



Social interactions, residential segregation and the dynamics of tipping

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Abstract

We develop an analytically tractable population dynamics model of heterogeneous agents to characterize how social interactions within a neighborhood determine the dynamic evolution of its ethnic composition. We characterize the conditions under which integration or segregation will occur, which depends on the majority's social externality parameter and net benefit from leaving, and the minority's leaving probability. Minority segregation may result from the process of tipping, which may arise from three possible channels: two are related to exogenous shocks (migration flows and changes in tipping points) and one is related to the endogenous probabilistic features of our framework (endogenous polarization). This characterization of integration and segregation conditions yields interesting policy implications for social and urban planning policies to mitigate segregation.

Keywords Residential segregation · Social interactions · Tipping

JEL Classification C60 · D70 · J15

1 Introduction

Despite significant improvements in the last few decades, minority segregation is still a pervasive aspect of our society today. Indeed, while segregation at the national level is falling, pockets of highly segregated neighborhoods continue to persist (Glaeser

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and Vigdor 2001; Logan et al. 2004). Segregation manifests in several ways ranging from labor market discrimination to residential segregation. From a historical perspective, residential segregation has received growing attention since (Schelling's 1969) seminal paper. Several works document the extent to which urban segregation occurs and negatively affects the wellbeing of the segregated groups by limiting their access to education, employment opportunities and health outcomes, along with favoring poverty and criminal behavior (Galster 1987; Orfield and Eaton 1996; Shihadeh and Flynn 1996; Cutler et al. 1999; Williams and Collins 2001; Card and Rothstein 2007; Shertzer et al. 2016; Rothstein 2017; Trounstein 2018; Aaronson et al. 2019; Troesken and Walsh 2019).

Theoretically, segregation outcomes are attributed to economic and social factors. Classical economic arguments going back to Tiebout (1956) and Rosen (1974) state that neighborhood sorting is a driver of residential segregation in the presence of heterogeneity in households' incomes and preferences, which ultimately determine their willingness to pay for location characteristics. According to this view, the provision of public goods in specific neighborhoods can generate segregation through its effects on prices and housing demand (McGuire 1974; Card et al. 2008; Kollmann et al. 2018). Social arguments originating in Schelling's (1969, 1971, 1978) works state that social interactions at microeconomic level are a cause of the segregation observed at the aggregate level. According to this view, even moderate individuals' preferences for ethnic isolation can yield segregation in the entire neighborhood (Zhang 2004a, b, 2011; Pans and Vriend 2007; Grauwijn et al. 2012).

Clearly, these alternative theories emphasize how different policies can be used to mitigate segregation: on the one hand, economic theory suggests that urban planning policy, taking the form of public investments in specific neighborhoods, is a natural answer; on the other hand, social interactions theory identifies in social policy, aiming to promote integration between different ethnic groups, as another solution.¹ Few attempts have also been made to bridge these two theories by developing a mixed approach in which some social interaction mechanism (i.e., taste for ethnic isolation) is introduced in a classical economic setup of neighborhood sorting (Becker and Murphy 2000; Sethi and Somanathan 2004; Banzhaf and Walsh 2013). By following this last branch of literature, the goal of this paper consists of developing a framework in which economic and social factors jointly contribute to determining single households' decisions which ultimately drive eventual segregation outcomes. Different to extant works, rather than extending an economic setting to account for some social elements, we introduce economic factors in a simple social interactions framework. Indeed, as pointed out by Arrow (1998), among the possible explanations of segregation, social interactions appear to provide the most convincing argument.

We build on the social interactions theory as outlined in Schelling's works (1969, 1971, 1978) by introducing some economic mechanisms which determine the extent to which economic and social factors may interact to determine segregation.

¹In practice, at least some urban planning policies and urban renewal projects have targeted minority-populated neighborhoods and led to increased segregation and increased discrimination (Highsmith 2009; Retzlaff 2020; Wyly and Hammel 2004). Therefore, social policy represents an important tool to reduce the undesirable consequences of public intervention aimed at mitigating segregation outcomes.

Schelling's main conclusion is that even if the majority and minority groups have some degree of preference for integration, individual decisions could lead to aggregate outcomes of predominantly segregated neighborhoods. In particular, following in-migration of a minority group, provided that the minority share exceeds a critical threshold the neighborhood will "tip" to being composed entirely of the minority population.² The existence of such non-linearities and tipping behaviors has been firstly demonstrated empirically by Card et al. (2008) who, by applying a regression discontinuity approach to analyze tract-level data within US cities between 1970–2000, show that the majority population flees when the minority share of a neighborhood exceeds 5–20%. A number of later studies applying the same methodology confirm the existence of tipping over different time periods and in other countries as well³ (Alden et al. 2015; Shertzer and Walsh 2019; Kollmann et al. 2018). However, very little is known about the determinants of such dynamic effects and only a few attempts have been made to characterize tipping. Most of the theories proposed thus far focus on simple static approaches based upon traditional economic arguments (Card et al. 2008; Heal and Kunreuther 2010; Banzhaf and Walsh 2013), while attempts to discuss the role of social interactions in dynamic settings are limited (see Zhang 2011). However, as suggested by Schelling (1971), "*the analysis of 'tipping' phenomena [...] requires explicit attention to the dynamic relationship between individual behavior and collective results*". Therefore, we aim to contribute to this latter literature in order to provide a dynamic explanation of tipping. Unlike previous works which present some modifications of Schelling's original model and are particularly complicated to the extent that they preclude the understanding of the main mechanisms in place (see Zhang (2011), who analyzes Schelling's setup from an evolutionary game point of view), we develop an alternative population dynamics model of heterogeneous agents which maintains the basic features of Schelling's model but gives rise to simpler, intuitive and analytically-tractable solutions.

To this end, we highlight the two main features of Schelling's model that need to be preserved. First, Schelling is often considered as one of the fathers of heterogeneous agents modeling, as his analysis represents one of the earliest agent-based model examples (Epstein and Axtell 1996; Zhang 2004a). Second, in Schelling's setup there exists some degree of preference for ethnic isolation, which is essential to yield segregation. These two features are maintained in our framework which relies upon social interactions models based upon a random utility setting (Brock and Durlauf 2001; Blume and Durlauf 2003). The presence of heterogeneous agents interacting with one another through a social externality gives rise to preferences for ethnic isolation (due to homophilious considerations), allowing us to understand whether and how social interactions may result in majority and minority segregation driven by individuals' decisions. In contrast to Schelling's model and its refinements (Zhang 2011),

²The phenomenon of tipping has been discussed firstly by Grodzins (1957), who claim that: "*for the vast majority of white Americans a tipping point exists*". He is also the first to suggest that the process of tipping is irreversible, a conclusion supported by Duncan and Duncan (1957) in their study of the experience of Chicago between 1940 and 1950, and by following theoretical works (Zhang 2011).

³Tipping points have been shown to exist also in other discrimination and segregation contexts, including schooling (Caetano and Maheshri 2017) and employment (Pan 2015).

our framework can be fully and analytically solved to give rise to closed-form results determining not only the long run equilibrium outcomes but also the tipping point, which to the best of our knowledge represents an important novelty in this literature. Specifically, our approach describes the dynamic evolution of the majority share in a neighborhood in which individuals are heterogeneous. This permits us to characterize the conditions under which either segregation or integration will occur, and in the former case, whenever segregation results from tipping, we identify three possible channels through which tipping may occur. Out of these three channels, two of them are associated with equilibrium outcomes (migration flows and changes in tipping points) and one with off equilibrium dynamics (endogenous polarization). To the best of our knowledge, the role of off equilibrium dynamics as a source of tipping has not been discussed in extant literature. Our neat characterization of integration and segregation conditions yields interesting policy implications for social and urban planning policies that mitigate segregation.

Note that our population dynamics setup with heterogeneous agents substantially differs from the agent-based frameworks typically employed in extant literature which discuss how the dynamics of complex evolving systems driven by local micro-level interactions in a spatial context may result in the emergence of a macro-level systemic outcome, such as segregation (see Tesfatsion 2006; Namatame and Chen 2016). Indeed, in our baseline model we rely on an a-spatial characterization of a neighborhood which implies that all individuals are located into a single point in space. This allows the network structure of individuals' interactions to be fully connected such that movements into or out of the neighborhood affect all individuals by the same degree, independent of the number of individuals considered. Such an abstraction from space allows us to derive closed-form solutions which clearly identify how the equilibrium value of the majority share depends on key parameters, which pinpoint mechanisms through which policy may affect the equilibrium outcomes. Nevertheless, we also present extensions of our baseline model showing that even by allowing for a spatial dimension (either in the form of a linear city or a random network) our main conclusions will continue to hold true. Therefore, we believe our modeling choices which permit analytical tractability represent mere simplifying assumptions and do not compromise the model's ability to characterize real world phenomena.

This paper proceeds as follows. Section 2 briefly reviews the sociological studies on racism discussing how our paper relates to this literature by finding in structural racism the ultimate cause of residential segregation outcomes. Section 3 introduces our a-spatial social interactions framework to describe how single households decide whether to continue residing in a certain neighborhood by taking into account both economic factors (i.e., individual benefits and costs), along with social factors (i.e., the behavior of other own-group households through a social externality). As a matter of expositional simplicity and in line with previous literature, we shall refer to the major ethnic group as "whites" and to the minority as "blacks". The aggregation of single households' decisions allows us to characterize the neighborhood outcome in terms of the white share. Section 4 analyzes the asymptotic-population dynamics, which is entirely summarized by a differential equation describing the evolution of

the white share. In this setting we can explicitly identify the long run equilibrium outcomes, which may represent either segregation or integration. In the case of minority segregation we determine under which conditions this may result from tipping and we identify two possible mechanisms related to exogenous shocks (migration flows and changes in tipping points) for tipping to occur. Section 5 analyzes the finite population dynamics, in which white flight is stochastic and characterized by some transition probabilities. In such a probabilistic setting, we show that our asymptotic long run predictions may not always come to pass and that a further potential tipping channel, related to endogenous polarization, may emerge. Section 6 presents some extensions of our baseline model to show that its qualitative results apply even in more complicated and general setups, confirming thus that our simple framework represents a good benchmark to characterize the dynamics of residential segregation. Section 7 presents some further extensions of our baseline setup aiming at introducing a spatial characterization, showing that despite the lack of analytical solutions from a qualitative point of view our main results continue to hold true even in a spatial context. Section 8 concludes and suggests directions for future research.

2 Residential segregation and racism

While our paper does not formally model racism⁴ and its conclusions are consistent even in the absence of racism, it is nonetheless critically important to relate our model to the racism literature as residential segregation of ethnic groups may be linked to racial discrimination.

To the best of our knowledge, there have been few attempts in the economics literature (Phelps 1972) to address racism's contentious issue but none of the works analyze the determinants of racism and its implications on socio-economic outcomes. In contrast, racism and its relation with racial discrimination and racial inequality have been widely discussed in sociology (see Clair and Denis (2015), for a concise and precise literature review).

It is not possible to talk about racism without first defining "race". The concept of race as applied to humans has no biological meaning, it is merely a social construct. Extensive sociological studies have attributed this social construction to two factors: individual attitudes at the micro-level and an aggregate social process at the macro-level (Clair and Denis 2015). Micro-level racism originates from prejudicial beliefs and attitudes among single individuals. Such prejudices may be either conscious or unconscious (due to an implicit bias, unaware negative beliefs and feelings about racial groups) and affect single individuals' expectations and actions (Lane et al. 2007). Macro-level racism results from a social process in which individual-level homophily and social norms reinforce each other, influencing aggregate-level

⁴Since it may be unfair to label individuals as "racists" if such intent is lacking, even if their actions may eventually lead to undesirable outcomes (Clair and Denis 2015), we do not characterize racism as in our setup individuals do not aim to harm others, and they do not voluntarily discriminate against people based on ethnic differences. We also stress that our model is not a rationalization of racism, it is both immoral and harmful for discriminated groups as discussed within the previous section.

attitudes. Such attitudes can give rise, even if without overtly racist intent, to broad-scale effects determining rules, opportunities and policies impacting different ethnic groups in a heterogeneous manner – a phenomenon referred to as structural (or institutional or systemic) racism (Stokely and Hamilton 1967; Denis 2015). These two starkly different views suggest that racism may emanate differently, and thus the strategies to address the problem may require different approaches. For the former, it is an individual-based problem that could be alleviated by favoring cross-racial interactions (Allport 1954). For the latter, this macro-structural process is sensitive to and could be aggravated by increasing cross-racial interactions (Nagel 1995).

Following a significant decline in overt racist attitudes witnessed since World War II, the structural racism theory has become the most widely spread view in contemporary sociological research, even if the individual racism theory remains appropriate for normative analysis.

Indeed, the goal of the individual-level approach to racism is “*not to blame the innocent but to improve understanding so that the policies, practices, and ideas that perpetuate racial inequality can be identified and dismantled. Conversely, institutional racism’s focus on the extraindividual might obscure and absolve the role of individual actors in maintaining racism*” (Clair and Denis 2015).

Against this sociological debate’s backdrop, our paper bridges the two sociological theories. We endogenize the structural nature of racism by providing micro-foundations through the homophilous behavior of single individuals.⁵ In our model setup, residential segregation may result from single individuals’ decisions regarding staying in or leaving a neighbourhood according to their homophilous preferences for ethnic isolation.⁶ Such preferences affect social norms and determine the desirability of conforming to the same group individuals’ behavior, which in turn affects others’ choices.

In so doing, homophilous considerations and social norms drive social interactions and determine residential outcomes, giving rise to a social process with heterogeneous effects on different ethnic groups at the aggregate level. Residential segregation can thus be attributed primarily to structural racism as individual-level homophily affects residential outcomes only to the extent that it gives rise to social effects at the macro level through the social interaction channel. Such a characterization of residential segregation follows Schelling’s (1969) view according to which micro motives translate into macro outcomes, and homophily is an important driver of individual behavior (Paolillo and Lorenz 2018).

Despite the critiques that it has received over the years (see Jost et al. (2004), for a reviews of such criticisms), homophily is a very well established fact of human nature (even among children), and the primary driver of homophilous relations is ethnicity as “*Homophily in race and ethnicity creates the strongest divides in our personal environments*” (McPherson and Cook 2001). Even in the housing market’s

⁵Homophily refers to the tendency of individuals to associate with others who are most similar to them (Currarini et al. 2009).

⁶As widely discussed in the sociological literature, ethnicity and race are perceived as two different concepts, but since they are both social constructs the two terms may be used interchangeably (Clair and Denis 2015).

specific context, some studies show that homophily drives residential choices (Bayer et al. 2014; Krysan and Farley 2002; Bakens and Pryce 2019).⁷ Building on this evidence, our approach recognizes that individual-level homophily is an important driver of human behavior, determining the structure of aggregate and social outcomes. As such, to mitigate segregation successfully, it is essential to focus on the determinants of both individual behavior and social interactions. Our normative analysis, as we are going to see later, confirms this prediction by showing that policies aiming to affect individuals' incentives and social phenomena can play both an important role in reducing ethnic inequalities.

3 The model

Consistent with previous literature, we consider a setting in which the housing supply is fixed. Specifically, we focus on a neighborhood D (for district) endowed with a large number N of dwellings, each of which can be occupied by either one white or one black household. We assume that both white and black households may leave the neighborhood and when they do so they are replaced by one household of either the same or the opposite ethnicity. In the former case the ethnic composition of the neighborhood does not change and this has no effects on single individuals' decisions, while in the latter the relative size of the different ethnic groups changes and this impacts individuals' residential choices. Since we are interested in understanding how population dynamics in the neighborhood may depend on the taste for ethnic isolation, which is clearly relevant only in situations in which the ethnic composition within the neighborhood changes, in the following we will focus only on the implications of the latter scenario in which the departure of a household is replaced by another of a different ethnicity.

For the sake of simplicity, while we model white household's behavior we assume that blacks' is exogenously given, and in particular with a given probability $0 < p < 1$ they will decide to leave the neighborhood.⁸ Let us assume that initially most dwellings are occupied by whites and each white household $i = 1, \dots, M$ with $M < N$ attempts to maximize the utility associated with its residential choice. White households are heterogeneous in their degree of preference towards own-group neighbors. The utility function of any white household i is associated with the choice $\omega_i = \{0, 1\}$ such that $\omega_i = 1$ ($\omega_i = 0$) denotes that the household leaves (stays in) the neighborhood. The decision to leave or stay is determined by the utility associated with residing in that neighborhood. This utility depends on three elements: a private and a social component, which are common to all households, and an idiosyncratic

⁷As Bakens and Pryce (2019) state: "In many studies, these socioeconomic differences between natives and ethnic minorities cannot fully explain the concentration and clustering of ethnic minority groups. Preferences for the own ethnic group in residential location choice is found to be part of the explanation".

⁸It may be possible to model black household's behavior similarly to whites', but this will just complicate the model without adding qualitatively any new insight on the neighborhood's composition dynamics. The same comments apply to the case in which dwellings are left vacant before being reoccupied by a newcomer. See Section 6 for further details.

component, which is household-specific. The private component is given by the net benefit of leaving the neighborhood $b - c \in \mathbb{R}$, determined by the difference between benefits $b > 0$ and costs $c > 0$. The social component is associated with the characteristics of the ethnic composition of the neighborhood. In particular, white households prefer to live in a white neighborhood, thus their propensity to leave increases with the prevalence of black individuals. This is captured by the term $J\tilde{b}_i^\varepsilon$, where \tilde{b}_i^ε is the expectations of (white) household i about the proportion of blacks in the neighborhood and $J > 0$ quantifies the importance of the social component relative to the private one in each household's utility.⁹ Since the share of blacks is complementary to the share of whites, the social component can be rewritten as $J(1 - \tilde{w}_i^\varepsilon)$ where \tilde{w}_i^ε is the expectations of household i about the proportion of whites in the neighborhood. Since blacks may move in the neighborhood only when some white dwelling is vacated (i.e., when some white household leaves the neighborhood), the social component represents the fact that whites tend to conform to the behavior of other whites: when one white household expects many whites to leave, that household will more likely decide to leave as well. To properly define \tilde{w}_i^ε , we introduce a vector of state variables s_j , $j = 1, \dots, N$ counting the number of white dwellings, by taking either the value 0 if the site j is occupied by a black or 1 if it is occupied by a white. This allows us to define the expectations about the proportion of whites as follows: $\tilde{w}_i^\varepsilon = \frac{1}{N-1} \mathbb{E}[\sum_{j \neq i} s_j]$. Therefore, the decision of the single (white) household depends on the expectations of other (white) households' choices. The idiosyncratic component is given by ε_i , which is household-specific random term independently and identically distributed (i.i.d.) across households with zero mean (i.e., $E(\varepsilon_i) = 0$), determining the specific type of each household. The utility function of each white household is, therefore, given by the following expression:

$$u_i(\omega_i) = \omega_i (b - c + \varepsilon_i + J(1 - \tilde{w}_i^\varepsilon)). \quad (1)$$

The utility associated with the decision to leave is $u_i(1) = b - c + \varepsilon_i + J(1 - \tilde{w}_i^\varepsilon)$ while the utility to stay is $u_i(0) = 0$. Clearly, as long as the utility associated with staying in the neighborhood is larger than the utility associated with leaving, the household will continue to stay in the neighborhood. In other words, whenever $u_i(1)$ is larger (smaller) than zero, the white household will leave (stay).

Note that the utility level depends on a number of factors. (i) The private component $b - c$ measures the net private utility associated with leaving the neighborhood D . This may be thought of as the net utility obtained from a wide range of sources, including the differential benefits between amenities in other neighborhoods and those in neighborhood D , and the costs related to relocation decisions. The differential benefits may take into account housing prices, population density, degree of safety, location, availability of public transport, school, parks or leisure facilities.

⁹In principle we can also consider the case in which $J < 0$, describing a situation in which the prevalence of black individuals in the neighborhood decreases the propensity of white households to leave the neighborhood itself. This scenario is probably not particularly interesting from a real world perspective and does not lead to outcomes qualitatively different from those associated with the $J > 0$ case. As a matter of expositional simplicity, it seems reasonable therefore to present the model for the $J > 0$ case and comment briefly on what will happen if J is negative.

The costs may include accommodation search, relocation fees (related to selling their property for homeowners, buying a new property, depositing bonds, or interrupting a current lease for tenants). We do not restrict a priori how benefits and costs compare such that net benefits can be positive, negative or null. Note that the private component term captures economic factors related to the availability of private and public goods, consistent with what is discussed by extant economics literature (McGuire 1974; Card et al. 2008; Kollmann et al. 2018). (ii) The social component $J(1 - \tilde{w}_i^e)$ captures the (negative) externality generated by the presence of individuals of a different ethnicity in the neighborhood on the residential decision of (white) households where J determines the magnitude of such an externality. Specifically, J quantifies the importance of a social norm measuring the extent to which conforming to the behavior of other whites is desirable from the point of view of the single (white) household. Indeed, the larger $(1 - \tilde{w}_i^e)$ the higher the expected number of whites leaving by household i , and thus the higher the number of incoming blacks. We can think of the utility arising from this social component as the support and benefits of informal networks in the local community shared by people of the same ethnic group: a larger share of whites leaving will reduce the strength of such informal networks providing each white household with a stronger incentive to leave the neighborhood as well. Therefore, the social component captures social factors similar to those discussed in the social interactions literature (Zhang 2004a, b, 2011; Pancs and Vriend 2007; Grauwin et al. 2012), along with factors capturing how institutions affect individual decisions (Rothstein 2017; Trounstein 2018; Troesken and Walsh 2019), indeed, such a social externality by incentivizing white households to mimic the behavior of other whites determines to some extent their preference for ethnic isolation. Thus, such a social externality determining preference for ethnic isolation may be viewed as a homophily-driven factor leading the majority population to reduce its interactions with minority groups (Durrett and Levin 2005)).¹⁰ Indeed, the primary driver of homophilous relations is ethnicity, and ethnic considerations have played an important role in the formation of social networks in modern societies (Wimmer and Lewis 2010). (iii) The idiosyncratic component ε_i captures any form of heterogeneity within the white population, which may be related to either the private or the social components. In particular, it may be useful to think of it as the individual degree of preference towards own-group neighbors due to ethnic prejudice. Similar to what is discussed in the mixed economic-social interactions literature (Becker and Murphy 2000; Sethi and Somanathan 2004; Banzhaf and Walsh 2013), we assume that such a degree of preference towards own-group neighbors is different from household to household. This suggests that, everything else equal, according to their specific type (the specific value of ε_i) different households may prefer leaving the neighborhood, while others staying. The type of each household, ε_i , is determined by the realization of i.i.d. random shocks drawn from a common distribution $\eta(z) = \mathbb{P}(\varepsilon_i \leq z)$.

¹⁰The concept of homophily has been firstly introduced by Lazarsfeld and Merton (1954) to study friendship network, and in the last decades it has been extended in a number of other social contexts. Homophily is usually introduced in network setups (Currarini et al. 2009; Jackson 2014), but it has also been brought in game-theoretic (Kets and Sandroni 2019) and mathematical (Durrett and Levin 2005) frameworks.

As in Brock and Durlauf (2001), we can show that the utility function (1) can be cast into a probabilistic choice model such that:

$$\mathbb{P}(\omega_i = 1 | s_i = 1, \tilde{w}_i^e) = \eta(b - c + \varepsilon_i + J(1 - \tilde{w}_i^e)). \quad (2)$$

By following the same approach as in Blume and Durlauf (2003), we can recast the above model into a dynamic probabilistic choice model. Let us define $w_t^N = \frac{1}{N} \sum_{j=1}^N s_{j,t}$ as the proportion of white households in the neighborhood at time t , which we will refer to as the “white share” for the sake of expositional simplicity. Under this dynamic version of the model, therefore, the number of whites in the neighborhood changes over time as follows: $M_t = Nw_t^N$. Also, note that the white share determines the stock of whites within the neighborhood and needs to be distinguished from the share of white households that leave the neighborhood at time t , which represents what can be referred to as the “white flight share” (see Banzhaf and Walsh, 2016). Then, as in the static model, white households decide whether to leave or stay in the neighborhood at any given time t with the following probability:

$$\mathbb{P}(\omega_{i,t+\Delta t} = 1 | \omega_{i,t} = 0, s_{i,t} = 1, w_t^N) = \eta(b - c + \varepsilon_i + J(1 - w_t^N)). \quad (3)$$

Recall that $s_{i,t}$ indicates whether the site i is occupied by a black ($s = 0$) or by a white ($s = 1$) individual, and when it is occupied by a black the household may decide to leave the neighborhood with probability $p \in [0, 1]$ as follows:

$$\mathbb{P}(\omega_{i,t+\Delta t} = 1 | \omega_{i,t} = 0, s_{i,t} = 0, w_t^N) = p. \quad (4)$$

The Markovian dynamics of the stochastic process $w^N \equiv (w_t^N)_{t \geq 0}$ in Eq. 3, representing the evolution of the white share, cannot be explicitly analyzed in a finite dimensional population framework. Therefore, in order to derive some analytical results in the next section we will focus on an infinite dimensional population version of the model, and we will get back to its finite population version in the following section. This will help understand the nature of neighborhood dynamics by clearly identifying when either integration or segregation due to tipping will occur, and in the latter case to pinpoint three alternative tipping channels, distinguishing between those associated with equilibrium and off equilibrium dynamics.

Note that our model is substantially different from Schelling’s (1969, 1971, 1978) original setup but at the same time it maintains its idiosyncrasies, even if in a simplified fashion. In particular Schelling has developed two alternative frameworks to discuss residential segregation, the spatial proximity model and the bounded-neighborhood model (see Zhang (2011), for a concise summary of their characteristics), both of which define the neighborhood from a spatial perspective. Such a spatial structure makes the dynamic analysis of segregation outcomes particularly cumbersome and precludes the possibility of obtaining intuitive closed-form results. In order to increase the degree of tractability, in our model the neighborhood is completely a-spatial, namely it is a point in space where the entire (white and

black) population resides.¹¹ By analyzing the evolution of the white share we can understand the population dynamics within the neighborhood.

4 Asymptotic dynamics

As mentioned above, even though analyzing the (stochastic) finite population dynamics is not possible, it is possible to analyze the (deterministic) dynamics associated with its asymptotic version in which the number of dwellings N is infinitely large. By following the same argument as in Blume and Durlauf (2003) and Marsiglio and Tolotti (2018), it is possible to derive the following convergence result.

Proposition 1 *Suppose that $\eta(\cdot)$ is absolutely continuous. Then, the sequence of stochastic processes w_N converges almost surely to $w \equiv (w_t)_{t \geq 0}$, where w_t is the solution of the following differential equation:*

$$\dot{w}_t = p(1 - w_t) - \eta(b - c + J(1 - w_t)) w_t, \tag{5}$$

with fixed and given initial condition $w_0 \in (0, 1)$.

Proposition 1 states that in such an asymptotic framework we can describe the evolution of the white share, which is entirely determined by two terms: the probability of blacks leaving the neighborhood, $p(1 - w_t)$, and the probability of whites leaving it, $\eta(b - c + J(1 - w_t)) w_t$, which quantifies the white flight and depends on whites' net benefits of leaving and the social externality parameter. Clearly, while the former term increases the share of whites (since each black leaving is replaced by a white individual), the latter decreases it.¹² Equation 5 implicitly determines also the evolution of the share of black households in the neighborhood, $b_t = 1 - w_t$, which since complementary to the white share is given by the following equation:

$$\dot{b}_t = \eta(b - c + Jb_t) (1 - b_t) - pb_t. \tag{6}$$

Moreover note that (5) describes the aggregate behavior of whites, and through aggregation we lose track of the behavior of each individual household (and their type). Since the aggregate outcome in the neighborhood is entirely described by a differential equation, this can be explicitly analyzed in order to characterize the long run equilibrium outcomes.

For ease of illustration, in the following we will focus on household types uniformly distributed over the unit interval. This allows us to explicitly derive a number

¹¹We will discuss later that, even if introducing a spatial dimension in our analysis leads to the loss of analytical tractability, through numerical simulations it is possible to show that our main results are qualitatively preserved. See Section 7 for further details.

¹²Note that in our model setup we have not ruled out the possibility that a household which leaves the neighborhood can be replaced by another of the same ethnicity. However, equation (5) describes the variation in the share of whites in the neighborhood and thus it accounts only for those situations in which a white household is replaced by a black household or viceversa. If a household is replaced by a household of the same ethnicity this does not give rise to a change the relative size of the two ethnic groups in the neighborhood and this does not show up in (5), similar to what is discussed in Barucci and Tolotti (2012).

of conclusions, different from previous literature which fails to provide a neat characterization of neighborhood outcomes. However, apart from the eventual loss of closed-form expressions for the equilibrium outcomes, the same qualitative results will hold true also whenever the distribution of types will be single-picked¹³. Under the uniform distribution assumption, $\eta(z)$ is given by the following expression:

$$\eta(z) = \begin{cases} 0 & \text{if } z < 0 \\ z & \text{if } 0 \leq z < 1 \\ 1 & \text{if } z \geq 1 \end{cases} \tag{7}$$

In such a setting it is straightforward to show that the equilibrium outcome depends on the size of blacks' leaving probability (p), the size of whites' social externality parameter (J) and net benefits ($b - c$). If J is small, there always exists a unique stable equilibrium, \bar{w}^* , while as soon as J is large enough there might exist either a unique stable equilibrium \bar{w}^* or a multiplicity of equilibria $\bar{w}_L < \bar{w}_M < \bar{w}_H$ with the middle being unstable and the extremes stable. In order to characterize the possible outcomes, it may be convenient to define the values w^+ and w^- as follows:

$$w^+ = \frac{(b - c + J + p) + \sqrt{(b - c + J + p)^2 - 4Jp}}{2J} \tag{8}$$

$$w^- = \max \left\{ \frac{(b - c + J + p) - \sqrt{(b - c + J + p)^2 - 4Jp}}{2J}; \frac{p}{1 + p} \right\}. \tag{9}$$

The above expressions deserve some explanations. Equation 8 represents the larger solution to the quadratic equation resulting from setting $\dot{w} = 0$ in Eq. 5, in the case in which the realization of w lies within the support of the uniform distribution η . Concerning (9), the first term represents the smaller solution of the same quadratic equation, while the second the lower bound for the share of whites driven by the presence of p in Eq. 5. Indeed, in the extreme case in which $b - c \rightarrow \infty$ the share of whites will naturally converge to $\frac{p}{1+p}$: even if all whites are keen to (and actually do) leave the neighborhood, a fraction $p(1 - w_t)$ of blacks is always exogenously leaving the neighborhood to be replaced by some white entrants. In this respect in the following we will interpret the outcome $w^- = \frac{p}{1+p} \equiv w_S^-$ as a situation of “minority segregation”, which may be either complete segregation (in the case in which $p = 0$ such that $w_S^- = 0$) or partial segregation (in the case in which $p > 0$ such that $w_S^- > 0$).¹⁴ Since it is also possible that not all white households choose to leave the neighborhood accepting thus to co-reside with blacks, and this

¹³In similar settings of social interactions, it is common to assume that the distribution of types is logistic since it describes social phenomena well (Anderson et al. 1992). More generally, by relying on any unimodal continuous distribution function, different from the uniform, the results will be qualitatively similar to those we will derive under a uniform assumption, apart from the lack of closed-form expressions for the equilibrium outcomes (Blume and Durlauf 2003; Marsiglio and Tolotti 2018). It seems convenient to discuss our model's implications in the simplest possible setup.

¹⁴We shall see in Section 6 that by modeling also blacks' residential choices, under certain circumstances, it will endogenously occur that blacks will never choose to leave the neighborhood (i.e., $p = 0$ in our baseline model's formulation) characterizing a situation of complete segregation (i.e., $w^- = w_S^- = 0$) in which the entire neighborhood will be populated by blacks. Note also that in the $p = 0$ case, the two terms in Eq. 9 will coincide and be equal to zero.

happens whenever $w^- = \frac{(b-c+J+p) - \sqrt{(b-c+J+p)^2 - 4Jp}}{2J} \equiv w_I^-$, we will interpret such an outcome as a situation of “ethnic integration”, even if whites and blacks may be co-residing in the neighborhood in different (proportionate or disproportionate) amounts. Equation 9 states that minority segregation and ethnic integration are mutually exclusive, and in particular segregation may be possible whenever $\Delta \equiv (1+p)\sqrt{(b-c+J+p)^2 - 4Jp} - (1+p)(b-c+J+p) - 2Jp > 0$ while integration whenever $\Delta < 0$. Therefore, in the case of convergence to the equilibrium w^- (both in the case of a unique equilibrium or multiple equilibria), we might observe either integration or minority segregation according to which of the above parameter conditions holds true.

The next two propositions summarize all the possible outcomes.

Proposition 2 *Assume that the social externality is small ($J \leq |b - c + p|$). If the net benefit is positive ($b - c > 0$) either integration ($\bar{w}^* = w_I^-$ provided that $\Delta < 0$ holds true) or minority segregation ($\bar{w}^* = w_S^-$ provided that $\Delta > 0$ holds true) will occur, while if the net benefit is negative ($b - c < 0$) majority segregation will occur ($\bar{w}^* = 1$).*

Proposition 3 *Assume that the social externality is large ($J > |b - c + p|$). If the net benefit is positive ($b - c > 0$) either integration ($\bar{w}^* = w_I^-$ provided that $\Delta < 0$ holds true) or minority segregation ($\bar{w}^* = w_S^-$ provided that $\Delta > 0$ holds true) will occur, while if the net benefit is negative ($b - c < 0$) there exists a tipping point ($\bar{w}_M = w^+$) determining whether majority segregation ($\bar{w}_H = 1$) or either integration ($\bar{w}^* = w_I^-$ provided that $\Delta < 0$ holds true) or minority segregation ($\bar{w}^* = w_S^-$ provided that $\Delta > 0$ holds true) will occur.*

Proposition 2 and Proposition 3 show that according to the specific parameter configurations the neighborhood might end up in a situation of either segregation (of either the majority or the minority) or integration, and which equilibrium outcome effectively occurs is determined by economic (i.e., net benefit), social (i.e., social externality) and behavioral (i.e., blacks’ leaving probability) factors.

Intuitively, independent of the size of the social externality, if the net benefit from leaving is positive whites will be keen to leave the neighborhood which becomes populated (either entirely or partially) by blacks. Alternatively, if the net benefit is negative, whites will generally not be keen to leave and the neighborhood will remain completely populated by whites unless the initial share of blacks is particularly large, since also in this case the neighborhood will become populated (either entirely or partially) by blacks. If the equilibrium is unique, which outcome is effectively achieved is entirely determined by how economic, social and behavioral factors compare, and the end results are to a large extent intuitive. If the equilibria are multiple, the possible results are more complicated (and interesting) since the outcome depends, other than the relative size of the different parameters, also on the initial share of whites (or, equivalently, of blacks) in the neighborhood. If the initial share of whites is sufficiently large ($w_0 > w^+$), then white segregation will always occur. If the initial share of whites is low ($w_0 < w^+$), then either integration will occur if $\Delta < 0$ or black

segregation will occur if $\Delta > 0$. In the case of integration, the equilibrium share of blacks will increase with whites' net benefit from leaving and the social externality parameter and decrease with the blacks' probability of leaving, while in the case of black segregation the equilibrium share of blacks depends inversely on the blacks' leaving probability.¹⁵

These different outcomes can be illustrated graphically using a specific parametrization. Specifically, we set blacks' leaving probability at $p = 0.1$, we let the social externality parameter vary as $J \in [0, 2]$ and we consider two different values for the net benefit from leaving, one positive ($b - c = 0.25 > 0$) and one negative ($b - c = -0.25 < 0$). Figure 1 shows how the equilibrium white share changes with the social externality parameter in the case in which the net benefit from leaving is either positive (panel A) or negative (panel B). We can observe that when the net economic benefit is positive, the equilibrium is unique and decreasing in J , and in particular it is associated with integration until Δ is negative while it is associated with minority segregation as soon as Δ becomes positive. When the net benefit is negative, for small J the equilibrium is unique and corresponds to majority segregation while for large J there coexist two stable equilibria (black curves), one associated with majority segregation and one with either minority segregation or integration, separated by the unstable equilibrium (red curve). Interestingly, there exists a discontinuity in the equilibrium value of the white share, which occurs at the point in which the majority segregation outcome collapses to either the minority segregation or integration outcome.

From the above analysis, we can conclude that tipping can eventually occur only in a situation in which the social externality is large ($J > |b - c + p|$), whites' net benefit from leaving is negative ($b - c < 0$) and $\Delta > 0$. Specifically, tipping refers to a situation in which the neighborhood starts from a white status-quo ($w_0 > \bar{w}_M$) and then it eventually tips to become predominantly black ($\bar{w}_L = w_S^-$). This requires that the white share changes basin of attraction by crossing the tipping point (see Zhang, 2011, for a similar discussion of the tipping process). The next proposition summarizes the situations under which this can occur.

Proposition 4 *Tipping may occur as a result of either (i) in-migration of the minority, or (ii) an increase in the tipping point.*

Proposition 4 outlines two channels through which tipping may occur, and in both cases this can only be due to exogenous shocks. (i) Clearly, an in-migration of blacks

¹⁵We have presented the model's outcomes only in situations in which the social externality parameter is positive. If this parameter is negative, that represents a scenario in which white households derive some positive utility from living in a neighborhood predominantly populated by blacks, it is possible to show that the equilibrium white share will always be unique. In particular, in the case of a negative net benefit ($b - c < 0$) the equilibrium outcome will be associated with majority segregation ($\bar{w}_H = 1$), while in the case of a positive net benefit ($b - c > 0$) the equilibrium outcome will be either minority segregation or integration ($\bar{w}^* = \max\{w_J^-; w_S^-\}$). Therefore, from a qualitative point of view the negative social externality scenario gives rise to an outcome equivalent to that described in Proposition 3, apart from the fact that multiple equilibria can never occur and thus the possibility of minority segregation induced by tipping is ruled out.

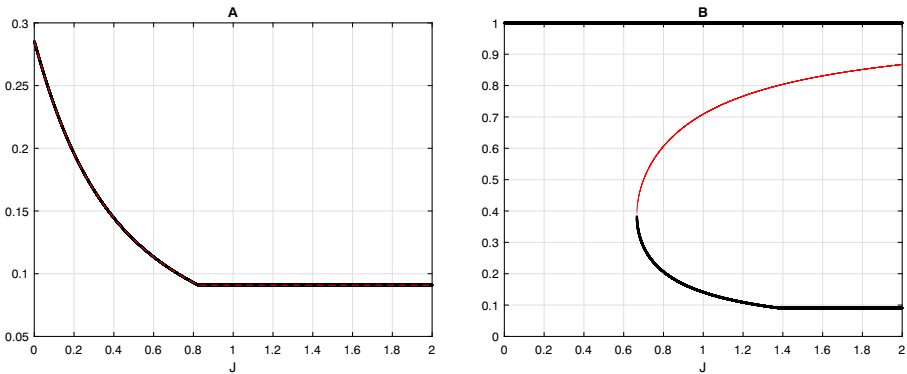


Fig. 1 Equilibrium values of the white share for different values of the social externality parameter, in the case in which the net benefit is either positive (panel A) or negative (panel B)

is accompanied by an out-migration of whites and thus, by reducing the overall share of whites in the neighborhood, it also increases the white flight. This is consistent with the interwar US experience where migration flows have been a major determinant of residential segregation (Wright 1980; Shertzer and Walsh 2019; Kollmann et al. 2018). (ii) Further, a rise in the tipping point, caused by either an increase in whites' net benefit from leaving or an increase in the social externality parameter or a decrease in the blacks' probability of leaving, can directly induce tipping. This is consistent with the post-war US experience where housing price considerations have played a major role in black segregation (Boustan 2010; Brooks 2011; Card et al. 2008). Note that these two channels for tipping are similar to those presented in the economics literature and in the original Schelling's (1971) work, even if they have been derived in more informal ways or in purely static frameworks. For example, Card et al. (2008) analyzing a local housing market model show that exogenous variations in the growth of blacks and thus in their demand for dwellings may induce whites to flee the neighborhood giving rise to tipping. Similarly, Banzhaf and Walsh (2013) show that in a neighborhood sorting model tipping may arise from exogenous changes in the level of provision of a local public good since it modifies the amenities differential between neighborhoods. Both mechanisms are consistent with our results in Proposition 4 suggesting that exogenous shocks may lead a neighborhood to tip.

The existence of these two channels (i.e., in-migration of blacks and increase in the tipping point) confirms that our theory is supported by empirical evidence which explains real world experiences. However, to the best of our knowledge, no other work has thus far explicitly characterized the tipping point, and its closed-form expression clearly shows that it depends on economic, social and behavioral factors which in turn explain the large variation in tipping point estimates across space and over time (Card et al. 2008; Kollmann et al. 2018) as driven by some parameter differences. Moreover, in the empirical literature on tipping (Card et al. 2008; Alden et al. 2015; Shertzer and Walsh 2019; Kollmann et al. 2018), a common assumption underlying the regression discontinuity approach is that tipping points are exactly the same in all neighborhoods within a city (Banzhaf and Walsh 2013). However, our model

suggests this is unlikely to be the case due to differential amenities across neighborhoods, and therefore in certain neighborhoods white flight might not take place till the black share exceeds particularly large values. A similar conclusion is presented in different contexts by Banzhaf and Walsh (2013), and Caetano and Maheshri (2017).

This is the first study to identify the conditions under which integration or tipping may occur in an intuitive and tractable approach. What our analysis suggests is that even a small difference in economic net benefit, social externality or behavioral factors like blacks' leaving probability can yield diametrically opposite segregation outcomes, which has important policy implications. For example, suppose that we observe a neighborhood in which growing black segregation occurs as a result of tipping, which is the case whenever $J > |b - c + p|$, along with $b - c < 0$ and $\Delta > 0$. Clearly, in this context understanding how to intervene with specific policies in order to mitigate segregation is a concern for local policymakers, and integration may be achieved by simply decreasing the value of Δ enough to turn it negative $\Delta < 0$. In order to effectively achieve integration two different types of policy intervention may be used, aiming at decreasing either the whites' net benefit or the social externality, respectively. A decrease in the net benefit from leaving for the majority can be implemented through specific urban planning policies aimed at increasing the provision of public goods (like safety, public transport, school, parks or leisure facilities) in the neighborhood, which in turn increases the attractiveness of a dwelling located in the neighborhood D disincentivizing households to leave. A decrease in the social externality can be implemented through social policies aimed at favoring multiculturalism and tolerance with respect to diversity in the local community, in order to lower the concerns of the majority for the presence of minor ethnic groups in the neighborhood. Provided that these policies affect the size of the net benefit and the social externality sufficiently to ensure that the inequality $\Delta < 0$ holds, then it will be possible for the neighborhood to experience integration. Our conclusions are consistent with extant literature which suggests that urban planning and social policies are natural solutions of residential segregation issues (Pancs and Vriend 2007; Banzhaf and Walsh 2013).

Some additional comments may be worthwhile at this point. Our theoretical model suggests that, despite the existence of preferences for ethnic isolation within a neighborhood, the dynamic evolution and equilibrium of its ethnic composition are not obvious at all. As we might expect the taste for own-group neighbors can naturally give rise to segregation (either of the majority or the minority) but this is only one of the possible outcomes. Indeed, another equally likely possibility is integration suggesting thus that the nature of preferences at individual level per se does not yield a specific residential pattern, but this is rather determined by how economic, social and behavioral factors combine and relate one another. Since integration can be considered as a desirable result which does not require the implementation of specific mitigation policies, in the above (and following) discussion we have emphasized segregation which appears more interesting from a normative perspective as it clearly requires careful consideration to understand how specific policy tools can be employed to improve the residential outcome favoring integration. Nevertheless, this does not mean that integration is not appealing or less likely than segregation, and in fact an important contribution of our paper involves showing that integration and segregation (eventually due to tipping) may alternatively arise within the same

framework from the same household preferences structure, and identifying clear cut conditions under which these two alternative scenarios may occur.

5 Finite dynamics

Thus far we have explicitly characterized the possible equilibrium outcomes within a neighborhood, but when studying social dynamic processes it is essential not to rely exclusively on equilibrium analysis but consider also off equilibrium dynamics (Pancs and Vriend 2007). Recall that we have focused only on the deterministic asymptotic version of the stochastic finite population framework characterized by transition probabilities as in Eq. 3, however such an approach may lead to the loss of important information in the presence of multiple equilibria as discussed in Marsiglio and Tolotti (2018).¹⁶ Indeed, in this case (which occurs whenever $J > |b - c + p|$ and $b - c < 0$), due to the probabilistic nature of the model and the presence of two locally stable equilibria, namely \bar{w}_L and \bar{w}_H , it may well be possible that, despite our deterministic theory suggests that as long as $w_0 > \bar{w}_M$ the long run equilibrium will be $\bar{w}_H = 1$, the equilibrium achieved will be $\bar{w}_L = w^-$ meaning that the neighborhood will endogenously tip (provided that $\Delta > 0$ holds true) giving rise to minority segregation (without any migration flow or change in the tipping point). This feature is related to the metastability properties of the two stable equilibria, whereby metastability is defined as in Benaim and Weibull (2003).¹⁷

We illustrate this outcome in Fig. 2 which represents the evolution of the white share as predicted by the asymptotic deterministic model (solid curve) and resulting from a simulation of the stochastic finite population model (dotted curve). We arbitrarily set the parameter values as follows: $J = 1.3$, $b - c = -0.25$, $x_0 = 0.806$, $N = 100$, implying that the tipping point is $\bar{w}_M \approx 0.8$, which is consistent with the estimate of Card et al. (2008). However, it is possible to show that the results will be qualitatively identical in any other parametrization giving rise to tipping (see Proposition 3). Note that since $w_0 > \bar{w}_M$ we would expect, consistent with our asymptotic theory, that white flight will not occur and the neighborhood will become entirely populated by whites, that is $\bar{w}_H = 1$. In panel A, we show the typical situation in which the white share converges straight towards the expected equilibrium of majority segregation. In panel B, in contrast, we show that the white share converges towards the unexpected equilibrium of minority segregation, $\bar{w}_L = w_S^- = 0.099$, and

¹⁶In presence of a unique equilibrium this type of problem does not occur, since in such a setting the stochastic finite population dynamics closely mimics the deterministic asymptotic dynamics (Marsiglio and Tolotti 2018).

¹⁷Metastability refers to a situation in which, even if a process starts in the basin of attraction of a particular equilibrium, there exists a “tunneling time” that is a time in which the process leaves this basin. However, a precise and unique definition in economics does not exist: according to Benaim and Weibull (2003) a process is metastable if there exists a unique tunneling time (i.e., when the process leaves its original basin of attraction, it then remains outside it forever), while according to Marsiglio and Tolotti (2018) a process is metastable if there exist infinite tunneling times (i.e., when the process leaves its original basin of attraction, it then keeps returning into and leaving it). Note that in our framework neighborhood tipping is due to metastability meant as in Benaim and Weibull (2003), but not as in Marsiglio and Tolotti (2018).

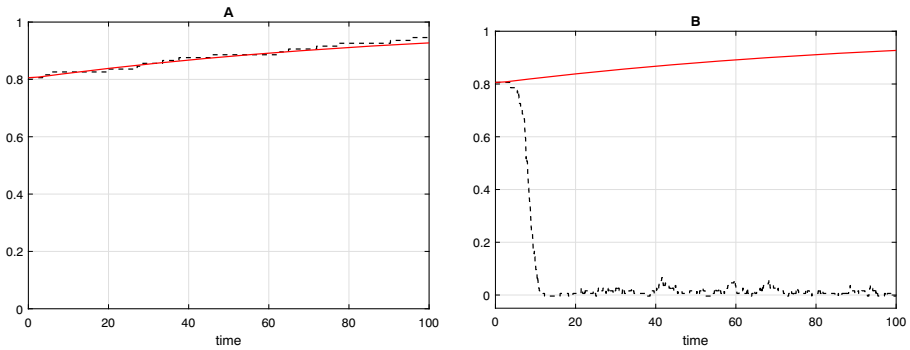


Fig. 2 White share dynamics in the asymptotic (in red) and finite population (in black, dashed) model. The simulated white share w_t^N shows different types of behavior: converging straight towards the expected equilibrium (A), or converging towards segregation (B)

such a shift from the basin of attraction of one equilibrium to the other occurs endogenously leading to polarization in agents' decisions: each agent decides whether to leave by forming his own expectations about the behavior of other agents, and when many agents agree on their expectations of a large number of agents leaving (even if the share of whites is higher than the tipping point) this will give rise to a process of white flight resulting in minority segregation (in which the actual white share falls short of the tipping point). By recalling that in our setup the white share does not necessarily coincide with the white flight share, to gain better understanding on the dynamics underlying the above figure, we report in Fig. 3 the fraction of whites arriving and leaving the neighborhood per unit of time¹⁸ associated with the two panels in Fig. 2. In panel A we can observe that convergence to the expected equilibrium $\bar{w}_H = 1$ occurs when the share of incoming whites (the black curve) exceeds the white flight share (the red curve). In contrast, in panel B we have the opposite situation in which the red curve is always above the black one, thus resulting in converge to the unexpected equilibrium of minority segregation. We can thus state the following result.

Proposition 5 *Tipping may also occur as a result of endogenous polarization in the majority's decisions.*

Proposition 5 outlines a third channel through which tipping may occur, but differently from the two identified in Proposition 4, this arises endogenously from off equilibrium dynamics. Note that endogenous polarization in whites' decisions is a peculiarity of our framework and deals with its probabilistic properties. As long as the trajectory of the stochastic process is still in its transient phase (i.e., out of equilibrium), it can leave the basin of attraction of the theoretically predicted equilibrium

¹⁸From a mathematical viewpoint, these figures can be monitored by two Poisson processes counting the number of departures and arrivals normalized over time.

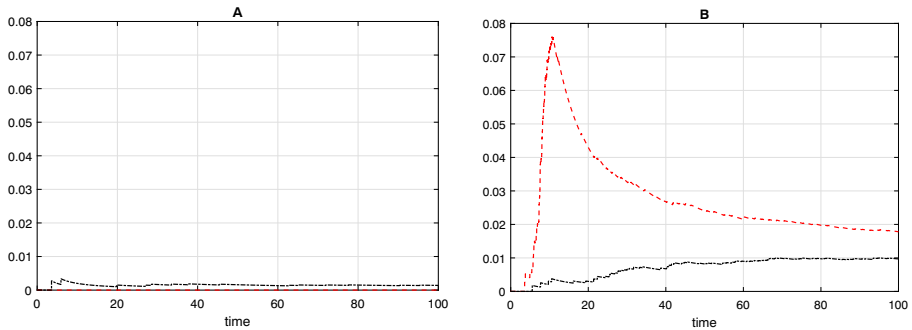


Fig. 3 Fraction of whites leaving (red, dashed) and arriving (black, dash-dot) per unit of time under the same situations in Fig. 2: converging straight towards the expected equilibrium (A), or converging towards the unexpected equilibrium with segregation (B)

and reach an unexpected equilibrium. In other words, due to the randomness induced by the households-specific random terms $(\varepsilon_i)_{i=1,\dots,N}$, the dynamics of the model with a finite number N of dwellings may behave differently from that predicted by the asymptotic model. In order to observe this phenomenon, N must be neither too large (otherwise a suitable law of large numbers would push the dynamics quickly towards its deterministic limit) or too small (otherwise pure noisy trajectories would emerge). In our simulations, we consider N in the order of hundreds as a suitable intermediate value to observe such a phenomenon.¹⁹

To the best of our knowledge, there is no other work in which tipping occurs endogenously except for Zhang's (2011), in which tipping is driven by a probabilistic framework similar to ours. However, in his model black segregation is the only possible outcome, while in our setting white segregation or integration of blacks and whites may also occur. Moreover, following Schelling's (1969, 1971, 1978) model his analysis is performed in terms of the pair of different-ethnicity neighbors, while ours is in terms of the white share which is the variable commonly employed in the empirical analysis of segregation (Card et al. 2008). However, note that unlike the other two channels, tipping due to endogenous polarization in whites' decisions cannot be easily captured in empirical works, and this raises concerns about the robustness of the regression discontinuity approach given that it precludes the possibility of tipping that results from endogenous polarization.

The possibility that a neighborhood will endogenously tip even in the absence of exogenous shocks further stresses the relevance of our previous policy discussion. The need to implement specific policies in order to mitigate segregation does not arise only when growing minority segregation occurs but also whenever there is multiplicity of equilibria such that tipping may occur. In fact, even if the white share is large enough to exceed the tipping point, black segregation may potentially occur via endogenous coordination. In this case relying on urban planning and social policies might be the best strategy in order to effectively mitigate segregation.

¹⁹For more details, we refer the reader to Marsiglio and Tolotti (2018) where such transitory behavior is analyzed explicitly characterizing the specific conditions under which a similar phenomenon may emerge.

6 Extensions

We now discuss a few extensions of our framework in order to show that, despite its simplicity, the conclusions of our baseline model are robust to a number of generalizations, and thus it represents a good benchmark to analyze issues related to residential segregation in an intuitive setup.

6.1 Private component depending on the minority share

We first consider a situation in which the private component in white households' utility is partly affected by the size of the minority share. Indeed, the private component may capture housing and rental prices which are likely to change when the demographic composition of the neighborhood changes. One of the main driver of the white flight during the postwar period in certain US cities was the white population's concern about a potential fall in real estate value as a large number of blacks moved from the southern to the northern states (Wright 1980; Shertzer and Walsh 2019; Kollmann et al. 2018). This may be captured in our setup by modifying the private utility component as follows: $b - c = (b - c)(1 + \alpha \tilde{b}_i^e)$, stating that the net benefit from leaving increases with the expected share of blacks in the neighborhood, and the parameter $\alpha \geq 0$ quantifies the extent to which the net benefit depends on the demographic composition of the neighborhood. By recalling that $\tilde{b}_i^e = 1 - \tilde{w}_i^e$, the utility function of white household i can be written as follows:

$$u_i(\omega_i) = \omega_i (b - c + \varepsilon_i + [J + \alpha(b - c)](1 - \tilde{w}_i^e)). \quad (10)$$

By defining $\bar{J} = J + \alpha(b - c)$, it straightforward to note that the model boils down to our benchmark model in which the social externality parameter is now given by \bar{J} . Therefore, all our conclusions from Proposition 1 to Proposition 5 still hold true (by replacing J with \bar{J} eventually).

6.2 Endogenous minority dynamics

We now consider a situation in which blacks' behavior is no longer exogenously given, but it is endogenous and mimics whites'. Blacks, rather than leaving the neighborhood with a given probability p , decide whether to leave or stay in the neighborhood by following a utility maximization process. The utility function of black household k with $k = 1, \dots, M - N$ is given by the following expression:

$$u_k(\omega_k) = \omega_k (b^b - c^b + \varepsilon_k^b + J^b(1 - \tilde{b}_k^e)), \quad (11)$$

where the parameters with the superscript b denote the fact that blacks' benefit, cost and social externality parameter may differ from whites'.

By assuming that ε_k^b are i.i.d. with absolutely continuous distribution function η^b , it is not difficult to show that, under this more general setting, Eq. 5 reads as follows:

$$\dot{w}_t = \eta^b(b^b - c^b + J^b w_t)(1 - w_t) - \eta(b - c + J(1 - w_t)) w_t. \quad (12)$$

The above equation states that, exactly as in our baseline model, the share of whites in the neighborhood depends on the probability of blacks leaving the neighborhood and the probability of whites leaving it. The only difference is related to the fact that blacks' leaving probability is given by $\eta^b(\cdot)$ and is determined by blacks' choices. By assuming an uniform distribution for η , it is clear that we can expect that the behavior of Eq. 12 will closely resemble that of Eq. 5, even if it is computationally more involved, and so our main conclusions from Proposition 4 and Proposition 5 will still hold true. One peculiarity of this more general setting is related to the possibility that complete segregation of the minority may occur as a result of an endogenous optimization problem: if $b^b - c^b$ is negative and very low, then η^b will be zero (corresponding to the $p = 0$ case in Eq. 5) and thus one of the possible equilibrium outcomes (both in the case of uniqueness and multiplicity) will be $w_S^- = 0$. In order to get a better sense of the possible outcomes, we present a specific example in which the parameters are set as follows: $b - c = 0.25$, $J = 0.5 = J^b$, with $b^b - c^b$ free to vary. It is possible to show that for $b^b - c^b \leq 0$ the only equilibrium for the white share is $w_S^- = 0$. For $0 < b^b - c^b < 0.25$, the equilibrium w_S^- increases with the net benefit in the range $[0, 0.5]$, and by symmetry when $b^b - c^b = 0.25$, the equilibrium is exactly 0.5. When $b^b - c^b > 0.25$, the situation is completely reversed and the equilibrium white share is larger than 0.5. In order to allow for multiple equilibria (and hence tipping, eventually), either J or J^b needs to increase. For example, with $b^b - c^b = b - c = -0.25$ and $J^b = J = 1$, the two extreme equilibria, $\bar{w}_H = 1$ and $w_S^- = 0$, coexist and are both stable, with their basin of attraction separated by the tipping point.²⁰

6.3 Two white subpopulations

We now consider a situation in which the white population is heterogeneous in its taste for ethnic isolation. Some studies show that individuals may differ in their willingness to interact with individuals of a different ethnicity, simply because they may have been differently exposed to diversity (Christ et al. 2014). Individuals interacting frequently with people from diverse ethnic groups tend to develop more tolerance towards diversity than individuals who seldom interact with them. In our setting, this suggests that it may be possible that the majority may be divided into multiple subgroups in which individuals are characterized by a different value of the social externality parameter. In order to keep the analysis as simple as possible, similar to Barucci and Tolotti (2012), let us assume that there exist two subpopulations of whites, W^1 and W^2 , in which the social externality parameter is given by J^1 and J^2 , respectively. Without loss of generality, we assume that $J^1 \leq J^2$, that is subpopulation 1 is less concerned than subpopulation 2 about the eventual presence of blacks in the neighborhood. We also assume that at time 0, $\alpha \in (0, 1)$ is the proportion of whites of type 1 and $1 - \alpha$ the proportion of type 2, such that $W_0 = \alpha W_0^1 + (1 - \alpha) W_0^2$.

²⁰The phase diagram analysis in the case of multiple equilibria is much richer here due to the presence of two behavioral terms in Eq. 12 and is out of the scope of this paper. We leave this intriguing issue for future research.

We generalize the site indicator s_j to account for three different types of residents: $s_j \in \{0, 1, 2\}$, in which $s_j = 0$ denotes that site i is occupied by a black, $s_j = 1$ that it is occupied by a white of type 1, and $s_j = 2$ that it is occupied by a white of type 2. When a black household leaves the site (again with exogenous probability p), now it may be replaced by a white of type 1 (with probability $0 < q < 1$) or by a white of type 2 (with probability $1 - q$). By defining $w_t^{1,N}$ and $w_t^{2,N}$ as the share of whites of type 1 and type 2, respectively, the probabilistic choice model can be stated as follows:

$$\begin{aligned} \mathbb{P}(\omega_{i,t+\Delta t} = 1 \mid \omega_{i,t} = 0, s_{i,t} = 1, w_t^{1,N}, w_t^{2,N}) \\ = \eta(b - c + \varepsilon_i + J^1(1 - w_t^{1,N} - w_t^{2,N})) \end{aligned} \tag{13}$$

$$\begin{aligned} \mathbb{P}(\omega_{i,t+\Delta t} = 1 \mid \omega_{i,t} = 0, s_{i,t} = 2, w_t^{1,N}, w_t^{2,N}) \\ = \eta(b - c + \varepsilon_i + J^2(-1w_t^{1,N} - w_t^{2,N})) \end{aligned} \tag{14}$$

$$\mathbb{P}(\omega_{i,t+\Delta t} = 1 \mid \omega_{i,t} = 0, s_{i,t} = 0) = p. \tag{15}$$

Similar to what we have discussed in our baseline model, it is possible to prove the following result.

Proposition 6 *Suppose that $\eta(\cdot)$ is absolutely continuous. Then, the sequence of stochastic processes $w_t^{1,N}$ and $w_t^{2,N}$ converges almost surely to $w \equiv (w_t^1, w_t^2)_{t \geq 0}$, where*

$$\dot{w}_t^1 = pq(1 - w_t^1 - w_t^2) - \eta(b - c + J^1(1 - w_t^1 - w_t^2))w_t^1, \tag{16}$$

$$\dot{w}_t^2 = p(1 - q)(1 - w_t^1 - w_t^2) - \eta(b - c + J^2(1 - w_t^1 - w_t^2))w_t^2, \tag{17}$$

with fixed and given initial conditions $w_0^1 = \alpha w_0$ and $w_0^2 = (1 - \alpha)w_0$, with $w_0 \in (0, 1)$.

Proposition 6 shows that the determinants of the evolution of the share of type 1 and type 2 whites are the same as the determinants of the evolution of white share in the case of a unique white population (economic, social and behavioral factors). The dynamics of the type 1 and type 2 shares is characterized by the difference between inflows in and outflows from the neighborhood of the relevant group. It may thus be possible to observe whites of type 1 fleeing, whites of type 2 fleeing, both white groups fleeing, or none fleeing out of the neighborhood. Again, relying on the assumption of an uniform distribution for η , the behavior of the system (16) and (17) will closely resemble that of Eq. 5, and so our main conclusions from Proposition 4 and Proposition 5 will still hold true.

In order to visualize this, we present a numerical simulation in the case of tipping in Fig. 4. In panels A and B we show the time evolution of three different populations: the (total) white share (in black), the share of type 1 whites (in red) and the share of type 2 (in blue). Parameters are chosen arbitrarily in order to make the figure as clear as possible. In particular, we set $b - c = -0.1, J^1 = 0.5, J^2 = 1, p = 0.22, \alpha = q =$

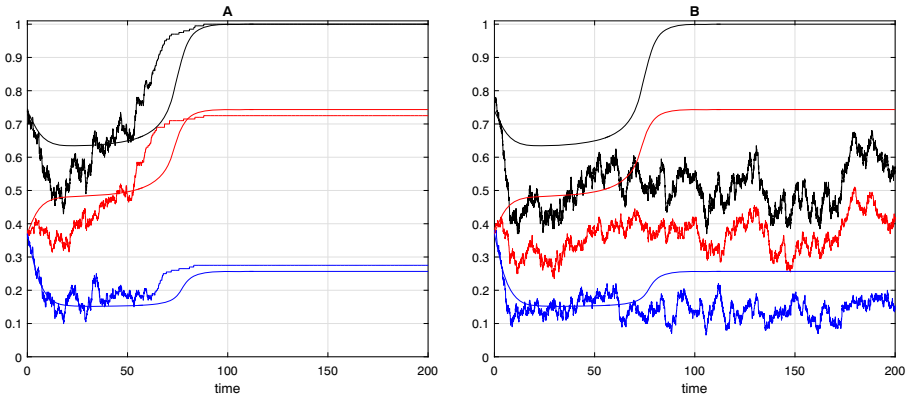


Fig. 4 White share dynamics for the two subpopulations model both for the asymptotic and finite dimensional model, with $N = 200$. The simulated white shares show different types of behavior: converging straight towards the expected equilibrium (A), or converging towards segregation (B)

0.5 such that $w_M \approx 0.735$ and $w_L = w_S^- \approx 0.51$, and the initial condition, $w_0 = 0.74$, is above the tipping point such that the expected equilibrium will be of majority segregation, $w_H = 1$. In panel A we can observe that the expected equilibrium is effectively achieved, after an initial transient period in which the dynamics seem to depart from the expected outcome. In contrast, in panel B the finite dimensional model converges to the unexpected equilibrium of minority segregation, $w_L = w_S^-$. These results are qualitatively identical to those discussed for our baseline model, confirming the robustness of our conclusions. Note, however, that the presence of two whites subpopulations allows for more complicated dynamics: by looking at the total share of whites (black curve) in panel A, we can see that the dynamics start decreasing because of a large flight of type 2 whites, but the persistence of type 1 whites makes the dynamics flip behavior and at some point also the type 2 whites return; the end result of this process is represented by the equilibrium of majority segregation in which the neighborhood becomes populated entirely by whites.

6.4 Vacant dwellings

Finally, we extend our baseline model to allow for the possibility that dwellings remain vacant. Specifically, when an occupant (white or black) decides to leave, the dwelling may be occupied by an occupant of opposite ethnicity (with probability $0 \leq q \leq 1$) or it remains empty (with probability $1 - q$). In turn, a vacant site may become occupied by a newcomer with probability $0 \leq \beta \leq 1$. For the sake of simplicity, we assume that q and β are homogeneous across the ethnic groups and thus take the same values for both whites and blacks. Similar to previous extensions, Eq. 5 can be easily modified to account for this new generalization. For this purpose, we introduce a new variable, $\xi_t = 1 - w_t - b_t$, counting the proportion of vacant dwellings such that $0 \leq \xi_t \leq 1$. Under the same technical assumption employed in

our previous settings, it is possible to prove that the evolution of the share of whites, blacks and vacant dwellings is given by the following expressions, respectively:

$$\dot{w}_t = p q b_t - \eta (b - c + J b_t) w_t + \beta \xi_t, \quad (18)$$

$$\dot{b}_t = -p b_t + \eta (b - c + J b_t) q w_t + \beta \xi_t, \quad (19)$$

$$\dot{\xi}_t = p(1 - q) b_t + \eta (b - c + J b_t) (1 - q) w_t - 2\beta \xi_t, \quad (20)$$

with fixed and given initial conditions $\xi_0 \in [0, 1]$, $w_0 = \alpha(1 - \xi_0)$ and $b_0 = (1 - \alpha)(1 - \xi_0)$, where $0 < \alpha < 1$ represents the initial proportion of whites in occupied dwellings. Note that the third equation is redundant since, at each point in time, $\xi_t = 1 - w_t - b_t$, thus it is possible to analyze the dynamic of our neighborhood by focusing only on the evolution of the white and black shares, which will in turn determine the evolution of the vacant dwellings. Note also that the model above is a mere generalization of our baseline setup, since whenever $q = 1$, $\beta = 0$, and $\xi_0 = 0$, it boils down to our original model summarized by Eq. 5, thus it should come as no surprise that our main conclusions will also apply in this extended framework.

We now present a numerical simulation to illustrate the implications of introducing the possibility of vacant dwellings in our analysis. In Fig. 5, we show that, similar to our baseline model, multiple equilibria still emerge in the asymptotic population model and thus tipping may occur in its finite population version. In order to push the new vacant dwellings feature as far as possible we set $q = 0$, meaning that when an occupant leaves a dwelling, this will always remain vacant before getting reoccupied by a newcomer with probability β . We set all parameters as follows: $b - c = -0.25$, $J = 1.3$, $p = 0.2$, $q = 0$, $\beta = 0.5$ and $\xi_0 = 0.1$, and consider two different values for α , quantifying the initial proportion of whites: $\alpha = 0.6$ (panel A) and $\alpha = 0.8$ (panel B). In this scenario initially 10% of dwellings are vacant, and the proportion of whites occupying non-vacant dwelling is either 60% or 80%. We can observe that in panel A the initial values of the white, black and vacant dwelling shares are $(w_0, b_0, \xi_0) = (0.54, 0.36, 0.1)$ and their final values are $(w_T, b_T, \xi_T) = (0.24, 0.54, 0.22)$ giving rise to an integration outcome in which the shares of whites and blacks are both positive. In panel B, instead, their initial and final values are $(w_0, b_0, \xi_0) = (0.72, 0.17, 0.1)$ and $(w_T, b_T, \xi_T) = (1, 0, 0)$ respectively giving rise to a majority segregation outcome in which the share of blacks is completely null. Given the same parameter values, the convergence to different equilibria when the initial conditions change confirms that multiple locally stable equilibria exist, which is the essential feature (also in our baseline model) for tipping through endogenous polarization to occur. Indeed, minority segregation may occur if the social externality parameter takes a higher value (set at $J = 2$) while all other parameters remain constant (panel C); we can see that the white share converges to a very low value and the neighborhood becomes predominantly populated by black households, characterizing thus a minority segregation outcome.

7 Spatial extensions

One of the most important features of Schelling's (1969, 1971, 1978) works, which is missing in our baseline setup, is some spatial characterization of neighborhoods. We

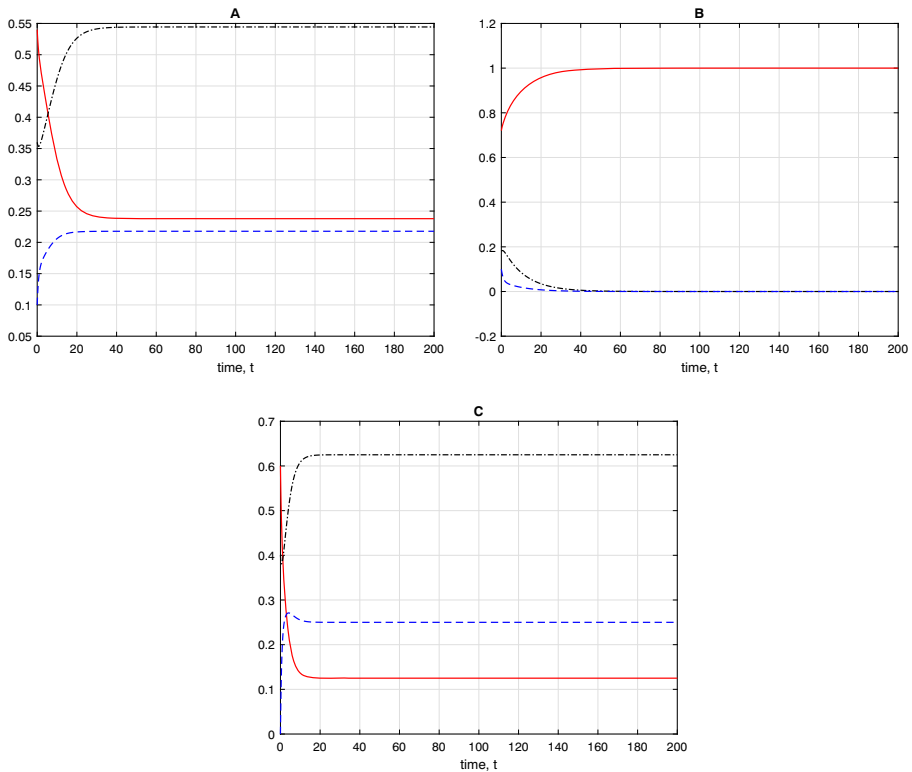


Fig. 5 Dynamics of the white share (red curve), black share (black dash-dotted curve) and vacant dwellings (blue dashed curve) in the asymptotic model with vacant dwellings. The panels show different types of behavior: integration (panel A), majority segregation (panel B) and minority segregation (panel C)

thus return to our baseline setup summarized by Eq. 5 and discuss further extensions of such an a-spatial model in order to show that, even by introducing some spatial dimension, from a qualitative point of view the main conclusions of our a-spatial analysis still apply. In particular, we model an urban agglomerate by considering two different characterizations of space, one in terms of neighborhoods interconnected within a linear city and one in terms of neighborhoods interconnected within a city through a random network, showing that both setups give rise to results consistent with those outlined in our previous analysis.

7.1 A linear City

We start by extending our previous analysis to allow for the existence of multiple interconnected neighborhoods within a city. Specifically, we now consider an urban agglomerate, which we refer to as a “city”, that is composed of a multiplicity of interconnected neighborhoods. We assume that the city is not a single point in space where all dwellings are located, but it develops along a line (Hotelling 1929) and thus white and black households may be heterogeneously distributed along the city. This

implies that in different neighborhoods the initial distribution of the ethnic groups may be different, but individuals may move from one neighborhood to another and thus over time the ethnic distribution in the city might change. Specifically, we denote with $W_{t,x}$ the number of white households in the position x at date t , in a compact interval $[x_a, x_b] \subset \mathbb{R}$, where, for simplicity, we assume $x_a = 0$ and $x_b = 1$. We assume that there are no migration flows through the borders of $[x_a, x_b]$, namely the directional derivatives are null, $\frac{\partial W_{t,x}}{\partial x} = 0$, at $x = x_a$ and $x = x_b$. In this framework, any position x may be interpreted as a specific neighborhood while a set of adjacent locations is interpreted as a region in the spatial city. Since dwellings are spatially distributed, by defining $N = \int_{x_a}^{x_b} N_x dx$ where N_x are the dwellings in neighborhood x , we can compute the share of whites in location x as $w_{t,x} = \frac{W_{t,x}}{N}$. The behavior of white households in each single neighborhood is exactly as discussed in our a-spatial model, thus we can describe the spatio-temporal evolution of the share of whites through the following partial differential equation:²¹

$$\frac{\partial w_{t,x}}{\partial t} = d \frac{\partial^2 w_{t,x}}{\partial x^2} + p(1 - w_{t,x}) - \eta(b - c + J(1 - w_{t,x})) w_{t,x}, \quad (21)$$

where $d \geq 0$ is the diffusion coefficient measuring the extent to which individuals tend to move from one to another neighborhood. The term $d \frac{\partial^2 w_{t,x}}{\partial x^2}$ captures some form of preference for diversity: white households tend to move from neighborhoods highly populated by whites to neighborhoods less populated by whites. This term tends to counterbalance whites' preference for residing in neighborhoods predominantly populated by white households. The initial spatial distribution of whites households $w_{0,x}$ captures the eventual initial heterogeneity in the ethnic composition of neighborhoods, and our goal in this section is to understand whether such heterogeneity may tend to persist or die out over time.

Analytically investigating the model's outcomes would require discussing the solution and properties of partial differential equations, which is quite complicated and goes beyond the scope of this section. We therefore proceed via numerical simulations to illustrate the spatio-temporal dynamics of the white share in our linear city. Figure 6 presents a significant example in which the main parameters are set as in Section 5 ($J = 1.3$, $p = 0.1$, $b - c = -0.25$) while the initial distribution of whites is uniform along the city ($w_{0,x} = x$) and the diffusion parameter d takes three different values: $d = 0$ (panel A), $d = 0.0001$ (panel B) and $d = 0.001$ (panel C). We can observe that in the absence of spatial diffusion each neighborhood is independent from the others and thus its white share follows exactly the same dynamic pattern we have characterized in our a-spatial theory: neighborhoods in which the initial share is lower than the tipping point ($\bar{w}_M = 0.8$) converge toward a minority segregation outcome, $w_S = 0.1$, while those in which the initial share is larger than the tipping point the convergence is towards a situation of majority segregation, $\bar{w}_H = 1$ (panel A).

²¹A similar setup where the spatio-temporal evolution of some variable is described through a partial differential equation has been used to model the geographical implications of capital accumulation, pollution, climate change and financial contagion (Boucekkine et al. 2009; La Torre et al. 2015; Brock and Xepapadeas 2017; Bucci et al. 2019).

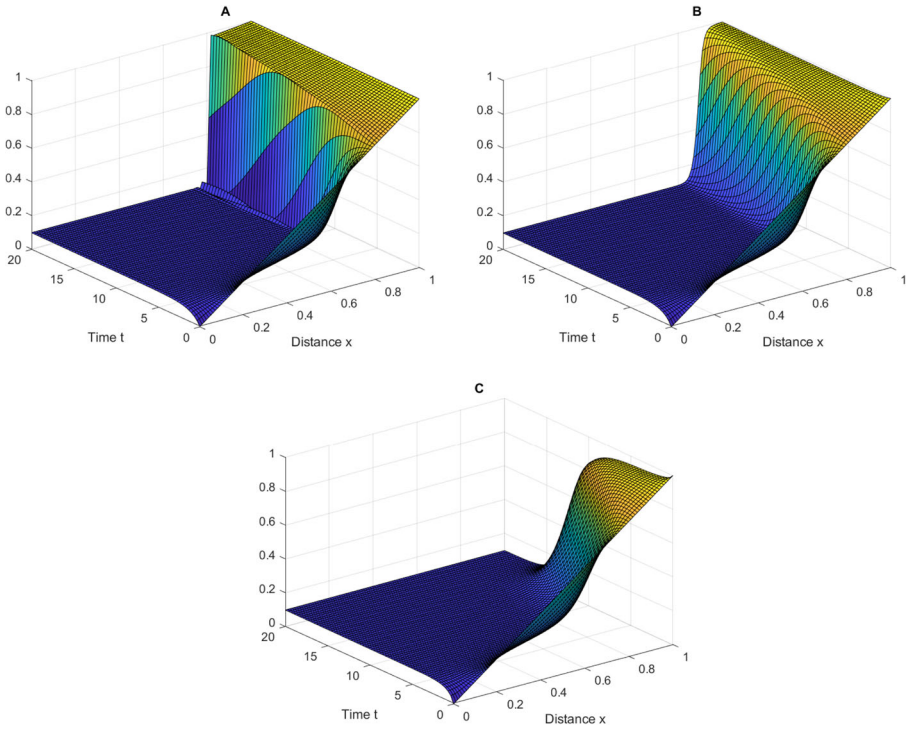


Fig. 6 Spatio-temporal dynamics of the white share within the linear city for different values of the diffusion coefficient d : $d = 0$ (panel A), $d = 0.0001$ (panel B), $d = 0.001$ (panel C)

In the presence of diffusion, all neighborhoods are interdependent and minority segregation becomes more prevalent, but the final outcome depends on whether spatial diffusion is fast or slow. We can see that there are some neighborhoods which even in the absence of diffusion are meant to converge to a majority segregation outcome, but because of diffusion these neighborhoods end up with minority segregation. In other words, these neighborhoods endogenously tip to become populated by the minority. However, if the diffusion parameter is low, there is still a small region that is predominantly populated by whites in which the equilibrium white share is $\bar{w}_H = 1$, while most of the city becomes populated by blacks where the equilibrium is $w_S^- = 0.1$ (panel B). If the diffusion parameter is high, such a region disappears and the entire city becomes predominantly populated by blacks giving rise to pervasive minority segregation and the equilibrium white share in every neighborhood is $w_S^- = 0.1$ (panel C).

These results can be explained as follows: by capturing preference for ethnic diversity, diffusion tends to increase the concentration of white households in neighborhoods which are initially and relatively less white-populated. If diffusion is fast most whites move rapidly to neighborhoods which are predominantly black-populated, but the evolution of the white share in those neighborhoods follows exactly the same pattern discussed in our a-spatial theory and, since there are

only few whites in those neighborhoods, white flight will occur giving rise to an urban outcome of minority segregation. If diffusion is slow a few whites move to neighborhoods that are predominantly black-populated, thus there will still be some predominantly white-population neighborhoods remaining. Therefore, in our setup the preference for ethnic diversity tends to smooth out initial spatial differences across neighborhoods which in the long run leads to more pronounced minority segregation outcomes at urban level. These results suggest that, exactly as in our a-spatial theory, a neighborhood initially populated by whites may tip to become predominantly populated by blacks and identifies preference for ethnic diversity as another possible driver of such an outcome.

7.2 A City as a random network

We now consider a spatial framework where a city forms a network. Each neighborhood (a node on the graph) is characterized by a *degree* which measures the number of interconnected neighborhoods for a referenced neighborhood. Specifically, different neighborhoods are located on a graph characterized by a certain degree distribution p^k , whose average is $\lambda > 0$ which measures the average number of connections between neighborhoods. Accordingly, this parameter captures the intensity of the interconnections between individuals across neighborhoods and the extent to which social interactions at urban level matter in driving households' residential choices. In particular, the probability that a white household will leave a neighborhood of degree k (and thus will be replaced by a black household) depends on the average probability that the city is predominantly black-populated, measured by $v_t = \sum_{k=0}^{\infty} q^k (1 - w_t^k)$, where q^k represents the *excess degree distribution* related to the distribution p^k .²² The idea is that all households located in a neighborhood of degree k have the same likelihood of leaving, independent of their exact location. Specifically, the probability that a white household leaves is determined by the degree of the neighborhood, say k , multiplied by the probability that a directly connected peer does, and this latter probability is represented exactly by v_t .²³ Therefore, the higher is k (or v_t), the higher is the probability of leaving. Accounting for such urban interconnections, the evolution of the share of whites residing in a neighborhood with degree k is given by the following expression:²⁴

$$\dot{w}_t^k = p(1 - w_t^k) - \eta \left(b - c + J \frac{k}{\lambda} v_t \right) w_t^k, \quad (22)$$

²²This quantity is at the basis of the so called *degree-based approach* developed by Pastor-Satorras and coauthors in a series of papers (see, for example, Barthélemy et al. (2005) and Pastor-Satorras and Vespignani 2001). Similar approaches have been employed to describe the evolution of communicable diseases, rumor and information spreading, opinion formation, and traffic and congestion models. We refer to Barrat et al. (2008) for a detailed survey of such important applications.

²³See Newman (2010), for more details on the interpretation and the calculation of v_t .

²⁴It is possible to characterize different network structures to model the interconnections between neighborhoods, but the scope of this section is to show that tipping effects exist also in specific networks (see Newman 2010, for a general presentation of network theory).

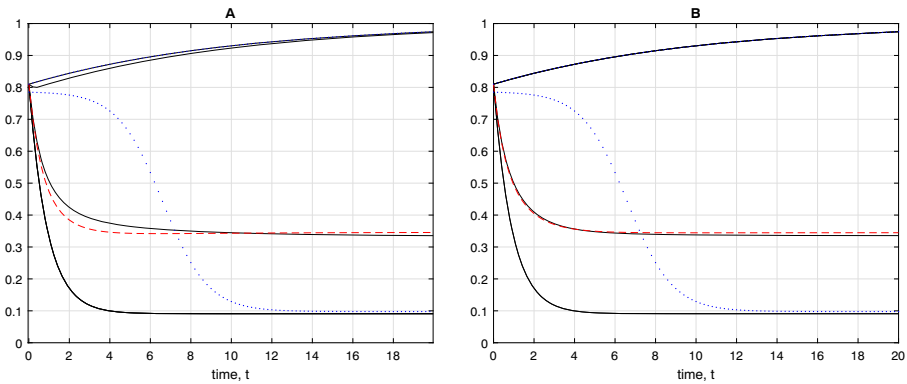


Fig. 7 Dynamics of the white share within the random network for different values of the average degree λ : $\lambda = 3$ (panel A), $\lambda = 30$ (panel B)

where the term $\frac{k}{\lambda} v_t$ multiplied to the social externality parameter generalizes the term $(1 - w_t) = b_t$ in our a-spatial model to account for the influence of the city ethnic composition in the white households’ decision process. The only difference with respect to our baseline a-spatial model is the “city composition effect”, which depends on k and λ . The higher the degree, k , the higher the probability that individuals’ decisions are influenced by the near urban context. The higher the average degree of the network, λ , the smaller the effect of a single neighborhood on the individual household’s residential decision. Indeed, if all neighborhoods have the same degree ($k \equiv \lambda$), our extended model in Eq. 22 reduces to our a-spatial model in Eq. 5; in fact, in this case, $J \frac{k}{\lambda} = J$ and $v_t = 1 - w_t$, such that the city is an exact replica of all the identical neighborhoods that compose it. The proportion of neighborhoods populated by whites can be obtained as the weighted average of all w^k , that is $w_t = \sum_{k=0}^{\infty} p^k w_t^k$. Note that we now have one equation as Eq. 22 for each k , thus with potentially infinitely many equations it is not possible to explicitly analyze our model’s dynamics. Therefore, since there is no closed-form solution we will proceed with numerical simulations to illustrate the model’s possible outcomes.

Consider the case of a specific distribution function, namely the Poisson distribution with parameter λ .²⁵ Figure 7 presents a significant example in which the main parameters are set out as in Section 5 ($J = 1.3$, $p = 0.1$, $b - c = -0.25$) with initial condition $w_0^k = 0.81$ above the tipping point $\bar{w}_M = 0.8$, and the parameter λ taking two different values: $\lambda = 3$ (panel A) and $\lambda = 30$ (panel B). The black curves represent the dynamics of w_t^k , that is the white share in neighborhoods characterized by different values of k , and for the purpose of exposition, we have only depicted the curves for $k \in \{0, 1, 3, 30, 100, 300\}$; the dashed red curve represents the dynamics of w_t , the white share in the city, whereas the two dotted blue curves show the dynamics of our a-spatial model with two initial conditions leading either to minority

²⁵In principle, we could choose any discrete distribution with non-negative support. Since in our example we consider degrees ranging from 0 to 300 and λ is a measure of both the mean and the variance of the Poisson distribution, this number of equations covers largely the entire spectrum of possible degrees.

or majority segregation. We can observe that because of the city composition effect some neighborhoods endogenously tip to become predominantly black-populated, while others do not. In particular, neighborhoods characterized by a high value of k ($k = 100, k = 300$) tend to converge towards a situation of minority segregation in which the white share is low (in our example, $w_S^- = 0.1$), while those characterized by a low degree ($k = 0, k = 1$) converge towards a situation of majority segregation in which the white share is close to one (remember that $\bar{w}_H = 1$) and those characterized by a degree close to λ show a less extreme behavior and their dynamics is similar to the white share dynamics in the city. Although the dynamics of the white share at urban level is similar in the two simulations, we stress that the proportion of neighborhoods characterized by a low k (and thus they do not experience tipping) have a different impact and outcome in the two simulations: for $\lambda = 3$, the proportion of neighborhoods whose white share ends up close to $\bar{w}_H = 1$ is about 6% (panel A), while in the case of $\lambda = 30$, this proportion is close to zero (panel B). This means that, if the city is less interconnected (small λ), a majority segregation outcome is more likely to occur in some regions in the city, at least in regions which are characterized by poorly interconnected neighborhoods. If the city is more interconnected (high λ), the equilibrium is characterized by pervasive minority segregation in which almost all neighborhoods are predominantly populated by black households.

These results can be explained as follows. The degree of interconnections between neighborhoods determines the extent to which the decision of a single household located in a specific neighborhood influences the residential outcome in that neighborhood as well as the other neighborhoods that it is connected to. If the degree of interconnections amongst neighborhoods is high there exist numerous interrelated neighborhoods, and the white flight in one neighborhood will induce a similar white flight in other neighborhoods leading to minority segregation in a substantial part of the city, which could expand to the entire city. If the degree of interconnections is low there exist some neighborhoods which might be almost independent from others (as it could be in the case of *elite* neighborhoods) and so the white flight in another neighborhood might not have any impact on the residential choices of whites residing in those neighborhoods, which might remain predominantly white-populated. Thus, our setup suggests that tight interconnections between neighborhoods might lead in the long run to more pronounced minority segregation outcomes at urban level. These results suggest that, exactly as in our a-spatial theory, a neighborhood initially populated by whites may tip to become predominantly populated by blacks and identifies interconnections between neighborhoods as another possible driver of such an outcome.

8 Conclusion

There is an established literature arguing that social interactions may be an essential determinant of residential segregation. We build on Schelling's works (1969, 1971, 1978) by introducing some economic mechanisms in a social interactions framework to determine the extent to which economic and social factors jointly determine segregation. We develop a simple analytically tractable population dynamics

model of heterogeneous agents to explicitly characterize and identify the conditions under which integration or minority segregation via tipping might occur as a result of optimizing households' behavior when choosing whether to leave or stay in a neighborhood. Their decision crucially depends on the relative size of economic (net benefits from leaving), social (social externality) and behavioral (other group's leaving probability) factors. We explicitly analyze the asymptotic population model to characterize the long run equilibrium and obtain a closed-form expression for the tipping point, which depends on economic, social and behavioral factors. We also analyze the finite population model to identify the possibility of crossing the basin of attraction of the (locally) stable equilibria. This allows us to determine three possible channels, two related to exogenous shocks (migration flows and changes in tipping points) and one to the model's endogenous probabilistic features (endogenous polarization in the majority's decisions), through which tipping may occur. The characterization of integration and segregation conditions point to the need for intervention through urban planning or social policies to mitigate segregation. We also present few extensions of our baseline model showing that from a qualitative point of view its main conclusions apply also in more sophisticated and realistic setups. Despite our baseline model abstracts completely from space, we present some further extensions that introduce some characterizations of space (in terms of a linear city and of a random network) which demonstrate that our conclusions are robust even to the incorporation of a spatial dimension.

Despite its simplicity we believe that our model is striking and intuitive in capturing the possible determinants of residential segregation, but further research is needed to clarify the mechanisms underlying tipping. Our closed-form expression for the tipping point has the potential to inform future empirical research, since it provides some testable predictions which permit testing of our model's implications without relying on the regression discontinuity approach. Moreover, as discussed in Schelling (1971) and in the later social interactions literature, residential decisions often give rise to segregation clusters, meaning that there are some spatial patterns which need to be further examined. Bringing theory to the data in order to validate our conclusions and better explaining the spatial dynamics of residential segregation are left for future research.

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Declarations

Conflict of Interests The authors declare that they have no conflict of interest.

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