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inequality of opportunity, bandit problem; unobservable talent, competitive screening

JEL Codes
D53, D82, O31

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I investigate whether wealth inequality hinders the discovery of novel alternatives in a competitive screening model. Agents can engage in experimentation, which may lead to the discovery of superior technologies, while wasting time with inferior ones. Talented agents are better at weeding out inferior actions, but talent is unobservable by lenders. When agents are poor, this causes an adverse selection problem and experimentation is also pursued by untalented agents. As economies become wealthier, this misallocation problem weakens. Higher inequality worsens the misallocation problem when the economy is rich, but can increase efficiency in poor economies.

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1 Introduction

Inventing something novel involves the exploration of untested approaches that are likely to fail. For example, after his patent was granted in 1880, Thomas Edison supposedly stated that he knew a thousand ways *not to make* a lightbulb, emphasizing the years of work and thousands of experiments needed for the creation of the incandescent lightbulb. In turn, the discovery of new and improved products, processes, and machines, through experimentation and learning, is a major source of economic growth, which makes understanding how and which individuals embark on these activities crucial. Edison's multiple attempts were made possible in part because a group of financiers, including J.P. Morgan, were willing to provide loans to Menlo Park Lab, the pioneering research laboratory funded by Edison in 1876 after he relocated to New Jersey. Nevertheless, failures in the credit markets may make access to finance difficult for individuals willing to experiment with novel ideas; if these missing explorers are talented, this has negative implications for growth at the aggregate level.

In this paper, I provide a theoretical model embedding a bandit problem into a two-period competitive screening framework. The economy is populated by agents who are heterogeneous in two dimensions, observable wealth and unobservable talent. Agents can choose between exploring a new technology that involves a small change of a breakthrough and a high risk of failure, or exploiting a conventional technology with a predictable outcome. This bandit problem highlights that experimentation may lead to the discovery of a superior technology, but it is likely than an experimenting agent wastes time with an inferior action. Talented agents are assumed to be better at weeding out inferior actions, so that, if capital markets worked perfectly, only talented agents would explore, whereas untalented ones would always rely on the conventional project for production. Unfortunately, agents need loans to setup firms but talent is private information, so that collateral is used to screen borrowers.

In equilibrium, inequalities in wealth translates into a problem of misallocation, such that some untalented agents may find it profitable to explore. Indeed, the contractual structure of the credit market endogenously introduces different wealth classes that represent different pool of workers. Relatively poorer agents are offered a pooling exploration contract, where the talented agents pay an interest rate on the loan that is higher than what would be consistent with their probability of success, effectively cross-subsidising the untalented ones in exploration. However, this adverse selection problem weakens for relatively richer agents, who can self-finance a larger proportion of the setup cost: as a consequence, the separating contracts offered by financial intermediaries are able to convince agents to self-select in the efficient production method for their talent.

Ceteris paribus, wealthier economies have more efficient credit markets. Indeed, as economies become wealthier, more agents will find themselves in the upper classes, where the problem of misallocation of untalented agents in experimentation weakens and eventually disappears. Conversely, the effect of increasing inequality on the quantity and quality of an economy's experimentation efforts is more nuanced. In particular, equality-enhancing redistributions are always associated with a decrease in the number of untalented innovators when the economy is relatively rich, as they reduce the mass of agents in the lower-classes. On the contrary, more inequality can reduce the adverse selection problem when the economy is relatively poor, as it moves at least some individuals to the upper classes, where the misallocation disappears.

These results are robust to a series of generalizations of the baseline model, such as assuming an unobservable choice between experimentation and relying on the conventional technology, adding the possibility of misallocation of talented agents into suboptimal activities, and considering a general equilibrium framework (where agents can additionally choose to deposit their wealth in banks as an outside option to entrepreneurship, and the interest rate adjusts to clear the credit market).

The remainder of this paper is organised as follows. Section 2 quickly reviews previous literature. Section 3 presents the model. Section 4 solves for the equilibrium, provides the main theoretical findings on the relationship between inequality and exploration, and presents a quantitative example. Section 5 shows that the main results are robust to a range of extensions to the baseline model. Finally, Section 6 concludes.

2 Related Literature

This paper is connected to different strands of literature. First, it relates to many theoretical papers studying the incentives for experimentation using principal–agent models (e.g. Bergemann and Hege, 1998, 2005, Manso, 2011, Hörner and Samuelson, 2013, Bouvard, 2014, Drugov and Macchiavello, 2014, Halac *et al.*, 2016, Gomes *et al.*, 2016, Spiganti, 2020a). I share with Drugov and Macchiavello (2014), Halac *et al.* (2016), and Gomes *et al.* (2016) a focus on adverse selection on the agent's ability to experiment: whereas we all highlight that inefficiencies in the credit market may lead to misallocation in experimentation, our aims are different. Indeed, these papers analyse the problem of a principal hiring an agent to work on a project of uncertain quality, whereas this paper focus on a multiple agents economy to focus on the consequences of misallocation at the aggregate level. Similarly to Manso (2011), Drugov and Macchiavello (2014), and Spiganti (2020a), this paper focuses on the tension between exploration and exploitation, and its implications in a principal-agent frameworks. In the spirit of Weitzman (1979), these models study innovation, interpreted as the discovery of superior actions through experimentation and learning. Once again, one fundamental difference between my model and these papers is that I focus on the economy-level problem, whereas these papers analyse an individual relationship between a financier and an inventor.

Second, the competitive screening framework of this paper shares many features with the occupational choice models by Grüner (2003), Jaimovich (2011), Inci (2013), and Spiganti (2020b),[1] where heterogeneous agents must choose between entrepreneurship and an outside option. Similarly to this paper, the contractual structure of the credit market introduces endogenous wealth classes, which receives different credit contracts. The novelty is the focus on experimentation, since agents can choose to produce through a conventional technology with a known probability of success or a new technology that involves a large risk of initial failure but the small chance of a breakthrough.

[1]There is a rich literature, too vast to be summarised here, that uses occupational choice models to study the relationship between wealth and entrepreneurship. See, among many others, Aghion and Bolton (1997), Meh (2005), Ghatak *et al.* (2007), and Coco and Pignataro (2014).

Finally, one focus of this paper is on the combination of credit market imperfections with the choice of technologies pursued. This has been studied from different angles by e.g. Acemoglu and Zilibotti (1999), Jaimovich (2011), and Legros *et al.* (2014), who are mainly interested in the evolution of informational asymmetries and sectoral differentiation over the development path. However, none of these papers focus on how credit market imperfections may curb incentives of talented agents to experiment with newer and riskier technologies, and this on the intertemporal externalities, in terms of information creation, that are lost due to lack of technological experimentation.

3 The Model

Consider a two-period economy populated by a continuum of agents of mass one. Agents live for both periods, they are risk neutral, and they maximise their end-of-life income; the discount factor is normalised to one.

Agents are heterogeneous in two dimensions. First, they are endowed with different observable wealth, A, which is distributed according to the continuously differentiable cumulative distribution function $G(A)$. The corresponding probability density function is $g(A)$; its support is $[0, I]$, where $I > 0$ is the setup cost of starting a firm. Total wealth in the economy, which is equal to average wealth, is $\bar{A} = \int_0^I AdG(A)$. Second, agents differ in their innovative talent: a proportion $\lambda \in (0, 1)$ of agents is talented, the remaining proportion $1 - \lambda$ is untalented. Agents privately observe their talent at the beginning of the first period. For simplicity, talent and wealth are assumed independent. The talent of the agents and their effort level are known only by them, but the distribution of talent in every wealth level is public information.

3.1 Technology

To setup a firm, agent needs to pay the setup cost I. As entrepreneurs, each agent takes an action $i \in \mathcal{I}$ in each period, producing a positive amount $Y > 0$ (a success) with probability p_i or zero output (failure) with probability $1-p_i$. Since this probability may be unknown, the entrepreneur may need to engage in experimentation. Let $E[p_i]$ denote the unconditional expectation of p_i, $E[p_i|S, j]$ the conditional expectation of p_i given a success on action j, and $E[p_i|F, j]$ the conditional expectation of p_i given a failure on action j. Taking an action i is only informative on the probability of success of the same action, p_i, i.e.

$$E[p_i] = E[p_i|S, j] = E[p_i|F, j] \qquad \text{for} \quad j \neq i. \tag{1}$$

The key trade-off that arises when learning happens though experimentation is the tension between exploiting well-known actions and exploring novel ones. To focus on the tension between exploration and exploitation, I thus assume that in each period an agent can choose between two actions. Action C represents a conventional project with a known probability of success p_C, such that

$$p_C = E\left[p_C\right] = E[p_C|S, C] = E[p_C|S, C]. \tag{2}$$

Conversely, action N represents a novel approach whose probability of success p_N is unknown but such that

$$E\left[p_N|F, N\right] < E\left[p_N\right] < E\left[p_N|S, N\right], \tag{3}$$

thus formalising the natural idea that an agent becomes more optimistic (pessimistic) about the probability of success of the novel approach after having observed a success (failure).

I make two assumptions about the novel approach. First, in line with the classic two-armed bandit problem with one known arm, I assume that the novel approach is of exploratory nature. This means that, when the entrepreneur experiments with the novel approach, she is not as likely to succeed as when she exploits the conventional method. While the experimenting entrepreneur is likely to waste time with an inferior action, this may lead to the discovery of a superior one: after a success with the novel method, the entrepreneur updates her belief about its probability of success p_N, so that it becomes perceived as better than the conventional work method. Second, I assume that agents differ in their ability to weed out inferior actions. In particular, talented agents have higher unconditional probability of success using the novel approach than untalented agents. Formally,

$$E\left[p_{N,L}\right] < E\left[p_{N,H}\right] < p_C < E\left[p_N|S,N\right], \tag{4}$$

where $E\left[p_{N,L}\right]$ and $E\left[p_{N,C}\right]$ are the unconditional probabilities of success of a untalented (indexed by L, mnemonic for low) and talented (indexed by H, for high) agent, respectively.[2]

3.2 A Contingent Plan of Action

After setting up a firm, an entrepreneur chooses a contingent plan of action $\langle m_f^s \rangle$, i.e. an action m in the first period, an action s in the second period after a success in the first period, and an action f in the second period in case of failure in the first period.

Only two action plans need to be considered. The first is the repetition of the conventional work method in any contingency $\langle C_C^C \rangle$, which is usually referred to as "exploitation" in the bandit problem literature (e.g. Manso, 2011, Spiganti, 2020a). I refer to such an entrepreneur as an "exploiter" and to such a firm as a "conventional firm". The second action plan consists in trying the novel work method in the first period, and sticking to it in case of success but resorting to the conventional work method following a failure in the first period, $\langle N_C^N \rangle$. Since this action plan is traditionally referred to as "exploration" (e.g. Manso, 2011, Spiganti, 2020a), I refer to an entrepreneur who sets up an "explorative firm" to engage in exploration as an "explorer". Hereafter, exploiters are indicated with an index C (mnemonic for conventional), talented explorers are denoted by H, whereas untalented explorers are denoted by L.

3.3 Credit Market

By assumption, no agent has wealth greater than I, so everyone needs financing for that part of the setup cost not covered by personal wealth,

[2] One may argue that talented agents are also more likely to discover better technologies, i.e. $E\left[p_{N,H}|S,N\right] > E\left[p_{N,L}|S,N\right]$. However, in the current specification of the model where banks live only for one periods, results would be identical because obtained for general expressions of the expected payoffs from talented and untalented exploration, later defined as $\mathcal{Y}_H$ and $\mathcal{Y}_L$. We thus avoid this inconsequential complication in the baseline model, but this case is covered in the Online Appendix A.

i.e. to borrow an amount $I - A > 0$; note that imposing maximum self-finance is without loss of generality (see e.g. DeMeza and Webb, 1987). In the first period, agents have access to several banks competing à la Bertrand.[3] Banks are willing to lend money to agents: banks can observe the wealth of a borrower, the action plan selection, and the outcome of her firm, but talent is unobservable (see Section 5.1 for unobservable action plans). I assume that the economy is small and there is perfect international capital mobility, meaning that banks can raise funds from international credit markets at an exogenous risk-free rate of return, R.

Banks take the risk-free rate of return as given and can offer a distinct menu of contracts for every wealth level. They hold the same beliefs, which they form simultaneously, about how agents decide when offered a given menu of contracts. This menu of contracts consists of a repayment schedule, contingent on the outcome of the firm and the type of borrowers. As explained above, three types of borrowers need to be considered: talented explorers (H), untalented explorers (L), and exploiters (C). I assume that agents are protected by limited liability, in the sense that they cannot end up with negative cash-holdings at the end of the contract: thus, they can pay back a positive amount only in the case of success.[4] A loan contract offered by a given bank then takes the following form (since it does not generate any confusion, I shall drop the subscript indicating a given bank), $\boldsymbol{\sigma}(A) = [\tau_H(A), \tau_L(A), \tau_C(A)]$, where $\tau_i(A)$ is the repayment to the bank by the i-type entrepreneur with wealth A in the success state. Given limited liability, $\tau_i(A) \leq Y$, $\forall i \in \{H, L, C\}$ and $\forall A$.

[3]For simplicity, banks only live for the first period, otherwise, the optimal menu contract would have to keep track of possible repayments in the second period, possibly contingent on the outcome of the first period and action plan. However, since agents are risk-neutral and have the same discount factor, having long-lived banks should not qualitatively change the results.

[4]In principle, the repayment in case of failure could be negative, i.e. banks could pay failed entrepreneurs. Since agents are risk-neutral, imposing zero repayments in the failure state is, however, without loss of generality.

Let the total expected output produced by an i-type entrepreneur be

$$\mathcal{Y}_i = E\left[p_{m,i}\right]\{Y + E[p_s|S,m]Y\} + \left(1 - E\left[p_{m,i}\right]\right)E[p_f|F,m]Y,$$
$$\text{where } \langle m_f^s \rangle, E\left[p_{m,i}\right] = \begin{cases} \langle C_C^C \rangle, p_C & \text{if } i = C \\ \langle N_C^N \rangle, E\left[p_{N,L}\right] & \text{if } i = L \\ \langle N_C^N \rangle, E\left[p_{N,H}\right] & \text{if } i = H. \end{cases} \tag{5}$$

Therefore, the expected payoff of a talented explorer with wealth A is $\mathcal{V}_H(A) = \mathcal{Y}_H - E[p_{N,H}]\tau_H(A)$, and of an untalented explorer is $\mathcal{V}_L(A) = \mathcal{Y}_L - E[p_{N,L}]\tau_L(A)$. Conversely, the expected payoff of an exploiter is $\mathcal{V}_C(A) = \mathcal{Y}_C - p_C\tau_C(A)$.

I take the standard assumption that exploration done by talented agents is efficient, whereas the one done by untalented agents is not. This is formalised as follows,

Assumption A. *(Efficiency) Absent the need for financing, the net presented value of a project run by a talented explorer is strictly greater than the net present value from exploitation, which is strictly greater than the net present value from untalented exploration, i.e.* $\mathcal{Y}_H > \mathcal{Y}_C > \mathcal{Y}_L$.

As a consequence of Assumption A, in first best all talented agents should become explorers, whereas untalented agents should exploit. However, due to the informational asymmetry between agents and financiers, untalented agents may find it profitable to pretend to be talented individuals.

4 The Equilibrium

In this section, I derive the set of credit contracts offered by the banks and the resulting occupational choices of the agents. I impose a Bertrand-Nash equilibrium concept in the credit market. In particular, I assume that banks are Bertrand-Nash players following pure strategies, offering loan contracts, and paying a rate of return R when raising funds, which they take as given. An equilibrium of this model then consists of entrepreneurial decisions and an individually rational and incentive compatible menu of contract, $\boldsymbol{\sigma}^\star(A)$, for each wealth class, such that (i) banks earn non-negative profits at every wealth level, (ii) the menu of contracts is a Bertrand-Nash equilibrium, (iii) individuals choose the action plan that maximises their end of life wealth, and (iv) talented agents choose exploration if indifferent between exploration and exploitation, whereas untalented individuals choose exploitation when indifferent.

4.1 Credit Contracts

Following the adverse selection literature originating with Akerlof (1970), one should expect two types of equilibrium contract: either a pooling contract, in which types remain undistinguishable, or a separating contract, in which types reveal their unobservable characteristics by selecting different terms.

Here, one can easily exclude that there exists a zero-profit separating menu of contract that induces both talented and untalented agents with the same amount of wealth A to become explorers. Indeed, each contract in such menu must satisfy the corresponding zero-profit conditions,

$$E\left[p_{N,H}\right]\tau_H(A) = R(I - A) \tag{6a}$$

$$E\left[p_{N,L}\right]\tau_L(A) = R(I - A), \tag{6b}$$

where the left-hand sides represent expected repayment in case of success and the right-hand sides are the cost for banks of raising the amount to loan. If negative profits are made on any of these contracts, a deviating bank could just cancel the contract incurring losses; if positive profits are made on a contract, a deviating bank could attract all the agents of the corresponding type by slightly reducing the corresponding repayment. This, however, cannot be incentive compatible, since the zero-profit conditions in (6) entail different repayments: agents can misreport their type, and so they would just choose the contract associated with the lower repayment.

Hence, an equilibrium contract must be a pooling contract or a separating contract that only talented agents accept.[5] In a zero-profit pooling contract, the repayment of a random borrower with wealth A in the success

[5] It is easy to prove, given the probabilities of success, that it can never be the case that a loan contract, for a given wealth level, attracts the untalented agent but not the talented agent.

state is given by $\tau = R(I - A)/\bar{\rho}$, where $\bar{\rho} = \lambda E[p_{N,H}] + (1 - \lambda)E[p_{N,L}]$ is the Bayesian probability of success of a random explorer. An untalented agent would accept this contract if her participation constraint is satisfied, i.e. if $\mathcal{Y}_L - E[p_{N,L}]\tau(A) > \mathcal{Y}_C - p_C\tau_C(A)$, where $\tau_C(A) = R(I - A)/p_C$ is the repayment under a zero-profit exploitation contract. Solving this participation constraint for A reveals that untalented agents accept this pooling contract only if their wealth is lower than a threshold A_L, given by

$$A_L \equiv I - \frac{\bar{p}\,(\mathcal{Y}_C - \mathcal{Y}_L)}{R\,(\bar{p} - E\,[p_{N,L}])}. \tag{7}$$

Untalented agents only accept pooling contracts if they can enjoy large cross-subsidies.

The other possibility is that, for a given wealth class, only the talented agents become explorers. A putative separating contract on the zero-profit condition entails $\tau_H(A) = R(I - A)/E[p_{NH}]$: I refer to this contract as "the zero-profit separating contract". Obviously, this contract can be offered only if an untalented agent with the same wealth does not have any incentive to imitate the talented agent, i.e. if $\mathcal{Y}_C - p_C\tau_C(A) \geq \mathcal{Y}_L - E[p_{N,L}]\tau_H(A)$. This condition requires that her initial wealth is higher than a threshold A_{HH} given by

$$A_{HH} \equiv I - \frac{E\,[p_{N,H}]\,(\mathcal{Y}_C - \mathcal{Y}_L)}{R\,(E\,[p_{N,H}] - E\,[p_{N,L}])}. \tag{8}$$

Note that $A_{HH} \in (A_L, I)$ given Assumption A and the relative ranking of the probabilities of success.

For $A \in (A_L, A_{HH})$ the zero-profit separating contract does not satisfy the incentive compatibility constraint of an untalented agent with identical wealth, but the pooling contract cannot be offered because it does not satisfy her participation constraint. As a consequence, the optimal separating contract for this wealth class involves raising the interest rate demanded of the talented agents in such a way that makes the untalented agents indifferent between becoming explorers or exploiters. This is achieved by imposing $\mathcal{Y}_C - p_C\tau_C(A) = \mathcal{Y}_L - E[p_{N,L}]\tau_H(A)$, or, equivalently, $\tau_H(A) = (\mathcal{Y}_L - \mathcal{Y}_C + RI - RA)\,/E[p_{N,L}]$. I refer to this contract as "the profitable separating contract".[6]

4.2 The Equilibrium Action Plans

Given the set of contracts offered to each wealth class, one can derive the resulting equilibrium choices of the agents. Indeed, the contractual structure of the credit market endogenously introduces wealth classes which represent different pool of borrowers. The following Proposition summarises the equilibrium that ensues.

Proposition 1. *Banks offer the following exploration contracts: pooling contracts to agents with wealth between* $[0, A_L)$*, profitable separating contracts to agents with wealth between* $[A_L, A_{HH})$*, and zero-profit separating contracts to agents with wealth* $[A_{HH}, I]$*. Banks also offer zero-profit exploitation contracts to all agents. All agents in* $[0, A_L)$ *become explorers, talented agents in* $[A_L, I]$ *become explorers, whereas their untalented counterparts become exploiters.*

Lower-class agents have wealth between $[0, A_L)$. They are offered a pooling exploration contract, which they accept: all agents in this class become explorers. The probability of success of a random agent in this class is $\bar{p}$, thus the interest rate on these loans is $R/\bar{p}$. Talented agents in this class cross-subsidise untalented explorers by paying an interest rate higher than the one consistent with their risk level.

The upper-class agents have wealth in $[A_{HH}, I]$. Banks can offer separating exploration contracts to agents in this class, and thus talented agents in this class become explorers, whereas untalented ones prefer to become exploiters. When wealth is sufficiently high, untalented individuals earn

[6] This means that, in the resulting Bertrand-Nash equilibrium, banks will make positive profits on the contracts offered to a particular wealth class. Nevertheless, this is an equilibrium if one assumes that banks cannot lure in additional explorers by lowering the interest rates on their loans. This restriction on the set of feasible contracts prevents the well-known problem of non-existence of a competitive equilibrium (Rothschild and Stiglitz, 1976); a similar characterization of the equilibrium credit contracts at different wealth classes is present in Martin (2009), Jaimovich (2011), and Spiganti (2020b), among others. Alternatively, I could enlarge the set of feasible contracts (so that banks could offer more attractive exploitation contracts) and either impose a Bertrand-Wilson (1977) equilibrium concept where banks are non-myopic (as in e.g. Inci, 2013), or allow two rounds of play (like in Hellwig, 1987). In any case, the qualitative results are unchanged.

enough from exploitation to not be interested in the zero-profit contract offered to talented explorers.

Middle-class agents have wealth levels in $[A_L, A_{HH})$. If talented agents were to be offered a separating contract on the zero-profit condition for talented exploration, untalented agents would pretend to be talented and banks would make negative profits. Banks must thus raise the interest rate demanded to these agents in such a way that makes untalented agents indifferent between exploitation and pretending to be talented.

4.3 Inequality and Exploration

Given the equilibrium choices by banks and agents described in Proposition 1, all talented agents λ become explorers (see Section 5.2 for the problem of misallocation of talented agents), whereas the number of untalented explorers and the equilibrium average probability of success of first-period exploration are, respectively,

$$n_L = (1-\lambda)\, G\,(A_L) \tag{9a}$$

$$p = \frac{\lambda E[p_{N,H}] + n_L E[p_{N,L}]}{\lambda + n_L}. \tag{9b}$$

In the following proposition, we explore how initial conditions related to the wealth distribution influence the amount and average quality of exploration and thus the likelihood of discover novel, more productive, technologies.

Proposition 2. *Consider two identical economies but for the initial wealth distributions, $G(A)$ and $G'(A)$. (i) Let $G(A)$ first-order stochastically dominate $G'(A)$. Then $n_L \leq n'_L$. (ii) Let $G'(A)$ be obtained by a single mean-preserving spread of $G(A)$: thus, $G(A)$ crosses $G'(A)$ only once, and from below. Denote this crossing as $\tilde{A}$. Then, if $A_L < \tilde{A}$, $n_L < n'_L$; if $A_L = \tilde{A}$, $n_L = n'_L$; if $A_L > \tilde{A}$, $n_L > n'_L$.*

Proof. If everything is the same across economies but the wealth distributions, the wealth thresholds are the same. (i) If $G(A)$ first-order stochastically dominates $G'(A)$, then by definition $G(A) \leq G'(A)$ for all A, with strict equality for some A. Therefore, $G(A_L) \leq G'(A_L)$, and thus $n_L \leq n'_L$. (ii) If $G'(A)$ is obtained by a single mean-preserving spread of $G(A)$, $G\,(A) >$

$G'(A)$ for $A > \tilde{A}$, $G(A) = G'(A)$ for $A = \tilde{A}$, and $G(A) < G'(A)$ for $A < \tilde{A}$. □

The intuition for Proposition 2 is straightforward. Part (i) focus on the total wealth of an economy. It says that, other things equal, wealthier economies suffer less from the problem of misallocation of untalented agents. Indeed, as an economy becomes wealthier, more agents will find themselves in the upper classes, where the adverse selection problem turns into an efficient redistribution (middle-class) or disappears (upper-class).

Instead, part (ii) focuses on the initial level of inequality, by considering a single mean-preserving spread. Mean-preserving spreads á la Rothschild and Stiglitz (1970) are common when comparing inequality levels as they amount to ranking distributions with the same average wealth by second-order stochastic dominance (Atkinson, 1970). In turn, this is equivalent to (generalized) Lorenz dominance, probably the most commonly used ordering for the comparisons of income (and wealth) distributions. It shows that equality-enhancing redistributions are always associated with a decrease in the number of untalented innovators when the economy is relatively rich, as they reduce the mass of agents in the lower-class. On the contrary, more inequality can reduce the adverse selection problem when the economy is relatively poor, as it moves at least some individuals to the upper classes, where the misallocation disappears.

4.4 A Quantitative Example

I now report the results of a simple quantitative example to highlight the comparative statics analysed above, rather than providing a comprehensive quantitative evaluation. Parameter values, which respect all the assumptions, are arbitrary chosen as: $p_C = 0.40$, $E[p_{NH}] = 0.35$, $E[p_{NL}] = 0.20$, $E[p_N|S, N] = 0.70$, $\lambda = 0.50$, $I = 1$, $Y = 1.84$, and $R = 1.40$. With these parameters, $A_L = 0.33$ and $A_{HH} = 0.57$.

Wealth is distributed according to a Beta distribution, $A \sim \beta(a, b)$.[7] I let the shape parameter a vary between $(0, 10)$, and set $b = a/\bar{A} - a$.

[7] The Beta distribution has been used to fit income and wealth distributions at least since Thurow (1970) and Podder and Kakwani (1976). Its bounded domain and great flexibility make it very convenient for this application.

This means that, by varying a, I am considering different degree of wealth inequality but the same aggregate (and average) wealth, $\bar{A}$. I present results for two levels of aggregate wealth, $\bar{A} = \{0.16, 0.45\}$. The difference is that for the former, the average agent is in the middle of the lower-class, whereas for the latter, she belongs to the middle-class.

Table 1 presents a comparison between the equilibrium for different economies with different levels of wealth inequality, but the same total wealth, $\bar{A} = 0.16$. As shown in the second column, the Gini coefficient of the sample decreases (i.e. there is an increase in equality) as the shape parameters of the Beta distribution increase (first column), since mass is moved from the tails of the distribution to the centre. As inequality decreases in an economy where the majority of agents are in the lower-class, agents move from the middle- and upper-class towards the lower-class. Keeping other things constant, this implies the inefficient reallocation of untalented agents from exploitation (fourth column) to untalented exploration (third column). Since untalented agents are much more likely to succeed in exploitation than exploration, the average probability of success of the economy in the first period decreases (fifth column), as does the total output across the two periods (sixth column).

Table 1: A Poor Economy

(a, b)	Gini	n_L	n_C	Success in $t = 1$	Total Output
(0.1, 0.51)	0.78	40.3%	9.7%	29.4%	1.419
(0.5, 2.56)	0.58	41.4%	8.6%	28.9%	1.416
(1.0, 5.12)	0.46	43.4%	6.6%	27.9%	1.411
(2.5, 12.81)	0.31	47.1%	2.9%	26.3%	1.401
(7.5, 38.44)	0.18	49.7%	0.3%	25.2%	1.394

Table 2 presents the same analysis but for a rich economy, with $\bar{A} = 0.45$. Now, the average agent is in the middle-class, so that when inequality increases, agents are moved towards the lower-class, where the adverse selection problem manifests itself. In the rich economy, a decrease in inequality thus implies the efficient reallocation of untalented agents from exploration (third column), where they are misallocated, to exploitation (fourth column). As a consequence, both the average probability of success of the economy in the first period (fifth column) and total output across the two periods (sixth column) increase.

Table 2: A Rich Economy

(a, b)	Gini	n_L	n_C	Success in $t = 1$	Total Output
(0.1, 0.51)	0.78	25.7%	24.3%	32.4%	1.456
(0.5, 2.56)	0.58	21.9%	28.1%	33.1%	1.466
(1.0, 5.12)	0.46	19.0%	31.0%	33.7%	1.474
(2.5, 12.81)	0.31	14.7%	35.3%	34.6%	1.485
(7.5, 38.44)	0.18	7.8%	42.2%	35.9%	1.503

5 Extensions

I made many simplifying assumptions to keep the model simple and the exposition as clear as possible. In this section, I provide intuitive explanations about how the results would qualitative change under a range of natural extensions, thus showing that the implications of this relatively simple model are quite robust. A comprehensive model adding all the extensions considered in this section to the baseline model is in Online Appendix A.

5.1 Unobservable Action Plans

Here, we change the baseline model of Section 3 by assuming that the action plan is unobservable by banks, which thus need to make the equilibrium exploration contract incentive compatible with respect to exploitation. This is in line with a partial equilibrium literature that focuses on how to design optimal incentives for exploration (e.g. Manso, 2011, Klein, 2016, Spiganti, 2020b)

Also in this case, it can be easily exclude that there exists a zero-profit separating menu of contract that induces both talented and untalented agents with the same amount of wealth A to enter entrepreneurship. Indeed, each contract in such menu must satisfy the corresponding zero-profit condition. This, however, cannot be incentive compatible, since the zero-profit conditions entail different repayments: agents can misreport their type and action plan, and so they would always just choose the exploitation contract, since this is associated with the lower repayment, $\tau_C(A) = R(I - A)/p_C$.

Hence, an equilibrium contract must be a zero-profit pooling contract, i.e. with $\tau(A) = R(I - A)/q$, where q is the Bayesian probability of success of a random applicant (which depends on the equilibrium choices of the agents in that wealth level). There are two possibilities: a pooling exploration contract, where talented explorers cross-subsidise untalented explorers, and a mixed pooling contract, where untalented exploiters are pooled with talented explorers.

Talented agents always prefer to explore with a pooling contract, but this must be incentive compatible for the untalented agents. In equilibrium, banks can offer pooling exploration contracts only to poor agents, and a mixed pooling contract only to wealthy individuals. This is because untalented agents prefer exploration when poor and exploitation when wealthy. Indeed, exploration is associated with a higher long-term payoff but a lower short-term payoff, as compared with exploitation. Relatively poorer agents thus prefer exploration because the likelihood of having to repay the larger loan is lower under exploration than under exploitation (and thus the second period payoff has more weight), given the initial probabilities of success; for relatively richer agents, the amount they need to borrow is lower, thus they prefer exploitation, since they can keep a relatively bigger share of the first period payoff.

Given the set of contracts offered by banks, two wealth classes arise: a lower-class where all agents become explorers with a pooling exploration contract and an upper-class where talented agents explore while untalented ones exploit, thanks to a mixed pooling contract. Even if the term of the contracts are different from the baseline model, the equilibrium occupations are the same and the same conclusions attain.

5.2 The Misallocation of Talented Agents

In the baseline model presented in Section 3, there is no misallocation of talented agents, as these are always free to choose their first best option (even if under different contracts, i.e. pooling, profitable separating, and zero-profit separating). The only inefficiency thus comes from the misallocation of untalented agents as explorers, rather than exploiters, that happens in the lower-class. Indeed, one cannot get talented agents to choose an inefficient allocation with pure adverse selection in this model; nevertheless, we show here that this can be achieved with a combination of adverse selection and moral hazard (see also Grüner, 2003, Inci, 2013, Spiganti, 2020b).

In particular, assume that everyone in this economy is born untalented but a proportion λ can exert *unobservable* effort at a fixed monetary cost $e > 0$ to increase their talent; for the remaining proportion $1 - \lambda$, the effort cost is prohibitively high. As a consequence, one needs to check the following additional incentive compatibility constraint to the existence of the pooling equilibrium, $\mathcal{Y}_H - e - E[p_{NH}]\tau(A) > \mathcal{Y}_C - p_C\tau_C(A)$, i.e. whether potentially talented agents are willing to exert effort. Solving this for A introduces an additional wealth threshold A_e below which agents are not willing to exert effort for a pooling contract: when the amount they need to borrow is large, talented agents do not want to cross-subsidise untalented ones under a pooling exploration contract, which therefore cannot be offered by banks. Assuming $A_e \in (0, A_L)$, in equilibrium there will be both adverse selection, as all agents with wealth in (A_e, A_L) become explorers thanks to a pooling exploration contract, and misallocation of poor potentially talented agents, as all agents with wealth lower than A_e become exploiters since they do not have access to credit. The equilibrium number of talented and untalented explorers, and the number of exploiters, are

$$n_H = \lambda \left[1 - G\left(A_e\right)\right] \tag{10a}$$

$$n_L = (1 - \lambda) \left[G\left(A_L\right) - G\left(A_e\right)\right] \tag{10b}$$

$$n_C = G\left(A_e\right) + (1 - \lambda) \left[1 - G\left(A_L\right)\right]. \tag{10c}$$

The resulting average probability of success of first-period exploration is

$$p = \frac{n_H E[p_{N,H}] + n_L E[p_{N,L}]}{n_H + n_L}. \tag{11}$$

Given the equilibrium choices of banks and agents, consider two identical economies but for the initial wealth distributions, $G(A)$ and $G'(A)$. If $G(A)$ first-order stochastically dominate $G'(A)$, then $G(A_e) \leq G'(A_e)$ and thus $n_H \geq n'_H$; whether this is associated with fewer untalented explorers depend on the particular wealth distribution. Conversely, let $G'(A)$ be obtained by a single mean-preserving spread of $G(A)$: thus, $G(A)$ crosses $G'(A)$ only once, and from below. Denote this crossing as $\tilde{A}$. Then, if $\tilde{A} < A_e$, $n_H < n'_H$; if $\tilde{A} = A_e$, $n_H < n'_H$ and $n_L > n'_L$; if $A_e < \tilde{A} \leq A_L$, $n_H > n'_H$ and $n_L >'_L$; if $\tilde{A} > A_L$, $n_H > n'_H$ but $n_L \lesseqgtr n'_L$ depending on the shape of the distribution.

The implications of the model are less sharp, but overall consistent with the current ones: richer economies have more agents in the upper classes, where the problems of misallocation disappears. Likewise, more inequality is potentially beneficial when the economy is poor, since it allows some talented individuals to escape the lower class; but when the total wealth of the economy is above a certain threshold, equality-enhancing redistributions are always associated with an increase in the number of talented explorers.

5.3 A General Equilibrium Model

The model in Section 3 does not consider a potentially important source of general equilibrium repercussions on the credit market (see e.g. Inci, 2013), since the risk-free interest rate is assumed fixed. This can be incorporated into the model by assuming that agents can use investment as an outside opportunity to entrepreneurship, by depositing their wealth in the banks. Banks then use these funds to provide explorers with loans. Depositors, banks, and explorers take the interest rate as given, but in general equilibrium this adjust to clear the credit market.

In particular, assume that agents, after having privately observed their ability, decide their occupation between entrepreneurship (which consists of borrowing to pay the setup cost and then choosing an action plan, like in the baseline model) and investment. Investment consists in depositing the wealth in a bank for a risk-free rate of return R, so that the payoff of an investor is $\mathcal{W}(A, R) = RA$. Similarly, the expected payoffs of the entrepreneurs now also depends on the endogenous rental rate, $\mathcal{V}_i(A, R)$, where $i = \{H, L, C\}$. As a consequence, the total number of entrepreneurs also depends on the equilibrium rental rate: the total number of entrepreneurs is $n(R) = n_H(R) + n_L(R) + n_C(R)$ and the total demand of funds is

$I \times n(R)$. The total availability of funds is given by aggregate wealth, $\bar{A}$. Thus, the clearing condition is $\bar{A} = I \times n(R^\star)$. Note that the number of entrepreneurs (and thus of investors) is uniquely identified by the ratio of total wealth to the setup cost, and thus positively related to how wealthy the economy is.

In this version of the model, the participation constraint of the untalented agents in the pooling exploration contract must thus consider this alternative use of funds, $\mathcal{Y}_L - E[p_{N,L}]\tau(A,R) \geq \max\{\mathcal{W}(A,R), \mathcal{Y}_C - p_C\tau_C(A,R)\}$. Likewise, an untalented agent does not have an incentive to accept a separating exploration contract designed for a talented agent if $\max\{\mathcal{Y}_C - p_C\tau_C(A,R), \mathcal{W}(A,R)\} \geq \mathcal{Y}_L - E[p_{N,L}]\tau_H(A,R)$.

Similarly to the baseline model, different wealth classes arise in equilibrium: a lower-class where all agents become explorer with a pooling exploration contract, a middle-class where untalented agents invest and talented agents explore (with a separating contract, which is of the profitable kind for relatively poorer agents and zero-profit for the relatively richer agents in this class), and an upper-class where talented agents explore whereas untalented agents exploit. The intuition for the existence of different wealth classes is unchanged: poor agents have a harder time getting funds, given the existence of adverse selection, whereas for richer agents (with more collateral) this problem lessens, so that they can choose their preferred occupation (for untalented agents, this will depend on their initial wealth).

The main difference with the baseline model is that now, obviously, the wealth thresholds become endogenous, as they also depend on the endogenous rental rate. It can be proven that it exists a partial equilibrium where the number of entrepreneurs is strictly decreasing in the rental rate; since the total supply of funds is fixed, a general equilibrium where the credit market clears not only exists but it is also unique.

In an economy where the average agent is in the lower-class, a mean-preserving spread initially increases the number of investors and thus the interest rate decreases, as more agents enter the middle-class. In general equilibrium, however, the number of entrepreneurs must remain constant, and thus the wealth thresholds adjust. In particular, the reduction in the number of untalented and talented explorers leaving the lower-class is partly compensated by a relatively bigger upper-class, which means that the number of talented explorers and untalented exploiters increases. These new entrepreneurs are richer and thus require smaller loans, justifying the reduction in the interest rate. On the contrary, in an economy where the average agent is in the middle-class, a similar effect is caused by a mean-preserving contraction. Thus, in the general equilibrium version of the model, the relationship between inequality and exploration is more nuanced, but the main results are consistent with the baseline model.

6 Conclusions

Does inequality hinder or foster the discovery of novel alternatives to production? In this paper, I offered a two-period competitive screening model, where agents differ in observable wealth and unobservable talent. Agents can engage in experimentation by giving up the possibility of using a well-known approach to production. Experimentation may lead to the discovery of a superior technology, but it is likely than an experimenting agent wastes time with an inferior one. Agents need loans to setup firms. Since talented agents are better at weeding out inferior actions, banks would like to finance experimentation only for talented agents. However, talent is private information which means that an adverse selection problem plagues the credit market. Indeed, when agents are poor, untalented agents find it profitable to experiment thanks to cross-subsidising pooling contracts; this is inefficient. Conversely, the adverse selection weakens for richer agents, who can self-finance a larger proportion of the setup cost, and thus tend to self-select in the right occupation.

Therefore, as economies become wealthier, more agents will find themselves in the upper classes, where the problem of misallocation of untalented agents in experimentation weakens. Conversely, the effect of increasing inequality on the quality of an economy's experimentation efforts is more nuanced. In particular, equality-enhancing redistributions are always associated with a decrease in the number of untalented innovators when the economy is relatively rich, as they reduce the mass of agents in the lower-classes. On the contrary, more inequality can reduce the adverse selection problem when the economy is relatively poor, as it moves at least some individuals to the upper classes, where the misallocation disappears.

These results are robust to a series of generalizations of the baseline model, such as assuming an unobservable choice between experimentation and relying on the conventional technology, adding the possibility of misallocation of talented agents, and considering a general equilibrium framework, when agents can additionally choose to deposit their wealth as an outside option to entrepreneurship, and the interest rate adjusts to clear the credit market.

References

Acemoglu, D., and F. Zilibotti. (1999), Information accumulation in development, *Journal of Economic Growth 4*, 5 – 38.

Aghion, P., and P. Bolton. (1997), A theory of trickle-down growth and development, *Review of Economic Studies 64*, 151 – 172.

Akerlof, G. A. (1970), The market for 'lemons': Quality, uncertainty, and the market mechanism, *Quarterly Journal of Economics 84*, 488 – 500.

Atkinson, A. B. (1970), On the measurement of inequality, *Journal of Economic Theory 2*, 244 – 263.

Bergemann, D., and U. Hege. (1998), Venture capital financing, moral hazard, and learning, *Journal of Banking & Finance 22*, 703 – 735.

Bergemann, D., and U. Hege. (2005), The financing of innovation: Learning and stopping, *RAND Journal of Economics 36*, 719 – 752.

Bouvard, M. (2014), Real option financing under asymmetric information, *Review of Financial Studies 27*, 180 – 210.

Coco, G., and G. Pignataro. (2014), The poor are twice cursed: Wealth inequality and inefficient credit market, *Journal of Banking & Finance 49*, 149 – 159.

DeMeza, D., and D. Webb. (1987), Too much investment: A problem of asymmetric information, *Quaterly Journal of Economics 102*, 281 – 292.

Drugov, M., and R. Macchiavello. (2014), Financing experimentation, *American Economic Journal: Microeconomics 6*, 315 – 349.

Ghatak, M., M. Morelli, and T. Sjöström. (2007), Entrepreneurial talent, occupational choice, and trickle up policies, *Journal of Economic Theory 137*, 27 – 48.

Gomes, R., D. Gottlieb, and L. Maestri. (2016), Experimentation and project selection: Screening and learning, *Games and Economic Behavior 96*, 145 – 169.

Grüner, H. P. (2003), Redistribution as a selection device, *Journal of Economic Theory 108*, 194 – 216.

Halac, M., N. Kartik, and Q. Liu. (2016), Optimal contracts for experimentation, *Review of Economic Studies 83*, 1040 – 1091.

Hellwig, M. (1987), Some recent developments in the theory of competition in markets with adverse selection, *European Economic Review 31*, 319 –

325.

Hörner, J., and L. Samuelson. (2013), Incentives for experimenting agents, *The RAND Journal of Economics 44*, 632 – 663.

Inci, E. (2013), Occupational choice and the quality of entrepreneurs, *Journal of Economic Behavior & Organization 92*, 1 – 21.

Jaimovich, E. (2011), Sectoral differentiation, allocation of talent, and financial development, *Journal of Development Economics 96*, 47 – 60.

Klein, N. (2016), The importance of being honest, *Theoretical Economics 11*.

Legros, P., A. F. Newman, and E. Proto. (2014), Smithian growth through creative organization, *The Review of Economics and Statistics 96*, 796–811.

Manso, G. (2011), Motivating innovation, *The Journal of Finance 66*, 1823 – 1860.

Martin, A. (2009), A model of collateral, investment, and adverse selection, *Journal of Economic Theory 4*, 1572 – 1588.

Meh, C. A. (2005), Entrepreneurship, wealth inequality, and taxation, *Review of Economic Dynamics 8*, 688 – 719.

Podder, N., and N. C. Kakwani. (1976), Distribution of wealth in Australia, *Review of Income and Wealth 22*, 75 – 92.

Rothschild, M., and J. Stiglitz. (1970), Increasing risk I: A definition, *Journal of Economic Theory 2*, 225 – 243.

Rothschild, M., and J. Stiglitz. (1976), Equilibrium in competitive insurance markets: An essay on the economics of imperfect information, *Quarterly Journal of Economics 90*, 629 – 649.

Spiganti, A. (2020a), Can starving start-ups beat fat labs? A bandit model of innovation with endogenous financing constraint, *Scandinavian Journal of Economics 122*, 702 – 731.

Spiganti, A. (2020b), Inequality of opportunity, inequality of effort, and innovation, EUI Working Paper 2020/02.

Thurow, L. C. (1970), Analyzing the American income distribution, *American Economic Review 60*, 261 – 269.

Weitzman, M. (1979), Optimal search for the best alternative, *Econometrica 47*, 641–54.

Wilson, C. (1977), A model of insurance markets with incomplete information, *Journal of Economic Theory 16*, 167 – 207.

A Online Appendix: A More General Model

This section presents a comprehensive model where I add all the extensions considered in Section 5 to the baseline model of Section 3. First, I add an outside option to entrepreneurship. I assume that at the beginning of the first period, agents can choose to become either investors or entrepreneurs. Investors deposit their wealth in a bank for a riskless return; conversely, an entrepreneur undertakes a risky investment in the form of starting a firm and can then choose to explore or exploit (as in the baseline model). Second, I add moral hazard to the pure adverse selection of the baseline model. I assume that a proportion λ of agents is talented, which means that they can exert unobservable effort at a fixed monetary cost $e > 0$ to increase their unconditional probability of success using the novel approach to $E[p_{NH}] \in (E[p_{NL}], p_C)$ and their conditional probability of success to $E[p_{NH}|S, N] \in (E[p_N|S, N], 1)$; relatively to the baseline model, talented agents are not only better at weeding out inferior actions but also at discovery more productive actions. For the remaining proportion $1-\lambda$ (the untalented agents), the effort cost is prohibitively high. Third, I assume that banks can observe the wealth of a potential entrepreneur and the outcome of her firm, but effort and project selection are unobservable. Fourth, I consider the general equilibrium case, where the rate of return adjust to clear the credit market. Since the analytical results are more notation-intensive than the in baseline model, for ease of reading, I shorten the notation as follows: $p_C = p$, $E[p_{NH}] = q_H$, $E[p_{NL}] = q_L$, $E[p_{NH}|S, N] = q_H^S$, and $E[p_N|S, N] = q_H^L$.

The assumption of efficiency is that, absent the need for financing, the net presented value of a project run by a talented explorer is strictly greater than the net present value from exploitation and investment, both of which are strictly greater than the net present value from untalented exploration, i.e. $\mathcal{Y}_H - e > \max\{\mathcal{Y}_C, RI\} > \min\{\mathcal{Y}_C, RI\} > \mathcal{Y}_L$. This implies that the equilibrium risk-free interest rate must be such that

$$R^\star \in \left(\frac{\mathcal{Y}_L}{I}; \frac{\mathcal{Y}_H - e}{I}\right) \equiv \left(\underline{R}; \bar{R}\right)$$

I do not take a stand on whether exploitation is preferred to investing in first best (this will depend on the equilibrium interest rate).

As in the baseline model, only three types of borrowers need to be considered: talented explorers (H), untalented explorers (L), and exploiters (C). I assume that agents choose to become investors if indifferent between becoming investors or entrepreneurs, and exploiters if indifferent between exploitation and exploration. The tie-breaking assumptions rule out mixed strategy equilibria. Thus, for a given wealth level and menu of contract, all talented agents make the same decision as each other, and all untalented agents make the same decision as each other. As a consequence, for any given wealth class, we only need to consider pair-wise combinations of exploration (with or without effort), exploitation, and investing. Equilibrium contracts can be of two types: pooling equilibrium, in which both types receive an identical contract, and separating equilibrium, in which contracts induce self-revelation of their unobservable ability.

As in the baseline model, it cannot exist a zero-profit separating menu of contract that induces two types to enter entrepreneurship, since this cannot be incentive compatible. Hence, an equilibrium contract must be either a pooling contract or a separating contract that only one type accepts. There are three possible pooling contracts: one pooling effort-exerting explorers with untalented explorers $\sigma^{\star}_{HL}$, one pooling effort-exerting explorers with untalented exploiters $\sigma^{\star}_{HC}$, and one pooling shirking explorers with untalented explorers $\sigma^{\star}_{L}$.[8] Each of these contracts must lie on the corresponding zero-profit condition, otherwise banks could undercut each other. These three putative contracts are then given by $\tau_{HL}(A,R) = R(I-A)/\bar{q}_{HL}$, $\tau_{HC}(A,R) = R(I-A)/\bar{q}_{HC}$, and $\tau_{L}(A,R) = R(I-A)/q_{L}$, where $\bar{q}_{HL} = \lambda q_H + (1-\lambda)q_L$ and $\bar{q}_{HC} = \lambda q_H + (1-\lambda)p$ are the corresponding Bayesian probability of success of a random applicant.

I start by considering the putative contract pooling effort-exerting talented and untalented explorers. Let $V(i,j,\tau)$ be the expected payoff of an agent of realized talent $i = \{H,L\}$ (i.e. talent after effort choice), choosing the project $j = \{N,C\}$, and under a repayment τ. This contract can be accepted by the untalented agent if the participation constraint, $V\left(L,N,\tau_{HL}\right) > RA$, and the incentive compatibility constraint, $V\left(L,C,\tau_{HL}\right) < V\left(L,N,\tau_{HL}\right)$, are both satisfied. Solving these for A leads to

$$A < -\frac{q_L}{\bar{q}_{HL}-q_L}I + \frac{\left(q_L + q_L q_L^S + p - q_L p\right)\bar{q}_{HL}}{R(\bar{q}_{HL}-q_L)}Y =: \phi_{HL}(R) \qquad \text{(A.12a)}$$

$$A < I - \frac{\left(p + q_L p - q_L - q_L q_L^S\right)\bar{q}_{HL}}{R(p-q_L)}Y =: \phi^{IC}_{HL}(R). \qquad \text{(A.12b)}$$

Likewise, consider the putative contract with both effort-exerting talented explorer and untalented exploiters. This contract can be offered to the untalented agent if the participation constraint, $V\left(L,C,\tau_{HC}\right) > RA$, and incentive compatibility constraint (versus exploration), $V\left(L,C,\tau_{HC}\right) <$

[8] Indeed, the assumption on efficiency ensures that talented agents always prefer effortful exploration to exploitation.

$V(L, N, \tau_{HC})$, are satisfied. Equivalently,

$$A > \frac{p}{p - \bar{q}_{HC}} I - \frac{2p\bar{q}_{HC}}{R(p - \bar{q}_{HC})} Y =: \phi_{HC}(R) \tag{A.13a}$$

$$A > I - \frac{\left(p + q_L p - q_L - q_L q_L^S\right) \bar{q}_{HC}}{R(p - q_L)} Y =: \phi_{HC}^{IC}(R). \tag{A.13b}$$

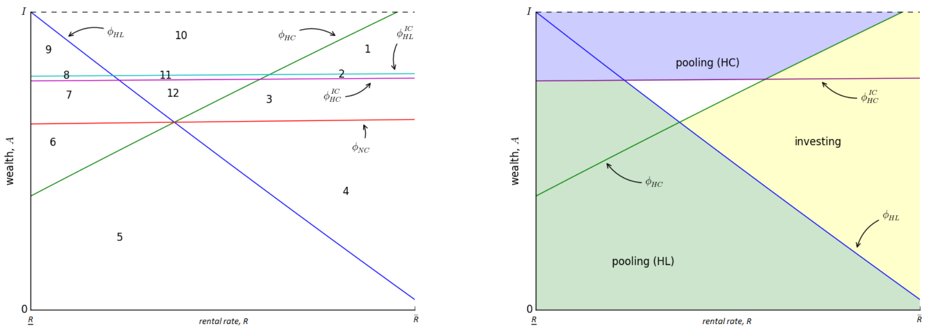
Imagine the bank offering both $\sigma_{HL}^\star$ and $\sigma_{HC}^\star$. A low ability agent will prefer the former to the latter if

$$A < I - \frac{\left(p - q_L - q_L q_L^S + q_L p\right) \bar{q}_{HL} \bar{q}_{HC}}{\lambda R q_H (p - q_L)} Y =: \phi_{NC}(R).$$

The wealth levels ϕ_{HL}, ϕ_{HC}, ϕ_{NC}, ϕ_{HC}^{IC}, and ϕ_{HL}^{IC} naturally divide the (R, A) space into twelve areas, as shown in Panel (a) of Figure A.1. In the areas (1)-(4), the low ability agent prefers investing to both pooling contracts, and thus a pooling contract is not offered in equilibrium. In the areas (5) and (6), the low ability agent prefers $\sigma_{HL}^\star$ to both investing and $\sigma_{HC}^\star$. Since $\sigma_{HL}^\star$ is also incentive compatible, it can be offered. In area (7), L would like to be offered $\sigma_{HC}^\star$ but this is not incentive compatible; however, $\sigma_{HL}^\star$ is both incentive compatible and preferred to investing, and thus it can be offered. In areas (8)-(11), L prefers $\sigma_{HC}^\star$ to both $\sigma_{HL}^\star$ and investing, and it is incentive compatible. Finally, in area (12), L prefers $\sigma_{HC}^\star$, which is not incentive compatible; however, $\sigma_{HL}^\star$ cannot be offered because L would prefer to become an investor (the equilibrium contract is derived below).

The relevant conditions are summarized in Panel (b) of Figure A.1. In the area labelled "investing", low ability agents prefer investing to accepting a pooling contract. In the blue area labelled "pooling (HC)", the pooling contract that can be offered by banks is $\sigma_{HC}^\star$; in the green area labelled "pooling (HL)", the equilibrium pooling contract is $\sigma_{HL}^\star$. In the remaining white area, a pooling contract cannot be offered, because $\sigma_{HC}^\star$ is not incentive compatible, whereas $\sigma_{HL}^\star$ does not satisfy the participation constraint.

When untalented agents prefer investing to any pooling contract, talented agents can be offered a separating contract. Given the assumption on efficiency, the only separating contract that we need to consider is the one inducing talented agents to become effort-exerting explorers. The first-best zero-profit exploration contract is $\tau_H(A, R) = R(I - A)/q_H$. An untalented agent with equal wealth A does not accept τ_H if she prefers investing to exploring using this contract, $V(L, N, \tau_H) \leq RA$, and to exploiting using this same contract, $V(L, C, \tau_H) \leq RA$. Solving these two conditions for A



(a) All Conditions (b) Relevant Conditions

Figure A.1: Contracts Offered

results in

$$A \geq -\frac{q_L}{q_H - q_L} I + \frac{\left(q_L + q_L q_L^S + p - q_L p\right) q_H}{R\left(q_H - q_L\right)} Y =: \phi_I(R) \tag{A.14a}$$

$$A \leq \frac{p}{p - q_H} I - 2\frac{p q_H}{R(p - q_H)} Y =: \phi_C(R). \tag{A.14b}$$

In Figure A.2, I update Figure A.1 (b) with the new threshold $\phi_I(R)$.[9] This shows that for certain wealth and risk-free interest rate combinations, a separating exploration contract with effort is incentive compatible, as shown in the area labelled "separating". We have also shown that there is an area where the only contract that can be offered is a pooling contract. In the area labelled "pooling (HC)", the equilibrium pooling contract is the one pooling effort-exerting explorers with exploiters; in the area labelled "pooling (HL)", the equilibrium pooling contract is the exploration one.

We still need to determine which contract is offered in the area between ϕ_{HL}, ϕ_I, and ϕ_{HC}^{IC}. The separating exploration contract requiring effort is not incentive compatible, as it is also accepted by low ability agents, but a pooling contract cannot be offered. To graphically show the equilibrium for these wealth class, it will prove useful to consider also possible repayments in case of failure τ^F; note that these be negative, given the limited liability assumption. This is represented in Panel (a) of Figure A.3, where R and

[9] Note that by the assumption on efficiency, $\phi_I > \phi_{HL}$. Also, ϕ_C is greater than ϕ_{HC} if $R > 2pY/I$, i.e. if investing is more efficient than exploiting. For $A \in [\phi_I, \phi_C]$, a separating contract is in principle incentive compatible, as low ability prefers investing to improperly accept it. However, above ϕ_{HC}, the pooling exploitation and exploration contract is preferred to investing. Thus, whenever τ_{HC} can be offered, the separating exploration contract cannot be an equilibrium because banks can undercut each other by offering this pooling contract.

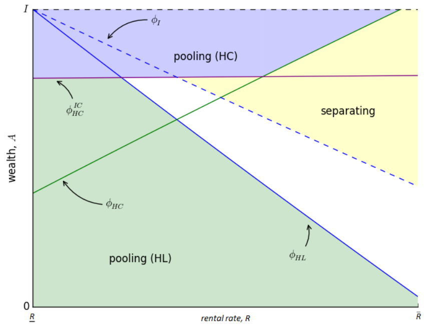


Figure A.2: Separating and Pooling Contracts

A have been chosen so that in the first panel we are in the white area on the left of ϕ_{HC}, while on the second panel we are in the white area on the right of ϕ_{HC} from of Figure A.2.[10] The pooling contract cannot be offered either because low ability agent prefers investing to both pooling contracts (second panel), or because the pooling exploration contract does not satisfy the participation constraint while the exploitation and exploration pooling contract is not incentive compatible (first panel).[11] In both cases, however, the banks can offer a separating pair of contracts, like $\{y_L, y_H\}$. Effort-exerting high ability agents strictly prefer y_H,[12] while low ability agents are indifferent among y_L, y_H, and investing: thus, they choose to become investors by assumption. This is a Bertrand-Nash equilibrium where banks make positive profits, since no profitable deviations are available.[13]

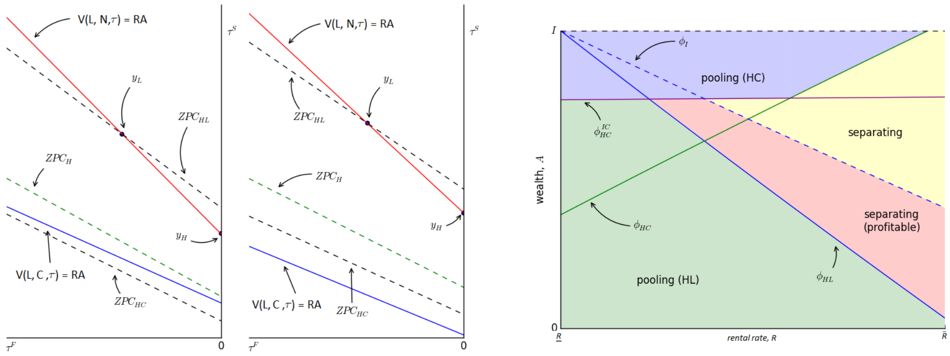
What are the terms of the profitable separating menu of contract? From Panel (a) of Figure A.3, the contract offered to the low ability agent (when neither the zero-profit separating contract nor a pooling contract can be offered) is given by the intersection of $V(L, N, \boldsymbol{\tau}) = RA$ and the zero-profit

[10] Qualitatively, the only difference between the two panels is the relative position of the iso-payoff of the exploiter at the outside opportunity level rA and the zero-profit condition from an exploitation and exploration pooling contract.

[11] That the exploitation and exploration pooling contract is not incentive compatible in the left panel is not immediate from the graph, but it means that, given a contract on ZPC_{HC}, the indifference curve $V(L, N, \boldsymbol{\tau}) = V_L^N$ passing through this contract is associated with a higher utility than the indifference curve $V(L, C, \boldsymbol{\tau}) = V_C$ passing trough the same contract. This can be shown analytically.

[12] Moreover, the assumption on efficiency ensures that the participation constraint is satisfied.

[13] Alternatively, one could follow Inci (2013) by enlarging the set of feasible contracts that the bank can offer and imposing a Bertrand-Wilson equilibrium concept. Banks would be willing to offer contracts to low ability depositors on which they make a loss, and this would exactly compensate the positive profits they make with high ability entrepreneurs: profits would be driven to zero.



(a) Profitable Separating (b) Contracts Offered

Figure A.3: Contracts offered

condition from a pooling exploration contract. Solving the corresponding system of equations,

$$\begin{cases} Y\left(q_L + q_L q_L^S + p - q_L p\right) - q_L \tau^S - (1-q_L)\,\tau^F = RA \\ \tau^S = \frac{R(I-A)}{\bar{q}_{HL}} - \frac{1-\bar{q}_{HL}}{\bar{q}_{HL}}\tau^F, \end{cases}$$

reveals that the contract offered to the low ability agent, y_L, is

$$y_L(A,R) = \begin{bmatrix} \frac{R(1-q_L)}{\bar{q}_{HL}-q_L} I - \frac{\left(q_L+q_L q_L^S+p-q_L p\right)(1-q_L)}{\bar{q}_{HL}-q_L} Y - RA \\ -\frac{R q_L}{\bar{q}_{HL}-q_L} I + \frac{\left(q_L+q_L q_L^S+p-q_L p\right)\bar{q}_{HL}}{\bar{q}_{HL}-q_L} Y - RA \end{bmatrix}^T.$$

The terms of the contract offered to the high ability agents, y_H, are derived by the intersection of $V(L,N,\boldsymbol{\tau}) = RA$ with the vertical axis, $\tau^F = 0$, i.e.

$$y_H(A,R) = \begin{bmatrix} \frac{Y\left(q_L + q_L q_L^S + p - q_L p\right) - RA}{q_L} & 0 \end{bmatrix}.$$

The high ability agent's expected utility associated with this contract is

$$Y\left(q_H + q_H q_H^S + p - q_H p\right) - \frac{q_H}{q_L}\left[Y\left(q_L + q_L q_L^S + p - q_L p\right) - RA\right]. \quad \text{(A.15)}$$

The assumption on efficiency ensures that high ability agents prefer this contract to investing. Setting (A.15) equal to $Y\left(q_H + q_H q_H^S + p - q_H p\right) - \hat{R} q_H (I-A)$ and solving for $\hat{R}$ reveals that the interest rate on this contract is

$$\hat{R} = \frac{Y\left(q_H + q_H q_H^S + p - q_H p\right) - RA}{q_L\left(I - A\right)}.$$

This completes the set of contracts offered by the banks, as summarised in Panel (b) of Figure A.3.

We now consider the agent's decision given that banks offer one of the pooling contracts above. An untalented individual with wealth A has three options: she can become an investor, she can become an exploiter, or she can become an explorer. Panel (b) of Figure A.1 above gave us the preferred option for each T and A combination.

Likewise, a talented individual with wealth A has four options: (a) she can become an investor, (b) she can become an exploiter, (c) she can become a shirking explorer, or (d) she can become an explorer who exerts effort. By the assumption on efficiency, effortful exploration is always preferred to exploitation, and pooling with low ability exploiters is always preferred to investing.[14] A shirking high ability agent is no different from a low ability agent, and thus the analysis is the same as above. In addition to those conditions, effortful exploration is preferred to shirking if

$$A \geq I - \frac{\bar{q}_{HL}}{r(q_H - q_L)}\left\{\left[q_H q_H^S - q_L q_L^S + (1-p)(q_H - q_L)\right] Y - e\right\} =: \phi_{HL}^H(R) \tag{A.16a}$$

$$A \geq I - \frac{\bar{q}_{HC}}{r(q_H - q_L)}\left\{\left[q_H q_H^S - q_L q_L^S + (1-p)(q_H - q_L)\right] Y - e\right\} =: \phi_{HC}^H(R) \tag{A.16b}$$

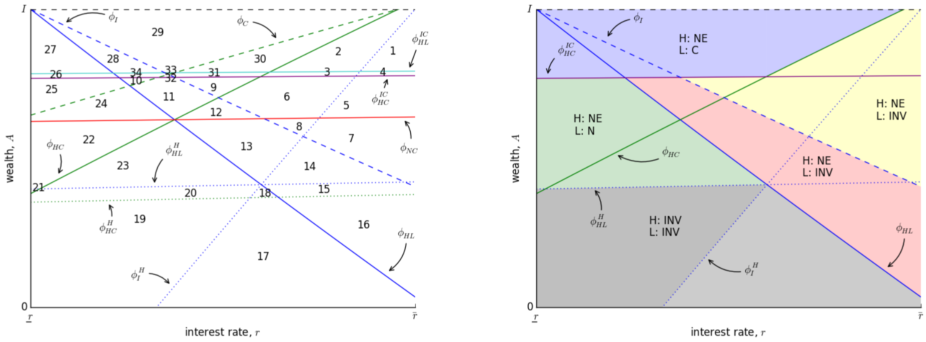
and to investing if

$$A > \frac{q_H}{q_H - \bar{q}_{HL}} I - \frac{\bar{q}_{HL}}{r(q_H - \bar{q}_{HL})}\left[\left(q_H + q_H q_H^S + p - q_H p\right) Y - e\right] =: \phi_I^H(R). \tag{A.17}$$

Panel (a) of Figure A.4 summarises all participation and incentive compatibility constraints in the (R, A) space. This allows us to complement the analysis regarding the banks' problem by adding the decisions of the agents.

In the areas (1)-(9), banks offer the zero-profit separating exploration contract to high ability agents, high ability agents accept it and exert effort, whereas low ability agents prefer investing. In the areas (10)-(16), the

[14] Indeed, effortful exploration is preferred to exploitation if the wealth level is lower than $I + \bar{q}\left[\left(q_H + q_H q_H^S - q_H p - p\right) Y - e\right]\left[R(p - q_H)\right]^{-1}$, for $\bar{q} = \{q_{HL}, q_{HC}\}$. Effort-exerting high ability explorers prefer to pool with low ability exploiters to investing if A is lower than $\left[r(\bar{q}_{HC} - q_H)\right]^{-1}\left\{-q_H RI + \bar{q}_{HC}\left[\left(q_H + q_H q_H^S + p - q_H p\right) Y - e\right]\right\}$. Both conditions are always satisfied given the assumption on efficiency.



(a) All Conditions

(b) Relevant Conditions

Figure A.4: Occupational choices

Notes. INV stands for investing, C for setting up a conventional firm, N for untalented exploration, and NE for effortful exploration.

profitable separating exploration contract is offered, high ability agents accept it and exert effort, whereas low ability agents become investors. In the areas (17)-(18), only the pooling exploration contract can be offered, but this does not satisfy the participation constraint of the high ability agents, who thus decide to become investors. Low ability agents do not explore because they would be identified as low ability, and thus they prefer to invest. In the areas (19)-(21), the pooling exploration contract is offered, but high ability agents do not exert effort. Therefore, they are no different from low ability agents and bank, to break even, must ask for the interest rate consistent with the low ability applicants. Thus, all agents prefer to invest. In the areas (22)-(25), the pooling exploration contract is offered, high ability agents exert effort and low ability agents become explorers. In the areas (26)-(34), the exploitation and exploration pooling contract is offered, high ability agents exert effort and low ability agents become exploiters.

These insights are summarised in Panel (b) of Figure A.4. In the yellow area, the zero-profit separating exploration contract is offered by the banks, high ability agents accept it and exert effort, whereas low ability agents become investors. Similarly, in the red area, low ability agents become investors, and high ability agents become effort-exerting explorers, but banks make positive profits. In the grey area, only the pooling exploration contract can be offered, but high ability agents do not become explorers either because they do not want to (light-grey area), or because they are unwilling to provide effort (dark-grey area): everyone would be treated as a low ability agent by the banks, and thus all agents prefer to become investors given the efficiency assumption. In the green area, the pooling exploration contract is offered, high ability agents provide effort and low ability agents become explorers. Finally, in the blue area, low ability agents become exploiters and high ability agents become effort-exerting explorers, thanks to the pooling exploration and exploitation contract.

Figure A.5 shows the possible general equilibria that could ensue. Up to the risk-free interest rate R_1 (defined by $\phi_{HL} = \phi_{HC}^{IC}$), there are only three wealth classes: a lower-class where everyone become an investor, a (lower) middle-class where high ability agents exert effort and cross-subsidise low ability explorers, and an upper-class where low ability agents become exploiters who cross-subsidise the high ability effort-exerting explorers. Since ϕ_{HL}^{H} is strictly increasing in R, the number of entrepreneurs decreases as we increase the risk-free interest rate in $[R, R_1]$.

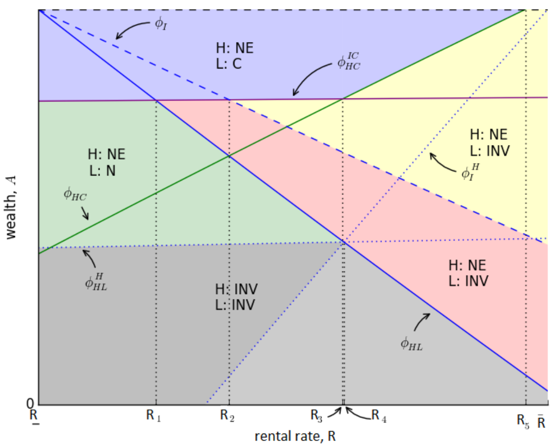


Figure A.5: General equilibrium

Between R_1 and R_2 (defined by $\phi_I = \phi_{HC}^{IC}$), we have four wealth classes, since (part of) the upper middle-class shows up, where low ability agents become investors and banks make positive profits on the effortful exploration contract accepted by the high ability agents. The number of entrepreneurs is strictly decreasing in the interest rate both because ϕ_{HL}^{H} is strictly increasing in R and because the richest low ability agents (that were previously lower middle-class and are now middle middle-class) switch from entrepreneurship to investing.

Consider a rental rate R_2 and R_3 (defined by $\phi_{HC} = \phi_{HC}^{IC}$). The number of entrepreneurs is still strictly decreasing in R since ϕ_{HL}^{H} is strictly increasing in R (and thus agents move from the lower middle-class to the lower-class where they become investors), ϕ_{HL} is strictly decreasing in R (more and more agents move from the lower middle-class to the middle middle-class where low ability agents become investors), and ϕ_{HC}^{IC} is strictly increasing in R (the upper-class shrinks in favour of the newly formed upper middle-class, and thus the poorest low ability agents of the upper-class move from non-innovative entrepreneurship to investing). For rates between R_3 and R_4 (defined by $\phi_{HL} = \phi_{HL}^{H}$), the case is similar to the previous one, with the difference that ϕ_{HC} is steeper than ϕ_{HC}^{IC} and thus the number of entrepreneurs decreases even faster as R increases.

For rates between R_4 and R_5 (defined by $\phi_{HC} = I$), the lower middle-class disappears. Since ϕ_{HL} is strictly decreasing in r, the richest members of the lower-class start entering the (upper) middle-class, and thus the number of entrepreneurs increases. However, since ϕ_{HC} is strictly increasing in r, the poorest members of the upper-class enter the upper middle-class, pushing the number of entrepreneurs down. Which of these two effects dominates depends on the wealth distribution.

Finally, for rates above R_5, the upper-class disappears. The number of entrepreneurs is strictly increasing in R as the lower-class shrinks.

To summarise, the number of entrepreneurs is strictly decreasing in R up to R_4, it is potentially non-monotonic between R_4 and R_5, and strictly increasing above R_5. Thus, depending on the wealth distribution (and parameters), we could have multiple equilibria. If we restrict our attention to rates below R_4, the equilibrium, if it exists, is unique.

Numerical simulations for this extended model are available on request.