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Leibniz on the Empty Term ‘Nothing’

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Abstract This paper discusses Leibniz’s treatment of the term ‘nihil’ that appears in some logical papers about the notion of Real Addition. First, the paper argues that the term should be understood as an empty (singular) term and that sentences with empty terms can be true (§2). Second, it sketches a positive free logic to describe the logical behaviour of empty terms (§3). After explaining how this approach avoids a contradiction that threatens the introduction of the term ‘nihil’ in the Real Addition calculus (§4), and how this approach should be understood within Leibniz’s philosophy (§5), the paper assesses the prospects of such an approach with regard to two fundamental issues in Leibniz’s thought: the fictional nature of infinitesimals (§6), and the occurrence of the term ‘nothing’ in the proof of the existence of God that we find in the New Essays (§7).

Keywords Leibniz. Empty terms. Real Addition. Mereology. Nothingness. Positive Free Logic.

Summary 1 Introduction. – 2 The Empty Term ‘Nihil’. – 3 Another Characterization of ‘nihil’. – 4 A Logic for Nothing! – 4.1 Language of PFL. – 4.2 Syntax of PFL. – 4.3 Semantics of PFL. – 4.4 Discriminating Actual from Merely Possible Objects. – 5 The Formal Machinery at Work 1: Avoiding the Contradiction. – 6 Some Comments about (Weak) Identity. – 7 The Formal Machinery at Work 2: The Case of Infinitesimals and Other Empty Notions. – 8 The Formal Machinery at Work 3: The Term ‘Nothing’ in the Proof of the Existence of God. – 9 Conclusion.

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1 Introduction

In §153 of the *Generales Inquisitiones*, Leibniz writes:

> This, however, presupposes that every proposition in which there enters a term which is not a thing, is denied; so it remains the case that every proposition is either true or false, whereas every one is false which lacks an existent subject [*Constantia Subjecti*], i.e. a real term. This, however, is to some extent remote from the way we usually speak about existential propositions. But this is no reason for concern, because I am seeking appropriate signs, and I do not intend to apply usually accepted names to them.[1] (A VI 4, 781/Leibniz 2021, 121-3)

For Leibniz a proposition consists in attributing a predicate to a subject. Since the law of bivalence holds, every proposition is either true or false. But a proposition which contains a non-denoting subject-term cannot be true, because there is no object to which we can attribute the property expressed by the predicate. So it will be false.

However, it is less clear what Leibniz had in mind with non-denoting terms. Is he speaking of terms that refer to something which is not actual, but still possible, or he is speaking of terms which are empty by logical necessity, i.e. terms which imply a contradiction such as 'the greatest velocity' or 'the infinite number'? It seems to me that the latter is the right interpretation.[2] First, the adjective 'real' is usually used by Leibniz as indicating something possible: for instance, a definition is said to be *real* (and not simply nominal) when we have a proof of the *possibility* (i.e. of the internal consistency) of the object defined; second, in the above passage, Leibniz uses the Latin term '*constantia subjecti*', which refers to a specific discussion within the Scholastic tradition, as the following passage from the *New Essays* explains:

> The Scholastics hotly debated *de constantia subjecti*, as they put it, i.e. how a proposition about a subject can have a real truth if the subject does not exist. The answer is that its truth is a merely conditional one which says that if the subject ever does exist it will be found to be thus and so. But it will be further asked what the ground is for this connection, since there is a reality in it which does not mislead. The reply is that it is grounded in the linking together of ideas. (A VI 6, 447-8/Leibniz 1996, 447-8).

1 When Leibniz says "there enters a term which is not a thing", he clearly intends "there enters a term whose referent does not exist". Based on this passage, Mates 1972 argues that Leibniz considered sentences with non-denoting terms as simply false.

2 Mates 1972 took the first interpretation; Mugnai, commenting on the text of Leibniz (see Leibniz 2008, 177), took the second. My defence here of the second interpretation is indebted to Mugnai's discussion.

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The passage explicitly states that there are true propositions whose subject-term refers to something which does not actually exist. A sentence of the form *P(a)* with '*a*' denoting a non-existent but possible subject has thus the form: if *a* exists, then *P(a)*. With *a* referring to an individual, the sentence is true if the property *P* is contained in the complete concept[3] of *a*, false otherwise. Therefore, non-denoting but possible terms are not a threat for the principle of bivalence. The general picture that emerges is thus as follows: every proposition is either true or false in accordance with the principle of bivalence; propositions with subject-terms whose referent is not actual but *possible* can be either true or false. Propositions that contain contradictory terms, such as 'the greatest velocity' or 'the infinite number', are always false.

2 The Empty Term 'Nihil'

To the picture just sketched there seems to be an exception: the term 'nihil' (nothingness). This is in fact a term that Leibniz employs quite a lot.[4] In particular I shall focus here on two logical essays, *Specimen Calculi Coincidentium* (A VI 4, 816-22) and *Non Inelegans specimen demonstrandi in abstractis* (A VI 4, 845-55), where Leibniz employs the term 'nihil' in relation to the notion of Real Addition. The notion of Real Addition is similar to that of mereological sum or fusion of contemporary mereology: the idea is that we can add or fuse different things and so obtain aggregates of those objects.[5] We shall use the symbol '⊕' employed by Leibniz in a further essay, *Calculus coincidentium et inexistentium* (A VI 4, 830-45) to formalize the notion. There are two axioms that regulate how Real Addition works:

1. $\forall x(x \oplus x = x)$
2. $\forall x \forall y(x \oplus y = y \oplus x)$

Axiom 1 states the Idempotence of Real Addition (which is of course a property not shared by arithmetical addition); axiom 2 expresses Commutativity. Moreover, Leibniz does not state but presupposes a third axiom (associativity):

3. $\forall x \forall y \forall z\ x \oplus (y \oplus z) = (x \oplus y) \oplus z$

3 The complete concept of an individual substance is the concept that contains every predicate of that substance. The notion eminently appears in the *Discourse on Metaphysics* (1686) and is discussed at length in the correspondence with Arnauld (see for instance GP II, 47-9).

4 A famous example can be found in the *New Essays* (A VI 6, 435-6), where Leibniz discusses Locke's proof of the existence of God. I shall analyse that discussion in §8.

5 On Real Addition see, for instance, Swoyer 1994; Lenzen 2000 and Mugnai 2019.

In these papers, it is by means of the notion of real addition (and identity) that Leibniz defines the containment relation (in what follows C(x,y) must be read as *x* contains *y*, or *y* is contained in *x*). Leibniz's definition uses indefinite letters as A, B, etc., i.e. letters that stand for variables, and so allow us to express general statements. Leibniz writes that "B⊕N=L means that *B* is (contained) in *L* or *L* contains *B*" ("B⊕N=L *significat B esse in L seu L continere B*". A VI 4, 832). In what follows, we shall avail ourselves of quantification theory[6] instead of indefinite letters. So Leibniz's definition becomes:

$$C(x,y)\equiv_{def}\exists z(y\oplus z=x)$$

which can be read as '*y* is contained in *x* if there is a z (contained in *x*) such that *y* plus *z* is equal to *x*'. Thanks to the relation of containment, Leibniz also develops a subtraction operation, clearly presented as the inverse of the operation of Real Addition. Leibniz writes:

> Def. 5. If *A* is in *L* in such wise that there is another term, *N*, in which belongs everything in *L* except what is in *A*, and of this last nothing belongs in *N*, then *A* is said to be subtracted (*detrahi*) or taken away (*removeri*), and *N* is called the remainder (*residuum*). Charact. 4. *L*−*A*=*N* signifies that *L* is the container from which if *A* be subtracted the remainder is *N*.[7]

The idea is simply that if C(x,y) is the case (which means that ∃z(y⊕z=x) is the case), then x−y=z is defined, where *z* is the reminder or the complement of *y* in *x*. However, as it stands, this definition must be amended. If we want real subtraction to be the inverse of real addition, the terms *y* and *z* must have nothing in common.[8] In fact, suppose otherwise, and consider the special case in which they have something in common because they are identical: z=y. Then from x−y=z by substitution of *z* with *y*, we obtain x−y=y,which is equivalent to: y⊕y=x. By idempotence, we

6 The choice of quantification theory is useful and elegant; however, one should bear in mind that Leibniz thought of his logical calculus mainly in *intensional* terms, i.e. as a calculus of concepts.

7 A VI 4, 848; the English translation comes from Lewis 1918, 374.

8 As Leibniz himself recognized in §29 of *Specimen Calculi Coincidentium* (A VI 4, 819): "if A+B=C, then A=C−B, and A is called the reminder [Residuum]. But it is necessary that A and B have nothing in common. In fact for example if A+A=A, then A=A−A. But from §30 we have that A−A=nihil, so A=nihil, which is against the hypothesis". (Author's translation). One has to notice that the requirement that A and B have nothing in common is a necessary condition in order to define subtraction, and does not apply to (Real) addition. In other words, from A+A=A we have (by definition of the containment relation) that C(A,A), i.e. the reflexivity of the containment relation. There is nothing problematic with this case of containment, and more generally with the definition of containment (thanks to a referee to ask for a clarification of this point).

have y⊕y=y, so x=y. By substituting the latter into x−y=y, we finally have y−y=y. But the latter is unacceptable, because it contradicts the only axiom that Leibniz states for the subtraction operation:

∀x(x−x)=nihil[9]

What the axiom says is that if you take something and subtract it from itself, you get nothing. This is rather intuitive, particularly if one thinks of subtraction as the inverse of real addition. Subtracting just means leaving out something from something else. The axiom is important because it can be seen as introducing into the calculus the delicate notion of nihil. Clearly, as subtraction is thought of in comparison to arithmetical subtraction, so nihil plays a part similar to that played by the number 0 in arithmetic.

The term 'nihil' is thus introduced in the calculus in order to define subtraction in cases where a thing is subtracted from itself. Since real addition and subtraction are thought of in comparison to arithmetical addition and (arithmetical) subtraction, and the 'nihil'-term plays a role analogous to the number 0, one might think that the 'nihil'-term is not really empty, but that it refers to something, much as the term 'zero' refers to a specific number, the number 0, and the term 'empty-set' refers to a particular set in set-theory. However, this is problematic, not only because this hypothesis seems to graft onto Leibniz some posteriors ideas,[10] but also because the idea that the calculus allows the presence of a nihil-*object* is immediately self-contradictory. Let us see why this is the case.

The notion of subtraction brings with it a principle known as Weak Supplementation (from now on: WS):

C(x,y) → ∃z(C(x,z)∧¬O(z,y))

What the principle says is that if *y* is contained in *x*, there is a *z* which is also contained in *x* but it is disjoint from *y*: *z* and *y* have nothing in common – the predicate 'O(x,y)' indicates the overlapping relation: O(x,y)=$_{def}$ ∃z(C(x,z)∧C(y,z)). That this principle is implicitly accepted when one accepts subtraction can be seen by noticing that when we subtract *y* from *x* what remains is a remainder that has nothing in common with *y*: the remainder is everything which is in *x* and not in *y*.

9 Again, Leibniz uses indefinite letters. So he writes: A-A=nihil. This is considered as an axiom in *Calculus coincidentium et inexistentium*; however in *Specimen Calculi coincidentium* (A VI, 4, nr. 173, 819), Leibniz assumes that A⊕nihil=A (§28) and concludes with A-A=nihil (§30), in virtue of the fact that (Real) subtraction is the inverse operation of (Real) addition (§29).

10 As Mugnai 2019 rightly acknowledges.

That WS is a valid principle within Leibniz's calculus is clear from how he defines subtraction.[11] The problem is that WS contradicts the existence of an empty-object, the supposed referent of the term 'nihil'. In fact, in Leibniz's calculus we have the following:

$\forall x(x \oplus nihil = x)$

By definition of the containment relation, this is equivalent to $\forall x C(x, nihil)$: nihil is contained in everything. In particular, this implies that there are no disjoint things: given any two things, they will have something in common: the object referred to by the term 'nihil'. We have therefore a contradiction with Weak Supplementation.[12]

In contemporary mereology, the standard way to avoid this situation is to get rid of the empty-object. Subtraction is defined in such a way that there must always be a positive remainder: 'A−A' is not a defined operation. However, this goes against what Leibniz did, and since the term 'nihil' often appears in Leibniz's writings, this standard option is not available. The only solution available is to consider 'nihil' an empty term: a term with no reference at all.

3 Another Characterization of 'nihil'

In these essays we find another characterization of nihil. For example, we can read that

> Not-nihil is something, and not-something is nihil. (A VI 4, 817, §17)[13]

> If *N* is not *A*, and *N* is not *B*, and *N* is not *C*, and so on; *N* is said to be Nothingness [nihil]. (A VI 4, 551)[14]

Nihil is characterized here as what is different from everything, and in this sense is not something.[15] As Lenzen (2000, 91) suggests, commenting on

11 This can be easily appreciated when looking back at the last quotation. Definition 5 and what follows clearly presuppose the validity of WS.

12 The contradiction can be derived even without appealing to Weak Supplementation. It is enough to notice that Leibniz exploits the existence of disjoint terms, i.e. terms that do not overlap and so have nothing in common (as we saw earlier in the definition of subtraction). But since nihil is contained in everything, the latter implies that no terms are disjoint. This way of formulating the problem can be found in Lenzen 2000, §5.1.

13 *Non nihil est aliquid, et non aliquid est nihil.*

14 *Si N non est A, et N non est B, et N non est C, et ita porro; N dicetur esse nihil.*

15 These characterizations go along with other two characterizations of nihil that we can find in Leibniz's texts. The first is a metaphysical characterization of nihil according to which it has no properties ("*nihil nulla esse attributa*": A VI 4, 570). The second

the second of these two passages, '*N* (i.e. nihil) is not *A*' can be translated by the claim that *N* does not contain *A*: ¬C(nihil, A). Since *A* is arbitrary, we have it that nihil does not contain anything: ∀x¬C(nihil, x). However, the containment relation is reflexive,[16] and so we have C(nihil, nihil) which implies that ∃xC(nihil, x). And this contradicts the previous claim.

4 A Logic for Nothing!

To vindicate Leibniz's idea that there are true (atomic) propositions with empty terms, we need a logic that allows such terms. In the literature there are different logical systems that allow for empty terms; in our case the system known as Positive Free Logic (PFL) will do.[17] I shall briefly expose PFL by considering, respectively, the language, the syntax, and the semantics.

4.1 Language of PFL

The language L of PFL does not differ much from a standard first-order language. It is composed of the following elements:

- variables: $x_1,\dots, x_n,\dots$
- individual constants: $c_1,\dots, c_n,\dots$
- constant function symbols: $f_1,\dots, f_n,\dots$
- n-place predicates: $P^n{}_1,\dots, P^n{}_n,\dots$
- propositional connectives: ¬, → (the others are defined as usual)
- the quantifier: ∀ (with $\exists \equiv_{def} \neg\forall\neg$)
- the 2-place weak identity predicate: ≈

Terms are defined as follows:

- variables and constants are terms;
- if $t_1,\dots,t_n,\dots$ are terms, then $f_1(t_1),\dots, f_n(t_n),\dots$ are terms;
- nothing else is a term.

Formulas are defined as follows:

- if $t_1,\dots,t_n,\dots$ are terms, then $P^n{}_1(t_1),\dots, P^n{}_n(t_n),\dots,$ are formulas;
- if t_1, t_2 are terms, then $t_1 \approx t_2$ is a formula;

is an epistemological characterization: nihil is what remains when we remove everything that can be known ("*a quo removetur quicquid cogitari potest*": A VI 4, 938). On these two further points, see the introduction by Schupp to Leibniz (2000, lxx-lxxiii).

16 The reflexivity of the containment relation is proved by Leibniz in proposition 7 of *Calculus coincidentium et inexistentium*. Here we can read that "A is (contained) in A. Everything is (contained) in itself [*A est in A. Unumquodque est in se ipso*]" (A VI 4, 835).

17 For a good presentation of PFL together with other systems that allow some terms to be empty, see Nolt's entry on Free Logic in the *Stanford Encyclopedia of Philosophy* (Nolt 2000). I have used this article as a basis for my exposition of PFL.

- if α,β are formulas, then $\neg\alpha$, $\alpha\rightarrow\beta$, $\forall x\alpha$ are formulas;
- nothing else is a formula.

4.2 Syntax of PFL

I shall here formalize PFL by means of Natural Deduction Rules. PFL diverges from standard first-order logic only concerning the rules governing quantifiers, while all other rules remain as usual. It will be useful for the clarification of the exposition to introduce an existence predicate E(x) defined as follows: $E(x)\equiv_{def}\exists x(x\approx x)$. I shall just focus here on those rules that differ from the classical ones:

Introduction of universal quantifier (∀I)

$$\frac{\begin{array}{c}[E(t)]\\ \vdots\\ \phi(t/x)\end{array}}{\forall x\phi(x)}$$

where $\phi(t/x)$ is the result of replacing every occurrence of *x* in ϕ with a variable *t* that is free for *x* in ϕ; *t* is new and does not occur in ϕ; ϕ does not depend on some non-discharged assumption where the variable *x* is free. The rule tells us that if we have derived $\phi(x)$ from the assumption that *t* exists – E(t)– we can conclude with $\forall x\phi(x)$ and discharge E(t). The only difference with the classical ∀I rule is in the requirement that *t* exists. If E(t) is not the case, from $\phi(t/x)$ we cannot introduce the universal quantifier. This means that the universal quantifier ranges *only* over 'existing' objects.

Elimination of the universal quantifier (∀E)

$$\frac{\begin{array}{c}[E(t)]\\ \vdots\\ \forall x\phi(x)\end{array}}{\phi(t/x)}$$

where *t* must be free for *x* in $\phi(x)$, i.e. *t* must not be bounded by a quantifier in ϕ after the substitution. Again, the only difference with the classical rule is in the requirement that E(t) is the case. This means that the universal quantifier ranges over *all* existing objects.

Since the existential quantifier is defined in the usual way, the rules that regulate it depart from the classical rules for requiring, as a premise, E(t):

Introduction of the existential quantifier ($\exists$I)

$$\frac{\begin{matrix}[E(t)]\\ \vdots \\ \phi(t/x)\end{matrix}}{\exists x\phi(x/t)}$$

(where *t* is free for *x* in ϕ);

Elimination of the existential quantifier ($\exists$E)

$$\frac{\exists x\phi(x) \quad \begin{matrix}[\phi(t/x), E(t)]\\ \vdots \\ \gamma\end{matrix}}{\gamma}$$

In this case, *x* need not be free in $\phi(x)$; *t* is new and does not occur in ϕ or γ; *x* is not free in the non-discharged assumption used to derive γ.

What these rules tell us is that quantifiers are restricted to ‘existing objects’, i.e. we can apply the rules governing them only in those cases where the terms involved denote. If we have an empty term, we cannot introduce or eliminate a quantifier. The rationale of such a restriction should be clear: from a sentence with an empty term $\varphi(t)$, I cannot conclude with $\exists x\varphi(x)$ which has existential commitments.

Concerning the weak identity predicate, the rules that govern it are just the classical rules for identity; and the reason is that weak identity is defined even for empty terms. As such the notion of weak identity is similar to the standard notion of identity, with the only difference being that in a weak identity statement ‘s≈t’, one or both of *s*and *t* may be empty. Standard identity may be defined in the following way:

$a=b \equiv_{def} (a\approx b)\wedge E(a)\wedge E(b)$.[18]

4.3 Semantics of PFL

Concerning the semantics for PFL, since we need some atomic sentences with non-denoting terms to be true, we need a positive seman-

18 Since the identity relation requires the relata to exist, we could replace E(a) with a=a. Notice that we could have taken identity as primitive and defined weak identity as follows: $a\approx b \equiv_{def}(a=b)$ and a, b may not refer.

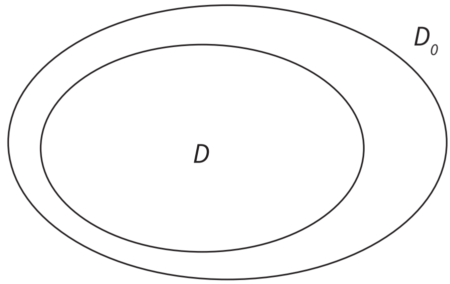

Figure 1
Representation of the relationship between the standard domain D and the enlarged domain D_0

tics (the term ‘positive’ in PFL just denotes this fact). There are at least two ways of providing a positive semantics for a free logic: the first is to consider a single domain D of objects over which the quantifiers range, and over which the interpretation of denoting terms is defined. To accommodate non-denoting terms, one takes the interpretation function to be partial, i.e. non-defined for these terms. This captures the idea that such terms are empty in a literary way, but makes the semantics complicated: while sentences with denoting terms can be evaluated in the usual Tarskian way, sentences with non-denoting terms must receive a special treatment. For this reason, I prefer a dual domain-semantics. There will be two domains: D,which is the standard domain containing everything, and a further domain D_0 which is a larger domain containing everything that is in D plus further objects which are associated with non-denoting terms (which are therefore the ‘non-existing’ things).[19] To keep things as simple as possible, we may imagine that there is a unique ‘non-existing’ thing; in other words, every empty term is associated with the same object. The picture is as shown **[fig. 1]**.

I shall call D the inner domain, while the difference D_0/D (the part of D_0 not contained in D) the outer domain. The basic idea of the semantics consists in letting singular terms and predicates be defined over D_0. At this point the dual domain semantics may be defined as follows: a model is a triple $<D, D_0, I>$, where D, D_0 are as above, and I is an interpretation function such that

- If t is a term $I(t)\in D_0$;
- If P^n is a predicate $I(P^n)\subseteq D_0{}^n$ (in particular $I(s\approx p)\subseteq D_0{}^2$);
- If f^n is a n-place function, $I(f^n)$ is a function defined over D_0.

The valuation function V assigned truth-value to formulas as follows:

- $V(Pt_1,\dots,t_n)=1$ if and only if $<I(t_1),\dots,I(t_n)>\in I(P)$, otherwise it is 0;

19 For simplicity, I shall consider D as a subset of D_0.

- V(s≈p)=1 if and only if *I(s)=I(t)*;[20]
- V(¬A)=1 if and only if V(A)=0;
- V(A→B)=1 if and only if either *I(A)*=0 or *I(B)*=1;
- V(∀xA)=1 if and only if for all $d \in D$, $V_{d,t}$(A(t/x))=1 (with *t* not in A and $V_{d,t}$ the valuation function on the model <*D*, D_0, *I**> such that *I** is like *I* except that *I***(t)=d*).

Notice that the semantics for the quantifier is quite standard; however, the clause is given with reference to *D* and not to D_0. This matches what we saw above: quantifiers only 'work' with denoting terms. Before proceeding, a word on the basic idea of dual-domain semantics is needed. We said that the semantics associated the empty terms with objects from the outer domain D_0/D, i.e. 'non-existing objects' (or, better, with the unique object in the outer domain D_0/D). This must not be taken literally, as implying that we are accepting both existent and non-existent objects, as happens in Meinongian ontologies. On the contrary, this is only *a technical fiction* that allows us to give a uniform Tarskian semantics both for denoting and non-denoting terms, but no ontological *morale* must be derived from this merely technical fact. It is interesting to note that a similar approach was championed by Leibniz himself concerning fictional entities like infinitesimals, infinite wholes, and others. Leibniz's idea was that we could use them to discover new truths, even though they do not exist or even in the case that they are contradictory notions. We can use them as if they existed, provided that in more rigorous contexts we can dismiss them in favour of some other method. Similarly, we can take empty terms as denoting non-existent objects for the sake of keeping the semantics simple and intuitive, provided that, when drawing philosophical conclusions, we dismiss any talk of non-existent objects in favour of talk about terms that do not refer at all.

4.4 Discriminating Actual from Merely Possible Objects

The semantics that we have just presented does not discriminate what actually exists from what is merely possible, and thus what exists in our world from what exists, according to Leibniz, in another possible world *in mente Dei*. It is not difficult to amend this situation. What we need to do is introduce a relation *comp(x,y)* to be read as '(the individual) *x* is compossible with (the individual) *y*', and show that it is an equivalence relation: in this way *comp*(*x*,*y*) partitions the domain *D* into different equivalent classes that correspond to different possi-

20 Notice that the identity sign between *I(s)* and *I(t)* is not the same identity sign which we defined by means of weak identity, because the latter belongs to the object language, while the former belongs to the metalanguage in which we are presenting the semantics.

ble worlds.[21] A possible world is thus a maximal series of compossible individuals. A consequence is that it cannot happen that two compossible states belong to different possible worlds. Between the possible worlds, the one that maximizes the amount of goodness is the actual world.

What this partition requires is a Kripke-style semantics, where formulas are evaluated with regard to possible worlds. The details are standard and since they will not play any role in what follows, I will not present them here. However, one has to bear in mind that terms referring to possible objects are not considered to be empty; rather they refer to some object in the inner domain *D*.

5 The Formal Machinery at Work 1: Avoiding the Contradiction

With this formal machinery in play, we can go back to the contradiction that emerges in the 'Real Addition' calculus as soon as Leibniz admitted the empty term 'nihil'. From WS, we have the claim that there are (at least) two disjoint things; but the admission of the term 'nihil' implies the truth of

4. $\forall x(x \oplus nihil = x)$

Which is equivalent (by definition of the containment relation) to:

5. $\forall x C(x, nihil)$

Which says that everything (in the sense of every object) contains the nothingness. In a classical setting, from 2 we could derive

6. $\forall x \exists y C(x,y)$

By applying the classical existential introduction rule. However, within PFL we cannot apply ∃I, because 'nihil' is an empty term, and E(t) (where I(t)=nihil) is false. In this way, one of the requirements necessary to apply ∃I fails, and we cannot derive the contradiction.

The same reasoning applies to the characterization of nihil given in §2.1 (nihil as what is different from everything). There the contradiction was between the claim that nothing is contained in the object nihil: $\forall x \neg C(nihil,x)$, and the claim that something is contained in it: $\exists x C(nihil,x)$, which was a consequence of the reflexivity of the containment relation applied to the notion of nihil: C(nihil,nihil). Clearly, within PFL, we cannot derive $\exists x C(nihil,x)$ from C(nihil,nihil), because this would require an application of ∃I; but since 'nihil' is an empty term, the rule cannot be applied.

21 The details of this construction can be founded in Arthur 2021, Appendix 1, A1.3.

To sum up, our setting allows us to commit ourselves to the claim that nihil is contained in everything,[22] and at the same time to reject the claim that there is something contained in everything, simply because 'nihil' is an empty term. Since we reject the latter, we are not committing ourselves to the idea that every two things have something in common. We are thus not forced to accept that there are no disjoint things. This shows that a positive free logic would allow Leibniz to have his cake and to eat it too: he can have the notion of nothingness, and at the same time accept the existence of disjoint terms.

6 Some Comments about (Weak) Identity

Above, we have defined identity through the notion of weak identity:

$$a=b \equiv_{def} (a \approx b) \wedge E(a) \wedge E(b)$$

While the standard identity predicate requires that both *a* and *b* are not empty (and for this reason is a *strong* predicate), weak identity is defined also in the case that one or both terms are empty. For this reason, from a=b we can derive ∃x(x=b), but the same cannot be derived from a≈b. The intuitive reading of a≈b is that *a* and *b* are the same, or that '*a*' refers to the same object as '*b*'. The sentence is false when the two terms refer to different objects, or one refers to something, while the other is empty. As such, in the case in which both are empty, they do not refer at all (i.e. they refer to the object in D_0/D), and so in particular it is not the case that they refer to different objects: the sentence will consequently be true.

The introduction of the term nihil is due to the will of defining a subtraction operation as the converse operation of Real Addition. Recall that Leibniz introduced the following axiom: ∀x(x−x)=nihil. Clearly, in our PFL as defined above, the identity symbol must be replaced with the weak identity symbol, the subtraction operation is a function symbol, and so the truth-conditions of this axiom can be interpreted as follows:

'∀x(x−x)=nihil' if and only if for all $d \in D$, $V_{d,t}(t-t \approx \text{nihil})=1$ if and only if for all $d \in D$, $I(t-t)=I(\text{nihil})$ (where $I^*(t)=d$).

Semantically the axiom says that the referent of any expression of the form *t*−*t* is the same as the referent of the term 'nihil'. This referent

[22] Even though Leibniz does not explicitly state that nihil is contained in everything (as far as I know), this is a direct consequence of his axiom governing subtraction and his definition of containment.

will belong to D_0/D. However, one has to notice that this formulation of the axiom only regards objects d such that $d \in D$, since quantifier rules in PFL are restricted to denoting terms. In order to extend the axiom to also cover empty terms, we need a schematic formulation such as $\alpha - \alpha =$nihil, where α is a meta-variable.

At this point, I would like to draw the reader's attention to two important points. The first regards sentences such as 'The current King of Italy is (≈) nihil', where the first term denotes a possible, but not actual object,[23] while the second is an empty term. Since our domain D comprises both actual and possible objects, the semantics will make all these sentences false.[24] Second, as we outlined above, the interpretation function I associates every empty term with the unique object in D_0/D, i.e. every empty term has the same reference. This makes every (weak) identity statement between empty terms true. This feature exactly captures an idea that we find in *Specimen Calculi Coincidentium* (§20) wherein Leibniz writes that 'if A is nihil and B is nihil, then A=B, i.e. two nothingness coincide' (A VI 4, 817, Author's translation): that 'two nothingness coincide' exactly means that every identity statement between two empty terms is true, as our semantics delivers.

Following a suggestion of Oliver and Smiley (2013), we can generalize the distinction between weak and strong identity to any predicate: Fx is strong if and only if the truth of Ft (where 't' is a term) implies the existence of t. If this is not the case, then the predicate is weak. For instance, 'walk' is a strong predicate, because the truth of 'Mark walks' implies the existence of Mark. But the predicate 'is not different from' is weak: the truth of '$t-t$ is not different from nihil' does not imply the existence of nihil (in fact, 'is not different from' is a good way of reading the ≈ predicate). Clearly, the extension of strong predicates is restricted to the domain D, while weak predicates have extensions in D_0.

23 This is not completely true: the definite description 'the actual King of Italy' is incomplete, and may denote different objects in different possible worlds. What one should do is pick up a complete concept which will denote a unique object in exactly one possible world.

24 This is a major difference between the present approach and the one developed by Oliver, Smiley 2013. According to their proposal, a sentence such as 'The current King of Italy is nihil' would be true, because their domain does not comprehend possible objects, but only actual ones, and so both terms turn out to be empty. In other words, if the sentence 'The current King of Italy is nihil' were false, the terms 'the current King of Italy' and 'nihil' would refer to different objects. But since, in their semantics, the terms do not refer, that sentence is true.

7 The Formal Machinery at Work 2: The Case of Infinitesimals and Other Empty Notions

It is interesting to look at how the present proposal performs with regard to a famous issue concerning Leibniz's philosophy of mathematics, i.e. the nature of infinitely small quantities. As is well-known, Leibniz considered infinitesimals to be useful *fictions* to discover mathematical truths, but at the same time always dispensable:

> Speaking philosophically, I maintain that there are no more infinitely small magnitudes than there are infinitely large ones, that is, no more infinitesimal than infinituples. For I hold both to be fiction of the mind thorough an abbreviated way of speaking […]. [They] are very useful for abbreviating thought and thus for discoveries, and cannot lead to an error, since it suffices to substitute for the infinitely small something as small as one wishes, so that the error is smaller than any given, whence it follows that there can be no error. (GP II, 305/Leibniz 2007, 33)

What Leibniz is claiming is that infinitesimals do not exist *in rerum natura* and that every mathematical sentence in which an infinitesimal term appears can be translated into a sentence that makes no reference to it. Clearly this very last sentence represents the most correct way of stating the truth in question; however, working with infinitesimals has some technical advantages. The question that I would like to raise is the following: what is the status of the sentence that contains an infinitesimal term? For example, consider a sentence φ(t), where '*t*' is a term referring to an infinitesimal. The sentence ascribed the property φ(x) to an infinitesimal *t*. Let us suppose that φ(t) is a mathematical theorem. Should we count it as true or false? Since it is a theorem, we take it for granted that its translation into a sentence with no reference to infinitesimal consists in a true proposition. Let us suppose that this translation is given by the sentence ∀xA.[25] However, the status of the sentence φ(t) is less clear. One might suggest that the sentence should be read as a conditional: if *t* existed, then φ(t). However, Leibniz believed that infinitesimals were contradictory objects,[26] so *t* can never exist, and this path is not viable. Another option would be to consider the sentence as false, since its subject-term does not refer. But then we would end up in the awkward position of

25 I have not chosen a universal sentence by chance; rather Leibniz proposed to paraphrase away reference to infinitesimals by means of general sentences to the effect that no matter how small a quantity can be, there will always be a smaller quantity. On this point, see Ishiguro 1990, 87 and Arthur 2013.

26 As Arthur 2013 and Rabouin, Arthur 2020 have strongly argued.

claiming that φ(t) is false, but its translation ∀xA is true. In this scenario it is difficult to understand how this could be possible: a good translation should preserve the meaning of the sentence, which implies that at least the truth-value of the sentence should not change. How could we assert that ∀xA is a good translation of φ(t) if they have different truth-values?

I think that the best way to solve this difficulty is to admit that sentences with empty terms might be true. The sentence φ(t) is true, even though '*t*' is empty. Clearly this requires a positive free logic as the one we present here, which does not allow to conclude that t exists on the ground that φ(t) is true. Moreover, the translation is now truth-preserving: we translate a true sentence (with infinitesimal terms) into a true sentence with no infinitesimal term. In this context, the translation succeeds in showing that the truth, which we may have discovered by means of infinitesimals, does not really depend on them, and can (and, from a philosophical point of view, should) be expressed without recurring to them.

This approach can be extended to other empty terms, such as 'infinite number', 'greatest velocity' or 'perpetual mechanical motion'. For instance, concerning the latter, Leibniz writes:[27]

> […] for when we speak of perpetual mechanical motion, for example, we know what we are saying, and yet such motion is an impossibility and so we can only appear to have an idea of it. (A VI 6, 438/Leibniz 1996, 438).

With the help of a PFL, we can interpret this passage literally: we know what we usually attribute to such a motion, because there are true subject-predicate sentences about it, even if its existence would imply a contradiction.

8 The Formal Machinery at Work 3: The Term 'Nothing' in the Proof of the Existence of God

In the *New Essays*, commenting on the proof of the existence of God provided by Locke, Leibniz/Theophilus says:

> I assure you perfectly sincerely that I'm most distressed to have to find fault with this demonstration; but I do so only so as to get you

27 Similar considerations can be found in different places; for instance, in a letter to Malebranche we can read: 'But one can also reason about the greatest of all numbers, an idea which nevertheless implies a contradiction, as does also the greatest of all velocities' (GP I, 327-8). The English translation follows that of Loemker (Leibniz 1969, 211). On Leibniz's argument against infinite number see Costantini (2020).

> to fill the gap in it. It is mainly at the place where you infer that 'something has existed from all eternity'. I find an ambiguity there.[28] If it means that there has never been a time when nothing existed, then I agree with it, and it really does follow with entirely mathematical rigor from the preceding propositions. For if there had ever been nothing, there would always have been nothing, because a being can't be produced by nothing; and if nothing had been produced we ourselves wouldn't have existed, which conflicts with the first truth of experience. (A VI 6, 436/Leibniz 1996, 436)

In this critique, the term 'Nothing' compares different times. The first three occurrences can be translated by means of a quantifier phrase. For instance, when Leibniz says "there has never been a time where nothing existed [*il n'y a jamais eu un temps, où rien n'existoit*]", the sentence is naturally understood as 'there has never been a time when *no thing* existed'; or when Leibniz adds "if there had ever been nothing, there would always been nothing [*si jamais il y a avoit eu rien, il y auroit toujours eu rien*]", the sentence is naturally understood as 'if there had been *no thing* at all, there would always be *no thing* at all'. However, the sentence "a being can't be produced by nothing [*le rien ne pouvant point produire un Etre*]"[29] cannot be directly translated – without altering its meaning – by a quantifier phrase, such as 'a being cannot be produced by no thing'. This can be appreciated by considering the equivalent

a. Nothing comes from nothing

where the first occurrence of 'nothing' is a quantifier, while the second a noun-phrase. If we tried to translate both occurrences with a quantifier, for instance

b. $\forall x \neg \exists y$(x comes from y)

we obtain a different sentence. Sentence (b) claims that no object comes from any other objects, which is not what (a) says. In fact, (a)

28 The ambiguity which Leibniz refers to can be expressed by the position of the quantifiers. The sentence 'something has existed from all eternity' can be translated either as $\forall t \exists x(x=x,t)$ or as $\exists x \forall t(x=x,t)$, where *t* is a variable for time. The former claims that in every time there exists something, while the latter claims that there is something that exists in all times. Only the latter implies the existence of an eternal entity, while the former is compatible with the idea that in every time there are only contingent entities.

29 The literal translation of Leibniz's sentence is "Nothing can produce no thing", where the first occurrence of 'nothing' must be a noun-phrase; otherwise, if it were a quantifier, the sentence would become 'there is no thing that can produce no thing', which is clearly false.

just excludes that something comes from nothing, but it is silent on the possibility that something comes from something else (a possibility explicitly denied by b). This implies that we must look for a different interpretation of the term 'nothing' which cannot be paraphrased away in quantificational terms. The reading of 'nothing' as an empty term seems to be perfect for this situation: 'nothing comes from nothing' is true because 'comes from' (in the sense of being produced by) requires the existence of a producer (and so it is a strong predicate); but 'nothing' is an empty term, and so, in this case, we have no producer. Since we have no producer, *there is no thing* that can be a product, and so nothing comes from nothing.

In the passage quoted above, Leibniz claims that, once the ambiguity affecting Locke's argument has been removed, the conclusion of the argument 'does really follow with mathematical reason from the premises'. However, the argument employs at the same time the same linguistic term 'nothing' both as a quantifier and as a noun-phrase, and this might be enough to suggest a certain ambiguity in it. But having accepted a positive free logic, one can accept 'nothing' as a noun-phrase and develop a valid argument which combines both readings of 'nothing'.[30]

The argument is based on the implicit assumption that everything has a reason (Principle of Sufficient Reason). Moreover, according to Leibniz's theory of time, if *a* is a reason for *b* (and they both are in time), then a must temporally precede *b*. The Principle of Sufficient Reason can be formalized as follows:

$$\exists x(x=b,t_1)\rightarrow[\exists y(y=a,t_0)\wedge R(a,b)] \quad \text{(PS1)}$$

Where *a* and *b* are two arbitrary constants, t_0, t_1 are two constants for time such that $t_0<t_1$ (t_0 precedes t_1) and R(a,b) means that *a* is a reason for *b*. This says that if there is an entity *b* in a time t_1, then there is a different entity *a* in a preceding time t_0 which is the reason of *b*. However, this will not do: in PFL, quantifiers range only over denoting terms, which implies that the sentence is silent with regard to empty terms, and in particular to nihil. To account for the latter, we might rewrite it as follows:

30 Concerning Leibniz's use in the *New Essays* of the terms '*rien*' and '*neant*', we should observe what follows: the term '*neant*' is used few times (I was able to find 5 occurrences of it) and always as a noun-phrase. Moreover, it is used twice in the expression 'to produce from nothing' (*tirer du neant*). The term '*rien*' occurs many times, sometimes as a quantifier, others as a noun-phrase (as in the example discussed in the main text above). A further occurrence of it as a *noun-phrase* is in the fundamental question of the *Principles of Nature and Grace: Pourquoi il y a plutôt quelque chose que rien? Car le rien est plus semple et plus facile que quelque chose*. The sentence 'nothing is simpler and easier than something' is one more example of an occurrence of the term 'nothing' that cannot be paraphrased away in quantificational terms.

$$\exists x(x=b,t_1)\rightarrow[\exists y(y=a,t_0)\wedge R(a,b)]\vee R(nihil,b) \quad \text{(PS2)}$$

But nihil cannot be a reason for the existence of any entities, because it has no properties.[31] Therefore we have

$$\neg R(nihil,b)$$

Supposing that the antecedent of PS2 is true, we can detach the consequent, and by an application of Disjunctive Syllogism, we obtain $\exists y(y=a,t_0)\wedge R(a,b)$. Since this depends on the antecedent, we obtain PS1 (which is not an assumption, but a truth of reason that can be derived by principle of reason, PS2, and a definition). What this shows is that, within PFL, the existence of a reason indeed follows with ‘mathematical rigor’, as Leibniz claims.

9 Conclusion

Based on the passage of the *Generales Inquisitiones* quoted at the beginning of this paper, Mates (1972) argues that sentences with non-denoting terms are always considered false by Leibniz. Contrary to this position, we have here developed a different approach, according to which Leibniz holds that some sentences with empty terms can be true. Our main reason in support of this view is Leibniz’s use of the term nihil in different logical essays concerning the notion of Real Addition. The term nihil can be seen as a counterexample to Mates’ position. After having considered the idea that the presence of empty terms does not exclude truth, we sketched a positive free logic that describes a possible way of understanding the logic of such terms. We then proceeded to show that different theses held by Leibniz (the fictional nature of infinitesimals, the fact that we ‘know what we say’ when speaking of contradictory notions, and the use of ‘nothingness’ in the proof of the existence of God) can be easily interpreted and vindicated within such an approach.

In conclusion, it must be borne in mind that our proposal consists in treating as empty all those terms that do not refer by means of logical necessity, and not those terms that refer to possible but not actual things. In this sense, the admission of empty terms does not contradict the strategy expressed in the *New Essays* consisting in the translation of sentences with terms denoting merely possible objects into conditional sentences. We argued that this strategy is not applicable to terms such as nihil, the greatest velocity, the infinite number, the perpetual mechanical motion, infinitesimal, etc. For these terms a PFL seems an apt tool that harmonizes perfectly well with the rest of Leibniz’s views.

31 See footnote 15.

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Abbreviations

A = Leibniz, G.W. (1923ff.). *Sämtliche Schriften und Briefe*. Berlin: Akademie Verlag.
GP = Leibniz, G.W. (1875-90). *Philosophische Schriften*. Bearb. von C.I. Gerhardt. 7 Bde. Berlin: Weidmann.

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