varsigma}{1+m}\right) + (q^\star + \phi)J_{KK}\right]\frac{K^\star}{B^\star}$$

$$J_{BB} = R + (q^\star + \phi)J_{KB}\frac{K^\star}{B^\star} \qquad J_{Kq} = \frac{R^2\pi}{\phi + (K^\star - \nu)}\frac{(K^\star - \nu)}{R-1}\left[1 + \frac{K^\star(1-\lambda+\pi\lambda)}{\phi + (K^\star - \nu)}\right]^{-1}$$

$$J_{KB} = -\frac{R\pi}{\phi + (K^\star - \nu)}\left[1 + \frac{K^\star(1-\lambda+\pi\lambda)}{\phi + (K^\star - \nu)}\right]^{-1}\frac{B^\star}{K^\star}$$

$$J_{KK} = \left[1 + \frac{K^\star(1-\lambda+\pi\lambda)}{\phi + (K^\star - \nu)}\right]^{-1}$$
$$\left[(1-\pi)\lambda + \frac{\pi\left(q^\star + \phi\lambda + a + \frac{\gamma\tau-(1-\gamma)\varsigma}{1+m}\right)}{\phi + (K^\star - \nu)} - R\pi\frac{(K^\star - \nu)K^\star}{\phi + (K^\star - \nu)}\frac{K^\star}{(K^\star - \nu)}\right].$$

By renaming the variables accordingly, we can refer the interested reader to Kiyotaki and Moore (1995, Appendix) for the analysis of the stability of the system around the steady state.

A.8 Shooting Algorithm and Renegotiation Mechanism

The simulations are obtained using the shooting algorithm. By using the equations of motion for capital price in (11), for aggregate entrepreneurs' asset holdings in (12), and for entrepreneurs' borrowings in (13), we can find (K_t, B_t, q_{t+1}) as function of (K_{t-1}, B_{t-1}, q_t).

We then have the following system of "transition equations" that we can iterate:

$$\begin{aligned}
K_{t+s} =& \frac{1}{2}\left[\nu - \phi + (1-\pi)\left(\lambda_{t+s}\rho_{t+s}\gamma_{t+s-1} + \lambda(1-\gamma_{t+s-1})\right)K_{t+s-1}\right] + \\
&+ \frac{1}{2}\Bigg\{\left[\phi - \nu - (1-\pi)\left(\lambda_{t+s}\rho_{t+s}\gamma_{t+s-1} + \lambda(1-\gamma_{t+s-1})\right)K_{t+s-1}\right]^2 + \\
&+ 4\Bigg[(\phi-\nu)(1-\pi)\left(\lambda_{t+s}\rho_{t+s}\gamma_{t+s-1} + \lambda(1-\gamma_{t+s-1})\right)K_{t+s-1} + \\
&+ \pi K_{t+s-1}\left[q_{t+s} + \phi\left(\lambda_{t+s}\rho_{t+s}\gamma_{t+s-1} + \lambda(1-\gamma_{t+s-1})\right) + a\right] + \\
&- \pi R B_{t+s-1} + \pi\frac{\gamma_{t+s-1}\tau_{t+s} - (1-\gamma_{t+s-1})\varsigma_{t+s}}{1+m}K_{t+s-1}\Bigg]\Bigg\}^{0.5} && \text{(A2a)} \\
q_{t+s} =& R(q_{t+s-1} - K_{t+s-1} + \nu) && \text{(A2b)} \\
B_{t+s} =& q_{t+s}(K_{t+s} - K_{t+s-1}) + \phi(K_{t+s} - \left(\lambda_{t+s}\rho_{t+s}\gamma_{t+s-1} + \lambda(1-\gamma_{t+s-1})\right)K_{t+s-1}) + \\
&+ RB_{t+s-1} - aK_{t+s-1} - \frac{\gamma_{t+s-1}\tau_{t+s} - (1-\gamma_{t+s-1})\varsigma_{t+s}}{1+m}K_{t+s-1}. && \text{(A2c)}
\end{aligned}$$

When the available carbon budget is announced, the amount of entrepreneurs' energy capital, after depreciation, reduces to $[\rho_0\gamma_0 + (1-\gamma_0)]\lambda K_F^\star$, where $(1-\rho_0)$ is the percentage of the stock of high carbon energy capital that it is optimally written off by the entrepreneurs.[5] When this shock hits, Equation (A2b) does not hold because the asset price jumps in response to the shock and entrepreneurs experience a loss on their asset holdings. In the original Kiyotaki and Moore's (1997) model, a shock of the magnitude we are interested in would throw the economy out of the basin of attraction of the interior steady state (see Section A.5). To prevent this, not only do we use a different parametrization, we also follow Cordoba and Ripoll (2004) and allow for renegotiation of the debt.

In Cordoba and Ripoll (2004), who introduce renegotiation in the basic Kiyotaki and Moore's (1997, Chapter II) model, markets are open during the day, shocks occur at dusk, and then there is a window of opportunity for debt renegotiation to take place, before production occurs overnight. If entrepreneurs want to, they can default on the debt, crucially, before production takes place: the lender gets the ownership of the fixed capital but loses the outstanding value of the debt. They may be able to do better by renegotiating the outstanding value of the debt repayments, RB_{t+s-1}, down to the new value of the collateral, $q_{t+s}K_{t+s-1}$, and incentivising the entrepreneurs to engage in production. This shares the burden of the fall in fixed capital values with the lenders and

---

[5] Note that λ and ρ have a time subscript here, to reflect that the actual depreciation rate must take into account the optimal path of withdrawing the remaining stock of high carbon energy capital. It turns out that $\lambda_{t+s} = \lambda$ and $\rho_{t+s} = 1$ for all $s \geq 1$. Also γ has a time subscript: in the simulation, the social planner has banned investment in high carbon energy capital, therefore depreciation and the optimal path of withdrawing the remaining high carbon energy capital imply that the share of fixed capital used with high carbon energy capital will change over time and eventually go to zero. In particular, $\gamma_{t+s} = \lambda_H^{t+s-1}\lambda\rho\gamma_0 K_0/K_{t+s}$.

ultimately limits the decrease in fixed capital prices and output with respect to Kiyotaki and Moore (1997).

Conversely, we use the full Kiyotaki and Moore's (1997, Chapter III) model, where the aggregate value of debt is not exactly aligned with the value of the collateral, since a fraction of entrepreneurs cannot invest and thus repay part of their debt obligations. Moreover, rather than allowing renegotiation for any negative shock as in Cordoba and Ripoll (2004), we allow for renegotiation only if the economy cannot converge back to the interior steady state following a shock, as in this case both parties have an incentive to renegotiate the outstanding value of the debt.[6] However, for any level of the shock, there are infinite combinations of changes in prices and debt levels, such that the economy converges back to its steady state. See, for example, Figure A7 where we plot these combinations for a write-off of high carbon energy capital equal to 35% of the current stock. The debt level retained by the entrepreneurs and the price of capital in our simulations are the maximum value for debt levels, B^{+}_{t+s-1}, and corresponding price q^{+}_{t+s}, consistent with the economy being able to reach its interior steady state. These correspond to the dot in Figure A7. This choice is the one that makes the downturn less severe (and thus reduces the welfare increase induced by our policies). Analytically, when renegotiation takes place, debt repayments RB_{t+s-1} are pushed down to RB^{+}_{t+s-1}, and prices jump to q^{+}_{t+s}.

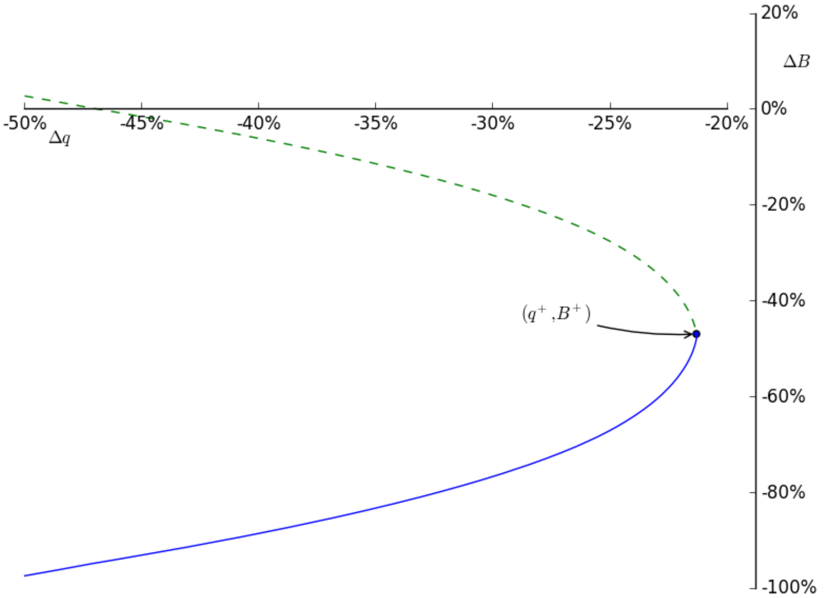

Figure A7: The (q, B) arms for a write-off of 35% of high carbon energy capital

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[6] This is because, while entrepreneurs would always like to reduce their debt, this is not always the case for savers. If the economy cannot converge back to the interior steady state, then the outside option for the representative saver is to accept the economy converging to the steady state with no fixed capital in the hands of the entrepreneurs.

Given the transversality condition in Assumption E, we know that $q_t = q^\star$ for large t. But since Equation (11) defines the asset price as a function of K_t, we can project the asset values back from steady state. So, the rough idea is to guess the initial asset price given the shock and then iterate the economy forward through time to see if it converges again to the steady state. If the level of asset price eventually explodes, the initial guess is revised downward; if it is forever smaller then the initial guess it is revised upward. This "guess and check" procedure is repeated until the asset price is close to the steady state (i.e. within the arbitrary level of tolerance).

When we allow the social planner to take over a fraction ω of debt from the entrepreneurs, the following additional changes are required in the transition equations. Between the period in which the shock is announced and the following period, the value of the entrepreneurs' debt is further reduced to $(1-\omega)B^+_{t+s-1}$. If the transfer of entrepreneurs' debt is funded with a constant tax τ^G over T periods, for T periods we add τ^G in the right-hand side of (A2c) and subtract $\pi\tau^G$ in the right-hand side of (A2a) (inside the square root). Additionally, the budget constraint of the saver now includes debt repayments and new debt from the social planner, RB^G_{t+s-1} and B^G_{t+s}. While this does not directly influence the transition equations, it changes the consumption of the savers in each period, thus influencing the social welfare level reached by the economy. Finally, at $t = T+1$, there is no longer any tax and the social planner holds no debt, so for $t \geq T+1$, the system of transition equations in (A2) holds.[7]

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[7] Simulations are run using Numpy (Walt et al., 2011); graphs are drawn in Matplotlib (Hunter, 2007). We used Python 2.7.

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