APPLYING BCMP MULTI-CLASS QUEUEING NETWORKS FOR THE PERFORMANCE EVALUATION OF HIERARCHICAL AND MODULAR SOFTWARE SYSTEMS

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KEYWORDS
Performance evaluation, Software engineering, Queueing networks, Product-form solutions, BCMP

ABSTRACT

Queueing networks with multiple classes of customers play a fundamental role for evaluating the performance of both software and hardware architectures. The main strength of product-form models, in particular of BCMP queueing networks, is that they combine a flexible formalism with efficient analysis techniques and solution algorithms. In this paper we provide an algorithm that starting from a high-level description of a system, and from the definition of its components in terms of interacting sub-systems, computes a multiple-class and multiple-chain BCMP queueing network. We believe that the strength of this approach is twofold. First, the modeller deals with simplified models, which are defined in a modular and hierarchical way. Hence, we can carry on sensitivity analysis that may easily include structural changes (and not only on the time parameters). Second, maintaining the product-form property allows one to derive the average system performance indices very efficiently. The paper also discusses the application of the algorithm for the performance evaluation of Web Sites with modular architectures, such as those based on Content Management Systems.

INTRODUCTION

Performance analysis of modular and hierarchical systems has always been an important topic for the performance evaluation and software engineering research communities (see, e.g., Smith (1990)). In particular, a good approach to software design requires the definition of a modular and hierarchical architecture. From a high-level point of view, the software may be seen as the interaction of several black-box components. The definition of these sub-components follows the same approach in a hierarchical fashion until the very low-level layer of the architecture is reached. Performance evaluation of such models is important since the earlier stages of development as shown in Smith and Williams (2006). In this context, the main problem consists in the definition of efficient algorithms capable of deriving the required performance indices efficiently.

The class of models we consider in this paper is the well-known class of Markovian models. In particular we focus on those models whose underlying stochastic process is a Continuous Time Markov Chain (CTMC). Particular attention will be devoted to BCMP queueing networks introduced in Baskett et al. (1975), i.e., a class of queueing networks with separable solution and for which efficient analysis algorithms have been introduced for instance in Buzen (1973), Resiser and Lavenberg (1980), Brueff and Lavenberg (1980), Bruell et al. (1984), Conway and Georgonas (1986), Conway et al. (1989). One of the main features of BCMP queueing networks is the possibility of characterising the customers of the system by assigning them a class (temporary characterisation) and a chain (permanent characterisation). Under a set of assumptions, the class and the chain of a customer determines its probabilistic routing among the queueing stations and the service time distributions.

In this paper we propose a methodology, supported by a novel algorithm, which aims to simplify the performance evaluation of systems designed according to a hierarchical and modular architecture. This methodology is based on the definition of a high level model.
of the system consisting of several components. Each of these may be further specified in a hierarchical fashion. Under some assumptions that will be detailed later, we provide an algorithm which transforms this abstract model into a BCMP queueing network. Figure 1 illustrates the steps of this analysis. Let us consider an example. Modern Web Sites are often built on Content Management System (CMS) applications. CMSs are flexible re-programmable software systems consisting of a set of modules that are specialised in some task, e.g., rendering the web page, forum or wiki management, news and comments, user management. Modules are programmed by communities of developers who often work autonomously and must respect the interface given by the core system. Examples of modern CMSs are Drupal, Joomla!, PostNuke, Typo3 just to mention a few. Users who visit the web sites based on CMS are usually not aware of such a modular architecture. Nevertheless, a log analysis may reveal their behaviour among the site modules and clustering techniques may be adopted to distinguish user habits. We aim to provide a modelling approach that allows the system administrator to predict the performance of its web portal under different scenario from the knowledge of:

- the customer behaviours among the modules
- the resource requirements of each module
- the mapping of a required resource to a physical device.

This kind of analysis is not trivial and a hierarchical and modular approach should be adopted as observed for instance in Smith (1990). On the other hand we aim to provide a methodology that is compatible with known exact analysis algorithms to avoid the need of simulation or approximated technique as usually done in Woodside et al. (1995).

The main contribution of this paper consists in providing a methodology supported by an original algorithm that allows the modeller to specify the system in terms of a multiple-class and multiple-chain queueing network (QN) in which each station is itself a multiple-class and multiple-chain QN. The peculiarity of this hierarchical approach is that each station at a given level of abstraction is defined in isolation but, given two or more stations, they may share one of their components at a lower level of abstraction. In the example of the CMS one may think that at the top level of the CMS one has the routing of customers among the site modules (stations at the top level). Each module is then defined in terms of usage of resources (e.g., database, CPU, etc.). However, when these modules are combined one should be able to specify whether a resource is shared among different modules or the modules have distinct resources available. Obviously, this may have great impact on the overall performance of the system (e.g., are the DB and the multimedia resources stored in the same hard disk?).

The goal of the algorithm that we introduce is to transform a QN defined at a top level into one defined at a lower level until the lowest level is reached. Once this is done, under some assumptions that will be described in the following sections, we obtain a product-form BCMP QN that may be analysed by the well-know algorithms for the computation of the average performance indices in steady-state.

The paper is structured as follows. First, we briefly recall the theoretical background on multiple-class BCMP queueing networks. Then, we describe the proposed methodology and we define of the algorithm. The last section provides an application example of the proposed approach. Some final remarks conclude the paper.

THEORETICAL BACKGROUND

This section aims to briefly recall the fundamental theorem on multiple-class product-form queueing networks, i.e., the BCMP theorem (Baskett et al. (1975)). Informally, we can say that it states sufficient conditions for a QN with multiple classes of customers to yield a product-form solution. Its importance is not only theoretical because several algorithms have been defined to compute the average performance indices in steady-state efficiently (e.g., Buzen (1973), Resiser and Lavenberg (1980), Conway and Georganas (1986)). This section first briefly illustrates the BCMP theorem and then lists the algorithms for the analysis with their computational complexity.

The BCMP theorem

BCMP queueing networks consist of a set of queueing centers and a (possibly infinite) set of customers. At a given epoch, each customer in the network has a class which may determine its routing probabilities or the ser-
Queueing stations must belong to one of the following types:

**Type 1**: The queueing discipline is First Come First Served (FCFS) and the service time distribution is exponential and class-independent,

**Type 2**: The queueing discipline is Processor Sharing (PS),

**Type 3**: The station has infinite servers (IS), hence customers never wait in queue (Delay Stations),

**Type 4**: The service discipline is Last Come First Served with Preemptive Resume (LCFSPR).

Stations of type 2, 3 or 4 may have a Coxian distributed service time that depends on the customer class. Moreover, the station service time may depend on the queue length at a given epoch (see Balsamo and Marin (2007) for a recent survey). A chain may be open or closed. In the former case, customers arrive from the outside according to a Poisson process with a given rate, while in the latter the number of customers for that chain must be specified. The network is called open if all its chains are open, closed if they are all closed or mixed otherwise. Queueing stations must belong to one of the following types:

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**Type 3**: The station has infinite servers (IS), hence customers never wait in queue (Delay Stations),

**Type 4**: The service discipline is Last Come First Served with Preemptive Resume (LCFSPR).

Theorem 1 (BCMP (salient results)) Let us consider a multiple-class and multiple-chain QN, open, closed or mixed, whose queueing stations are of type 1, 2, 3 or 4. Then, if the underlying stochastic process is ergodic, the steady-state probabilities are in product-form with respect to the queueing stations, i.e., let \( n = (n_1, \ldots, n_M) \) be the vector representing the state of the network, where component \( n_i \) is the state of station \( S_i \), then the following relation holds:

\[
\pi(n) = \frac{1}{G} \prod_{i=1}^{M} g_i(n_i),
\]

where \( \pi \) is the steady-state distribution of the QN, and \( g_i(n_i) \) is the steady-state distribution of station \( S_i \) considered in isolation, with arrival rates \( \mathbf{e}_i \), and \( G \) is a normalising constant.
Solution algorithms

Theorem 1 and the class of BCMP networks have been widely applied for system performance analysis, because several efficient solution algorithms have been defined to compute the stationary state distribution \( \pi \) and a set of average performance indices. Such algorithms specifically apply to analyse closed or mixed networks, where we have to compute the normalising constant, as stated by Theorem 1. Note that for open networks we have \( G = 1 \), the solution \( e^{(c)}_r \) of the traffic equations (1) already gives the throughputs of each node \( S_i \) for classes \( r \) in chain \( c \). Then one can easily derive the other average performance indices by classical queuing system results.

Various solution algorithms have been defined for closed and mixed BCMP networks. Some algorithms, such as the Convolution Algorithm, directly compute the normalising constant \( G \) in equation (3) and hence a set of mean performance indices, such as the mean response time, the average queue length, and the throughput of each queuing station. Other algorithm, such as MVA (Mean Value Analysis) avoid the computation of the normalising constant \( G \) and iteratively (over the number of customers) directly compute a set of average performance indices. For multi-class and multiple-chain BCMP networks some algorithms apply special recursive scheme on the number of chains, and/or take advantage of the possible sparsity of the chains (e.g., chains that contain few classes) to derive efficient solution. Although it is out of the scope of this paper to describe these well-known algorithms, we just cite them and recall their computational complexity. Several tools for the analysis of queuing networks have been implemented over the last decades. A recent work, called qnetworks toolbox, is described in Marzolla (2010). Such an implementation of several algorithms is given in terms of library of functions for Octave, i.e., a programmable environment for numerical computation. This allows one to integrate easily the algorithms of networks with new ones, for instance that presented by Algorithm 1. Hereafter, we consider a queuing network with multiple chains but where each station has just one class per chain (single-class, multiple-chain QN). One can show that for each multiple-class and multiple-chain BCMP QN it is possible to define another BCMP QN with single-class and multiple-chain with the same average performance indices (see, e.g., Kant (1992)). For the sake of clarity, we consider the QN consisting of only closed chains.

The Convolusion Algorithm computes the normalising constant from which the average performance indices may be derived. The computational complexity, given the solution of the traffic equations system (2), depends on the type of stations in the QN. In particular each iteration has a cost of \( O(CH) \) for load-independent stations and of \( O(H^2) \) for the others, where \( H = \prod_{c=1}^{K(c)} (K(c) + 1) \). If all the chains has the same population \( \kappa \) and no load-dependent stations are present, then the computational cost is \( O(MC\kappa^C) \).

The Mean Value Analysis algorithm (MVA) is based on the Arrival theorem that provides an efficient recursive scheme to compute the steady-state average performance indices. For a QN without load-dependent stations, and with identical chain populations, its complexity is identical to that of the Convolution. The Recursion by Chain Algorithm (RECAL), defined in Conway and Georganas (1986), computes the normalising constant and, in a similar fashion of Convolution, from this it derives the average performance indices. It is particularly interesting because despite of a greater complexity in the implementation, its computational complexity grows in a polynomial way with the number of chains, i.e., for high number of chains,
then we define the binary relation \(m\) consisting of the classes of the customers leaving the sub-model, \(d_1, \ldots, d_{\ell_1}\). The easiest way to interpret the model specification is seeing these components as the queueing stations of a multiple-class and multiple-chain BCMP QN. Hence, probabilistic routing and customer characteristics are allowed. Each of the components \(d_i\), with \(d_i = d_1, \ldots, d_{\ell_1}\), seen at the highest level of abstraction, may be defined as:

- A BCMP queueing station
- A sub-model consisting of components \(d_{(i)1}, \ldots, d_{(i)\ell_2}\). Note that it is not the case that \(d_{(i)j} \neq d_j\) for all \(1 \leq j \leq \ell_1\) and \(1 \leq k \leq \ell_2\), i.e., a component may use a resource which has already been described at a higher level. These components interact as stations of an open multiple-class and multiple-chain BCMP QN. Each sub-model from the outside can be seen as a black box, with a set of access points with some labels, i.e., the classes of the customers arriving from the outside (input classes) and the classes of the customers leaving the sub-model (output classes). We require that the set of input classes must be equal to the set of output classes of each component.

This recursive definition is the basis of the algorithm that follows.

We now introduce the concept of well-formed model.

**Definition 1 (Well-formed models)** Given a model consisting of \(m\) components (in any level of abstraction) then we define the binary relation \(>\) as follows:

- \(d > d'\) if and only if \(d'\) appear in the definition of \(d\) and the binary relation \(>\) as follows:
- \(d > d'\) if and only if \(d > d'\) or
- \(d > d'\) if there exists \(d''\) such that \(d > d''\) and \(d'' > d'\).

A model is well-formed if and only if relation \(>\) is a strict partial order.

Roughly speaking, a well-formed model does not have cycles in the definition of the components. However, it is possible, at a given level of abstraction, to refer to components specified at higher levels. Hereafter, we consider only well-formed models.

**Algorithm definition**

In order to better understand our approach to modular BCMP network design, we will first consider the algorithm, then we show how it could be further optimized. Let \(D\) be the set of the \(m\) components \(d_1, \ldots, d_m\) that form the model, and \(R\) be the set of the \(n\) classes \(r_1, \ldots, r_n\) for the component \(d_i\). In each component, we call \(EI\) (external input) the arrival streams from the outside, and \(EO\) (external output) the departure streams.

Binary relation \(d > d'\) given by Definition 1, means that \(d\) contains \(d'\), i.e., if \(d\) is not a simple QN station, then \(d'\) appears in the routing matrix of \(d\).

Let \(P_d\) be the routing matrix of component \(d\) and let be \(P_{d'}\), with \(d > d'\), the routing matrix of a subcomponent of \(d'\) of \(d\), then we aim to unfold \(d'\) in order to specify a routing matrix for its subcomponents. If \(d_i > d_j\), let \(A_{i,j} : R_i \rightarrow R_j\) be a partial function that associates class names of component \(d_i\) with class names of component \(d_j\). Notice that \(A\) is not necessarily injective, i.e., two or more classes of the container component can be mapped into the same class of the contained component. Whenever it happens, we should add a new class set to the submodel. \(A_{i,j}\) functions must be given by the user of the algorithm, and define how classes of a high level component maps on classes of its subcomponents. This allows the design of a low level component without knowledge of its future use in a high level one.

In Algorithm 1 we show a method for routing matrix unfolding. To keep the code simple, we use a single routing matrix that combines every chain of the component, but the algorithm preserves chains, i.e. (node, class) pairs that are in different chains in high-level model description remain in different chains in the solution computed by the algorithm.

Notice that here the insertion and replacement operators for rows and columns have a loose semantics, i.e., whenever a row or a column with less elements is assigned to
Algorithm 1: Algorithm UnfoldComponents to derive a BCMP QN from a model specified at a higher level of abstraction.

Input: routing matrix \( P_d \) of component \( d \), component counter array \( D_c \), functions \( A_{i,j} \).

Output: unfolded routing matrix \( P'_d \) of component \( d \).

if \( d \) is a station then /* base case */

forall class \( r \) of \( d \) do

| insert in \( P' \) rows and a columns for \( EI_d, r \) and \( EO_d, r \) of \( P \) |

else

forall \( d_i \) if \( d_i \rightarrow d_i \) do

\( D_{d_i} \leftarrow D_{d_i} + 1 \)

Let \( R_{c_d} \) be a class counter array

forall class \( r_k \) of \( d_i \) as named in \( d \) do

\( r_{i,j} \leftarrow \lambda_{d,d_i}(r_k) \)

\( R_{c_{i,j}} \leftarrow R_{c_{i,j}} + 1 \)

if \( R_{c_{i,j}} > \max R_{c_i} \) then

\( U = \text{UnfoldComponents}(P_{d_i}) \)

rename each class \( r_{i,j} \) of \( d_i \) in \( U \) as \( r_{i,j,D_{d_i},R_{c_{i,j}}} \)

insert in \( P' \) rows and columns of \( U \)

end

replace column \( EI_{d_i}, r_{i,j,D_{d_i},R_{c_{i,j}}} \) in \( P' \) with column \( d_i,r_k \) of \( P \)

replace row \( EO_{d_i}, r_{i,j,D_{d_i},R_{c_{i,j}}} \) in \( P' \) with row \( d_i,r_k \) of \( P \)

end

end

return \( P' \)

Illustrating example

We now provide an example that aims to illustrate the modelling methodology and the application of Algorithm 1 to a case-study. For the sake of brevity we keep the modelled system rather simple even if, obviously, the algorithm usefulness is enhanced by larger systems.

System description. We consider just a single module of a CMS, i.e., a database-indexed file archive. This is a typical feature of many websites that provide downloadable resources (e.g., multimedia or documents). Suppose that this module serves two classes of customers, one which models the file upload, and the other the file download. A download request passes through the database and then accesses the disk. An upload request, passes through the database and then accesses the disk to be written. If the operation is successfull then a message for the database is generated to confirm the correct operation.

Model definition. The black-box model of the CMS module is shown in Figure 2. We can see the two incoming and outgoing classes of customers. Let this component name be \( d_1 \). Figure 3 represents the internal structure of \( d_1 \), where \( d_2 \) is the database component and \( d_3 \) the disk component. Hence, we clearly have \( d_1 \succ d_2 \), \( d_1 \succ d_3 \) and \( A_{1,2}(r_{1,1}) = r_{2,1}, A_{1,2}(r_{1,2}) = r_{2,2}, A_{1,3}(r_{1,3}) = r_{3,1}, A_{1,3}(r_{1,3}) = \)

![Figure 2: A black-box model of a database-indexed file archive CMS module.](image-url)
Let us suppose, for the sake of brevity, that component $A$ that is a multiple-class station, and that $(\text{component}, \text{class})$ combinations, a 9 queueing station, in this case. The algorithm invokes recursively the UnfoldComponents function of Algorithm 1 until it finds a BCMP queueing station, in this case $d_4$, then, assuming it is encountered for the first time, it renames its classes $r_{4,1}$ and $r_{4,2}$ in $r_{4,1,1,1}$ and $r_{4,2,1,1}$. At the end of the invocation of UnfoldComponents($P_{d_2}$), the resulting routing matrix is like in Table 2. Notice that rows $EO_{r_{2,1}}$ and $EO_{r_{2,2}}$ are both zero, because they represent departures from the component, that have yet to be connected to a higher level sub-model.

During the invocation of UnfoldComponents($P_{d_3}$) both $A(r_{1,1})$ and $A(r_{1,2})$ return $r_{2,1}$, and therefore $Rec_{2,1}$ is incremented twice. The algorithm, then, inserts the elements of $P_{d_{2}}$ twice in $P'$, with different class names, e.g., $r_{4,1,2,1}$ and $r_{2,1,1,2}$.

As previously stated, the usefulness of the algorithm become more noticeable when the model is complex, e.g., if the Database module, instead of being made of a single queueing station, was described in terms of interacting submodels, like a CPU, one or more disks, a caching system, et cetera. All this subsystems may be also used by other modules.

$\text{CONCLUSION}$

This paper addresses the problem of combining a modular and hierarchical modelling technique with an efficient analysis method. The main theoretical contribution is an algorithm which allows the transformation of models defined at a higher level of abstraction into models defined at a lower one. A recursive application of this algorithm produces, under a set of conditions, a product-form multiple-class and multiple-chain BCMP QN. Although to obtain this we must limit the formalism expressivity, e.g., fork and join constructs are not permitted, our aim is to provide a methodology supported by efficient algorithms for the exact analysis. Other more expressive hierarchical approaches, such as those defined in Woodside et al. (1995), may require approximate algorithms for deriving the average performance indices in equilibrium. Future works have several directions. From a theoretical point of view, an extension of the class of models tractable by such a formalism would be important. Another important aspect is the integration of this framework with well-known and recent advances in web mining, particularly in log analysis. These techniques provide a partition of application users according to their behaviour. Although these techniques have been traditionally applied to predict the customer interests (e.g., for proposing context aware advertisements) we claim they may be very useful also for performance evaluation purposes.

\textbf{REFERENCES}


\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{The internal definition of the module of Figure 2.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{Minimal DB module design.}
\end{figure}

$r_{3,2}$. The routing matrix $P_1$ is, ignoring impossible (component, class) combinations, a 9 × 9 square matrix.

Let us suppose, for the sake of brevity, that component $d_2$, as in Figure 4, is made of a single component, $d_4$, that is a multiple-class station, and that $A_{2,4}(r_{2,1}) = r_{4,1}, A_{2,4}(r_{2,2}) = r_{4,2}$. Then we show how an application of the algorithm to $d_1$ transforms $d_2$. Note that we limit the observation to this part of the system for the sake of readability, since the number of classes arising from this simple example may be difficult to represent graphically.
\[ P' = \begin{bmatrix}
EI, r_{2,1} & EI, r_{2,2} & EO, r_{2,1} & EO, r_{2,2} & d_{4, r_{4,1,1,1}} & d_{4, r_{4,2,1,1}} \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
\end{bmatrix} \]

Table 2: Routing table \( P' \) at the end of UnfoldComponents(\( P_{d_2} \)).


**BIOGRAPHY**

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